Searching for continuous gravitational waves: data analysis strategies in LIGO/Virgo Collaboration

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Plan

- Motivation
- Astrophysical sources of continuous gravitational waves (CGW)
- Search strategies in LIGO-Virgo Collaboration (LVC)
- Detector characterisation (DetChar) and CGW synergy
- CGW data analysis: future insights
Motivation
New era: gravitational waves astronomy

10 BH-BH mergers, 1 NS-NS merger

→ O1/O2 Catalog: https://www.gw-openscience.org
Motivation
New era: gravitational waves astronomy

Upgrade of the existing detectors + new methods in data analysis + new detectors = detections of the more subtle signals

Reviews: K. Riles (2013), Andersson et al. (2009)
According to Einstein’s quadrupole formula, time-varying (mass) quadrupole moment is needed to produce GW.
Mountains (elastic, magnetic, viscosity stresses)
\[ f_{GW} = 2f_{rot} \]

Oscillations (r-modes)
\[ f_{GW} = \frac{4}{3}f_{rot} \]

Free precession
\[ f_{GW} \propto f_{rot} + f_{prec} \]

Accretion (thermal gradients)
\[ f_{GW} \approx f_{rot} \]

Reviews
Bejger (2018)
Lasky (2015)
Andersson et al. (2011)
CGW radiation model

Commonly used model

Non-axisymmetric rotating NS (described as a triaxial ellipsoid) radiating purely quadrupolar CGW.

Strain amplitude

\[ h_0 = 4 \times 10^{-25} \left( \frac{\epsilon}{10^{-6}} \right) \left( \frac{l_3}{10^{45} \text{ g cm}^2} \right) \left( \frac{f}{100 \text{ Hz}} \right)^2 \left( \frac{100 \text{ pc}}{d} \right) \]

Compare GW 150914: \( h_0 \sim 10^{-21} \) (Abbott et al. 2016)

\[ \epsilon = (I_1 - I_2)/I_3 \]
\[ I = I_3 \]
\[ f = \Omega/2\pi \]
\[ d - \text{distance} \]

Target:
rapidly (or slowly for ET) spinning NS in our Galaxy
(\( \sim 2600 \text{ known, potentially } 10^8 \text{ objects} \))
Signal-to-noise ratio (SNR)

\[ SNR \propto \frac{h_0}{\sqrt{S_n}} \sqrt{T} \]

- \( S_n \) - strain noise
- (aLIGO: \( \sqrt{S_n} \sim 10^{-23}\text{Hz}^{-1/2} \))
- \( T \) - observational time

Network of the detectors

\[ SNR \propto \sqrt{N} \]

- \( N \) - number of detectors with comparable sensitivity

- **GW150914**: \( h_0 \sim 10^{-21}, T \sim 0.2\text{s} \rightarrow SNR \sim 24 
- **CGW**: \( h_0 \lesssim 10^{-25}, T \sim \text{days, months, years...} \)

Regimbau et al. (2017)
Spin-down

NS is losing energy and spinning-down, due to the CGW emission, magnetic braking, neutrino emission, accretion (e.g. Greenstein & Cameron 1969, Illarionov & Kompaneets 1990, Dvornikov & Dib 2009, Staff et al. 2012).

We can measure it e.g. from radio-observations.

Spin-down limit (assumption: NS loses energy only due to the CGW)

\[ h_{\text{spindown}} = 2.5 \times 10^{-25} \left( \frac{1\text{kpc}}{d} \right) \sqrt{ \left( \frac{1\text{kHz}}{f_{\text{GW}}} \right) \left( \frac{-\dot{f}_{\text{GW}}}{10^{-10}\text{Hz/s}} \right) \left( \frac{I_z}{10^{38}\text{kg}\cdot\text{m}^2} \right) } \]
No CGW signal? Set upper limits!

With the known sensitivity of the detectors we can put constraints on the $f_{GW}$, $\dot{f}_{GW}$ and $\epsilon$.

**GW strain limit, spin-down limit, sensitivity:**
S5, aLIGO, ET
(Lasky 2015; Aasi et al. 2014; Dupuis & Woan 2005)

O1 run
Abbott et al. (2017)
No CGW signal? Set upper limits!

