

FLECS: a finite elements simulation tool for prototyping GAS spring blades

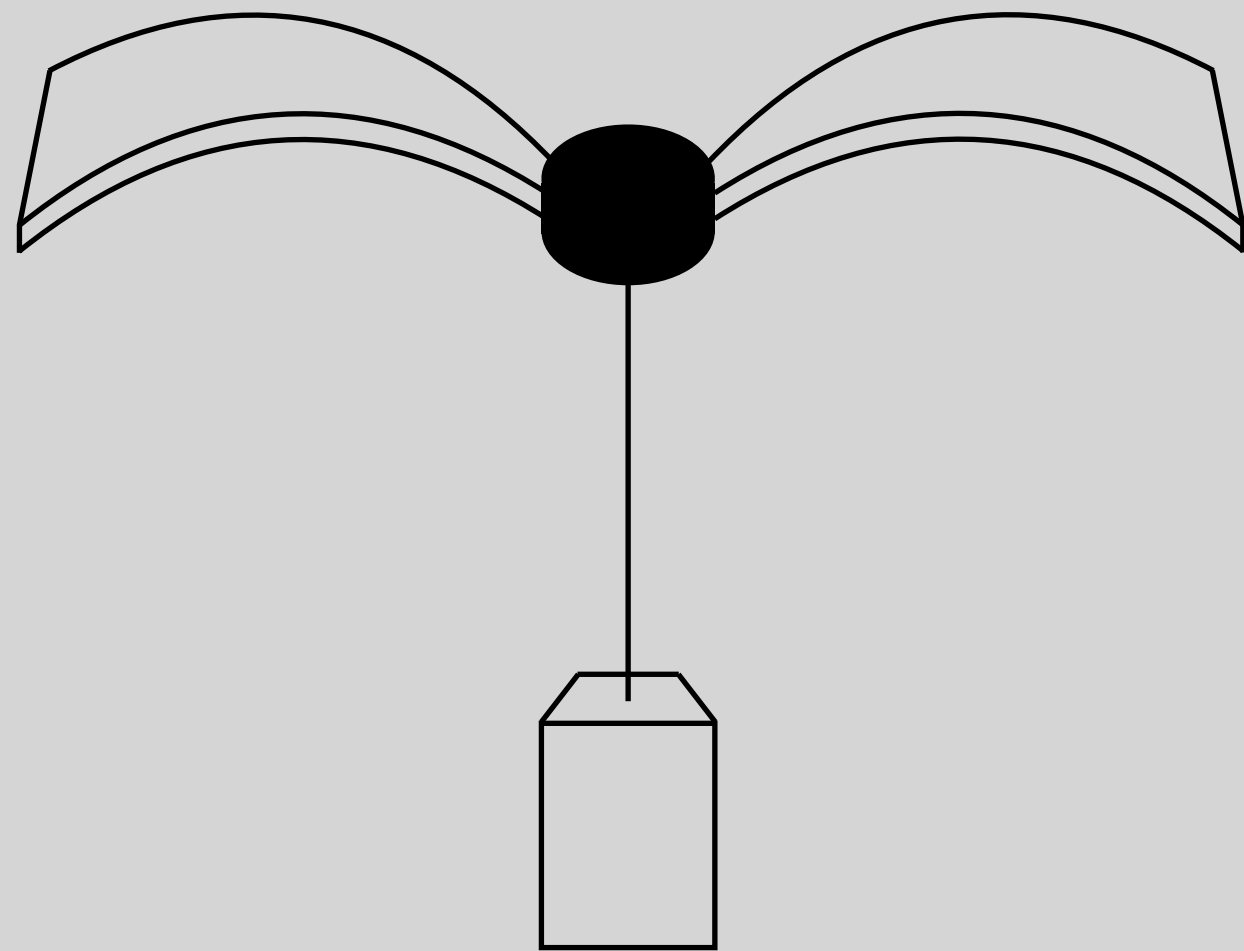
Nicolò Baldicchi, Giancarlo Cella, Piero Chessa

The 14th KAGRA International Workshop - Perugia, 16th May 2026

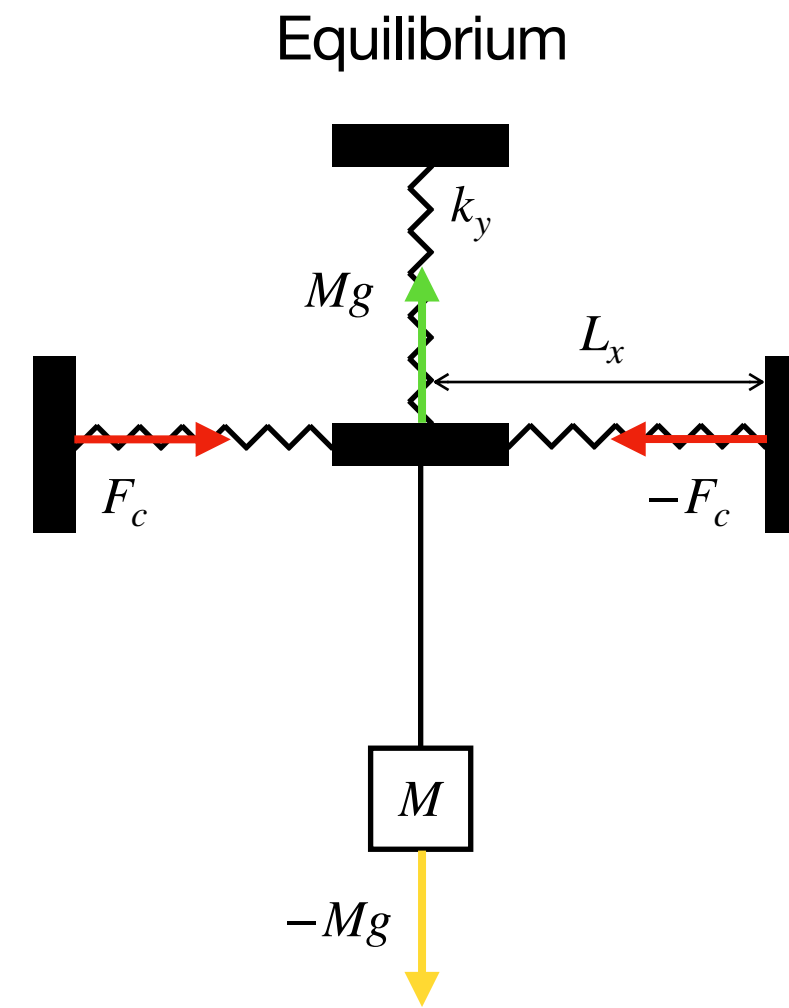


A linear model to describe GAS

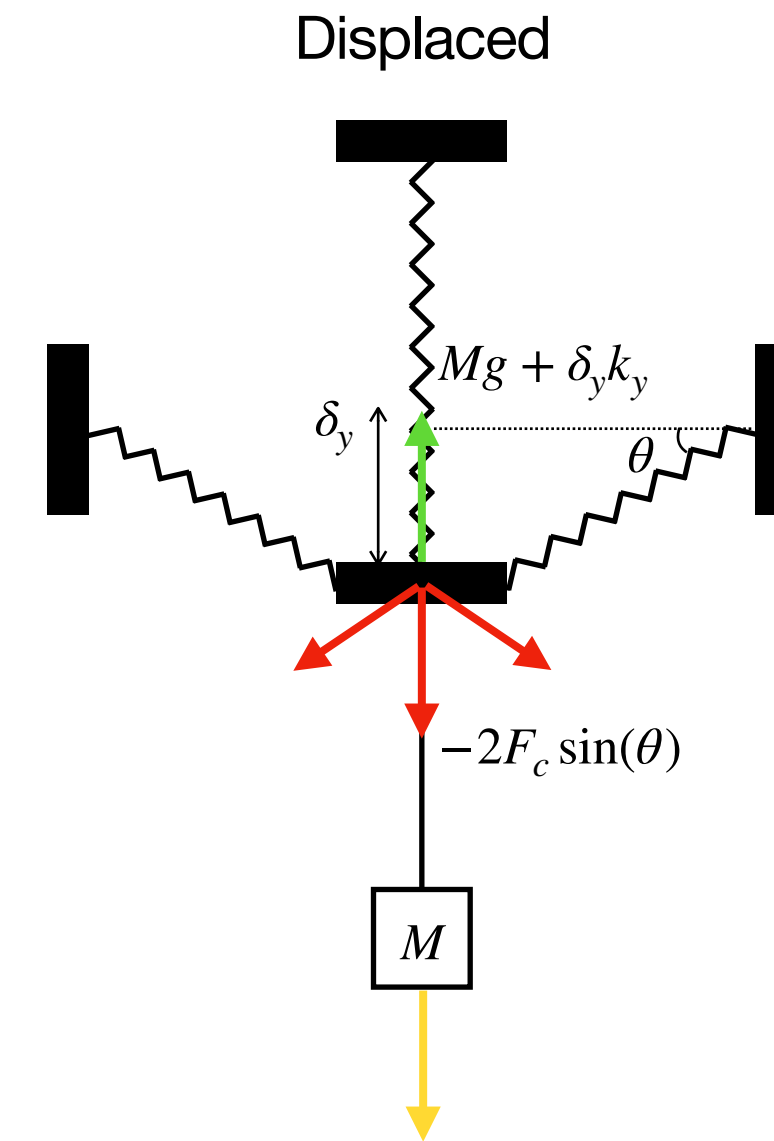
Cryogenic geometric anti-spring vibration isolation system, L.Feenstra et al., arXiv:2509.09010



A GAS system can be described in an intuitive way with a linear model: a vertical spring and two horizontal and opposite **compressed springs** exerting a force F_c



