FRONTIERS

Bringing Nobel Prize Physics in the Classroom

"Finding black-holes in a chirp"

Co-funded by the Erasmus+ Programme of the European Union

Background information for teachers

Overview of this lesson pack:

Name of the activity	Gravitational Waves at Action
Topics introduced	Black holes, Gravitational waves detection, inferring black holes properties from data, graphs, scientific methods
Curriculum Connection	CROATIA: Physics (Newton's law of gravity, Kepler's laws, space-time, perturbations, experiment); Math (mathematical description of Laws of Physics) SWEDEN: Methods for the study of the universe. Electromagnetic radiation from stars and interstellar space. The significance of the experimental work for testing, re-evaluating and revising hypotheses, theories and models. etc. CHILE: Physics. (Waves, Modern Physics). Planning and development of research questions in science ROMANIA: Physics (Kepler's and Newton's laws, Oscillations, Waves, Energy). Mathematics (calculus, graphs, slope, linearization), general knowledge about scientific methods, numerical simulations.
Reference Demonstrator	"Finding black-holes in a chirp": how to understand the first gravitational-wave detection
Age of students	15-19
Duration	2 hours/4 hours

Overview of this lesson pack:

Type of activity	Workshop, Problem solving, Collaboration, CLIL
Description of activity	Teacher activities: ● provides and prepares all the necessary materials (worksheets, videos, Powerpoint presentations, materials necessary for experiments, etc)i ● explains the new notions (space-time entity, gravitational waves, black holes, etc.) ● reinforce previous knowledge ● guides the students to succeed by their own ● assess the work of the students Student activities: ● Hands on activities - experiments ● Drawing graphs ● Calculations ● filling in Activity Sheets ● peer assessment
Equipment requirements	pc, internet connection, PowerPoint presentations, worksheets, an elastic sheet, hula hoop, balls of different sizes, weights and bodies of different densities.
Prior knowledge for students	Physics (Newton's laws, Oscillations, Waves, Energy). Mathematics (graphs, slope, linearization of nonlinear graphs, fitting tools, statistics, Measurement error theory, general knowledge about scientific methods, numerical simulations)

Background and overview of the "Finding black-holes in a chirp" demonstrator:

The activities we propose are based on the "Finding black-holes in a chirp" demonstrator.

Our students are 15-19 years old and in our national curricula many of the theoretical notions (at Physics and Mathematics) necessary to our students to understand and actually carry on these activities are learned mainly in the last 2 years of Secondary School.

This is the reason why we have decided to make 2 presentations, one for the younger students, to introduce them in this fascinating topic, with informative and hands-on activities, and another one, based on all activities proposed in the demonstrator, for the older students, who are able to do all the tasks. (as a matter of fact, this demonstrator is recommended to >17 yo students)

In every version the students have the opportunity to better understand the importance of Gravitational Interferometry field in the development of fundamental research, in unravelling the mysteries of the Universe, and equally, in the technological development necessary to increase the sensitivity of gravitational waves detection.

They also can find out more about the methods used in advanced science, that crucial discoveries are made as a result of extended collaborations, not only between research institutes, but also at international level.

Another valuable lesson for them is to discover the richness of information hidden in a signal, to follow the process through which the scientists infer from it so much information about a remote, very old, catastrophic, but spectacular event, to learn how to listen to the messages sent by the Universe.

Inspiration and for prior knowledge

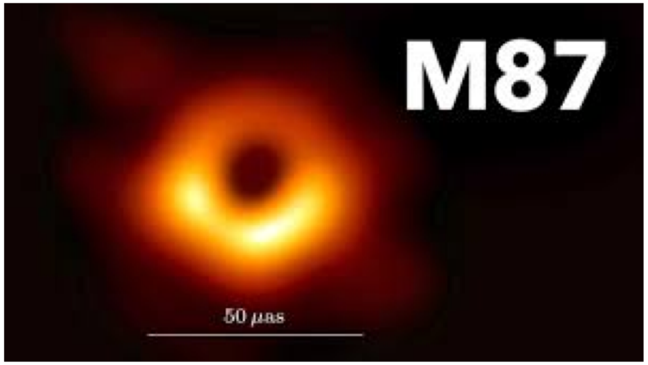
- Gravitation - https://en.wikipedia.org/wiki/Gravity
- Spacetime - https://en.wikipedia.org/wiki/Spacetime
- Black hole - https://en.wikipedia.org/wiki/Black_hole
- https://www.nasa.gov/audience/forstudents/k-4/stories/nasa-knows/what-is-a-black-hole-k4.html

- Gravitational waves - https://www.gw-openscience.org/path/

- Electro magnetic spectrum - https://imagine.gsfc.nasa.gov/science/toolbox/emspectrum1.html

- General Relativity theory - https://en.wikipedia.org/wiki/Introduction_to_general_relativity

