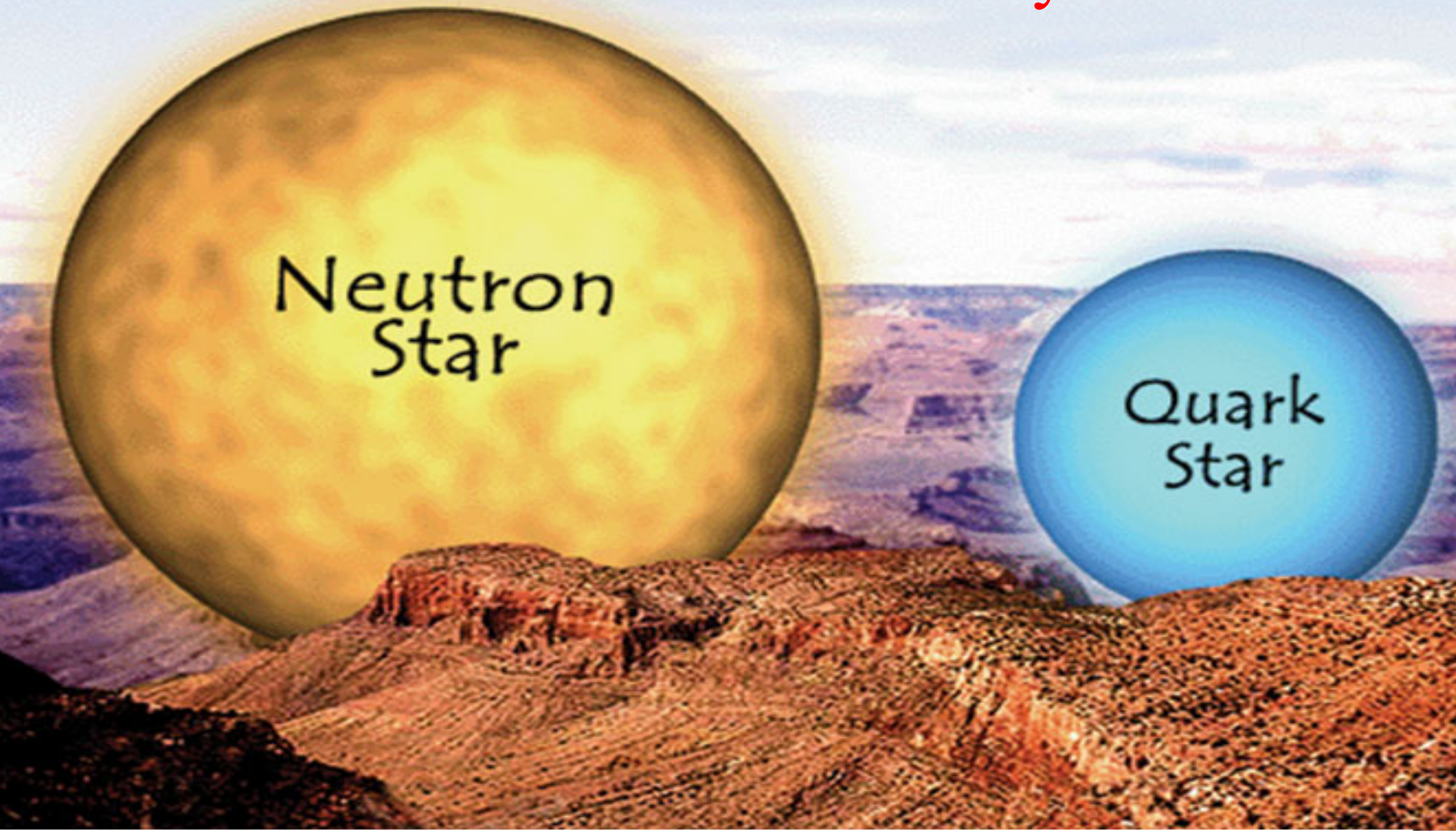
Alessandro Parisi 16 May 2026



The 14th KAGRA International Workshop

INFN
Istituto Nazionale di Fisica Nucleare

STUDIUM GENERALE CIVITATIS PERUSII
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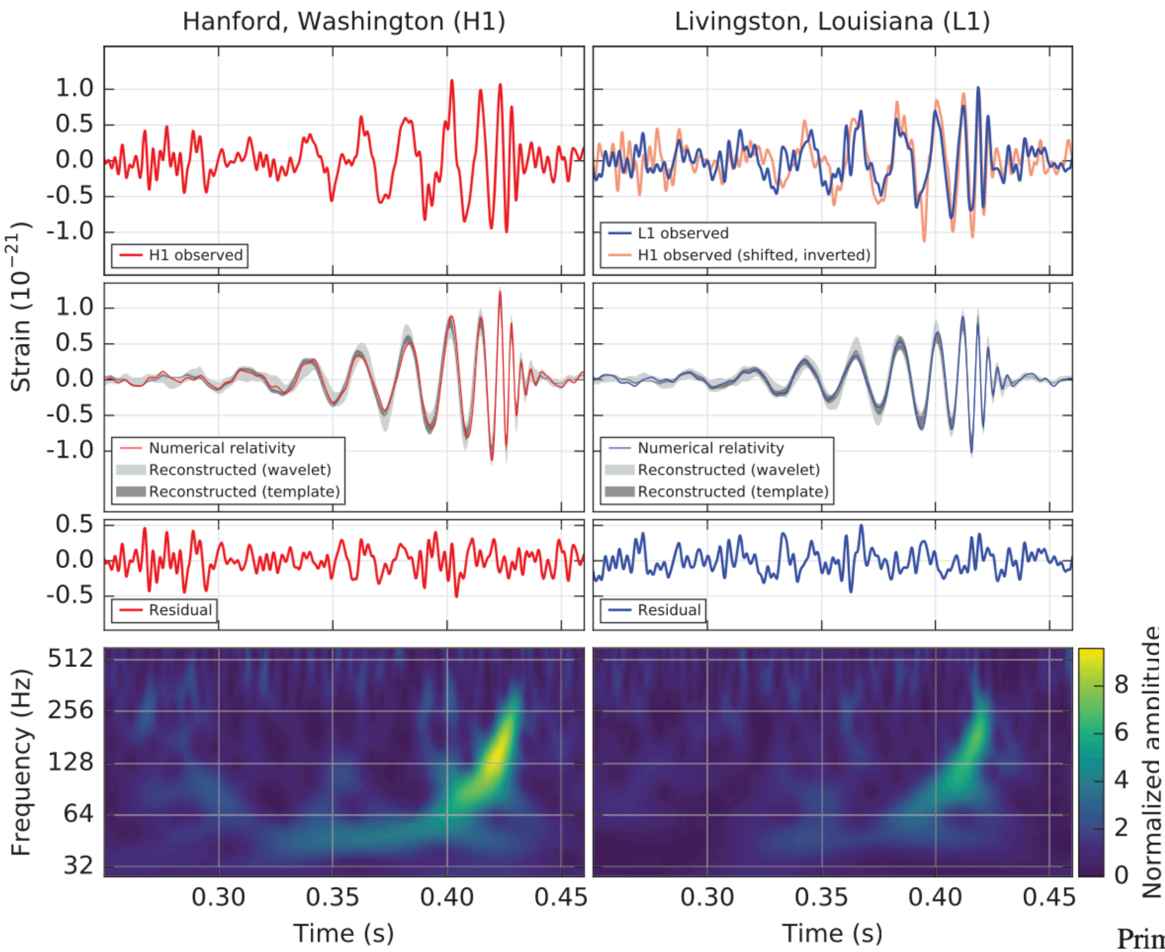
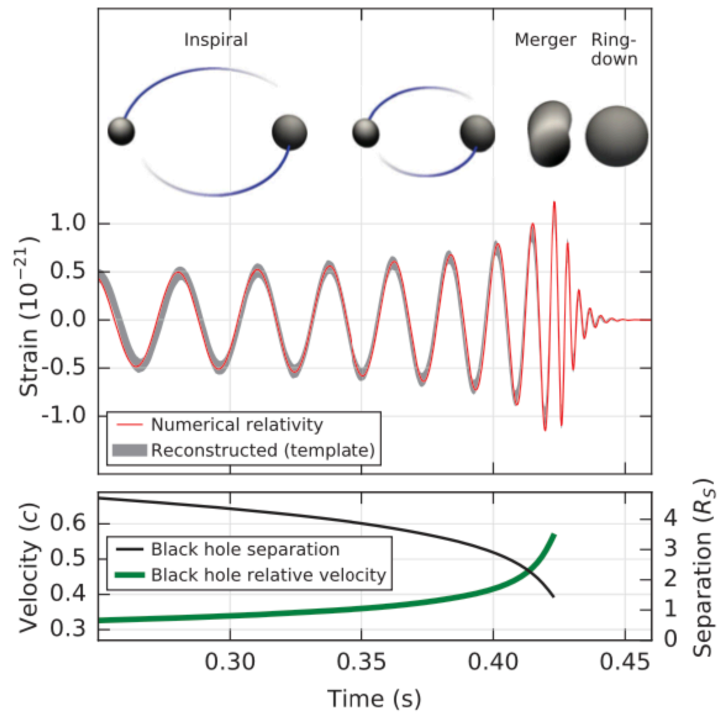
Motivation:First LIGO detection during OI:GW150914



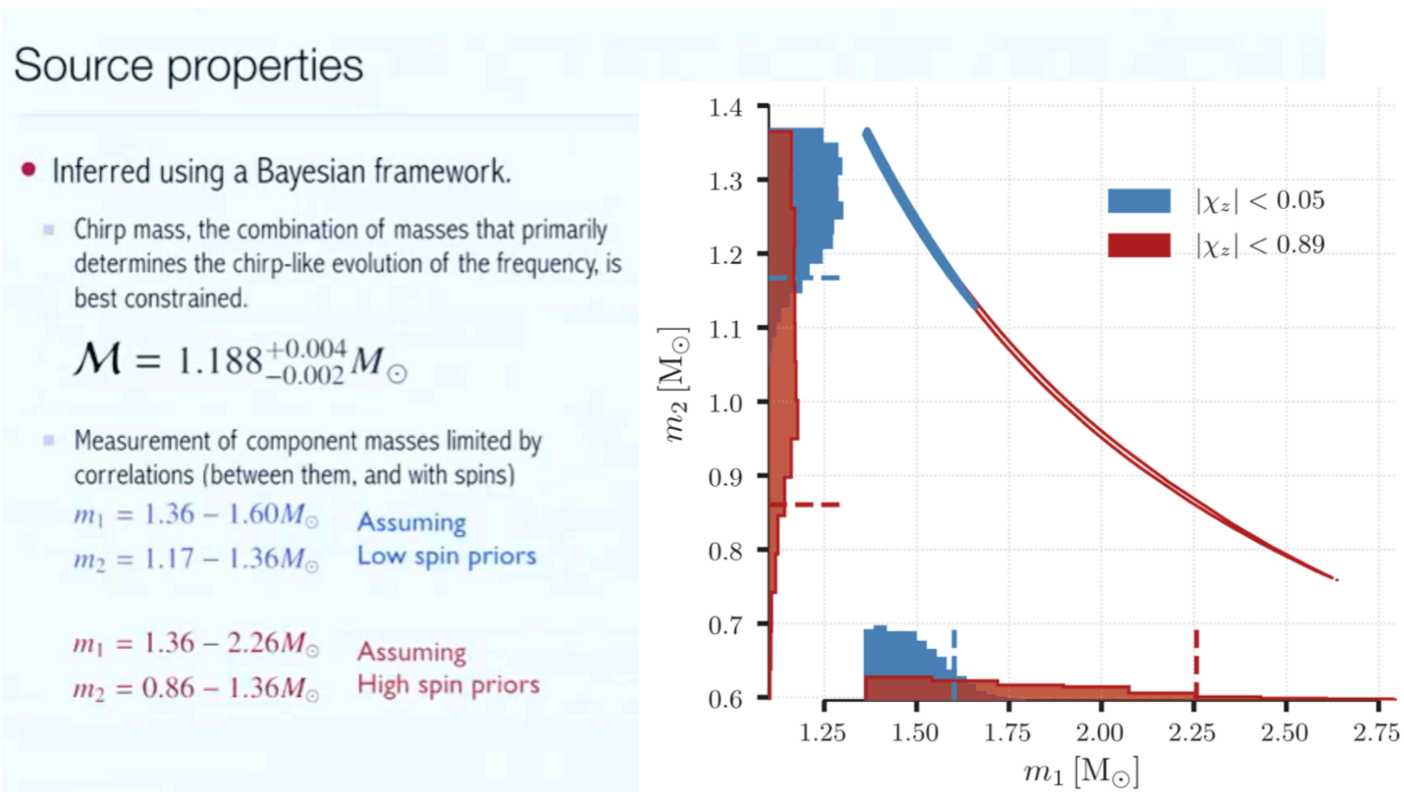


$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5}$$

Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

	Low-spin priors ($\lvert\chi\rvert \leq 0.05$)	High-spin priors ($\lvert\chi\rvert \leq 0.89$)
Primary mass m_1	1.36–1.60 $M_\odot$	1.36–2.26 $M_\odot$
Secondary mass m_2	1.17–1.36 $M_\odot$	0.86–1.36 $M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0	0.4–1.0
Total mass $m_{\rm tot}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\rm rad}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance $D_{\rm L}$	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55°$	$\leq 56°$
Using NGC 4993 location	$\leq 28°$	$\leq 28°$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800	≤ 1400



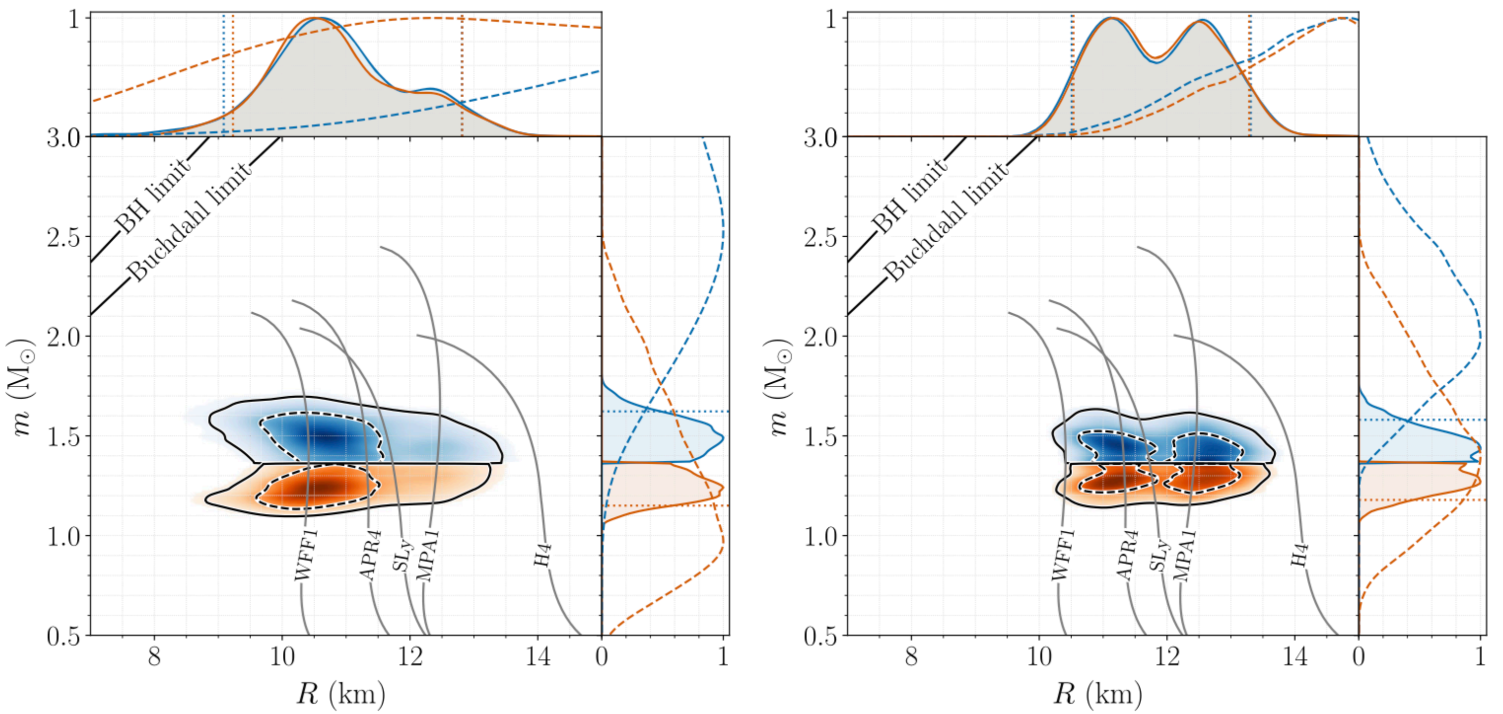


FIG. 3. Marginalized posterior for the mass m and areal radius R of each binary component using EOS-insensitive relations (left panel) and a parametrized EOS where we impose a lower limit on the maximum mass of 1.97 $M_\odot$ (right panel). The top blue (bottom orange) posterior corresponds to the heavier (lighter) NS. Example mass-radius curves for selected EOSs are overplotted in gray. The lines in the top left denote the Schwarzschild BH ($R = 2m$) and Buchdahl ($R = 9m/4$) limits. In the one-dimensional plots, solid lines are used for the posteriors, while dashed lines are used for the corresponding parameter priors. Dotted vertical lines are used for the bounds of the 90% credible intervals.

Open Questions

Test of GR

Primordial BH

DM Spike or ultralight boson cloud

Dark star

Phase transition

Pulsar oscillation

EoS

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

Hybrid stars in the light of the merging event GW170817

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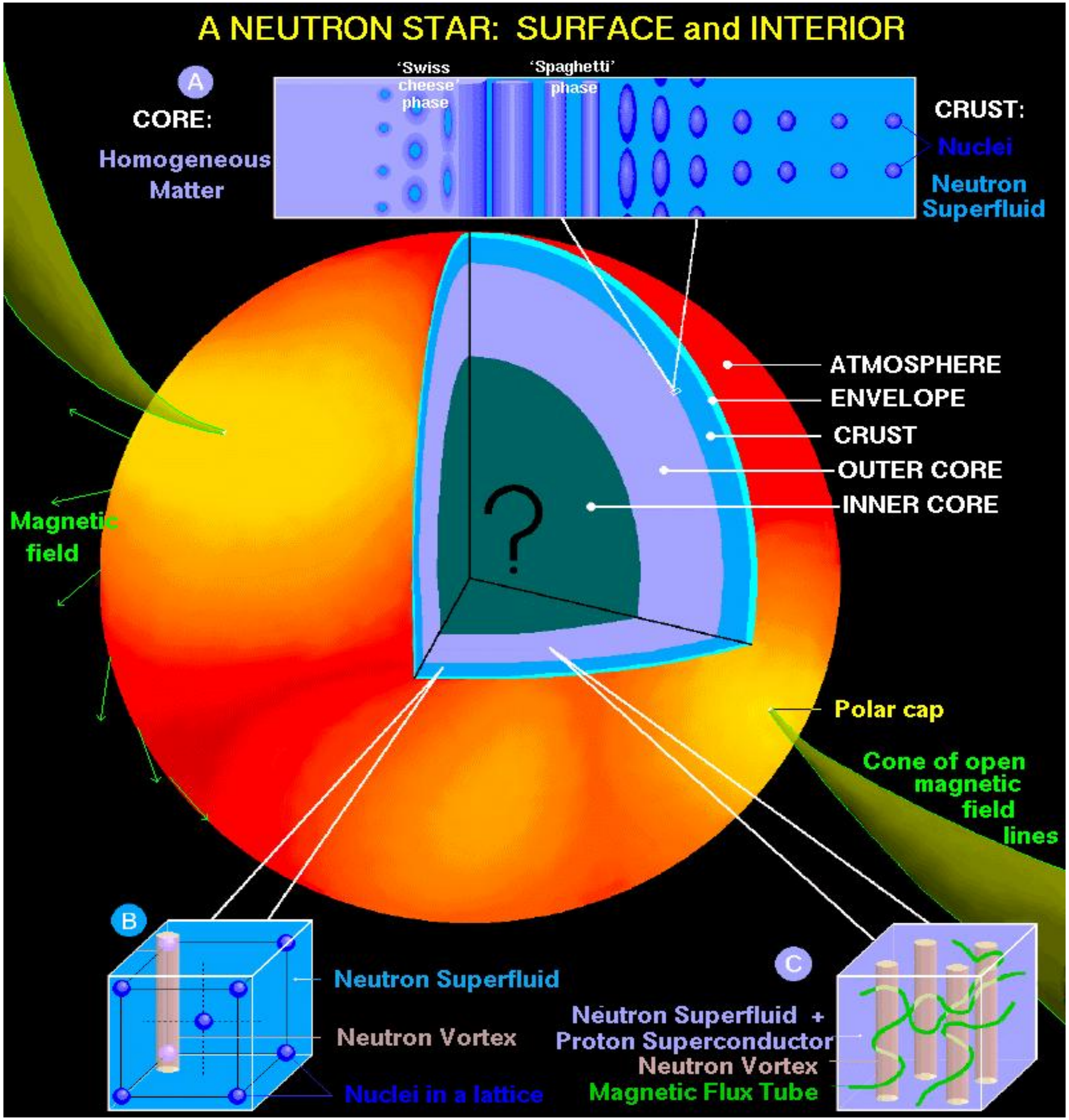
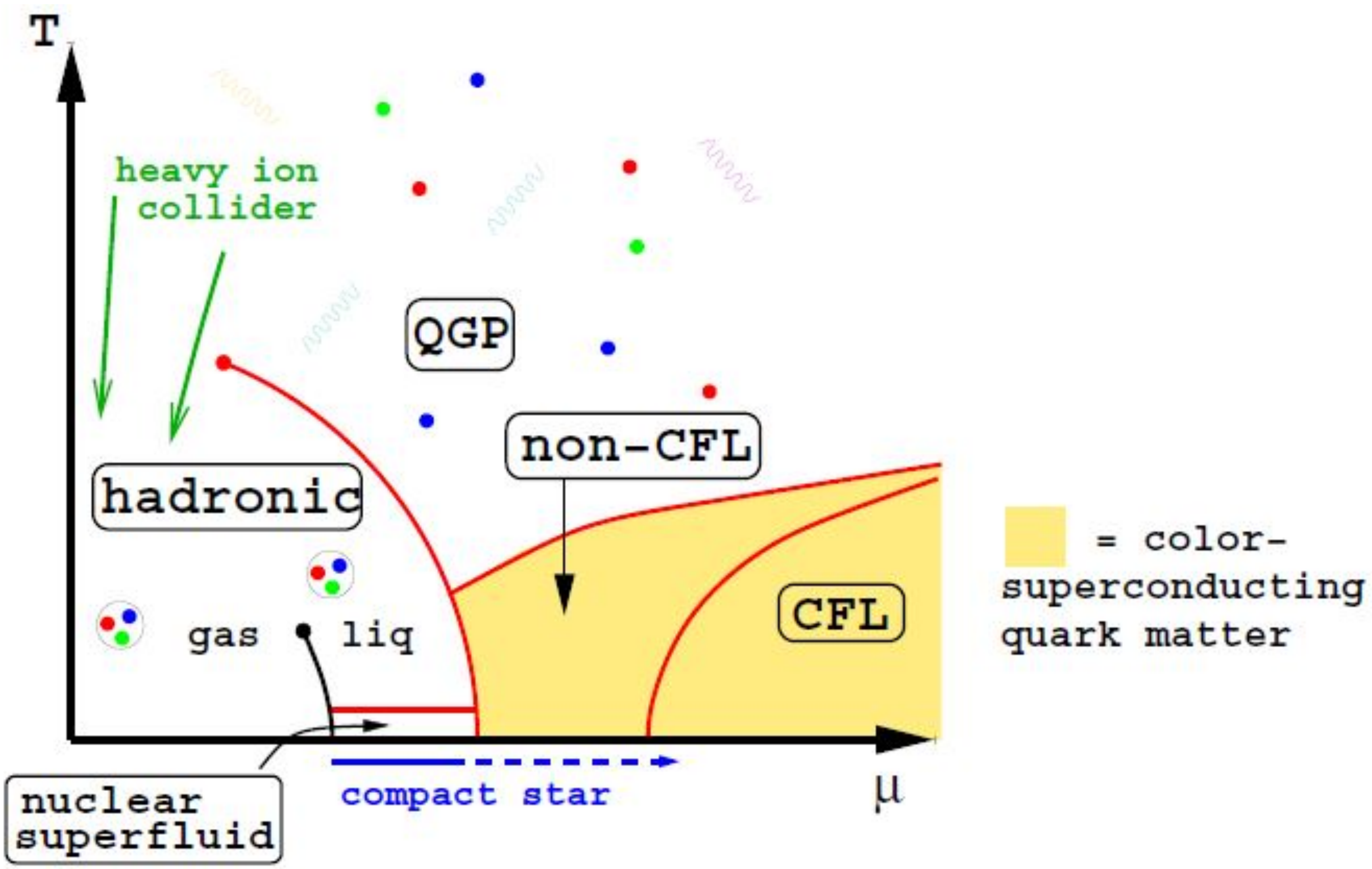
Neutron Star Structure

