

A pendulum and seismic isolation

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Eva Pankova, Gymnasium Postova 9, Kosice, Slovakia

Learning outcomes:

1. To introduce students to the concept of a pendulum.
2. To investigate the relationships between the variables that affect the period of a pendulum
3. To graph and analyse data and form conclusions about how a pendulum operates.

Age of students: 16 - 17 years old

Activity time: 60 min

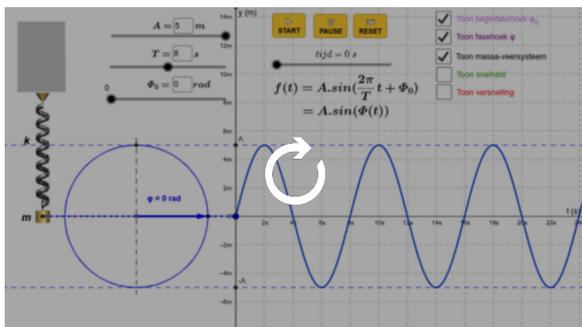
Prior knowledge:

- Basis about harmonic motion
- Kinematics of harmonic motion
- Coupled harmonic oscillators and qualitative description of the result amplitude of two oscillators under superposition

Students **have done** investigation on following simulations

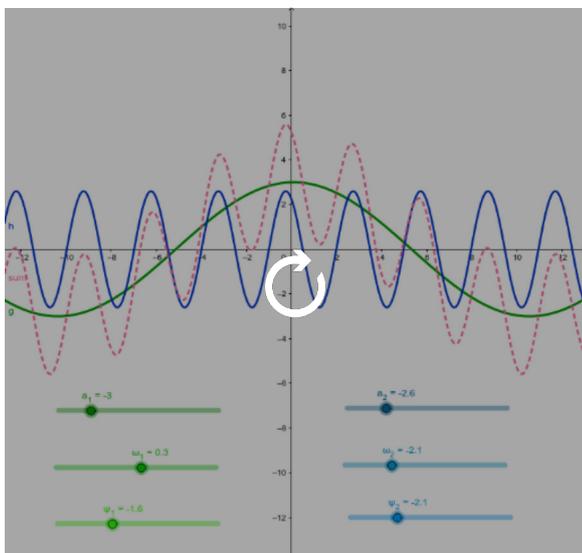
Kinematics of the harmonic motion:

[Harmonic vibration](#)



Superposition - the resulting amplitude:

[Superposition](#)



Motivation - Provide contact with the content and/or provoke curiosity:

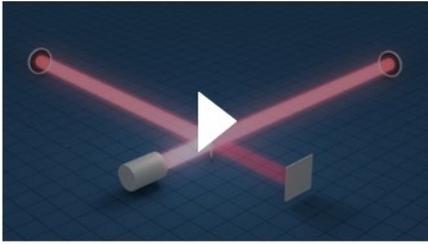
Question: **What is the role of the pendulums in the VIRGO detector for gravitational waves?**

Before you come up with the answer investigate:

Videos:

<https://www.ligo.caltech.edu/LA/video/IFO-response>

[Most Precise Ruler Ever Constructed](#)



Text:

Virgo

nipslab.org/virgo

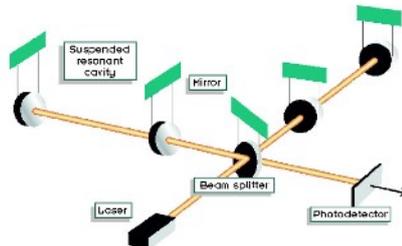
NiPS Laboratory

Noise in Physical Systems



The VIRGO collaboration was set up between Italian and French research teams, for the realization of an interferometric gravitational wave detector.

The Virgo detector for gravitational waves consists mainly in a Michelson laser interferometer made of two orthogonal arms being each 3 kilometers long. Multiple reflections between mirrors located at the extremities of each arm extend the effective optical length of each arm up to 120 kilometers. Virgo is located within the site of EGO, European Gravitational Observatory, based at Cascina, near Pisa on the river Arno plain.