- Ligo - https://www.ligo.caltech.edu/
- Virgo - https://www.virgo-gw.eu/



Inspiration

- https://youtu.be/e-P5IFTqB98

- https://www.stem.org.uk/esero/resources

- https://youtu.be/tzQC3uYL67U

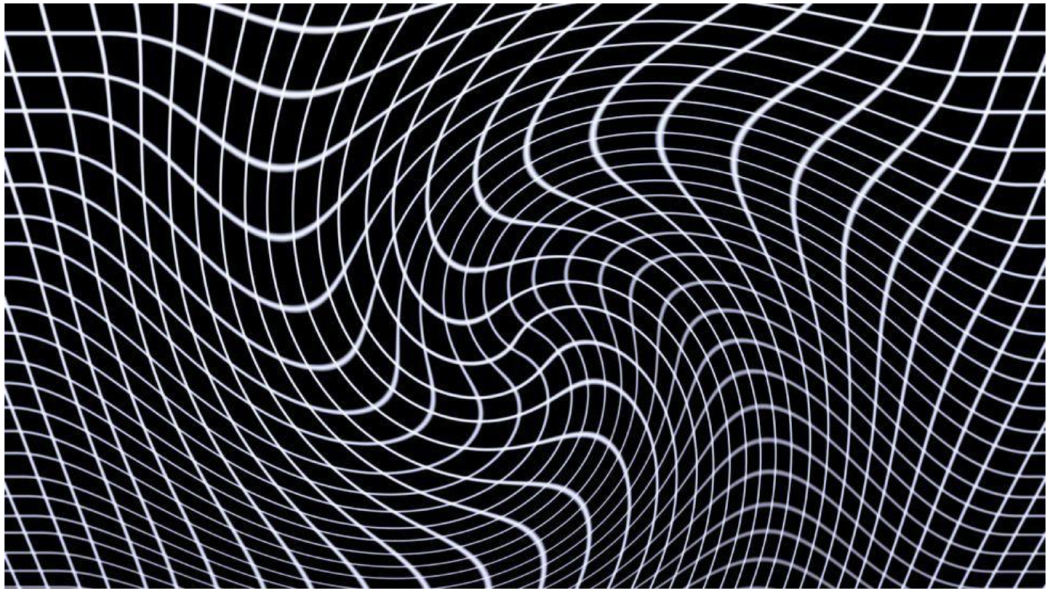
- Gravitational Waves hit the Late Show

Presentation for students

Introduction to the topic

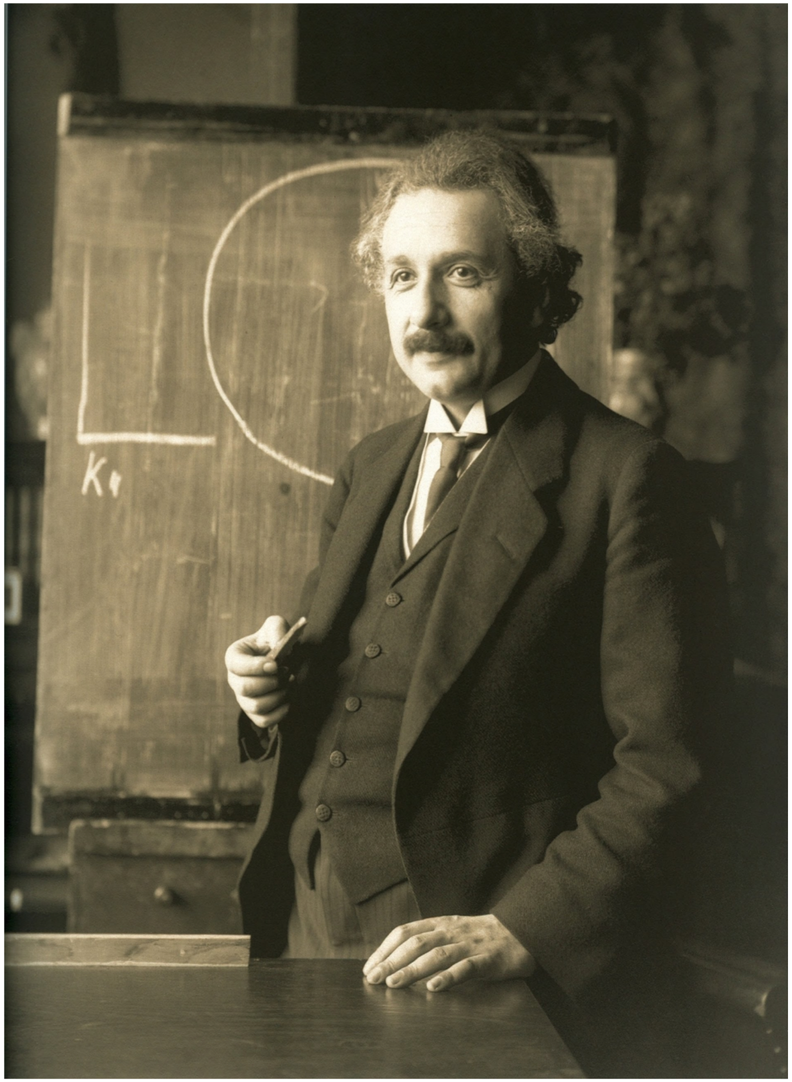
Teacher guidelines can be found in the notes attached to each slide

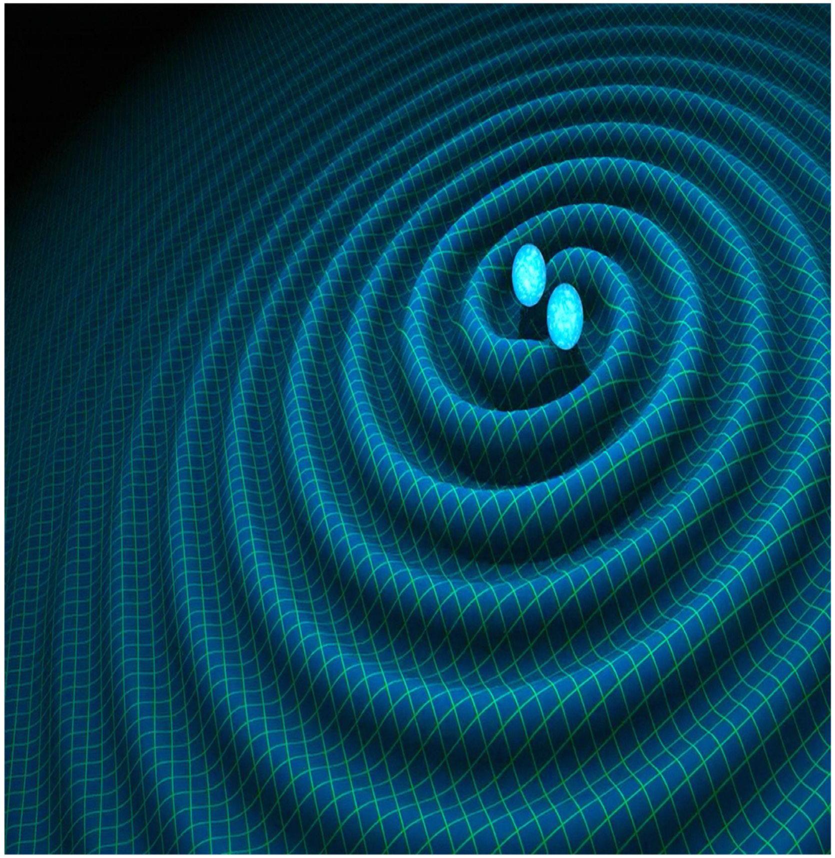
Historical introduction to Gravitational Waves



- 1915. A. Einstein published the General Theory of Relativity

- Space - time becomes an entity: it can be curved by matter and energy and waves can propagate through it





- A hundred years later, in 2015, gravitational waves have been detected
- the merger of two black holes radiate gravitational waves detected by LIGO and Virgo
- What do you think, why do we needed a hundred years to confirm existence of gravitational waves?

What do we “feel” on Earth when gravitational waves pass by us?

So, how can we measure it?

