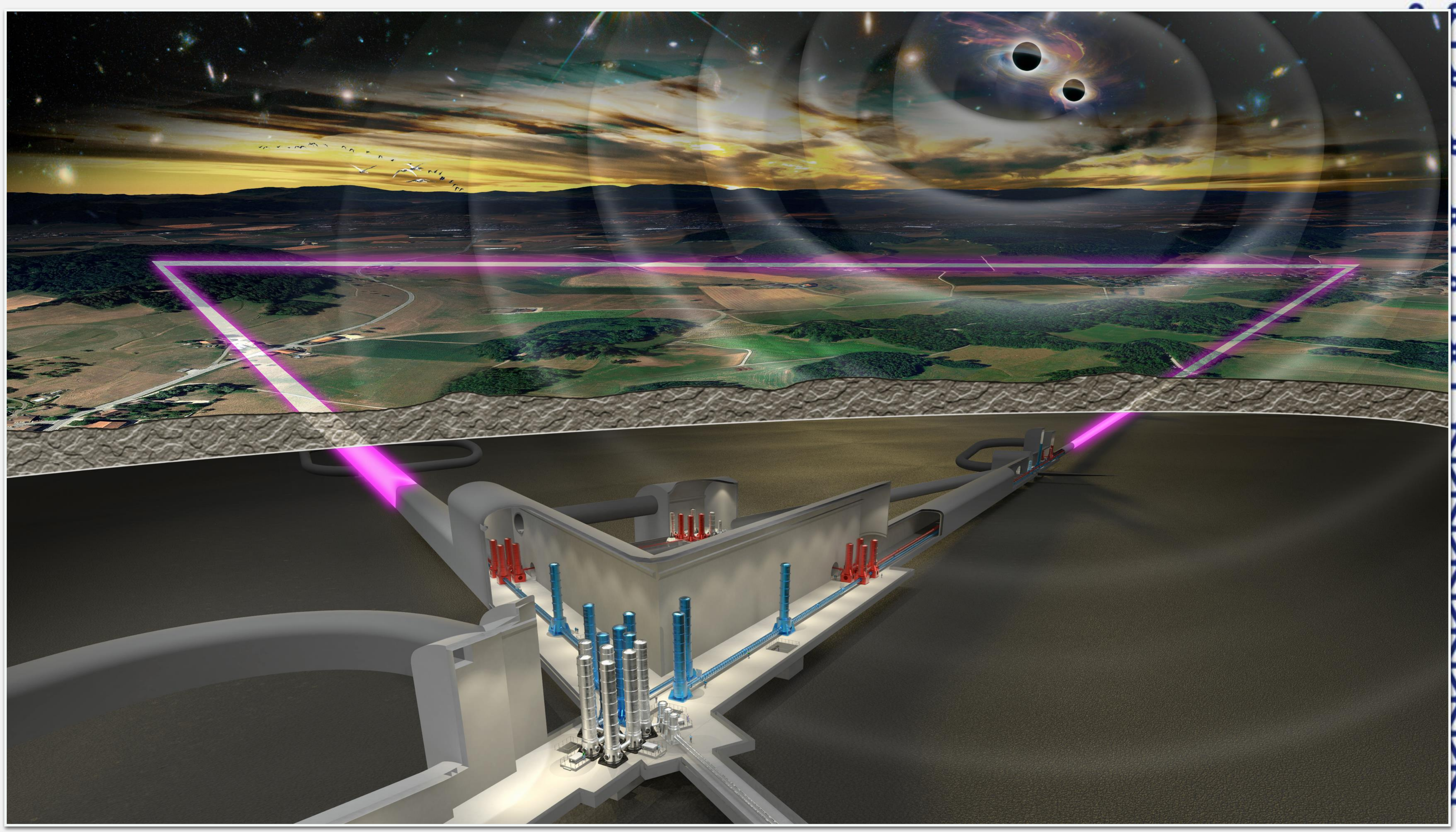

COMPUTING NEWTONIAN NOISE: THE IMPACT OF GEOLOGY AND CAVERNS

SCB meeting, April 14, Rome

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Seismic Newtonian Noise
Governing equations

- Perturbation $\delta\hat{\mathbf{a}}(\mathbf{x}_0)$ [m/s^2/Hz] in the gravitational acceleration or total Newtonian Noise $\delta\hat{\mathbf{a}}(\mathbf{x}_0)$ at the mirror position $\mathbf{x}_0$ due to the seismic displacement field $\hat{\mathbf{u}}(\mathbf{x})$.

$$\begin{aligned}\delta\mathbf{a}_{\mathrm{t}}(\mathbf{x}_0,t) &= -\nabla_0\delta\phi(\mathbf{x}_0,t)\\ &= -G\int_{\bar{\Omega}}\rho(\mathbf{x},t)\nabla_0\left(\mathbf{u}(\mathbf{x},t)\cdot\boldsymbol{\chi}(\mathbf{x},\mathbf{x}_0)\right)\,\mathrm{d}v\\ &= -G\int_{\bar{\Omega}}\rho(\mathbf{x},t)\nabla_0\boldsymbol{\chi}(\mathbf{x},\mathbf{x}_0)\cdot\mathbf{u}(\mathbf{x},t)\,\mathrm{d}v\\ &= +G\int_{\bar{\Omega}}\frac{\rho(\mathbf{x},t)}{\|\mathbf{x}-\mathbf{x}_0\|^3}(\mathbf{I}_3-3\mathbf{e}_{\mathrm{r}}\otimes\mathbf{e}_{\mathrm{r}})\mathbf{u}(\mathbf{x},t)\,\mathrm{d}v\end{aligned}$$

with $\mathbf{e}_{\mathrm{r}} = \frac{\mathbf{x}-\mathbf{x}_0}{\|\mathbf{x}-\mathbf{x}_0\|}$ and $\boldsymbol{\chi}(\mathbf{x},\mathbf{x}_0) = \frac{\mathbf{x}-\mathbf{x}_0}{\|\mathbf{x}-\mathbf{x}_0\|^3}$.

- Bulk contribution to the total Newtonian noise:

$$\delta\mathbf{a}_{\mathrm{b}}(\mathbf{x}_0,t) = -G\int_{\bar{\Omega}}\boldsymbol{\chi}(\mathbf{x},\mathbf{x}_0)\nabla\cdot\left(\rho(\mathbf{x},t)\mathbf{u}(\mathbf{x},t)\right)\,\mathrm{d}v$$

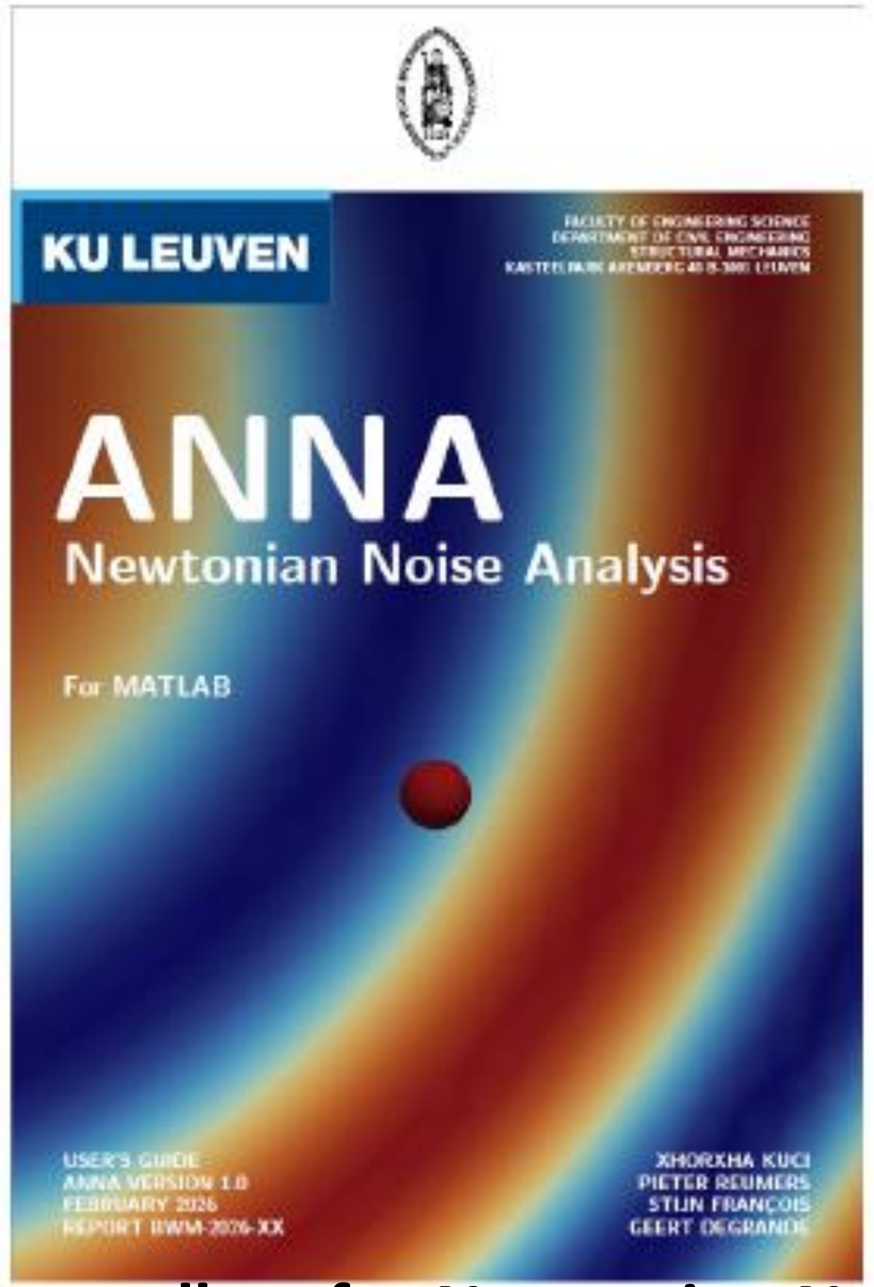
- Surface contribution to the total Newtonian noise:

$$\delta\mathbf{a}_{\mathrm{s}}(\mathbf{x}_0,t) = +G\int_{\bar{\Gamma}}\boldsymbol{\chi}(\mathbf{x},\mathbf{x}_0)\rho(\mathbf{x},t)\mathbf{u}(\mathbf{x},t)\cdot\mathbf{n}(\mathbf{x},t)\,\mathrm{d}a$$

Seismic Newtonian Noise

ANNA Newtonian Noise Analysis toolbox

- **ANNA N**ewtonian **N**oise **A**nalysis is a toolbox that computes Newtonian Noise from a seismic wave field.
- Finite element formulation based on the functionalities of the Stabil toolbox.
- Implementations available in MATLAB, GNU Octave and Python.



P. Reumers, X. Kuci, S. François, and G. Degrande. ANNA: a toolbox for Newtonian Noise Analysis. https://arxiv.org/abs/2603.15157

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S. François, M. Schevenels, D. Dooms, M. Jansen, J. Wambacq, G. Lombaert, G. Degrande, and G. De Roeck. Stabil: an educational Matlab toolbox for static and dynamic structural analysis. Computer Applications in Engineering Education, 29(5):1-18, 2021.

Seismic Newtonian Noise

Finite element formulation

- Total Newtonian Noise $\delta\hat{\mathbf{a}}_\mathrm{t}(\mathbf{x}_0)$ at the mirror position $\mathbf{x}_0$ due to the seismic displacement field $\hat{\mathbf{u}}(\mathbf{x})$:

$$\delta\hat{\mathbf{a}}_\mathrm{t}(\mathbf{x}_0) \simeq \sum_{e=1}^{n_e} G \int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1} \frac{\rho_e}{\|\mathbf{x}-\mathbf{x}_0\|^3} \left(\mathbf{I}_3 - 3\mathbf{e}_\mathrm{r}\otimes\mathbf{e}_\mathrm{r}\right)\hat{\mathbf{u}}^e(\boldsymbol{\xi})\det(\mathbf{J}^e(\boldsymbol{\xi}))\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2\,\mathrm{d}\xi_3 \tag{11}$$

- Gaussian quadrature with n_G Gauss points with local coordinates $\boldsymbol{\xi}_j$ and weights w_j:

$$\delta\hat{\mathbf{a}}_\mathrm{t}(\mathbf{x}_0) \simeq \sum_{e=1}^{n_e}\left[G\sum_{j=1}^{n_\mathrm{G}} w_j \frac{\rho_e}{\|\mathbf{x}_j-\mathbf{x}_0\|^3}\left(\mathbf{I}_3 - 3\mathbf{e}_\mathrm{r}\otimes\mathbf{e}_\mathrm{r}\right)\mathbf{N}^e(\boldsymbol{\xi}_j)\det(\mathbf{J}^e(\boldsymbol{\xi}_j))\right]\underline{\hat{\mathbf{u}}}^e = \sum_{e=1}^{n_e}\mathbf{A}_\mathrm{t}^e\underline{\hat{\mathbf{u}}}^e = \mathbf{A}_\mathrm{t}\underline{\hat{\mathbf{u}}} \tag{12}$$

- The $3\times 3n$ matrix $\mathbf{A}_\mathrm{t}^e$ yields the contribution of the nodal displacements $\underline{\hat{\mathbf{u}}}^e$ of element e to the total NN.
- Element matrices $\mathbf{A}_\mathrm{t}^e$ are assembled in a matrix $\mathbf{A}_\mathrm{t}$ of dimension $3\times 3N$, with N the number of nodes in V.
- $\mathbf{A}_\mathrm{t}$ is independent of the seismic wave field $\hat{\mathbf{u}}(\mathbf{x})$ and needs to be computed only once (post-processing).
- Similar procedure is followed for the bulk (matrix $\mathbf{A}_\mathrm{b}$) and surface (matrix $\mathbf{A}_\mathrm{s}$) contribution (not shown).

Seismic Newtonian Noise

Validation: plane P-wave

- Plane P-wave in a homogeneous full space:

$$\hat{\mathbf{u}}(\mathbf{x}) = \exp(-\mathrm{i}k_\mathrm{p}\mathbf{e}_k \cdot \mathbf{x})\mathbf{e}_k \tag{13}$$

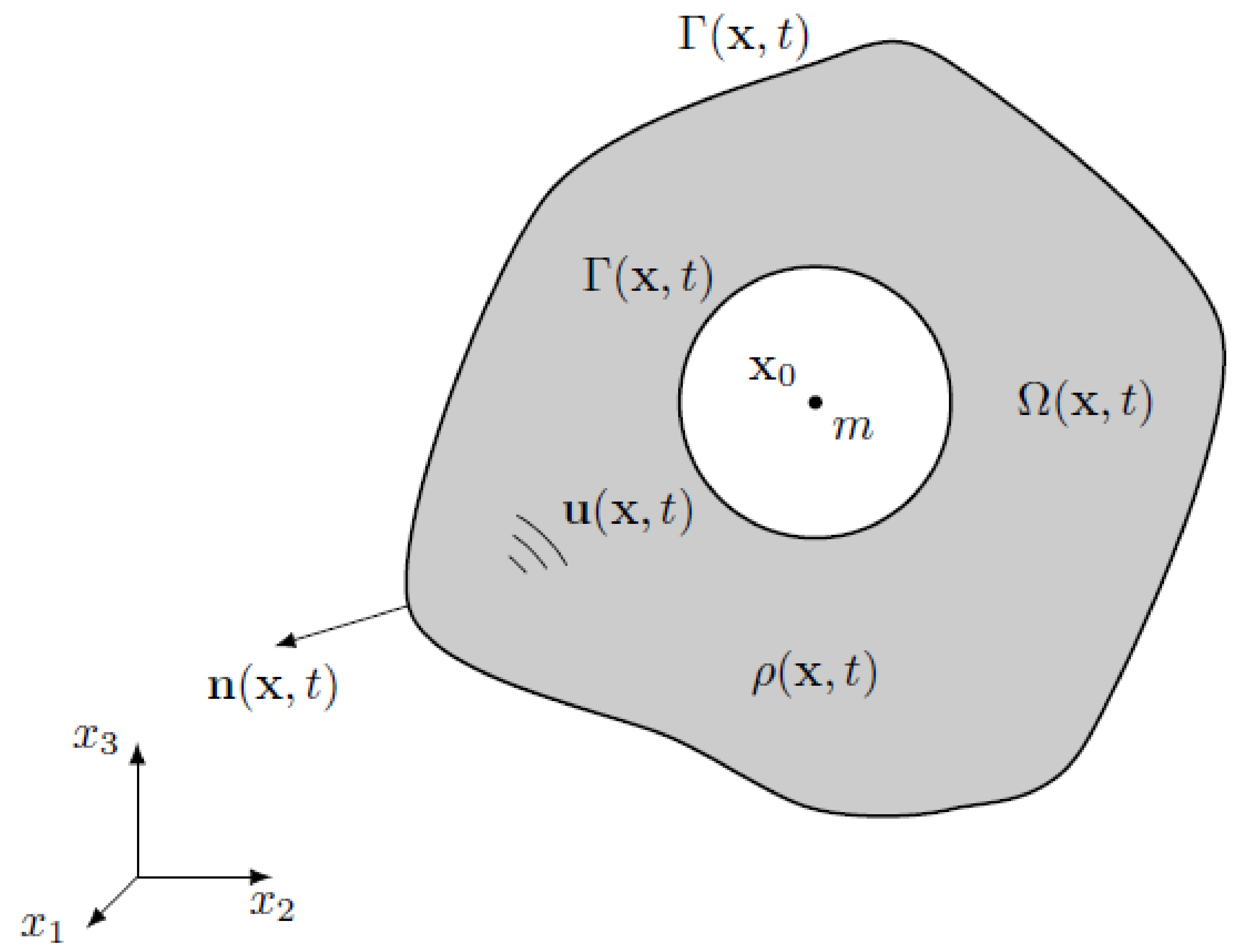
- Total Newtonian Noise inside a cavity with radius r_0 [Harms (2019)]:

$$\delta\hat{\mathbf{a}}_\mathrm{t}(\mathbf{x}_0) = 8\pi\rho G\left(\frac{j_1(k_\mathrm{p}r_0)}{k_\mathrm{p}r_0} - \frac{j_1(k_\mathrm{p}R)}{k_\mathrm{p}R}\right)\hat{\mathbf{u}}(\mathbf{x}_0) \tag{14}$$

  with $j_1(x)$ the spherical Bessel function of the first kind of order 1.

