Tests of GR and the Nature of Gravity

Marc Favata
Abhirup Ghosh
Walter Del Pozzo
Anuradha Samajdar
Huan Yang

Physics and Astrophysics at the extreme VI, Pisa, May 28, 2019

[link to discussion slides]

Overview

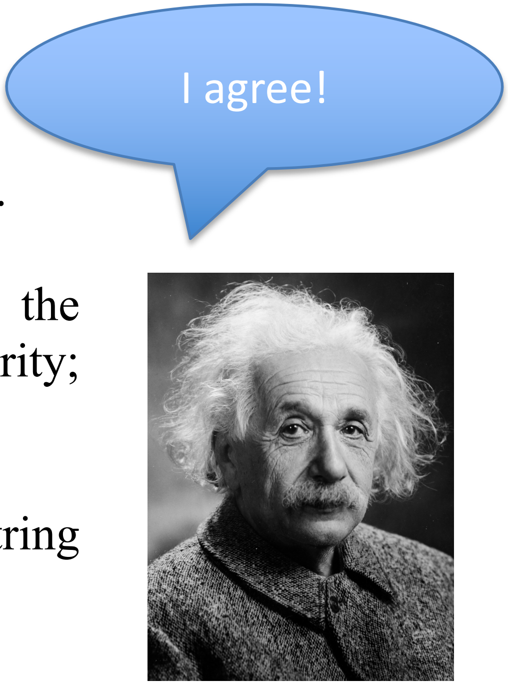
- The necessity of testing gravity at various regime.

- Where we are in tests of GR?
  - Non-GW tests
  - GW tests

- Future aspects of gravity tests:
  - Multi-messenger/multi-band tests
  - Consistency tests of GR/Parametrized tests for deviations/Searches for interesting new signatures
  - Difficulties in consistent modelling of modified gravity models
  - Environmental effects

- Topics for discussions

General Relativity is Awesome!



- GR is beautiful in its mathematical form.
- GR has passed all previous experimental/observational tests.
- Problems arise when quantum mechanics is included in the game: black hole information paradox; the nature of singularity; the quantumness/quantization of gravity, etc.
- Extension/modification of GR from different motivations: string theory, extra degrees of freedom, extra dimension…
- High risk/high pay-off business. Deviation from GR @ what scale and system not clear. Necessary and important to push the boundary of experimental/observational test in the strong gravity regime and different length scales. How do we assess the theoretical prior of different modified models (or should we)?

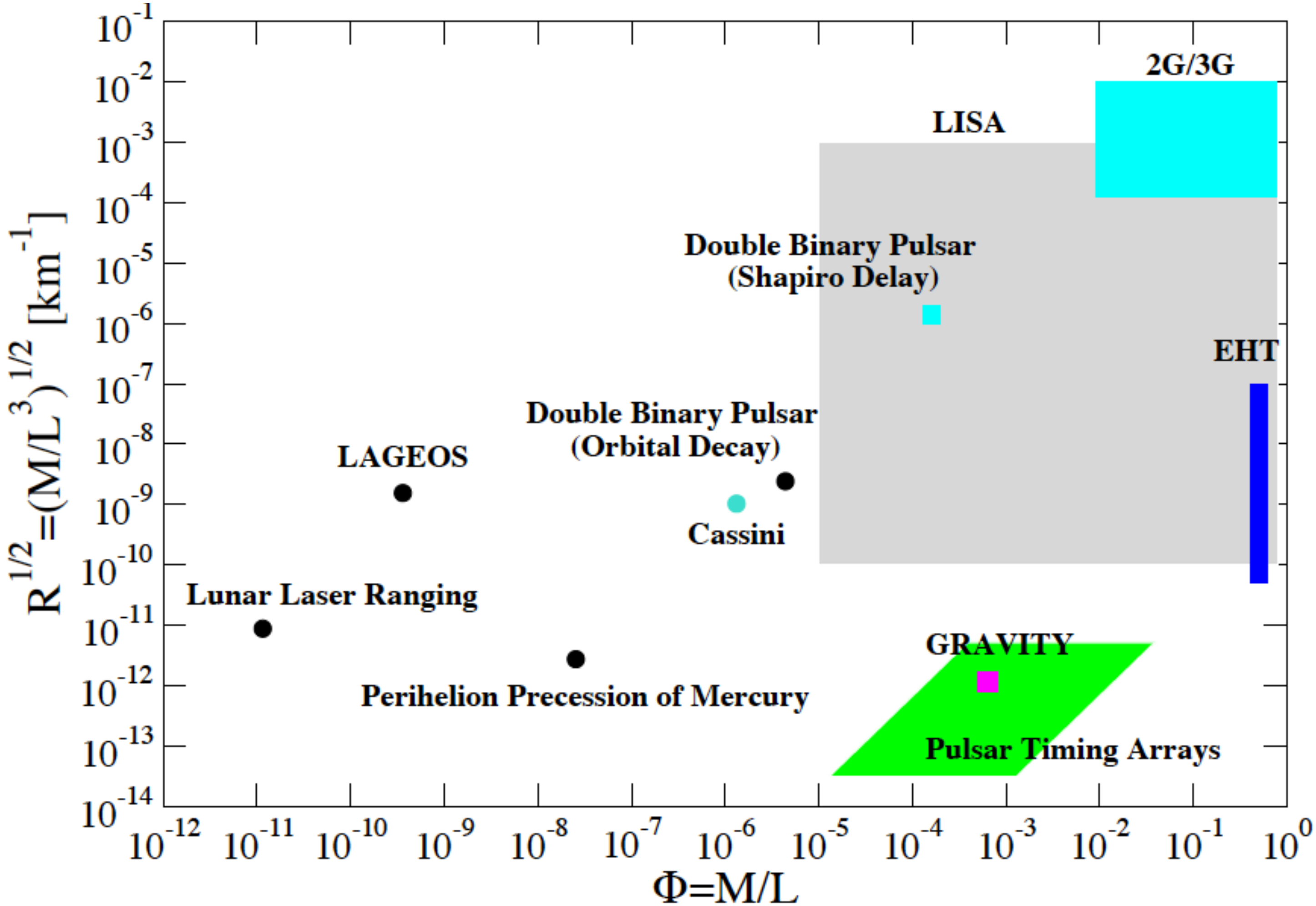
A Survey in "Fundamental Physics with LISA" Workshop

Table 1: Ideas underlying possible routes to new physics. Grades indicate the relative importance of searching for evidence of the idea (higher grade = more important). The first column indicates whether there is a viable theory supporting the idea. Column two ranks the theoretical motivation for the idea, and column three asks whether we know where in parameter space to look for signals of this physics.

	Theoretically Sound?	Why?	Where?	Net Grade
Gravitational Waves	10	20*	10	40
Near weak-scale physics (e.g. EDMs, Flavor, g-2)	10	10	5	25
Neutrino masses (neutrinoless double beta decay)	10	10	5	25
QCD axion	10	15*	5	30
Axiverse, Photiverse	10	5	0	15
Moduli, Extra Dimensions	10	5	0	15
Lorentz, CPT Violation	5	0	0	5
Gravitational Decoherence	-5	-10	0	-15
Quintessence, Chameleons, Galileons	-5	-10	0	-15
Vacuum Energy, Holography	-10	-10	0	-20

Slide from Emanuele Berti, survey done by Diego Blas

Non-GW Tests



$R^{1/2}=(M/L^3)^{1/2}$ [km^{-1}]
Φ=M/L
2G/3G
LISA
Double Binary Pulsar
(Shapiro Delay)
EHT
Double Binary Pulsar
(Orbital Decay)
LAGEOS
Cassini
Lunar Laser Ranging
GRAVITY
Perihelion Precession of Mercury
Pulsar Timing Arrays

BNS GW Tests

- Constraint on the speed of GW:

$$-3 \times 10^{-15} \leqslant \frac{\Delta v}{v_{\mathrm{EM}}} \leqslant +7 \times 10^{-16}$$

- Lorentz invariance violation limits:

Constraints on the Dimensionless Minimal Gravity Sector Coefficients

ℓ	Previous Lower	This Work Lower	Coefficient	This Work Upper	Previous Upper
0	-3×10^{-14}	-2×10^{-14}	$\bar{s}^{(4)}_{00}$	5×10^{-15}	8×10^{-5}
1	-1×10^{-13}	-3×10^{-14}	$\bar{s}^{(4)}_{10}$	7×10^{-15}	7×10^{-14}
	-8×10^{-14}	-1×10^{-14}	$-\mathrm{Re}\ \bar{s}^{(4)}_{11}$	2×10^{-15}	8×10^{-14}
	-7×10^{-14}	-3×10^{-14}	$\mathrm{Im}\ \bar{s}^{(4)}_{11}$	7×10^{-15}	9×10^{-14}
2	-1×10^{-13}	-4×10^{-14}	$-\bar{s}^{(4)}_{20}$	8×10^{-15}	7×10^{-14}
	-7×10^{-14}	-1×10^{-14}	$-\mathrm{Re}\ \bar{s}^{(4)}_{21}$	2×10^{-15}	7×10^{-14}
	-5×10^{-14}	-4×10^{-14}	$\mathrm{Im}\ \bar{s}^{(4)}_{21}$	8×10^{-15}	8×10^{-14}
	-6×10^{-14}	-1×10^{-14}	$\mathrm{Re}\ \bar{s}^{(4)}_{22}$	3×10^{-15}	8×10^{-14}
	-7×10^{-14}	-2×10^{-14}	$-\mathrm{Im}\ \bar{s}^{(4)}_{22}$	4×10^{-15}	7×10^{-14}

- Shapiro delay:

$$-2.6 \times 10^{-7} \leqslant \gamma_{\mathrm{GW}} - \gamma_{\mathrm{EM}} \leqslant 1.2 \times 10^{-6}$$