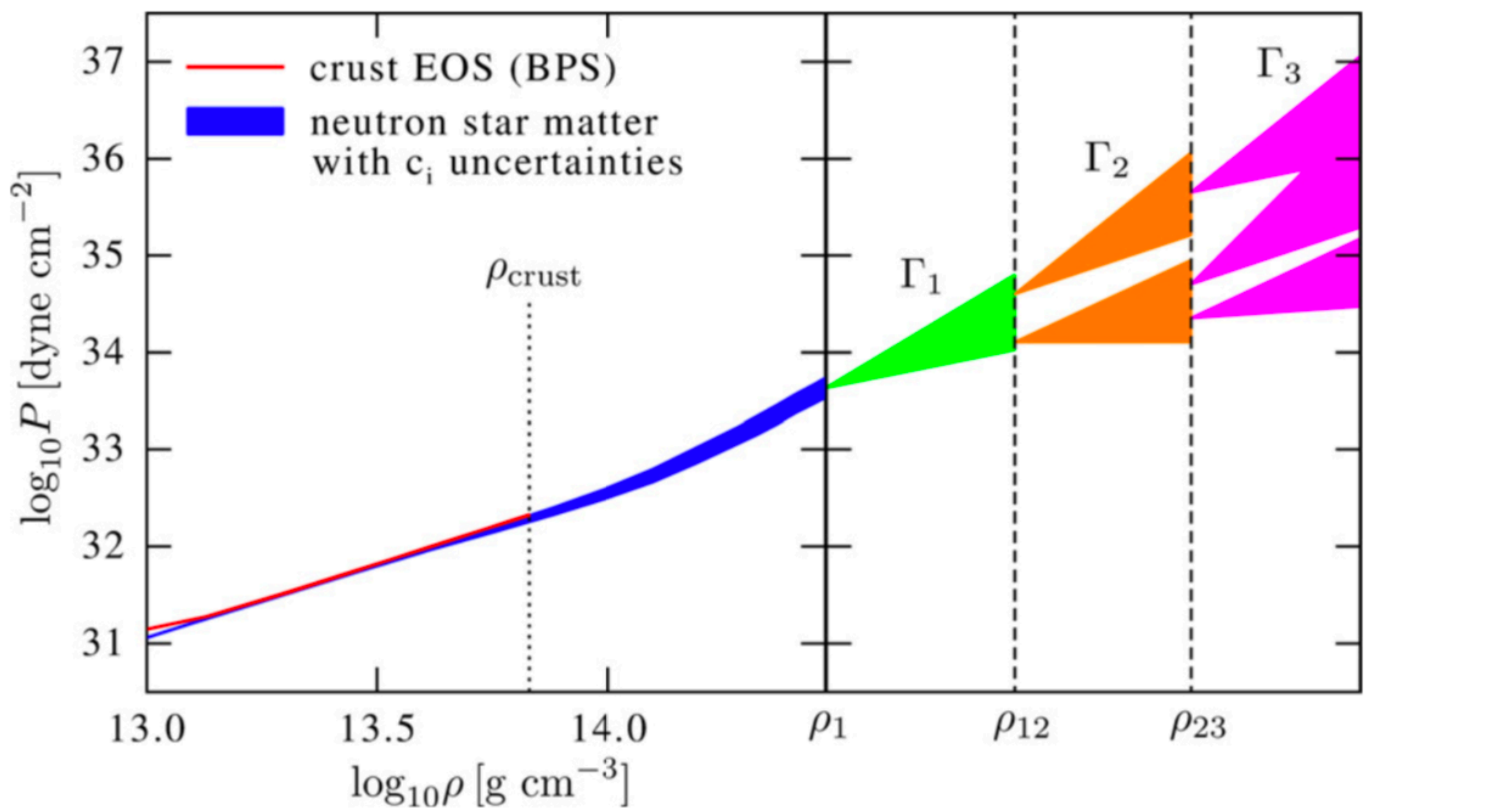


Conjectured QCD phase diagram

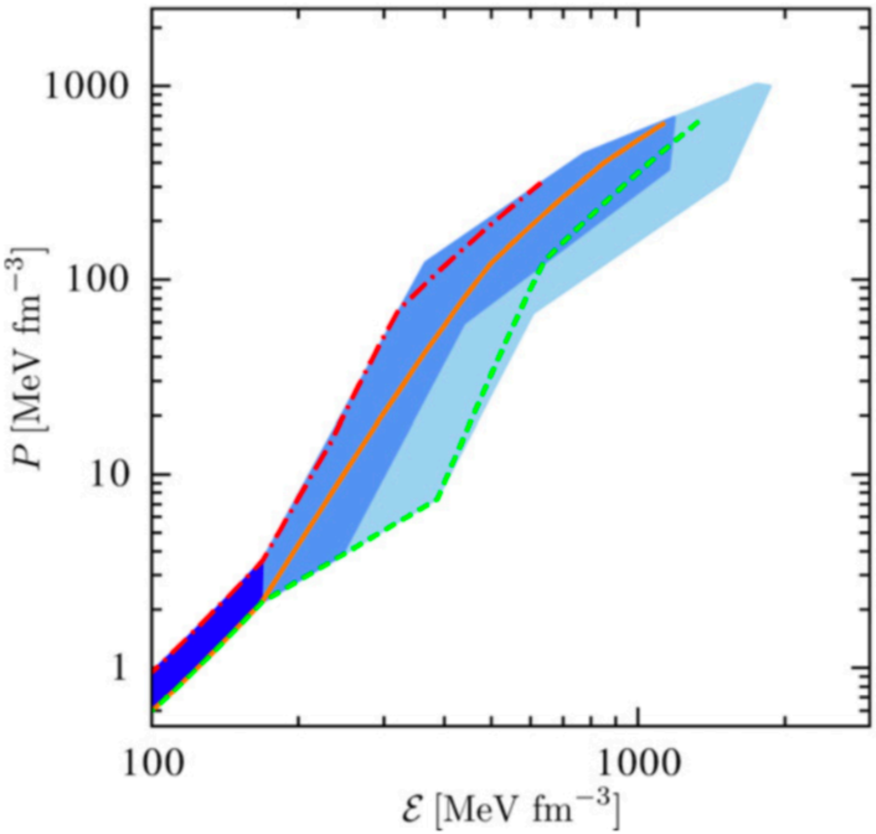


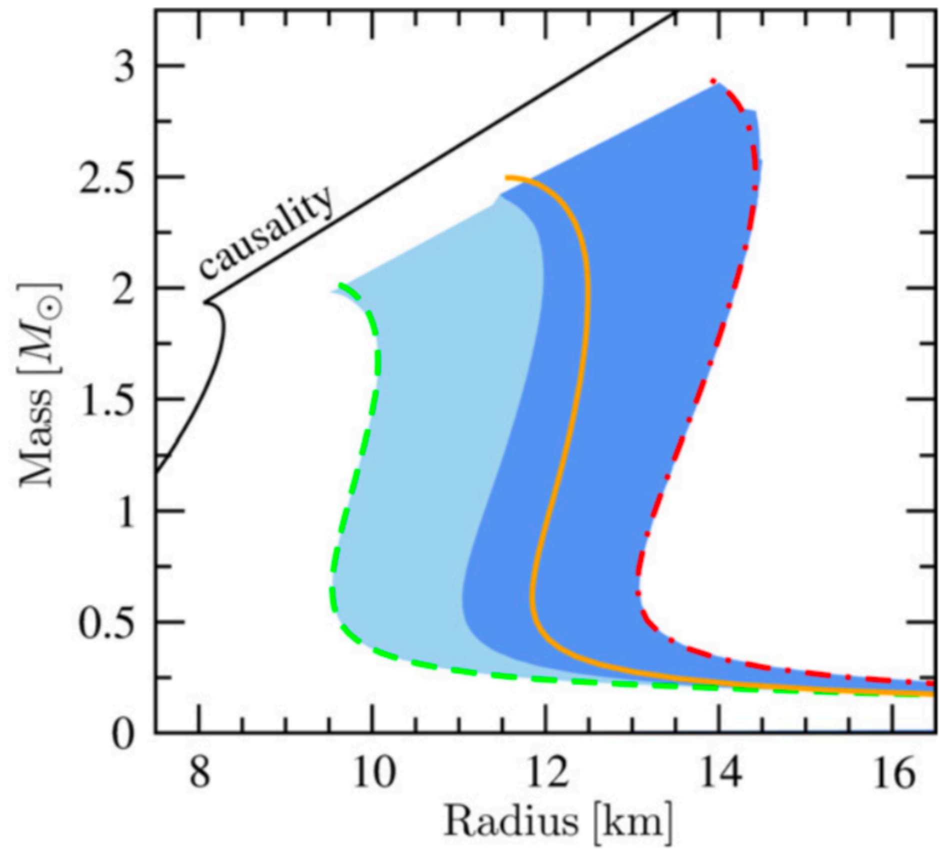
M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, arXiv:0709.4635 (RMP review)
A. Schmitt, arXiv:1001.3294 (Springer Lecture Notes)

Hebeler, Lattimer, Pethick, Schwenk EOS



Hebeler, Lattimer, Pethick, Schwenk EOS





EOS of the Crust

- Outer crust: densities below neutron drip $\rho \lesssim 10^{-4}$ fm^{-3}
  - Baym-Pethick-Sutherland (BPS) (1971)
  - Haensel & Pichon (1994)
  - Ruester, Hempel, Schaffner-Biellich (2006)
- Inner crust: from neutron drip to crust-core transition
  - Baym-Bethe-Pethick (CLDM) 1971
  - Negele-Vautherin (HF) 1973
  - Douchin & Haensel (CLDM) 2001
  - Crust-core transition density (Ducoin et al 2011)
  - Inner crust EOS within TF (Grill et al 2014)
    NL3, TM1, FU, IUFSU, DDME2,DDHδ,NL3$\omega\rho$
- Which construction is generally used?
  - Glendenning: (*Compact stars* 2000)
    BPS + BBP + homogeneous above $\rho = 0.01$ fm^{-3}
- Does the radius of NS depend on the crust EOS?

Alford-Brady-Paris-Reddy Quark EoS

$$\Omega_{\rm QM} = -\frac{3}{4\pi^2} a_4\, \mu^4 + \frac{3}{4\pi^2} a_2\, \mu^2 + B_{\rm eff} + \Omega_e$$

$$a_4 \equiv 1 - c$$

$$a_2 = m_s^2 - 4\Delta^2$$

$$n = -\frac{1}{3}\frac{\partial \Omega_{\rm QM}}{\partial \mu} = \frac{1}{2\pi^2}(2a_4\mu^3 - a_2\mu),$$

$$\mu \equiv (\mu_u + \mu_d + \mu_s)/3$$

$$\begin{aligned}\epsilon = 3n\mu - p &= \frac{9}{4\pi^2} a_4 \mu^4 - \frac{3}{4\pi^2} a_2 \mu^2 + B_{\rm eff} \\ &= 3p + 4B_{\rm eff} + \frac{3a_2}{2\pi^2}\mu^2.\end{aligned}$$

$$p(\epsilon) = \frac{(\epsilon - 4B_{\rm eff})}{3} - \frac{a_2^2}{12\pi^2 a_4}\left[1 + \sqrt{1 + \frac{16\pi^2 a_4}{a_2^2}(\epsilon - B_{\rm eff})}\right]$$

Thermodinamical equilibrium conditions at the interface

$$p_{\rm tr}^{\rm QP} = p_{\rm tr}^{\rm HP} \qquad \epsilon_{\rm tr}^{\rm QP} = \epsilon_{\rm tr}^{\rm HP} + \Delta\epsilon$$

$$g_{\rm tr}^{\rm QP} = g_{\rm tr}^{\rm HP} \qquad 3\mu_{\rm tr} = \frac{p_{\rm tr}^{\rm HP} + \epsilon_{\rm tr}^{\rm HP}}{n_{\rm tr}^{\rm HP}}$$

$$\begin{cases} B_{\rm eff} &= \beta_0 + \beta_1 \Delta\epsilon + \beta_2 a_2 \\ a_4 &= \alpha_0 + \alpha_1 \Delta\epsilon + \alpha_2 a_2 \end{cases}$$

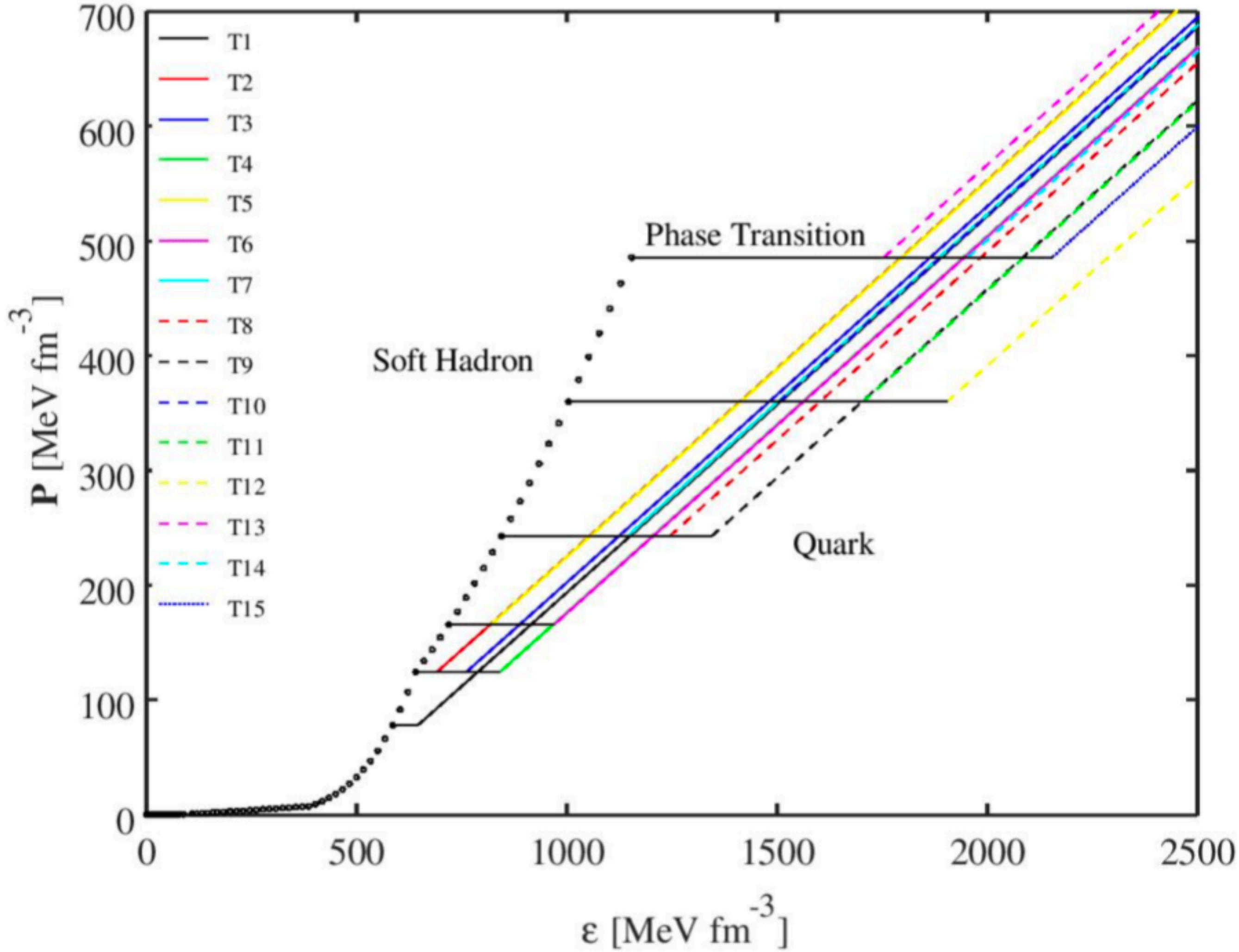
$$\beta_0 = \frac{1}{4}(\epsilon_{\rm tr}^{\rm HP} - 3p_{\rm tr}^{\rm HP}), \qquad \beta_1 = \frac{1}{4}, \qquad \beta_2 = -\frac{3\mu_{\rm tr}^2}{8\pi^2},$$

$$\alpha_0 = \frac{\pi^2}{3\mu_{\rm tr}^4}(\epsilon_{\rm tr}^{\rm HP} + p_{\rm tr}^{\rm HP}), \qquad \alpha_1 = \frac{\pi^2}{3\mu_{\rm tr}^4}, \qquad \alpha_2 = \frac{1}{2\mu_{\rm tr}^2}.$$

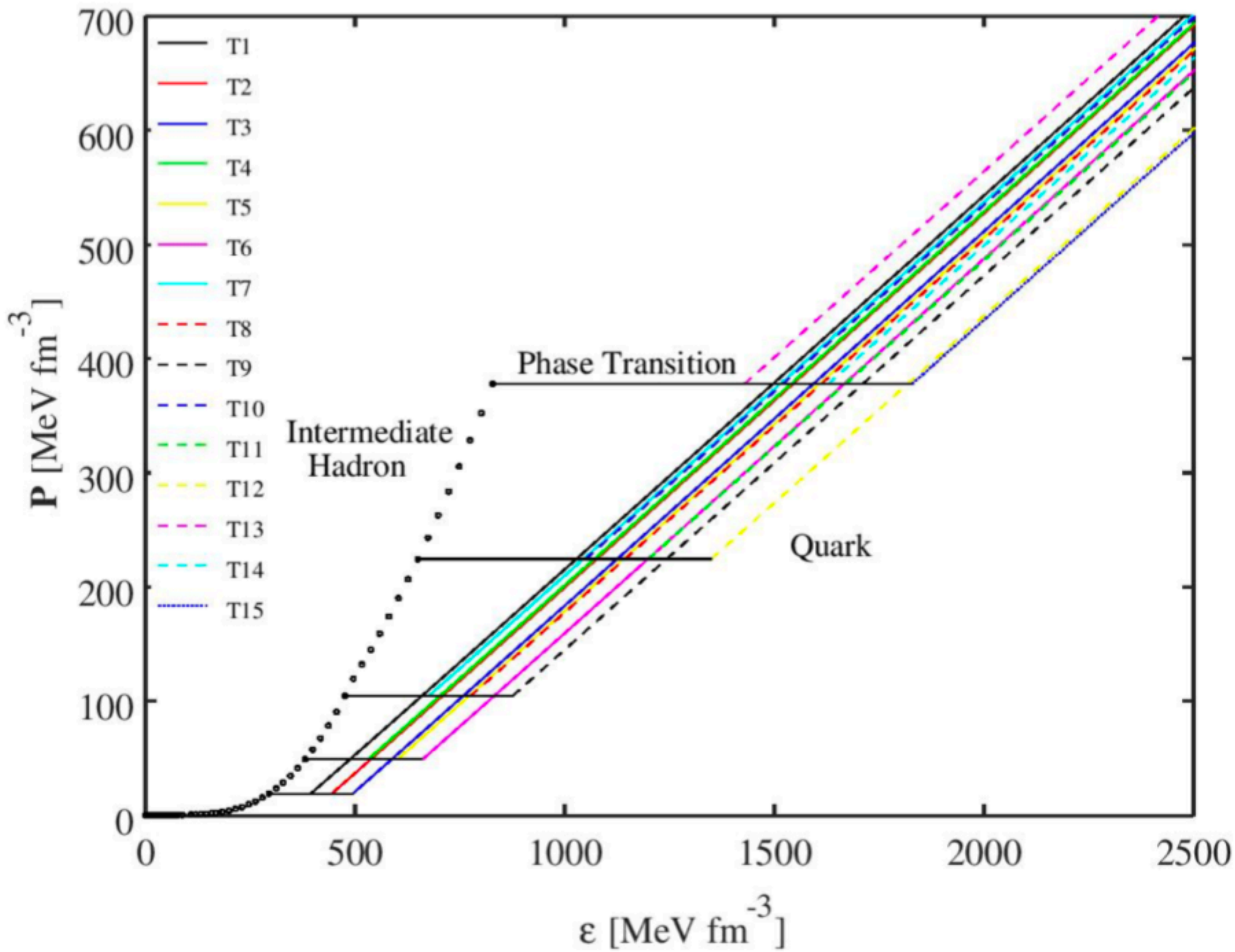
Table 1. We show the main parameters that characterize the phase transition, for Hebeler et al. (2013) soft EoS.

Transition	$p_{\rm tr}^{\rm HP}$(MeV fm^{-3})	$\epsilon_{\rm tr}^{\rm HP}$(MeV fm^{-3})	$n_{\rm B,tr}^{\rm HP}$(fm^{-3})	$\Delta\epsilon$(MeV fm^{-3})	$a_2^{1/2}$(MeV)	$B_{\rm eff}^{1/4}$(MeV)	a_4	$M_{\rm max}(M_\odot)$
1	77.9	585.2	0.592	60	100	164.79	0.976	1.497
2	124.3	640.7	0.639	50	100	153.15	0.848	1.666
3	124.3	640.7	0.639	120	100	161.75	0.919	1.640
4	124.3	640.7	0.639	200	100	170.15	0.999	1.565
5	165.5	719.1	0.704	100	100	153.35	0.837	1.490
6	165.5	719.1	0.704	250	100	170.30	0.960	1.564
7	242.8	844.6	0.799	300	100	163.89	0.856	1.724
8	242.8	844.6	0.799	400	100	173.85	0.916	1.714
9	242.8	844.6	0.799	500	100	182.34	0.976	1.712
10	359.8	1004	0.912	500	100	163.87	0.783	1.866
11	359.8	1004	0.912	700	100	182.33	0.865	1.865
12	359.8	1004	0.912	900	100	196.45	0.947	1.865
13	485.8	1154	1.008	600	100	146.29	0.671	1.947
14	485.8	1154	1.008	800	100	170.35	0.730	1.946
15	485.8	1154	1.008	1000	100	187.13	0.788	1.946

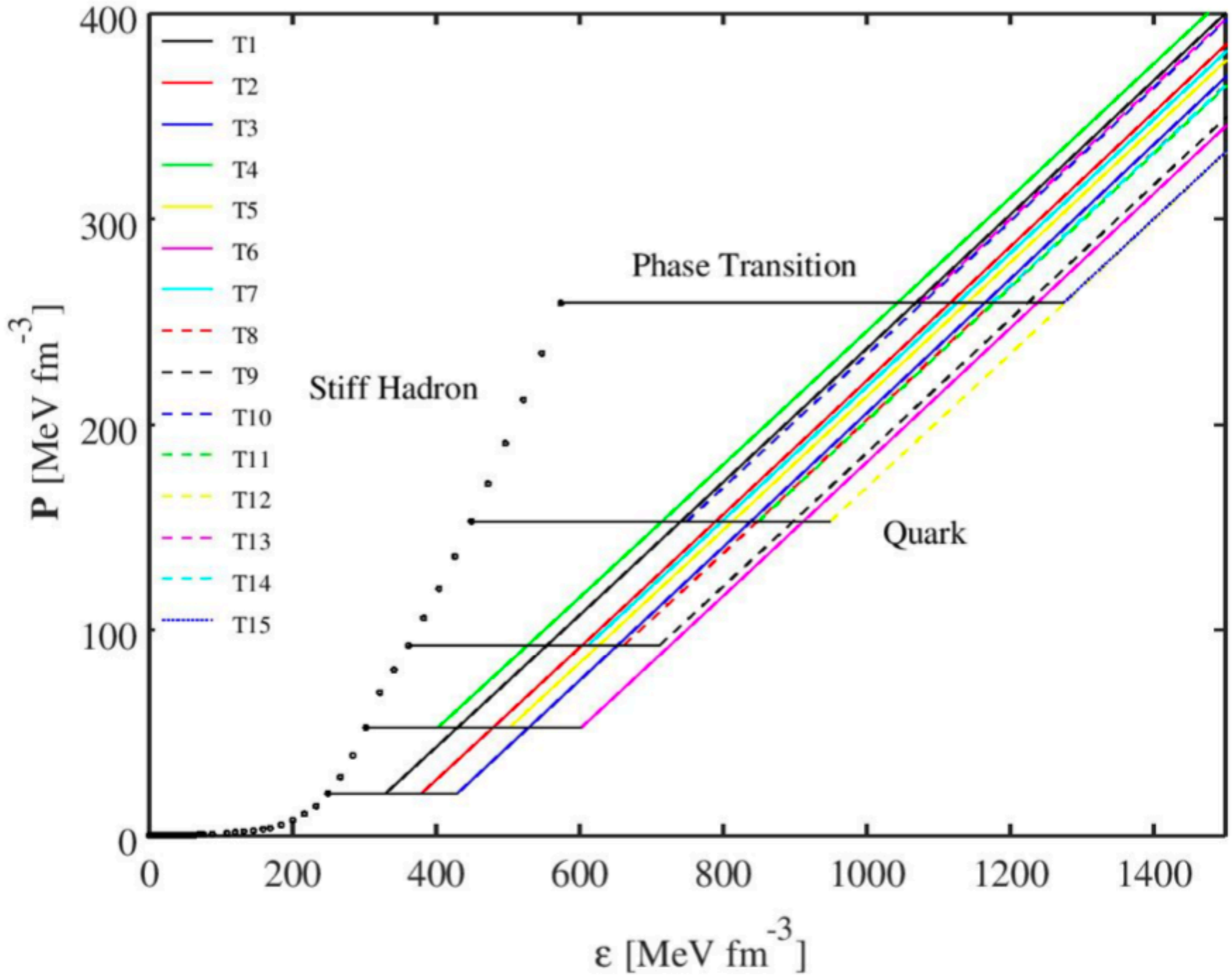
Soft EoS+Quark

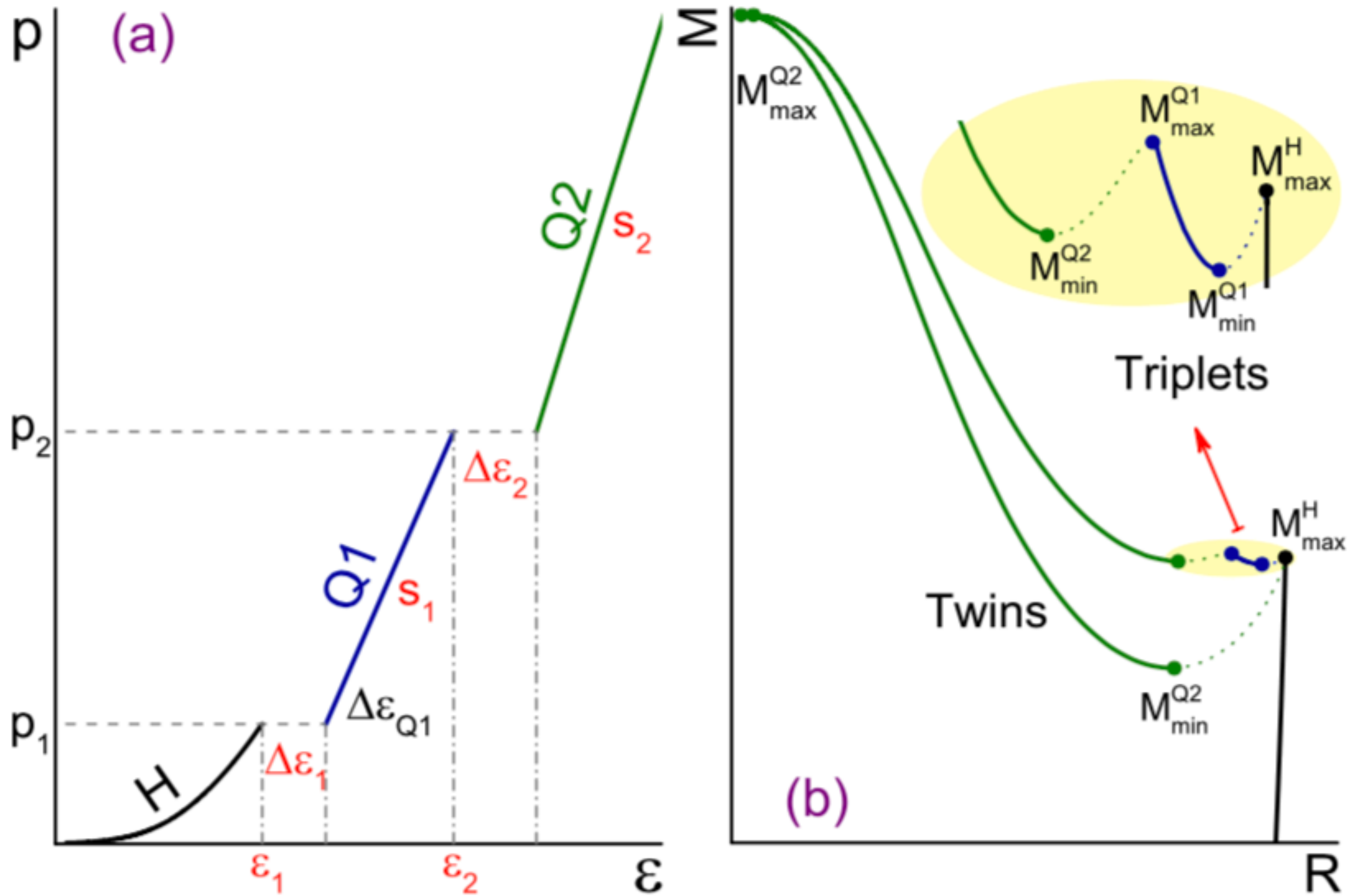


Intermediate EoS+Quark



Stiff EoS+Quark





$$
P(\varepsilon) = \begin{cases} P_1, & \varepsilon_1 < \varepsilon < \varepsilon_1 + \Delta\varepsilon_1, \\ P_1 + s_1[\varepsilon - (\varepsilon_1 + \Delta\varepsilon_1)], & \varepsilon_1 + \Delta\varepsilon_1 < \varepsilon < \varepsilon_2, \\ P_2, & \varepsilon_2 < \varepsilon < \varepsilon_2 + \Delta\varepsilon_2, \\ P_2 + s_2[\varepsilon - (\varepsilon_2 + \Delta\varepsilon_2)], & \varepsilon > \varepsilon_2 + \Delta\varepsilon_2. \end{cases}
$$

