# Deep Learning Taxonomy

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https://xkcd.com/1838/



# What is Deep Learning about?

Intuitively, machine learning (and by extension deep learning) studies how to teach machines (agents) to *solve tasks* with little to no human intervention by learning from interaction with the particular task (i.e. from data). The solution is not prescribed by the practitioner, just the learning strategy (and encoding of the solution).

Deep Learning focuses on a particular family of *parametric* approaches that has been extremely successful recently. This usually indicates a particular family of learning algorithms (typically gradient-based ones) and architectures (neural networks).



## Starting point: Supervised Learning

- Let  ${\mathcal X}$  denote the space of input values
- Let  ${\mathcal Y}$  denote the space of output values
- Given a data set  $D \subset \mathcal{X} \times \mathcal{Y}$ , find a function:

$$h: \mathcal{X} \to \mathcal{Y}$$

such that  $h(\mathbf{x})$  is a "good predictor" for the value of y.

- *h* is called a *hypothesis*
- Problems are categorized by the type of output domain
  - If  $\mathcal{Y} = \mathbb{R}$ , this problem is called *regression*
  - If  $\mathcal{Y}$  is a categorical variable (i.e., part of a finite discrete set), the problem is called *classification*
  - In general,  $\mathcal{Y}$  could be a lot more complex (graph, tree, etc), which is called *structured prediction*

### $h: \mathcal{X} \to \mathcal{Y}$

#### **non-parametric** k nearest neighbour classifier



#### **parametric** Linear models Neural Networks



A parameterized function is a function:

$$h: \theta \times \mathcal{X} \to \mathcal{Y}$$

for example a linear function of the form

$$h(\mathbf{w}, x) = \mathbf{w}x$$

Learning then boils down to finding the best  $\theta$  to minimize the distance between prediction and targets

$$\arg\min_{\boldsymbol{\theta}} L(\boldsymbol{\theta}) = \arg\min_{\boldsymbol{\theta}} \mathbb{E}\left[dist(h(\boldsymbol{\theta}, x_i), y_i)\right]$$

We will focus on two questions:

What is a good parametrization ?

How do we find these optimal parameters ?

#### What is a good parametrization?

The linear model 
$$\,h({old w},x)={old w}x$$

 $\begin{array}{c} \star_{2} \\ h(u, \mathbf{x}) > 0 \\ h(u, \mathbf{x}) > 0 \\ h(u, \mathbf{x}) < 0 \end{array}$ 

- Can straightforwardly be used for classification (and regression)
- Defines linear decision boundaries (and represents linear functions)

This (intuitively) is not enough

#### What is a good parametrization ?



Most problems are not linearly separable.

Neural networks to the rescue!

$$h(\mathbf{x}, \theta) = \mathbf{W}_{out} ReLU(\mathbf{W}_{in}\mathbf{x} + \mathbf{b}_{in}) + \mathbf{b}_{out}$$

$$ReLU(x) = \begin{cases} x & x > 0\\ 0 & \text{otherwise} \end{cases}$$

- Other non-linearities are possible
- Why have a non-linearity?





**Deep** in deep learning:

 $h(\mathbf{x},\theta) = \mathbf{W}_k ReLU(\mathbf{W}_{k-1}ReLU(\mathbf{W}_{k-2}\dots ReLU(\mathbf{W}_1\mathbf{x}+\mathbf{b}_1)+\ldots)+\mathbf{b}_{k-1})+\mathbf{b}_k$ 

The term *deep learning* or *deep neural networks* was introduce to highlight the importance of *depth*.

• What does depth provide? Why is it the secret sauce?





Partitioning into regions by second hidden layer











<u>Guido Montufar et al 2014, On the number of linear regions of Deep Neural</u> <u>Networks</u>

By folding the space, you gain expressivity! (exponentially more linear regions) without increasing the number of parameters!

- Is having exponentially more linear regions a good thing? Can this explain the success of DL?
- Is the limiting factor of previous methods expressivity?
- Do shallow model underperform because of lack of capacity?