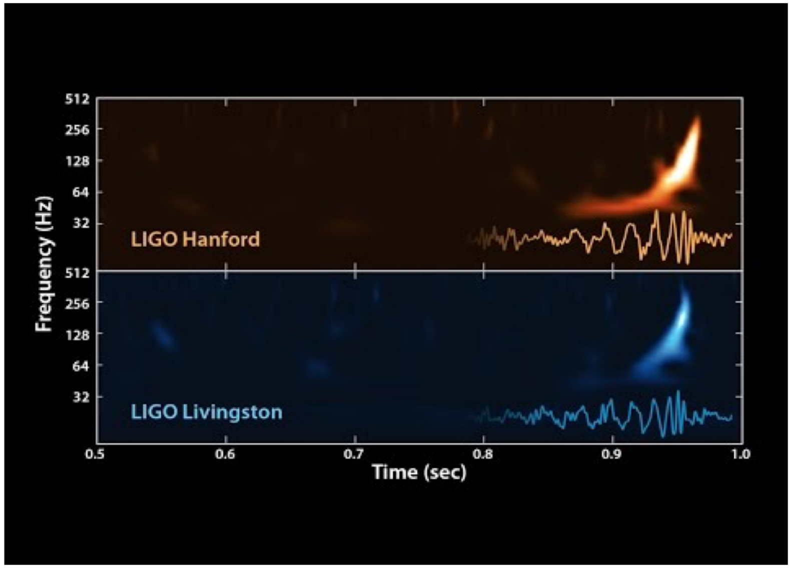
LIGO

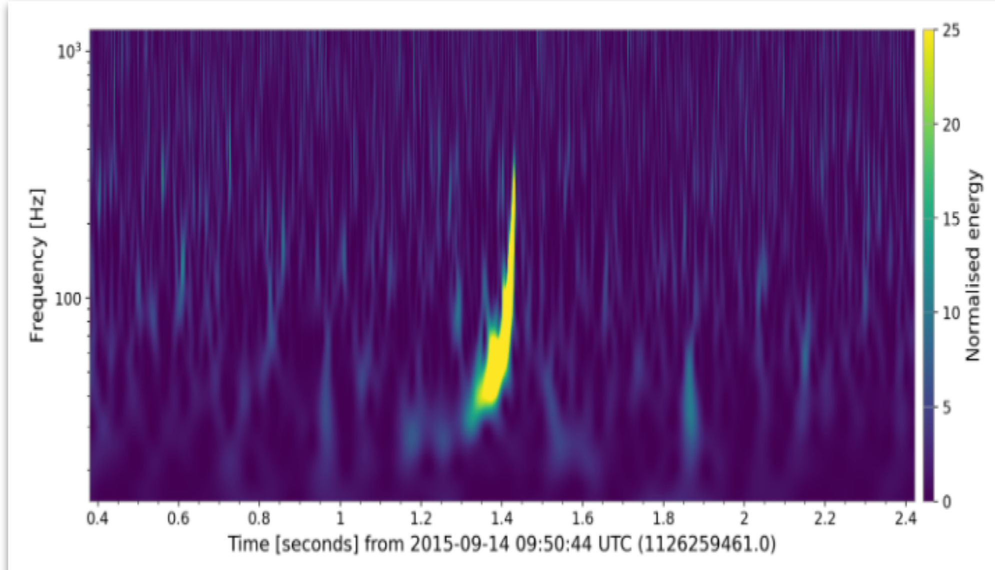
- Laser Interferometer Gravitational-Wave Observatory
- Two large observatories built in USA to detect gravitational waves by laser interferometry and develop gravitational-wave observations as an astronomical tool.
- The arms are 4 km long and have many (at least 10) mirrors which make possible the detection of a change in distance less than one ten-thousand the diameter of a proton.
- Built: 1994-2002

VIRGO

- A large interferometer designed to detect gravitational waves.
- Located in Pisa, Italy
- The interferometer has two 3 km long arms
- Works in the same way as LIGO
- Since 2007, Virgo and LIGO have agreed to share and analyze together the data recorded by their detectors and publish their results together.
- Why? The signals from a gravitational wave are so weak that the results are better if many instruments record them simultaneously because through triangulation the source of GW can be better localized.

The “Chirp” - The Sound of Two Black Holes Colliding





On September 14, 2015, LIGO observed gravitational waves from the merger of two black holes, each about 30 times the mass of our sun. The incredibly powerful event, which released 50 times more energy than all the stars in the observable universe, lasted only fractions of a second.
Gravitational waves sent out from a pair of colliding black holes have been converted to sound waves, as heard in this animation.

Play time!

In **Black Hole Hunter** game, your objective is to listen to gravitational wave detector data and determine whether or not you can hear the given gravitational wave signal in the sound file, or whether it is just noise. So click Play Now, and hear the sound track of the universe ...

Link to the game: https://blackholehunter.org/

Presentation for students
- Basic level-

Teacher guidelines can be found in the notes attached to each slide

Worksheet
Hands on activities

Activity 1.

For this activity, you will need a hula hoop, an elastic sheet, and balls of different sizes, weights and densities.

Performing experiments: Let students put an elastic sheet over a hula hoop and hold it in their hands. In the middle of the elastic sheet put a light ball. What happens with the elastic sheet?

Change balls on the sheet. What happens with a curvature of the elastic sheet when you put a heavier ball in a center of the sheet?

In the middle of the elastic sheet put a heavy ball and then from an edge of the sheet put to move a small ball. What happens?

What happens when you put to move with some speed small ball from the edge of the sheet along with the edge of the hula hoop? Can you describe the trajectory of the small ball?

Why the trajectory has that curvature? Which law explains that?

What happens when you put a small ball with greater velocity?

How can you explain that?

How does the velocity of a small ball on the curve around a large ball change? Which law explains that?

Activity 2.

On the same elastic sheet on the hula hoop put, one by one, balls with higher density. What happens with the curvature of the elastic sheet? When the curvature is the greatest?

Small ball with the highest density we can consider a black hole.

What happens when you spin two small balls with high density around the edge of the hula hoop?

What happens with the elastic sheet when you spin two small balls with high density around the edge of the hula hoop?

Do you see the perturbations on the elastic sheet or the students who hold the sheet feel the shaking in their hands? Why?

What can you conclude about the perturbation of the elastic sheet?

Can gravitational waves propagate without space-time?

Write 10 to 15 sentences about this exercise.

Conclusions

Write a conclusion in which you describe what you have learned from your work in these activities.

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What are some specific ideas or skills that you have learned or improved upon?