- Newtonian Noise for a full space and a cavity with radius r_0:

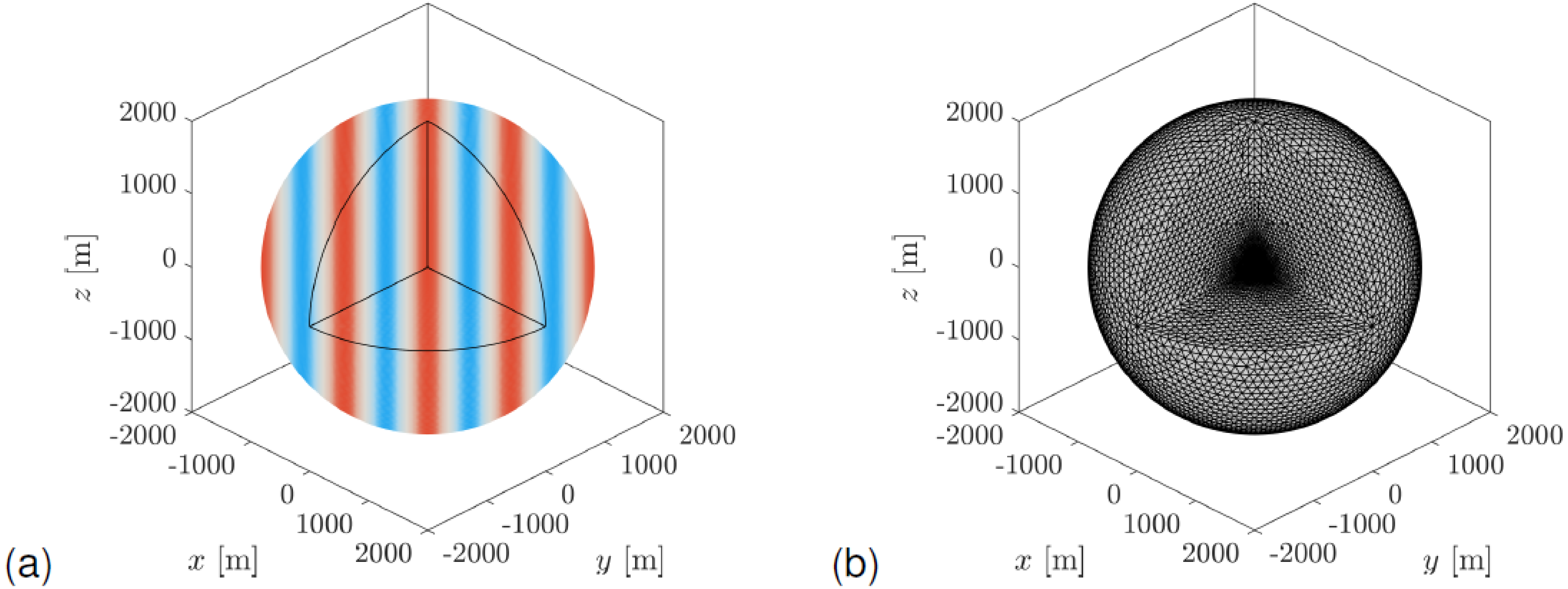
$$\lim_{R\to\infty} \delta\hat{\mathbf{a}}_\mathrm{t}(\mathbf{x}_0) = 8\pi\rho G\frac{j_1(k_\mathrm{p}r_0)}{k_\mathrm{p}r_0}\hat{\mathbf{u}}(\mathbf{x}_0) \tag{15}$$



Seismic Newtonian Noise

Validation: plane P-wave

- (a) $\hat{u}_x(\mathbf{x})$ at 5 Hz of a plane P-wave with unit amplitude propagating in the direction $\mathbf{e}_\mathrm{k} = \{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\}^\mathrm{T}$ in a full space with $C_\mathrm{s} = 2500\,\mathrm{m/s}$, $C_\mathrm{p} = 5000\,\mathrm{m/s}$, and $\rho = 2800\,\mathrm{kg/m^3}$;
  (b) FE mesh with $R = 2000$ m for NN calculation on a mirror in a cavity with centre $\mathbf{x}_0$ and radius $r_0 = 20$ m.

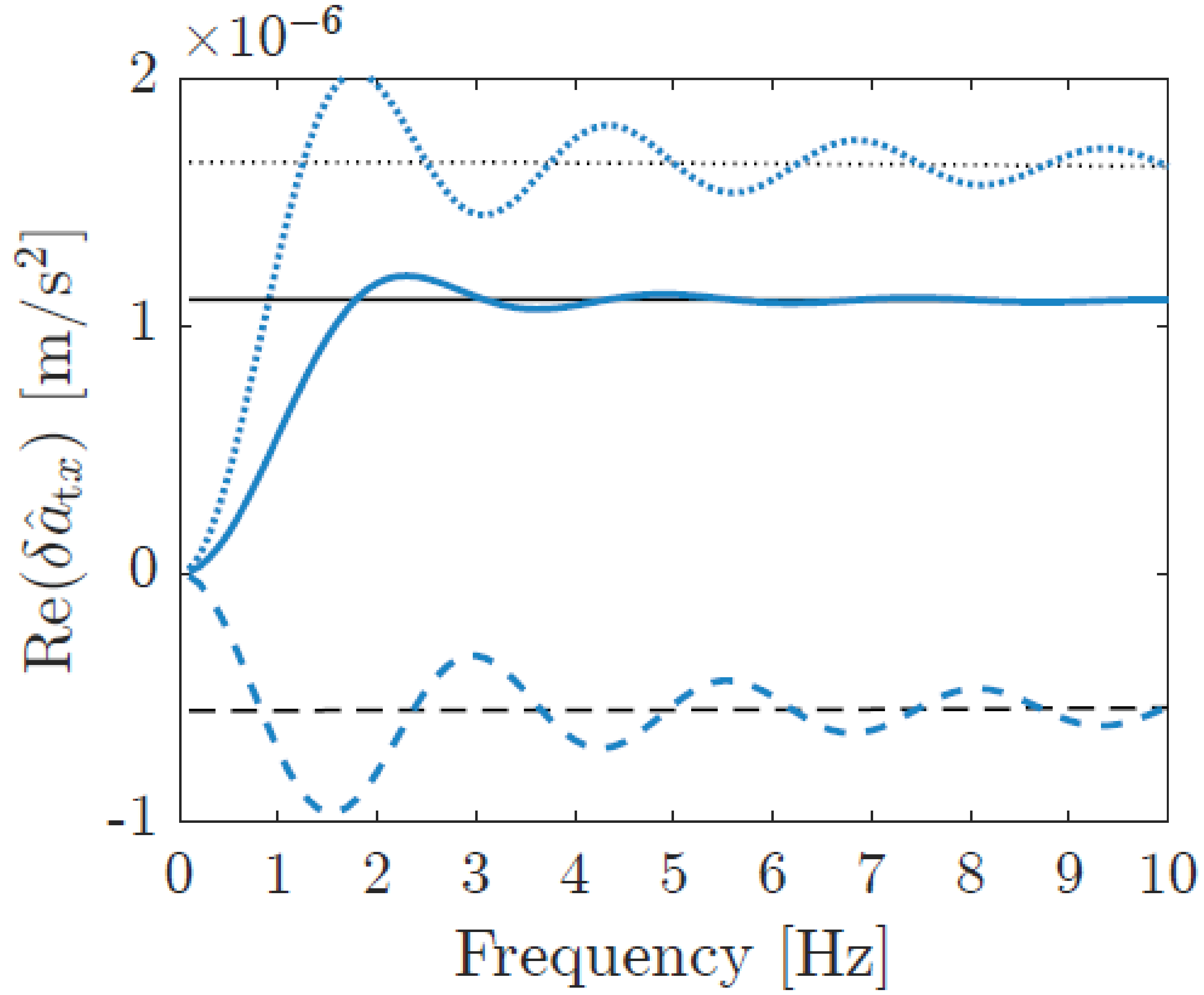


- Finite element mesh criteria:
  - **C1**: The domain radius R must be large enough to approximate the infinite medium solution.
  - **C2**: The element size l_e should be sufficiently small to well resolve the propagating waves.
  - **C3**: FE mesh refininement near cavity to capture the strong spatial variation of the kernel $1/\|\mathbf{x} - \mathbf{x}_0\|^3$.

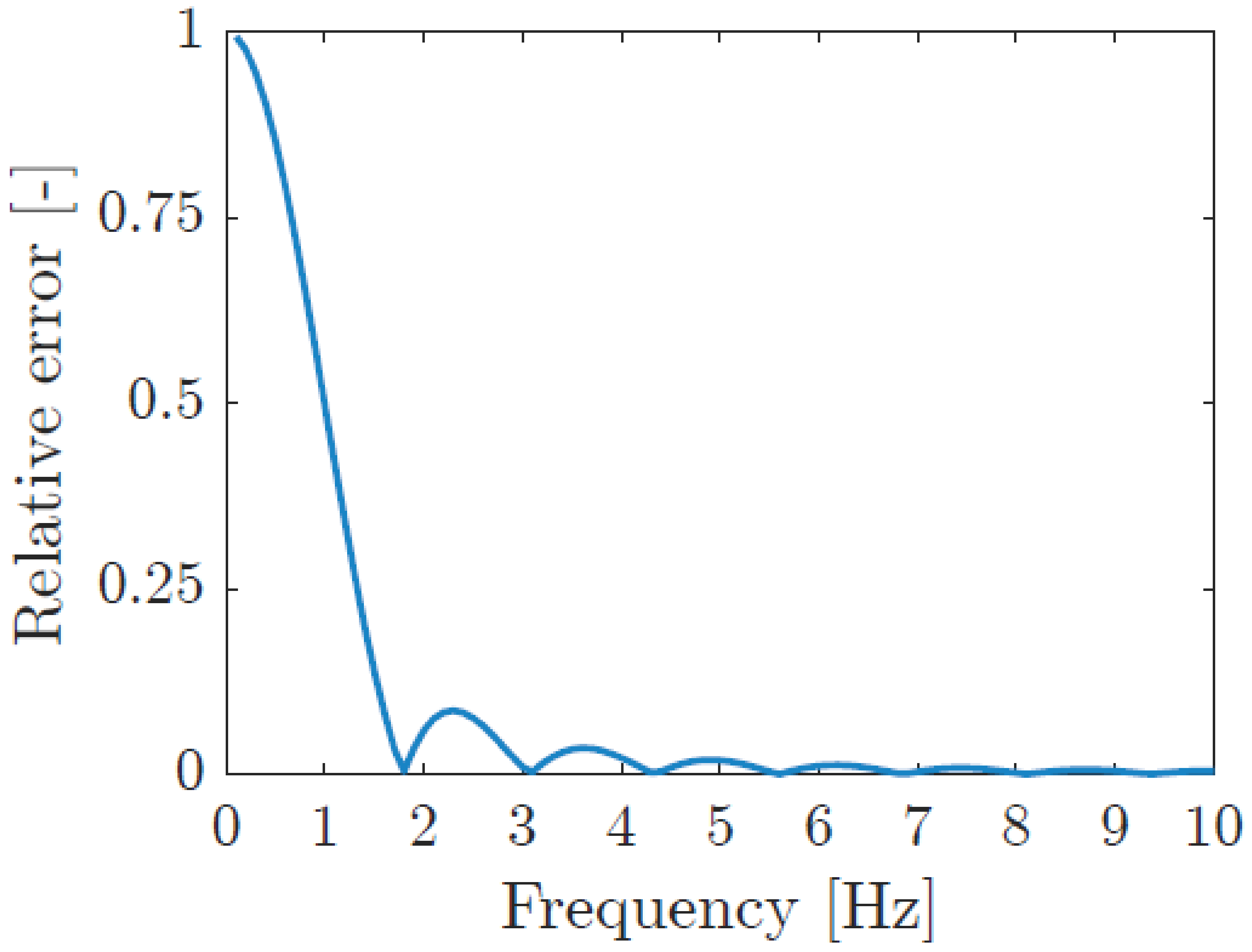
Seismic Newtonian Noise

Validation: plane P-wave

- (a) Real part of the total NN $\delta\hat{a}_{\mathrm{t}x}(\mathbf{x}_0, \omega)$ computed analytically (——) and with the numerical model (——), as well as the bulk (▪▪▪▪ and ▪▪▪▪) and surface contributions (- - - and - - -). (b) Relative error $\varepsilon_{\mathrm{t}x}(\omega)$ on the total NN.



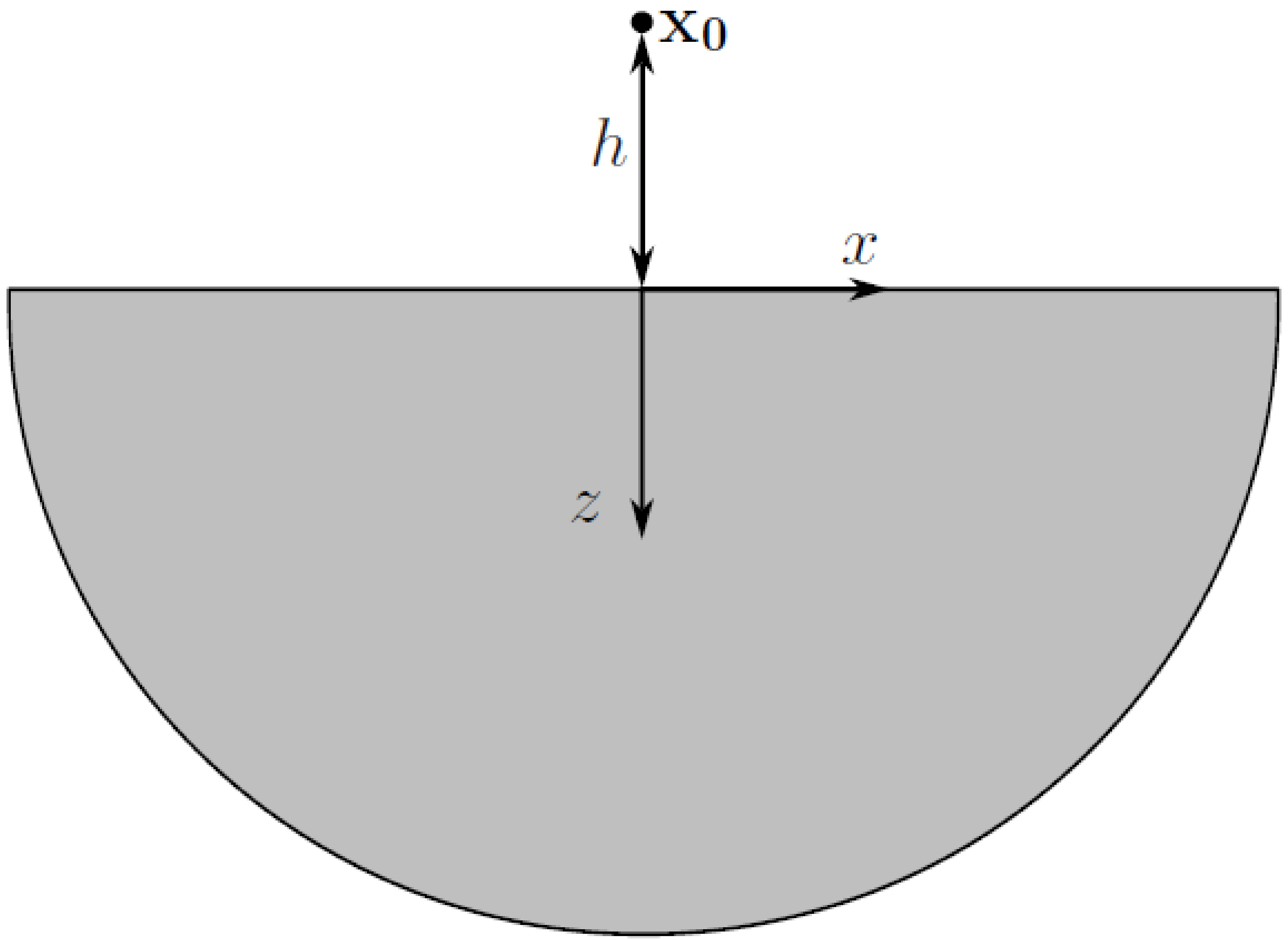
(a)



(b)

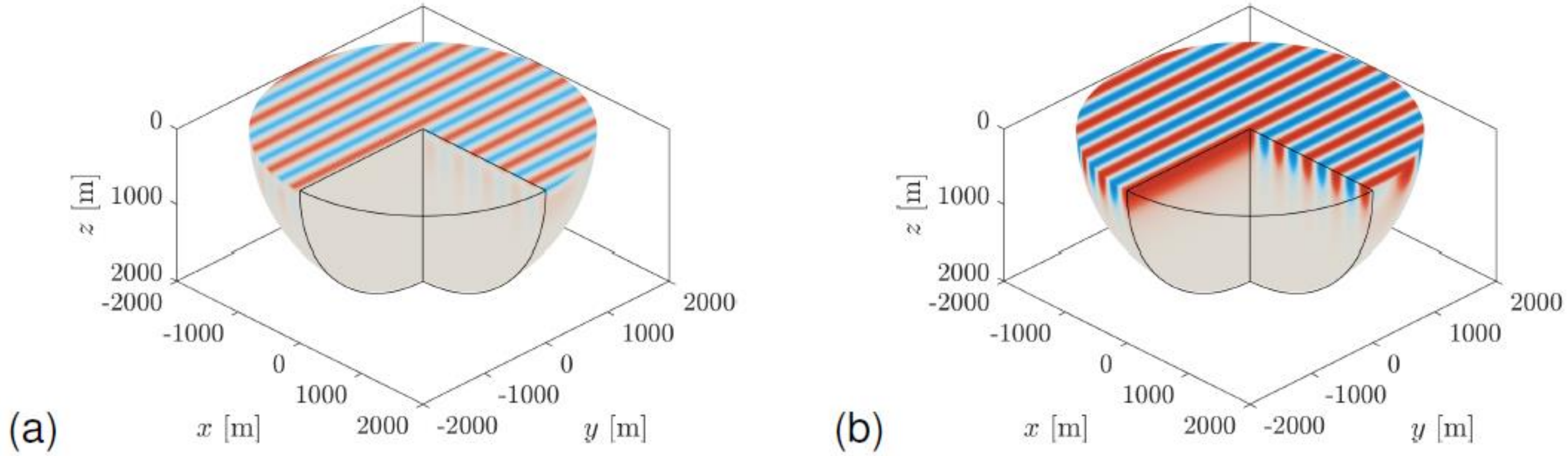
Seismic Newtonian Noise
Validation: Rayleigh wave

- Homogeneous halfspace with test mass at a height $h = 20$ m above the free surface.