- Is having exponentially more linear regions a • he answer is probably no good thing? Can this explain the success of DL?
- Is the limiting factor of previous method • expressivity?
- Do shallow model underpa lack of capacity?





**Convolutional Networks** 

- Depth acts as an inductive bias (constraints the solution).
- Do we understand what this inductive bias is?



Honglak Lee et al. 2009, Convolutional Deep Belief Networks for Scalable Unsupervised Learning of Hierarchical Representations How do we find the optimal parameters ?

$$\arg\min_{\theta} L(\theta) = \arg\min_{\theta} \mathbb{E} \left[ dist(h(\theta, x_i), y_i) \right]$$

$$Optimization \checkmark Learning/Generalization$$

Optimization: Gradient Descent









The celebrated **Backpropagation Algorithm** 



$$\theta_{t+1} = \theta_t - \eta \frac{\partial L}{\partial \theta}$$

- Why is it important to start from the output towards the input?
- How is it different from the chain rule?

The IID assumption in Gradient Descent









Andrew Saxe et al 2013, Exact solutions to the nonlinear dynamics in deep linear models

Learning has **tug-of-war** dynamics to resolve credit assignment

- This requires data to be **I.I.D.**
- No explicit knowledge composition
- Failure in credit assignment leads to catastrophic forgetting

Continual Learning studies credit assignment in learning, and in particular the effects of this tug-of-war dynamics.

Resolving continual learning means finding better/different mechanism of doing credit assignments. CL is about optimization/learning.

It has big implications for our understanding of the learning process for neural networks.













 Statistical physics (on random gaussian fields) [Bray and Dean, 2007, Fyodorov and Williams, 2007]





Wojciech Czarnecki, Simon Osindero, Razvan Pascanu, Max Jaderberg, A neural network's loss surface contains every low dimensional pattern





(c)



https://arxiv.org/pdf/1406.2572.pdf



Chaoyue Liu, Libin Zhu, Mikhail Belkin, Loss landscapes and optimization in over-parameterized non-linear systems and neural networks



Initialization (and gradient propagation)



He et al. ICCV 2015 adds a correction for ReLU.

It is always useful to step back, and don't think of your neural network as a function. Look at its structure, and at how gradients propagate.

Looking back at activations

- One can reason about gradient propagation (saturated regimes)
- Still an active area of research (e.g. <u>Gu et al. 2020</u>)



Stochastic GD vs Gradient descent: estimate true gradient by using a small subset of datapoints.

$$\nabla f \stackrel{\text{def}}{=\!\!=} \sum_i \nabla f_i$$
$$i \sim \mathcal{U}[1, N] \to \nabla f \approx \nabla f_i$$

$$\nabla f$$





Stochastic Gradient Descent





Learning / Generalization

**Example: Data and best linear hypothesis** y = 1.60x + 1.05



### **Order-2 fit**



Is this a better fit to the data?

### **Order-9 fit**



Is this a better fit to the data?

### Neural Net

Neural Net Neural Net  $h(x) = a \cdot x^2 + b \cdot x + c$  $h(x) = a \cdot x + b$ |W| small

Universal

approximator |W| -> ∞

/W/ large


Universal approximator

#### UNDERSTANDING DEEP LEARNING REQUIRES RE-THINKING GENERALIZATION

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• We want to be able **to generalize** 

#### Use

**Training set:** data used for finding the right parameters

Validation set: data used to estimate true loss on unseen data

Learning is about minimizing an intractable function via optimizing a tractable approximation of it





- Priors/regularization terms and inductive biases provide mechanisms to introduce knowledge in the learning problem
- It restricts the search space for the parameters of the model
- They can take various forms:
  - Parametrization restricts the search space, making certain solutions unrepresentable
  - Or it can make the loss surface such that certain solutions are easier to find given initial conditions (e.g. what is the role of depth in deep learning)





Double descent curve from *Reconciling modern machine learning practice and the bias-variance trade-off* Belkin et al. <u>https://arxiv.org/abs/1812.11118</u>