LVC Collaboration, ApJL 2017

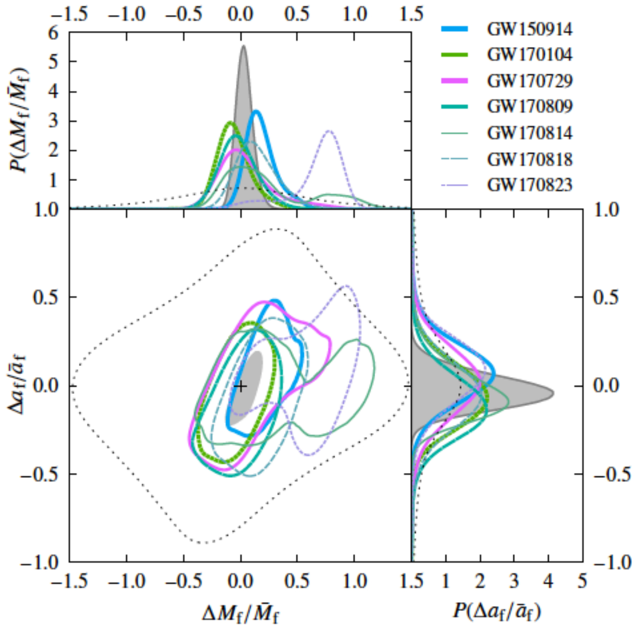
BBH GW Tests

Event	Properties				FAR			SNR	GR tests performed				
	D_L [Mpc]	M_{tot} [$M_\odot$]	M_f [$M_\odot$]	a_f	PyCBC [yr^{-1}]	GstLAL [yr^{-1}]	cWB [yr^{-1}]		RT	IMR	PI	PPI	MDR
GW150914[b]	430^{+150}_{-170}	$66.2^{+3.7}_{-3.3}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$< 1.5 \times 10^{-5}$	$< 1.0 \times 10^{-7}$	$< 1.6 \times 10^{-4}$	$25.3^{+0.1}_{-0.2}$	✓	✓	✓	✓	✓
GW151012[b]	1060^{+550}_{-480}	$37.3^{+10.6}_{-3.9}$	$35.7^{+10.7}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	0.17	7.9×10^{-3}	–	$9.2^{+0.3}_{-0.4}$	✓	–	–	✓	✓
GW151226[b,c]	440^{+180}_{-190}	$21.5^{+6.2}_{-1.5}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$< 1.7 \times 10^{-5}$	$< 1.0 \times 10^{-7}$	0.02	$12.4^{+0.2}_{-0.3}$	✓	–	✓	–	✓
GW170104	960^{+440}_{-420}	$51.3^{+5.3}_{-4.2}$	$49.1^{+5.2}_{-4.0}$	$0.66^{+0.08}_{-0.11}$	$< 1.4 \times 10^{-5}$	$< 1.0 \times 10^{-7}$	2.9×10^{-4}	$14.0^{+0.2}_{-0.3}$	✓	✓	✓	✓	✓
GW170608	320^{+120}_{-110}	$18.6^{+3.1}_{-0.7}$	$17.8^{+3.2}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$< 3.1 \times 10^{-4}$	$< 1.0 \times 10^{-7}$	1.4×10^{-4}	$15.6^{+0.2}_{-0.3}$	✓	–	✓	✓	✓
GW170729[d]	2760^{+1380}_{-1340}	$85.2^{+15.6}_{-11.1}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	1.4	0.18	0.02	$10.8^{+0.4}_{-0.5}$	✓	✓	–	✓	✓
GW170809	990^{+320}_{-380}	$59.2^{+5.4}_{-3.9}$	$56.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	1.4×10^{-4}	$< 1.0 \times 10^{-7}$	–	$12.7^{+0.2}_{-0.3}$	✓	✓	–	✓	✓
GW170814	580^{+160}_{-210}	$56.1^{+3.4}_{-2.7}$	$53.4^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$< 1.2 \times 10^{-5}$	$< 1.0 \times 10^{-7}$	$< 2.1 \times 10^{-4}$	$17.8^{+0.3}_{-0.3}$	✓	✓	✓	✓	✓
GW170818	1020^{+430}_{-360}	$62.5^{+5.1}_{-4.0}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	–	4.2×10^{-5}	–	$11.9^{+0.3}_{-0.4}$	✓	✓	–	✓	✓
GW170823	1850^{+840}_{-840}	$68.9^{+9.9}_{-7.1}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	$< 3.3 \times 10^{-5}$	$< 1.0 \times 10^{-7}$	2.1×10^{-3}	$12.1^{+0.2}_{-0.3}$	✓	✓	–	✓	✓

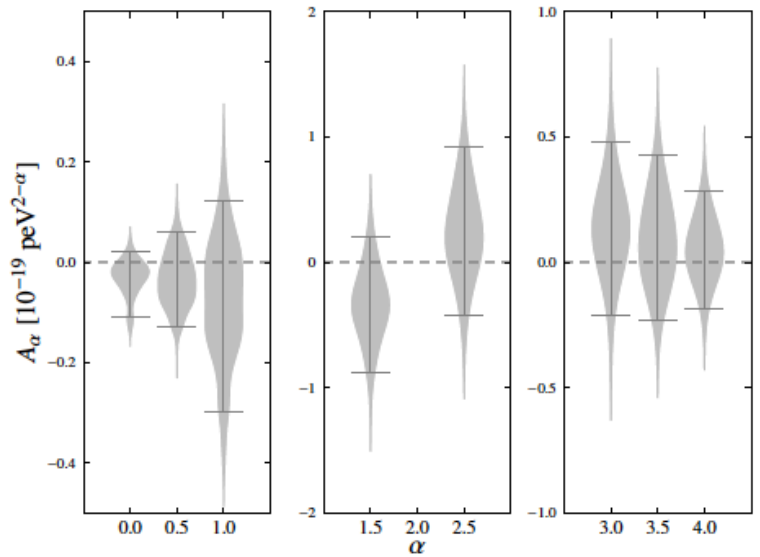
given event: RT = residuals test (Sec. V A); IMR = inspiral-merger-ringdown consistency test (Sec. V B); PI & PPI = parameterized tests of GW generation for inspiral and post-inspiral phases (Sec. VI); MDR = modified GW dispersion relation (Sec. VII). The events with bold names are used to obtain the combined results for each test.