The frequency range of Virgo extends from 10 to 6,000 Hz. This range as well as the very high sensibility should allow detection of gravitational radiation produced by supernovae and coalescence of binary systems in the milky way and in outer galaxies, for instance from the Virgo cluster.

In order to reach the extreme sensitivity required, the whole interferometer attains optical perfection and is extremely well isolated from the rest of the world in order to be only sensitive to the gravitational waves. To achieve it, Italian and French scientists involved in the project, have developed most advanced techniques in the field of high power ultrastable lasers, high reflectivity mirrors, seismic isolation and position and alignment control.

In the field of optics, Virgo uses a new generation of ultrastable lasers, and the most stable oscillator ever built. A specific optical coating facility has been built to produce extremely high quality mirrors combining the highest reflectivity (over 99,999 %), with nanometer surface control

To avoid spurious motions of the optical components due to seismic noise; each one of them is isolated by a 10m high, very elaborate system of compound pendulums. Because the presence of a residual gas would slightly perturb the measurements the light beam

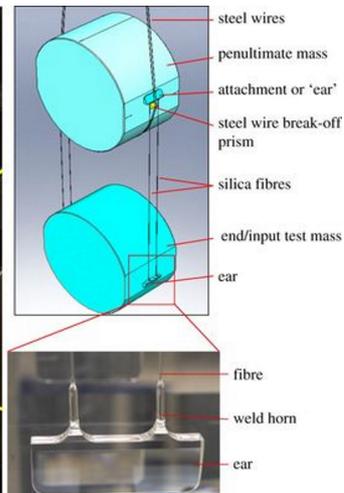
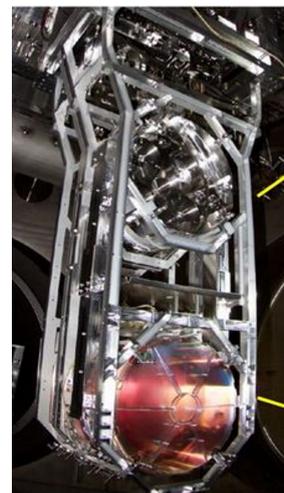
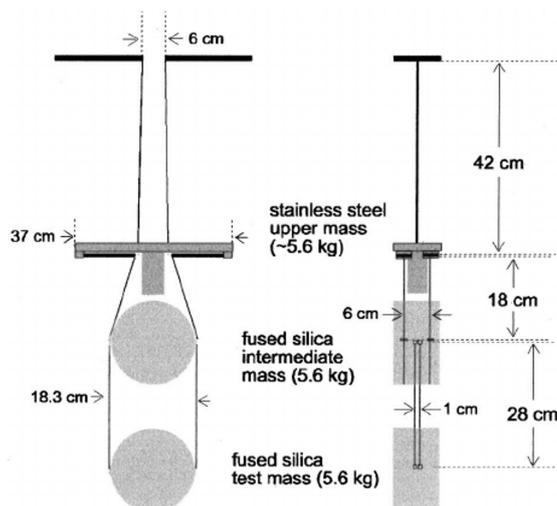
must propagate under ultra high vacuum. The two tubes, 3km long and 1.2m diameter each are actually the largest ultra high vacuum vessels in Europe and the second largest in the world. The environment of the Virgo interferometer is quieter than that of a spacecraft orbiting the earth.

Virgo, the construction of which was completed in June 2003 and is at present is in the commissioning phase, will run day and night listening to all gravitational signals which may arrive at any time and coming from any part of the Universe. The signals are detected, recorded and pre-analysed through an on-line computing system. These data will then be made available to the scientific community for further analysis

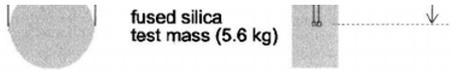
Monolithic Suspension Test

Perugia lab pictures

Research problem: The role of the coupled pendulum suspension in seismic isolation. Investigate this phenomenon and how it depends on relevant parameters.