Implicit regularization of GD & SGD (helpful noise)



More recently (ICLR'17) arXiv:1609.04836

We can change flatness (largest eigenvalue) without changing the function ! We need a more robust measure of flatness <u>https://arxiv.org/pdf/1703.04933.pdf</u>



#### Implicit bias of SGD / GD

https://arxiv.org/abs/2009.11162 https://openreview.net/forum?id=rq\_QrOc1Hyo

$$\widetilde{C}_{SGD}(\omega) = C(\omega) + \frac{\epsilon}{4m} \sum_{k=0}^{m-1} ||\nabla \widehat{C}_k(\omega)||^2.$$

Punch line: SGD optimizes a different objective than GD!

Not all noise is equal ! https://arxiv.org/abs/2105.13343 (Noise from data augmentation hurts generalization, while noise from data sampling helps)



• We need Out-of-Distribution Generalization in order to solve tasks (not fit datasets). However IMHO we need to define properly what we want as I believe there is a lot of confusion (and misunderstanding) in this space. But is at the core of understanding what learning can do!

# Summary

- *DL* corresponds to a family of parametric models (neural networks) optimized by gradient descent techniques
- NN are typically layers of linear projects (sometimes with additional constraints) followed by non-linearities
- They operate in an over-parametrized regime more often than note
- Magic sauce of DL is restricting the search in the learning process to get to solutions that generalize. This is done by architecture design, choice of loss function and alterations of the optimizer.

# Taxonomy of Deep Learning Architectures



- Abundance of resources online showing how to instantiate typical architectures using pytorch / jax / tensorflow
- Some examples:
  - EEML practical sessions:
    - https://github.com/eemlcommunity/PracticalSessions2021 https://github.com/eemlcommunity/PracticalSessions2020 https://github.com/eemlcommunity/PracticalSessions2019
  - https://github.com/deepmind/educational
  - <u>https://roberttlange.github.io/posts/2020/03/blog-post-10/</u>
  - 0



### $h(\mathbf{x}, \theta) = \mathbf{W}_{out} ReLU(\mathbf{W}_{in}\mathbf{x} + \mathbf{b}_{in}) + \mathbf{b}_{out}$

MLP

- Versatile and important/heavily used architecture
- No structure for the data
- Relies on the bias introduced by depth





Convolutional Network

## **Convolutional Neural Networks**



## **Convolutional Neural Networks**

- Structural prior: spatial neighbourhood defines the role of a pixel
- Apply **same function** at all position
- Induces translation invariance as features are computed independent of position

Can an MLP reproduce a ConvNet?



Stefan Carlsson (KTH): <u>https://arxiv.org/abs/1905.08922</u>



#### Can an MLP reproduce a ConvNet?

• Yes, and you end up with circular matrices

$w_1$	$w_2$		$w_k$	0					0
0	$w_1$	$w_2$		$w_k$	0				0
0	0	$w_1$	$w_2$		$w_k$	0			0
				:					
0			0	$w_1$	$w_2$				$w_k$
$w_k$	0			0	$w_1$	$w_2$			$w_{k-1}$
$w_{k-1}$	$w_k$	0			0	$w_1$	$w_2$		$w_{k-2}$
			:						
$w_2$		$w_k$	0					0	$w_1$



- One typically stays close to an existing architecture:
  - AlexNet
  - VGG
  - $\circ$  Inception
  - Resnet
  - NFNet
  - MobileNet
  - $\circ$  DenseNet
  - etc.