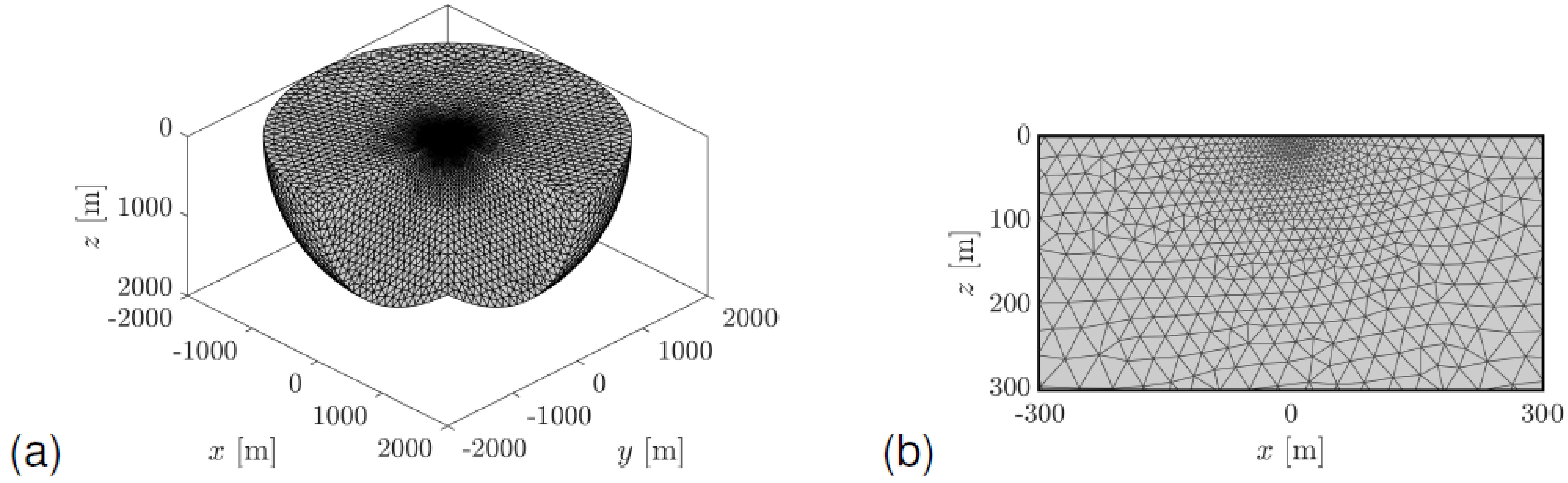


Seismic Newtonian Noise
Validation: Rayleigh wave

- Seismic displacements (a) $\hat{u}_x(\mathbf{x})$ and (b) $\hat{u}_z(\mathbf{x})$ at 5 Hz.



- (a) 3D FE mesh tailored for the homogeneous halfspace with test mass above surface and (b) 2D mesh visualization.



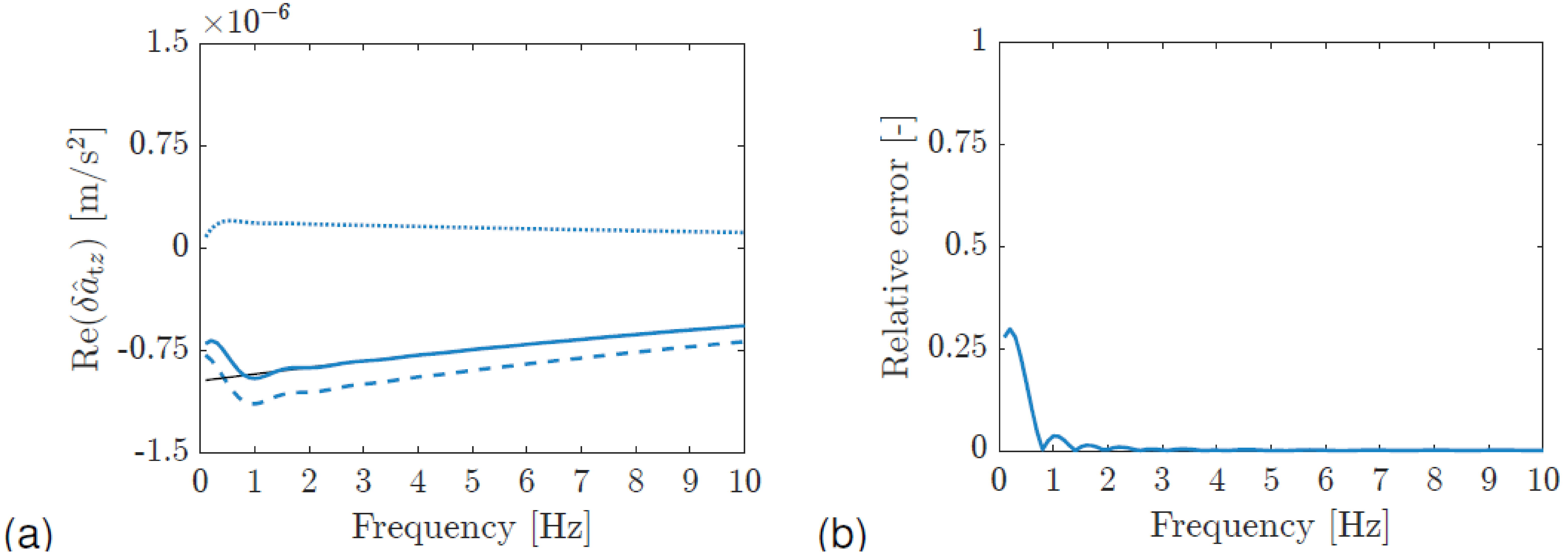
Seismic Newtonian Noise
Validation: Rayleigh wave

- Newtonian noise for a Rayleigh wave in a halfspace with test mass at a height h m above free surface [Harms (2019)]:

$$\delta\hat{\mathbf{a}}_{\mathrm{t}}(\mathbf{x}_0) = 2\pi G\rho\gamma \exp(-k_{\mathrm{R}}h)(\mathrm{i}\mathbf{e}_{\mathrm{k}} - \mathbf{e}_z)\exp(-\mathrm{i}\mathbf{k}_{\mathrm{k}}\cdot\mathbf{x}_0) \quad (16)$$

where $\gamma = \frac{k_{\mathrm{R}}(1-\sqrt{k_{z\mathrm{p}}/k_{z\mathrm{s}}})}{\mathrm{i}k_{z\mathrm{p}}-k_{\mathrm{R}}\sqrt{k_{z\mathrm{p}}/k_{z\mathrm{s}}}}$.

- (a) Real part of the total Newtonian noise $\delta\hat{a}_{\mathrm{t}z}(\mathbf{x}_0,\omega)$ for a Rayleigh wave in a halfspace with a test mass at a height $h = 20$ m above free surface computed analytically (▬▬) and with the numerical model (▬▬), as well as the bulk (▪▪▪▪ and ▪▪▪▪) and surface contributions (▬ ▬ ▬ and ▬ ▬ ▬). (b) Relative error $\varepsilon_{\mathrm{t}z}(\omega)$ on the total Newtonian noise.



ANNA Workflow

Newtonian noise computation

1. FE mesh

- Generated in Gmsh.
- Enforces the three mesh criteria.
- Supports linear and quadratic tetrahedral and hexahedral elements.

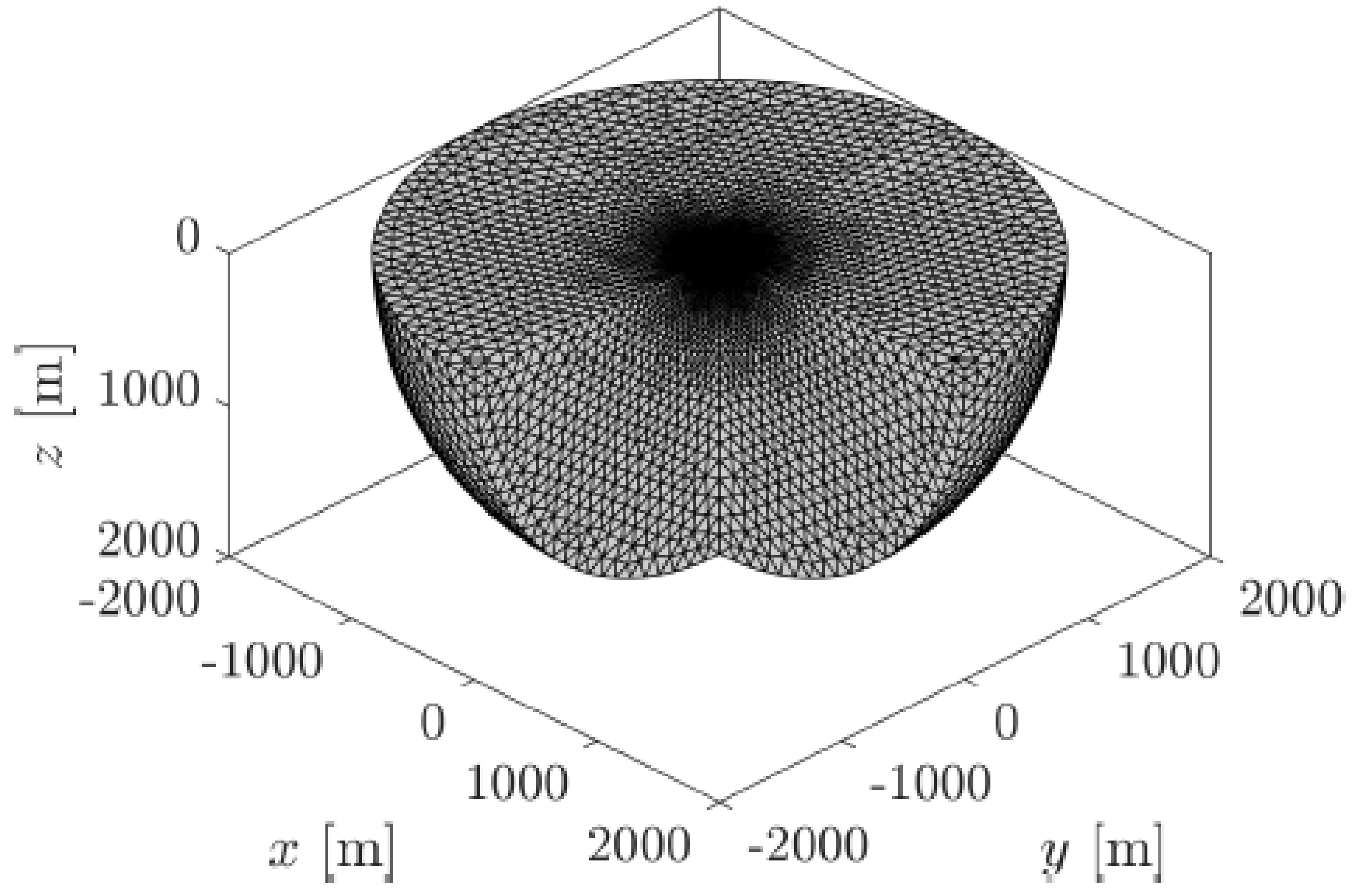
2. Seismic wave field

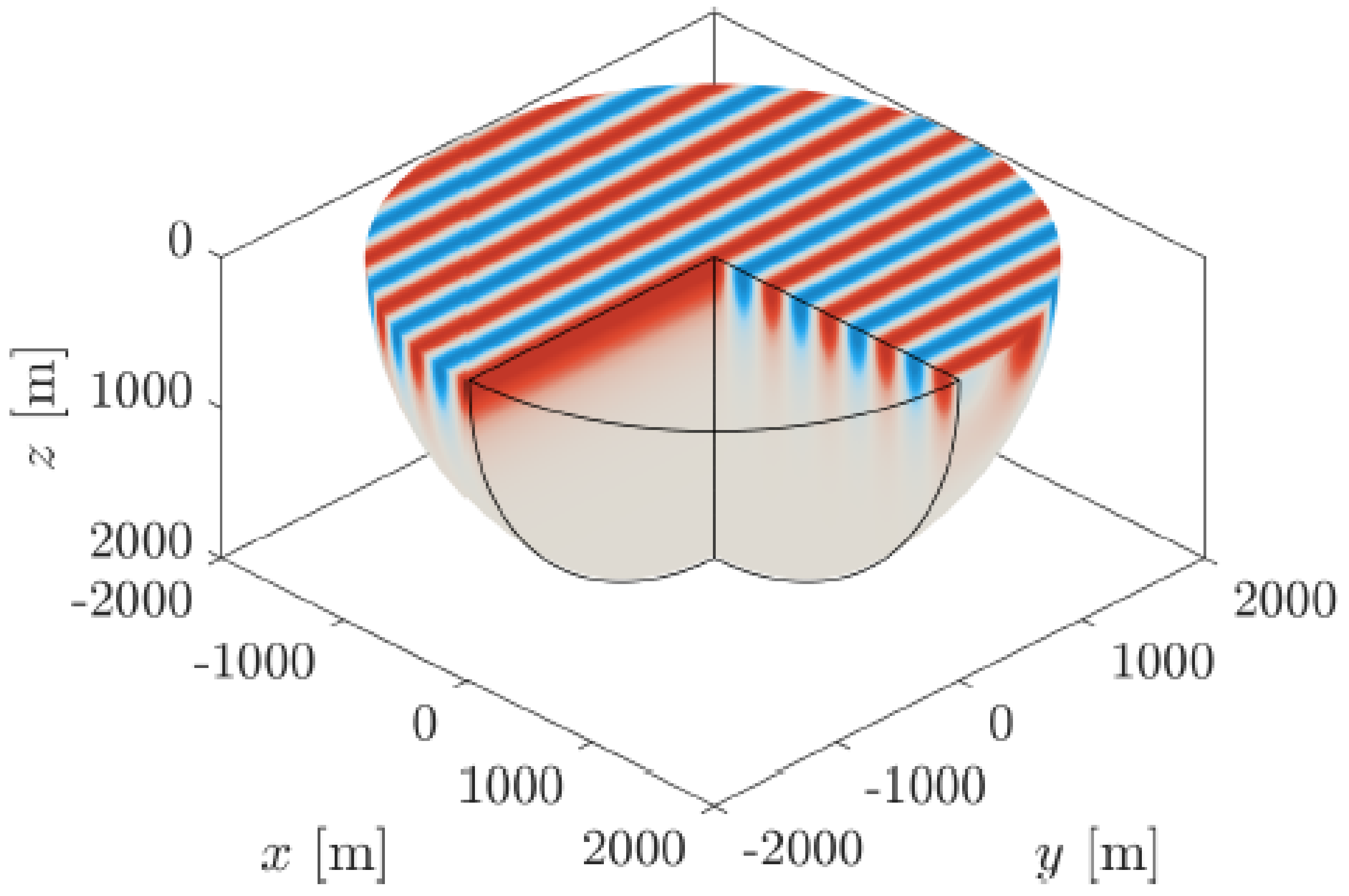
- Computed on a coarser mesh and interpolated, or
- Evaluated directly at FE mesh nodes.
- Stored as $\underline{\hat{\mathbf{u}}}(\omega)$ $(3N \times 1)$.

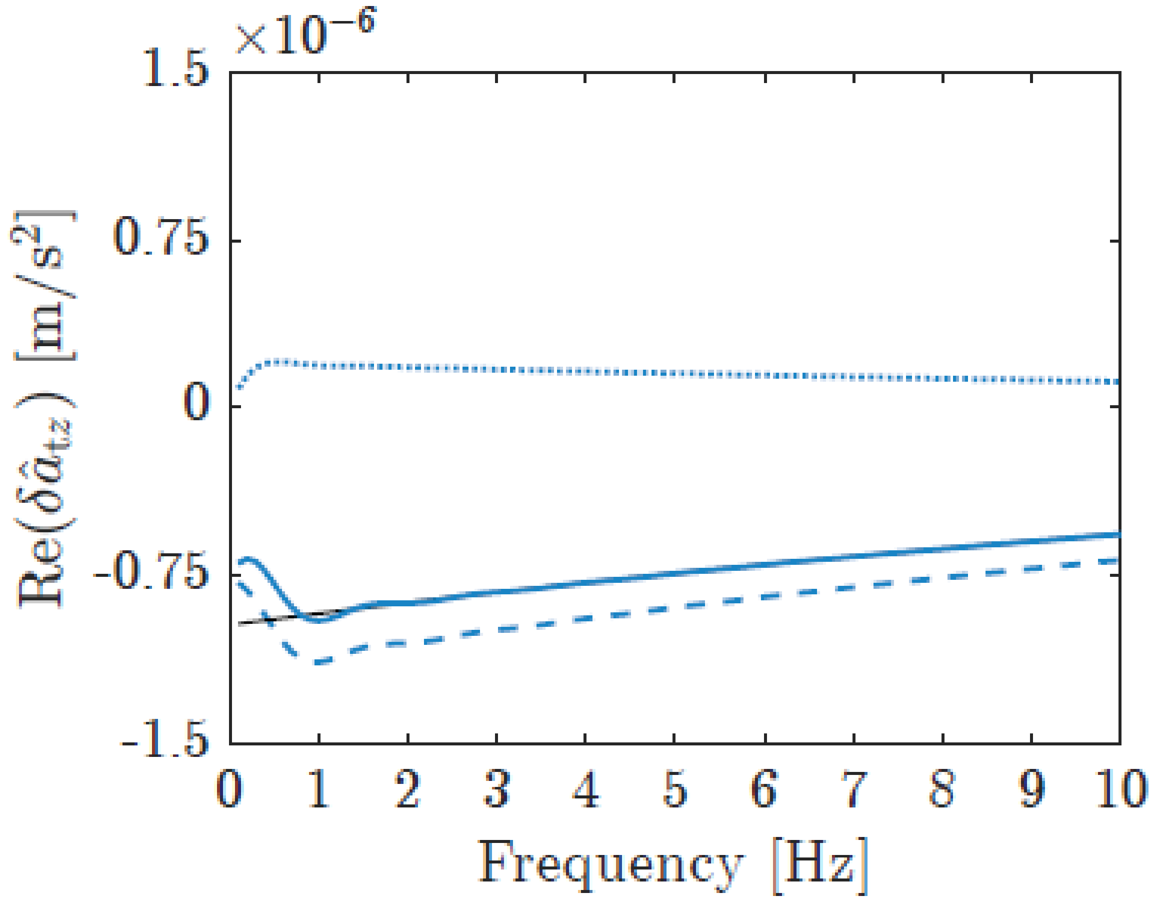
3. NN computation

- Input: FE mesh, ρ, $\mathbf{x}_0$
- Assemble NN matrices (`asmnn.m`): $\mathbf{A}_\mathrm{t}$, $\mathbf{A}_\mathrm{b}$, $\mathbf{A}_\mathrm{s}$ $(3 \times 3N)$
- Compute NN acceleration:

$$\delta\hat{\mathbf{a}}_\mathrm{i}(\omega) = \mathbf{A}_\mathrm{i}\,\underline{\hat{\mathbf{u}}}(\omega)$$