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What information did you learn by doing this experiments?

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What insights did you gain about the process of origin of gravitational waves?

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What would you like to learn/know more

____

Presentation for students
- Advanced level-

Teacher guidelines can be found in the notes attached to each slide

Worksheet for students

Key concepts, check on previous learning, transfer

Gravitational waves are disturbances in the curvature of space-time, generated by accelerated masses, that propagate as waves outward from their source at the speed of light.

Gravitational waves transport energy as **gravitational radiation**, a form of radiant energy similar to electromagnetic radiation.
As with other waves, there are a number of characteristics used to describe a gravitational wave:

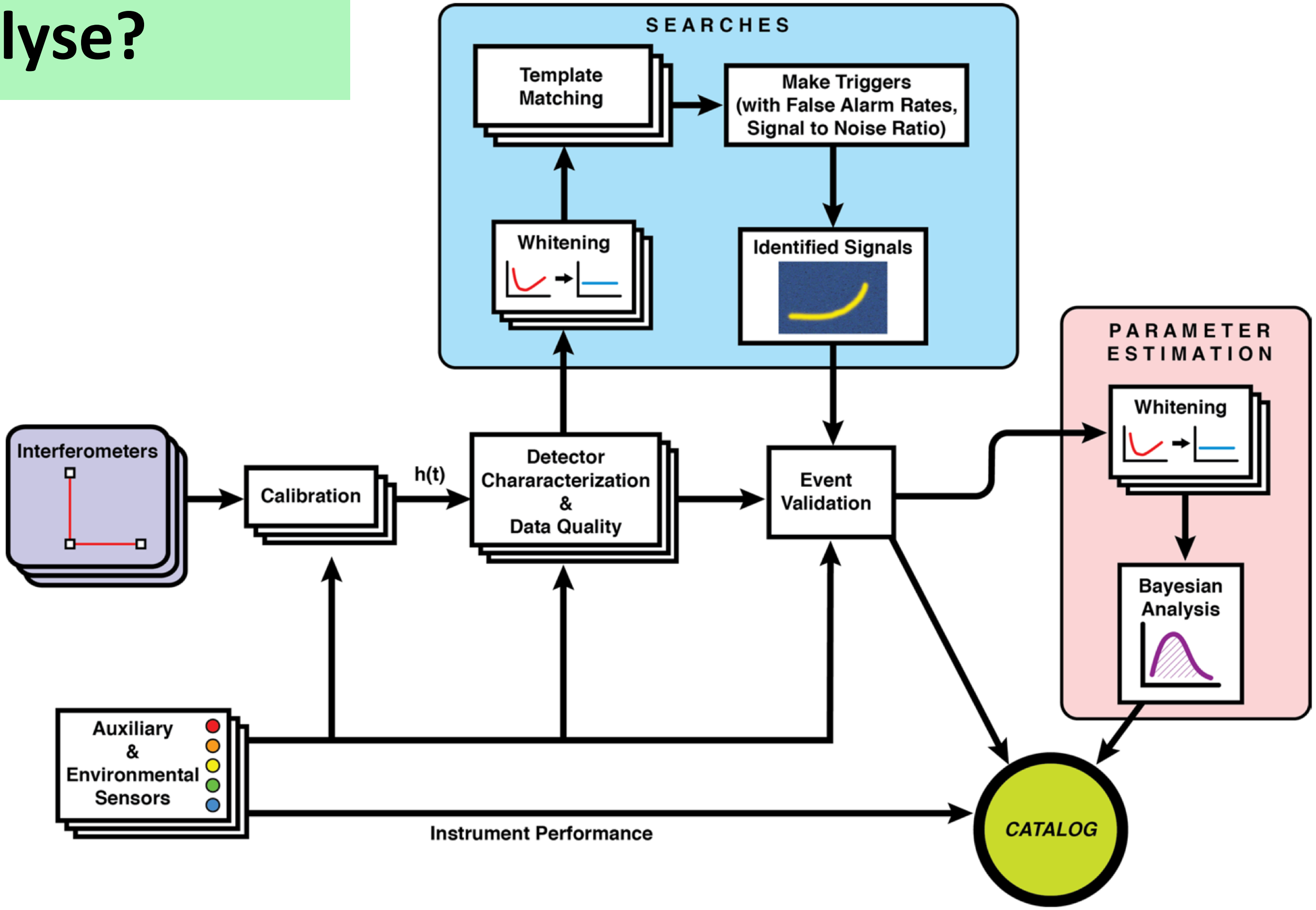
- **Amplitude, *h*, (**strain - dimensionless) this is the size of the wave: $h \approx 10^{-20}$.
- **Frequency**, **(*f*)**, this is the frequency with which the wave oscillates (1 divided by the amount of time between two successive maximum stretches or squeezes)
- **Wavelength, (*λ*),** this is the distance along the wave between points of maximum stretch or squeeze.
- **Speed, (c)**, This is the speed at which a point on the wave (for example, a point of maximum stretch or squeeze) travels. For gravitational waves with small amplitudes, this wave speed is equal to the speed of light.

The speed, wavelength, and frequency of a gravitational wave are related by the equation

$$c = \lambda f$$

just like the equation for a light wave.