- Pooling is not typically used, but is a great tool to reduce computation
- Large optimization space, usually tuned on ImageNet and relying on intuition
- Bigger is better (though one hits diminishing returns)

#### Resnets - powering computer vision

X

y

loffe & Szegedy

 $\overline{W}$ 



## Initialization in Deep Learning

De & Smith: Batch Normalization Biases Deep Residual Networks Towards shallow paths <u>https://arxiv.org/pdf/2002.10444.pdf</u> Stabilizing Transformer for RL: Parisotto et al. 2020



Reconciling modern machine learning practice and the bias-variance trade-off Belkin et al. https://arxiv.org/abs/1812.11118

Jonathan Frankle, Michael Carbin 2018, The lottery ticket hypothesis



3.0



MLP





Recurrent Network

Convolutional Network

### **Recurrent Neural Networks**



Pascanu et al. 2014

$$\begin{aligned} \mathbf{h}_t &= f_h(\mathbf{x}_t, \mathbf{h}_{t-1}) \\ \mathbf{y}_t &= f_o(\mathbf{h}_t), \end{aligned}$$



Pascanu et al. 2014

Deep RNNs..











#### **Exploding Gradients**



$$\frac{\partial C}{\partial \mathbf{W}} = \sum_{t} \frac{\partial C(t)}{\partial \mathbf{W}} = \sum_{t} \sum_{k=0}^{t} \frac{\partial C(t)}{\partial \mathbf{h}(t)} \frac{\partial \mathbf{h}(t)}{\partial \mathbf{h}(t-k)} \frac{\partial \mathbf{h}(t-k)}{\partial \mathbf{W}}$$
$$\frac{\partial \mathbf{h}(t)}{\partial \mathbf{h}(t-k)} = \prod_{j=k+1}^{t} \frac{\partial \mathbf{h}(j)}{\partial \mathbf{h}(j-1)}$$



The error is  $(h(50) - 0.7)^2$  for  $h(t) = w\sigma(h(t-1)) + b$ with h(0) = 0.5





#### Vanishing Gradients



$$\frac{\partial C}{\partial \mathbf{W}} = \sum_{t} \frac{\partial C(t)}{\partial \mathbf{W}} = \sum_{t} \sum_{k=0}^{t} \frac{\partial C(t)}{\partial \mathbf{h}(t)} \frac{\partial \mathbf{h}(t)}{\partial \mathbf{h}(t-k)} \frac{\partial \mathbf{h}(t-k)}{\partial \mathbf{W}}$$
$$\frac{\partial \mathbf{h}(t)}{\partial \mathbf{h}(t-k)} = \prod_{j=k+1}^{t} \frac{\partial \mathbf{h}(j)}{\partial \mathbf{h}(j-1)}$$



- Weights largest eigenvalue < 1, damping regime [fixed point attractor]
- Weights close or 1, information travels through the system
- Weights largest eigenvalue > 1, potentially in a chaotic regime

Echo State Network literature, e.g.: <u>http://www.scholarpedia.org/article/Echo\_state\_network</u> Ilya Sutskever et al 2013, <u>On the importance of initialization and momentum in deep learning</u>

#### Saxe et al. 2014 :

• Orthogonal weights as solution for deep linear models

<u>Henaff et al. 2016</u>; <u>Arjovski et al. 2016</u>

• Reparametrize RNN so recurrent weights stays orthogonal



But we do need to forget !?

$$\frac{\partial (x_1 + x_2 + x_3)}{\partial x_3} = 1$$

$$x_1 + x_2 + \frac{x_3}{3} = 10$$

 $x_3 = ?$ 

$$i_{t} = \sigma \left( W_{xi}x_{t} + W_{hi}h_{t-1} + W_{ci}c_{t-1} + b_{i} \right)$$

$$f_{t} = \sigma \left( W_{xf}x_{t} + W_{hf}h_{t-1} + W_{cf}c_{t-1} + b_{f} \right)$$

$$c_{t} = f_{t}c_{t-1} + i_{t} \tanh \left( W_{xc}x_{t} + W_{hc}h_{t-1} + b_{c} \right)$$

$$o_{t} = \sigma \left( W_{xo}x_{t} + W_{ho}h_{t-1} + W_{co}c_{t} + b_{o} \right)$$

$$h_{t} = o_{t} \tanh(c_{t})$$



Hochreiter et al. 1997 Graves 2013

The gates dilate and contracts time, similar to a low-pass filter in typical signal processing.