At the equilibrium the horizontal compression forces F_c balance



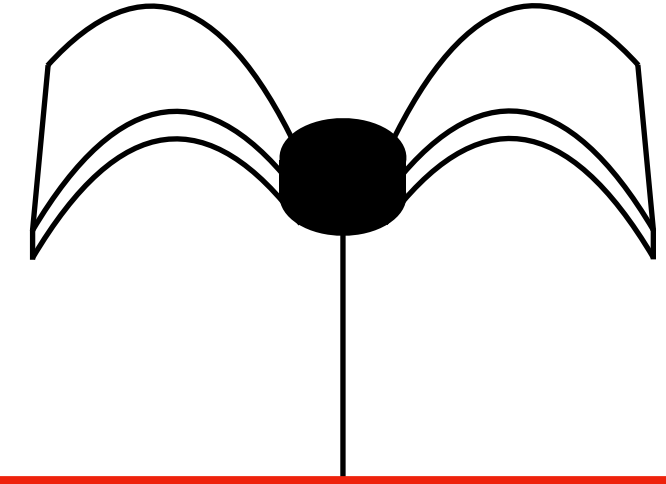
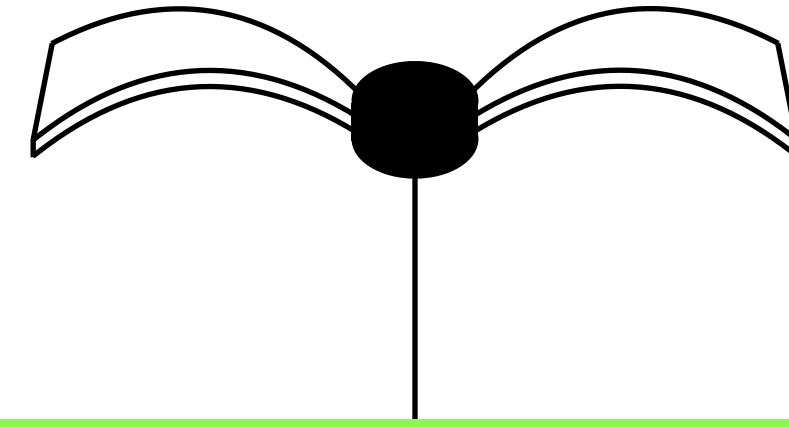
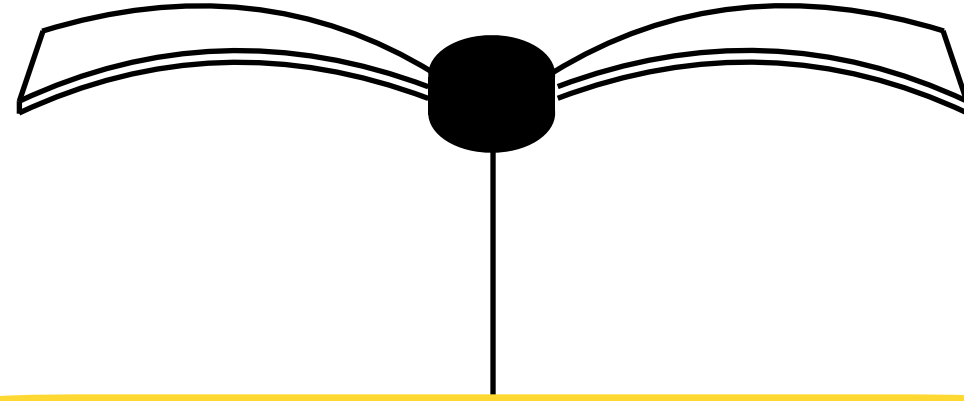
Outside of the equilibrium the **vertical component** of the compression force counteracts the elastic restoring force of the vertical spring

$$\delta F_y = k_y \delta_y - 2F_c \sin(\theta) \Big|_{L_x \gg \delta_y} = \left(k_y - \frac{2F_c}{L_x} \right) \delta_y$$

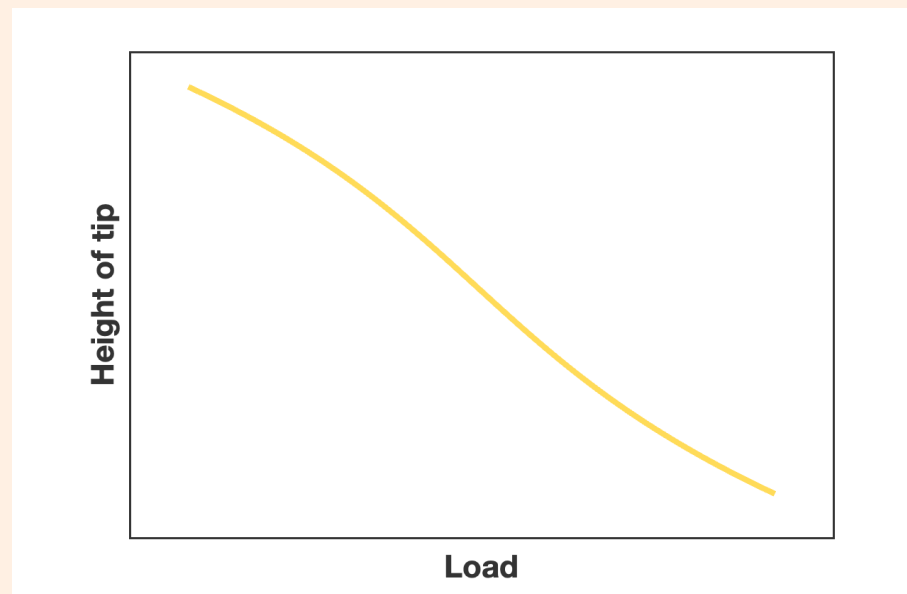
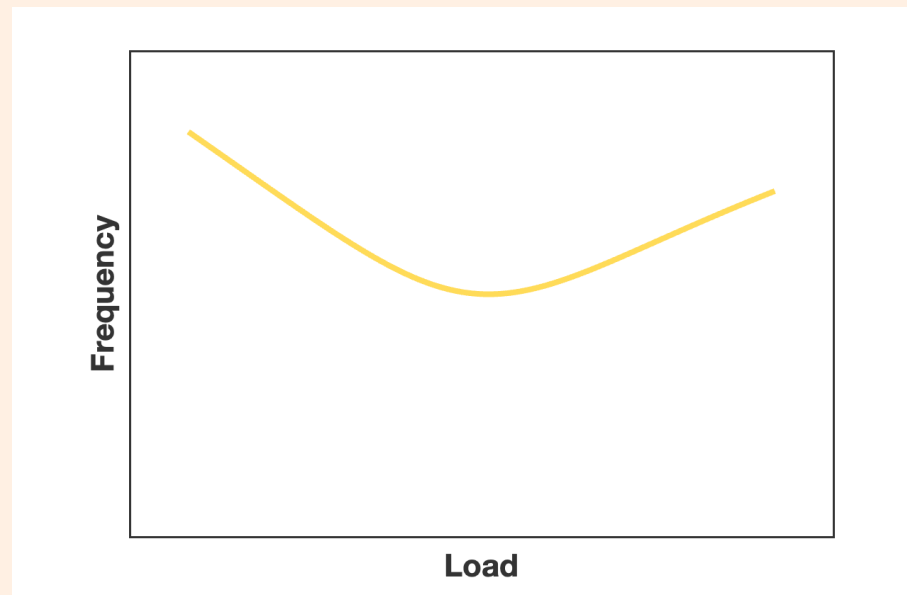
The **geometric anti-spring effect** consists in the reduction of the **effective stiffness** k_{eff} of the vertical spring:

$$\frac{d\delta F_y}{d\delta_y} \approx k_y - \frac{2F_c}{L_x} = k_{\text{eff}}$$

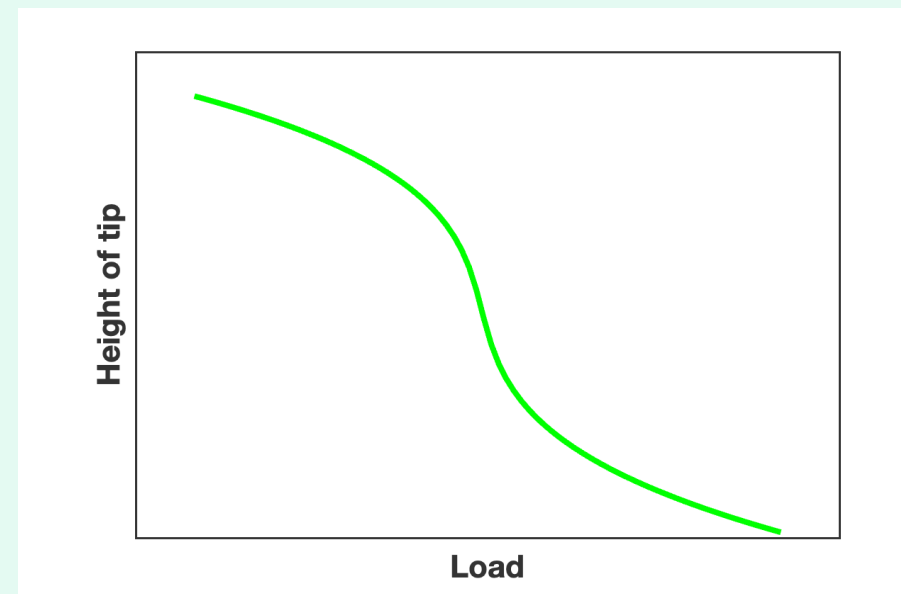
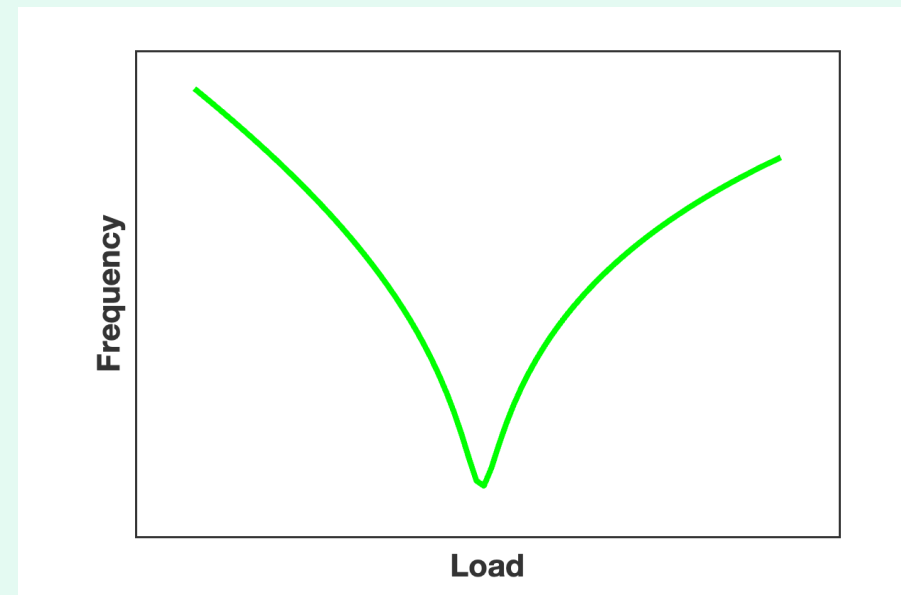
Tuning of a GAS system