Key assumptions: GR , Causality, EoS

$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

$$\frac{dp}{dr} = -\frac{G}{r^2} \frac{(\epsilon + p)(mc^2 + 4\pi r^3 p)}{\left(1 - \frac{2Gm}{r}\right)}$$

$$\frac{d\phi}{dr} = \frac{m + 4\pi r^3 p}{r(r - 2Gm)}$$

Radial Oscillations

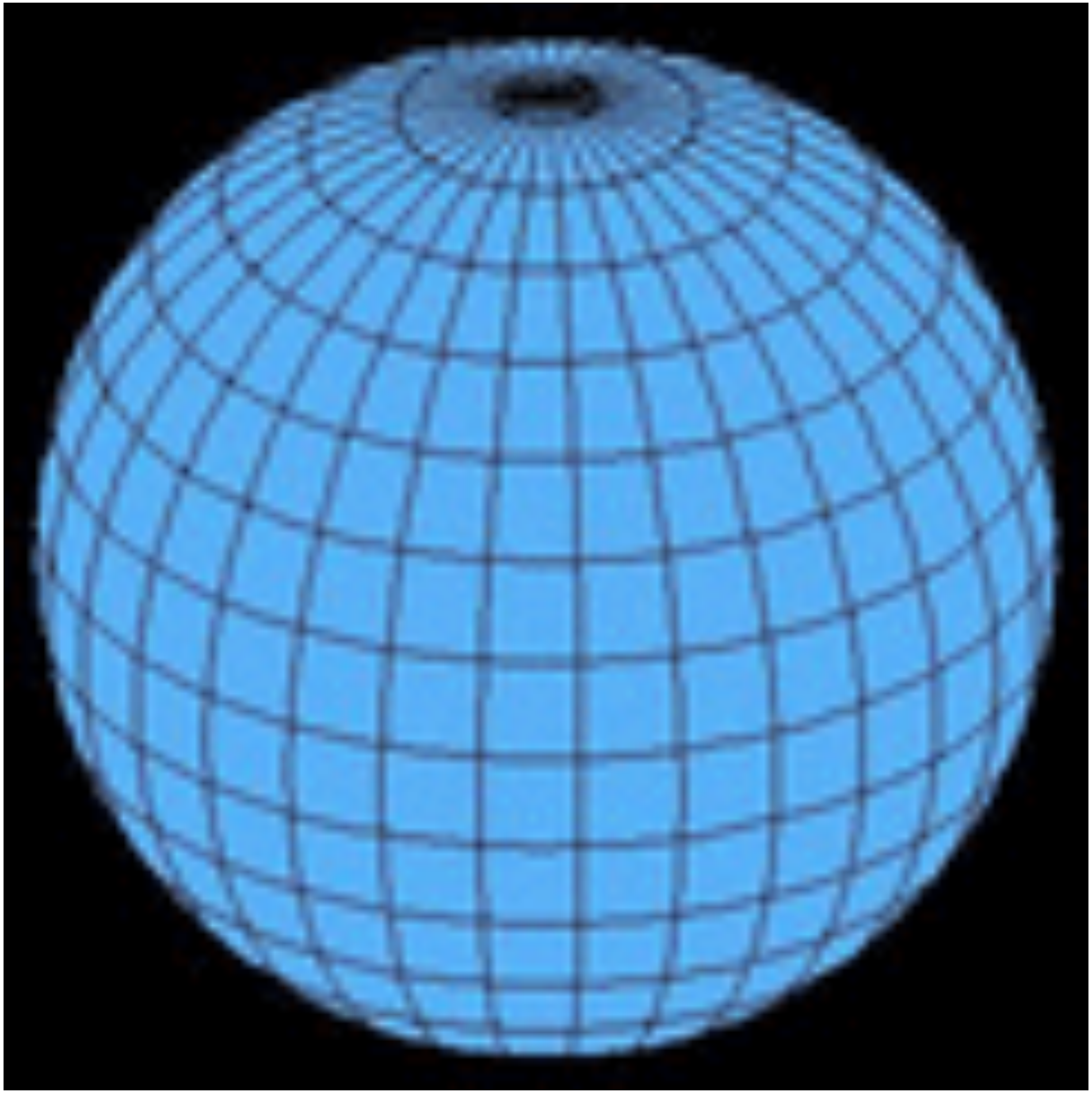
$$\frac{d\xi}{dr} = -\frac{1}{r}\left(3\xi + \frac{\Delta p}{\Gamma p}\right) - \frac{dp}{dr}\frac{\xi}{(p+\epsilon)}$$

$$\frac{d\Delta p}{dr} = \xi\left[\omega^2 e^{\lambda-\nu}(p+\epsilon)r - 4\frac{dp}{dr}\right] + \xi\left[\left(\frac{dp}{dr}\right)^2 \frac{r}{(p+\epsilon)}\right] - \xi\left[8\pi e^{\lambda}(p+\epsilon)pr\right] + \Delta p\left[\frac{dp/dr}{(p+\epsilon)} - 4\pi(p+\epsilon)re^{\lambda}\right]$$

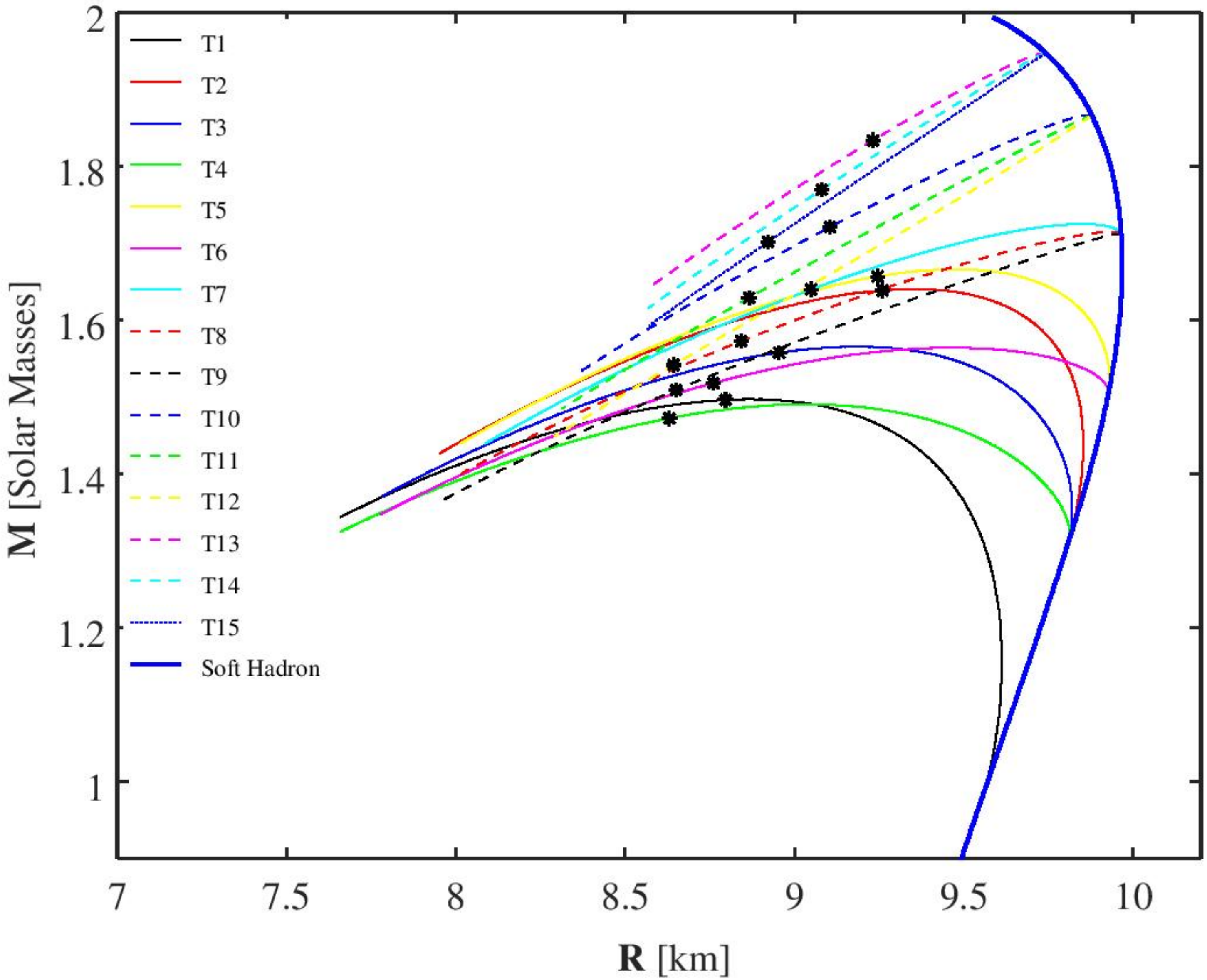
$$(\Delta p)_{\text{center}} = -3(\xi\Gamma p)_{\text{center}}. \qquad (\Delta p)_{\text{surface}} = 0.$$

$$[\xi] = \xi_+ - \xi_- = 0, \qquad [\Delta p] = (\Delta p)_+ - (\Delta p)_- = 0$$

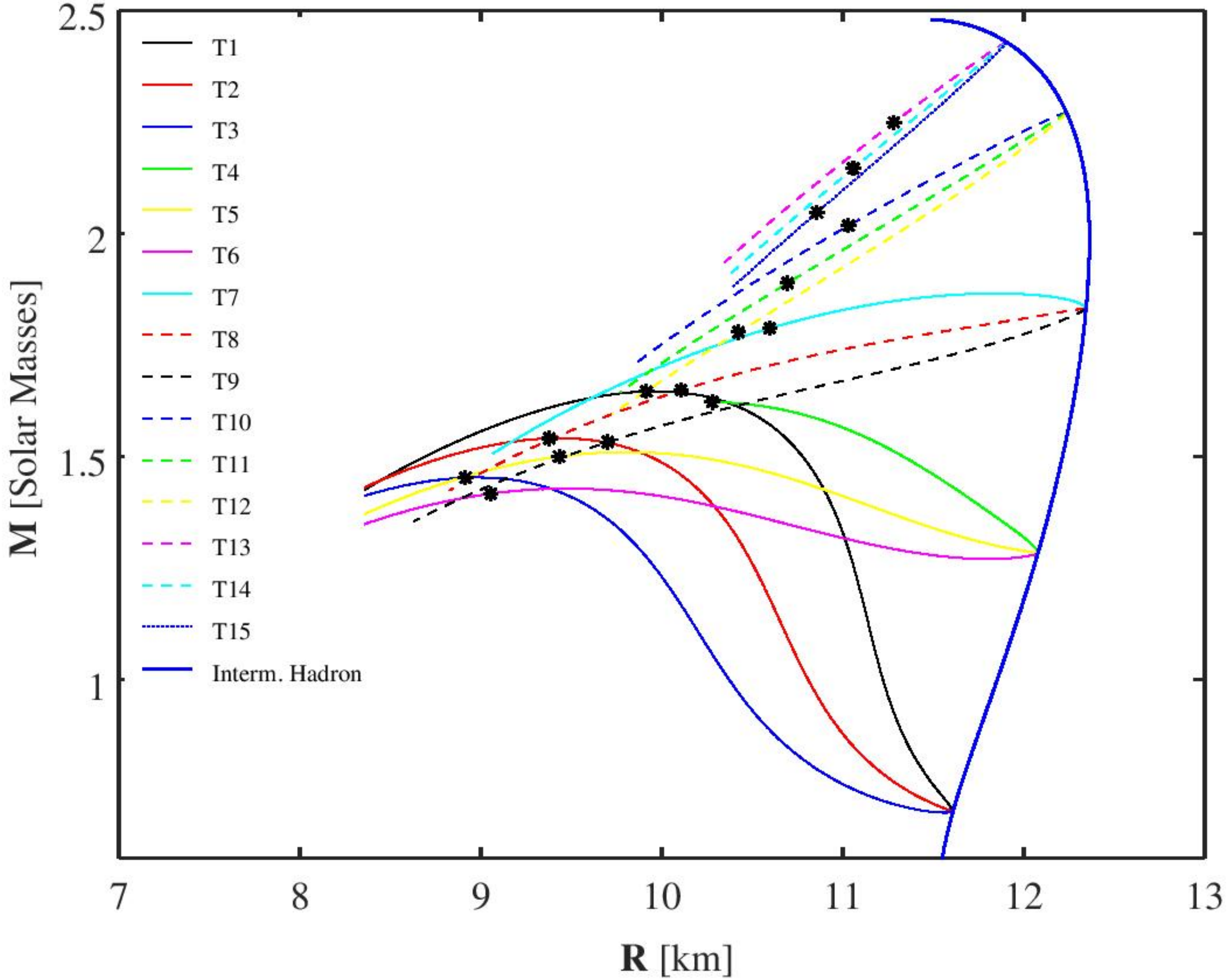
Stability Conditions - Radial Oscillations



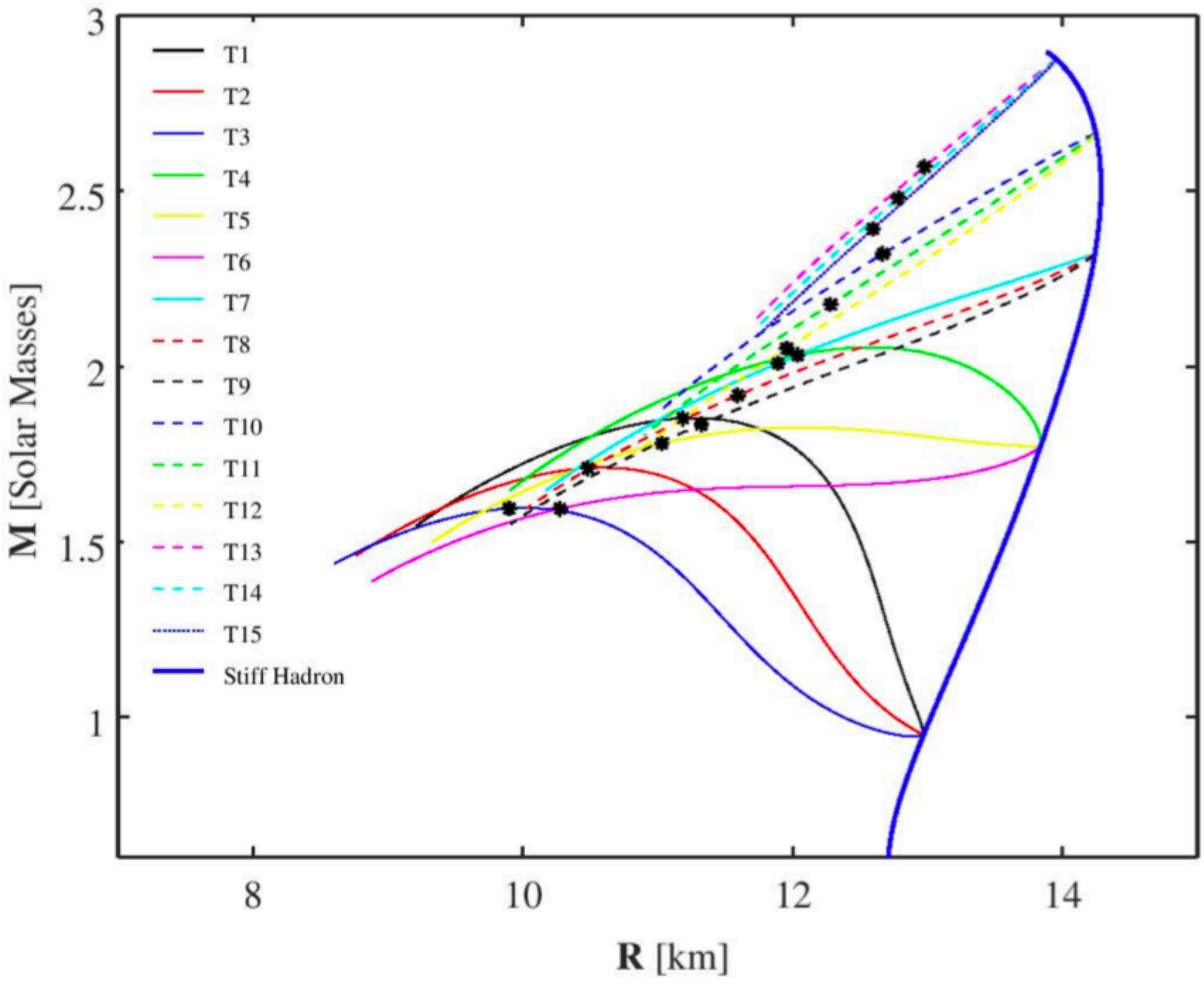
Mass Radius Relationship Soft EoS+Quark



Mass Radius Relationship Intermediate EoS+Quark



Mass Radius Relationship Stiff EoS+Quark



Tidal love numbers

When a static, spherically symmetric star is perturbed by an external stationary tidal field, it is deformed, developing a multipolar response. The main contribution is the quadrupolar one, $l = 2$:

$$Q_{ij} = -\lambda \mathcal{E}_{ij} \qquad \lambda = \frac{2}{3} k_2 R^5 \qquad \mathcal{E}_{ij} \equiv R_{i0j0}$$

Q_{ij}: mass quadrupole moment $\qquad$ $\mathcal{E}_{ij}$: tidal quadrupole moment

$R_{\alpha\beta\gamma\delta}$: Riemann tensor

λ: tidal deformability $\qquad$ k_2: Love number

In the Newtonian limit:

$$Q_{ij} = \int \rho \left(x^i x^j - \frac{1}{3} \delta^{ij} r^2 \right) d^3x \qquad \mathcal{E}_{ij} = -\partial_i \partial_j \phi_{ext}$$

All tidal Love numbers of a Schwarzschild black hole vanish exactly

The tidal Love numbers of neutron stars carry the imprint of the equation of state

Tidal effects in the gravitational wave signal emitted in NS-NS binary coalescence

$$h(f) = \mathcal{A}(f)e^{i\psi(f)} \qquad \psi(f) = \psi_{PP} + \psi_{\bar{Q}} + \psi_{\bar{\lambda}}$$

$x = (m\pi f)^{5/3}$ PN expansion parameter

point-particle contribution

$$\psi_{PP}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{1 + \left(\frac{3715}{756} + \frac{55}{9}\eta\right)x - (16\pi - 4\beta)x^{3/2} + \left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2 - 10\sigma\right)x^2 + \mathcal{O}(x^{5/2})\right\}$$

σ contains spin-spin and spin-orbit terms. Note that it appears in the 2-PN term (x^2)

Quadrupole contribution:

$$\psi_Q = \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{-50\left[\left(\frac{m_1^2}{m^2}\chi_1^2 + \frac{m_2^2}{m^2}\chi_2^2\right)(Q_S - 1) + \left(\frac{m_1^2}{m^2}\chi_1^2 - \frac{m_2^2}{m^2}\chi_2^2\right)Q_a\right]x^2\right\}$$

$$Q_S = \frac{\bar{Q}_1 + \bar{Q}_2}{2}, \quad Q_a = \frac{\bar{Q}_1 - \bar{Q}_2}{2}$$

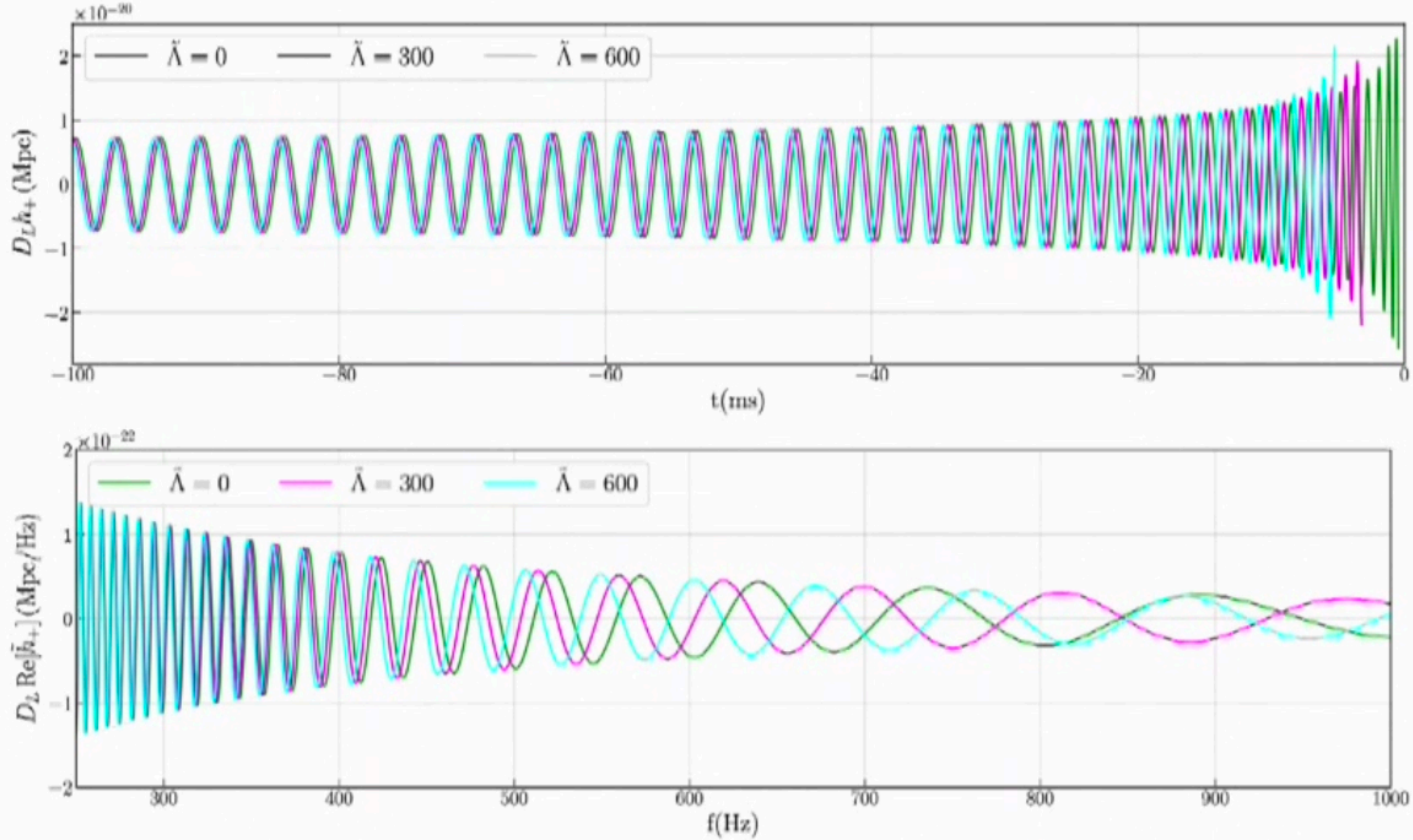
Tidal contribution: $\psi_{\bar{\lambda}} = -\frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{24[(1 + 7\eta - 31\eta^2)\lambda_S + (1 + 9\eta - 11\eta^2)\lambda_a\delta m]x^5 + \right\} + \mathcal{O}($

$$\lambda_S = \frac{\bar{\lambda}_1 + \bar{\lambda}_2}{2}, \quad \lambda_a = \frac{\bar{\lambda}_1 - \bar{\lambda}_2}{2} \qquad \delta m = \frac{m_1 - m_2}{m}$$

> degeracy can be removed by expressing the Q's in terms of λ using the universal relations
> NOTE THAT: λ is independent of the spins

In practice with current sensitivity we only measure:

$$\tilde{\Lambda} \equiv \frac{16}{13}\frac{(m_1+12m_2)m_1^4\Lambda_1+(m_2+12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$



Chatziioannou (arxiv:2006.03168)

Waveform: Dietrich+ (arxiv:1804.02235)

Tidal induced Quadrupole Moment and tidal deformability of a Neutron Star

Einstein field equation for perturbation

- Linearized perturbation

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

TOV Eqs.

$$\begin{aligned} h_{\mu\nu} = diag\big[& -e^{2\Phi(r)} H(r) Y_{20}(\theta,\phi), \\ & -e^{2\Lambda(r)} H(r) Y_{20}(\theta,\phi), \\ & -r^2 K(r) Y_{20}(\theta,\phi), \\ & -r^2 \sin^2\theta K(r) Y_{20}(\theta,\phi)\big] \end{aligned}$$

$$\delta T_0^0 = -\delta\epsilon(r) Y_{20}, \quad \delta T_i^i = \delta p(r) Y_{20}$$

$$\delta G_{\mu\nu} = 8\pi \delta T_{\mu\nu}$$

$$\begin{aligned} H'' & + \left(\frac{2}{r} + \Phi' - \Lambda'\right) H' \\ & + \left\{ 2(\Phi'' - \Phi'^2) - \frac{6}{r^2} e^{2\Lambda} + \frac{3}{r}\Lambda' \right. \\ & \left. + \frac{7}{r}\Phi' - 2\Phi'\Lambda' + \frac{f}{r}(\Phi' + \Lambda') \right\} H = 0 \end{aligned}$$

$$f(r) = \frac{d\epsilon}{dp}$$

Love Number

$$k_2 = \frac{8\mathcal{C}^5}{5}(1-2\mathcal{C})^2[2-y_R+2\mathcal{C}(y_R-1)]$$
$$\times\{2\mathcal{C}[6-3y_R+3\mathcal{C}(5y_R-8)]$$
$$+4\mathcal{C}^3[13-11y_R+\mathcal{C}(3y_R-2)+2\mathcal{C}^2(1+y_R)]$$
$$+3(1-2\mathcal{C})^2[2-y_R+2\mathcal{C}(y_R-1)]\log(1-2\mathcal{C})\}^{-1}$$