LVC Collaboration, 1903.04467

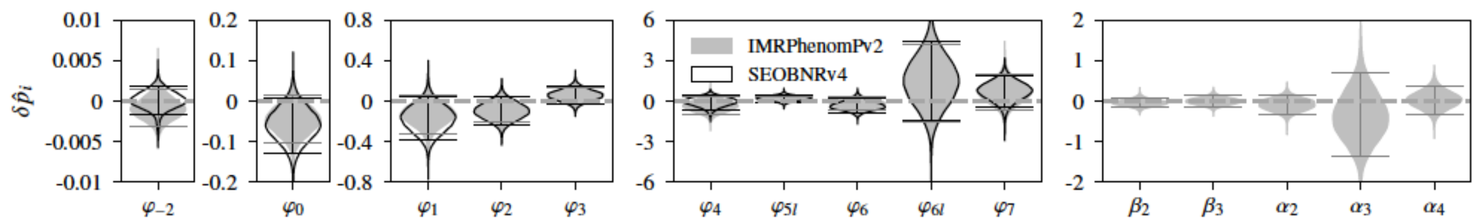
BBH GW Tests



IMR consistency test results

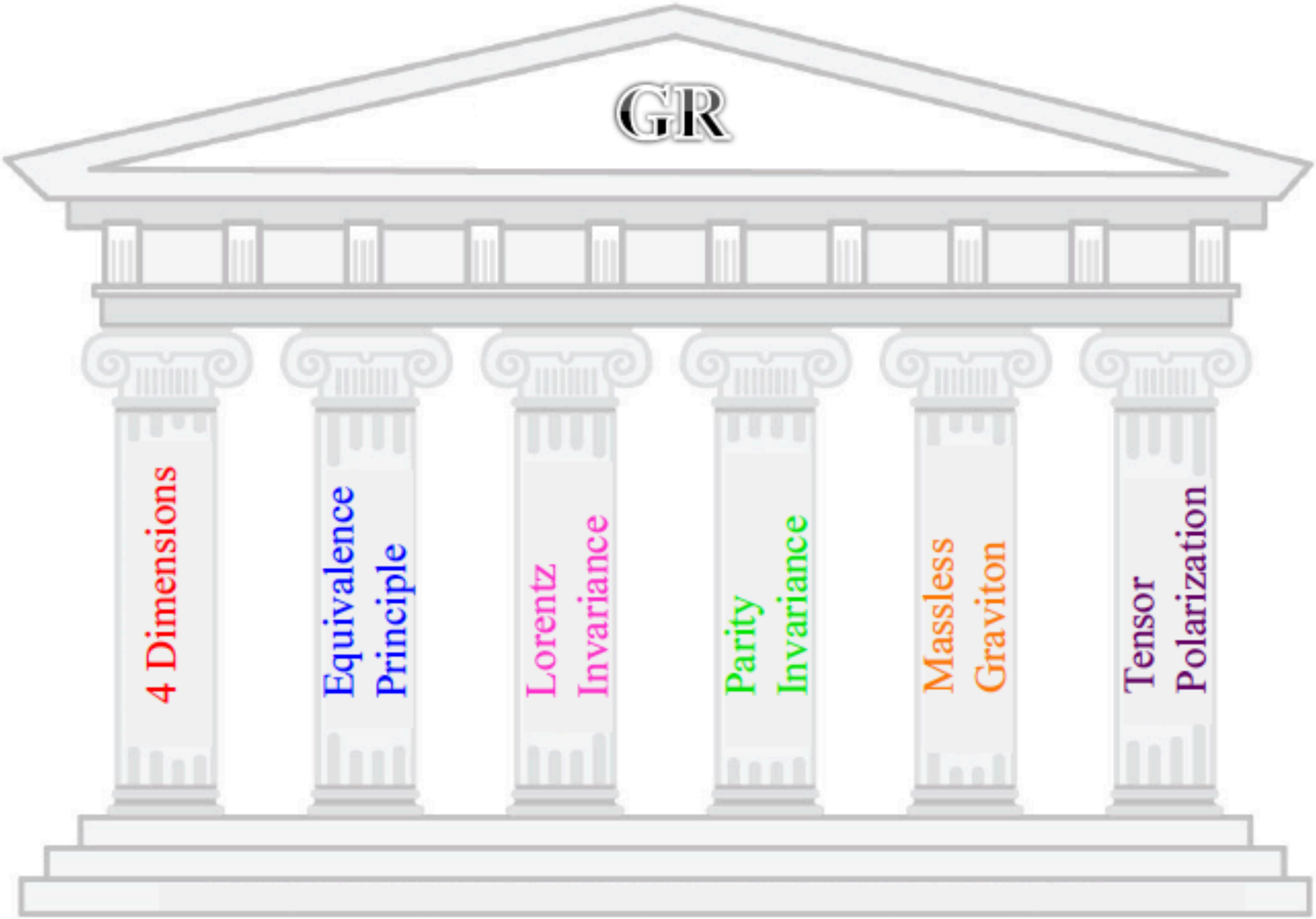


$$E^2 = p^2c^2 + A_\alpha p^\alpha c^\alpha$$



Combined posterior for inspiral PN violation and merger/ringdown violations

GW Tests



Slide from K. Yagi

GW Tests

Theoretical Mechanism	GR Pillar	PN	$\lvert\beta\rvert$ GW150914	$\lvert\beta\rvert$ GW151226	Repr. Parameters	Example Theory Constraints GW150914	Example Theory Constraints GW151226	Current Bounds
Scalar Field Activation	SEP	-1	$\mathbf{1.6 \times 10^{-4}}$	$\mathbf{4.4 \times 10^{-5}}$	$\sqrt{\lvert\alpha_{\rm EdGB}\rvert}$ [km] $\lvert\dot{\phi}\rvert$ [1/sec]	— —	— —	10^{7} [56], 2 [57–59] 10^{-6} [60]
Scalar Field Activation	SEP, PI	$+2$	$\mathbf{1.3 \times 10^{1}}$	$\mathbf{4.1}$	$\sqrt{\lvert\alpha_{\rm dCS}\rvert}$ [km]	—	—	10^{8} [61, 62]
Vector Field Activation	SEP, LI	0	$\mathbf{7.2 \times 10^{-3}}$	$\mathbf{3.4 \times 10^{-3}}$	(c_{+}, c_{-}) $(\beta_{\rm KG}, \lambda_{\rm KG})$	$\mathbf{(0.9, 2.1)}$ $\mathbf{(0.42, -)}$	$\mathbf{(0.8, 1.1)}$ $\mathbf{(0.40, -)}$	$(0.03, 0.003)$ [63, 64] $(0.005, 0.1)$ [63, 64]
Extra Dimensions	4D	-4	$\mathbf{9.1 \times 10^{-9}}$	$\mathbf{9.1 \times 10^{-11}}$	ℓ [μm]	$\mathbf{5.4 \times 10^{10}}$	$\mathbf{2.0 \times 10^{9}}$	10–10^{3} [65–69]
Time-Varying G	SEP	-4	$\mathbf{9.1 \times 10^{-9}}$	$\mathbf{9.1 \times 10^{-11}}$	$\lvert\dot{G}\rvert$ [10^{-12}/yr]	$\mathbf{5.4 \times 10^{18}}$	$\mathbf{1.7 \times 10^{17}}$	0.1–1 [70–74]
Massive graviton	$m_g = 0$	$+1$	$\mathbf{1.3 \times 10^{-1}}$	$\mathbf{8.9 \times 10^{-2}}$	m_g [eV]	$\mathbf{10^{-22}}$ [19]	$\mathbf{10^{-22}}$ [5]	10^{-29}–10^{-18} [75–79]
Mod. Disp. Rel. (*Multifractional*)	LI	$+4.75$	$\mathbf{1.1 \times 10^{2}}$	$\mathbf{2.6 \times 10^{2}}$	E_{*}^{-1} [eV^{-1}] (time) E_{*}^{-1} [eV^{-1}] (space)	$\mathbf{5.8 \times 10^{-27}}$ $\mathbf{1.0 \times 10^{-26}}$	$\mathbf{3.3 \times 10^{-26}}$ $\mathbf{5.7 \times 10^{-26}}$	— 3.9×10^{-53} [80]
Mod. Disp. Rel. (*Modified Special Rel.*)	LI	$+5.5$	$\mathbf{1.4 \times 10^{2}}$	$\mathbf{4.3 \times 10^{2}}$	$\eta_{\rm dsrt}/L_{\rm Pl} > 0$ $\eta_{\rm dsrt}/L_{\rm Pl} < 0$	$\mathbf{1.3 \times 10^{22}}$	$\mathbf{3.8 \times 10^{22}}$	— 2.1×10^{-7} [80]
Mod. Disp. Rel. (*Extra Dim.*)	4D	$+7$	$\mathbf{5.3 \times 10^{2}}$	$\mathbf{2.4 \times 10^{3}}$	$\alpha_{\rm edt}/L_{\rm Pl}^{2} > 0$ $\alpha_{\rm edt}/L_{\rm Pl}^{2} < 0$	$\mathbf{5.5 \times 10^{62}}$	$\mathbf{2.5 \times 10^{63}}$	2.7×10^{2} [80] —
Mod. Disp. Rel. (*Standard Model Ext.*)	LI	$+4$	—	—	$\mathring{k}_{(I)}^{(4)} > 0$ $\mathring{k}_{(I)}^{(4)} < 0$	— $\mathbf{0.64}$	— $\mathbf{19}$	6.1×10^{-17} [80, 81] —
		$+5.5$	$\mathbf{1.4 \times 10^{2}}$	$\mathbf{4.3 \times 10^{2}}$	$\mathring{k}_{(V)}^{(5)} > 0$ [cm] $\mathring{k}_{(V)}^{(5)} < 0$ [cm]	$\mathbf{1.7 \times 10^{-12}}$ [82]	$\mathbf{3.1 \times 10^{-11}}$	1.7×10^{-40} [80, 81] —
		$+7$	$\mathbf{5.3 \times 10^{2}}$	$\mathbf{2.4 \times 10^{3}}$	$\mathring{k}_{(I)}^{(6)} > 0$ [cm^2] $\mathring{k}_{(I)}^{(6)} < 0$ [cm^2]	$\mathbf{7.2 \times 10^{-4}}$	$\mathbf{3.3 \times 10^{-3}}$	3.5×10^{-64} [80, 81] —
Mod. Disp. Rel. (*Hořava-Lifshitz*)	LI	$+7$	$\mathbf{5.3 \times 10^{2}}$	$\mathbf{2.4 \times 10^{3}}$	$\kappa_{\rm hl}^{4}\mu_{\rm hl}^{2}$ [1/eV^2]	$\mathbf{1.5 \times 10^{6}}$	$\mathbf{6.9 \times 10^{6}}$	—
Mod. Disp. Rel. (*Lorentz Violation*)	LI	$+4$	—	—	c_{+}	$\mathbf{0.7}$ [83]	$\mathbf{0.998}$	0.03 [63, 64]

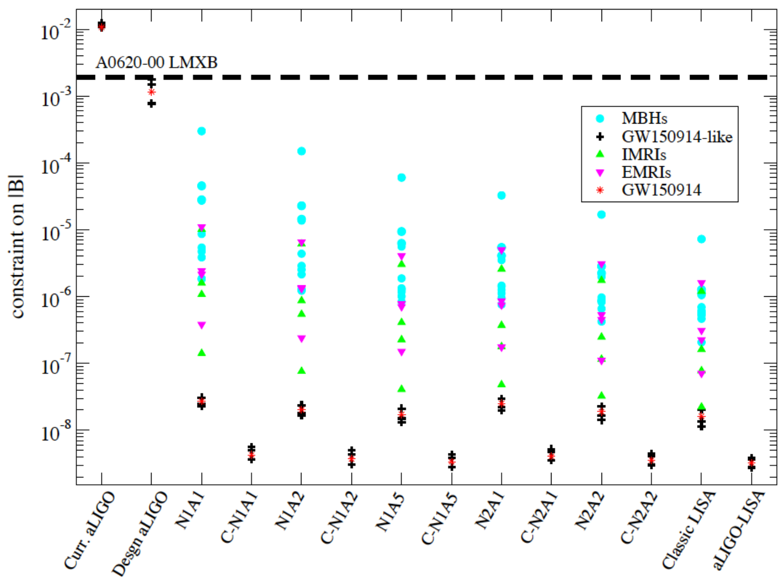
N. Yunes, K. Yagi, F. Pretorius. 2016

Future multi-messenger tests

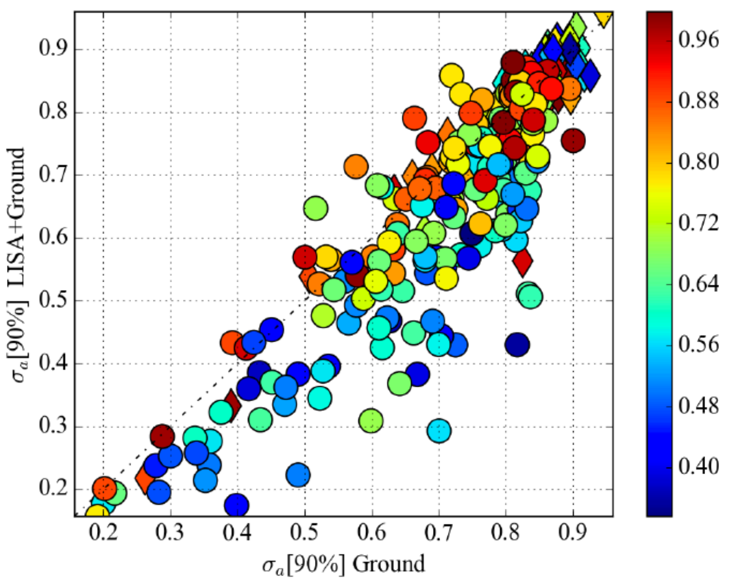
- GW170817 is an example for multi-messenger tests.
- In the era of O3/O4, and with 3G detectors, we will see many BNS mergers, many BH/NS mergers. Constraints to be greatly improved.
- May be able to put better constraints on scalar-tensor theory (and/or other modified gravity theories) by combining inspiral GW measurement with post-merger EM signal.
- Other sources: tidal description event of NS/BH binary, or star-massive BH merger, or supernovae. What else? What can we learn from these events?