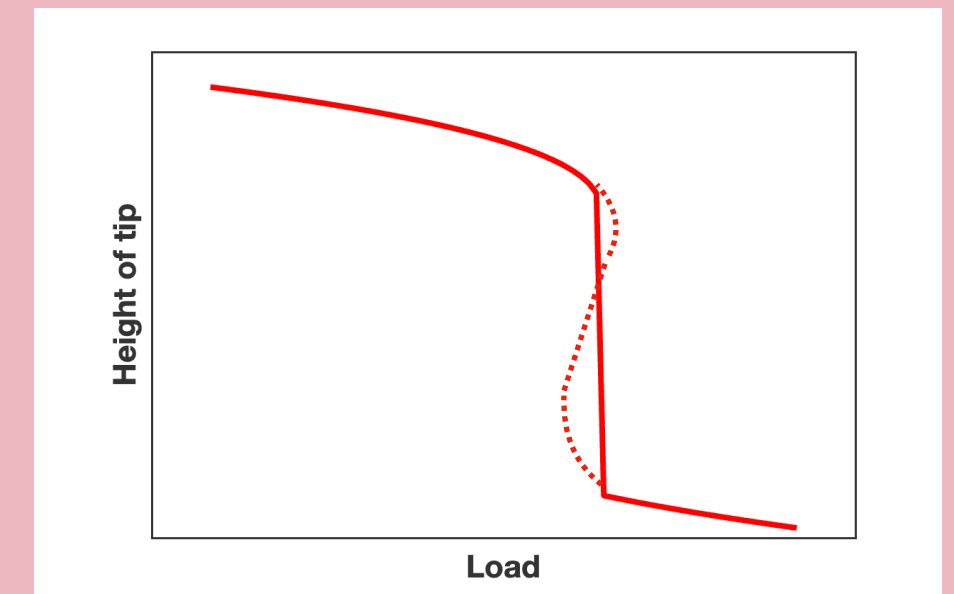
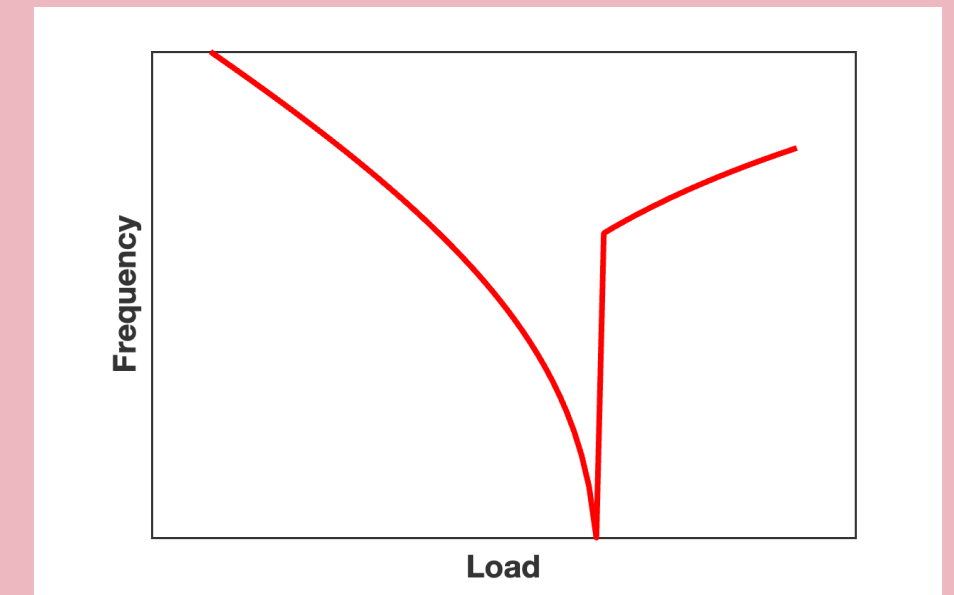
Low compression:
Stable system but the frequency is not minimized



Working point:
The frequency is minimized and the system is still stable



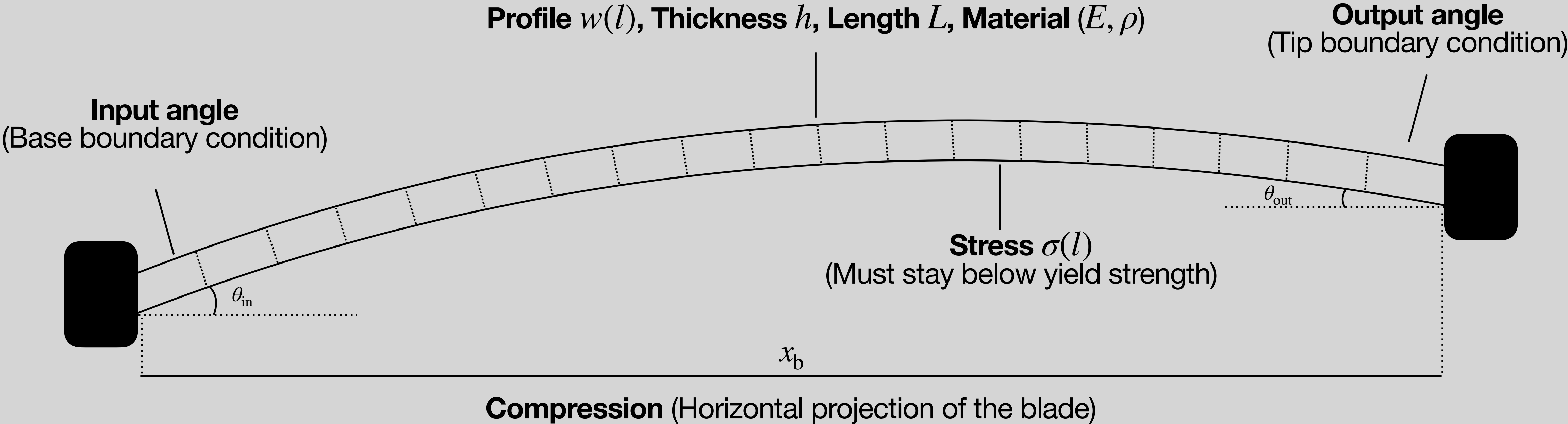
Too high compression:
Bistability region, the system has two equilibrium positions for a single load value



Compression of the blade is the tuning knob of a GAS system: once the geometry and the load are fixed, the horizontal load applied to the blades defines its resonant frequency $f = \sqrt{k_{\text{eff}}/M}/2\pi$

Flexible Linear Elements Connected in Series (FLECS)

In practice, a real blade introduces several coupled parameters. **FLECS** is a finite elements simulation tool that accounts for all of them in the **thin-blade** (Euler-Bernoulli) approximation.

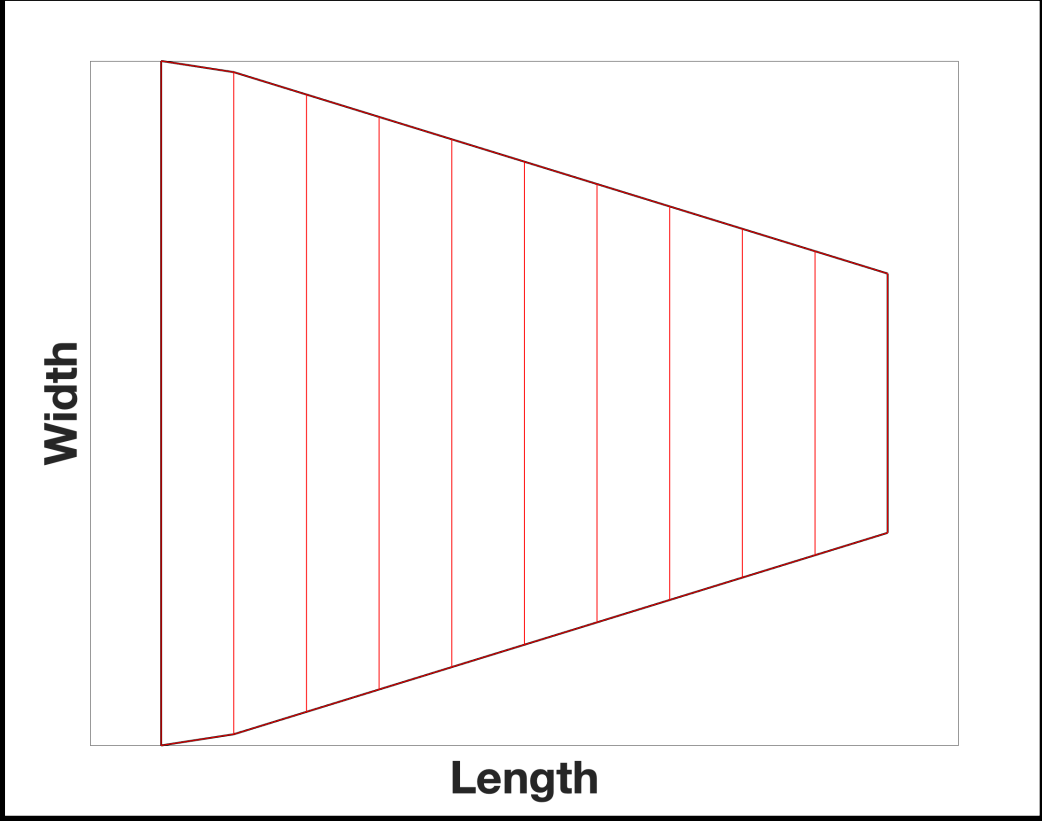


FLECS computes the profile at equilibrium, stress along the blade, frequencies and mode shapes **for any combination of these parameters**

How does FLECS work?

Discretization

- Division of the blade in N arc elements
- Each element forms an angle with the horizontal axis: $\theta \in \mathbb{R}^{N-2}$ are our free variables
- Setting of the projection constraint



Potential energy


- Compute total potential energy of the system as a function of the angles θ

$$U_{\text{tot}}[\theta] = U_{\text{el}} + U_{\text{grav}} + U_{\text{load}}$$

- Compute potential energy gradient $\nabla U_{\text{tot}}[\theta]$ for a fast optimization

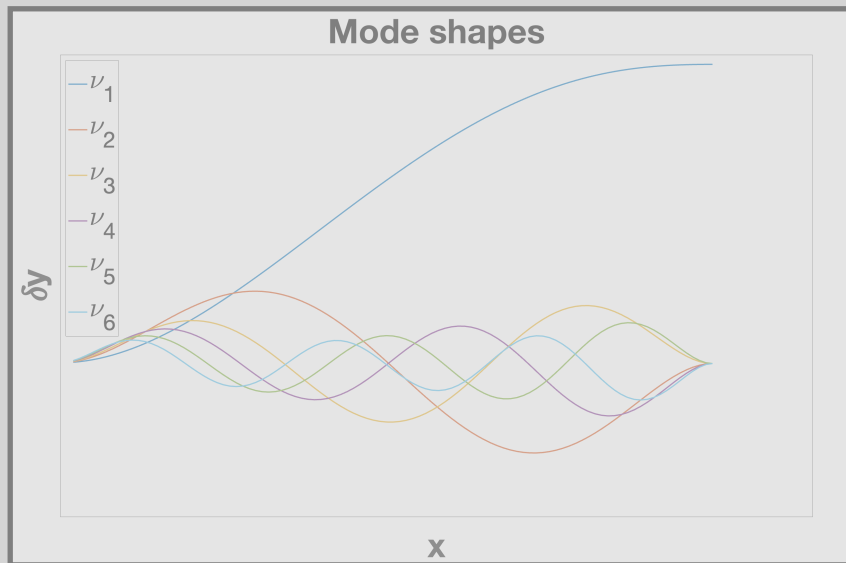
Equilibrium

- Constrained minimization:
 $\nabla U_{\text{tot}}[\theta^*] = 0 : X[\theta^*] = x_b$
- Outputs at θ^* : blade profile, stress $\sigma(l)$, horizontal load λ