- Most constraining ellipticity is $1.3 \times 10^{-8}$ for J0636+5129

- $\epsilon$ can be converted to a maximal 'mountain' size:
  - for Crab $\sim 10$ cm
  - for Vela $\sim 50$ cm

- $\epsilon$ can tell us about NS matter:
  - $\sim 10^{-5} - 10^{-7}$ for 'normal' NS
    - Ushomirsky et al. (2000)
  - $\sim 10^{-4} - 10^{-5}$ for 'strange' NS
    - Owen (2005)
Data analysis strategies

Is it isolated NS or binary system?
How well do we know the source?
How much computational power do we have?

- Targeted searches
  - Rotational parameters
  - Sky position

- Narrow-band Searches
  - Rotational parameters
  - Sky position

- Directed searches
  - Rotational parameters
  - Sky position
  - Other parameters

- Blind Searches
  - Rotational parameters
  - Sky position

- Computational cost
Measured signal strain $h(t; A, \lambda)$ depends on:

- Amplitude parameters $A \equiv \{ h_0, \cos \iota, \psi, \phi_0 \}$
- Phase-evolution parameters $\lambda \equiv \{ \vec{n}, f, \dot{f}, \ldots \}$

$\rightarrow$ One has to include extra modulations

Search on 4-dimensional \((f, \dot{f}, \alpha, \delta)\) optimal grid (Pisarski & Jaranowski 2015).

\[
\mathcal{F} = \frac{2}{\sigma^2} \left( \frac{|F_a|^2}{\langle a^2 \rangle} + \frac{|F_b|^2}{\langle b^2 \rangle} \right)
\]

\[
F_a = \sum_{t=1}^{N} x(t) a(t) \exp[-i\phi(t)], \quad F_b = \sum_{t=1}^{N} x(t) b(t) \exp[-i\phi(t)],
\]

\[
\langle a^2 \rangle = \sum_{t=1}^{N} a(t)^2, \quad \langle b^2 \rangle = \sum_{t=1}^{N} b(t)^2,
\]

\(\sigma^2\) - variance of the data \(x(t)\),

\(a(t), b(t)\) - amplitude modulation functions (depend on the location and orientation of the detectors on Earth and on the position of GW source on the sky \((\alpha, \delta))\),

\(\phi(t)\) - phase modulation function (like above + depends on frequency \(f\) and spindown \(\dot{f}\)).
Targeted searches
e.g. known radio, X-ray or $\gamma$-ray pulsars

**Heterodyne (Bayesian) method**
Dupuis & Woan (2005)
- Bayesian parameter-estimation for $A \equiv \{h_0, \cos \iota, \psi, \phi_0\}$
- Known $\lambda \equiv \{\vec{n}, f, \dot{f}, ...\}$

**$\mathcal{F}$-statistics method**

Only small range in $f$ and $\dot{f}$ around known values is explored.

**5-vector method** Astone et al. (2010, 2012)
- Fourier-domain
- amplitude modulation from the Earth’s sidereal rotation of each detector’s antenna pattern

### Table: 5-vector method

<table>
<thead>
<tr>
<th>Name</th>
<th>distance [kpc]</th>
<th>$h_{md} \cdot 10^{-25}$</th>
<th>$\epsilon_{md} \cdot 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>J0205+6449</td>
<td>2.0 $\pm$ 0.3$^b$</td>
<td>6.9 $\pm$ 1.1</td>
<td>14</td>
</tr>
<tr>
<td>J0554+2200 (Crab)</td>
<td>2.0 $\pm$ 0.5$^c$</td>
<td>14 $\pm$ 3.5</td>
<td>7.6</td>
</tr>
<tr>
<td>J0835-4510 (Vela)</td>
<td>0.28 $\pm$ 0.02$^c$</td>
<td>34 $\pm$ 2.4</td>
<td>18</td>
</tr>
<tr>
<td>J1400-6326</td>
<td>10 $\pm$ 3$^d$</td>
<td>0.90 $\pm$ 0.27</td>
<td>2.1</td>
</tr>
<tr>
<td>J1813-1246</td>
<td>&gt; 2.5$^a$</td>
<td>&lt; 1.8</td>
<td>&lt; 2.4</td>
</tr>
<tr>
<td>J1813-1749</td>
<td>4.8 $\pm$ 0.3$^f$</td>
<td>3.0 $\pm$ 0.2</td>
<td>7.0</td>
</tr>
<tr>
<td>J1833-1034</td>
<td>4.8 $\pm$ 0.4$^g$</td>
<td>3.1 $\pm$ 0.3</td>
<td>13</td>
</tr>
<tr>
<td>J1952+3252</td>
<td>3.0 $\pm$ 0.5$^b$</td>
<td>1.0 $\pm$ 0.2</td>
<td>1.1</td>
</tr>
<tr>
<td>J2022+3842</td>
<td>10 $\pm$ 2$^d$</td>
<td>1.0 $\pm$ 0.3</td>
<td>6.0</td>
</tr>
<tr>
<td>J2043+2740</td>
<td>1.5 $\pm$ 0.6$^l$</td>
<td>6.9 $\pm$ 2.8</td>
<td>23</td>
</tr>
<tr>
<td>J2229+6114</td>
<td>3.0 $\pm$ 2$^e$</td>
<td>3.4 $\pm$ 2.2</td>
<td>6.2</td>
</tr>
</tbody>
</table>