Chung et al. 2015  

$$z = \sigma(W_{z}x_{t} + U_{z}h_{t-1})$$

$$r = \sigma(W_{r}x_{t} + U_{r}h_{t-1})$$

$$\tilde{h} = tanh(W_{h}x_{t} + U_{h}(r \circ h_{t-1}))$$

$$h_{t} = (1 - z) \circ h_{t-1} + z \circ \tilde{h}$$

(b) Gated Recurrent Unit



#### E.g.:

Memory networks: https://arxiv.org/abs/1410.3916

Turing machines: https://arxiv.org/abs/1410.5401

Can be extended towards Complementary Learning Systems (mix of non-parametric & parametric models)





**Recurrent Network** 

Convolutional Network



Graph Neural Networks

## *"... infinite use of finite means"*







- Allows for adding inductive bias in the process through the structure of the graph
- Fast growing field of research with still many open questions
- Might provide a reliable connection to classical AI (e.g. logic) and to algorithms



- Similar to convnets consider your input divided into items (objects) -- pixels for CNN
- Define a neighborhood (which nodes are connected to which)
- The (new) state of the node is a function of its neighbourhood, **but** it should be order invariant wrt to the elements in the neighborhood (by using commutative operations like summation)


• Graph nets can be thought of in terms of RNN as well (unrolling over time)/





Graph Neural Networks and Attention

End-to-end translation systems (LSTM based):



## Attention mechanism





## Bahdanau et al. 2015





• Baidu/ UCLA: <u>Explain Images with Multimodal Recurrent Neural Networks</u>

- Toronto: <u>Unifying Visual-Semantic Embeddings with Multimodal Neural Language Models</u>
- o Berkeley: Long-term Recurrent Convolutional Networks for Visual Recognition and Description
- Google: <u>Show and Tell: A Neural Image Caption Generator</u>
- Stanford: <u>Deep Visual-Semantic Alignments for Generating Image Description</u>
- UML/UT: <u>Translating Videos to Natural Language Using Deep Recurrent Neural Networks</u>
- Microsoft/CMU: Learning a Recurrent Visual Representation for Image Caption Generation
- Microsoft: From Captions to Visual Concepts and Back





A woman is throwing a frisbee in a park.

A dog is standing on a hardwood floor.



A little girl sitting on a bed with a teddy bear.



A group of people sitting on a boat in the water.

- Treat inputs as a set
- Everytime you evaluate an item, attend to all other elements





## Self-Attention



### Self-attention (the core component of Transformer network)







https://arxiv.org/pdf/1806.01822.pdf



Figure 1: The Transformer - model architecture.



#### ViT 1st linear embedding filters



Class

Bird

Ball

Car

....

Patch + Position Embedding

\* Extra learnable [class] embedding

-

RGB embedding filters (first 28 principal components)



Graph Attention Nets (GATs)

$$h_{j}^{(t)} = \sigma\left(\sum_{i} g(h_{i}^{(t-1)}, h_{j}^{(t-1)}) f(h_{i}^{(t-1)})\right)$$

https://arxiv.org/abs/1710.10903

- Even if Transformers were not originally developed as a GraphNet, GraphNets are a natural formalism to describe transformers
- In this space, Transformers are close to GATs

## **Categorization of Machine Learning**



Splitting Machine Learning and positioning a set of approaches within the field can be messy.

## Many more topics I would have liked to bring up:

- Meta / Continual / Transfer / Multi-objective Learning
- Uncertainty
- Different domains: language, vision, etc.
- Different protocols: Unsupervised / Self-supervised
- Optimization (trust region methods, natural gradient, etc.)
- Non-parametric (and interaction with parametric)
- Bayesian DL and probabilistic framework for ML
- Compression/low-precision DL, sparsity
- ..



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  - https://github.com/deepmind/educational
  - <u>https://roberttlange.github.io/posts/2020/03/blog-post-10/</u>
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# THANK YOU!