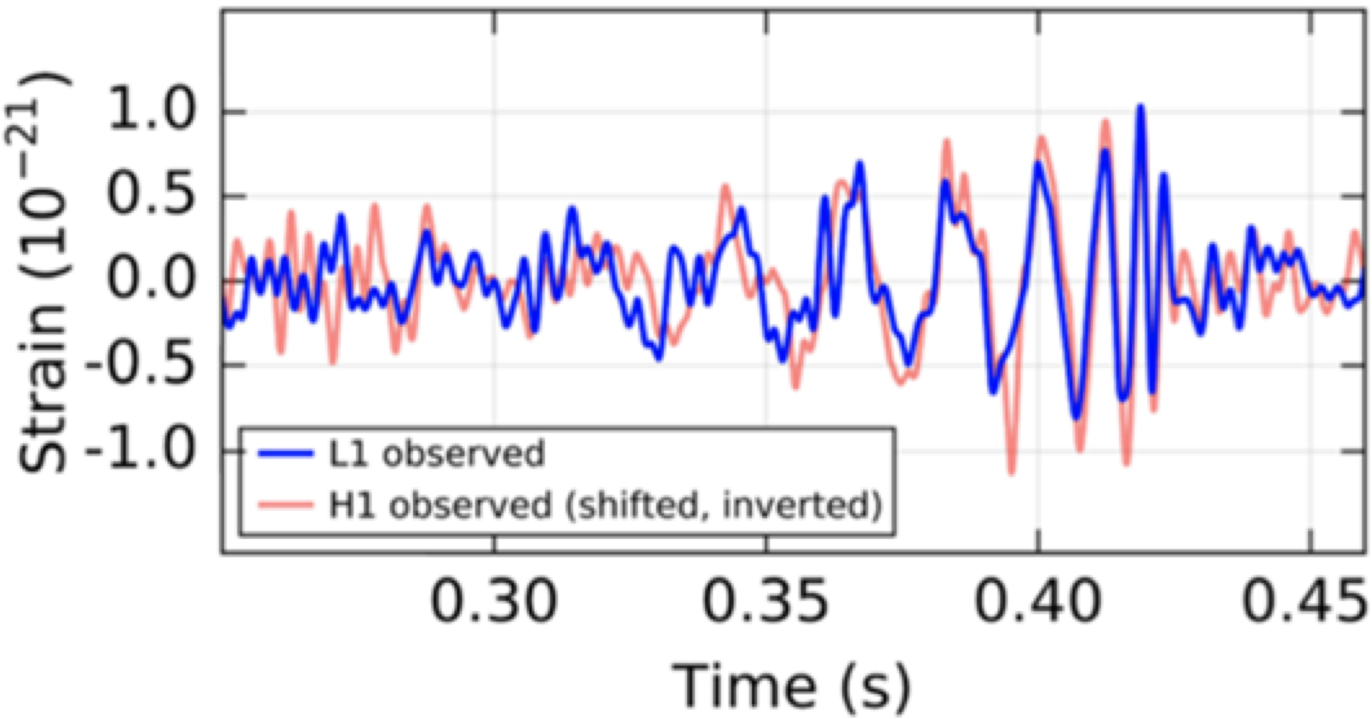
How was obtained the graph that you are going to analyse?



A simplified schematic summarizing the main steps in LIGO–Virgo data processing, from the output of the data to the results reported in a catalog of transient events.
B P Abbott et al 2020 Class. Quantum Grav. 37 055002

Activity 1

Analyse this graph of «a chirp » (a signal increasing in frequency and amplitude) and answer to the following questions:



1. What are the titles of the axis and their units:

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Is anything odd regarding the strain's order of magnitude?
Calculate the ratio between the size of a microbe (micrometre) and the Earth-Moon distance (400,000 km). Compare with strain's order of magnitude!
Can you find another example closer to the strain's order of magnitude?

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2. a. What happens with the strain value in time?

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b. What happens to the frequency?

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3. In your opinion what type of interaction can produce this kind of gravitational waves? Explain why.

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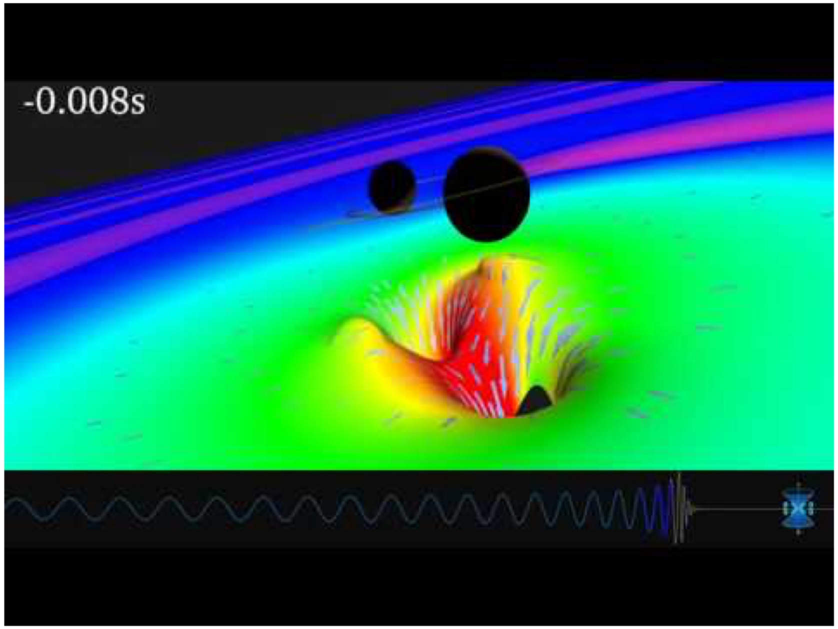
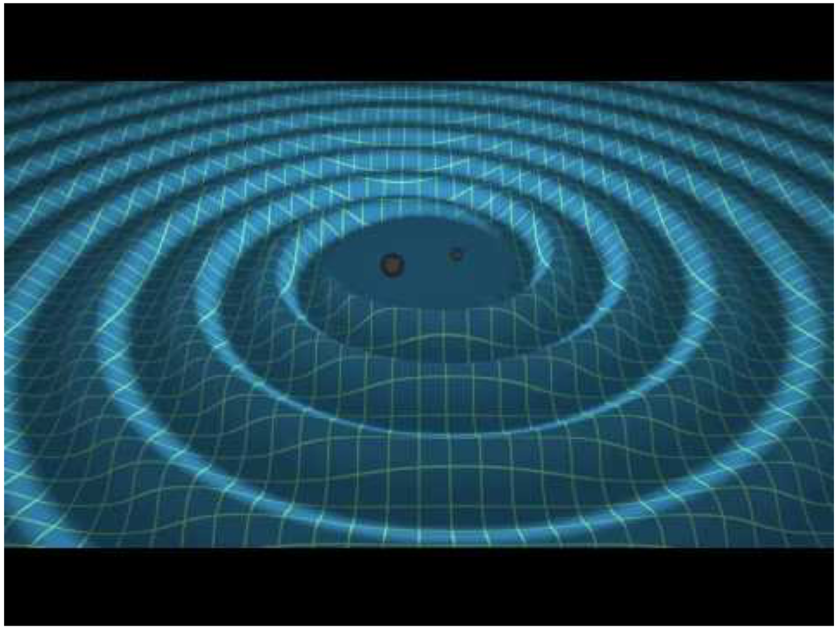
Watch the 2 videos! Is the model you imagined similar with what is presented in these 2 animations?





Time to investigate!