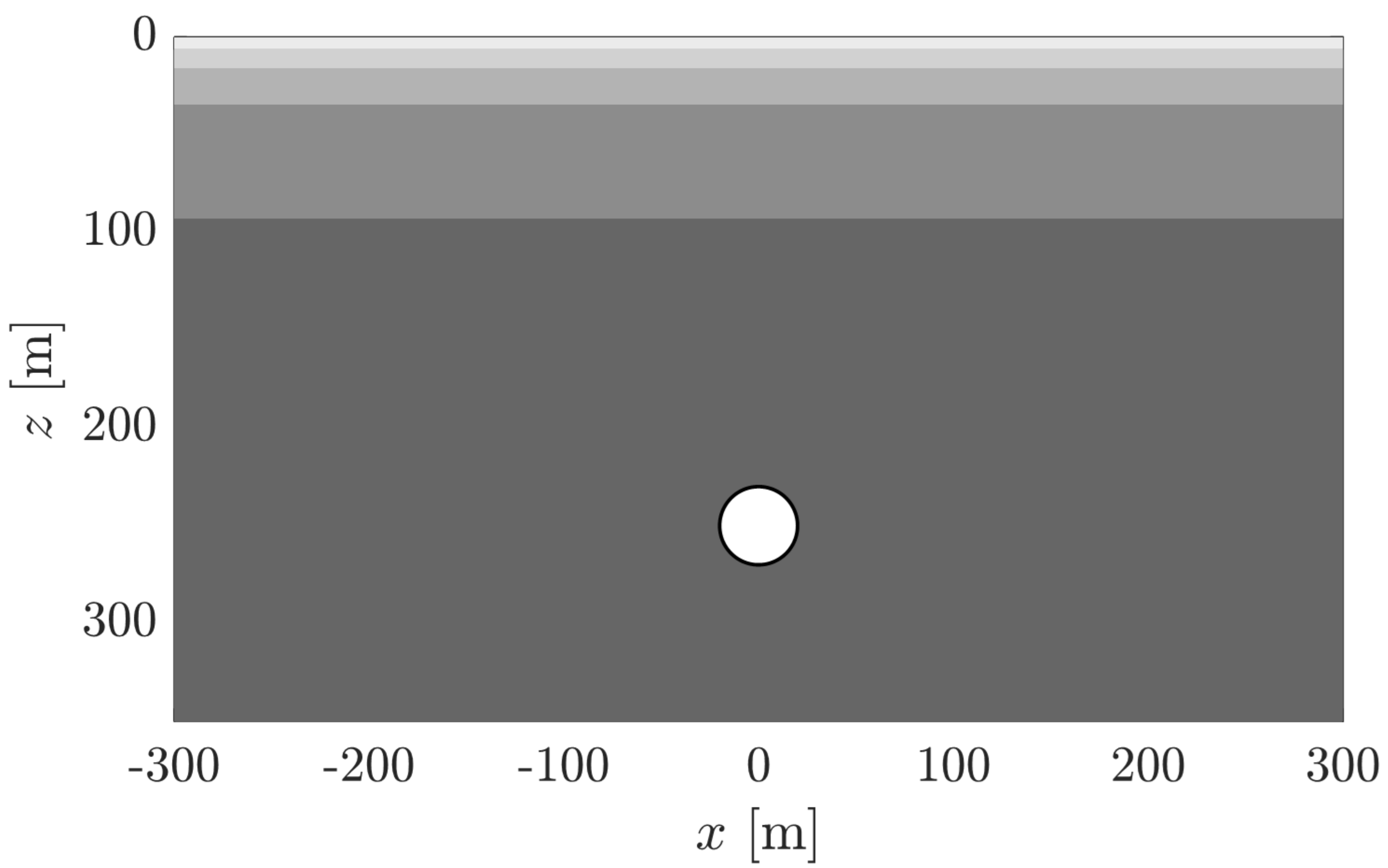


Layered geology

- Dynamic soil characteristics:

Layer	h [m]	C_s [m/s]	C_p [m/s]	ρ [kg/m^3]
1	5.7	165	385	1950
2	10.2	270	445	2250
3	18.9	335	685	2500
4	58.2	1240	2810	2800
5	∞	2430	4050	2800

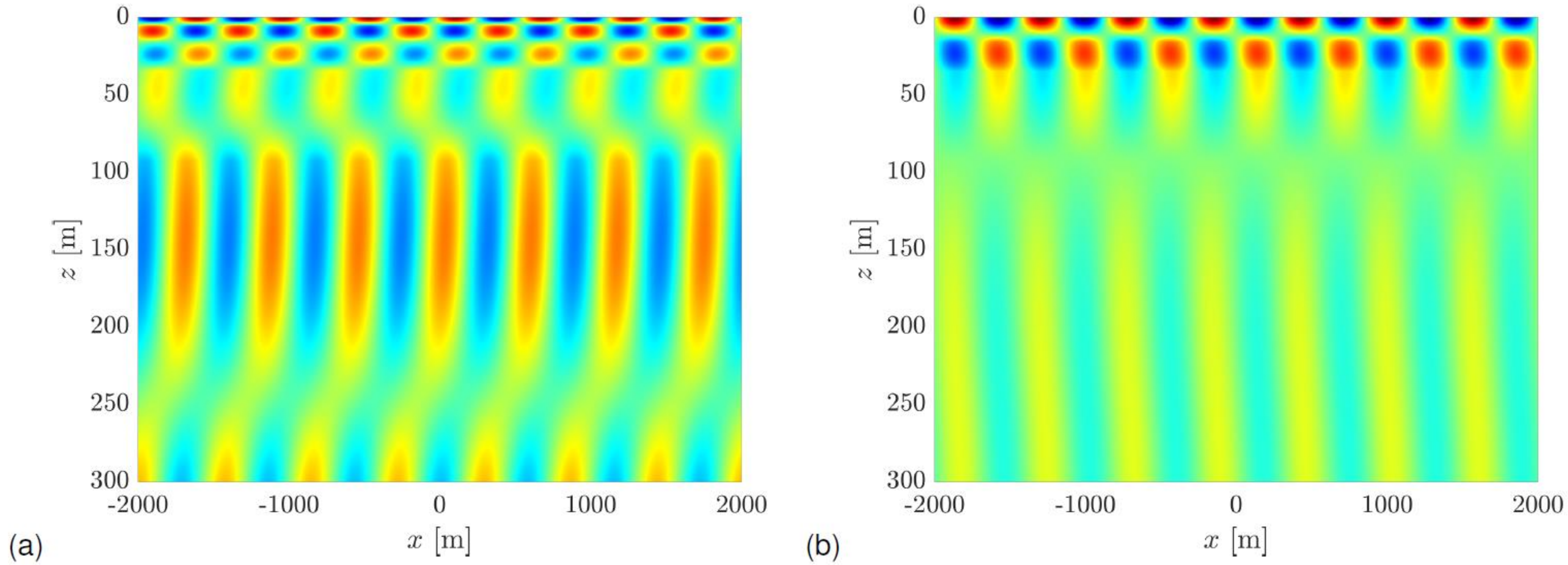
- Spherical cavern of radius $r_0 = 20$ m at 250 m depth.



Layered geology

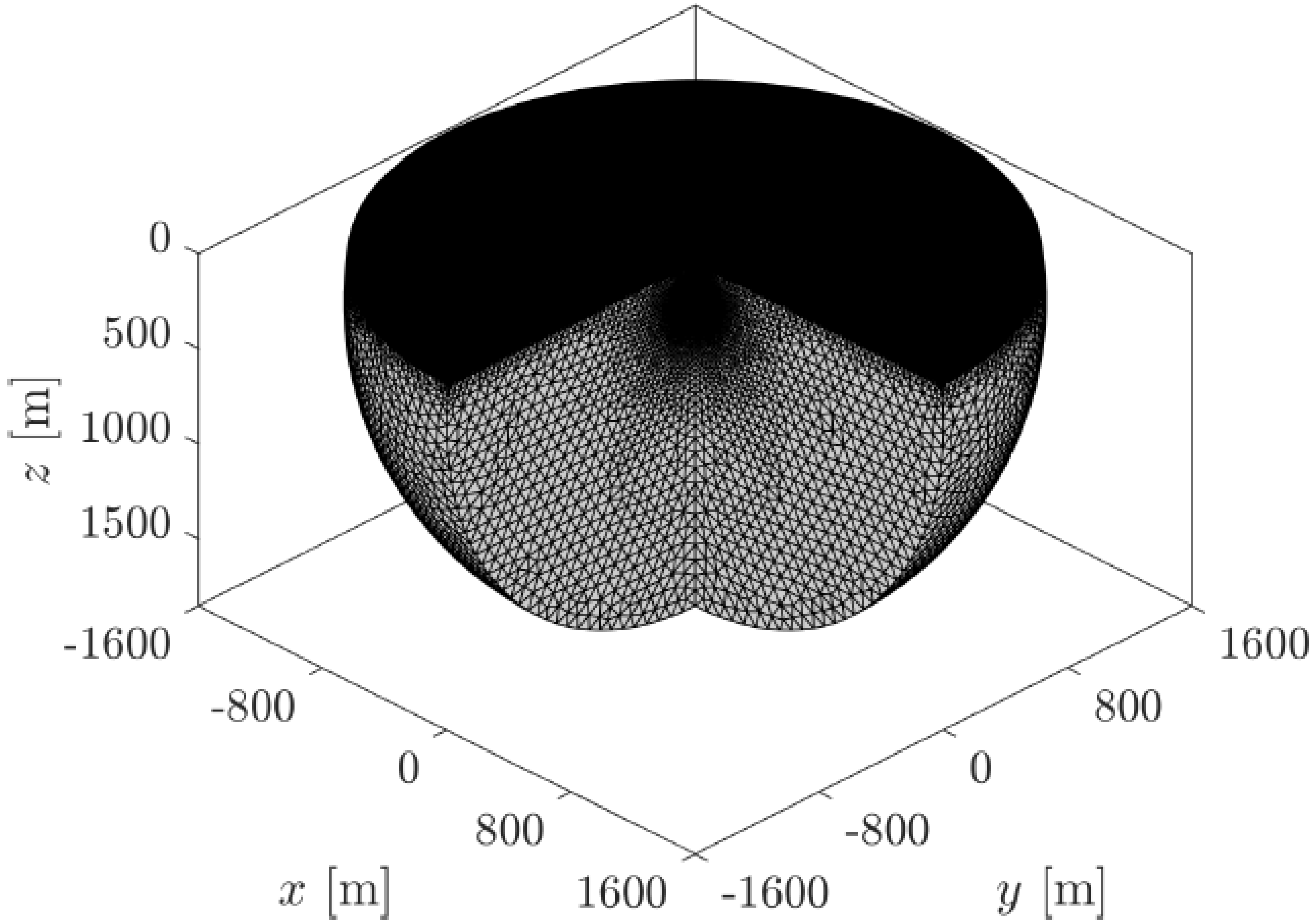
Seismic wave field

- Illustrative example: (a) $\hat{u}_x(\mathbf{x})$ and (b) $\hat{u}_z(\mathbf{x})$ at 10 Hz of a plane P-wave with incident angle $\phi = 45°$ in the layered halfspace.

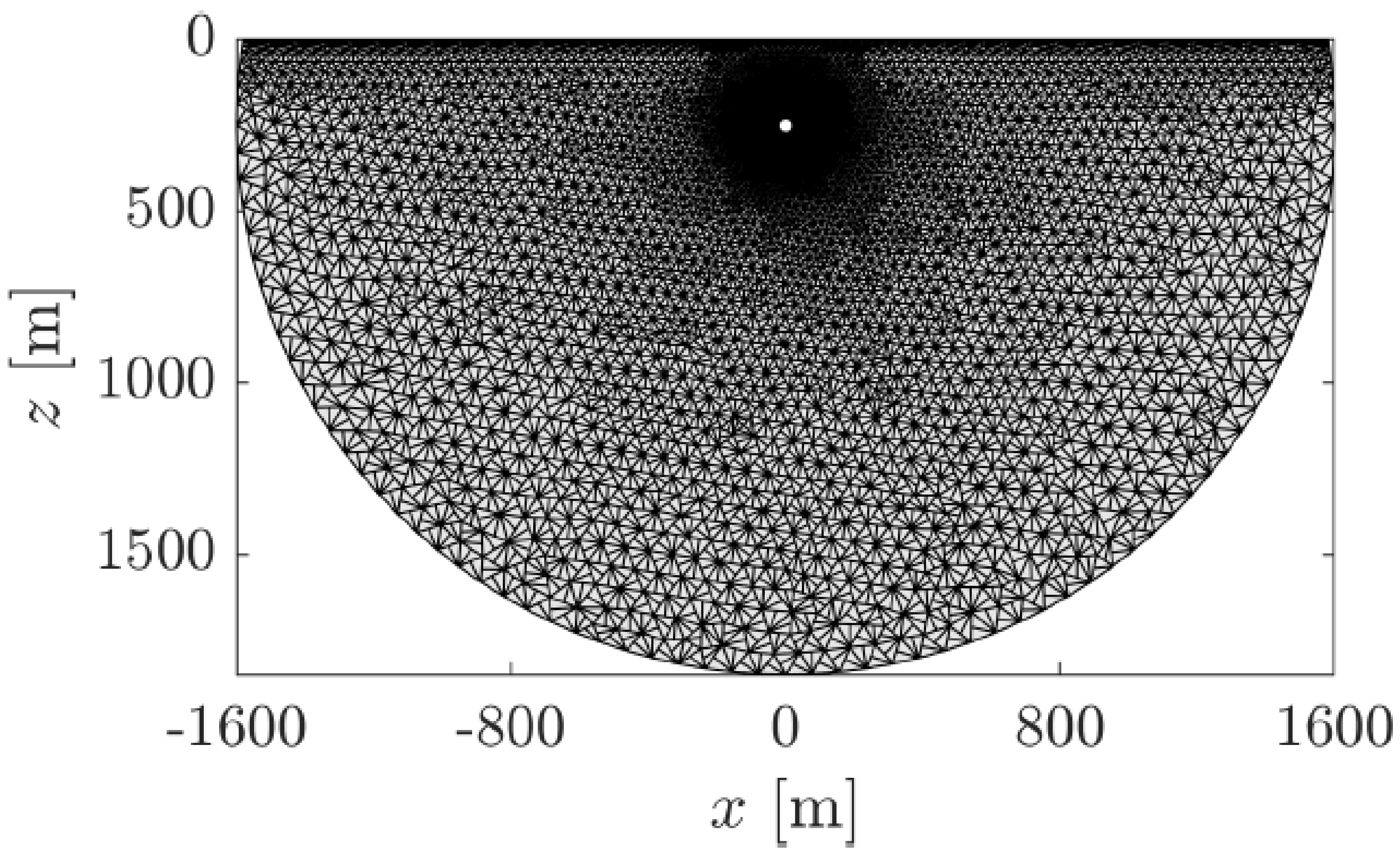


Layered geology
Mesh design strategy

- Finite element mesh criteria:
  - **C1**: Domain size is set by the largest wavelength, i.e. by $f_{\min}$ and the maximum dilatational wave speed C_{p}.
  - **C2**: Element size is set by the smallest wavelength, i.e. by $f_{\max}$ and the shear wave speed C_{s}.
  - **C3**: Additional refinement is imposed near the cavity.
- (a) 3D FE mesh and (b) 2D mesh visualization tailored for 5 Hz.



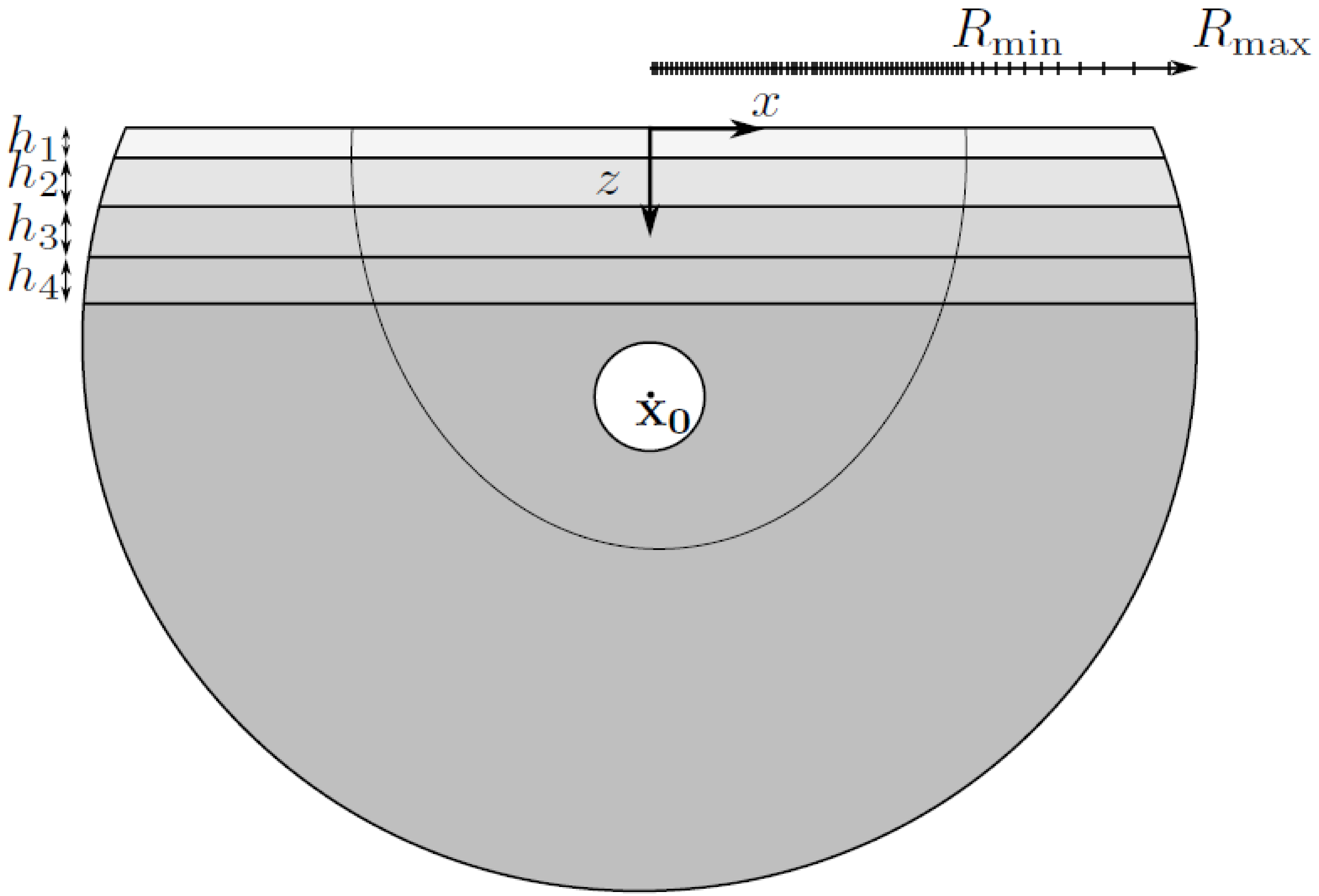
(a)



(b)

Layered geology
Mesh design strategy

- For a tailored mesh in $[f_{\min}, f_{\max}]$ the required domain size varies from $R_{\min}$ for $f_{\max}$, to $R_{\max}$ for $f_{\min}$.
- Both scales are combined in a single distance-dependent discretization.