Future multi-band tests

- GW observation from multiple frequency band greatly improves the ability of parameter estimation and testing gravity with BBHs.



E. Barausse, N. Yunes, K. Chamberlain. 2014



S. Vitale. 2016

- Future gravity constraints by comparing properties of BBH population at different frequencies? What about comparing SGWB at different band?

Consistency tests of GR

- Instead of searching for signatures of non-GR effects, it is always economical to test the foundation/prediction of GR.

- Future tests of GR can still be based on verifying “pillars” of GR: equivalence principle, local Lorentz invariance, etc.

- Alternatively, more tests can be performed for various GR observables: GW memory effect (see Marc’s discussion), consistency test of GR using higher harmonics of the entire inspiral-merger-ringdown model of a BBH waveform, no-hair theorem/BH spectroscopy (See Abhirup's discussion), pulsar systems, etc.

- Deviation v.s. no deviation. Null tests with Bayesian model selection (see discussion by Walter).

- Are we doing this in a case by case basis? Is it useful to combine events to get a better consistency check? Systematic errors in GR tests (see discussion by Marc and Abhirup).

Parametrized tests of GR deviations

- Post-Newtonian parametrization (not good for negative PN order effect)/Parametrized post-Einsteinian method/Parametrized phenomenological waveform… Not capturing possible resonance effect, spontaneous scalarization, etc. Better ways?

- Ringdown tests with multiple modes will be available with 3G detector and LISA, possibly with Advanced LIGO+ stacking. They should include QNM amplitudes and phases, in addition to QNM frequencies. The starting time issue needs to be better resolved, the role of higher overtones needs to be better understood. For a parametrized test, how do we map the deviations back to the deviation of Kerr/additional fields?

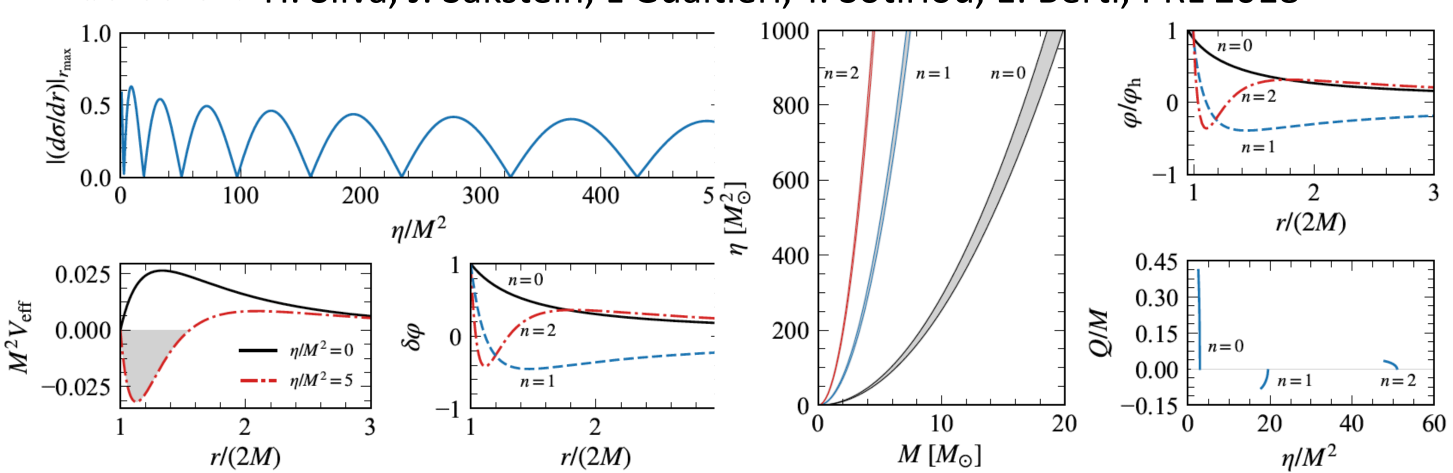
- Parametrized IMR waveform needed. Facing similar problems.

- Parametrized non-Kerr spacetime (bumpy BHs) for LISA EMRI tests needed. Do we understand the effect of non-Kerr deviations on EMRI orbits? Resonance? Chaos?

- Parametrized dispersion relation for GWs. Is the existing one good enough?

Interesting signatures of GR deviations

- In addition to generic gravity tests, it's interesting to look at unique signatures of different models or perform model-specific tests.
- Example: scalarization/resonances in the inspiral stage, echoes/extra mode in the ringdown stage, scalar-tidal love number in NS binaries, etc.
- Be prepared for un-expected signals (unmodeled search).
- Interesting propagation effects of GWs (see discussion by Anuradha).
- There are signatures that only turns on at specific scales – high frequency detector! H. Silva, J. Sakstein, L Gualtieri, T. Sotiriou, E. Berti, PRL 2018



“Well-posedness” issue

- Most of the modified gravity theories are ill-posed – essentially they fail to generate an unique/converging answer as an initial-value problem (Chern-Simon/Gauss Bonnet/… except scalar tensor theory).

- No consistent IMR waveform for ill-posed theories. Tests restricted to Post-Newtonian/ BH ringdown regime.

- One possible way out: EFT perspective.

    - Perturbative approach

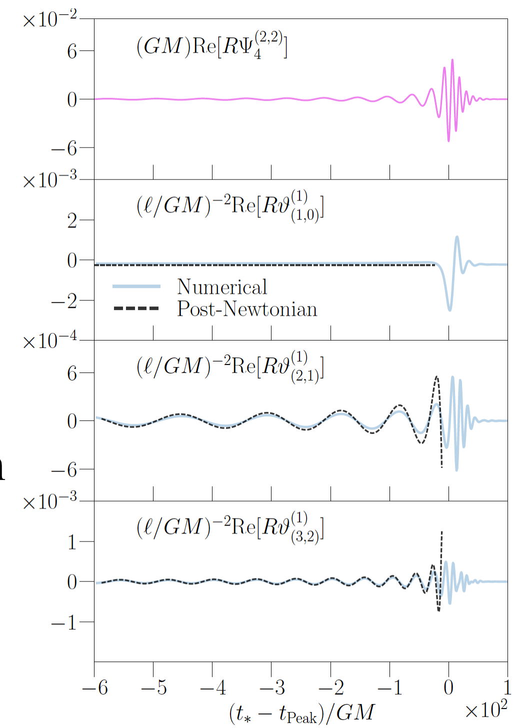
      $$g = g_{\rm GR} + \epsilon^2 h_1 + \epsilon^4 h_2 + \ldots$$
      $$\theta = \epsilon\theta_1 + \epsilon^3\theta_2 + \ldots$$

    - Israel-Stewart (hydrodynamical) approach

      J. Cayuso, N. Ortiz, L. Lehner. 2017

Can we get a working example?

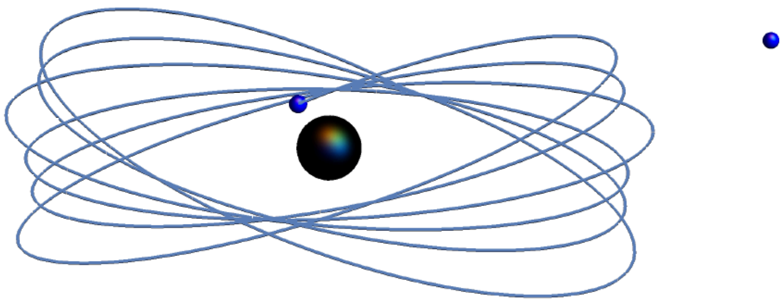
Is there a better way out?



Okounkova et al. 2018

Astrophysical/Environmental effects

- The astrophysical environmental effects themselves are interesting to understand. (3rd body to the EMRI system; disk effect to the ISCO measurement/EHT image; EOS variation for NS tests …)



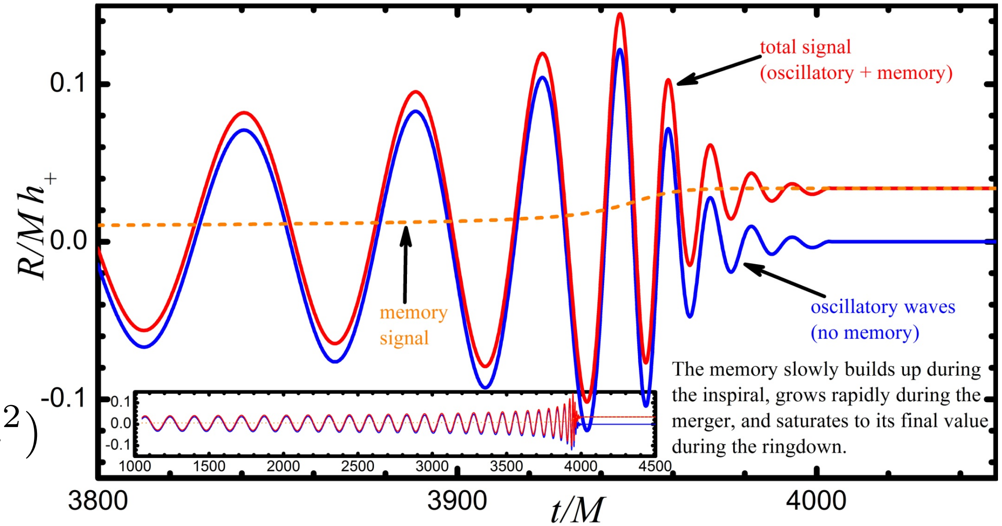
B. Bonga, HY, S. Hughes, 2019

EHT

- Environmental effects could limit the sensitivity for gravity tests, and/or generate false positive results.