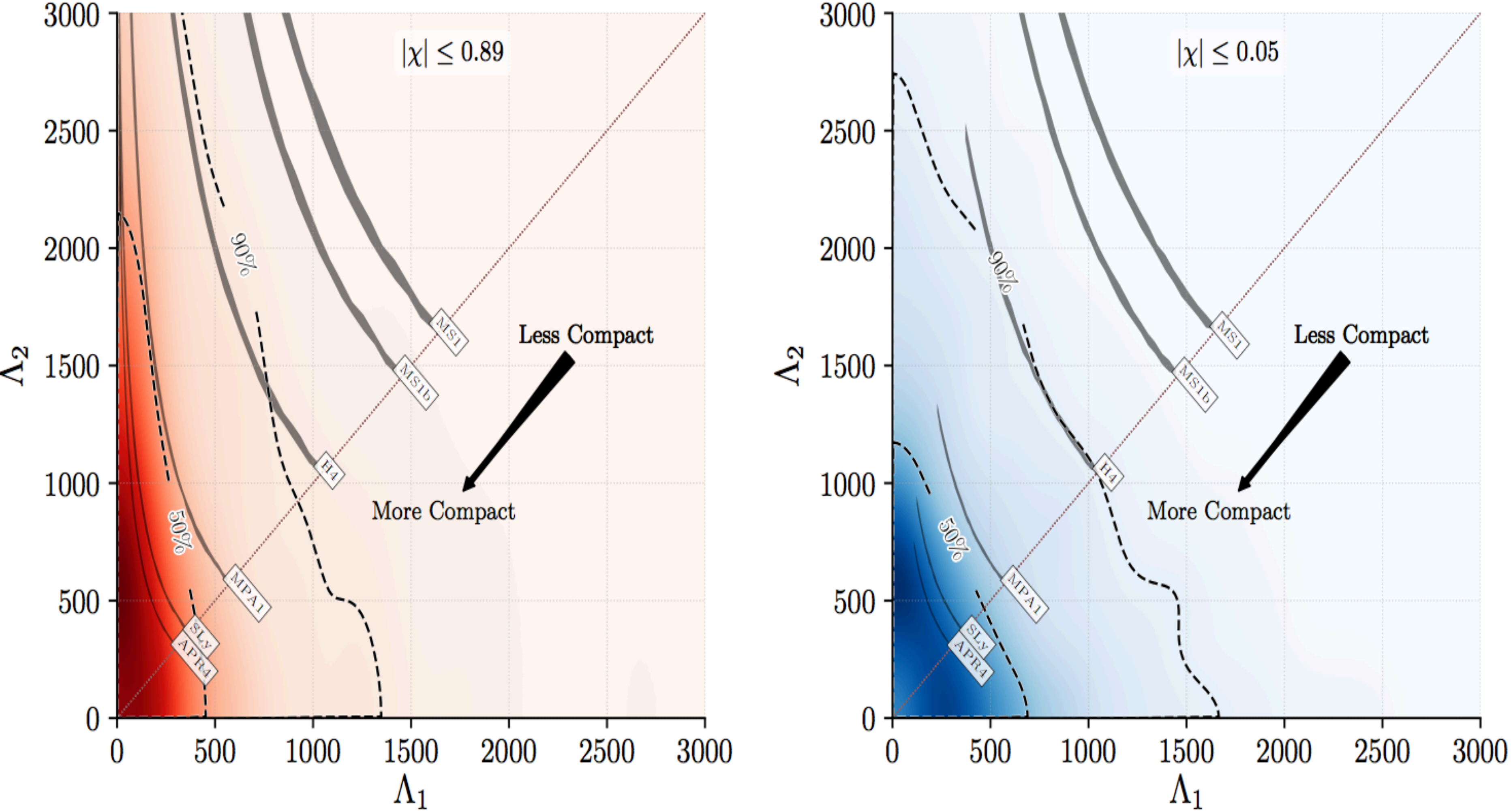
$$r\frac{dy(r)}{dr} + y(r)^2 + y(r)F(r) + r^2 Q(r) = 0.$$

$$F(r) = [1-4\pi r^2(\epsilon - p)]\left[1-\frac{2m}{r}\right]^{-1}$$

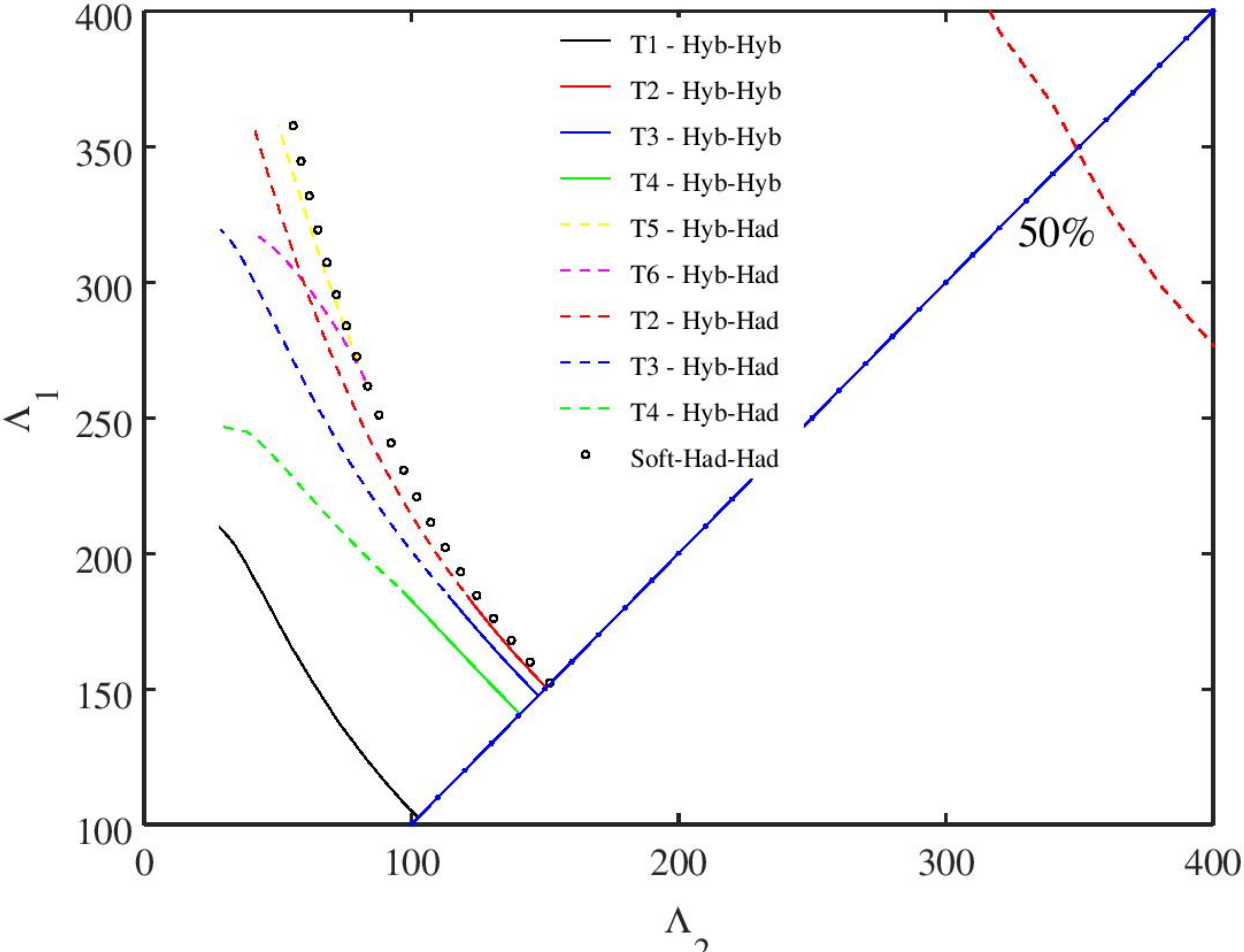
$$Q(r) = 4\pi\left[5\epsilon + 9p + \frac{\epsilon+p}{c_s^2} - \frac{6}{4\pi r^2}\right]\left[1-\frac{2m}{r}\right]^{-1} - \frac{4m^2}{r^4}\left[1+\frac{4\pi r^3 p}{m}\right]^2\left[1-\frac{2m}{r}\right]^{-2},$$

$$y(r_d+\epsilon) = y(r_d-\epsilon) - \frac{\Delta\rho}{\tilde{\rho}/3 + p\left(r_d\right)}$$

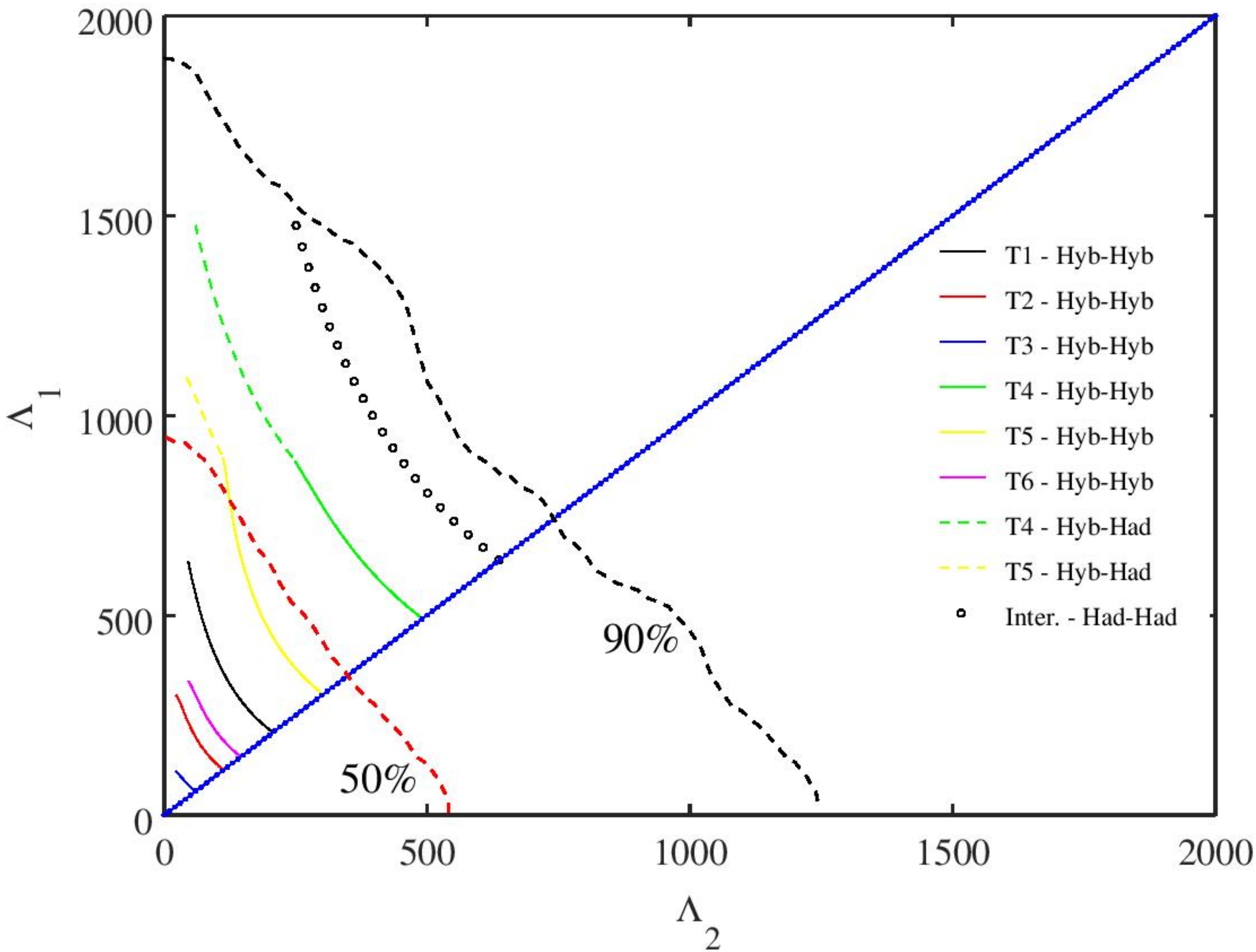
LIGO-Virgo PRL 119, 161101 (2017), GW170817



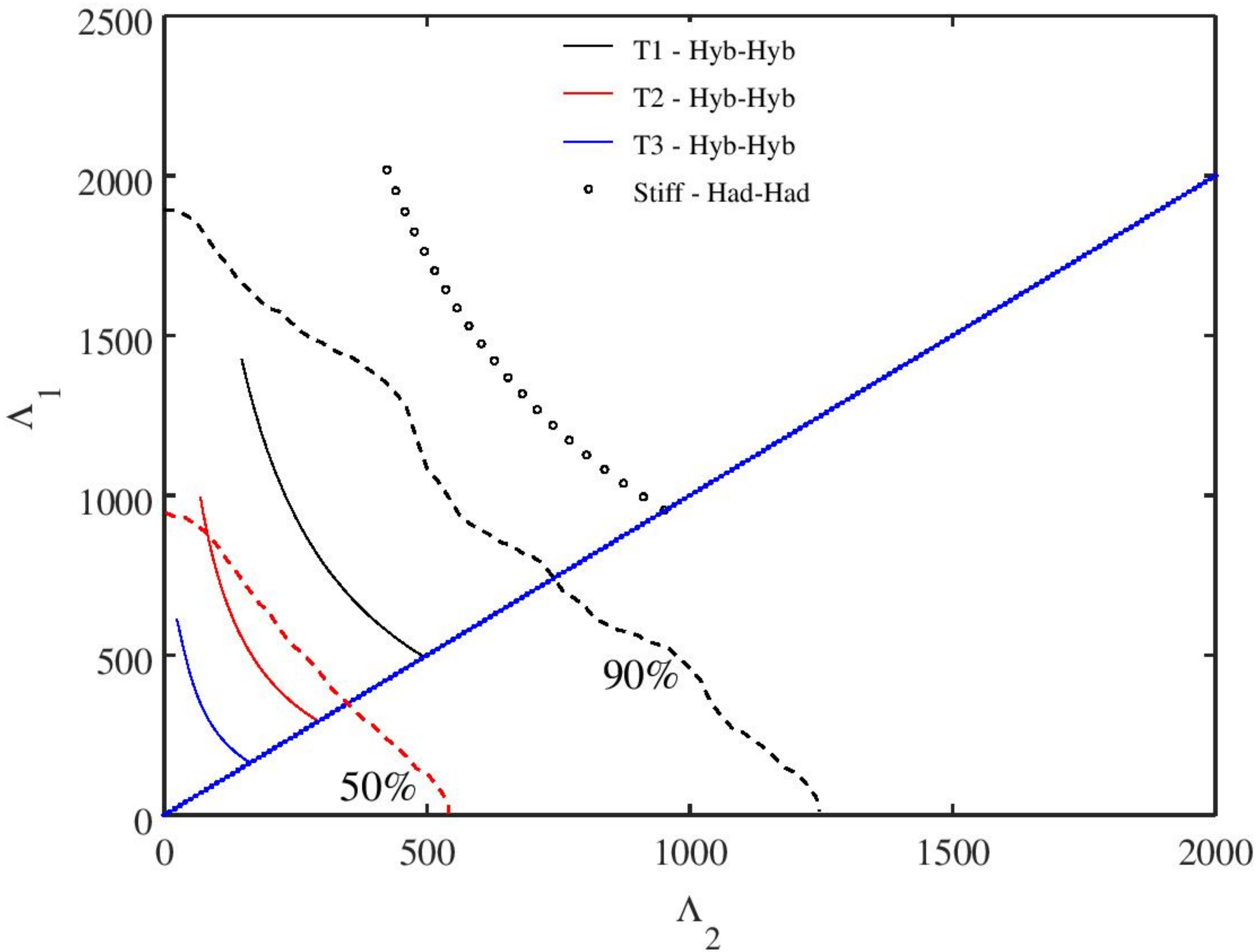
$$\Lambda_i = \frac{2}{3} k_2^{(i)} \left(\frac{c^2 R_i}{G M_i} \right)^5 \qquad \tilde{\Lambda} = \frac{16}{13} \left[\frac{(m_1 + 12 m_2) m_1^4 \Lambda_1 + (m_2 + 12 m_1) m_2^4 \Lambda_2}{(m_1 + m_2)^5} \right]$$



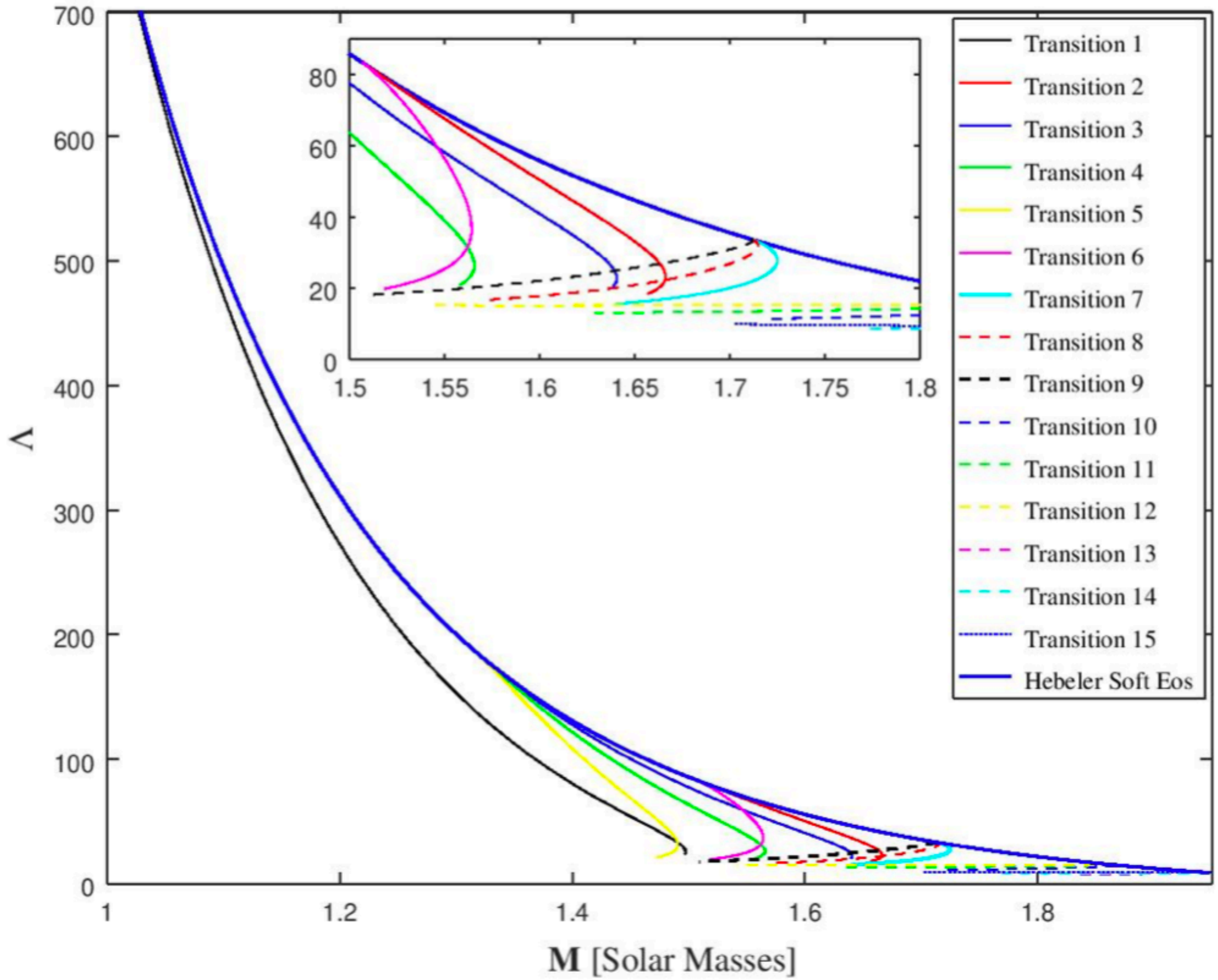
T1 - Hyb-Hyb
T2 - Hyb-Hyb
T3 - Hyb-Hyb
T4 - Hyb-Hyb
T5 - Hyb-Had
T6 - Hyb-Had
T2 - Hyb-Had
T3 - Hyb-Had
T4 - Hyb-Had
Soft-Had-Had
50%
Λ_1
Λ_2
100
150
200
250
300
350
400
0
100
200
300
400



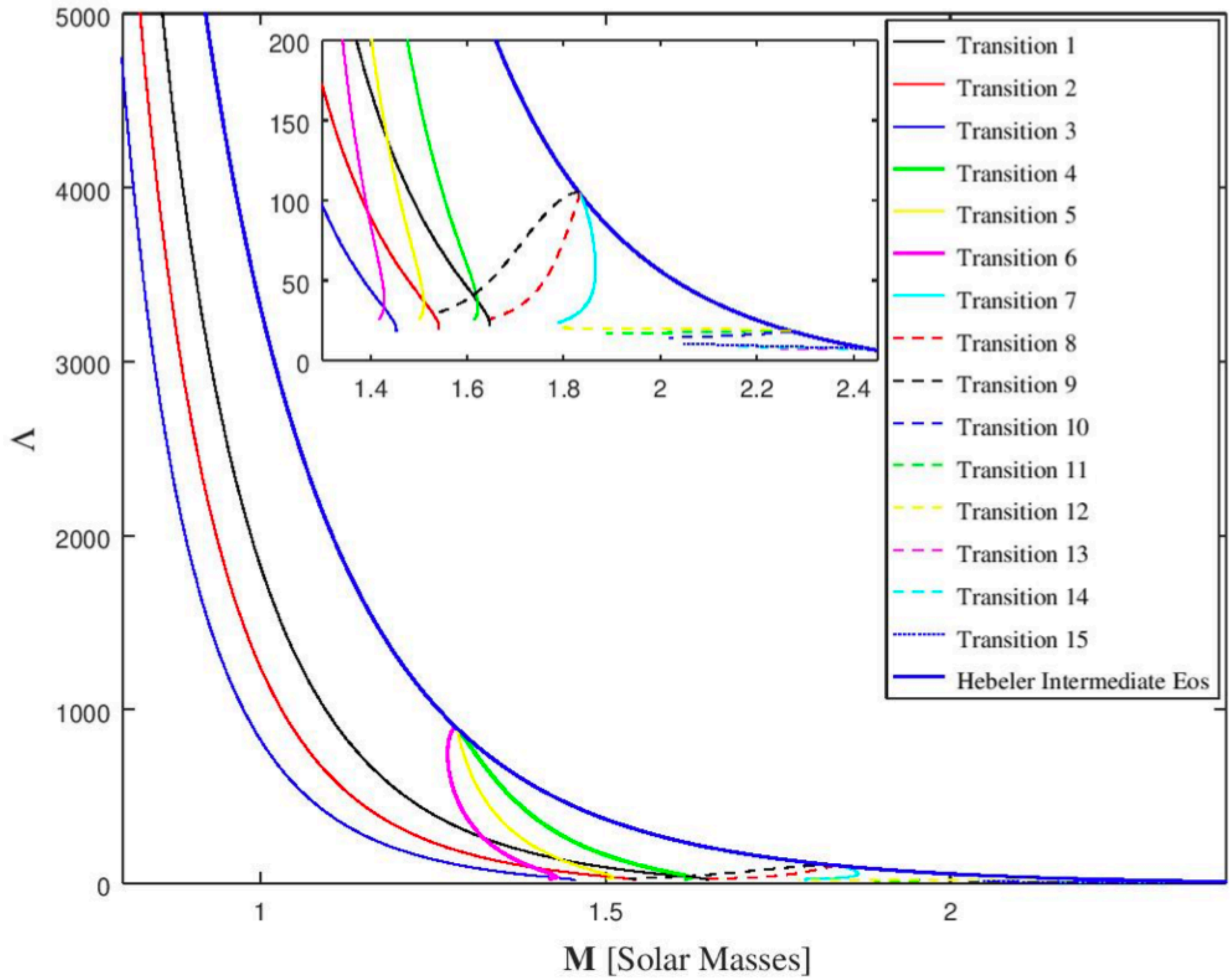
2000
1500
1000
500
0
0
500
1000
1500
2000
Λ_1
Λ_2
T1 - Hyb-Hyb
T2 - Hyb-Hyb
T3 - Hyb-Hyb
T4 - Hyb-Hyb
T5 - Hyb-Hyb
T6 - Hyb-Hyb
T4 - Hyb-Had
T5 - Hyb-Had
Inter. - Had-Had
90%
50%



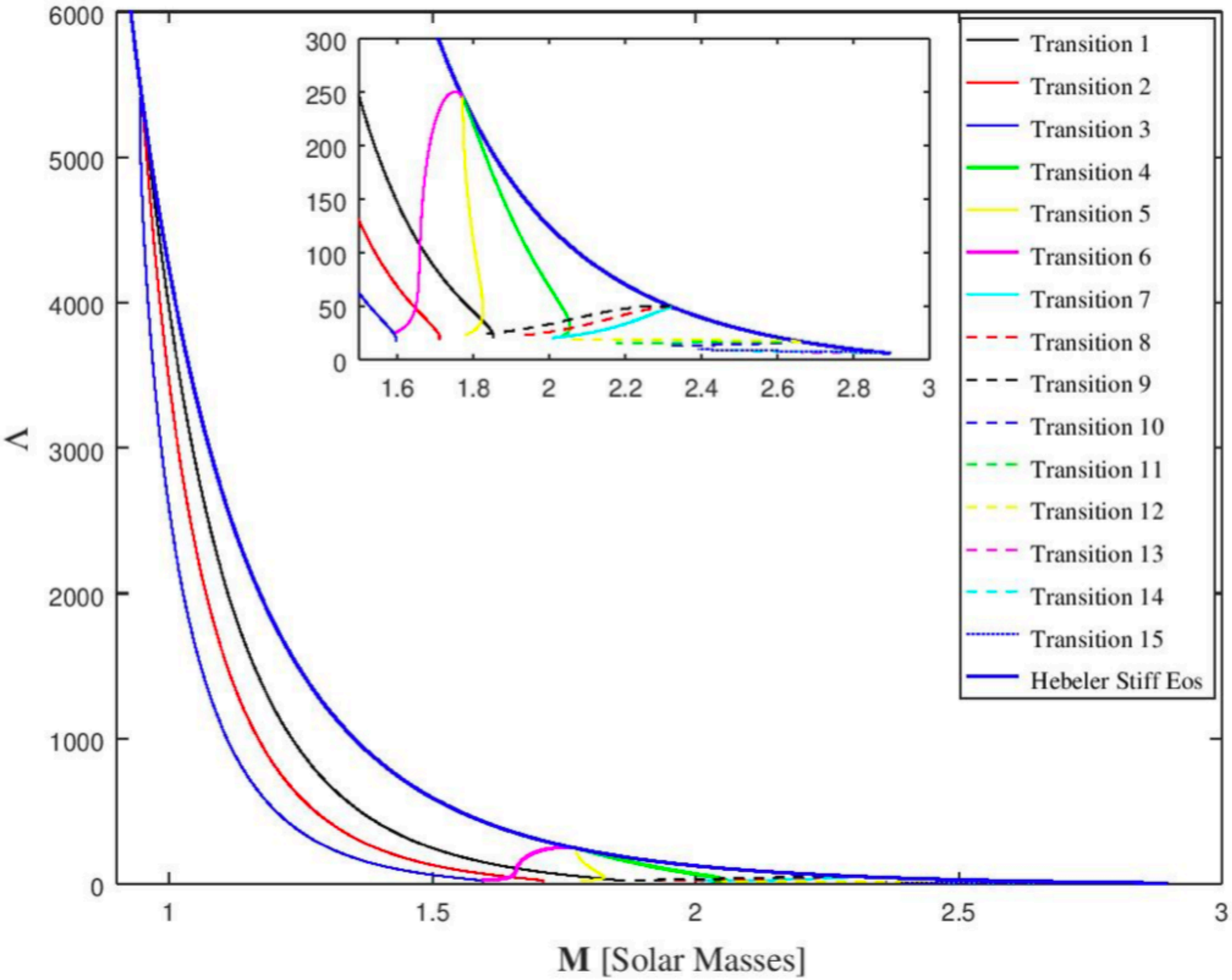
T1 - Hyb-Hyb
T2 - Hyb-Hyb
T3 - Hyb-Hyb
Stiff - Had-Had
90%
50%
Λ_1
Λ_2
0
500
1000
1500
2000
2500