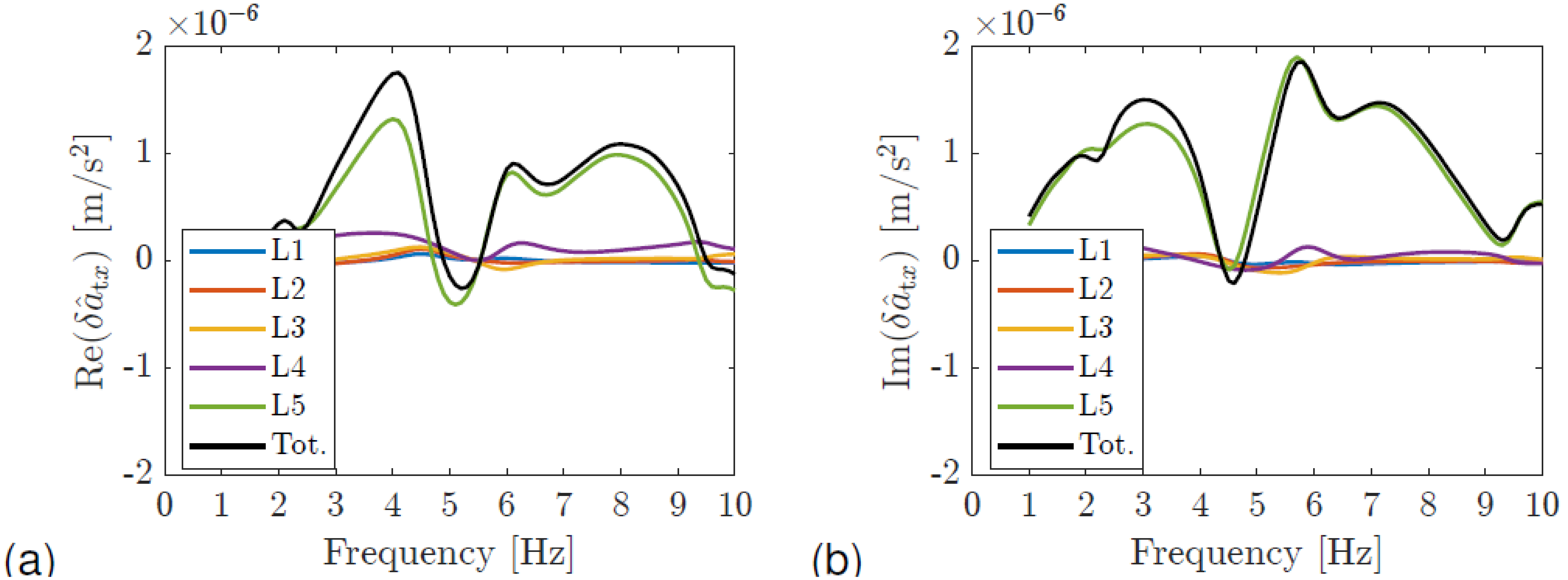


Layered geology
Newtonian noise

- The NN operator is assembled separately for each layer mesh.
- The contribution of each layer to the total Newtonian noise is quantified

$$\delta\hat{\mathbf{a}}_{\mathrm{t}}(\omega) = \sum_{\ell=1}^{n_{\mathrm{L}}} \mathbf{A}_{\mathrm{t}}^{(\ell)}\, \underline{\hat{\mathbf{u}}}^{(\ell)}(\omega)$$

- (a) Real part and (b) imaginary part of the contribution of each layer to the total NN $\delta\hat{a}_{\mathrm{t}x}(\mathbf{x}_0, \omega)$, due to a plane P-wave with incident angle $\phi = 45°$.



Summary and Outlook

- The ANNA toolbox efficiently computes Newtonian Noise from 3D seismic wave fields using a finite element formulation.
- The implementation was verified against analytical solutions and is available in MATLAB and Python.
- The methodology has been extended to account for layered geology with a consistent computational strategy.

- The approach will be applied to 3D geology model once available.

Acknowledgements

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  - the FWO-IRI project I002123N "Essential Technologies for the Einstein Telescope" (2023-2024);
  - the FWO-IRI project I000725N "ET-TECH: Empowering Tomorrow's Technological Horizons for Einstein Telescope" (2025);
  - the FWO Research Collaboration "Modelling external vibration sources" (2026).

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Cavern Wall Newtonian Noise Coupling

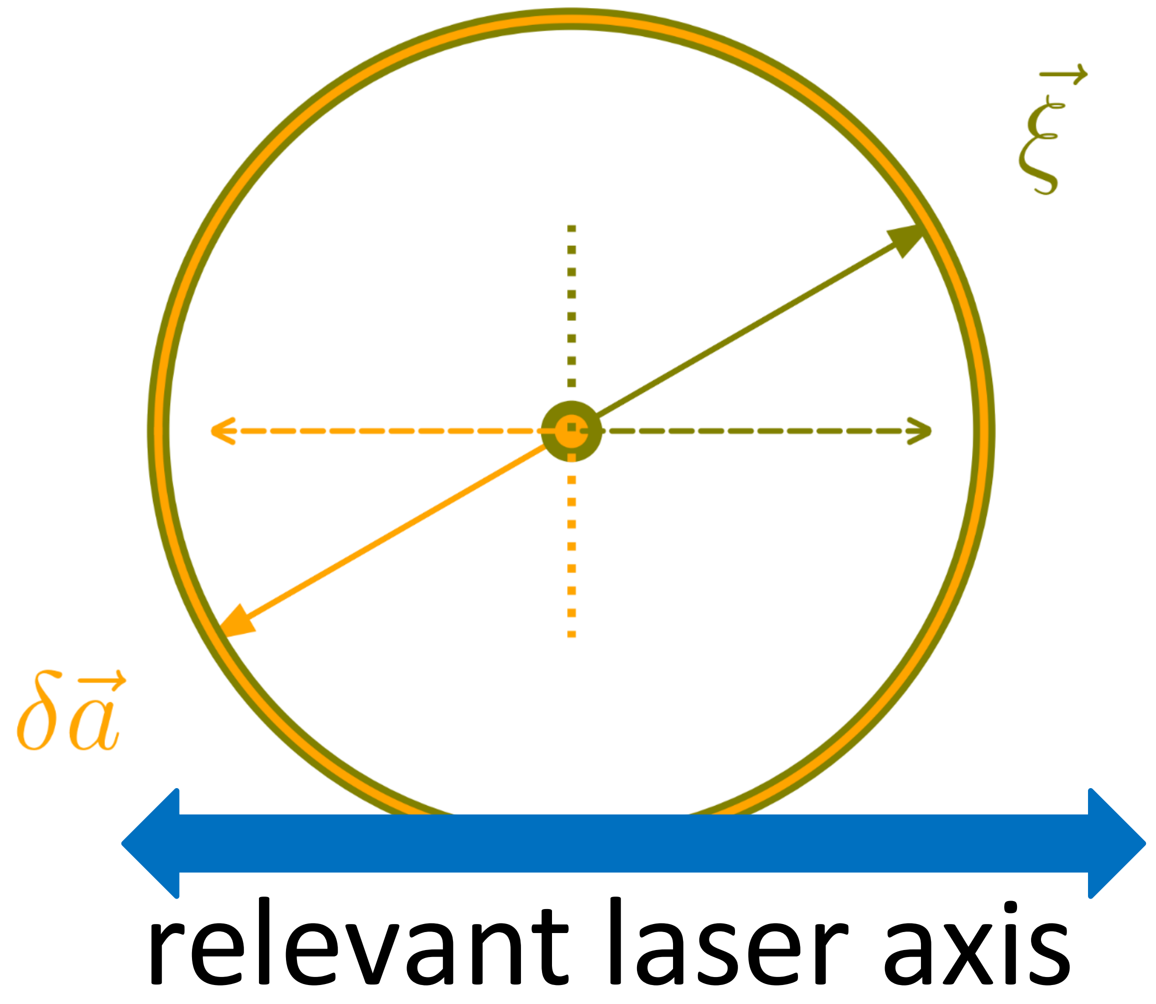
For **small caverns** (with respect to the seismic wavelenght) one can derive **analytical approximations for NN coupling** of mirror acceleration and seismic displacement:

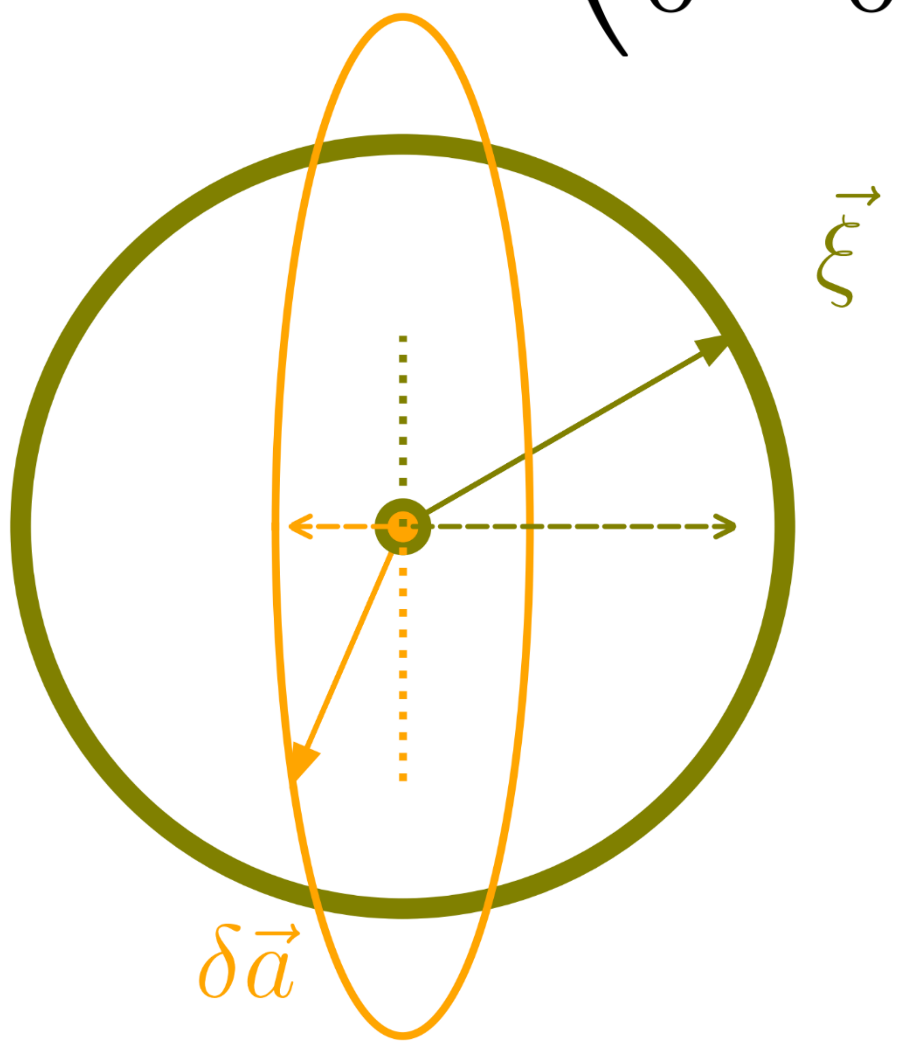
$$\delta\vec{a}^{\mathrm{SCA}}_{\mathrm{low}}(\vec{r}_0,t) = -\frac{4\pi}{3}G\rho_0\underline{\underline{C}}^{\mathrm{SCA}}\vec{\xi}(\vec{r}_0,t)$$

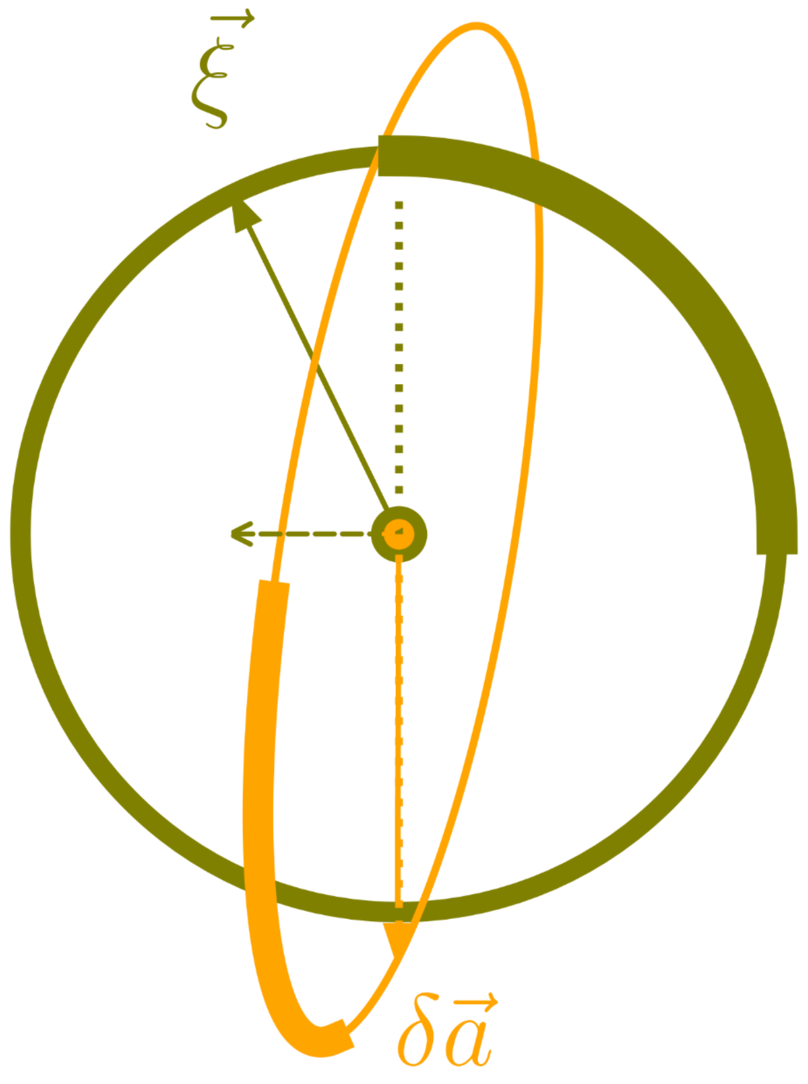
$$-\underline{\underline{C}}^{\mathrm{SCA}} = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$-\underline{\underline{C}}^{\mathrm{SCA}} = -\begin{pmatrix} \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$$-\underline{\underline{C}}^{\mathrm{SCA}} = -\begin{pmatrix} \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{3} \end{pmatrix}$$