- Have we identified relevant environment effects for the gravity tests proposed? What are the un-identified ones? Methods to fix the problem? Removal of the best guess of the environmental effect? Consistent incorporation of these effects in the gravity test scheme? More ideas?

Gravitational-Wave Memory



Nonlinear memory

[Blanchet-Damour-Christodoulou]

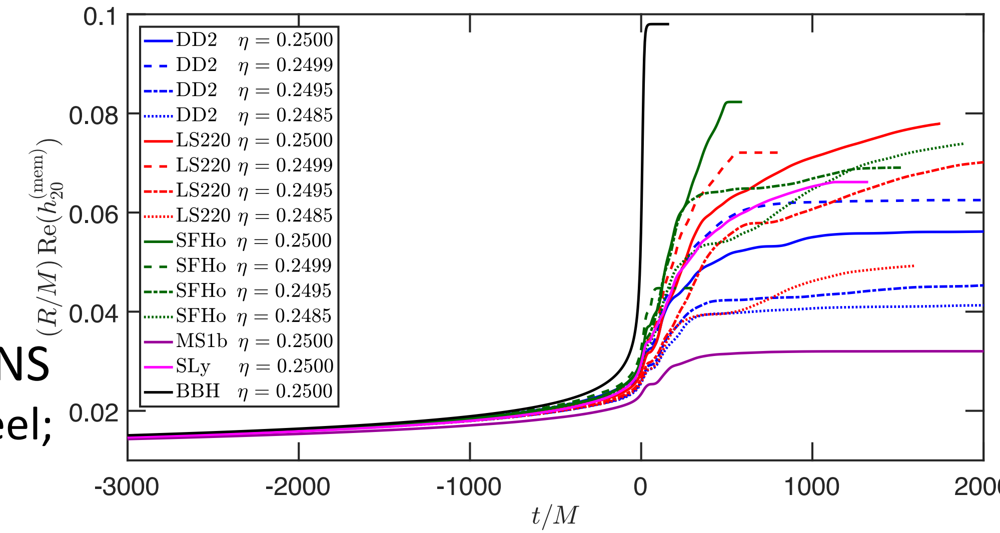
- GWs producing GWs:

$$\Box \bar{h}^{jk} = -16\pi(-g)(T^{jk} + T^{jk}_{\rm GW}[\bar{h},\bar{h}]) + O(\bar{h}^2)$$

- Produced by all GW sources; strongest for BBH.

- Unique waveform signature. [MF ApJL'09; MF, Karlson, Dojcinoski]

- Waveforms computed for BBH and BNS [Karlson & MF; Karlson, MF, Barkett, Scheel; Yang & Martynov]



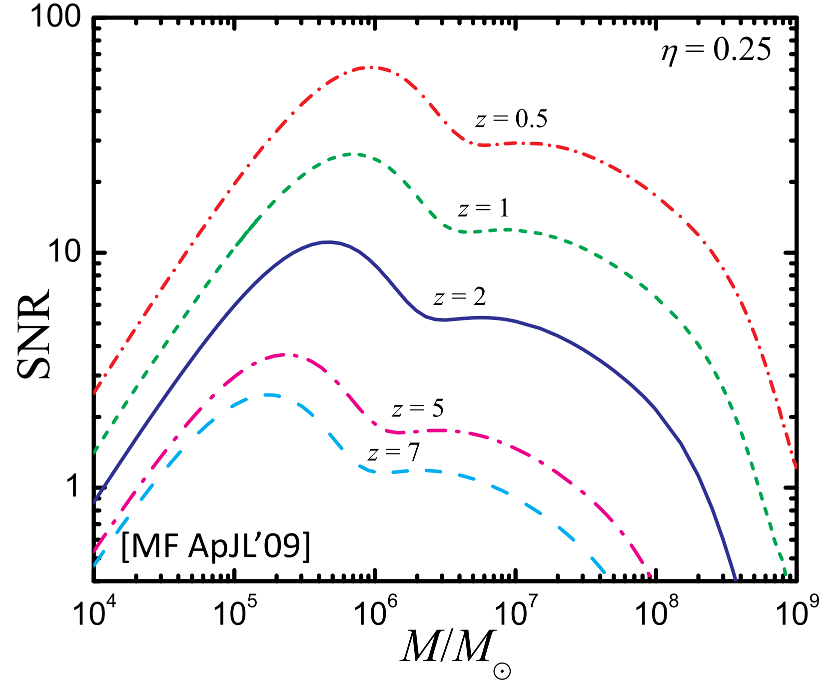
Other (subdominant) memory effects also possible; [e.g., ``spin'' memory, Nichols '18; MF PRD '09].

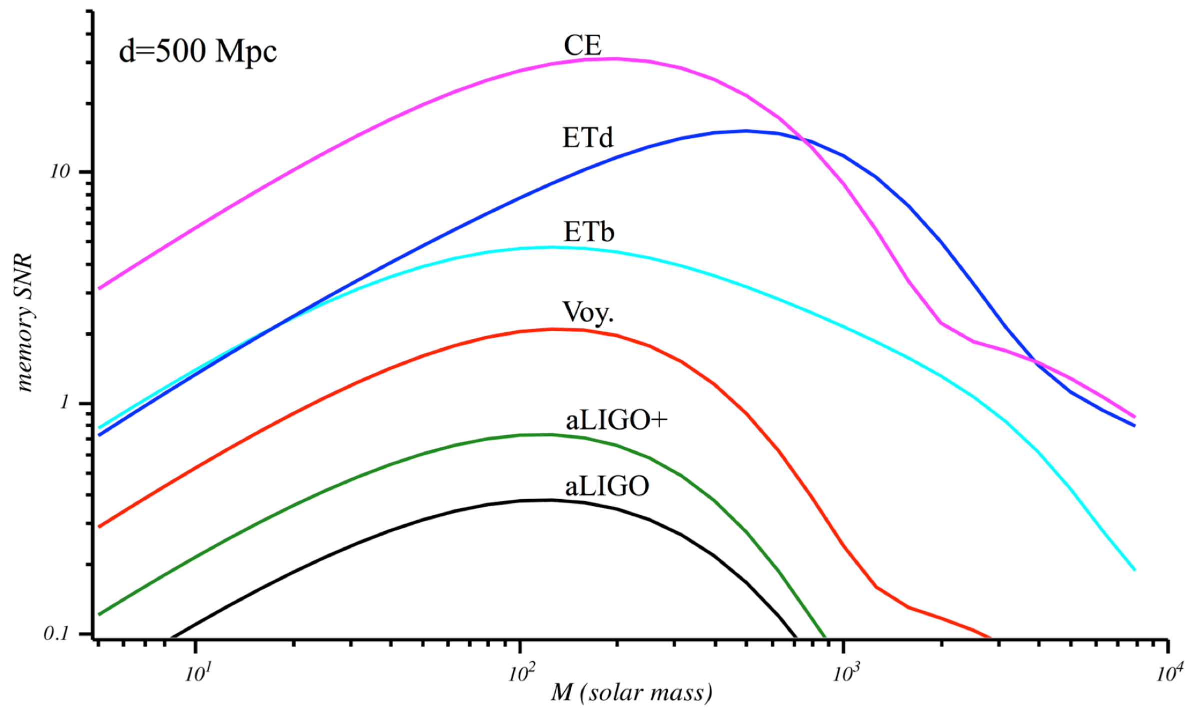
"Linear" memory produced by unbound sources; 2-body scattering, CCSN, GRB jets,...

Gravitational-Wave Memory

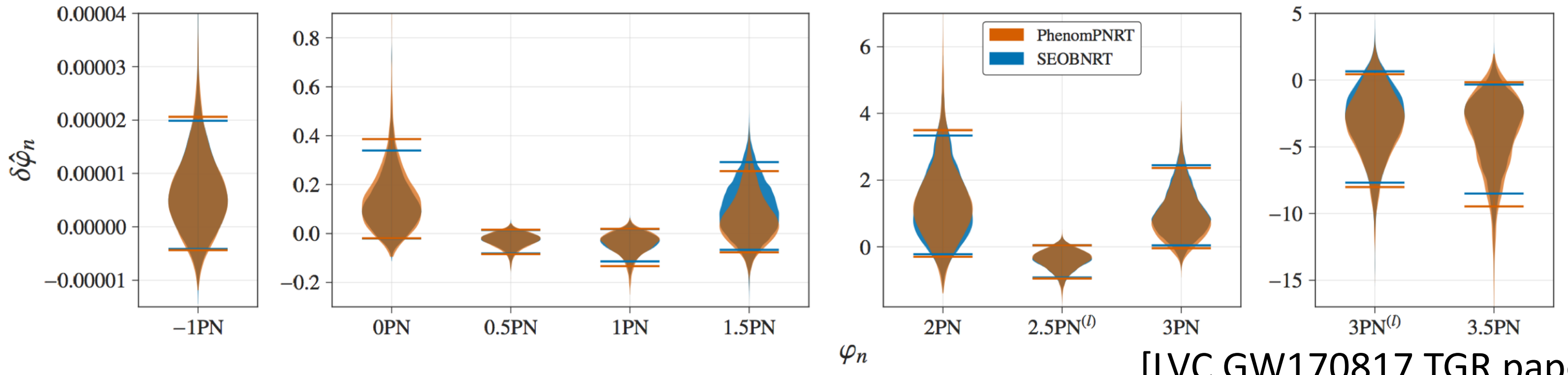
Detection prospects:

- poor for aLIGO for single events.
- Multiple GW159014 events [~O(100)] may reveal presence of memory [Lasky et al.]
- 3G and LISA have better prospects for detection of isolated events [MF; Yang & Martynov]
- PTA groups have set upper limits on rates of memory bursts [e.g., Arzoumanian et al. Wang et al]