Eigenvalue problem

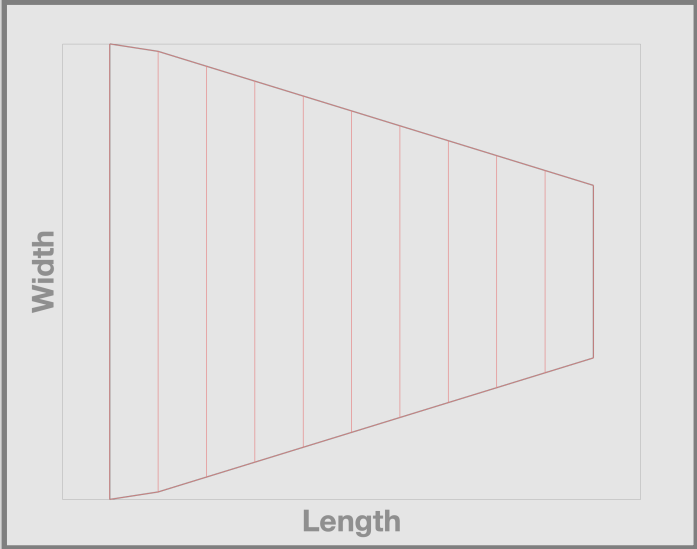
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 $K\nu = \omega^2 M\nu$
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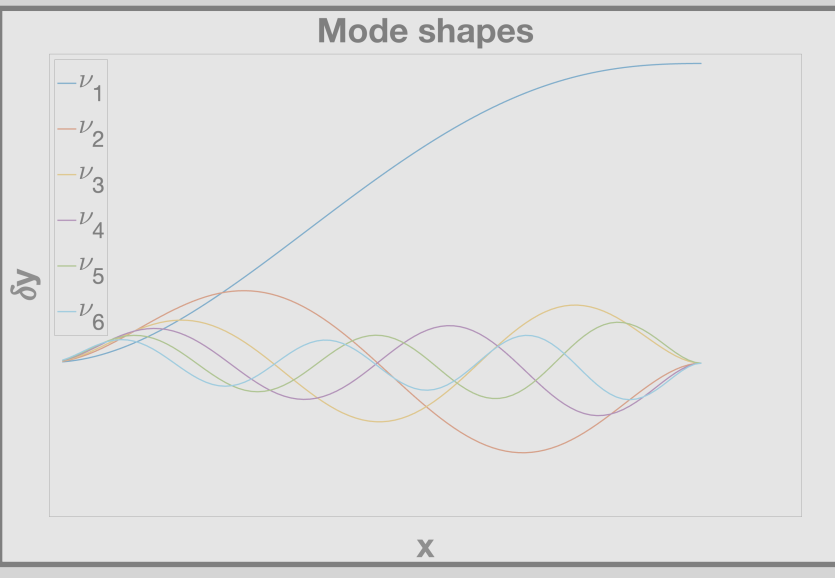
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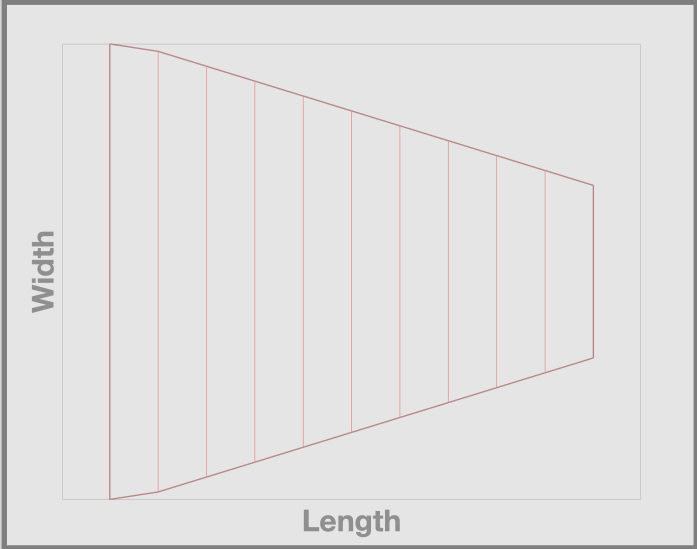
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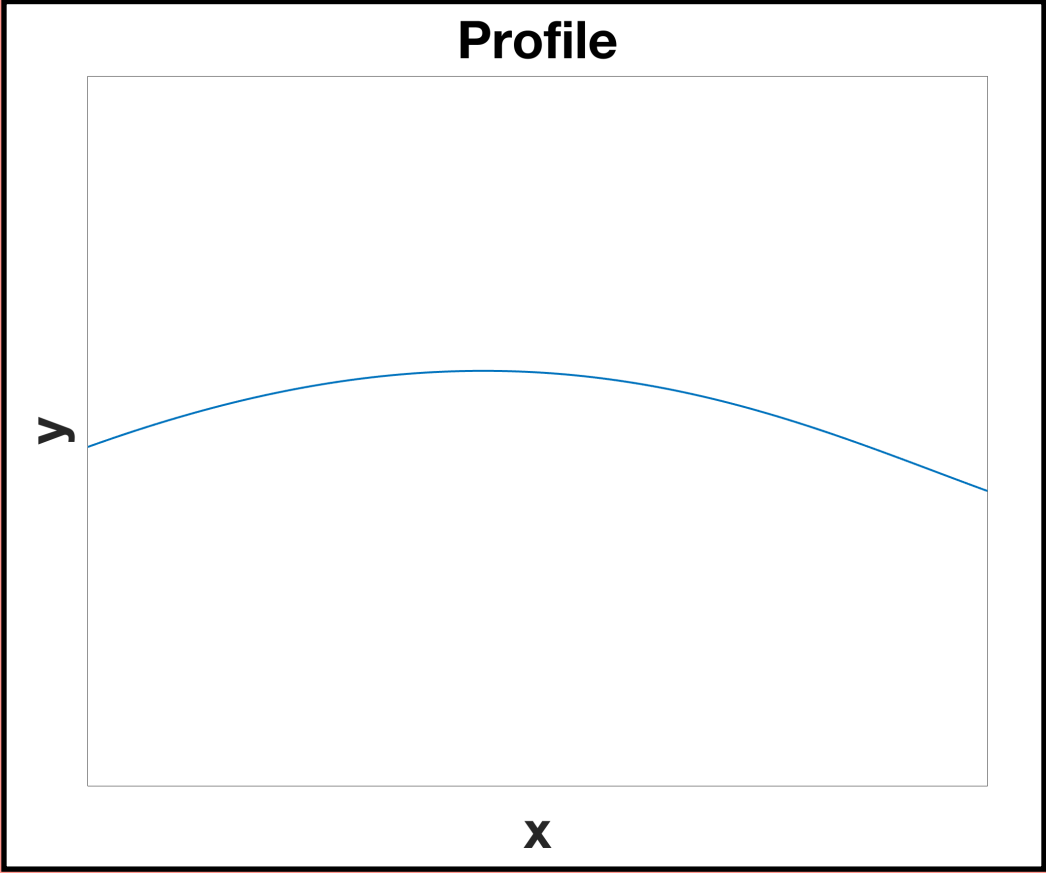
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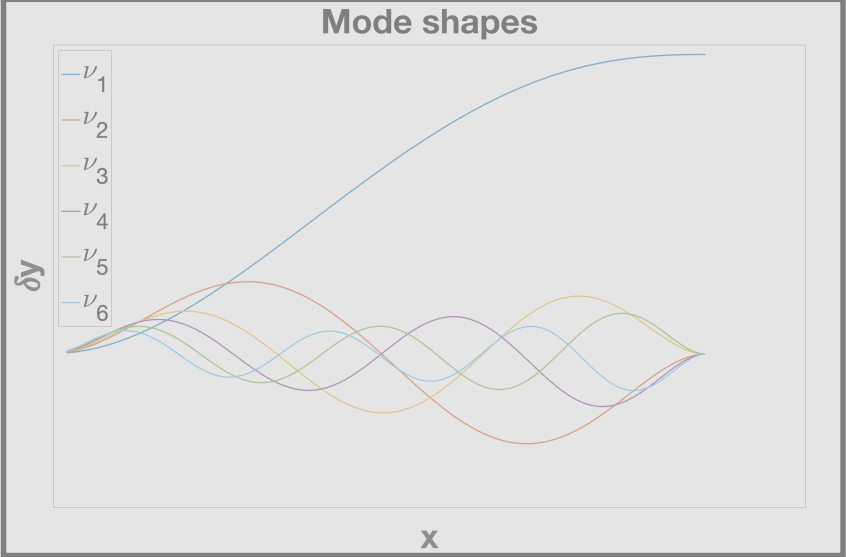
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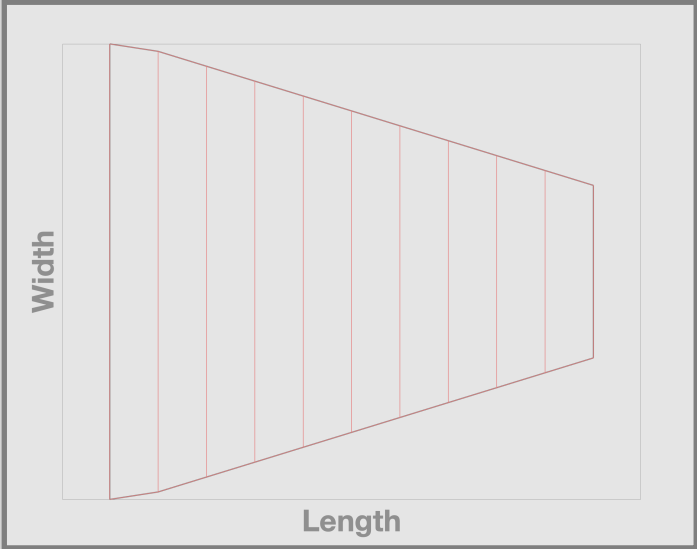
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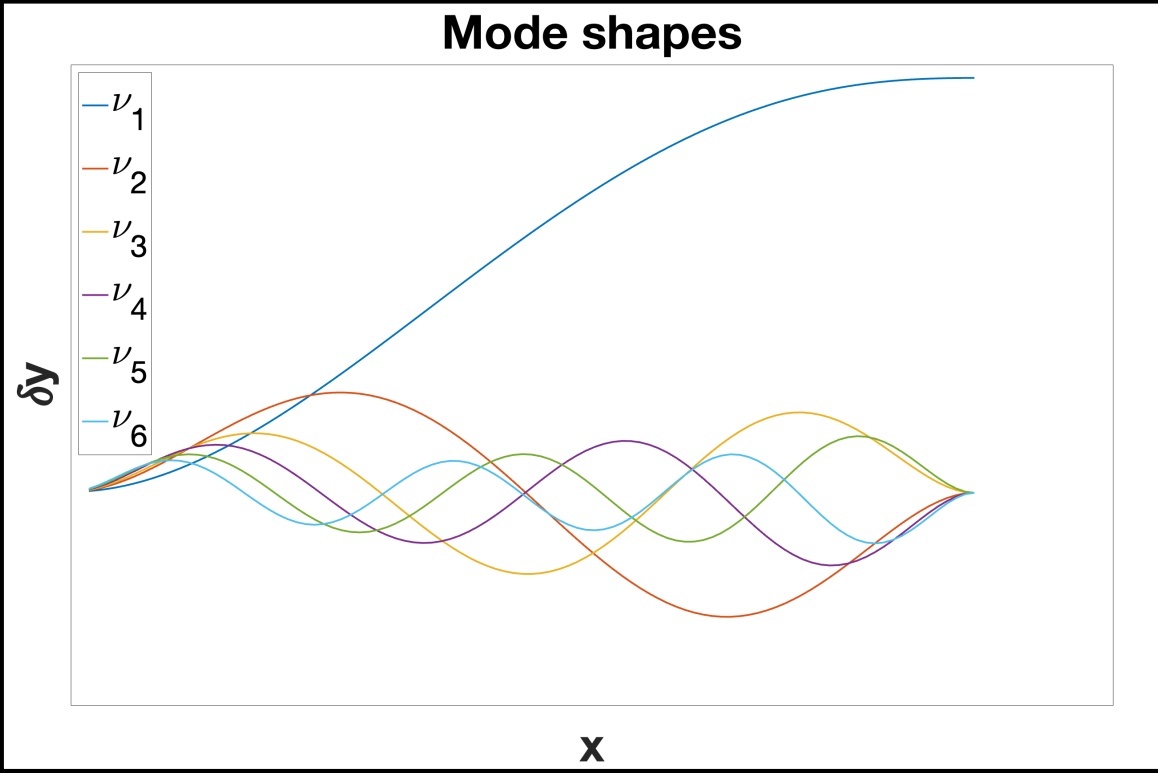
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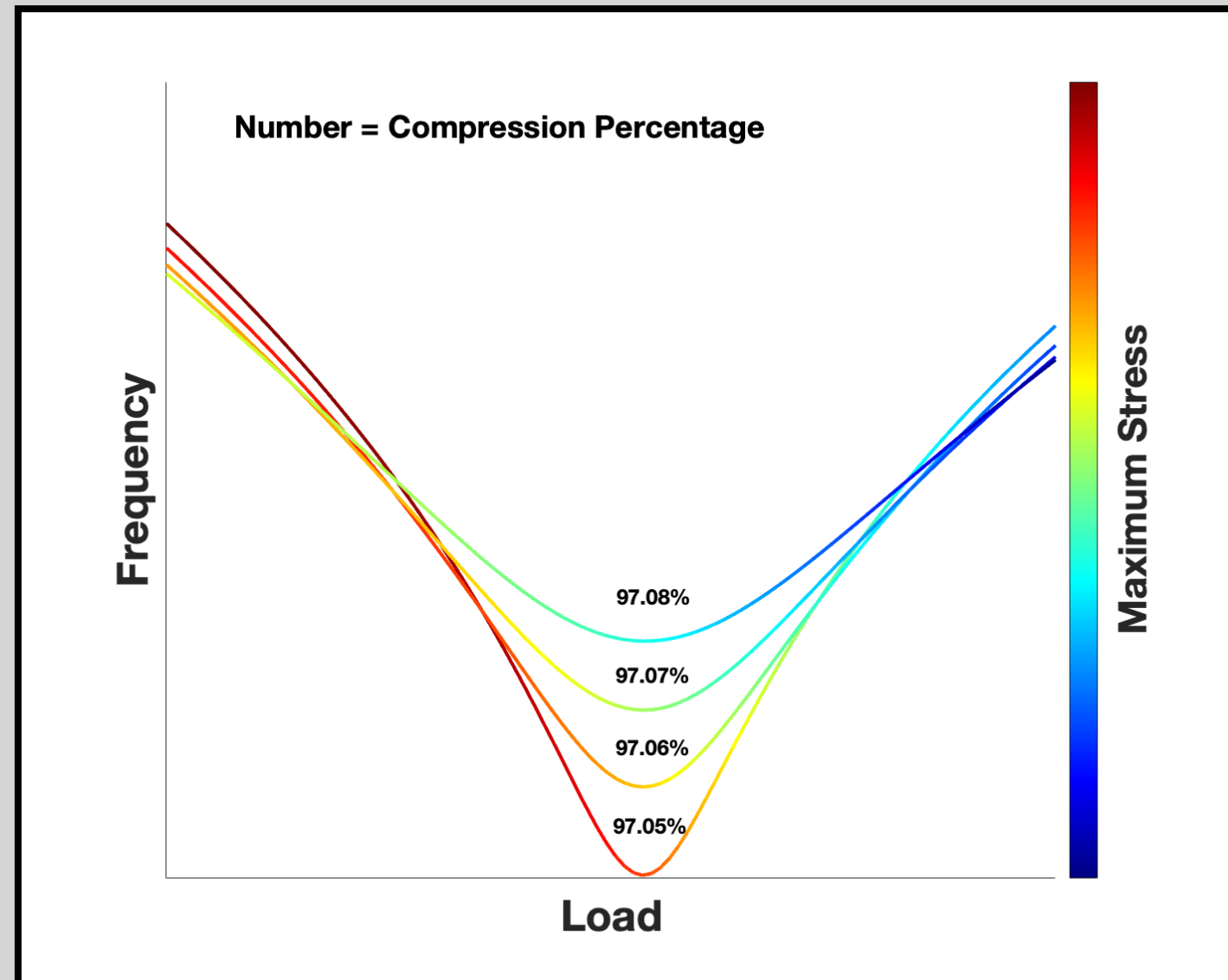
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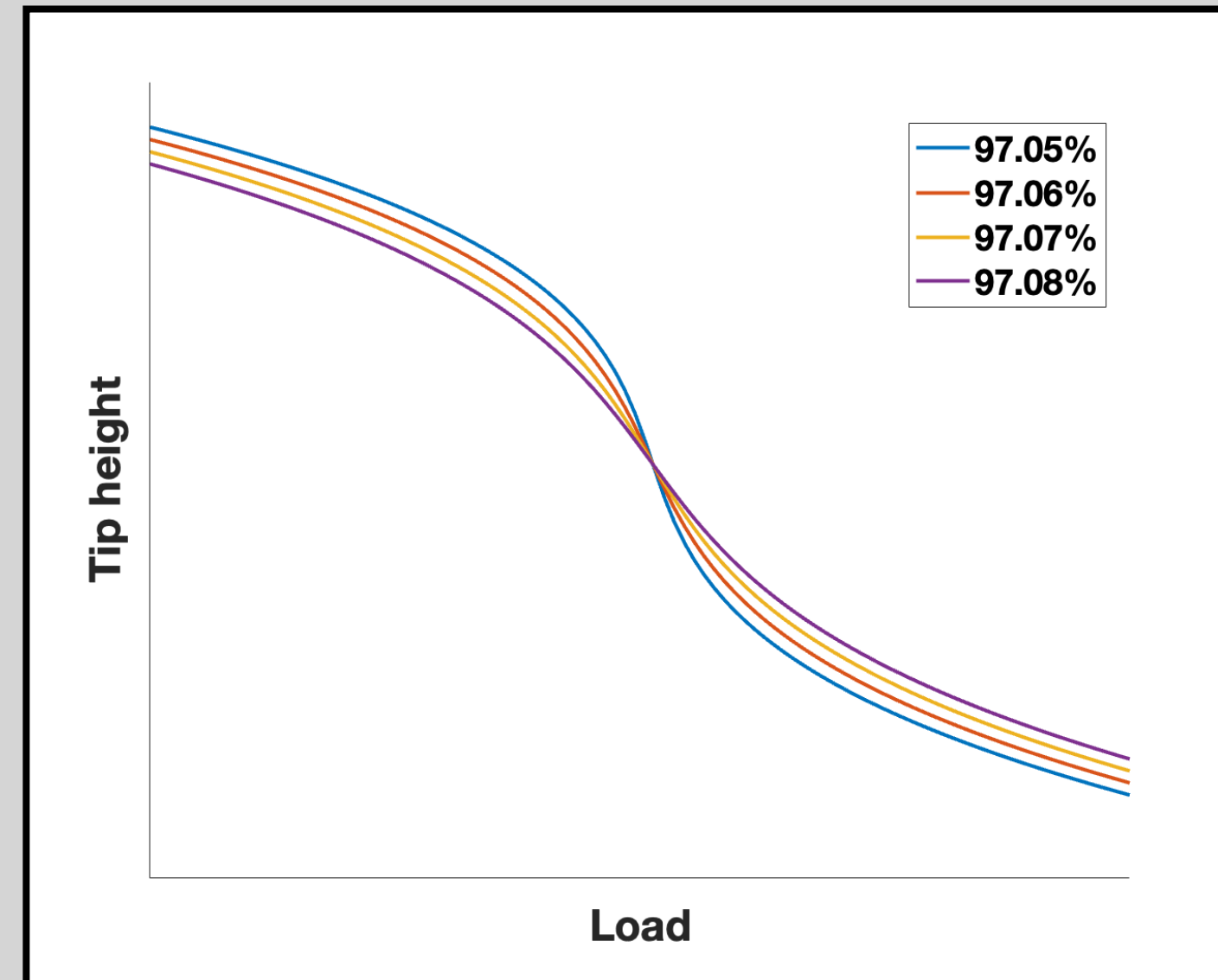
What can you do with FLECS?