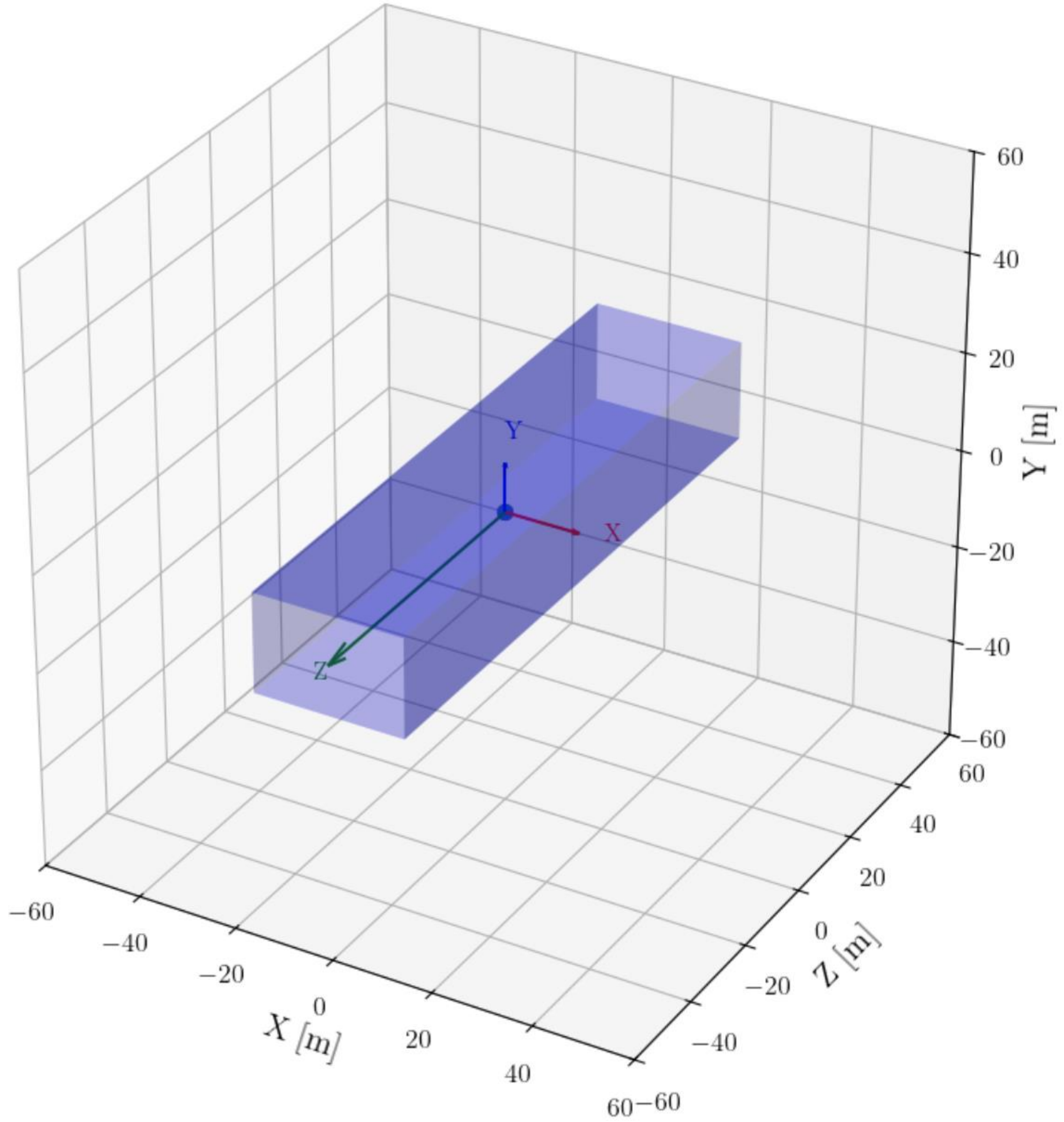






Cavern Wall Newtonian Noise Coupling

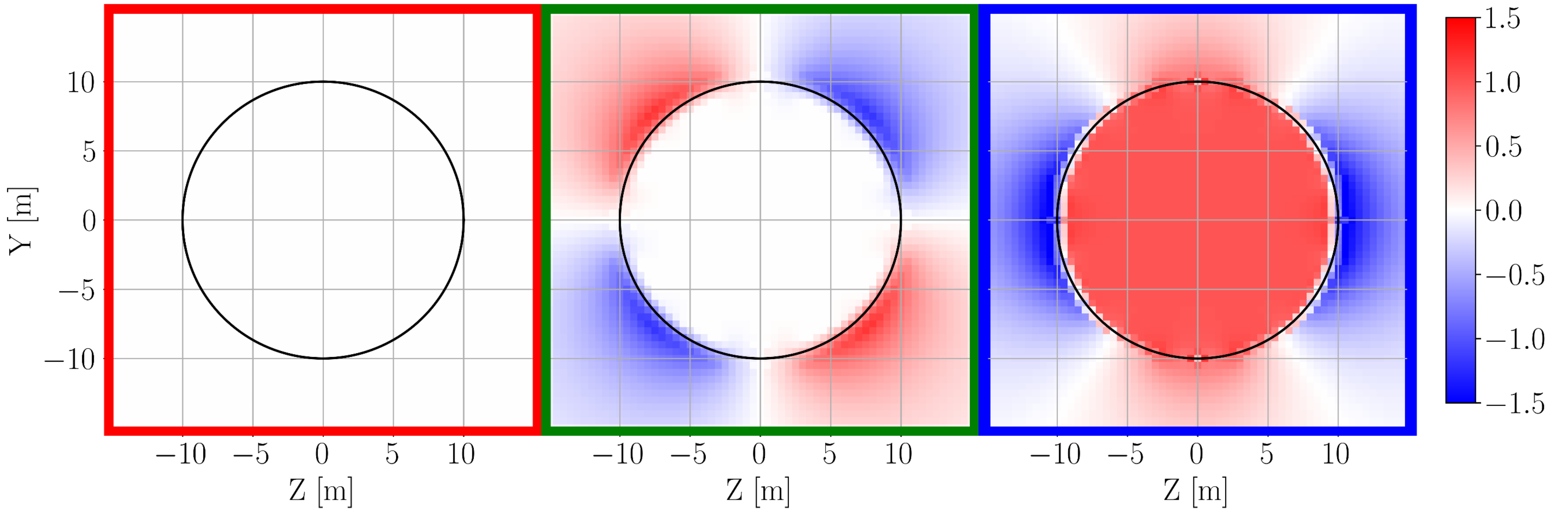
$$\underline{\underline{C}}^{\mathrm{SCA}} = \begin{bmatrix} c_{xx} & c_{xy} & c_{xz} \\ c_{xy} & c_{yy} & c_{yz} \\ c_{xz} & c_{yz} & c_{zz} \end{bmatrix}$$



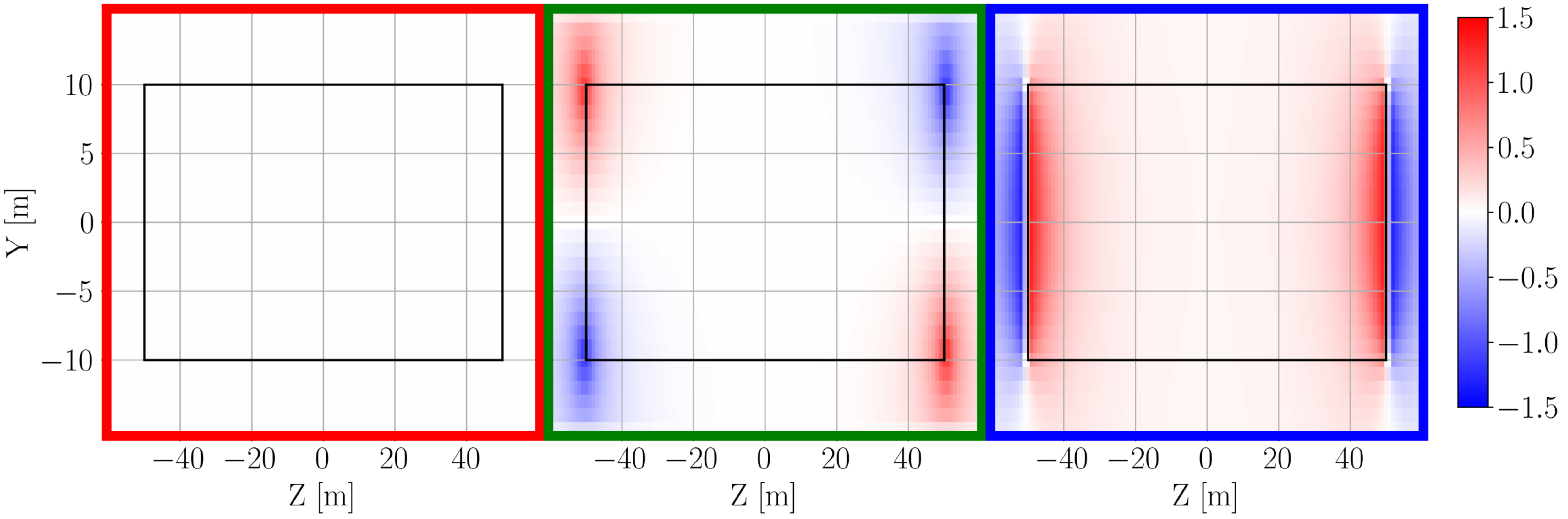
Coupling elements inside different caverns

$$\underline{\underline{C}}^{\mathrm{SCA}} = \begin{bmatrix} c_{xx} & c_{xy} & c_{xz} \\ c_{xy} & c_{yy} & c_{yz} \\ c_{xz} & c_{yz} & c_{zz} \end{bmatrix}$$

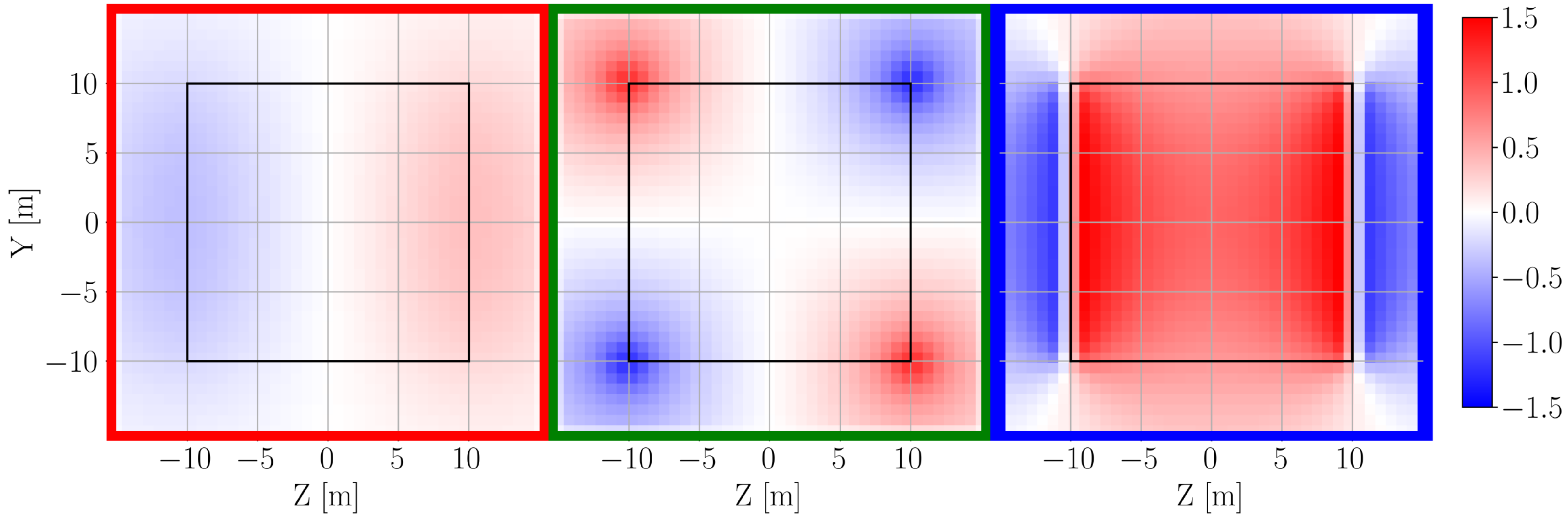
a) Sphere R=10m [cut through cavern center]



c) elongated cylinder L=100m [cut at center X]



b) cube [cut through cavern off center (-5m)]

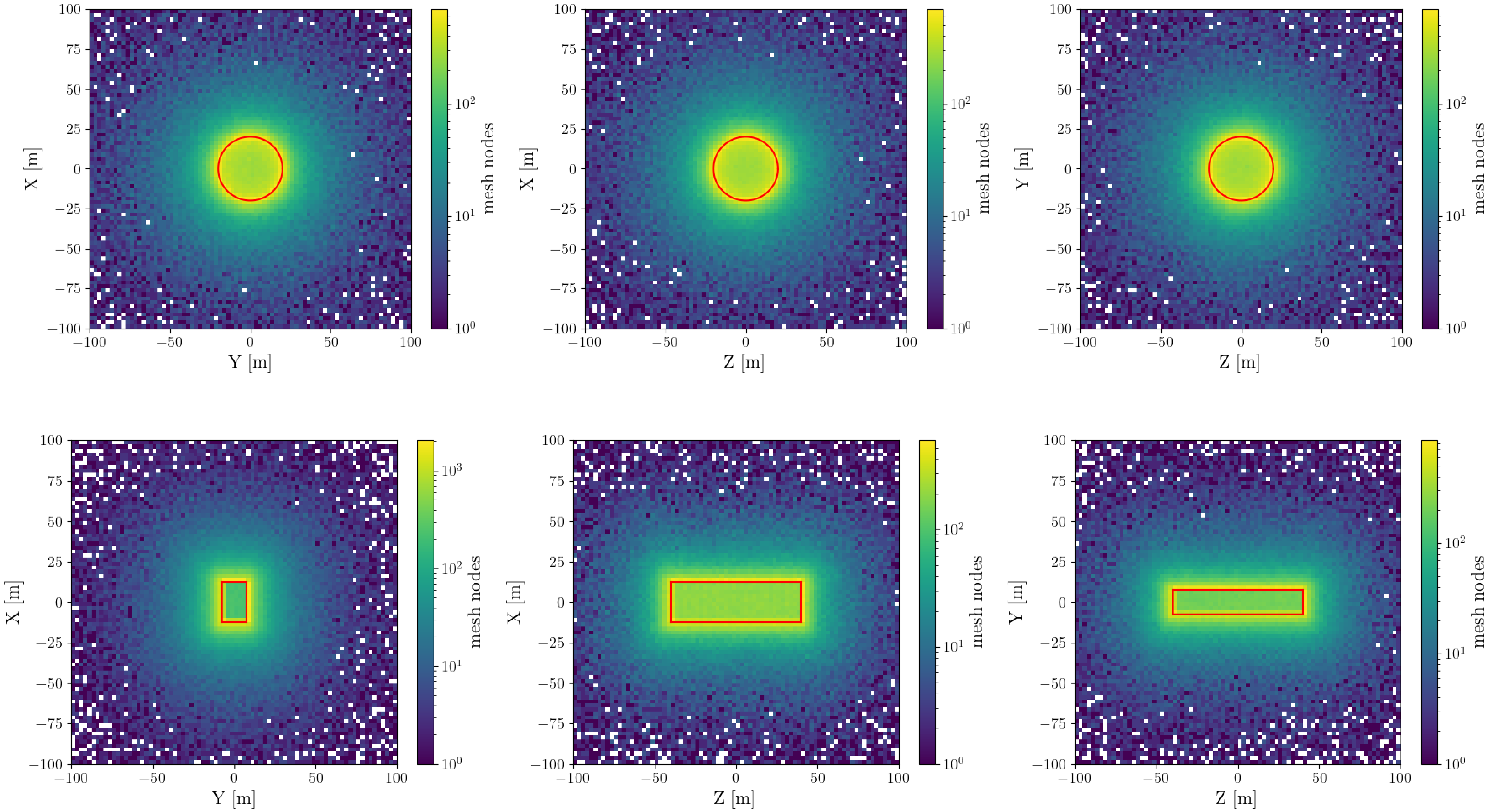


ANNA mesh examples

→ Validation of cavern wall effects in ANNA:

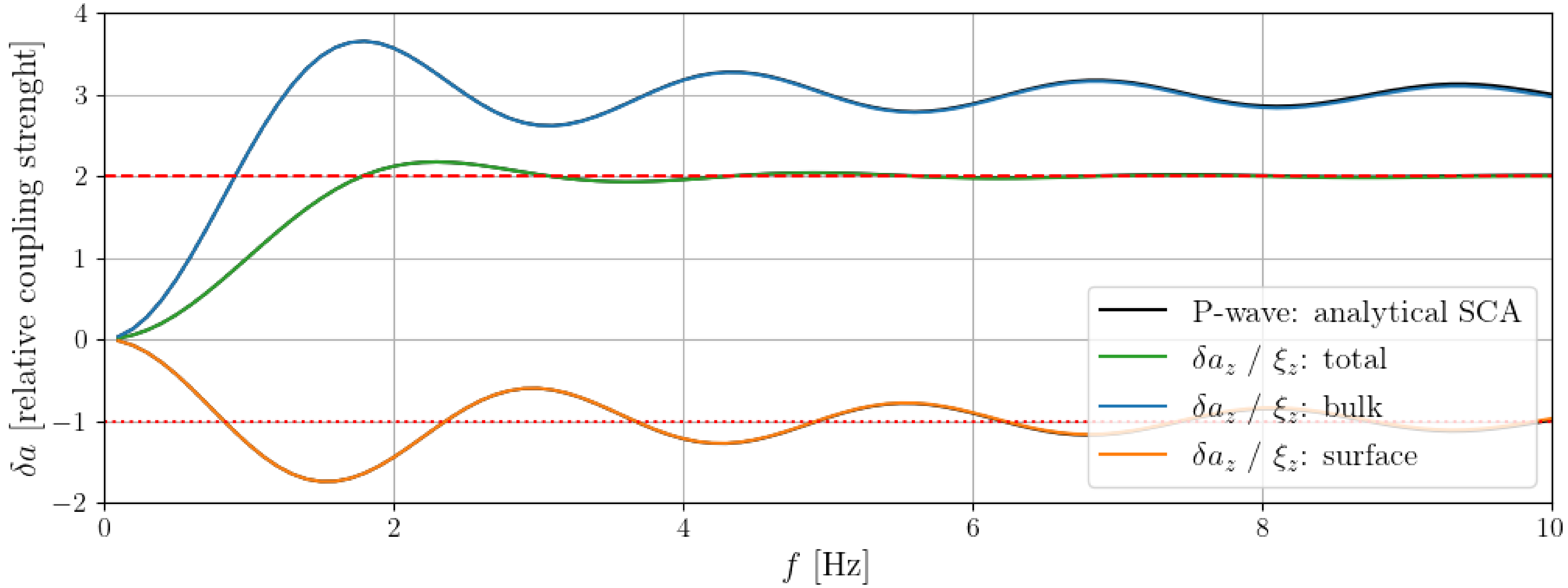
1. embed in homogenous space (R=2000m)
2. choose simple cavern geometries
3. choose simple seismic fields (P- and S- waves)