Credit: <https://royalsocietypublishing.org/doi/10.1098/rsta.2017.0281>



FACE VIEW

SIDE VIEW



Credit: <https://royalsocietypublishing.org/doi/10.1098/rsta.2017.0281>

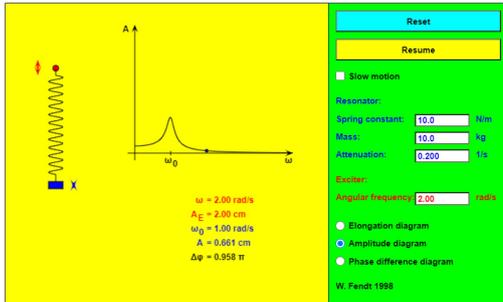
Credit: Plissi, M. & Torrie, C. & Husman, Matthew & Robertson, N. & Strain, K. & Ward, Hywel & Lück, Harald & Hough, James. (2000). GEO 600 triple pendulum suspension system: Seismic isolation and control. Review of Scientific Instruments. 71. 2539-2545. 10.1063/1.1150645.

Investigation of the FORCED oscillations.

Research questions:

- 1, What is the relation between exciter's angular frequency and the amplitude of the resonator's oscillation?
- 2, What is the relation between resonator's mass and resonator's angular frequency?

https://www.walter-fendt.de/html5/phen/resonance_en.htm



Analogy with Virgo pendulum suspension:

The top of a spring pendulum (red circle) is moved to and fro simulating harmonic motion, which can be describe by sine or cosine function - similar as produce earthquakes or human seismic sources (traffic, building activities), which could generate false alerts on interferometer.

Method:

Step 1: Set the mass of the resonator to 1 kg and exciter's angular frequency to 2 rad/s, $\omega < \omega_0$

Press the button *Resume*.

Observe the motion of the coupled oscillators then switch to *Elongation diagram*.

Step 2: Set exciter's angular frequency to 3,16 rad/s, $\omega = \omega_0$

Press the button *Resume*

Observe the motion of the coupled oscillators then switch to *Elongation diagram*.

Step 3: Set exciter's angular frequency to 10 rad/s, $\omega > \omega_0$

Repeat the procedure from the previous step

Step 4: Set the mass of the resonator to 6 kg (mirror's mass-article) and exciter's angular frequency to 10 rad/s

Repeat the procedure from the previous step

Step 4: Set the mass of the resonator to 40 kg (mirror's mass-VIRGO) and exciter's angular frequency to 10 rad/s

Repeat the procedure from the previous step

Conclusion: The aim of the investigation was to observe relation between exciter's angular frequency and natural angular frequency of the resonator, how the mass of the resonator influence the amplitude of the resonator in coupled oscillations. According observations we had made we conclude that minimum amplitude of the resonator was under the condition $\omega > \omega_0$ and with big mass of the resonator = 40 kg.

Recommendations for the construction of equipment for filtering seismic shocks: Coupled oscillators with the resonator with less natural angular frequency compared to frequency of seismic source and resonator with big mass.

Sources:

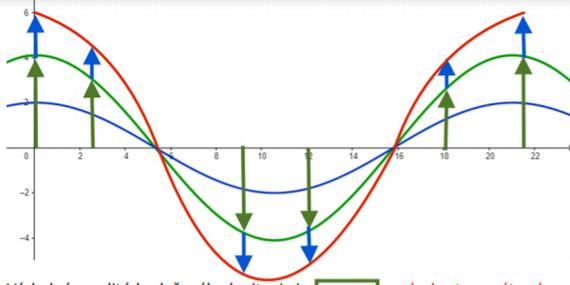
<http://public.virgo-gw.eu/frontier-technology/>