Frequency vs Load scans



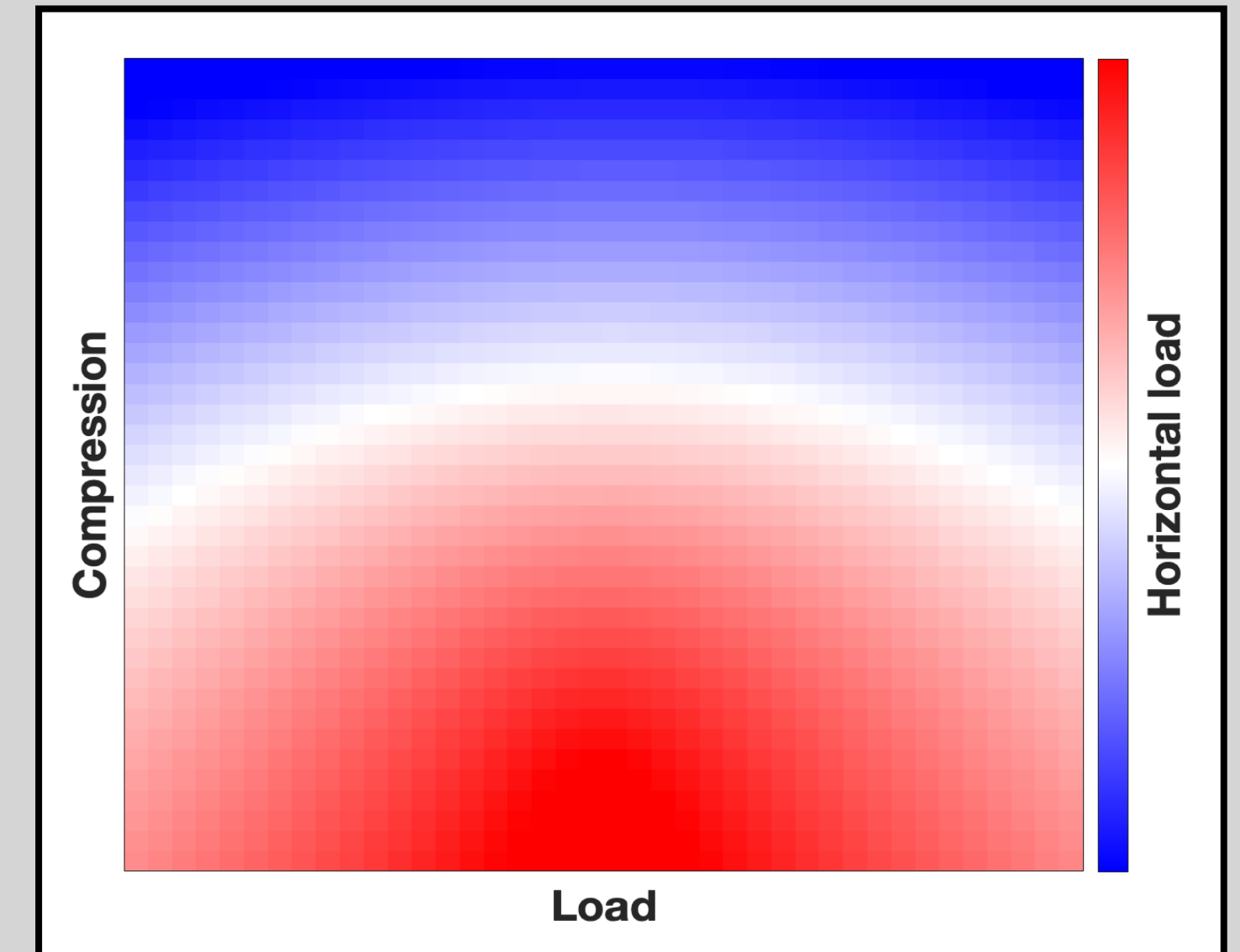
Turn the **compression** and **vertical load** knobs to find the **minimum frequency** reachable by your blade and the **maximum stress** to which it is subjected