→Analytical results for comparison available

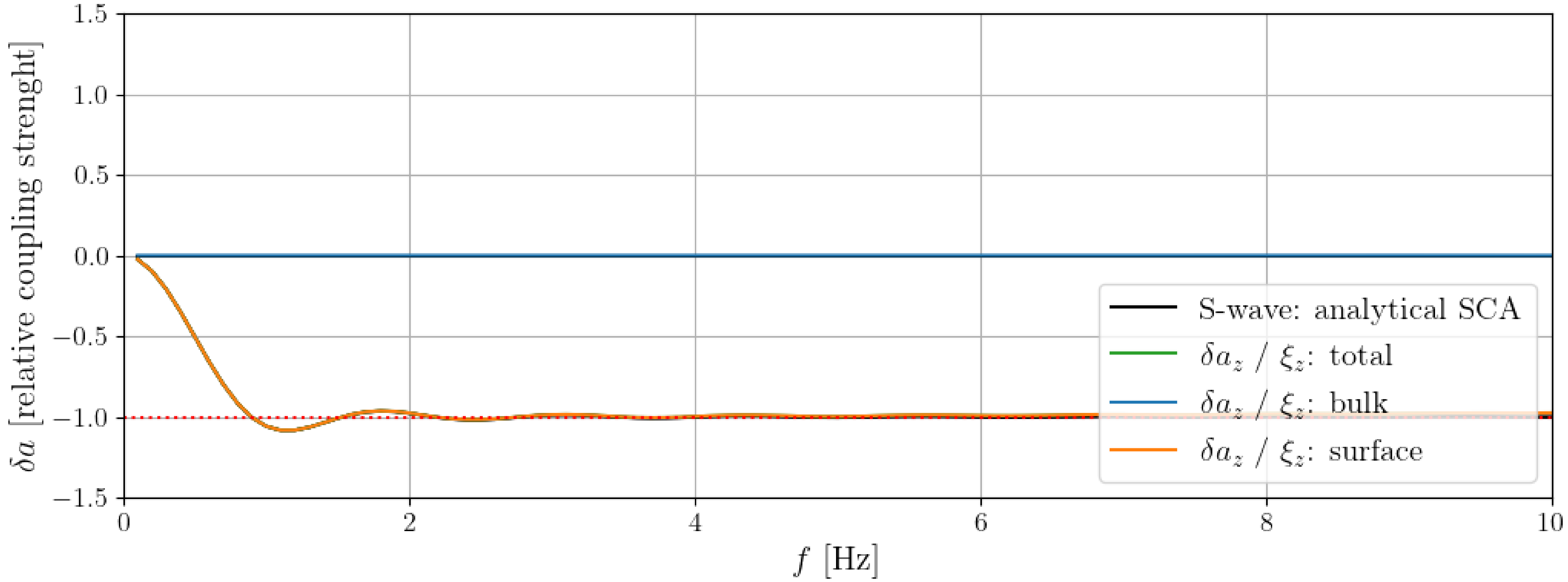


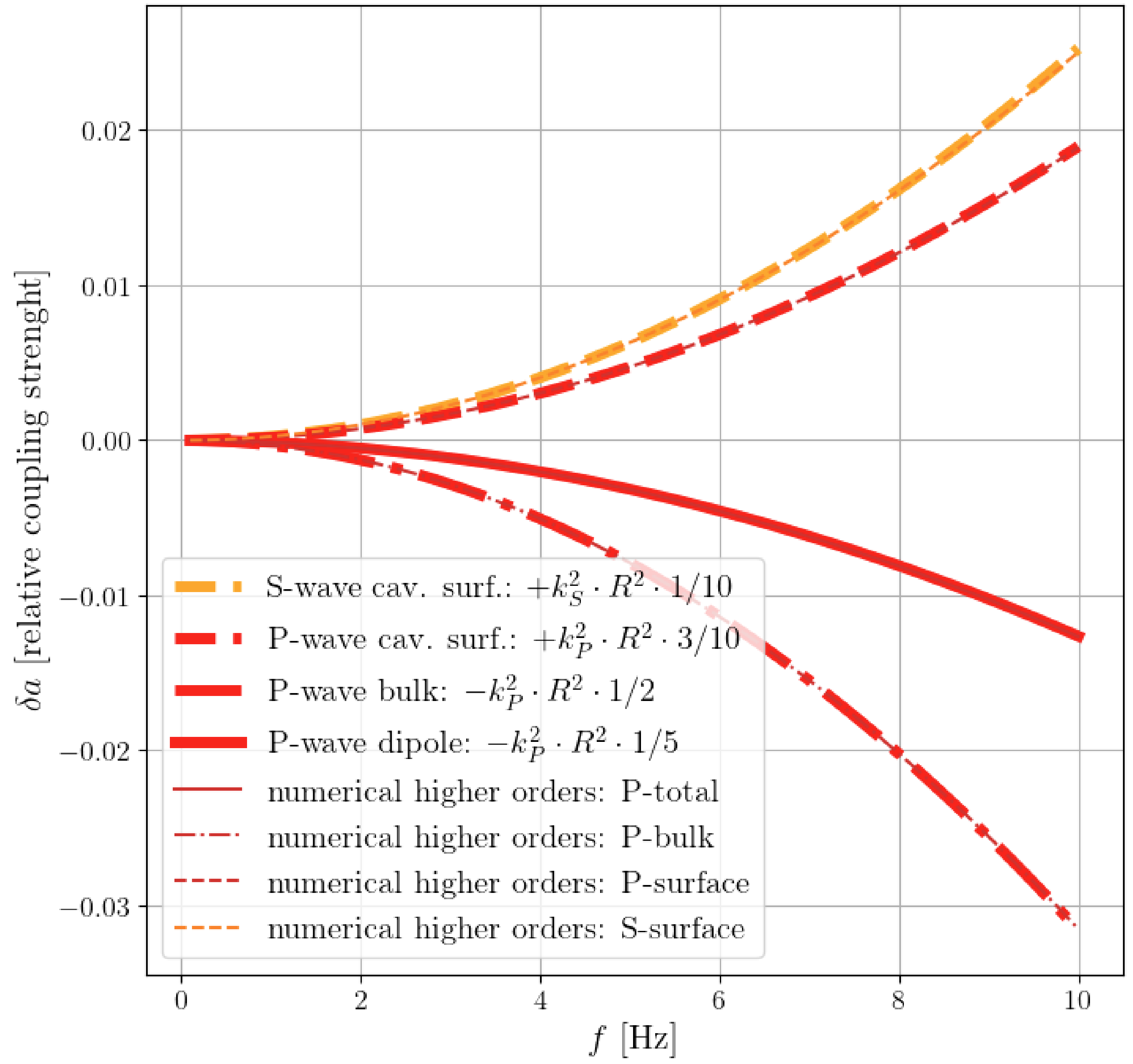
For both the surface integrals and dipole description: dense meshed around cavern needed

Validating numerical tools against analytic solutions

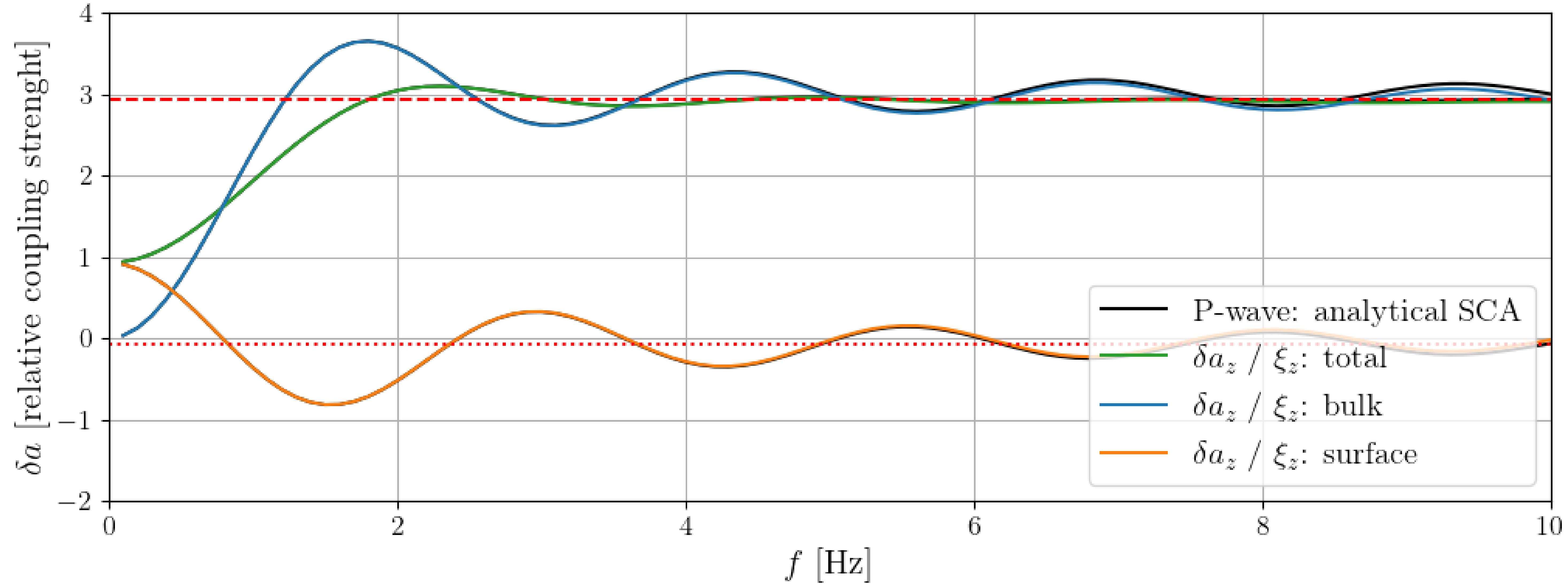


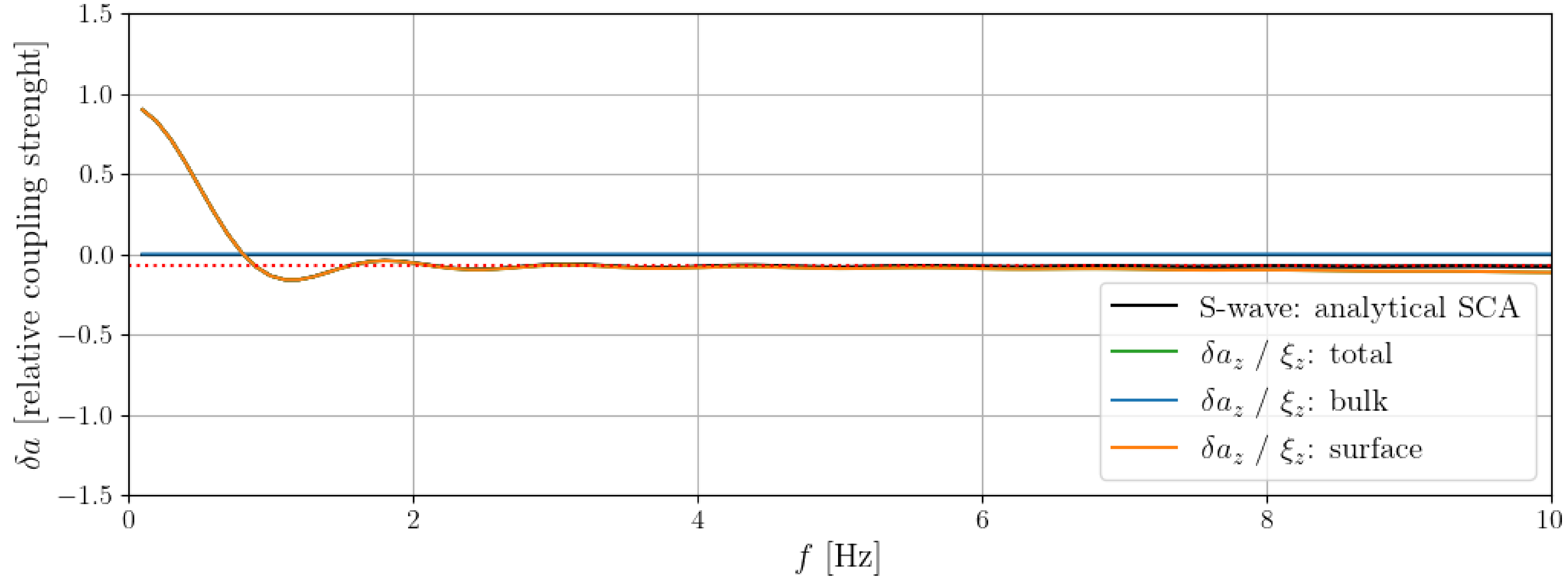
$$
\begin{aligned}
\delta\vec{a}_{\mathrm{P}}(\vec{r}_0,t) &= \delta\vec{a}_{\mathrm{P}}^{\text{cav. surf.}} + \delta\vec{a}_{\mathrm{P}}^{\text{rock. surf}} + \delta\vec{a}_{\mathrm{P}}^{\text{bulk}} = \delta\vec{a}_{\mathrm{P}}^{\text{dipole}} \\
&= -4\pi G\rho_0\vec{\xi}_P(\vec{r}_0,t)\cdot\left[j_0(k_{\mathrm{P}}r_{\mathrm{cav}}) - 2\frac{j_1(k_{\mathrm{P}}r_{\mathrm{cav}})}{k_{\mathrm{P}}r_{\mathrm{cav}}}\right] \\
&\quad + 4\pi G\rho_0\vec{\xi}_P(\vec{r}_0,t)\cdot\left[j_0(k_{\mathrm{P}}r_{\mathrm{rock}}) - 2\frac{j_1(k_{\mathrm{P}}r_{\mathrm{rock}})}{k_{\mathrm{P}}r_{\mathrm{rock}}}\right] \\
&\quad + 4\pi G\rho_0\vec{\xi}_{\mathrm{P}}(\vec{r}_0,t)\cdot\left[-j_0(k_{\mathrm{P}}r_{\mathrm{rock}}) + j_0(k_{\mathrm{P}}r_{\mathrm{cav}})\right] \\
&= 8\pi G\rho_0\vec{\xi}_{\mathrm{P}}(\vec{r}_0,t)\cdot\left[\frac{j_1(k_{\mathrm{P}}r_{\mathrm{cav}})}{k_{\mathrm{P}}r_{\mathrm{cav}}} - \frac{j_1(k_{\mathrm{P}}r_{\mathrm{rock}})}{k_{\mathrm{P}}r_{\mathrm{rock}}}\right] \\
\delta\vec{a}_{\mathrm{S}}(\vec{r}_0,t) &= \delta\vec{a}_{\mathrm{S}}^{\text{cav. surf.}} + \delta\vec{a}_{\mathrm{S}}^{\text{rock surf.}} + 0 = \delta\vec{a}_{\mathrm{S}}^{\text{dipole}} \\
&= -4\pi G\rho_0\vec{\xi}_{\mathrm{S}}(\vec{r}_0,t)\cdot\left[\frac{j_1(k_{\mathrm{S}}r_{\mathrm{cav}})}{k_{\mathrm{S}}r_{\mathrm{cav}}}\right] + 4\pi G\rho_0\vec{\xi}_{\mathrm{S}}(\vec{r}_0,t)\cdot\left[\frac{j_1(k_{\mathrm{S}}r_{\mathrm{rock}})}{k_{\mathrm{S}}r_{\mathrm{rock}}}\right] \\
&= -4\pi G\rho_0\vec{\xi}_{\mathrm{S}}(\vec{r}_0,t)\cdot\left[\frac{j_1(k_{\mathrm{S}}r_{\mathrm{cav}})}{k_{\mathrm{S}}r_{\mathrm{cav}}} - \frac{j_1(k_{\mathrm{S}}r_{\mathrm{rock}})}{k_{\mathrm{S}}r_{\mathrm{rock}}}\right]
\end{aligned}
$$