Link: http://data.cardiffgravity.org/waveform-fitter/

Play a little with the Waveform Fitter

4. Describe what happens with the signal characteristics when modify the parameters:

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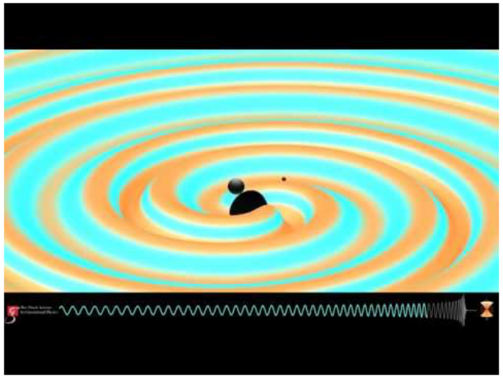


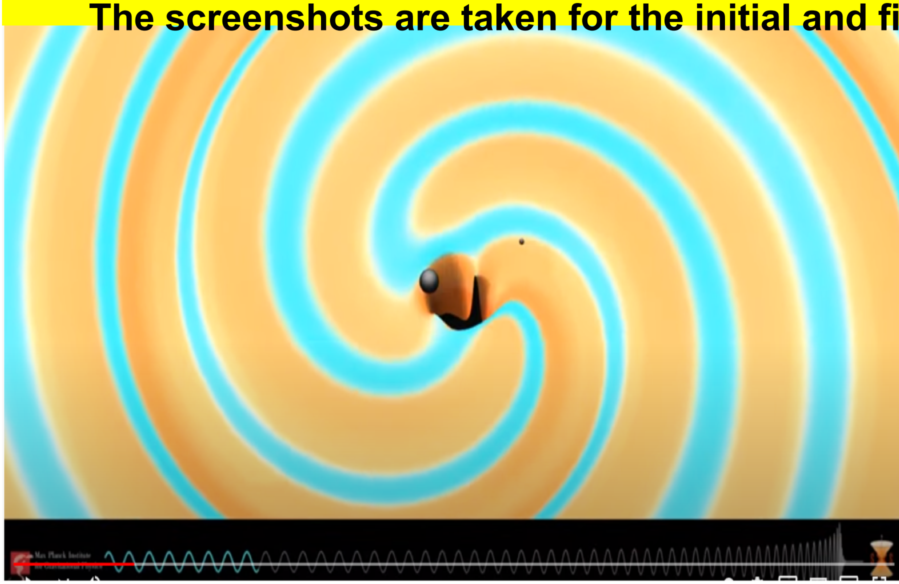
Look carefully at the video!

What can you say about the rotation frequencies of the 2 bodies?

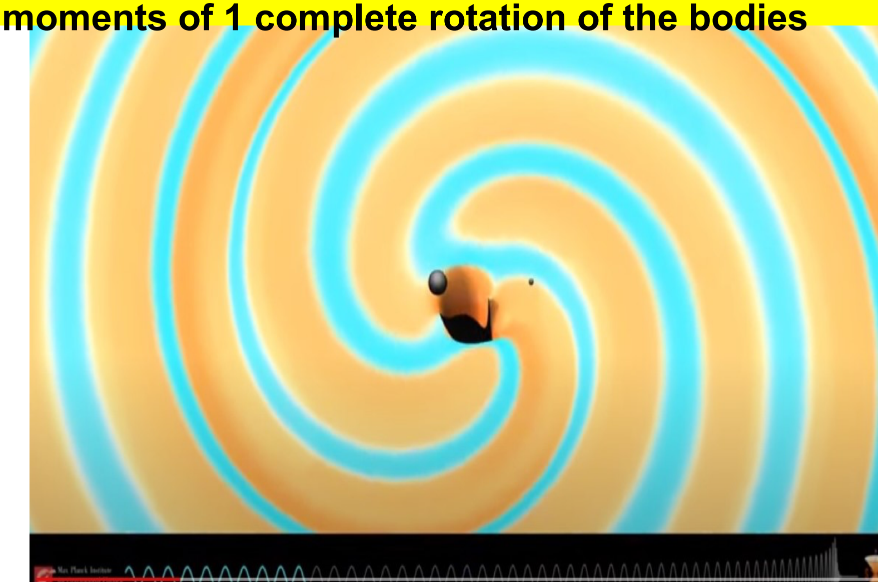
5. Find the relationship between the frequency of the gravitational wave (f_{GW}) and the orbital frequency (f_{orb}) of the sources?

f_{GW}= ______________________________

The screenshots are taken for the initial and final moments of 1 complete rotation of the bodies



Number of gravitational wavelengths=_________
(in blue, at the bottom of the page)



Number of gravitational wavelengths=_________
(in blue, at the bottom of the page)

Activity 2

How to determine Chirp mass from frequencies

Chirp mass (M) = is the effective mass of a binary system (black holes or neutron stars able to produce detectable gravitational waves), in the context of the Quadrupole Gravitational Radiation emitted by it.

The larger the mass of the objects, the larger is the gravitational-wave emission. The larger is the gravitational-wave emission, the larger is the gravitational-wave emission during the inspiral of the system (and thus the increase of the frequen [illegible] es).

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

Relationship between chirp mass and gravitational waves frequency (a rather complicated formula):

$$\mathcal{M} = \frac{c^3}{G}\left(\left(\frac{5}{96}\right)^3 \pi^{-8} (f_{\mathrm{GW}})^{-11} \left(\dot{f}_{\mathrm{GW}}\right)^3\right)^{1/5}$$

Where **c**=3 ·10⁸ m/s is the speed of light

f_{GW} is the frequency of the gravitational wave

$\dot{f}_{GW}$ is the frequency rate of change with time of the gravitational wave

G= 6.67408 × 10^{-11} m^3 kg^{-1} s^{-2} gravitational constant

Linearising the Chirp mass - frequency formula we get:

$$f_{\mathrm{GW}}^{-8/3}(t) = \frac{(8\pi)^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(t_c - t)$$

t_c is the coalescence time

Use Microsoft Excel or other software to perform the calculations.