Tip height vs Load scans



Visualize the expected lowering of the vertical load suspension system

Horizontal load scans



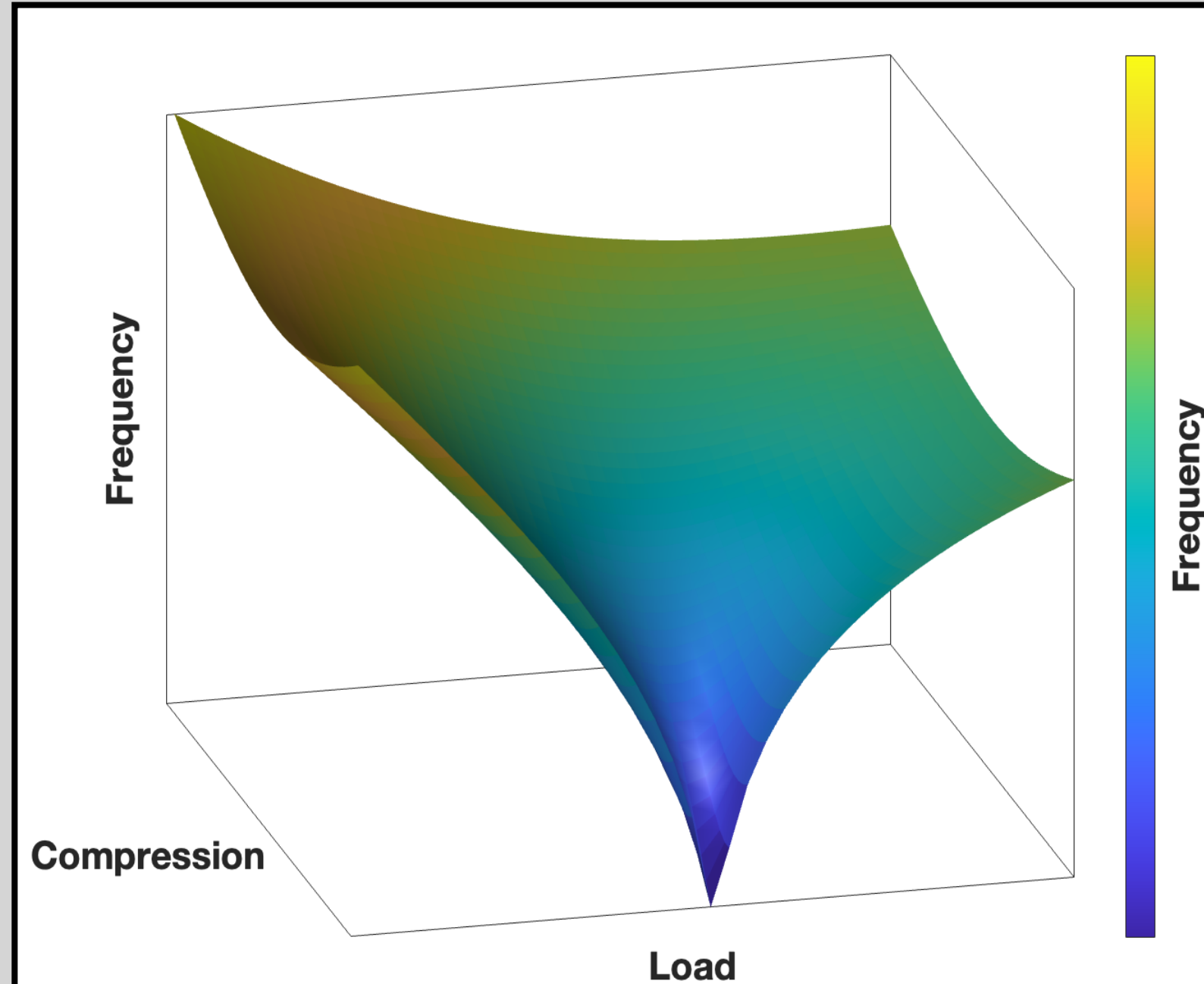
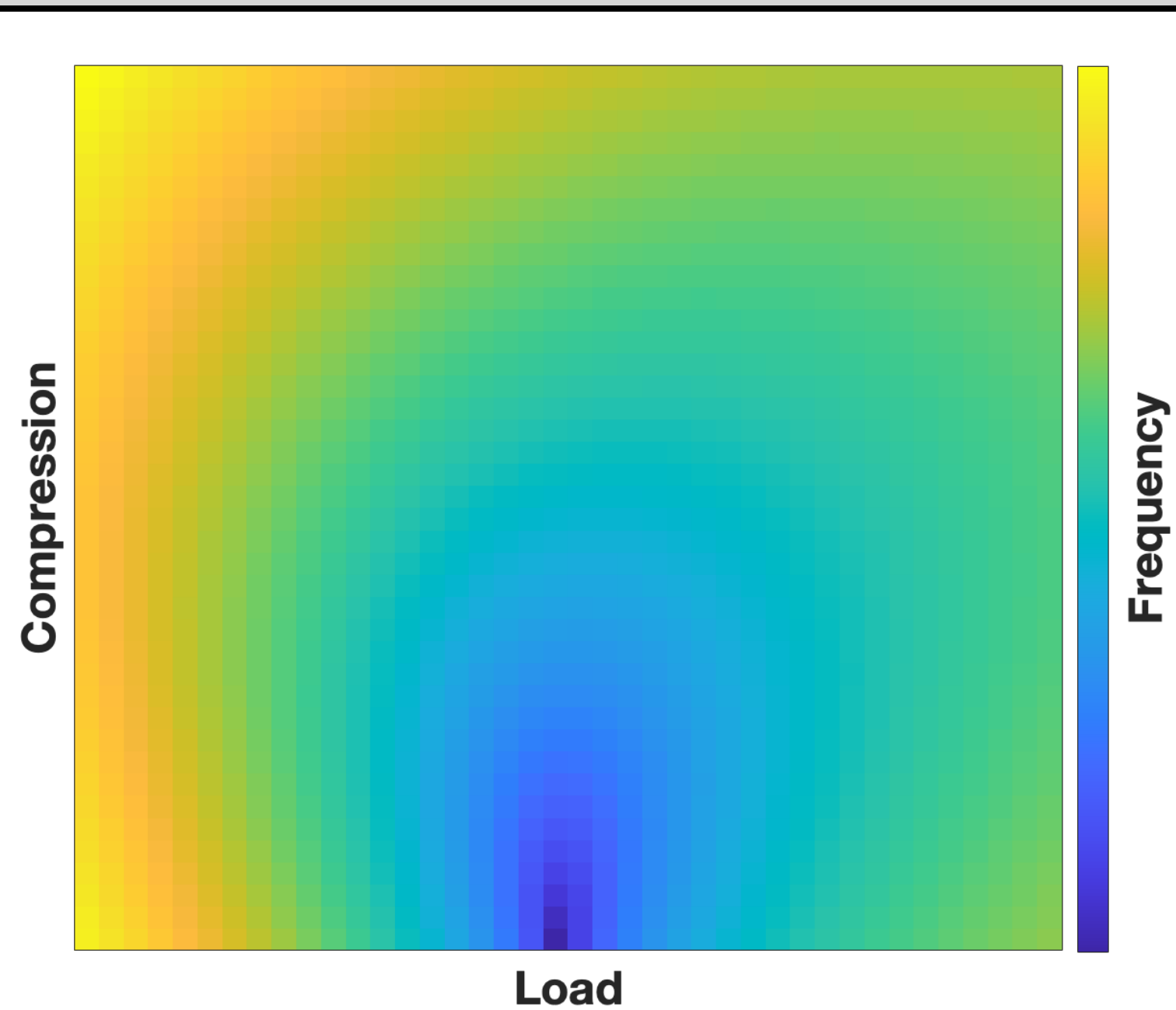
Evaluate the **force** needed to reach the degree of compression required to configure the GAS system at the working point

A 40×40 heatmap (1600 points, full eigenvalue problem at each with N=64 discretization elements) completes in ~180 s — **~112 ms per point**

FLECS: finding the working point

WORK IN PROGRESS

The goal: given a certain blade geometry and a $(\theta_{in}, \theta_{out})$ pair \longrightarrow find (x_b^*, M^*) that minimize the resonant frequency, i.e. find automatically the optimal compression and load for a given GAS configuration