Systematic effects in Tests of GR



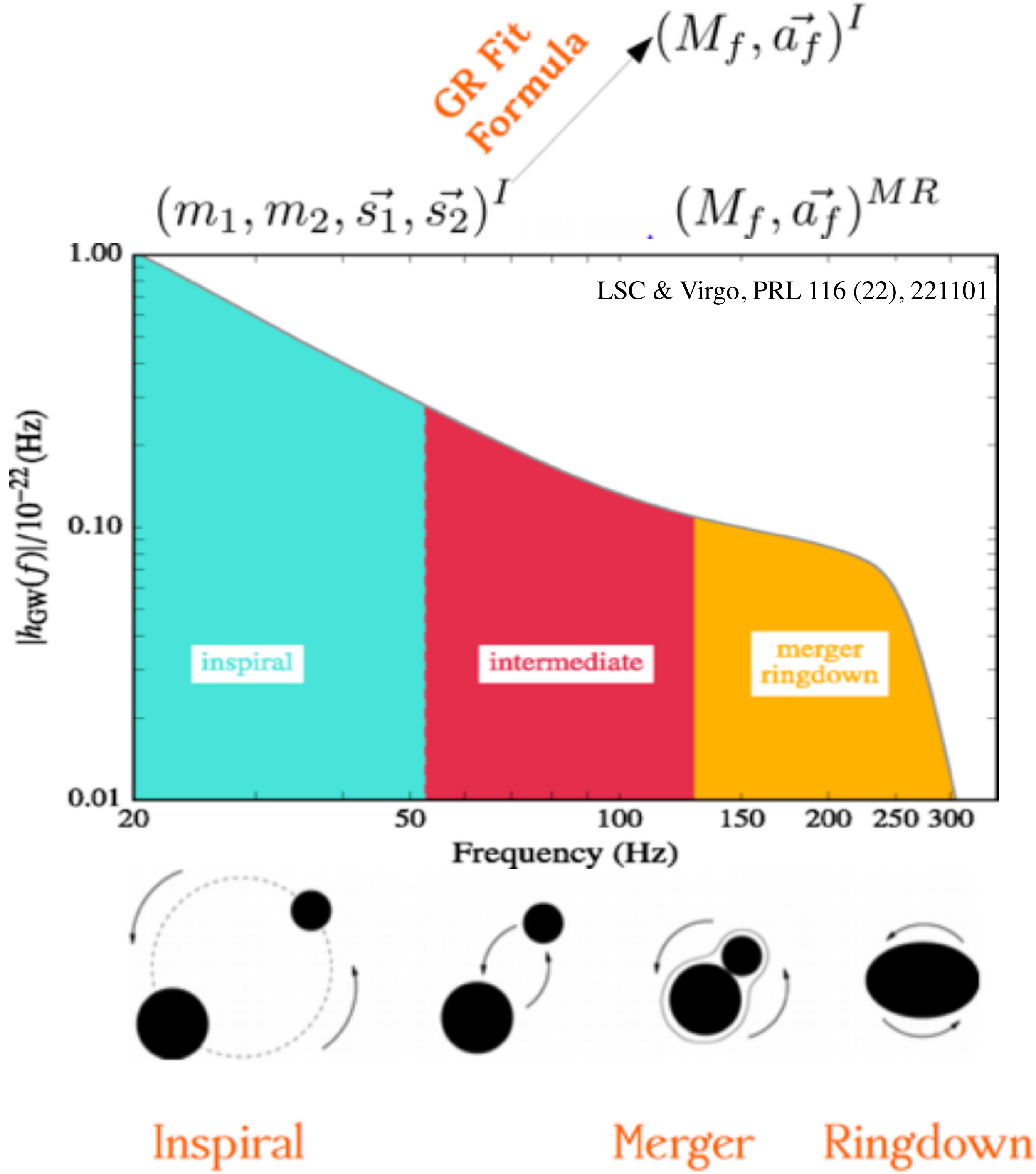
[LVC GW170817 TGR paper]

- Probing GR effects or GR modifications are vulnerable to systematic bias.
- Unmodeled GR effects or astrophysical perturbations can mimic a GR violation.
- Examples: higher modes, eccentricity, higher PN terms.
- Eccentricity bias on TGR PN coefficients: comparable to statistical errors for e_0(10Hz) $\gtrsim$ 0.008 (aLIGO), $\gtrsim$ 0.0001 (ET) [GW170817-like system; MF, KG Arun (prelim.)]
- 4PN+ terms: not significant for aLIGO design (except possible bias to 3PN, 3.5PN coefficients); significant for ET (need 6PN+ terms for all terms except 0PN and 2PN).

Inspiral-merger-ringdown consistency test

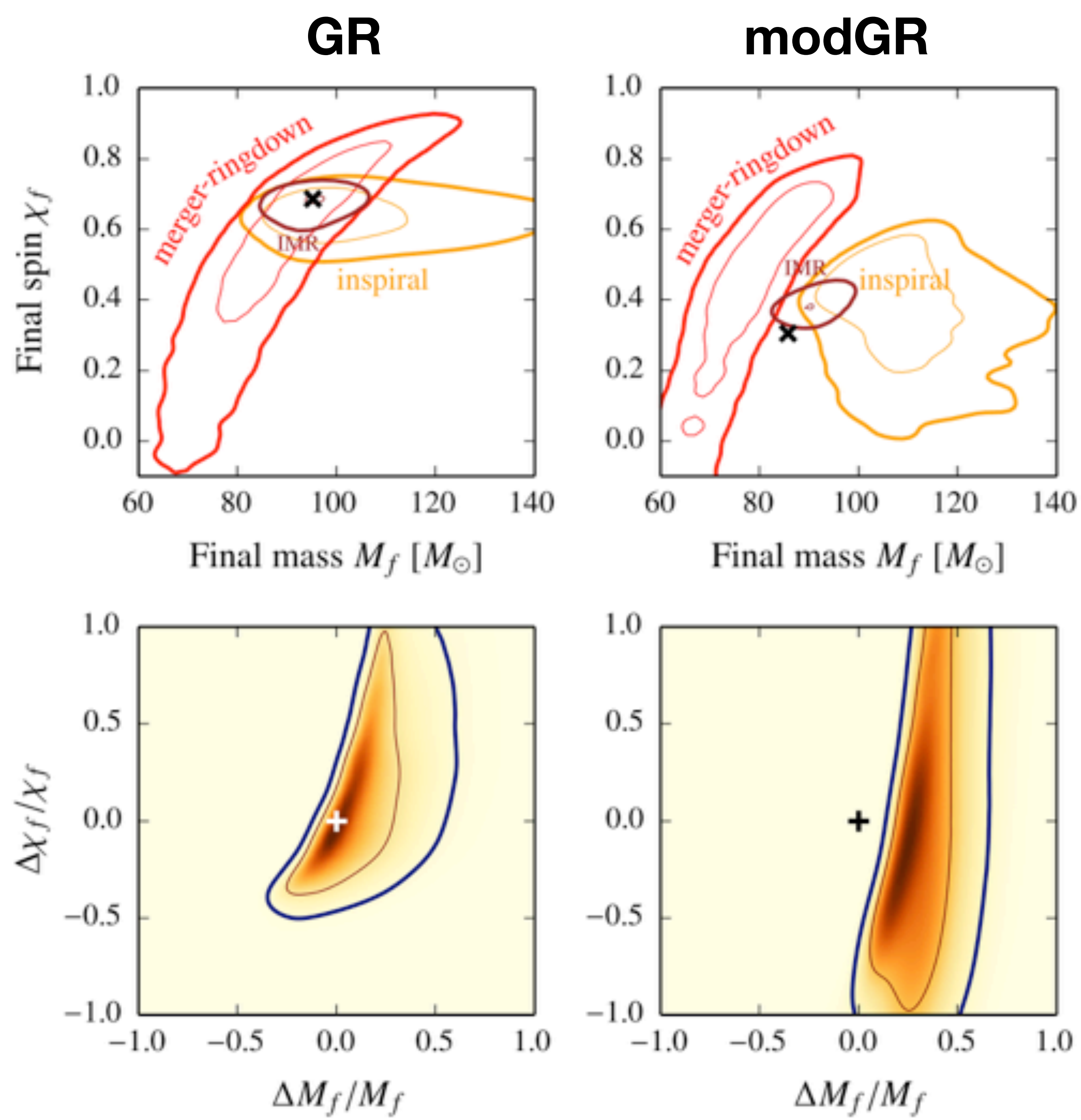
AG et al. (PRD 94, 021101)
AG et al. (2018 CQG **35** 014002)

- The IMR consistency test is based on estimating the final mass and spin from the initial (low frequency) and final (high frequency) stages of a BBH coalescence and checking their consistency.