Upper: S3+S4 runs, Abbott et al. (2008)
Bottom: O1 run, Abbott et al. (2017)
Directed searches
e.g. non-pulsating X-ray source at the center of a supernova remnant

- $f$, $\dot{f}$, ... are unknown, but sky location is known
- higher $f$ derivatives can be important for young (hot) and/or accreting NS
- hard to model
- strain depends on age $a$ and distance $d$ (Wette et al. 2008); additional factors like mass or equation of state increase $h_0^{age}$ uncertainty by 50%

**Strongest possible signal from supernova remnant**

$$h_0^{age} = 1.26 \times 10^{-24} \left( \frac{3.30\text{kpc}}{d} \right) \left( \frac{300\text{yr}}{a} \right)^{1/2}$$
Blind (all-sky) searches
Unknown sources

Computational cost
Exploring huge parameter space requires huge computational power
→ reduce number of parameter to the minimum

\[ f_i \quad (i, j) \quad (i, j + 1) \quad (i - 1, j) \quad T_0 \quad T_0 \quad B = \frac{1}{2\delta t} \]

\( F \)-statistic pipeline example
Computing power scales as \( \sim T^5 \log(T) \)
→ divide data into shorter segments (e.g. 6 days)
Blind (all-sky) searches
Unknown sources

Hough transform (Hough 1959, Hough 1962)

- detection statistic is compared to a threshold and given a weight
- weighting based on antenna pattern and detector noise
- antenna pattern depends on \( \{\alpha, \delta, f, \dot{f}, \ldots\} \)
- different parameter spaces chosen to accumulate weight sums:
  - Sky Hough (Krishnan et al. 2004, Aasi et al. 2014)
- sums of weights are accumulated in “maps”
Blind (all-sky) searches
Unknown sources

Generalized Frequency Hough \textsuperscript{Miller et al. (2018)}

Braking index

\[ n = \frac{f|\ddot{f}|}{f^2} \]

particle wind \( n = 1 \)
dipole (EM) radiation \( n = 3 \)
quadrupole (GW) radiation \( n = 5 \)
oscillations (r-modes) \( n = 7 \)
F-statistic method
Main goal: to find F-statistic maximum and $f, \dot{f}, \alpha, \delta$ associated with it.

Hierarchical pipeline allows for computational cost reduction.
Blind (all-sky) searches
Unknown sources

Lines and signals in $\mathcal{F}$-statistics method

Main goal: find $\mathcal{F}$-statistic maximum and $f, \dot{f}, \alpha, \delta$ associated with it.

$$SNR = \sqrt{2(\mathcal{F} - 2)}$$
DetChar and CGW synergy
Lines hunt - spectral density

DetChar team provides list of known, stationary lines → vetoing.

CW team during data analysis finds new lines, distinguishes lines and astrophysical signals and gives feedback to DetChar.
Some lines are problematic - they evolve or disappear in time.

Band 0047, 24-days

Frame 003

Frame 009
Future insights

- detections and/or better upper limits
- mountains as small as 10 cm can be detected (ET)
- exploring lower frequencies
- joint EM and GW observations will deliver unique information about NS (equation of state, environment, physical phenomena)
Future insights

Magdalena Sieniawska (CAMK)
Searching for continuous gravitational waves in LVC

Not only sensitivity improvement matters. The higher number of the detectors in the network, the bigger chance to detect CGW!