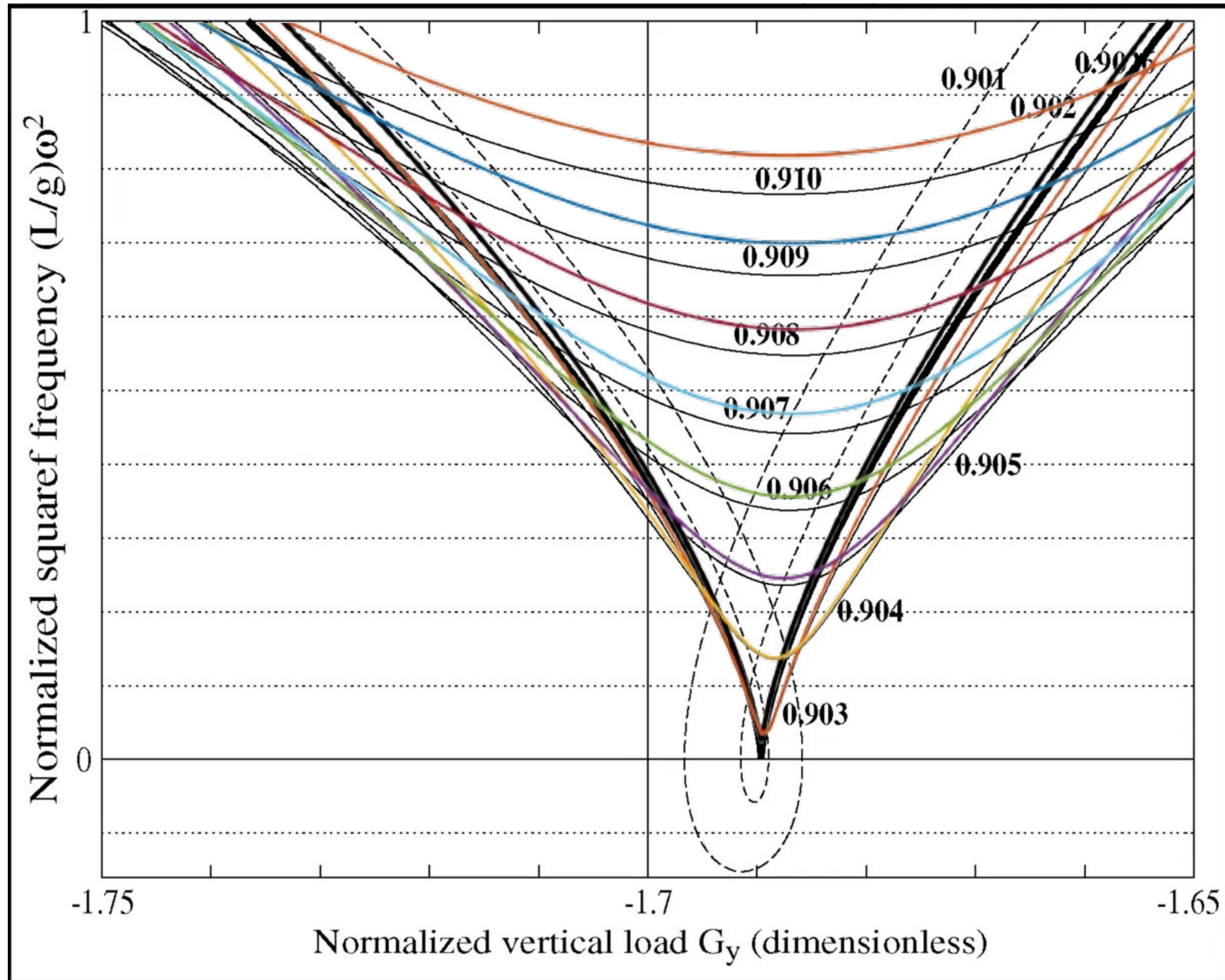
How:

- compute the numerical gradient of the $f(M, x_b)$ function
- Impose symmetry conditions on the $f_{x_b}(M)$ and $y_{tip, x_b}(M)$ that cancel $\nabla f(M, x_b)$



Stable configurations don't have discontinuities

Validation against G. Cella et al., Monolithic geometric anti-spring blades, NIM A



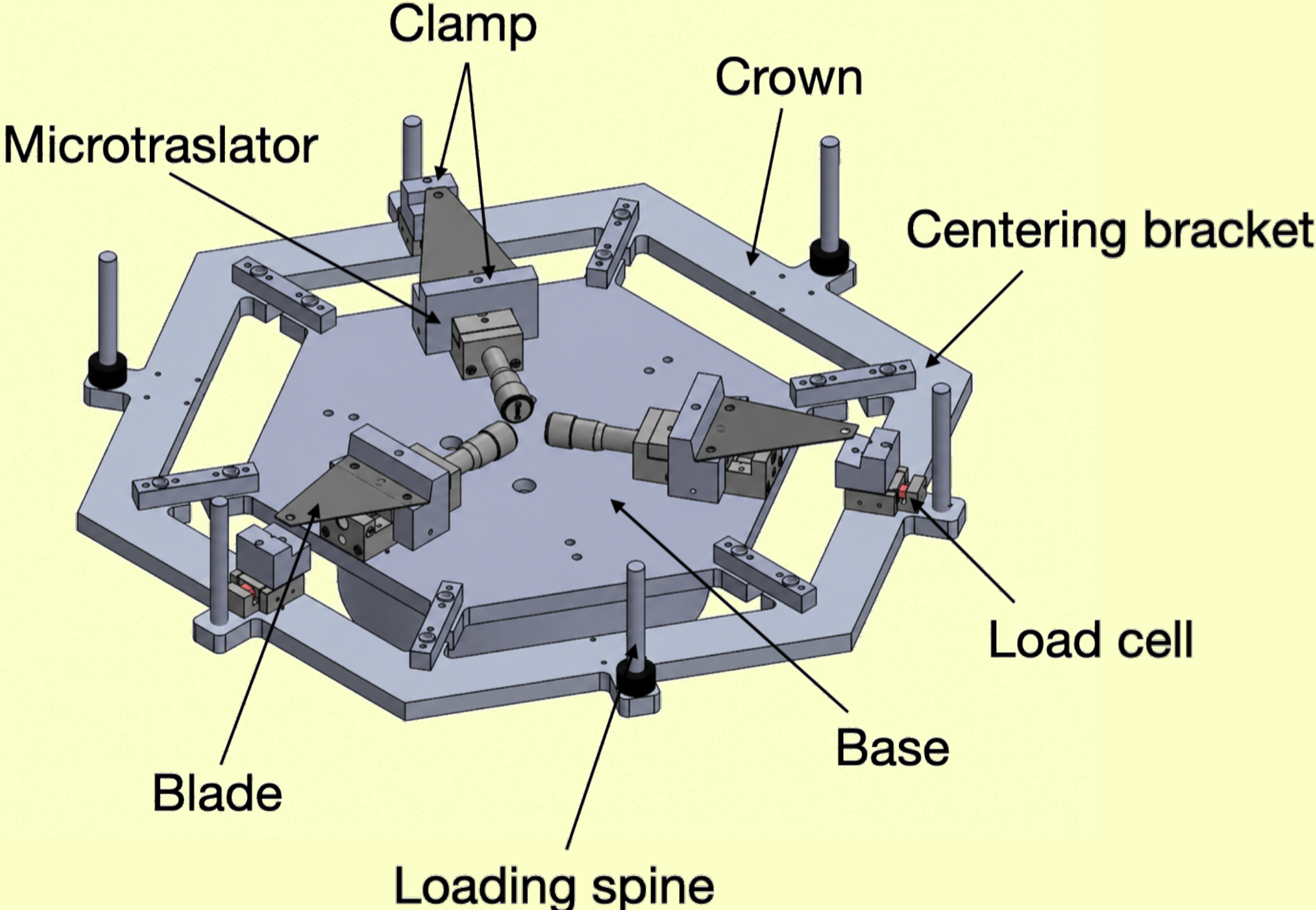
Cella et al. (black curves) solves the exact continuous thin-blade model numerically via AUTO 2000

FLECS (colored curves) discretizes **almost** the same physics (in Cella et al. the blade is massless) into N arc elements

Quantitative agreement on normalized frequencies for high compression curves. Lower compression curves discrepancy - origin under investigation

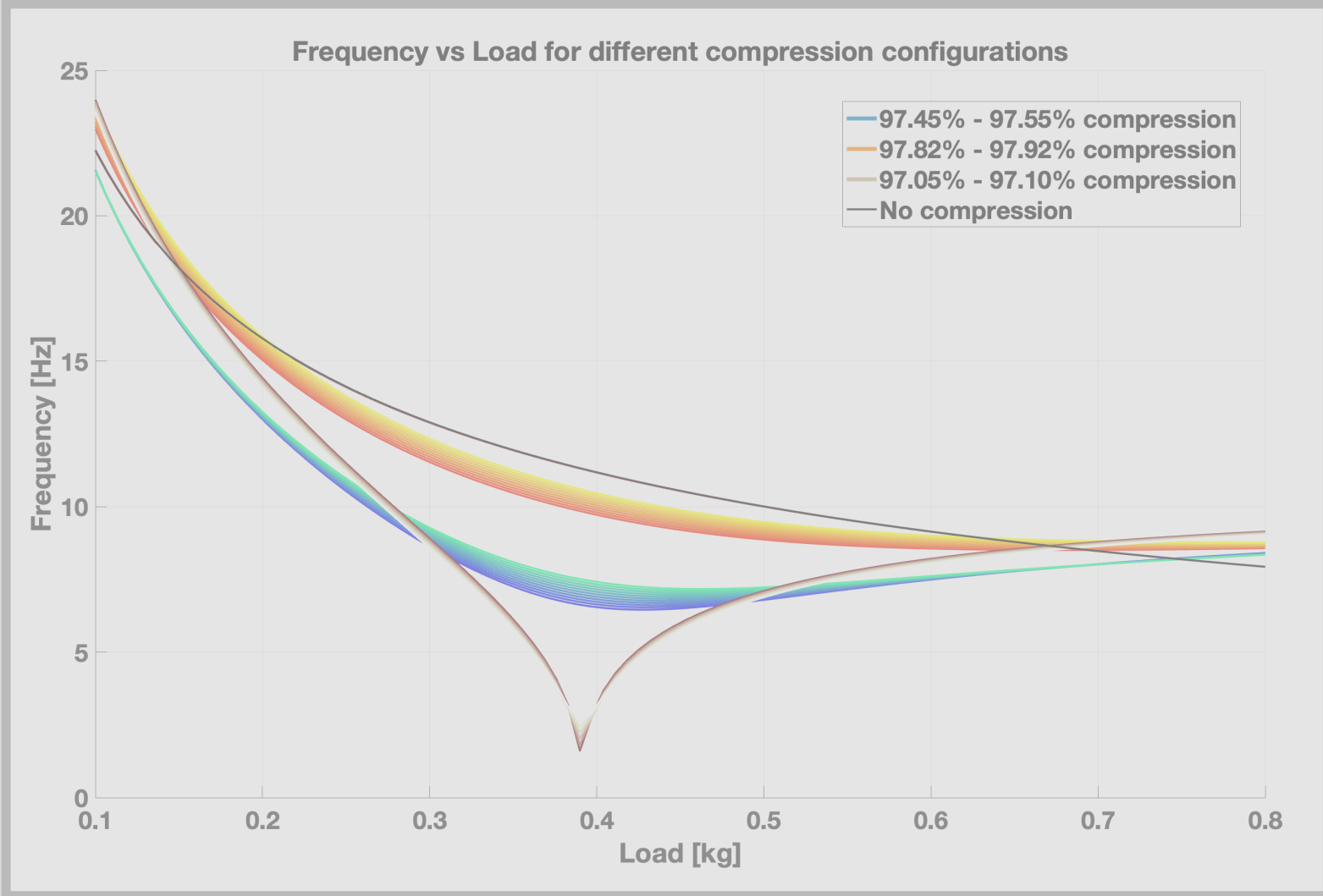
CROWN-GAS: an experiment to validate FLECS

Experimental setup



Crown diameter ≈ 30 cm **Blade thickness** ≈ 0.3 mm
Blade length after clamping ≈ 5 cm **Blade material** = Spring steel