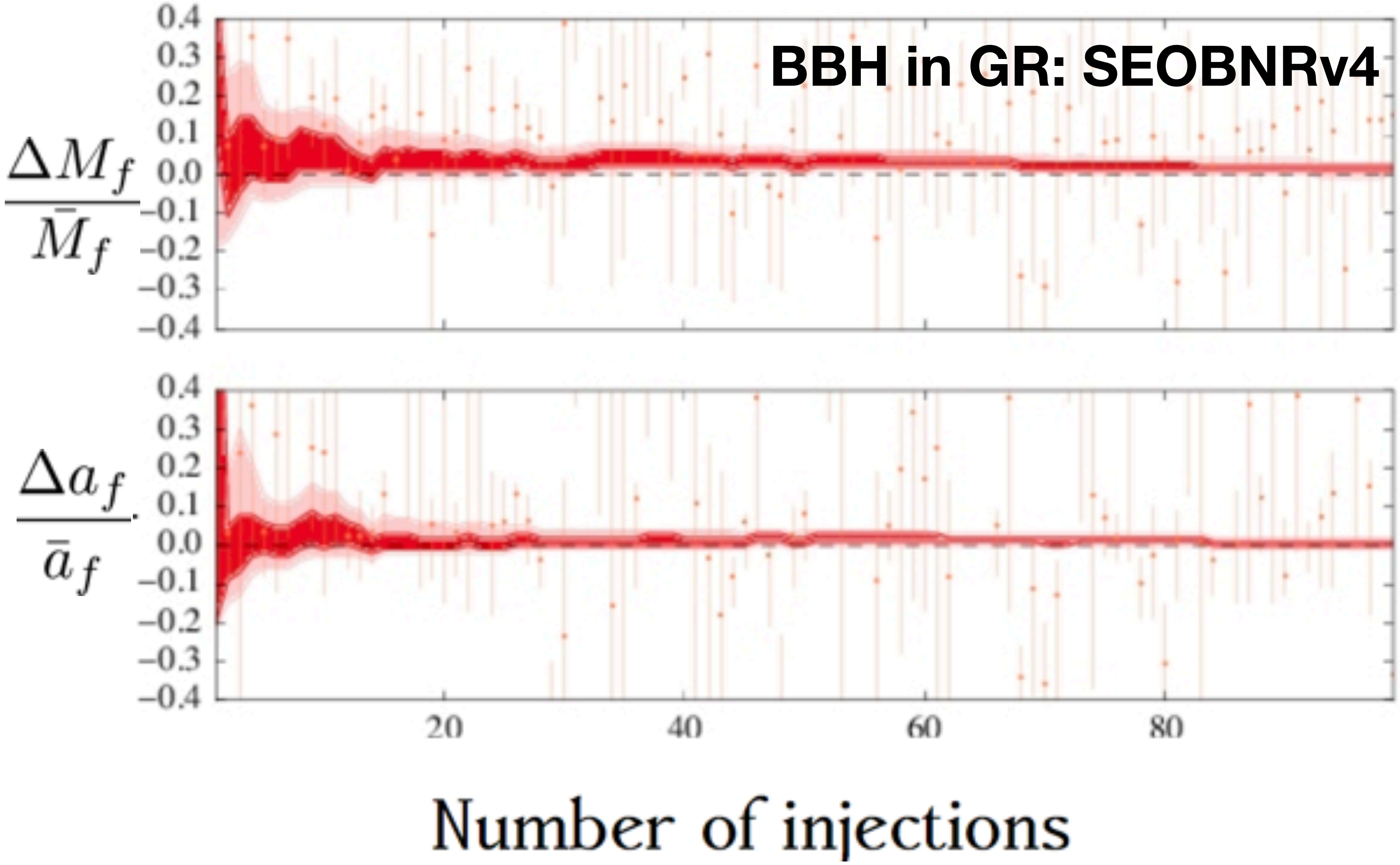


Inspiral-merger-ringdown consistency test

- The IMR consistency test is based on estimating the final mass and spin from the initial (low frequency) and final (high frequency) stages of a BBH coalescence and checking their consistency.
- If GR is right, then the independent estimates of the mass and spin of the final BH should be consistent with one another.



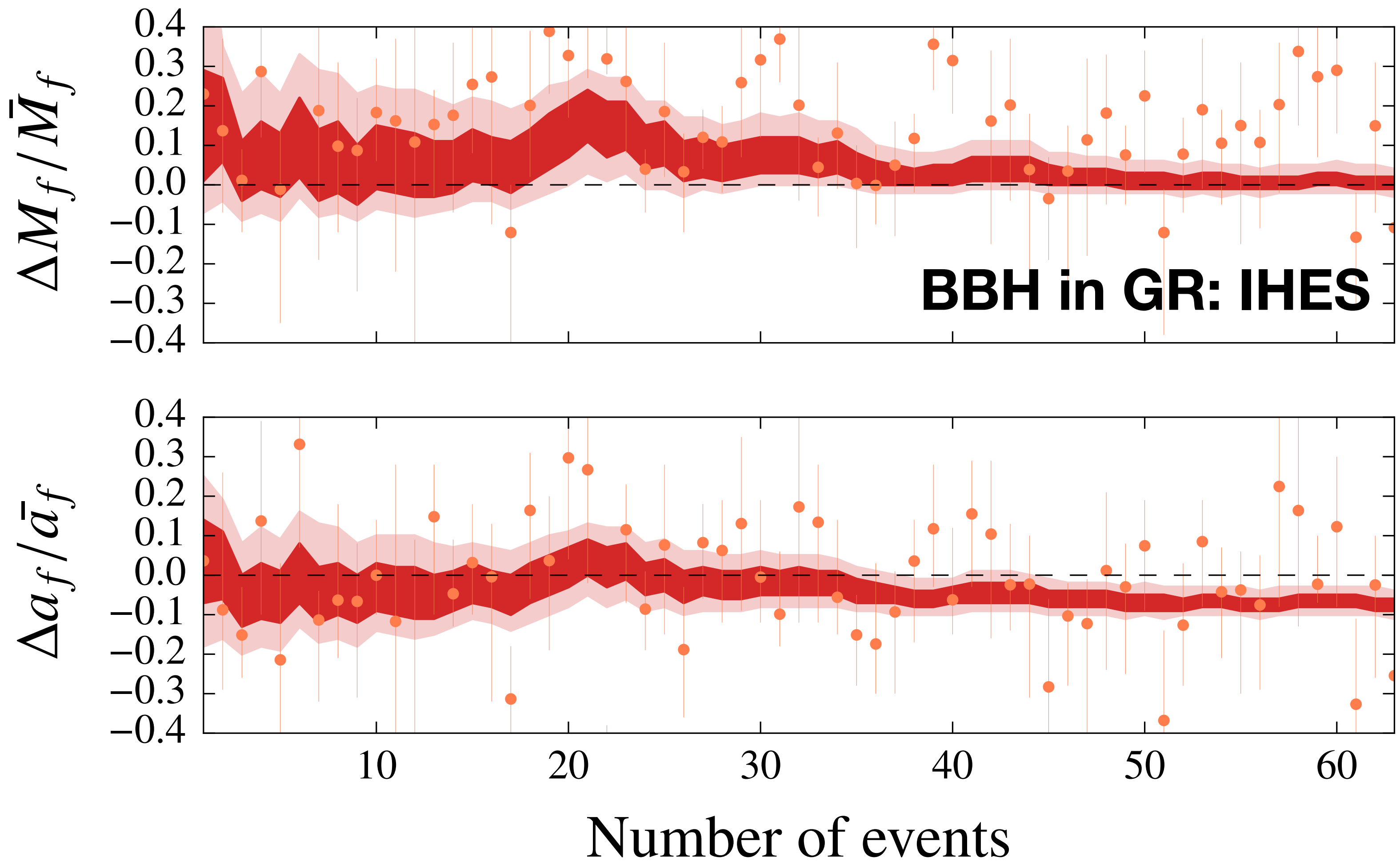
- The test can be used to detect certain kinds of deviations from GR
  - Energy and angular momentum loss differs from the predictions of GR.



- Parameters describing fractional deviations in the final mass and spin [should be consistent with the GR prediction (0,0)]

$$\frac{\Delta M_f}{\bar{M}_f} = \frac{2(M_f^I - M_f^{MR})}{M_f^I + M_f^{MR}}$$

$$\frac{\Delta a_f}{\bar{a}_f} = \frac{2(a_f^I - a_f^{MR})}{a_f^I + a_f^{MR}}$$



A no-hair test for binary black holes

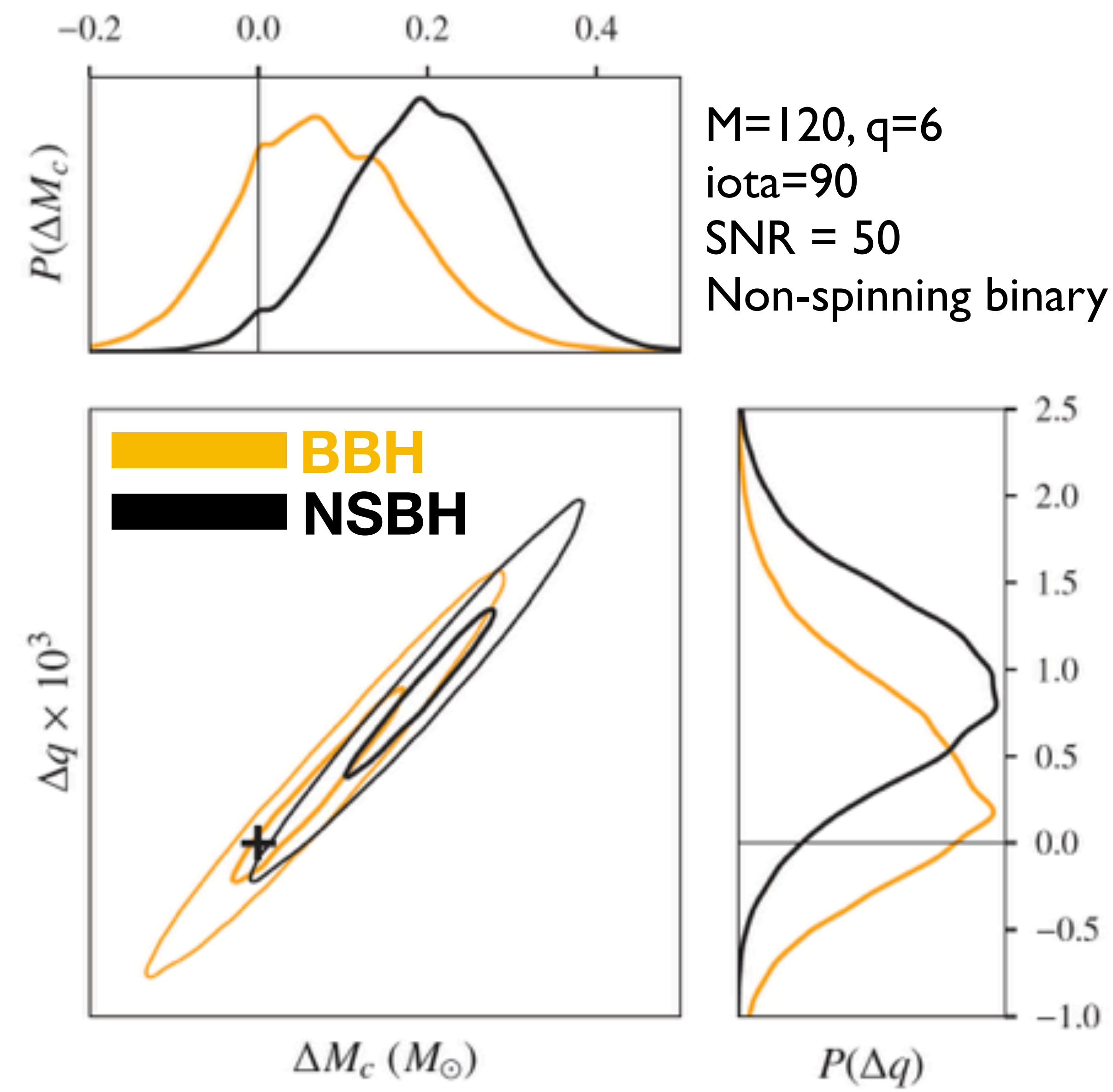
- Evolution of a BBH system through back-reaction of GWs depend on a few parameters:

$$\lambda = (m_1, m_2, \vec{s}_1, \vec{s}_2)$$

$$h(t;\mathbf{n},\boldsymbol{\lambda}) = \frac{1}{d_L}\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} Y_{\ell m}^{-2}(\mathbf{n})\, h_{\ell m}(t;\boldsymbol{\lambda}),$$

- Consistency between different modes of the observed signal: powerful test that the radiation is emanated from a binary black hole.