Use the given data in the previous slide and represent the f^(-8/3) with respect of time (use linear trendline):

1. Calculate the slope of the graph (K): K= ______________________________

2. Knowing the slope K calculate the chirp mass (suggestion: insert in excel the Chirp Mass formula)

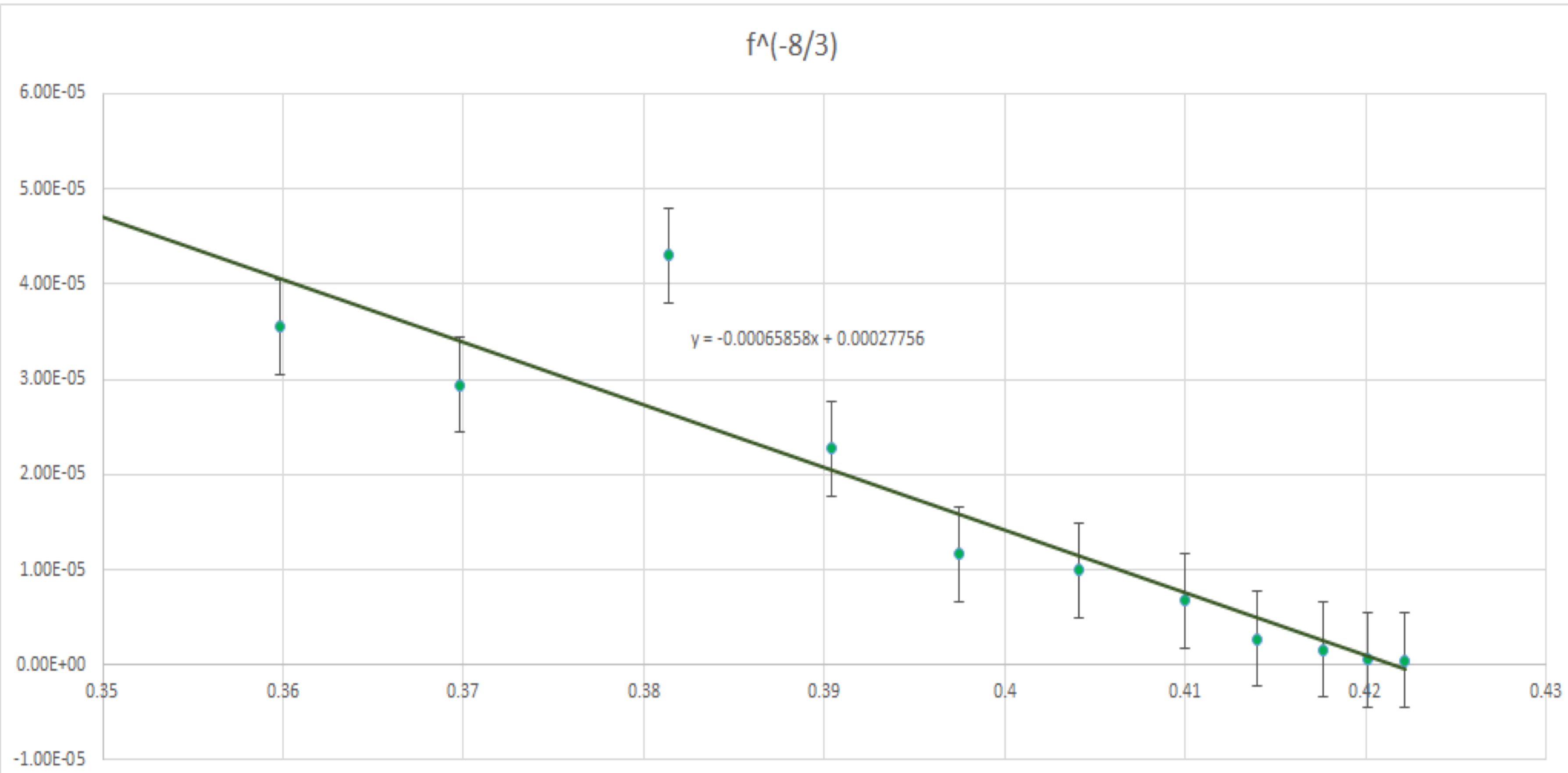
M= ____

K= $\frac{(8\pi)^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}$ $\rightarrow$ $\mathcal{M} = \left[\frac{5K}{(8\pi)^{8/3}}\right]^{3/5} \cdot \frac{c^3}{G} = \frac{c^3}{G} \cdot \frac{5K^{3/5}}{(8\pi)^{8/5}}$

3. Assuming the 2 bodies have the same mass (m_1=m_2=m) calculate the total mass of the system before merging

mtot=____

t(s)	0.349038	0.359789	0.369778	0.381385	0.390423	0.397463	0.404123	0.410021	0.414017	0.417632	0.420106	0.422199
f^(-8/3) Hz^(-8/3)	4.74E-05	3.55E-05	2.94E-05	4.30E-05	2.27E-05	1.16E-05	9.91E-06	6.75E-06	2.74E-06	1.62E-06	5.98E-07	5.13E-07



Activity 3

1.What is the distance (d) between the colliding objects?

Using the following formula (for 2 identical bodies, each of mass m, $\mathbf{f_{orb}}$ is the frequency of rotation of each body)

$$\mathbf{d} = \left(\frac{\mathbf{G} \cdot \mathbf{2m}}{(\mathbf{2\pi f}_{orb})^2}\right)^{\frac{1}{3}}$$

Using the mass of a body obtained in the Activity 2 calculate the distance between the 2 bodies before collision (let's assume $\mathbf{f_{orb} = f_{GW}/2}$=100 Hz - approximately the orbital frequency right before the merger)

d=__________________________________

- Facultative task:

Based on the Newtonian physics calculate the spatial separation for 2 rotating objects with different masses

2. Based on the results obtained for the distance between the 2 bodies and their masses what is the conclusion?

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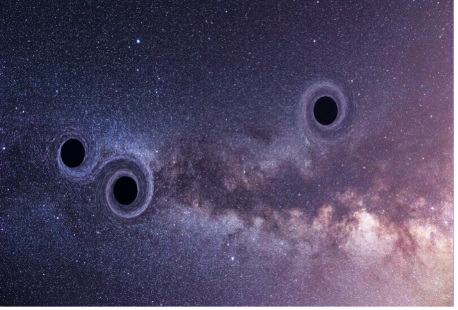
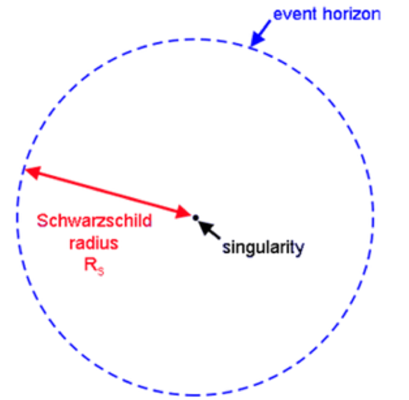


Are the collapsing bodies Black holes?



Short reminder: What is a Black Hole?