<http://inspiringscience.rdea.gr/delivery/view/index.html?id=a571ea3049c7469188c10e9a482b3576&t=p>

https://www.walter-fendt.de/html5/phen/resonance_en.htm

<https://www.nipslab.org/virgo/>

Samples of students work

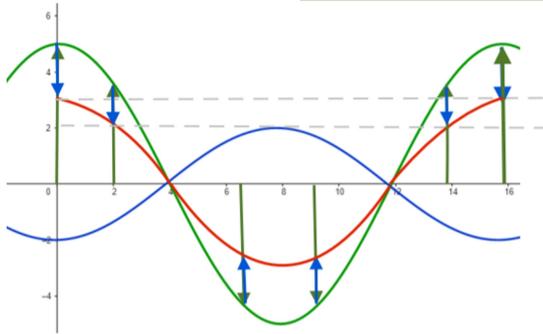


Výsledná amplitúda zloženého kmitania je **Väčšia** (vyberte zo zátvorky, nulová, väčšia, menšia) ako amplitúdy kmitov, z ktorých výsledné kmitanie vzniklo.

Napište podmienku pre veľkosti amplitúd výchylky a fázový rozdiel, za ktorých je výsledná amplitúda výchylky zloženého kmitania maximálna:

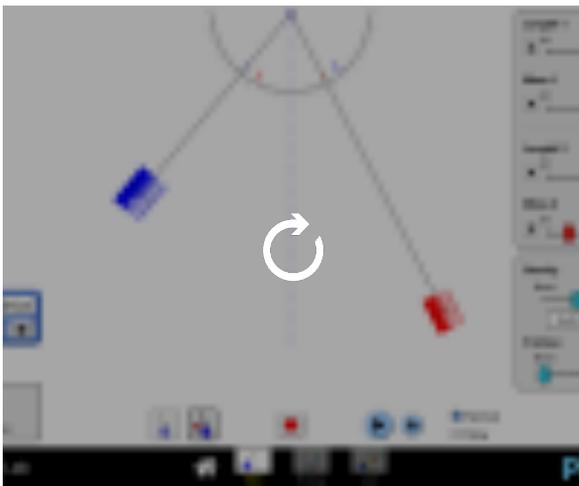
- majú rovnakú periódu a frekvenciu, prebiehajú v jednej priamke
- pre výchylky rovnakého smeru platí $y = y_1 + y_2$
- $\Delta\phi = \phi_2 - \phi_1 = 0$ rad

b) opačná začiatočná fáza



LC-24-
Overenie...

[Pendulum Lab](#)



LABORATÓRNE CVIČENIE č.

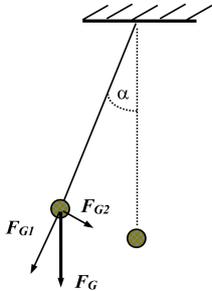
TÉMA: OVERENIE VZŤAHU PRE PERIÓDU MATEMATICKÉHO KYVADLA.

DÁTUM:

FYZIKÁLNY PRINCÍP:

Matematické kyvadlo je každé teleso s hmotnosťou m , ktoré je zavesené na pevnom závесе dĺžky l ; hmotnosť závesu je oproti hmotnosti telesa zanedbateľne malá.

Príčinou kmitavého pohybu kyvadla je zložka tiažovej sily, ktorej veľkosť a smer sa počas kmitavého pohybu mení.



F_G - tiažová sila pôsobiaca na teleso,
 F_{G1} - zložka tiažovej sily napínajúca záves,
 F_{G2} - pohybová zložka tiažovej sily,
 α - uhol odklonu kyvadla od zvislého smeru.

ak uhol $\alpha < 5^\circ$ pre periódu kyvadla platí

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \dots(1)$$

kde

g - je normálne tiažové zrýchlenie,
 l - je dĺžka pevného závesu.

ÚLOHA: Odmerajte periódu kyvadla s rôznou dĺžkou a overte vzťah pre periódu kyvadla.
Overte, že perióda kyvadla nezávisí od hmotnosti telesa.

Pomôcky: tri telesá s rozličnou hmotnosťou, stojan s držiakom, vlákno, dĺžkové meradlo, stopky.