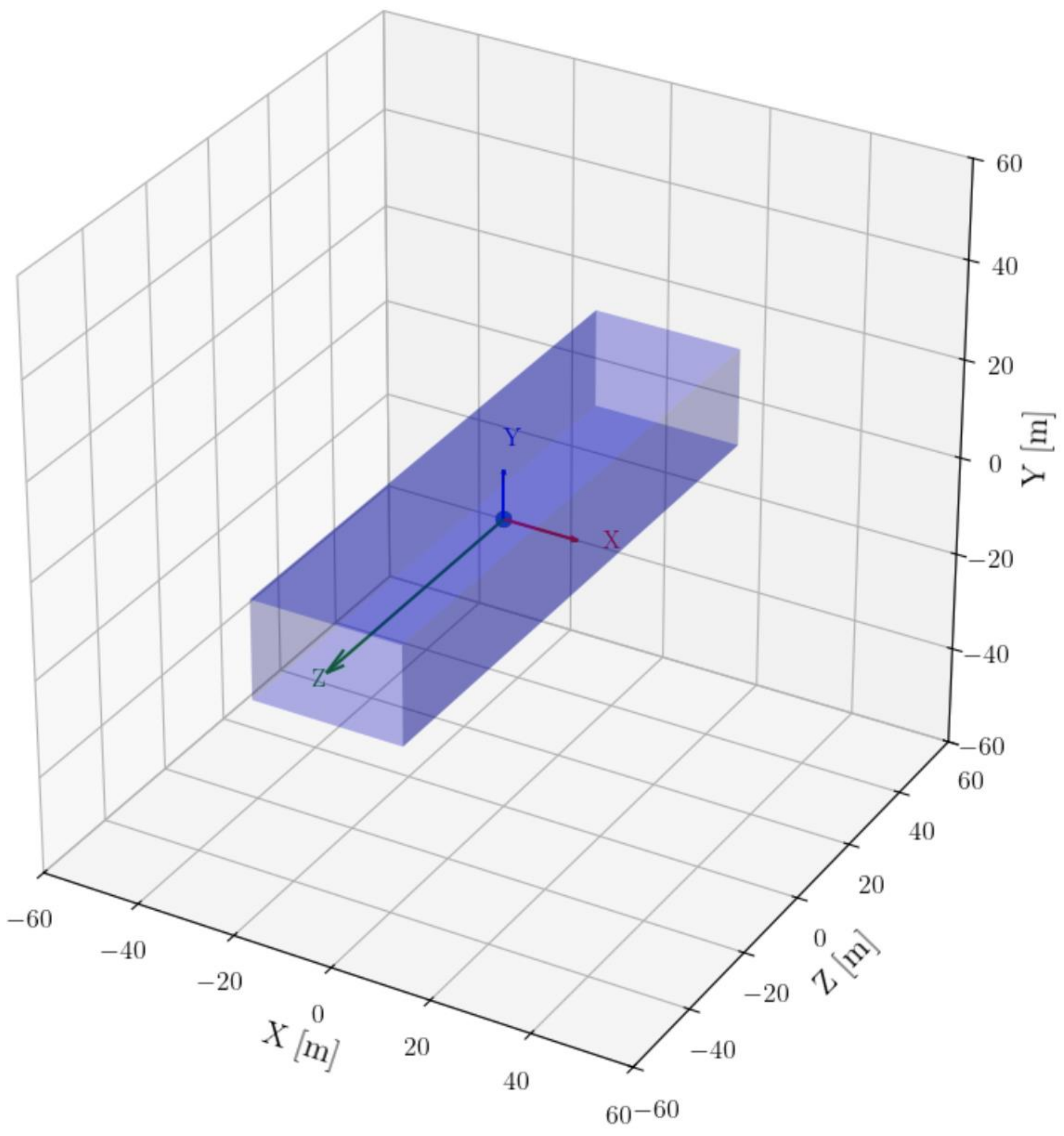




Validating numerical tools against analytic solutions





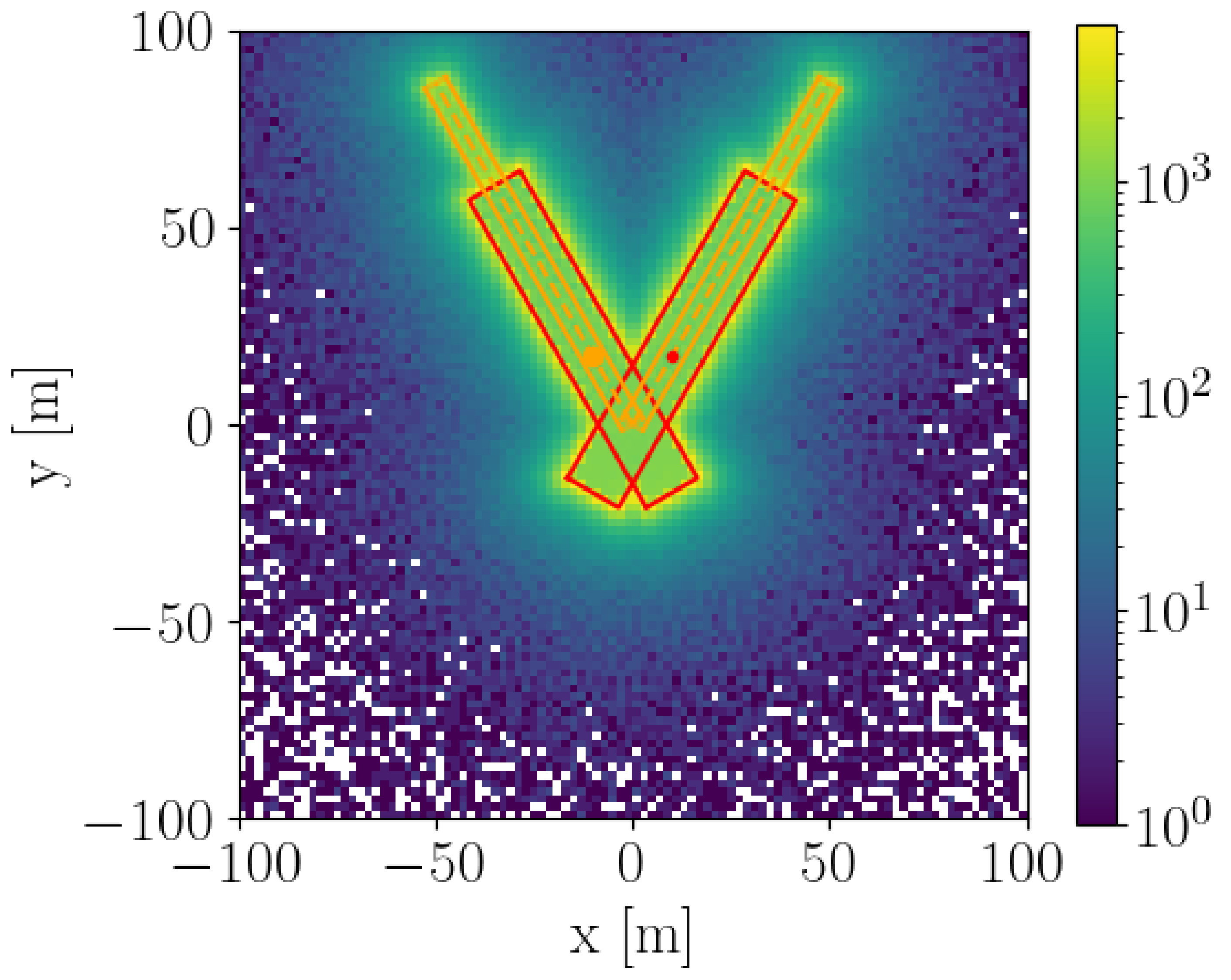


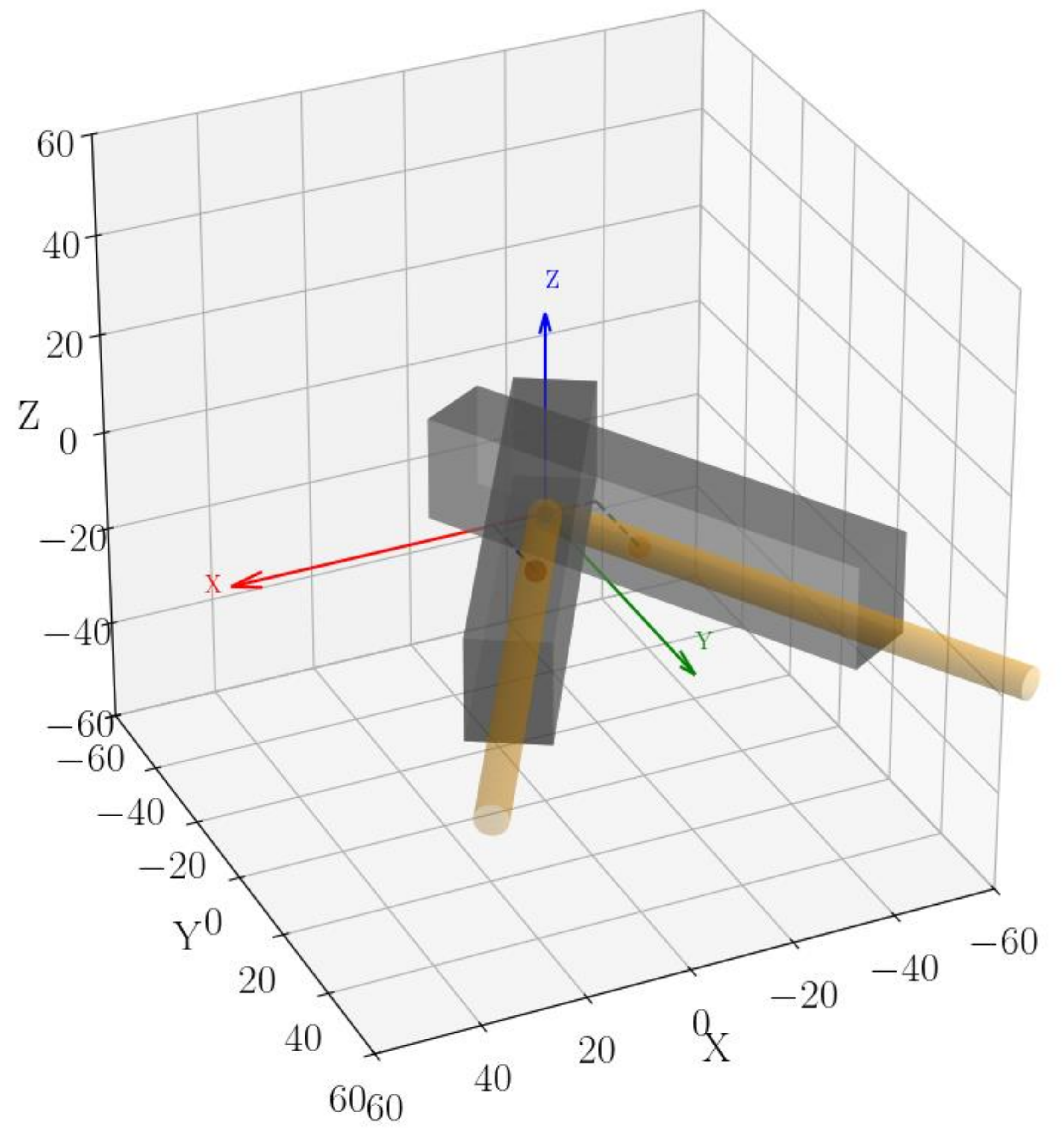
$$a_{\text{diag.}}(x_0, y_0, z_0, B, H, L) = +\frac{1}{\frac{4\pi}{3}} \sum_{i,j,k=-1,+1} \arctan\left(\left(\left(\frac{L+k\cdot 2z_0}{B+i\cdot 2x_0}\right)^2 + \left(\frac{L+k\cdot 2z_0}{H+j\cdot 2y_0}\right)^2 + \left(\frac{(L+k\cdot 2z_0)(L+k\cdot 2z_0)}{(B+i\cdot 2x_0)(H+j\cdot 2y_0)}\right)^2\right)^{-\frac{1}{2}}\right)$$

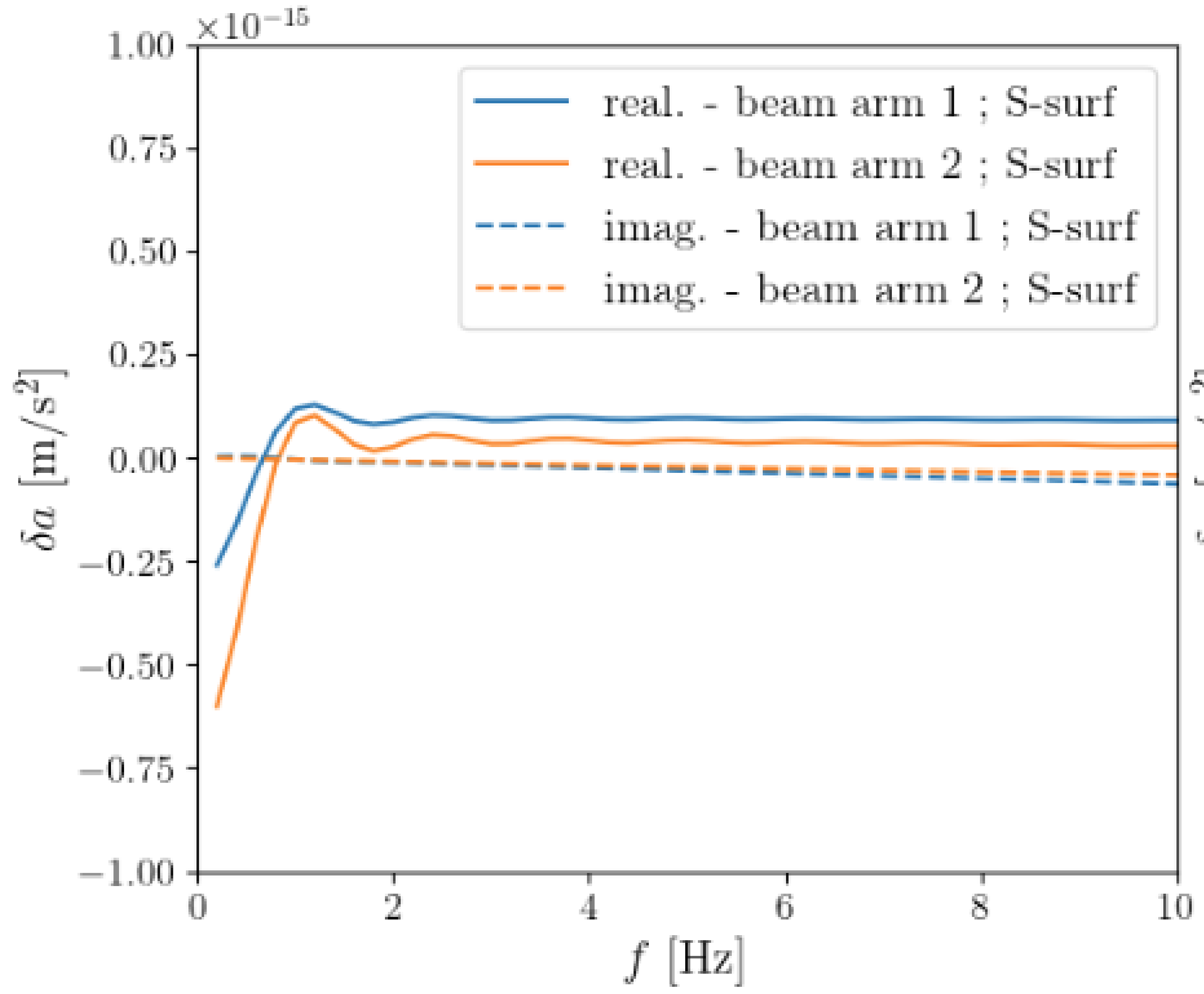
$$a_{\text{off-diag.}}(x_0, y_0, z_0, B, H, L) = -\frac{1}{\frac{4\pi}{3}} \sum_{i,j,k=-1,+1} j\cdot k \,\text{artanh}\left(\left(1 + \left(\frac{H+j\cdot 2y_0}{B+i\cdot 2x_0}\right)^2 + \left(\frac{L+k\cdot 2z_0}{B+i\cdot 2x_0}\right)^2\right)^{-\frac{1}{2}}\right)$$

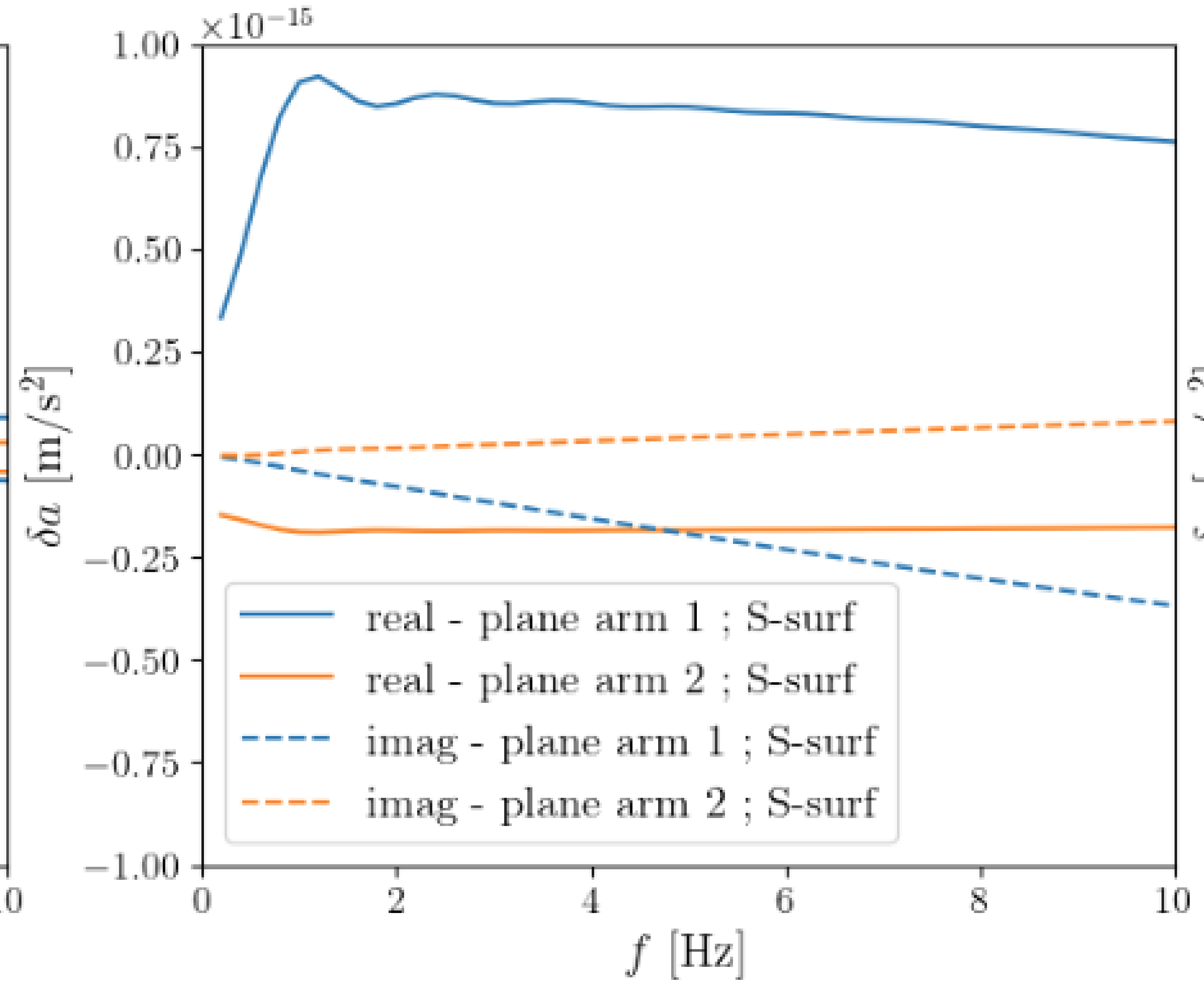
ANNA example for two mirrors in corner caverns

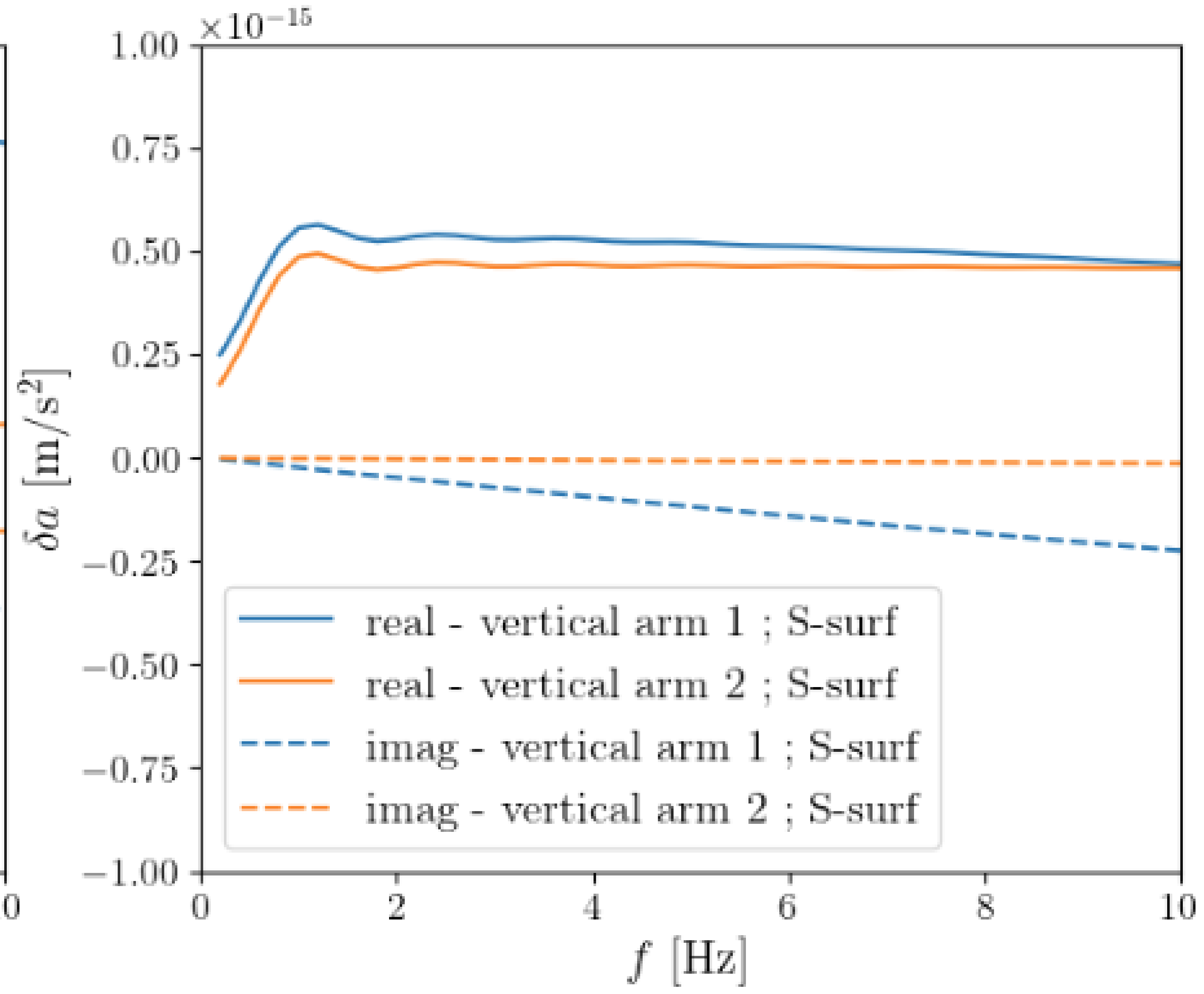
Testing ANNA: for incoming S-wave at ET corner cavern walls











Summary and Outlook

- We extensively validate numerical NN software:
  - **very good agreement with analytical theory** for NN accelerations in all scenarios checked so far → **confidence** in future results
  - both leading and higher order effects: resolved very well
  - **both modeling approaches**: "dipole" and "bulk + surface" **consistent** → allowing for cross-check of mesh quality even without analytical results

- NN response can significantly depend on local geometry close to the mirrors

- Ongoing work towards ET Symposium Aachen: EMR NN modeling with 3D model including geology, recent caverns models and topography