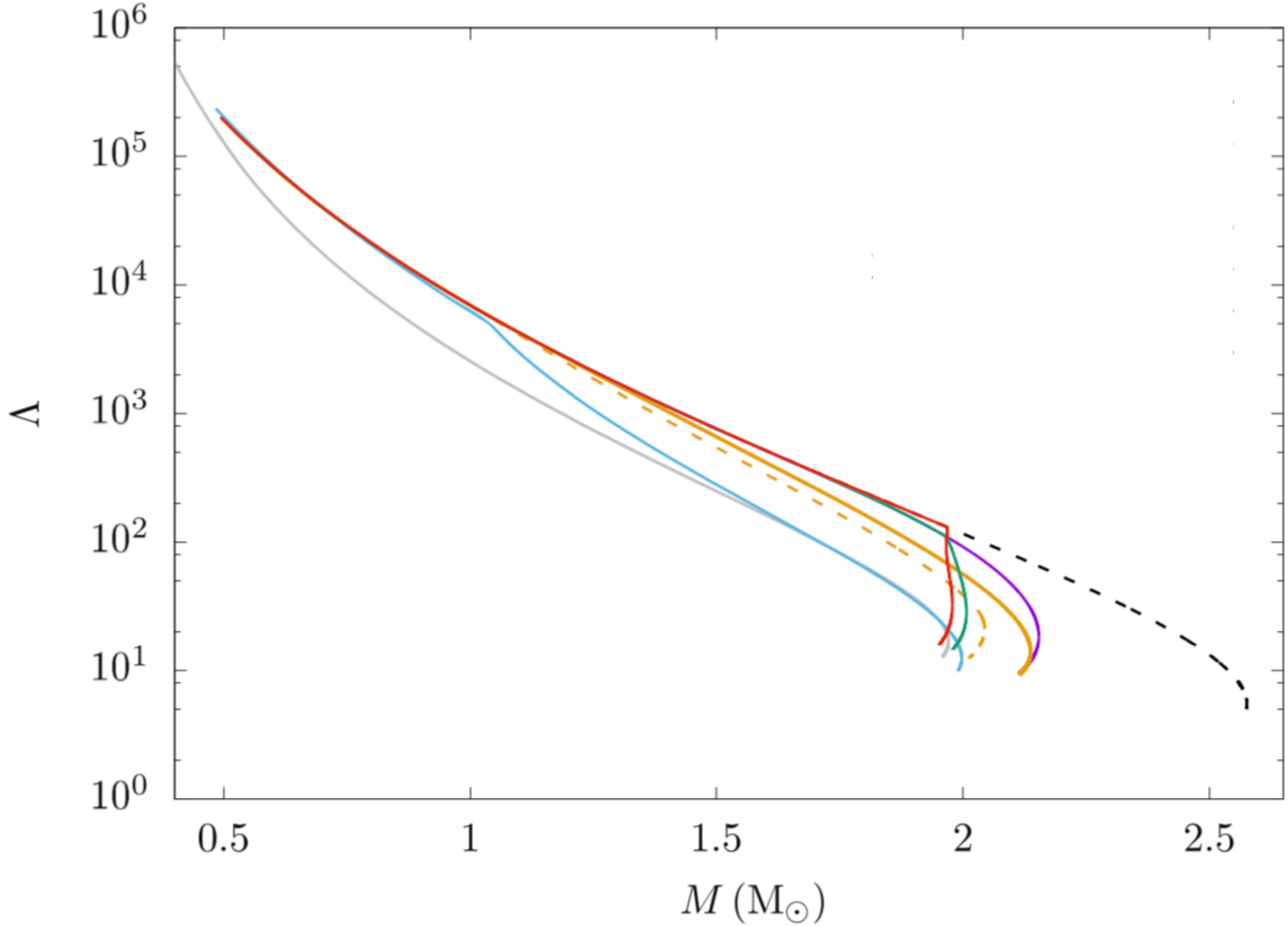
700
600
500
400
300
200
100
0
1
1.2
1.4
1.6
1.8
Λ
M [Solar Masses]
80
60
40
20
0
1.5
1.55
1.6
1.65
1.7
1.75
1.8
Transition 1
Transition 2
Transition 3
Transition 4
Transition 5
Transition 6
Transition 7
Transition 8
Transition 9
Transition 10
Transition 11
Transition 12
Transition 13
Transition 14
Transition 15
Hebeler Soft Eos



5000
4000
3000
2000
1000
0
1
1.5
2
Λ
M [Solar Masses]
200
150
100
50
0
1.4
1.6
1.8
2
2.2
2.4
Transition 1
Transition 2
Transition 3
Transition 4
Transition 5
Transition 6
Transition 7
Transition 8
Transition 9
Transition 10
Transition 11
Transition 12
Transition 13
Transition 14
Transition 15
Hebeler Intermediate Eos



6000
5000
4000
3000
2000
1000
0
1
1.5
2
2.5
3
Λ
M [Solar Masses]
300
250
200
150
100
50
0
1.6
1.8
2
2.2
2.4
2.6
2.8
3
Transition 1
Transition 2
Transition 3
Transition 4
Transition 5
Transition 6
Transition 7
Transition 8
Transition 9
Transition 10
Transition 11
Transition 12
Transition 13
Transition 14
Transition 15
Hebeler Stiff Eos



10^6
10^5
10^4
10^3
10^2
10^1
10^0
Λ
0.5
1
1.5
2
2.5
M ($M_\odot$)

Neutron star oscillations

When a NS is excited by some perturbation, it can be set into *non-radial oscillations* emitting GWs at the characteristic frequencies of its quasi-normal modes (QNMs). These oscillations are *damped*, due to GW emission, thus have *complex frequencies:*

$$\omega_n = \sigma_n + \iota / \tau_n$$

The quasi-normal modes frequencies (σ_n) and damping times (τ_n), at which the NS oscillates, are also the frequencies and damping times at which it emits gravitational waves.

Several possible excitation mechanisms:

- glitches
- gravitational collapse giving birth to the NS
- accretion from a companion star
- electromagnetic activity, as in magnetar giant flares
- phase transition of the matter composing the star
- compact binary inspiral and coalescence

Neutron star oscillations

The QNMs of NSs are classified depending to the *main restoring force*:

- g-modes: buoyancy force. Probe entropy/composition gradients. ν~few hundreds of Hz but τ very large => not efficient in radiating GW
- p-modes: pressure. ν~few *thousands* of Hz. Probe sound speed in the star
- f-mode: the fundamental mode, intermediate between g and p. ν~1-2 kHz it the most efficient in radiating GW (τ~1s). Scales ~ with average density.
- w-modes: pure space-time modes. ν ~ many kHz

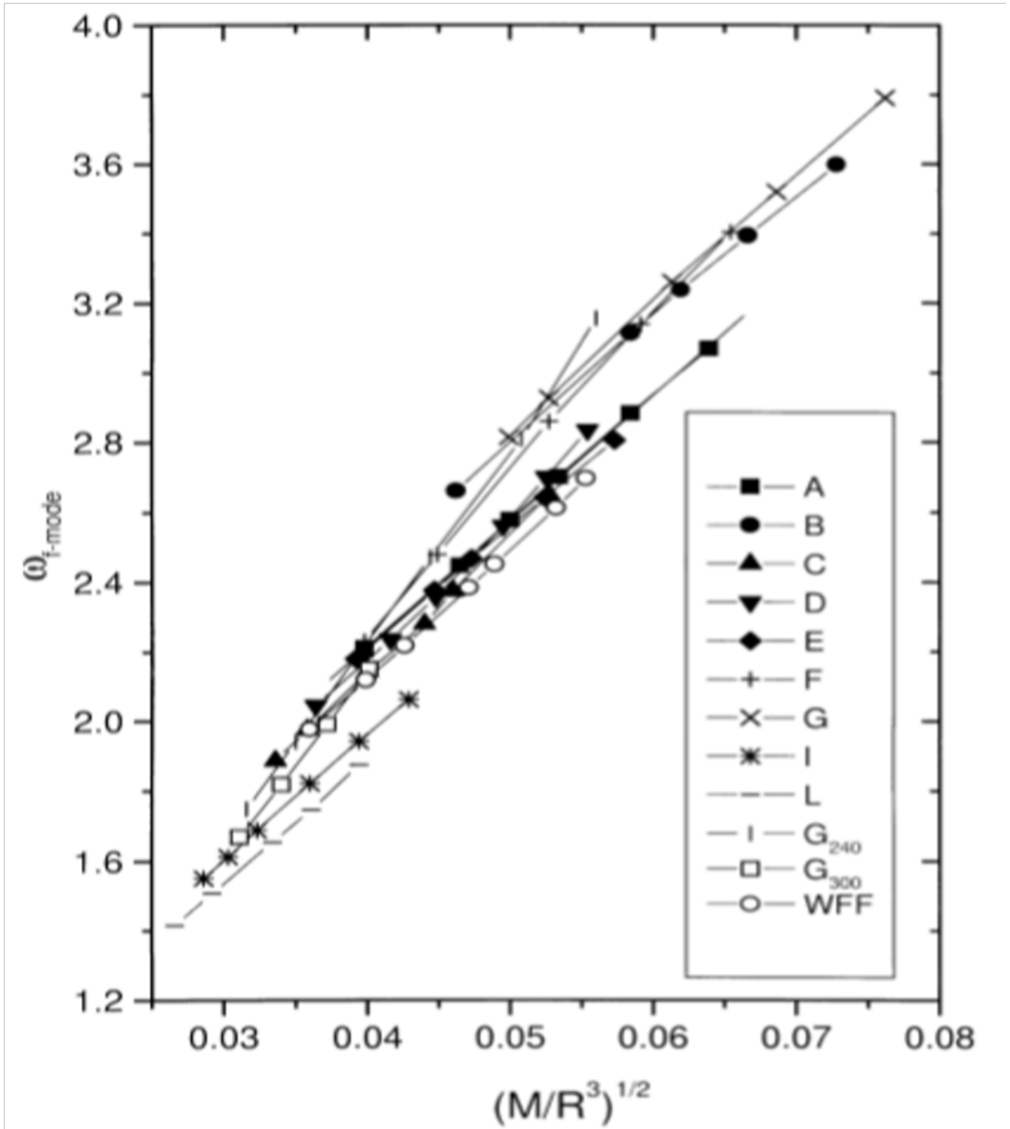
Including more physics, other families of modes appear:

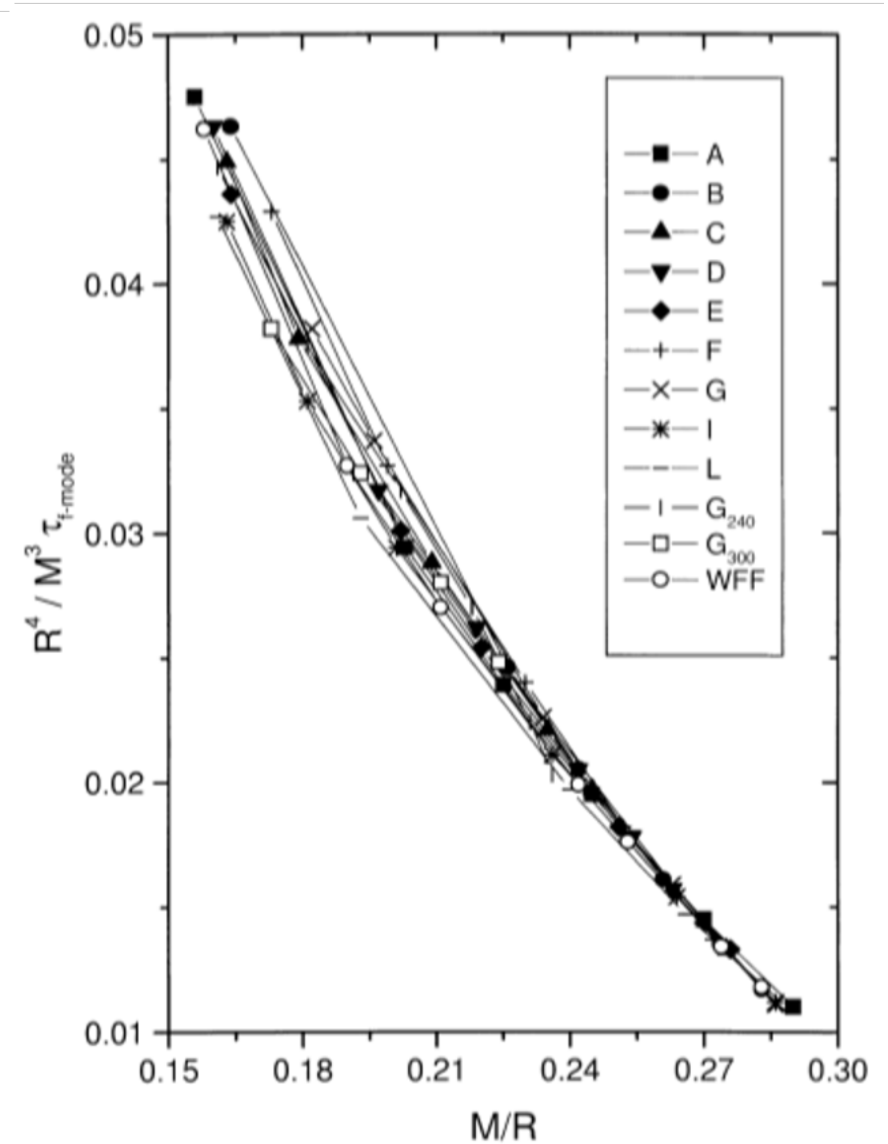
- r-modes: Coriolis force (for rotating stars only)
- magnetic modes
- elastic modes (in the crust)
- superfluid modes (when baryon superfluidity occurs)

Gravitational wave astreroseismology

$$\omega_f(\mathrm{kHz}) \approx 0.78 + 1.635\left(\frac{\bar{M}}{\bar{R}^3}\right)^{1/2}$$

$$\frac{1}{\tau_f(\mathrm{s})} \approx \frac{\bar{M}^3}{\bar{R}^4}\left[22.85 - 14.65\left(\frac{\bar{M}}{\bar{R}}\right)\right]$$





The perturbations of the compact fluid is described by the Lagrangian displacement vectors

$$\xi_r(t,r,\theta,\varphi) = e^{\lambda/2} r^{\ell-1} W^{\ell m}(r) Y_{\ell m}(\theta,\varphi) e^{i\omega t},$$
$$\xi_\theta(t,r,\theta,\varphi) = -r^\ell V^{\ell m}(r) \partial_\theta Y_{\ell m}(\theta,\varphi) e^{i\omega t},$$
$$\xi_\varphi(t,r,\theta,\varphi) = -r^\ell V^{\ell m}(r) \partial_\varphi Y_{\ell m}(\theta,\varphi) e^{i\omega t}.$$

The perturbed metric tensor is given by

$$ds^2 = -e^\psi(1 + r^\ell H_0^{\ell m} Y_{\ell m} e^{i\omega t})dt^2 + e^\lambda(1 - r^\ell H_2^{\ell m} Y_{\ell m} e^{i\omega t})dr^2$$
$$- 2i\omega r^{\ell+1} H_1^{\ell m} Y_{\ell m} e^{i\omega t} dt dr + r^2(1 - r^\ell K^{\ell m} Y_{\ell m} e^{i\omega t})(d\theta^2 + \sin^2\theta d\varphi^2),$$

We use the so-called Lindblom-Detweiler formulation

$$H_1'^{\ell m} = -\frac{1}{r}\left[\ell+1+\frac{2Me^{\lambda}}{r}+4\pi r^2 e^{\lambda}(p-\varepsilon)\right]H_1^{\ell m}+\frac{e^{\lambda}}{r}[H_0^{\ell m}+K^{\ell m}-16\pi(p+\varepsilon)V^{\ell m}],$$

$$K'^{\ell m} = \frac{1}{r}H_0^{\ell m}+\frac{\ell(\ell+1)}{2r}H_1^{\ell m}-\left[\frac{\ell+1}{r}-\frac{\psi'}{2}\right]K^{\ell m}-8\pi(p+\varepsilon)\frac{e^{\lambda/2}}{r}W^{\ell m},$$

$$W'^{\ell m} = -\frac{\ell+1}{r}W^{\ell m}+re^{\lambda/2}\left[\frac{e^{-\psi/2}}{(p+\varepsilon)c_s^2}X^{\ell m}-\frac{\ell(\ell+1)}{r^2}V^{\ell m}+\frac{1}{2}H_0^{\ell m}+K^{\ell m}\right],$$

$$X'^{\ell m} = -\frac{\ell}{r}X^{\ell m}+\frac{(p+\varepsilon)e^{\psi/2}}{2}\left[\left(\frac{1}{r}-\frac{\psi'}{2}\right)H_0^{\ell m}+\left(r\omega^2 e^{-\psi}+\frac{\ell(\ell+1)}{2r}\right)H_1^{\ell m}+\left(\frac{3}{2}\psi'-\frac{1}{r}\right)K^{\ell m}\right.$$
$$\left.-\frac{\ell(\ell+1)}{r^2}\psi' V^{\ell m}-\frac{2}{r}\left(4\pi(p+\varepsilon)e^{\lambda/2}+\omega^2 e^{\lambda/2-\psi}-\frac{r^2}{2}\left(\frac{e^{-\lambda/2}}{r^2}\psi'\right)'\right)W^{\ell m}\right]. \qquad (A.1)$$

$$0 = \left[3M+\frac{1}{2}(\ell-1)(\ell+2)r+4\pi r^3 p\right]H_0^{\ell m}-8\pi r^3 e^{-\psi/2}X^{\ell m}$$
$$+\left[\frac{1}{2}\ell(\ell+1)(M+4\pi r^3 p)-\omega^2 r^3 e^{-(\lambda+\psi)}\right]H_1^{\ell m}$$
$$-\left[\frac{1}{2}(\ell-1)(\ell+2)r-\omega^2 r^3 e^{-\psi}-\frac{e^{\lambda}}{r}(M+4\pi r^3 p)(3M-r+4\pi r^3 p)\right]K^{\ell m},$$

$$X^{\ell m} = \omega^3(\varepsilon+p)e^{-\psi/2}V^{\ell m}-\frac{p'}{r}e^{(\psi-\lambda)/2}W^{\ell m}+\frac{1}{2}(\varepsilon+p)e^{\psi/2}H_0^{\ell m},$$

$$H_0^{\ell m} = H_2^{\ell m}. \qquad (A.2)$$

The perturbation functions near r=0 are nonsingular and can be obtained by an asymptotic expansions

$$X^{\ell m}(0) = (\varepsilon_0 + p_0)e^{\psi_0/2}\left\{\left[\frac{4\pi}{3}(\varepsilon_0 + 3p_0) - \frac{\omega^2}{\ell}e^{-\psi_0}\right]W^{\ell m}(0) + \frac{1}{2}K^{\ell m}(0)\right\},$$

$$H_1^{\ell m}(0) = \frac{1}{\ell(\ell+1)}[2\ell K^{\ell m}(0) + 16\pi(\varepsilon_0 + p_0)W^{\ell m}(0)],$$

At the surface, we assume continuity of the perturbation functions and vanishing pressure perturbation

$$X^{\ell m}(R) = 0.$$

In the exterior, the metric perturbations are described by the Zerilli functions

$$Z^{\ell m} = \frac{r^{\ell+2}}{nr + 3M}(K^{\ell m} - e^{\psi}H_1^{\ell m}), \qquad \frac{d^2 Z^{\ell m}}{dr_\star^2} + [\omega^2 - V_Z(r)]Z^{\ell m} = 0,$$

Binary neutron star mergers: a review of Einstein's richest laboratory

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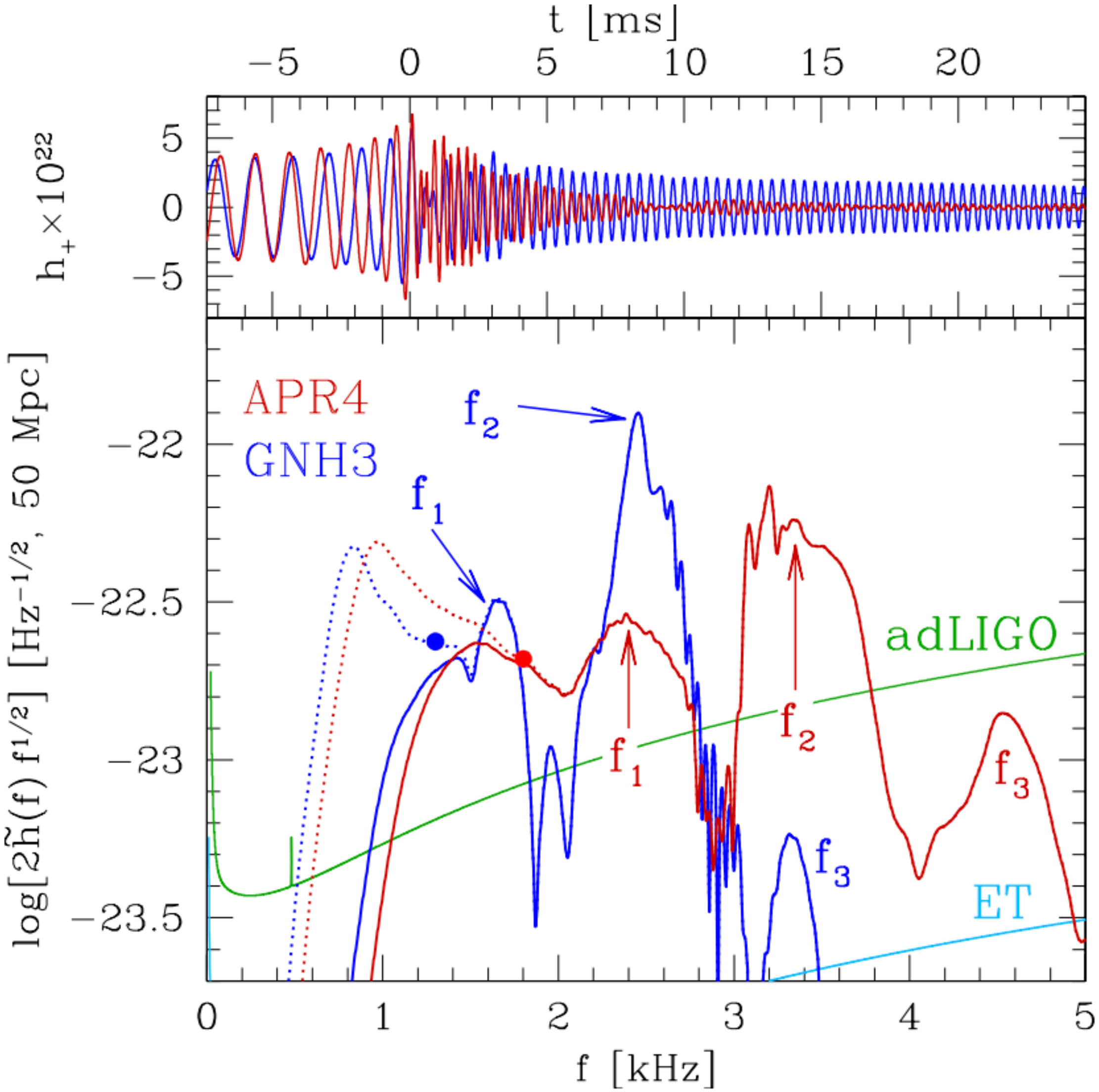
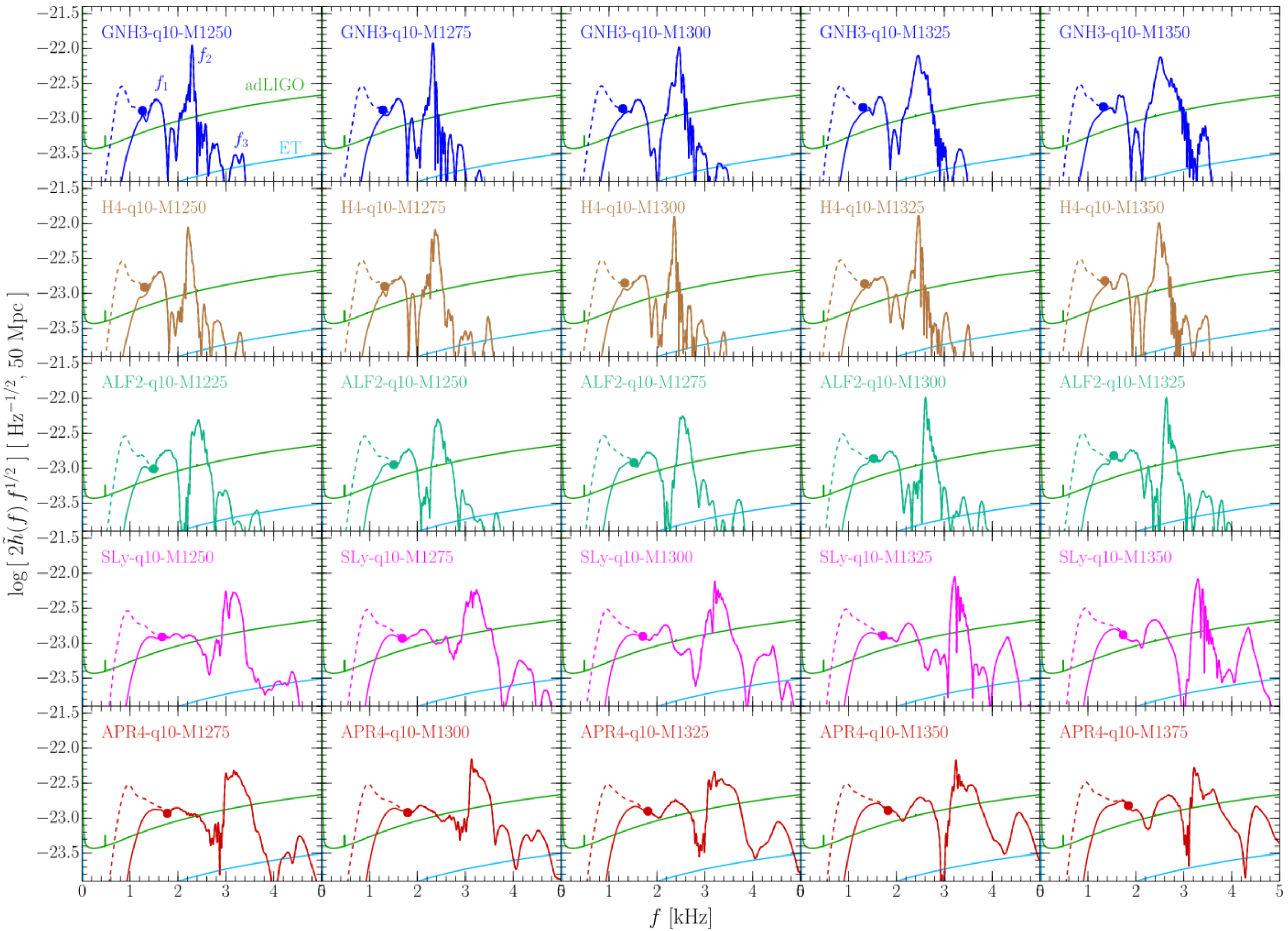




GNH3-q10-M1250
GNH3-q10-M1275
GNH3-q10-M1300
GNH3-q10-M1325
GNH3-q10-M1350
f_1
f_2
f_3
adLIGO
ET
H4-q10-M1250
H4-q10-M1275
H4-q10-M1300
H4-q10-M1325
H4-q10-M1350
ALF2-q10-M1225
ALF2-q10-M1250
ALF2-q10-M1275
ALF2-q10-M1300
ALF2-q10-M1325
SLy-q10-M1250
SLy-q10-M1275
SLy-q10-M1300
SLy-q10-M1325
SLy-q10-M1350
APR4-q10-M1275
APR4-q10-M1300
APR4-q10-M1325
APR4-q10-M1350
APR4-q10-M1375
$\log [\, 2\tilde{h}(f)\, f^{1/2}\,]$ [$\mathrm{Hz}^{-1/2}$, 50 Mpc]
f [kHz]

Tidal resonances in binary inspirals

$$\frac{1}{|\mathcal{D}-r|}=\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\frac{4\pi}{2\ell+1}\frac{r^{\ell}}{\mathcal{D}^{\ell+1}}Y^{*}_{\ell m}(\theta,\phi)Y_{\ell m}(\pi/2,\psi)$$

$$\frac{e^{im\psi}}{\mathcal{D}^{\ell+1}}=\frac{1}{a^{\ell+1}}\sum_{j=0}^{\infty}\{c_j^{(\ell,m)}(e)\cos(j\beta)+is_j^{(\ell,m)}(e)\sin(j\beta)\}$$

$$\ddot{x}_n(t)+2\frac{\dot{x}_n(t)}{\tau_n}+\omega_n^2x_n(t)=C_j\cos(\omega_j t)+S_j\sin(\omega_j t)$$

$$\begin{aligned}\tau_{\rm GW}=\left(\frac{\dot{\omega}_0}{\omega_0}\right)^{-1}&\simeq\frac{5}{96\eta}(G_NM)^{-5/3}\omega_0^{-8/3}g^{-1}(e)\\&\simeq 50\ \sec\left(\frac{\eta}{0.2}\right)^{-1}\left(\frac{x}{0.02}\right)^{-5/2}\\&\quad\times\left(\frac{\omega_0}{100\ \mathrm{Hz}}\right)^{-1}g^{-1}(e),\end{aligned}$$

$$\begin{aligned}\tau_c&\equiv\sqrt{\tau_{\rm GW}2\pi/\omega_0}\\&\simeq 2\ \sec\left(\frac{\eta}{0.2}\right)^{-1/2}\left(\frac{x}{0.02}\right)^{-5/4}\left(\frac{\omega_0}{100\ \mathrm{Hz}}\right)^{-1}g^{-1/2}(e)\end{aligned}$$

EoS

TABLE I. Data for the equation of state A (APR) [20] and [33] for the crust.