Postup:

1. Zhotovte kyvadlo z guľôčky a pevného vlákna. Kyvadlo upevnite na držiak stojana tak, aby ste pri meraní mohli dobre určiť bod závesu. Dĺžku kyvadla merajte dĺžkovým meradlom od bodu závesu po stred telesa.
2. Periódu kyvadla určte ako aritmetický priemer z meraní desiatich periód, ktoré päťkrát opakujte.
3. Opakujte meranie pre tri rôzne dĺžky kyvadla (60 cm až 160 cm).
4. Pre najväčšiu dĺžku kyvadla zopakujte meranie ešte dvakrát, použite telesá s inou hmotnosťou.

TABUĽKA:

i	Určenie závislosti periódy od dĺžky kyvadla									Nezávislosť od hmotnosti						Nezávis
	$m_1 = 0,16 \text{ kg}$									$l_1 = 0,83 \text{ m}$						
	$l_1 = 0,83 \text{ m}$			$l_2 = 0,72 \text{ m}$			$l_3 = 0,61 \text{ m}$			$m_2 = 0,18 \text{ kg}$		$m_3 = 0,2 \text{ kg}$				
	\overline{ST}_1 s	\overline{T}_1 s	$\overline{\Delta T}_1$ s	\overline{ST}_2 s	\overline{T}_2 s	$\overline{\Delta T}_2$ s	\overline{ST}_3 s	\overline{T}_3 s	$\overline{\Delta T}_3$ s	\overline{ST}_4 s	\overline{T}_4 s	$\overline{\Delta T}_4$ s	\overline{ST}_5 s	\overline{T}_5 s	$\overline{\Delta T}_5$ s	
1	7,64	1,528	0,0368	7,05	1,41	0,01	6,5	1,3	0,0216	7,52	1,504	0,0364	8,31	1,662	0,114	
2	8,1	1,62	0,0552	6,98	1,396	0,004	6,31	1,262	0,0164	7,63	1,526	0,0144	7,69	1,538	0,01	
3	7,79	1,558	0,0068	7,03	1,406	0,006	6,47	1,294	0,0156	8,01	1,602	0,0616	7,84	1,568	0,02	
4	8,31	1,662	0,0972	6,78	1,356	0,044	6,2	1,24	0,0384	7,89	1,578	0,0376	7,6	1,52	0,028	
5	7,28	1,456	0,1088	7,16	1,432	0,032	6,48	1,296	0,0176	7,46	1,492	0,0484	7,26	1,452	0,096	
Priemer	7,824	1,565	0,061	7	1,4	0,28	6,392	1,278	0,256	7,702	1,540	0,308	7,74	1,548	0,054	

Výsledky merania :

závislosť periódy kyvadla od jeho dĺžky

všeobecne
$$T = \left(\overline{T} - \overline{\Delta T}; \overline{T} + \overline{\Delta T} \right)_s$$

$$\delta T = \frac{\overline{\Delta T}}{\overline{T}}$$

z nameraných hodnôt:

$T_1 = 1,565; \delta T_1 = 3,89\%$

$T_2 = 1,400; \delta T_2 = 1,37\%$

$T_3 = 1,278; \delta T_3 = 1,71\%$

Výpočtom:

$T_{v1} = 1,828$

$T_{v2} = 1,702$

$T_{v3} = 1,567$

nezávislosť periódy kyvadla od hmotnosti závažia

z nameraných hodnôt:

$T_4 = 1,540; \delta T_4 = 2,57\%$

$T_5 = 1,548; \delta T_5 = 3,56\%$

Výpočtom:

$T_{v1} = 1,828$

Záver

Naším cieľom bolo overiť platnosť vzťahu pre výpočet periódy matematického kyvadla a porovnať výpočtové výsledky s nami nameranými údajmi. Ako kyvadlo som použil kĺbko špagátu, meranie sa uskutočnilo v interiéri, preto môžeme aerodynamický odpor a odpory iného druhu zanedbať.

Nakoľko výpočet používa aproximáciu a počas merania sme sa dopustili systematických chýb (oneskorenie ľudskej reakcie až o 0,2s; nehomogénnosť kyvadla...), dáva porovnanie vypočítaného a meraného výsledku odlišné hodnoty. V meraní sme sa dopustili malej relatívnej chyby merania (priemerná $\delta T = 2,60\%$).