A black hole is another consequence of the general theory of relativity. It is an object so compact that nothing can escape from it, even if it travels at the largest possible speed: the speed of light. The region inside which nothing can escape is defined by the "black-hole horizon". The horizon is a mathematical surface (not a hard surface, like the one of the Earth or the surface of a metal ball): entering the horizon of the black-hole means to be lost forever. The radius of the horizon is called the **Schwarzschild radius** and depends on the mass of the black hole.

$$R_S = \frac{2Gm}{c^2}$$

3. Using the mass calculated for the each body, (equal masses assumed), before collapsing, find the Schwarzschild radius, Rs:

R_S=______________________(km)

4. Based on the obtained results make a sketch of the 2 bodies annotating the distances, and the Rs:

Write a report on what you have done addressing the following points (context, methodology, results, discussions).

Report:

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Conclusions

Write a conclusion in which you describe what you have learned from your work in these activities.

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What are some specific ideas or skills that you have learned or improved upon?

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What information did you learn by doing the calculations?

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What insights did you gain about the process of analyzing gravitational wave data?

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What would you like to learn/know more?

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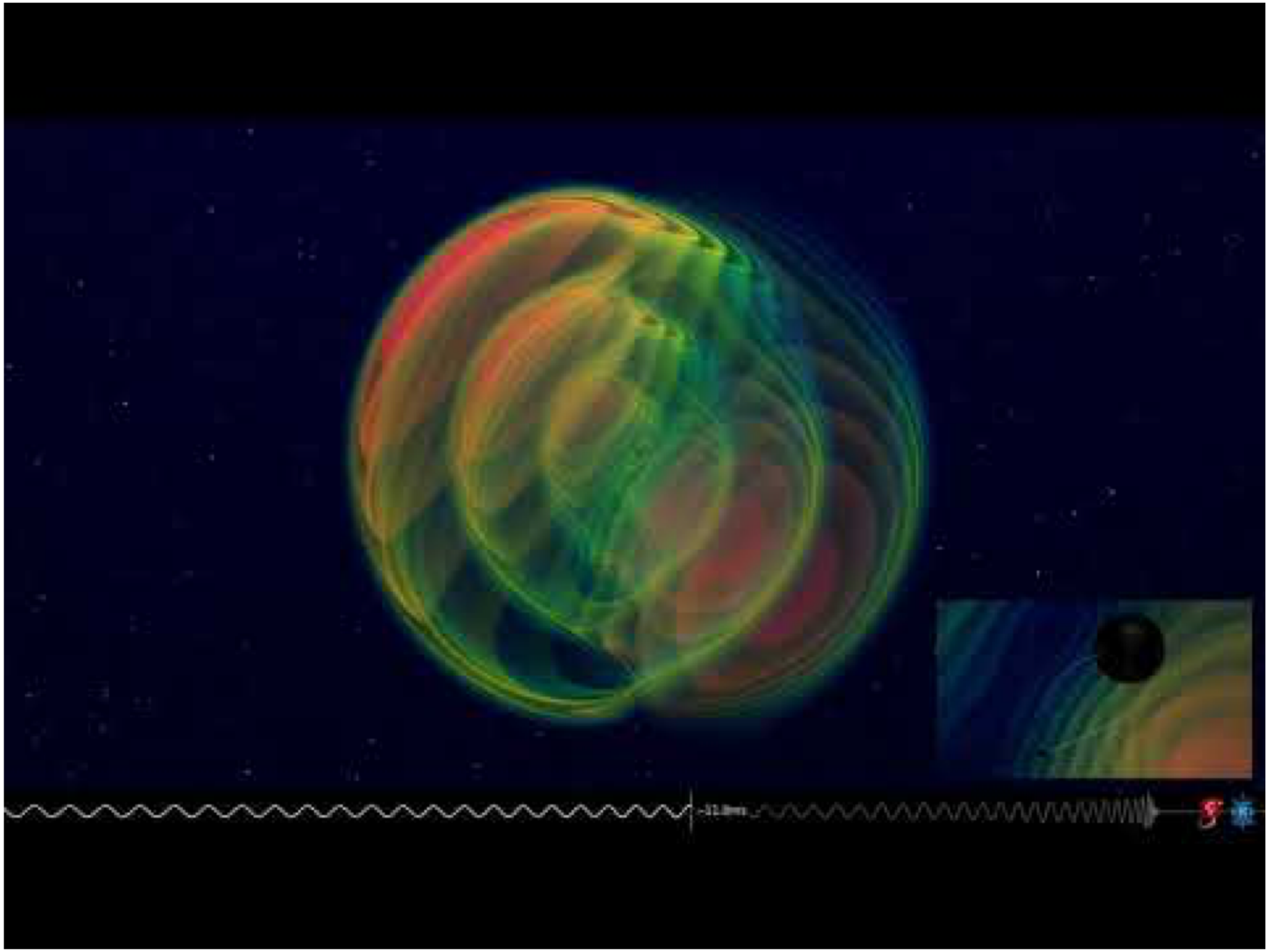
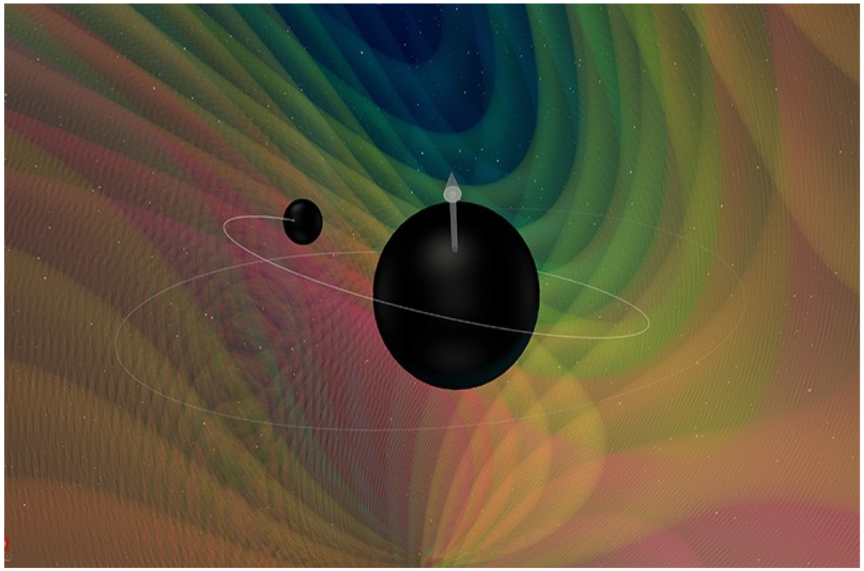


Thank you!

Chirp Mass Working group for Gravitational waves:



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