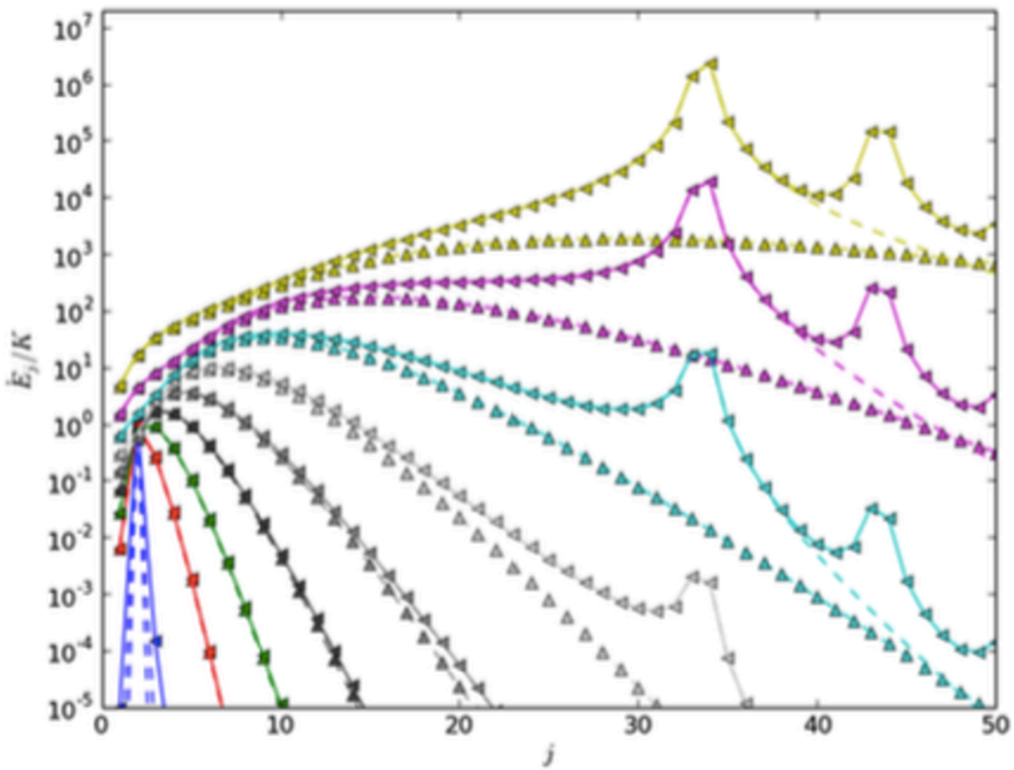
ρ_0 (gr/cm^3)	R (km)	M ($M_\odot$)	$\nu_{f,\ell=2}$ (kHz)	$\nu_{f,\ell=3}$ (kHz)	$\nu_{f,\ell=4}$ (kHz)	$\tau_{\ell=2}$ (s)	$\tau_{\ell=3}$ (s)	$\tau_{\ell=4}$ (s)	$\lvert Q_{02}\rvert$	$\lvert Q_{03}\rvert$	$\lvert Q_{04}\rvert$
1.5×10^{15}	11.132	1.965	2.888	3.742	4.420	0.153	4.494	67.1	2.321	2.437	2.613
1.2×10^{15}	11.433	1.704	2.741	3.456	4.033	0.158	3.28	181	2.258	2.482	2.501
9.9×10^{14}	11.603	1.408	2.384	3.071	3.602	0.204	2.939	152	2.323	2.594	2.653

TABLE II. Data for the equation of state B (SLy4) [21].

ρ_0 (gr/cm^3)	R (km)	M ($M_\odot$)	$\nu_{f,\ell=2}$ (kHz)	$\nu_{f,\ell=3}$ (kHz)	$\nu_{f,\ell=4}$ (kHz)	$\tau_{\ell=2}$ (s)	$\tau_{\ell=3}$ (s)	$\tau_{\ell=4}$ (s)	$\lvert Q_{02}\rvert$	$\lvert Q_{03}\rvert$	$\lvert Q_{04}\rvert$
2.0×10^{15}	10.615	1.994	3.300	4.143	4.829	0.152	4.48	60.2	2.148	2.372	2.443
1.6×10^{15}	11.017	1.884	3.024	3.808	4.461	0.149	3.17	196	2.180	2.439	2.446
1.2×10^{15}	11.435	1.634	2.654	3.372	3.967	0.168	8.53	130	2.270	2.989	2.508

$$\dot{E}_* = \sum_j \dot{E}_j = \rho_0 R_* \left(\frac{R_*}{a}\right)^4 \left(\frac{G_N M_{\rm BH}}{a}\right)^2$$
$$\times \sum_{j,n,\ell,m} \left(c_j^{(\ell,m)2} + s_j^{(\ell,m)2}\right)$$
$$\times \left(\frac{R_*}{a}\right)^{2\ell-4} Q_{n\ell}^2 W_{\ell m}^2 \frac{\omega_j^2/\tau_{n\ell}}{(\omega_j^2 - \omega_{n\ell}^2)^2 + 4\omega_j^2/\tau_{n\ell}^2}$$

$$x \equiv (G_N M \omega_0)^{2/3} = \frac{G_N M}{a}$$



Gravitational waves from neutron star excitations in a binary inspiral

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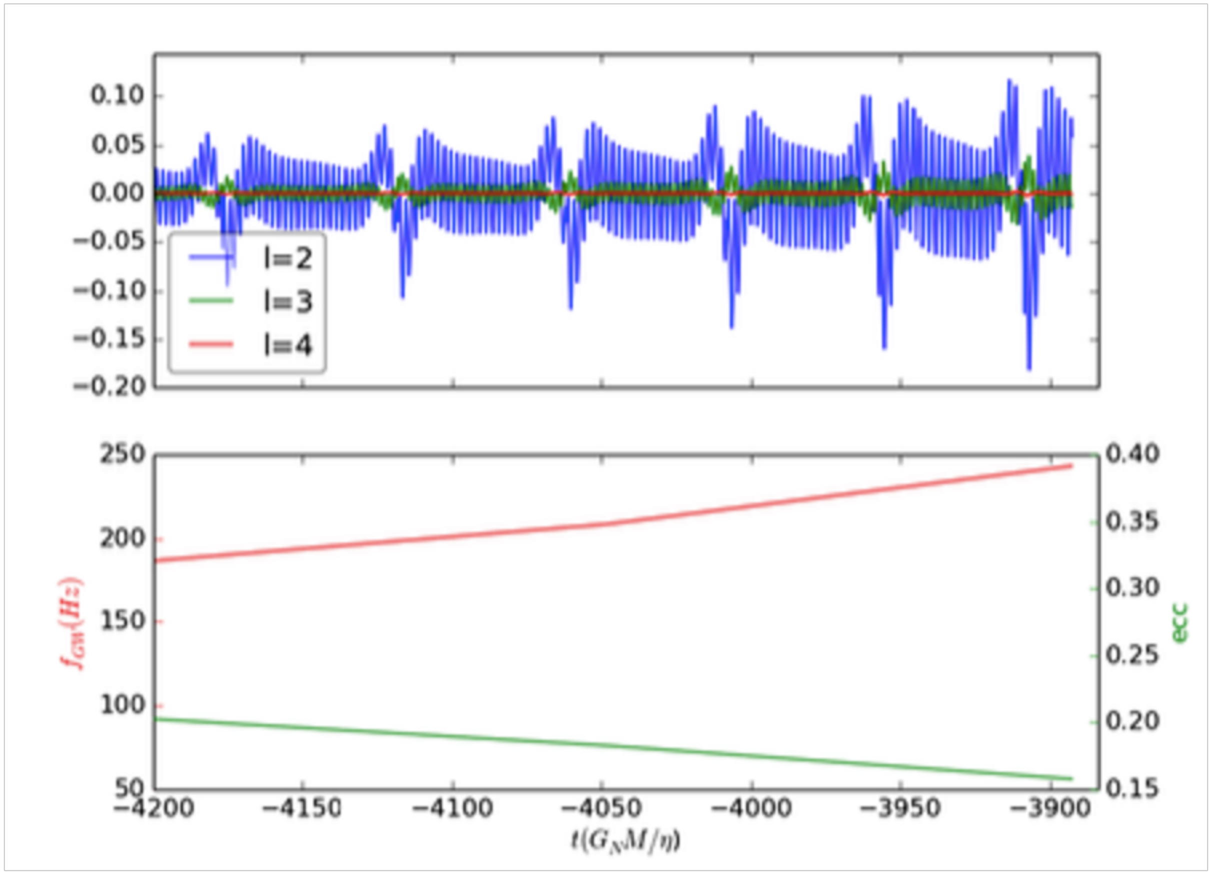
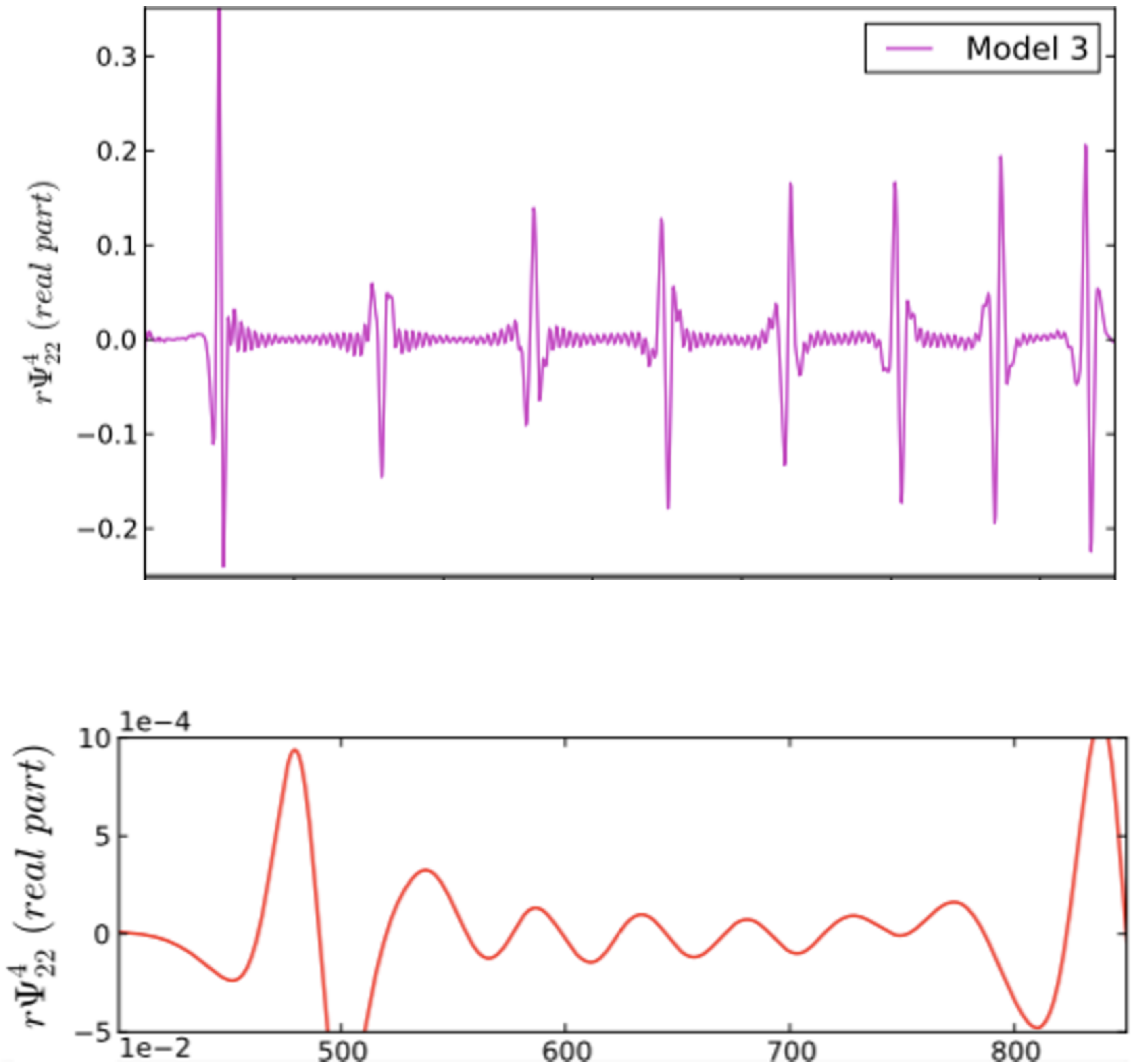




Model 3
0.3
0.2
0.1
0.0
−0.1
−0.2
$r\Psi^4_{22}$ (real part)
1e−4
10
5
0
−5
1e−2
500
600
700
800
$r\Psi^4_{22}$ (real part)

Which EoS can we use?

Phenomenological approaches

Based on effective density-dependent NN force with parameters fitted on nuclei properties.

- Liquid Drop models
  - ✧ BPS Baym et al, *ApJ* ***170***, *299 (1971)*
  - ✧ BBP Baym et al., *NPA* ***175***, *225 (1971)*
  - ✧ LS Lattimer&Swesty, *NPA* ***535***, *331 (1991)*
  - ✧ DH Douchin&Haensel, *A&A* ***380***, *151 (2001)*
- TF + RMF
  - ✧ Shen et al., *NPA* ***637***, *435 (1998)*
- ETFSI + Eff. Skyrme force
  - ✧ BSk Goriely et al.,*PRC* ***82***, *035804 (2010)*
- Hartree-Fock
  - ✧ NV Negele&Vautherin, *NPA* ***207***, *298 (1973)*
  - ✧ RMF Serot&Walecka, *Adv. NP 16, 1 (1986)*
  - ✧ RHF Boussy et al., *PRL* ***55***, *1731 (1985)*
  - ✧ QMC Guichon et al., *NPA* ***814***, *66 (2008)*
- Statistical models
  - ✧ NSE Raduta&Gulminelli. *PRC* ***82***, *065801 (2010)*
  - ✧ HS Hempel&Schaffner-Bielich, *NPA* ***837***, *210 (2010)*

Chiral SU(3) model

Dexheimer&Schramm, ApJ 683, 943 (2008)

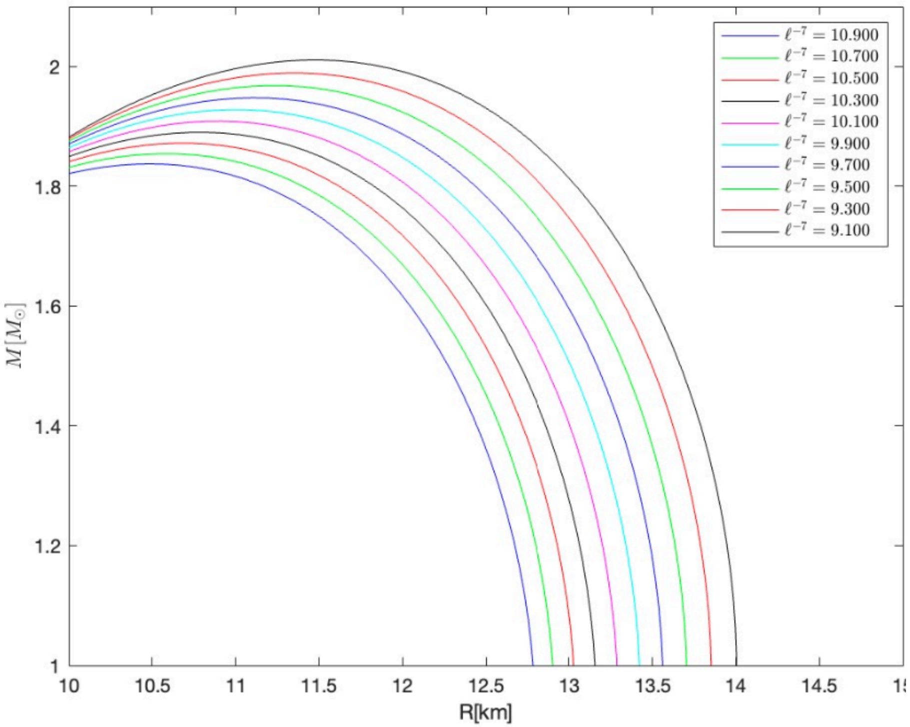
Ab initio approaches

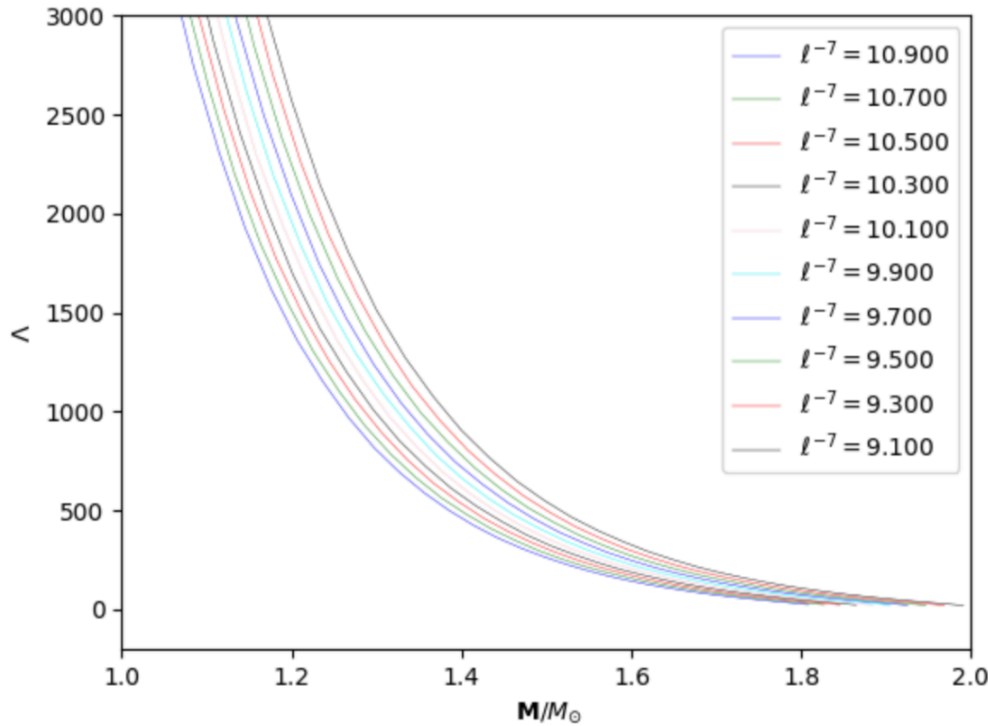
The nuclear problem is solved starting from the two- and three-body realistic nucleon interaction.

- Diagrammatic
  - ✧ BBG *Day*, *RMP* ***39***, *719 (1967)*
  - ✧ SCGF Kadanoff&Baym, Quantum Statistical Mechanics (1962)
  - ✧ DBHF Ter Haar&Malfiet, *Phys, Rep.* ***149***, *207 (1987);*
- Variational
  - ✧ APR *Akmal et al.*, *PRC* ***58***, *1804 (1998)*
  - ✧ FHNC Fantoni&Rosati, *Nuovo Cimento* ***A20***, *179 (1974)*
  - ✧ CBF Fabrocini&Fantoni, *PLB* ***298***, *263(1993)*
  - ✧ LOCV Owen et al., *NPA* ***277***, *45 (1978)*
- Monte Carlo
  - ✧ VMC Wiringa, *PRC* ***43***, *1585 (1991)*
  - ✧ GFMC Carlson, *PRC* ***68***, *025802 (2003)*
  - ✧ AFDMC Schmidt&Fantoni, *PLB* ***446***, *99 (1999)*

Holographic EoS: Witten-Sakai-Sugimoto Model

$$\frac{\epsilon_1}{\epsilon_\odot} = 0.140\mathcal{A}^{0.571}\left(\frac{p_1}{p_\odot}\right)^{0.429} + 3.896\mathcal{A}^{-0.335}\left(\frac{p_1}{p_\odot}\right)^{1.335} \qquad \mathcal{A} = 1.8 \times 10^{-5} \times \ell^{-7}$$





f-Mode oscillations and the gravitational responses of compact stars with analytic equations of state

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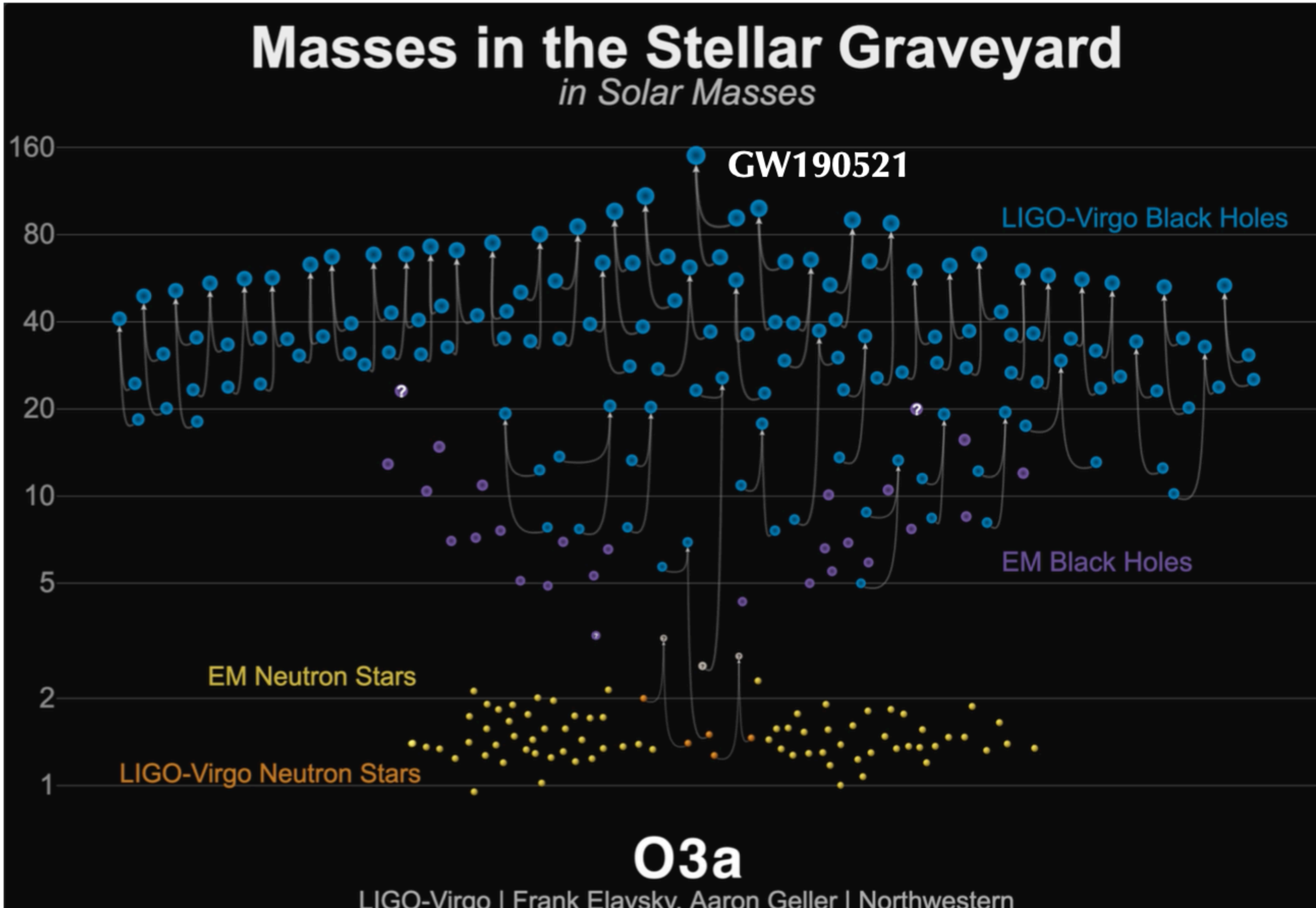
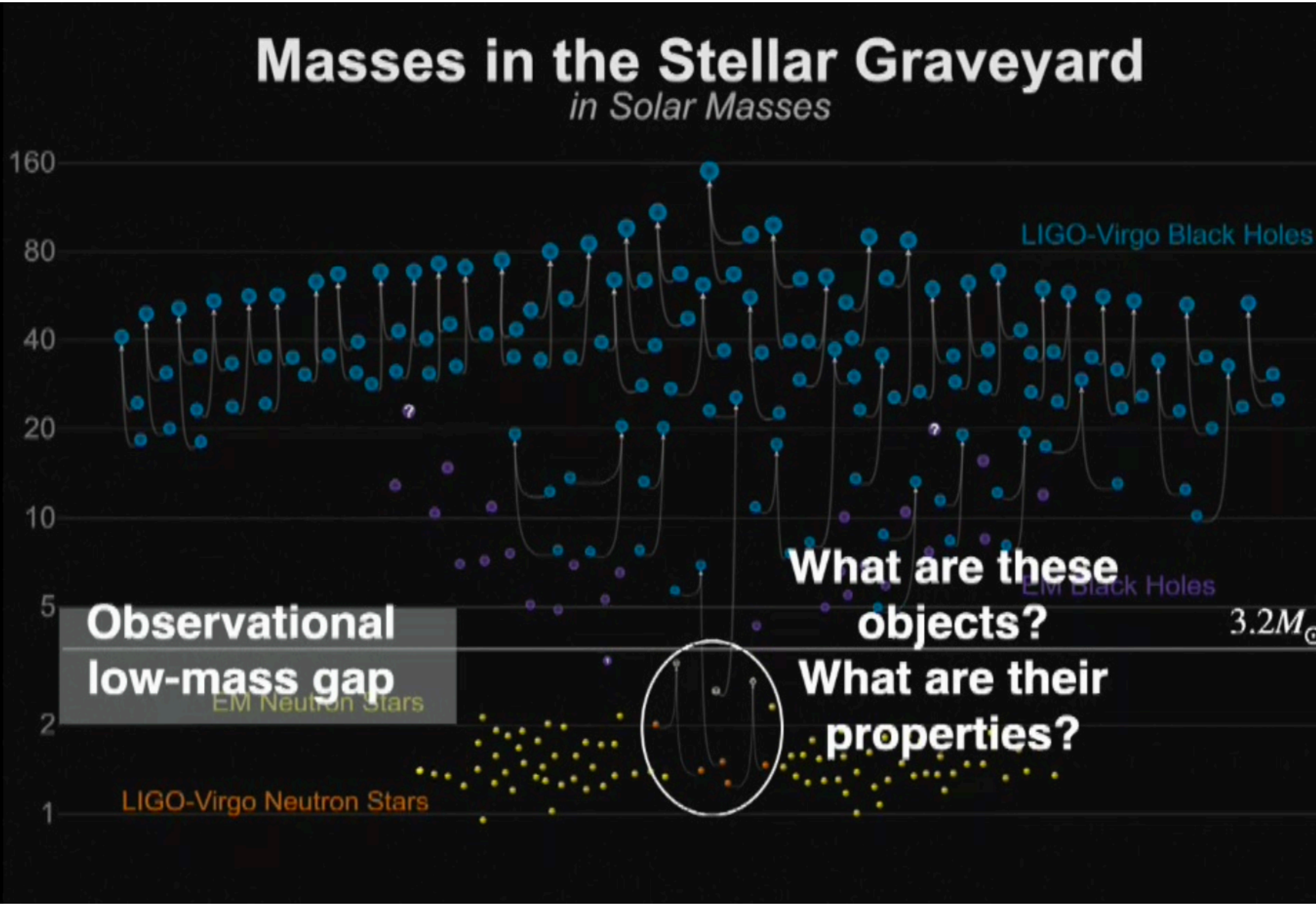
During O3a (~6 months) **50 events!**



Abbott+, arXiv:2010.14527 (2020)



Masses in the Stellar Graveyard
in Solar Masses
160
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
EM Black Holes
EM Neutron Stars
LIGO-Virgo Neutron Stars
Observational low-mass gap
What are these objects?
What are their properties?
$3.2M_{\odot}$

Boson star model

BS is described by a complex scalar field coupled to gravity.