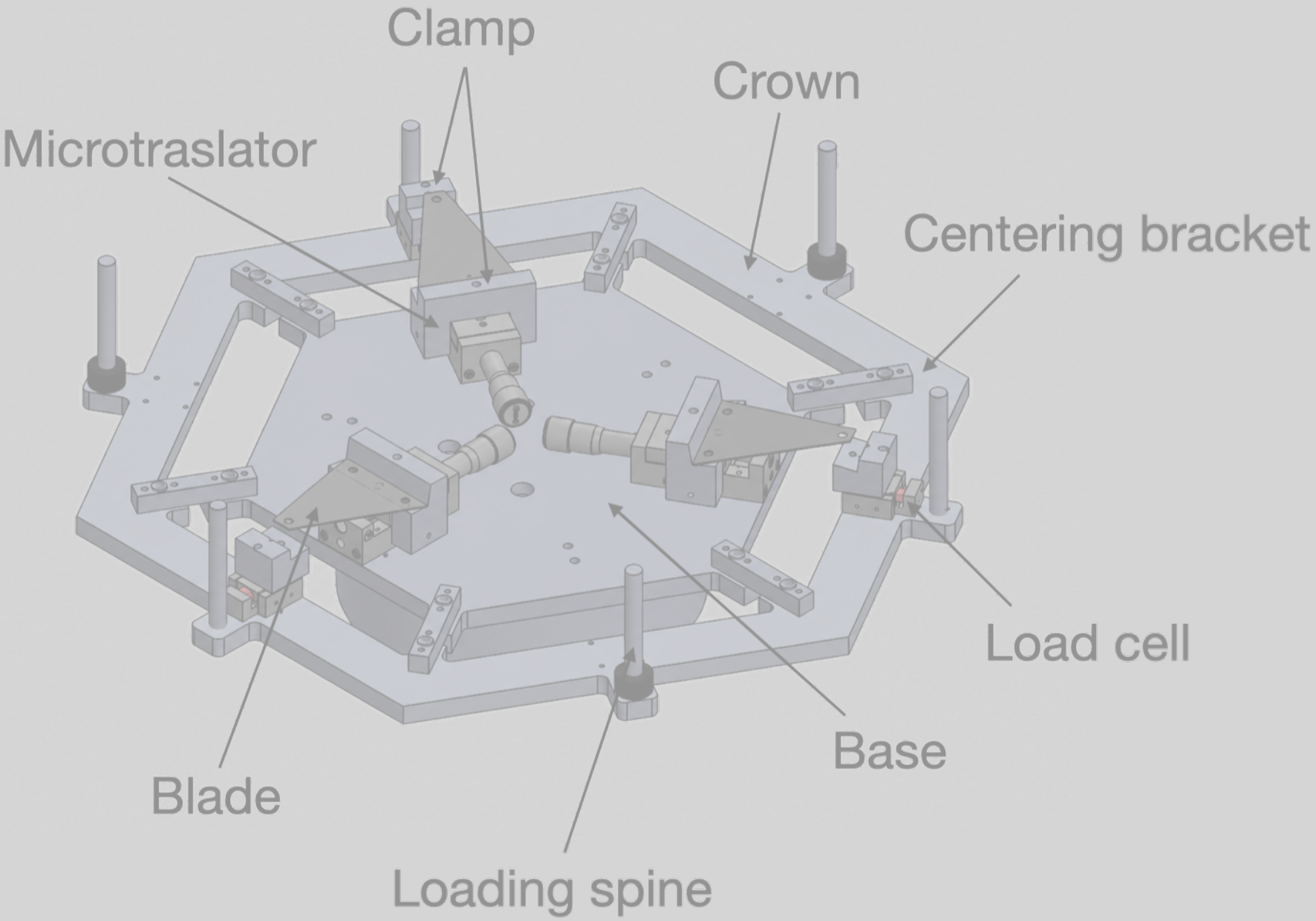
What do we measure?



We plan to investigate the first mode frequency vs vertical load curve in different horizontal compression configurations and compare the results with the predictions of FLECS

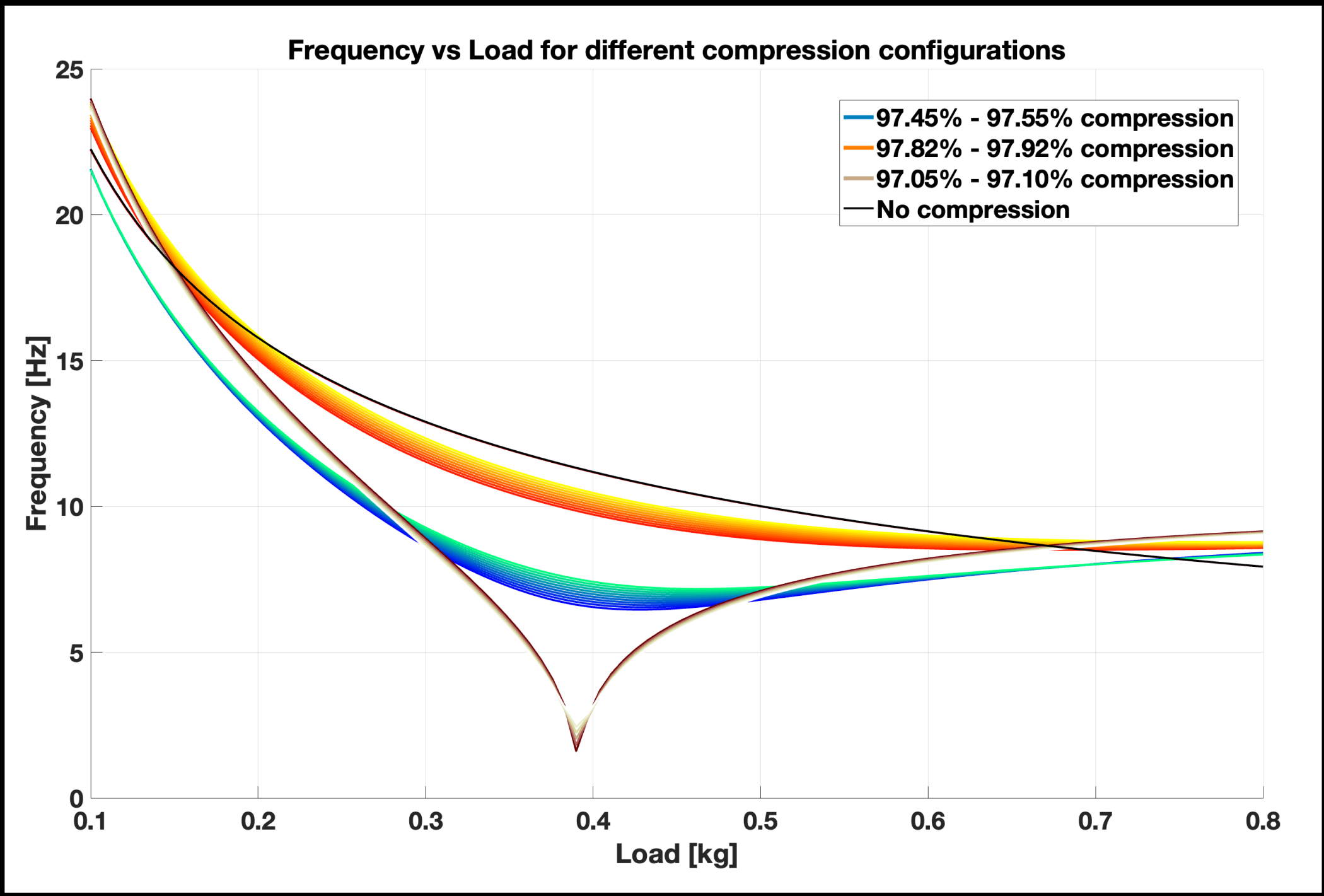
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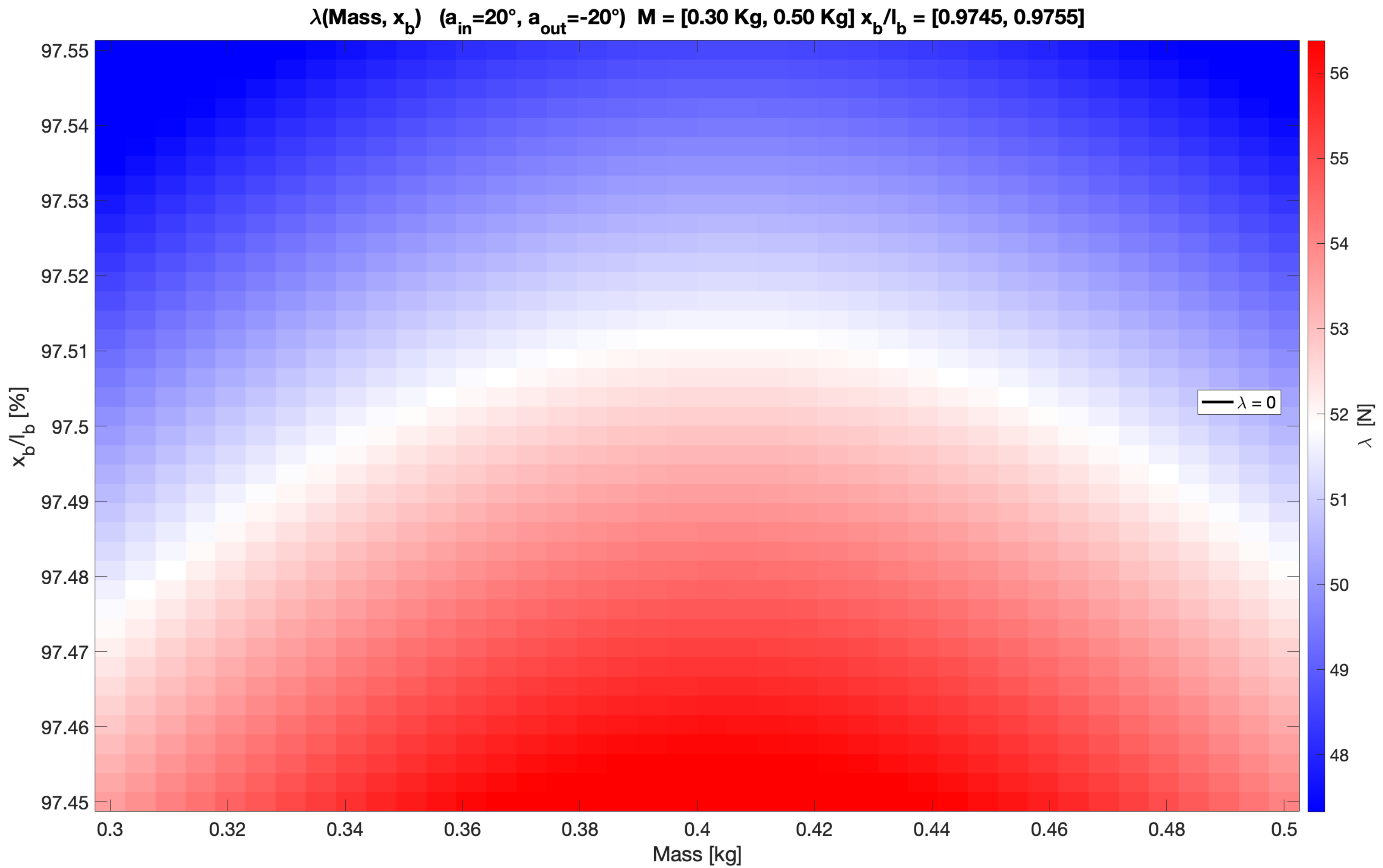


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Thank you for your attention!

Supplementary slides

Translating a $\pm 25 \mu\text{m}$ measurement into a $\pm 4 \text{ N}$ measurement with load cells



A variation of ± 2 K around room temperature corresponds to a variation of $\pm 1\mu\text{m}$ of the length of the blade