The Lagrangian of the matter is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \nabla^\alpha \Phi \nabla_\alpha \Phi^* - V(|\Phi|^2) \right]$$

$$\frac{1}{\sqrt{-g}} (\sqrt{-g} J^\mu)_{,\mu} = 0. \qquad J_\mu = i(\Phi^* \nabla_\mu \Phi - \Phi \nabla_\mu \Phi^*)$$

$$T^\Phi_{\alpha\beta} = \nabla_\alpha \Phi^* \nabla_\beta \Phi + \nabla_\beta \Phi^* \nabla_\alpha \Phi - g_{\alpha\beta}(\nabla^\gamma \Phi^* \nabla_\gamma \Phi + V(|\Phi|^2)).$$

Einstein-Klein-Gordon Equations

$$ds^2 = -B(r)\,dt^2 + A(r)\,dr^2 + r^2\,d\theta^2 + r^2\ \sin^2\theta\,d\psi^2$$

$$\phi \equiv \phi(r)e^{-i\omega t}$$

$$A' = xA^2\left[\left(\frac{\Omega^2}{B}+1\right)\sigma^2 + \frac{\sigma'^2}{A}\right] - \frac{A}{x}\Big(A-1\Big)\,,$$

$$B' = xAB\left[\left(\frac{\Omega^2}{B}-1\right)\sigma^2 + \frac{\sigma'^2}{A}\right] + \frac{B}{x}\Big(A-1\Big)\,,$$

$$\sigma'' = -\left(\frac{2}{x}+\frac{B'}{2B}-\frac{A'}{2A}\right)\sigma' - A\sigma\left(\frac{\Omega^2}{B}-1\right).$$

$$x = mr\,, \qquad \sigma(x) = \sqrt{8\pi G}\,\phi(x/m)\,, \qquad \Omega = \omega/m$$

EoS

$$V(|\Phi|) = \frac{1}{2}m^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4 \qquad \Lambda \equiv \lambda M_{\rm Pl}^2/(4\pi m^2)$$

$$p = \frac{c^4}{36K}\left[\left(1 + \frac{12K}{c^2}\rho\right)^{1/2} - 1\right]^2 \qquad K \equiv \frac{\lambda \hbar^3}{4m^4 c}$$

We impose the strength of self-interactions in the DM sector

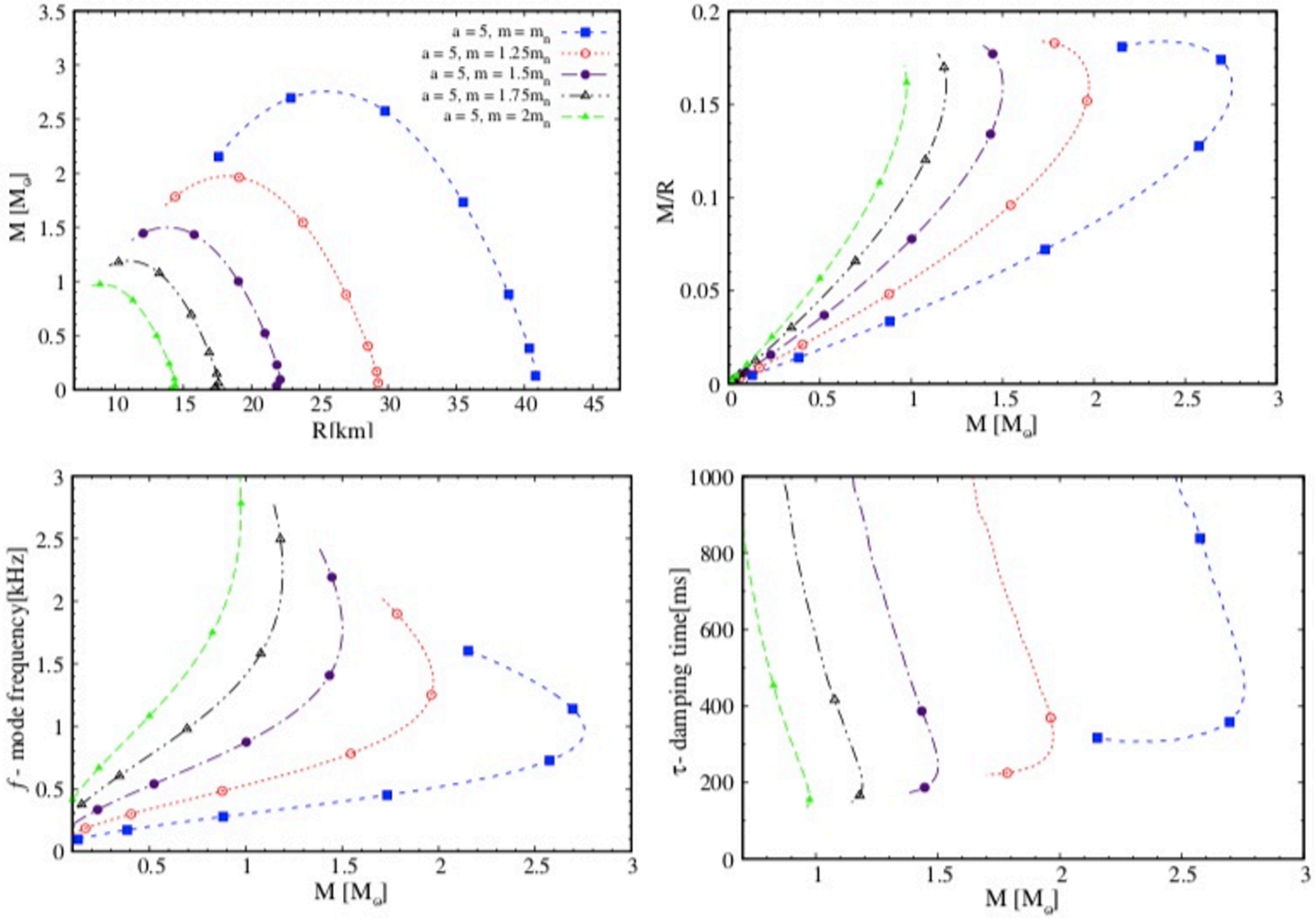
$$0.1\frac{\rm cm^2}{\rm g} \leq \frac{\bar{\sigma}}{m} \leq 10\frac{\rm cm^2}{\rm g}. \qquad \bar{\sigma} = \frac{\lambda^2}{64\pi m^2}. \qquad \frac{\lambda}{8\pi} \equiv \frac{a}{\lambda_c} = \frac{amc}{\hbar},$$

Benchmark points:

a=5fm, a=10fm, a=15fm, and a=20fm

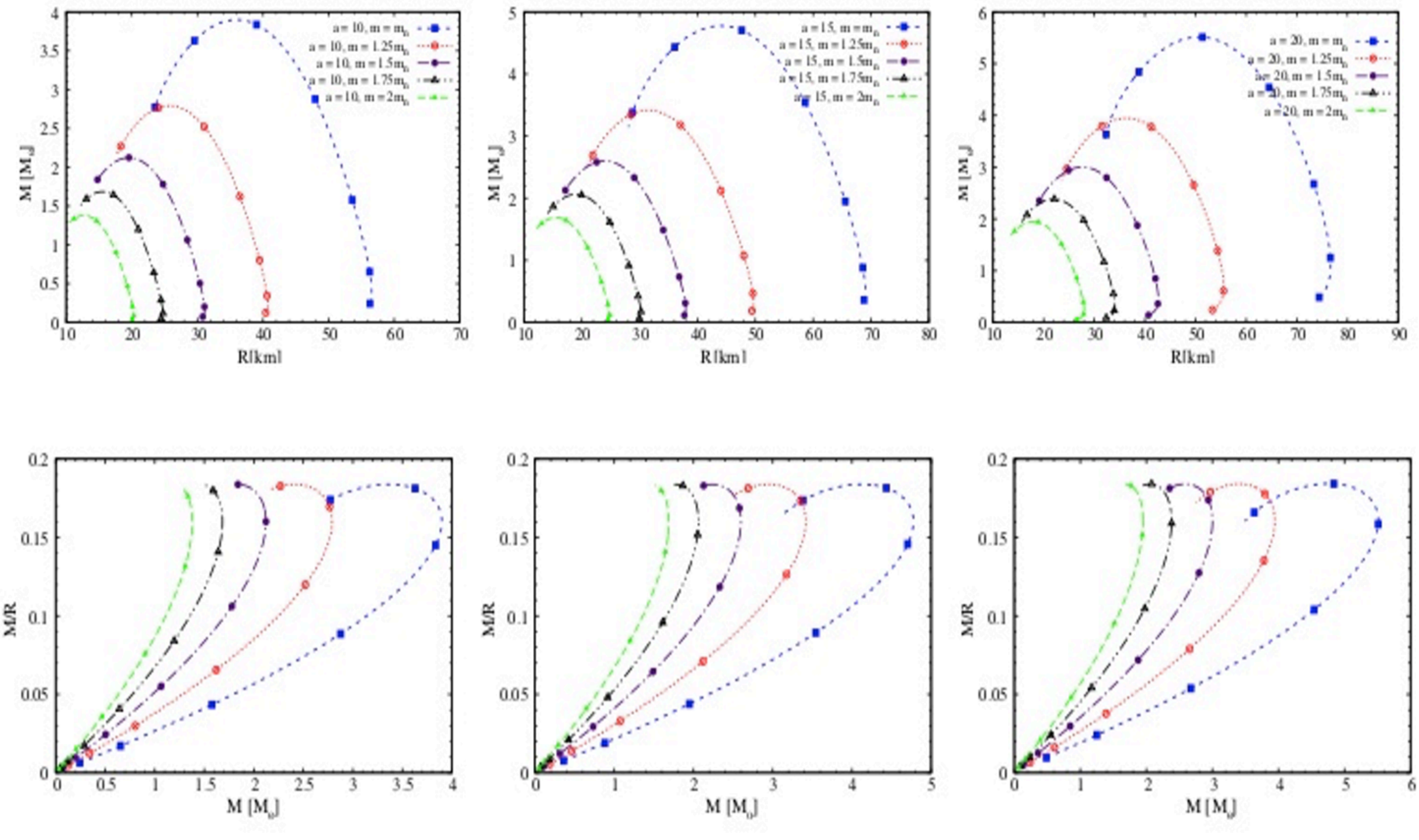
m=1m_n, m=1.25m_n, m=1.5m, m=1.75m_n, and m=2.0m_n

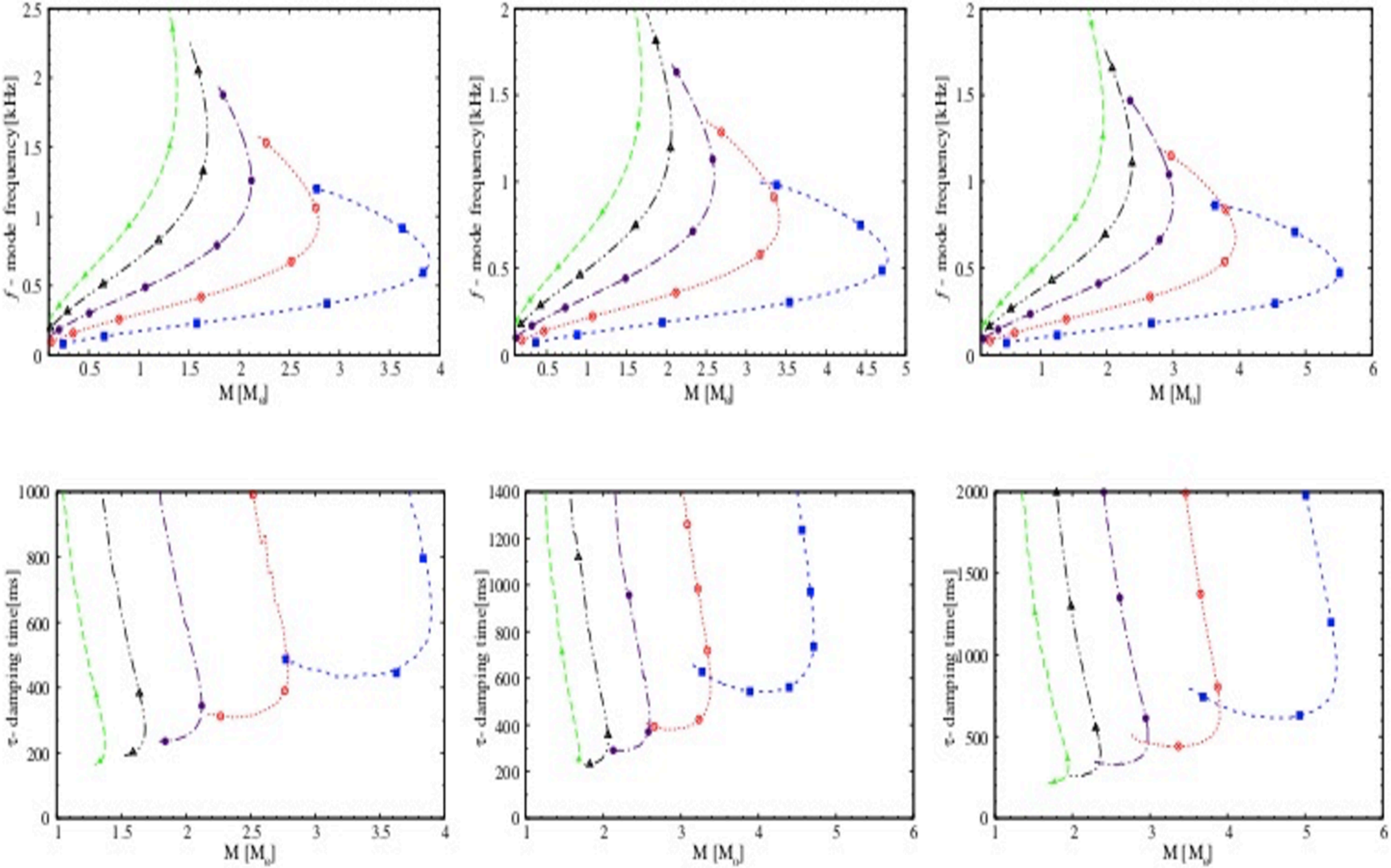
a=5fm



a = 5, m = m_n
a = 5, m = 1.25m_n
a = 5, m = 1.5m_n
a = 5, m = 1.75m_n
a = 5, m = 2m_n
M [M_⊙]
R[km]
M/R
M [M_⊙]
f - mode frequency[kHz]
M [M_⊙]
τ- damping time[ms]
M [M_⊙]

Other results (different sets of parameter a)





f - mode frequency [kHz]
M [M$_\odot$]
f - mode frequency [kHz]
M [M$_\odot$]
f - mode frequency [kHz]
M [M$_\odot$]
τ- damping time[ms]
M [M$_\odot$]
τ- damping time[ms]
M [M$_\odot$]
τ- damping time[ms]
M [M$_\odot$]

Journal of Cosmology and Astroparticle Physics

Fundamental oscillation modes of self-interacting bosonic dark stars

C. Vásquez Flores, Alessandro Parisi, Chian-Shu Chen and Germán Lugones

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Open Questions

Can we observe the phase transition in ET ?

Can we observe pulsar oscillations in ET?

Observer

Thanks for your attention

LIGO-Virgo PRL 121, 161101 (2018)

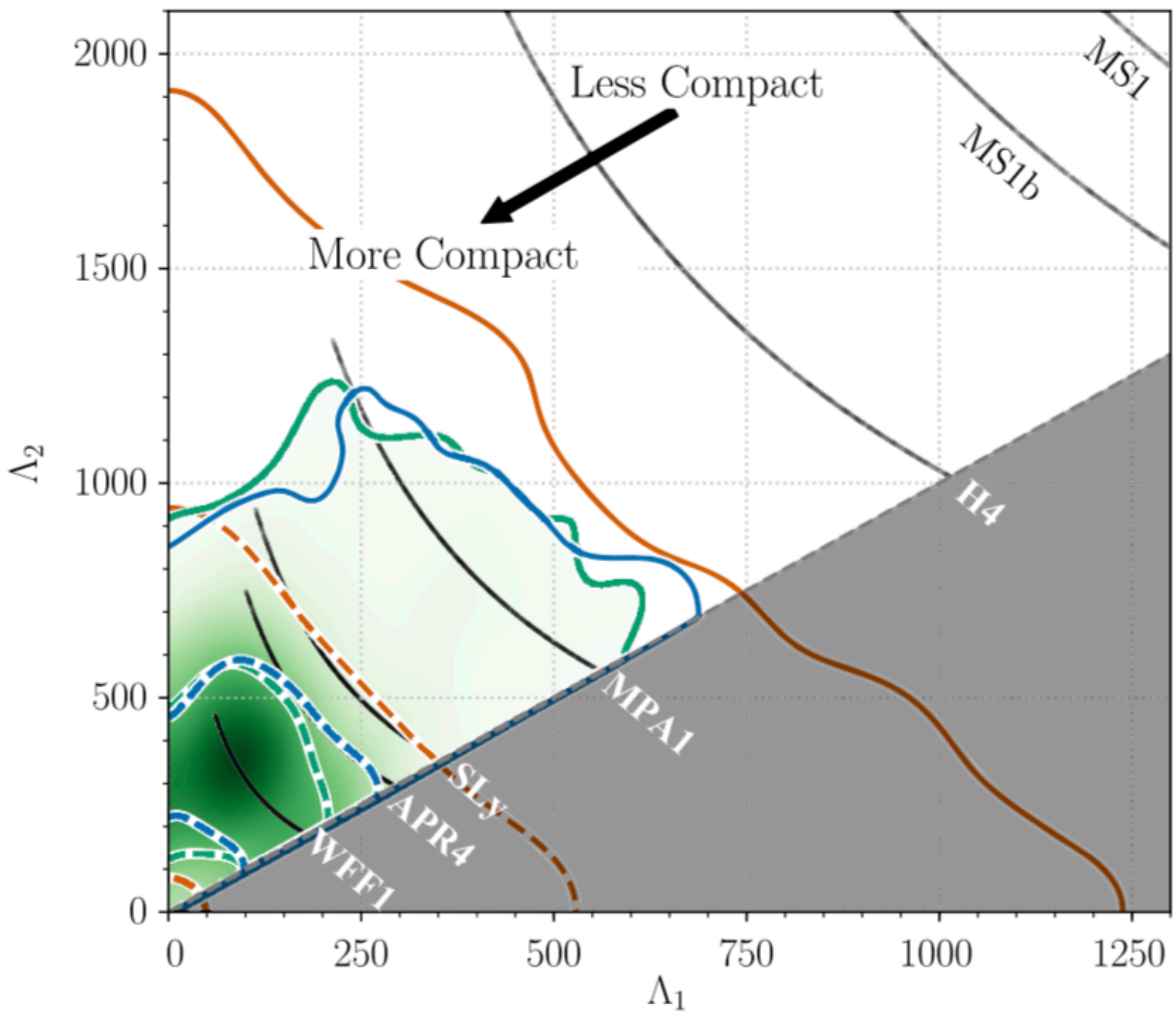


Table 3. We show the main parameters that characterize the phase transition, for Hebeler et al. (2013) stiff EoS.

Transition	$p_{\rm tr}^{\rm HP}$(MeV fm^{-3})	$\epsilon_{\rm tr}^{\rm HP}$(MeV fm^{-3})	$n_{\rm B,tr}^{\rm HP}$(fm^{-3})	$\Delta\epsilon$(MeV fm^{-3})	$a_2^{1/2}$(MeV)	$B_{\rm eff}^{1/4}$(MeV)	a_4	$M_{\rm max}(M_\odot)$
1	20.39	249.2	0.256	80	100	147.08	0.622	1.852
2	20.39	249.2	0.256	130	100	154.11	0.705	1.711
3	20.39	249.2	0.256	180	100	160.28	0.789	1.596
4	52.49	302.2	0.304	100	100	142.52	0.535	2.054
5	52.49	302.2	0.304	200	100	156.81	0.645	1.825
6	52.49	302.2	0.304	300	100	168.01	0.756	1.770
7	92.63	361.4	0.352	250	100	154.54	0.547	2.318
8	92.63	361.4	0.352	300	100	160.67	0.584	2.318
9	92.63	361.4	0.352	350	100	166.17	0.621	2.317
10	152.9	448.9	0.416	300	100	147.16	0.443	2.662
11	152.9	448.9	0.416	400	100	160.35	0.489	2.661
12	152.9	448.9	0.416	500	100	170.91	0.536	2.661
13	259.1	573.6	0.496	500	100	145.66	0.359	2.874
14	259.1	573.6	0.496	600	100	159.19	0.385	2.874
15	259.1	573.6	0.496	700	100	169.95	0.411	2.874

Tidal induced Quadrupole Moment and tidal deformability of a Neutron Star

In the star's local asymptotic frame (asymptotically mass-centered Cartesian coordinates) at large distance r, the metric coefficient g_{tt} can be written as (Thorne 1998)

$$\frac{1-g_{tt}}{2} = -\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + \mathcal{O}\left(\frac{1}{r^3}\right) + \frac{1}{2}C_{ij}x^i x^j + \mathcal{O}\left(r^3\right)$$

where $n^i = x^i/r$. For instance in Newtonian theory

$$C_{ij} = \frac{\partial^2 \Phi_{ext}}{\partial x^i \partial x^j}$$

In GR the tidal field C_{ij} is found by projecting the Riemann tensor associated to the external field which produces the star deformation, onto a parallely transported tetrad attached to the deformed star

$$C_{ij} = e^{\alpha}_{(0)} e^{\beta}_{(i)} e^{\gamma}_{(0)} e^{\delta}_{(j)} R_{\alpha\beta\gamma\delta}$$

Under the action of the external, quadrupolar tidal field the star's metric is perturbed

$$g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h_{\alpha\beta}$$

$g^{(0)}_{\alpha\beta}$ describes the geometry of the static star, $h_{\alpha\beta}$ is the metric perturbation.

Expanding $h_{\alpha\beta}$ in spherical harmonics (in the Regge-Wheeler gauge) allows to write $h_{\alpha\beta}$ as

$$h_{\alpha\beta} = diag\left[-e^{\nu(r)}H_0(r), e^{\lambda(r)}H_2(r), r^2K(r), r^2\sin\theta K(r)\right] Y_{2m}(\theta,\phi)$$

Einstein's equations linearized about $g^{(0)}_{\alpha\beta}$ give:

$$H_0 = -H_2 = H$$

and, by suitably combining the various components, a relation between H(r) and K(r), and a second order differential equation for H(r):

$$H'' + H'\left\{\frac{2}{r} + e^{\lambda}\left[\frac{2m(r)}{r^2} + 4\pi r(p-\rho)\right]\right\} + H\left[-\frac{6e^{\lambda}}{r^2} + 4\pi e^{\lambda}\left(5\rho + 9p + \frac{\rho+p}{dp/d\rho}\right) - (\nu')^2\right] = 0$$

T. Hinderer, ApJ 677,2008

The function H is the only function we need to find the perturbed metric

Tidal effects in the gravitational wave signal emitted in NS-NS binary coalescence

$$h(f) = \mathcal{A}(f) e^{i\psi(f)} \qquad \psi(f) = \psi_{PP} + \psi_{\bar{Q}} + \psi_{\bar{\lambda}}$$

$x = (m\pi f)^{5/3}$ PN expansion parameter

point-particle contribution

$$\psi_{PP}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{1 + \left(\frac{3715}{756} + \frac{55}{9}\eta\right)x - (16\pi - 4\beta)x^{3/2} + \left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2 - 10\sigma\right)x^2 + \mathcal{O}(x^{5/2})\right\}$$

σ contains spin-spin and spin-orbit terms. Note that it appears in the 2-PN term (x^2)

Quadrupole contribution:

$$\psi_Q = \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{-50\left[\left(\frac{m_1^2}{m^2}\chi_1^2 + \frac{m_2^2}{m^2}\chi_2^2\right)(Q_S - 1) + \left(\frac{m_1^2}{m^2}\chi_1^2 - \frac{m_2^2}{m^2}\chi_2^2\right)Q_a\right]x^2\right\}$$

$$Q_S = \frac{\bar{Q}_1 + \bar{Q}_2}{2}, \quad Q_a = \frac{\bar{Q}_1 - \bar{Q}_2}{2}$$

both the quadrupole moments and the spin terms appear at the 2-PN order and cannot be measured independently : in this sense we say that there is complete degeracy

External solution

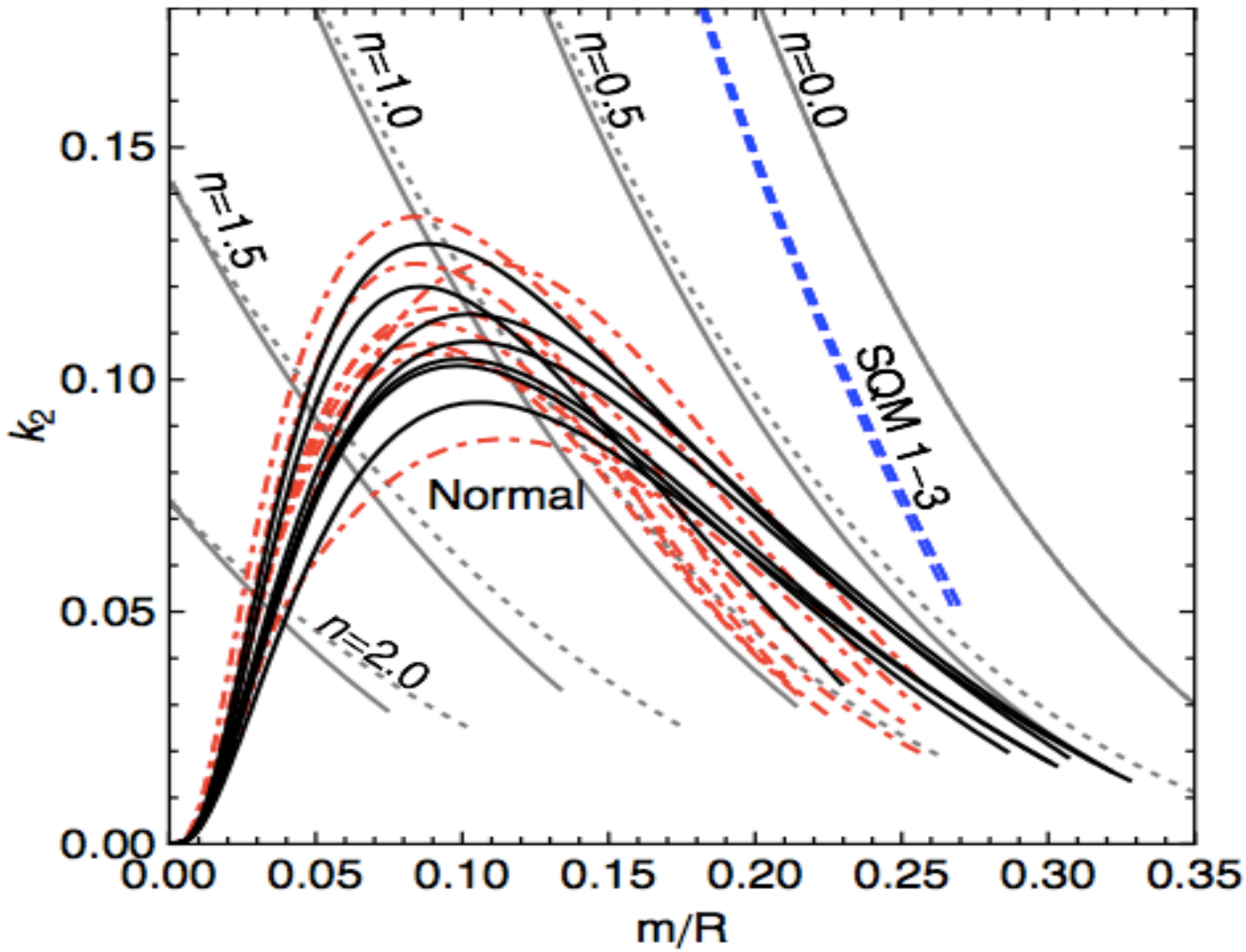
$$H'' + \left(\frac{2}{r} - \lambda'\right)H' - \left(\frac{6e^{\lambda}}{r^2} + \lambda'^2\right)H = 0$$

$$H = c_1 Q_2^2\left(\frac{r}{M} - 1\right) + c_2 P_2^2\left(\frac{r}{M} - 1\right)$$

$$H = \frac{8}{5}\left(\frac{M}{r}\right)^3 c_1 + O\left(\left(\frac{M}{r}\right)^4\right) + 3\left(\frac{r}{M}\right)^2 c_2 + O\left(\frac{r}{M}\right)$$

$$c_1 = \frac{15}{8}\frac{1}{M^3}\lambda\mathcal{E}, \quad c_2 = \frac{1}{3}M^2\mathcal{E}.$$

Result for k2 Love number



Love Number

$$k_2 = \frac{8C^5}{5}(1-2C)^2\left[2+2C(y-1)-y\right]\left\{2C\left[6-3y\ + \right.\right.$$
$$+3C(5y-8)\left.\right] + 4C^3\left[13-11y+C(3y-2)\ + \right.$$
$$\left.\left.+2C^2(1+y)\right] + 3(1-2C)^2\left[2-y+2C(y-1)\ln(1-2C)\right]\right\}^{-1},$$

T. Hinderer, ApJ 677,2008

where $C = M/R$ is the star compactness,

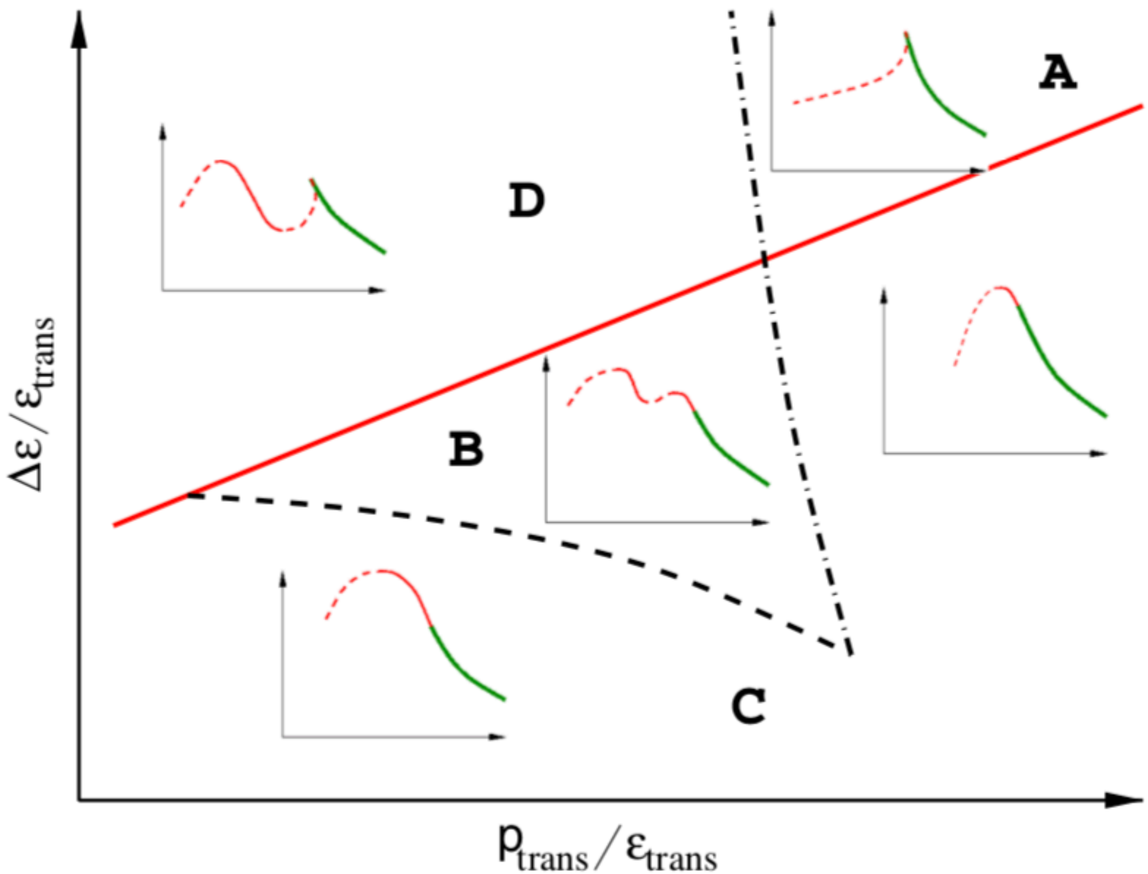
$$\lambda = \frac{2R^5}{3G}k_2$$

tidal deformability

NOTE THAT:

the Love number k_2 and the tidal deformability λ depend only on the stellar compactness (a quantity which depends on the equation of state) and on the value of H and H' at the surface, which again depends on the EoS through the pressure and density profiles in the unperturbed star

Stability Conditions



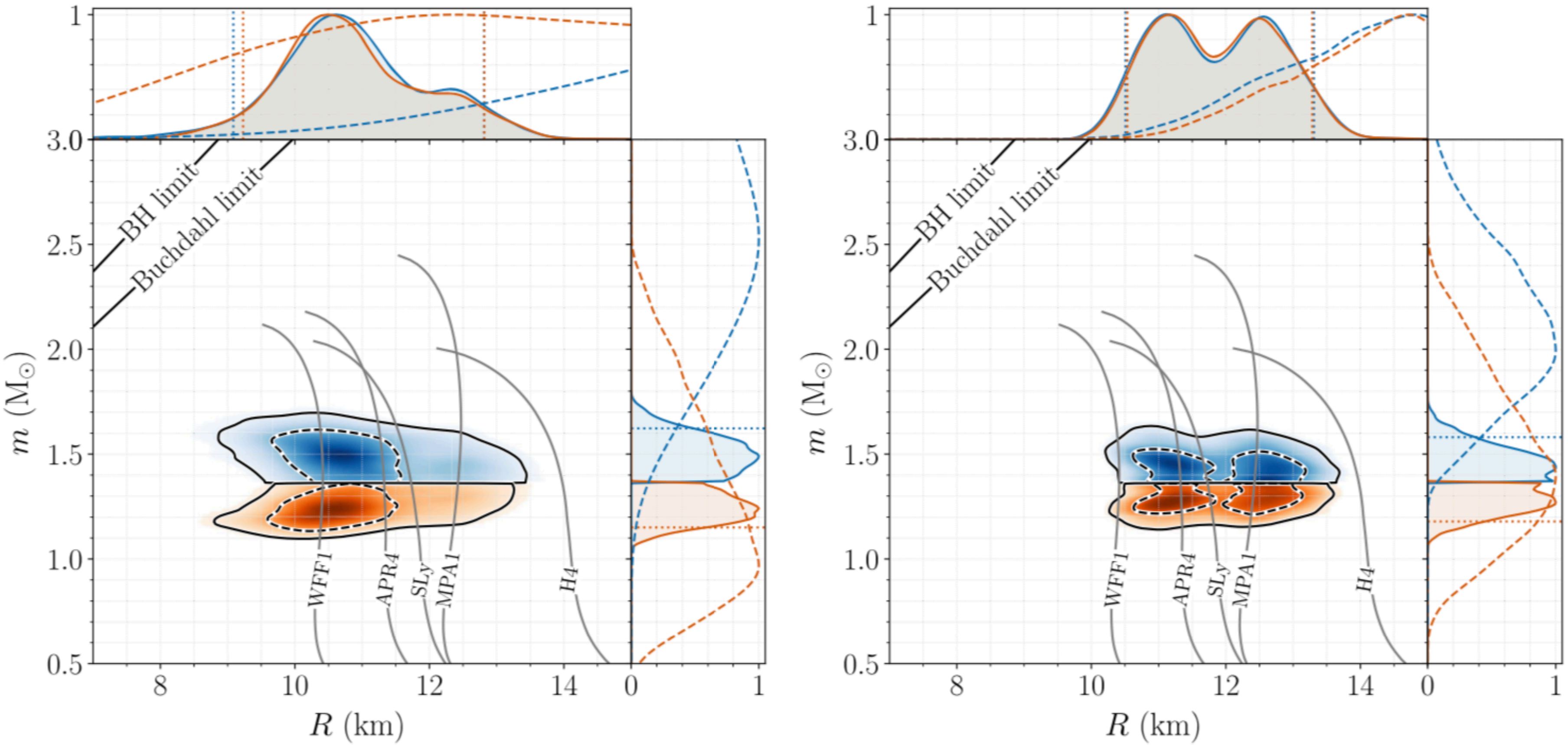
Above the red line ($\Delta\varepsilon > \Delta\varepsilon_{crit}$), connected branch disappears

$$\frac{\Delta\varepsilon_{crit}}{\varepsilon_{trans}} = \frac{1}{2} + \frac{3}{2}\frac{p_{trans}}{\varepsilon_{trans}}$$

(Seidov, 1971; Schaeffer, Zdunik, Haensel, 1983; Lindblom, gr-qc/9802072)

Disconnected branch exists in regions D and B.

LIGO PRL 121, 161101 (2018)



LIGO PRL 119, 161101 (2017)

Source properties

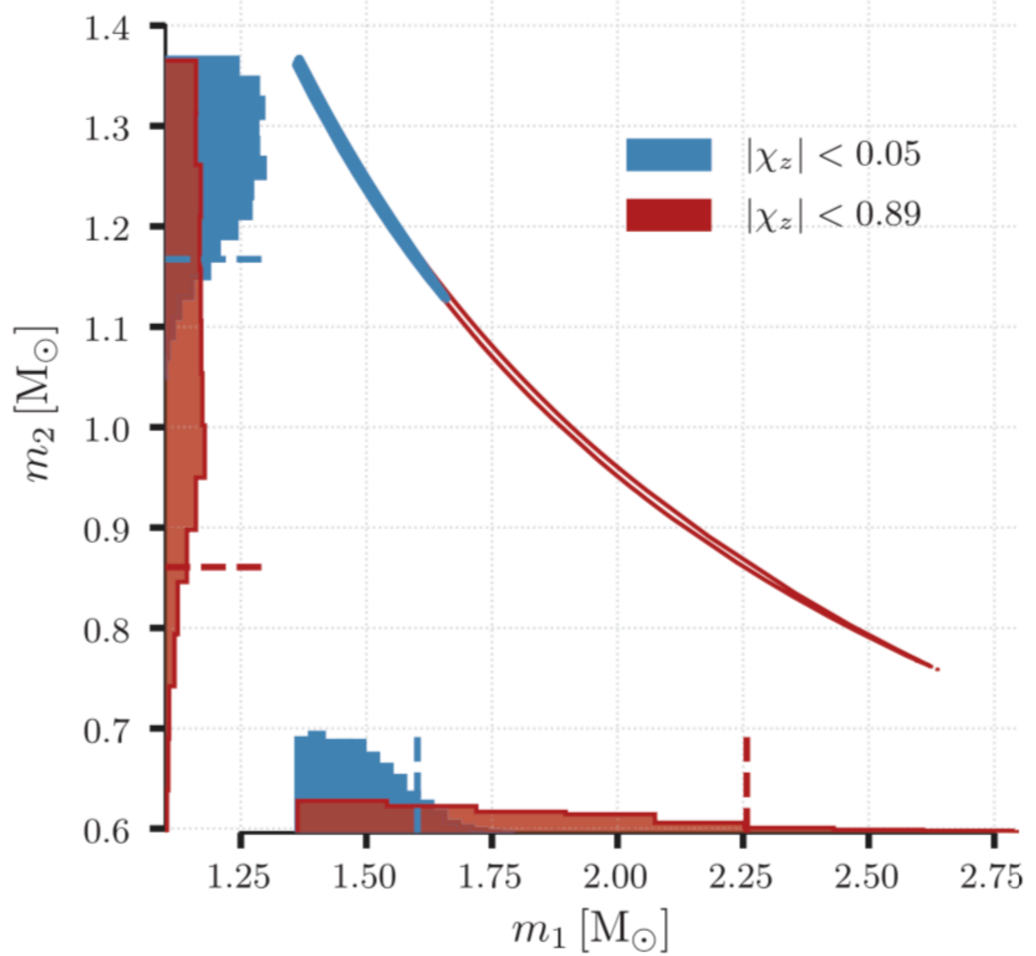
- Inferred using a Bayesian framework.
  - Chirp mass, the combination of masses that primarily determines the chirp-like evolution of the frequency, is best constrained.

$$\mathcal{M} = 1.188^{+0.004}_{-0.002} M_{\odot}$$

  - Measurement of component masses limited by correlations (between them, and with spins)

$m_1 = 1.36 - 1.60 M_{\odot}$
$m_2 = 1.17 - 1.36 M_{\odot}$
Assuming Low spin priors

$m_1 = 1.36 - 2.26 M_{\odot}$
$m_2 = 0.86 - 1.36 M_{\odot}$
Assuming High spin priors



Selected for a Viewpoint in *Physics*

PRL **119**, 161101 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

B. P. Abbott *et al.*[*]

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 26 September 2017; revised manuscript received 2 October 2017; published 16 October 2017)

On August 17, 2017 at 12:41:04 UTC the Advanced LIGO and Advanced Virgo gravitational-wave detectors made their first observation of a binary neutron star inspiral. The signal, GW170817, was detected with a combined signal-to-noise ratio of 32.4 and a false-alarm-rate estimate of less than one per 8.0×10^4 years. We infer the component masses of the binary to be between 0.86 and 2.26 $M_\odot$, in agreement with masses of known neutron stars. Restricting the component spins to the range inferred in binary neutron stars, we find the component masses to be in the range 1.17–1.60 $M_\odot$, with the total mass of the system $2.74^{+0.04}_{-0.01} M_\odot$. The source was localized within a sky region of 28 deg^2 (90% probability) and had a luminosity distance of 40^{+8}_{-14} Mpc, the closest and most precisely localized gravitational-wave signal yet. The association with the γ-ray burst GRB 170817A, detected by Fermi-GBM 1.7 s after the coalescence, corroborates the hypothesis of a neutron star merger and provides the first direct evidence of a link between these mergers and short γ-ray bursts. Subsequent identification of transient counterparts across the electromagnetic spectrum in the same location further supports the interpretation of this event as a neutron star merger. This unprecedented joint gravitational and electromagnetic observation provides insight into astrophysics, dense matter, gravitation, and cosmology.

DOI: 10.1103/PhysRevLett.119.161101

TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

	Low-spin priors ($\lvert\chi\rvert \leq 0.05$)	High-spin priors ($\lvert\chi\rvert \leq 0.89$)
Primary mass m_1	1.36–1.60 $M_\odot$	1.36–2.26 $M_\odot$
Secondary mass m_2	1.17–1.36 $M_\odot$	0.86–1.36 $M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0	0.4–1.0
Total mass $m_{\rm tot}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\rm rad}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance $D_{\rm L}$	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55°$	$\leq 56°$
Using NGC 4993 location	$\leq 28°$	$\leq 28°$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800	≤ 1400

Key assumptions:

Einstein's Theory of GR is correct

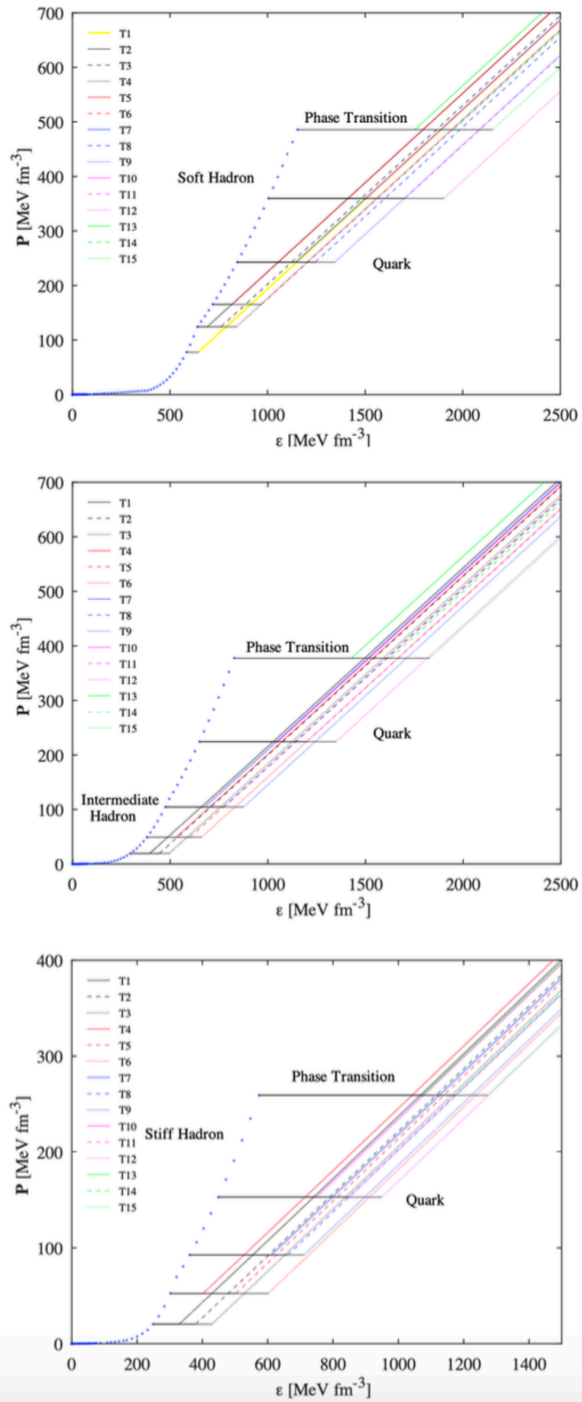
$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

Microscopic stability

$$\frac{dp}{dr} = -\frac{G}{r^2} \frac{(\epsilon + p)(mc^2 + 4\pi r^3 p)}{\left(1 - \frac{2Gm}{r}\right)}$$

Causality

$$\frac{d\phi}{dr} = \frac{m + 4\pi r^3 p}{r(r - 2Gm)}$$



P [MeV fm^-3]
ε [MeV fm^-3]
T1
T2
T3
T4
T5
T6
T7
T8
T9
T10
T11
T12
T13
T14
T15
Phase Transition
Soft Hadron
Quark
Intermediate Hadron
Stiff Hadron

Tidal induced Quadrupole Moment and tidal deformability of a Neutron Star

If a static, spherically symmetric star of mass M is placed in a static, external, quadrupolar tidal field C_{ij}, it develops a quadrupole moment Q_{ij}

To linear order in the tidal field C_{ij}, the "tidal - induced" quadrupole moment Q_{ij} can be written as

$$Q_{ij} = \lambda C_{ij}$$

where λ is the tidal deformability,

λ is related to the l=2 tidal Love number k_2 (or apsidal constant) by the equation

$$k_2 = \frac{3G}{2R^5}\lambda$$

How do we find λ ?

f-mode oscillations

$$r\frac{dy_1}{dr} = (V_g - 1 - \ell)y_1 + \left[\frac{\ell(\ell+1)}{c_1\omega^2} - V_g\right]y_2 + V_g y_3$$

$$r\frac{dy_2}{dr} = (c_1\omega^2 - A^*)y_1 + (3 - U + A^* - \ell)y_2 - A^* y_3,$$

$$r\frac{dy_3}{dr} = (3 - U - \ell)y_3 + y_4,$$

$$r\frac{dy_4}{dr} = A^* U y_1 + U V_g y_2 + [\ell(\ell+1) - UV_g]y_3 - (U + \ell - 2)y_4,$$

$$V_g = -\frac{1}{\Gamma_1}\frac{d\ln p}{d\ln r} = \frac{gr}{c_s^2},$$

$$A^* = \frac{1}{\Gamma_1}\frac{d\ln p}{d\ln r} - \frac{d\ln\rho}{d\ln r},$$

$$U \equiv \frac{d\ln m(r)}{d\ln r} = \frac{4\pi\rho r^3}{m(r)},$$

$$c_1 \equiv \frac{r^3}{R_*^3}\frac{M_*}{m(r)},$$

$$\Gamma_1 = \left(\frac{\partial\ln p}{\partial\ln\rho}\right)_s,$$

$$\omega^2 = \frac{R_*^3}{G_N M_*}\sigma^2.$$

$$y_1 = \frac{\xi^{(r)}_{n\ell}}{r}, \qquad y_2 = \frac{1}{gr}\left(\frac{p'}{\rho} + \Phi'\right) = \frac{\sigma^2}{g}\xi^{(h)}_{n\ell},$$

$$y_3 = \frac{\Phi'}{gr}, \qquad y_4 = \frac{1}{g}\frac{d\Phi'}{dr}.$$

$$c_1\omega^2 y_1 - \ell y_2 = 0 \qquad\qquad y_1 - y_2 + y_3 = 0$$

$$\ell y_3 - y_4 = 0 \qquad\qquad (\ell + 1)y_3 + y_4 = 0$$

$$\xi_{n\ell m} = \left[\xi^{(r)}_{n\ell}(r), \xi^{(h)}_{n\ell}(r)\frac{\partial}{\partial\theta}, \frac{\xi^{(h)}_{n\ell}(r)}{\sin\theta}\frac{\partial}{\partial\phi}\right]Y_{\ell m}(\theta,\phi)e^{i\sigma t}$$

GW170817 tides

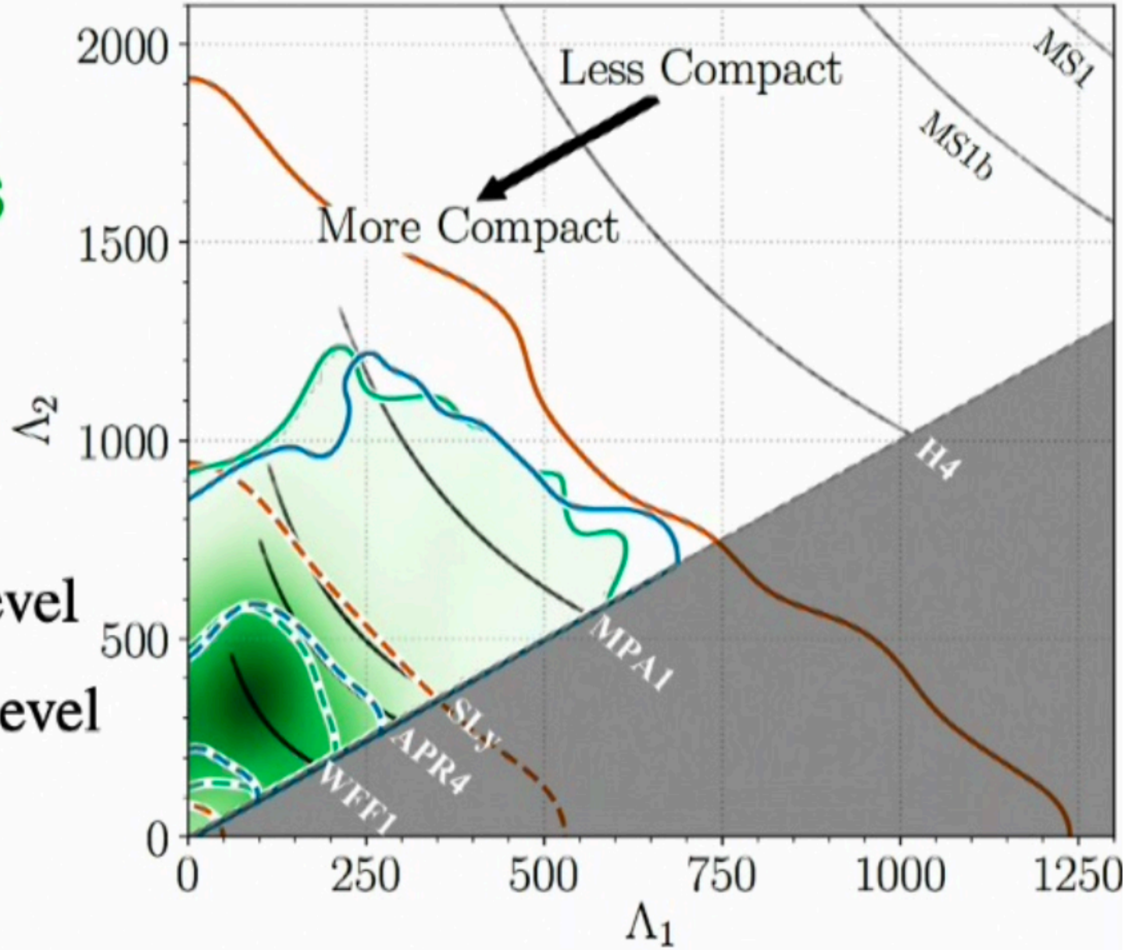
Independent EoSs

Same hadronic EoS

Spectral EoS parametrization

$\tilde{\Lambda} \lesssim 700$ at the 90 % level

$R \lesssim 13$km at the 90 % level



LVC (arxiv:1805.11581)
PE: Veitch+ (arxiv:1409.7215)
Waveform: Dietrich+ (arxiv:1804.02235)
Universal relations: Yagi and Yunes (arxiv:1512.02639), Chatziioannou+ (arxiv:1804.03221)
EoS Parametrization: Lackey and Wade (arxiv:1410.8866), Carney+ (arxiv:1805.11217)