- Inconsistency between different modes:
    - departure from GR, or
    - the non-black hole nature of the compact objects.



[Siddharth D., AG, et al.; arXiv: 1804.03297, accepted in PRD]

Null tests of GR

- No complete/viable alternatives to GR
  - H0: “GR is the correct theory of gravity”
  - H1: “GR is not the correct theory of gravity”
- Our tests are only as good as our knowledge of the theory
- Statement

$$B = \frac{p(D|H_1 I)}{p(D|H_0 I)}$$

- How do we define the alternative hypothesis?

Alternatives

- H1 is a perturbation around GR
  - Gravitational waves:
    - TIGER (Li et al) $\phi_{eff} = \phi_{GR}(1 + \delta\phi)$
    - Ringdown tests (Gossan et al) $\omega_{eff} = \omega_{GR}(1 + \delta\omega)$
  - Binary pulsars:
    - PPK (Damour & Taylor)
- Reduce to GR for a fixed value of the perturbation parameter

$$
\begin{aligned}
\dot{\omega} &= n_b \left(\epsilon - \frac{1}{2}\xi + \frac{1}{2}\right) \frac{\beta^2}{1-e^2}\,, \\
\gamma &= \frac{e}{n_b} X_B(\widetilde{G}_{\mathrm{OB}} + \kappa + X_B)\beta^2 \\
r_p &= \frac{1}{4n_b} X_B \widetilde{G}_{\mathrm{OB}}(\epsilon_{0B} + 1)\beta^3\,, \\
s_p &= \frac{n_b x_A}{\beta X_B}\,,
\end{aligned}
$$

Strategy

- Constrain the perturbation parameters and compute the Bayes factor (LVC, GW150914)

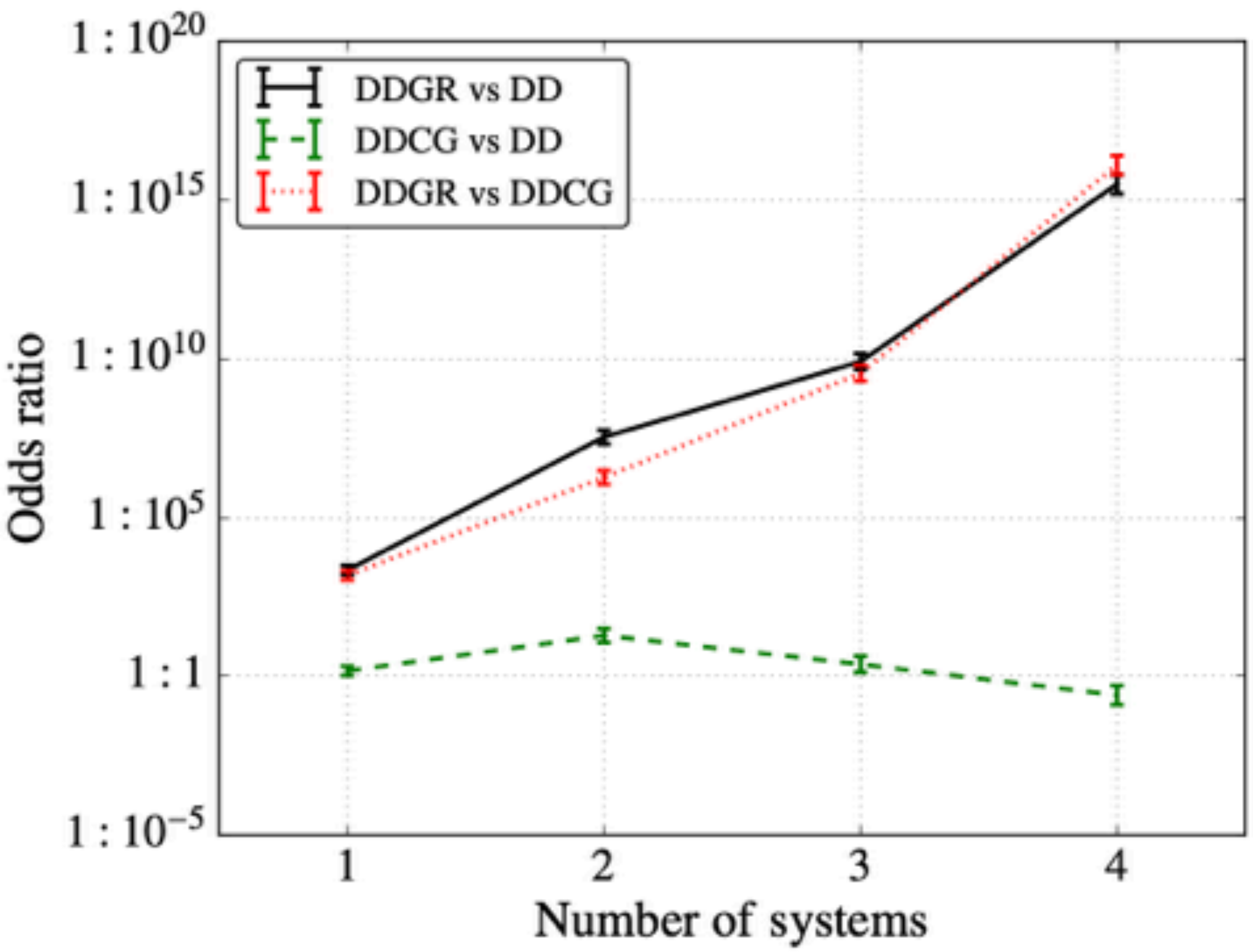
waveform regime			median		GR quantile		$\log_{10} B^{\mathrm{GR}}_{\mathrm{model}}$	
	parameter	f−dependence	single	multiple	single	multiple	single	multiple
early-inspiral regime	$\delta\hat{\varphi}_0$	$f^{-5/3}$	$-0.1^{+0.1}_{-0.1}$	$1.4^{+3.3}_{-3.0}$	0.94	0.21	1.9 ± 0.1	2.1 ± 0.6
	$\delta\hat{\varphi}_1$	$f^{-4/3}$	$-0.4^{+0.0}_{-0.9}$	$-0.6^{+17.7}_{-18.0}$	0.94	0.52	1.3 ± 0.3	
	$\delta\hat{\varphi}_2$	f^{-1}	$-0.35^{+0.3}_{-0.35}$	$-3.2^{+19.3}_{-15.2}$	0.97	0.60	1.2 ± 0.2	
	$\delta\hat{\varphi}_3$	$f^{-2/3}$	$0.2^{+0.2}_{-0.2}$	$2.6^{+13.8}_{-15.7}$	0.04	0.41	1.2 ± 0.1	
	$\delta\hat{\varphi}_4$	$f^{-1/3}$	$-2.0^{+1.6}_{-1.8}$	$0.5^{+17.3}_{-18.2}$	0.98	0.49	0.3 ± 0.1	
	$\delta\hat{\varphi}_{5l}$	$\log(f)$	$0.8^{+0.6}_{-0.55}$	$-1.5^{+19.1}_{-16.3}$	0.02	0.55	0.7 ± 0.1	
	$\delta\hat{\varphi}_6$	$f^{1/3}$	$-1.5^{+1.1}_{-1.1}$	$-0.6^{+18.2}_{-17.2}$	0.99	0.53	0.4 ± 0.1	
	$\delta\hat{\varphi}_{6l}$	$f^{1/3}\log(f)$	$8.9^{+6.8}_{-6.8}$	$-2.4^{+18.7}_{-15.2}$	0.02	0.57	-0.2 ± 0.1	
	$\delta\hat{\varphi}_7$	$f^{2/3}$	$3.7^{+2.6}_{-2.75}$	$-3.4^{+19.3}_{-14.8}$	0.02	0.59	-0.0 ± 0.2	
intermediate regime	$\delta\hat{\beta}_2$	$\log f$	$0.1^{+0.4}_{-0.3}$	$0.15^{+0.6}_{-0.5}$	0.29	0.35	1.2 ± 0.1	2.2 ± 0.1
	$\delta\hat{\beta}_3$	f^{-3}	$0.1^{+0.5}_{-0.3}$	$-0.0^{+0.8}_{-0.6}$	0.38	0.56	0.6 ± 0.1	
merger–ringdown regime	$\delta\hat{\alpha}_2$	f^{-1}	$-0.1^{+0.4}_{-0.4}$	$-0.0^{+1.0}_{-1.15}$	0.68	0.51	1.1 ± 0.1	2.1 ± 0.1
	$\delta\hat{\alpha}_3$	$f^{3/4}$	$-0.5^{+2.0}_{-1.5}$	$-0.0^{+4.4}_{-4.4}$	0.67	0.50	1.3 ± 0.1	
	$\delta\hat{\alpha}_4$	$\tan^{-1}(af+b)$	$-0.1^{+0.5}_{-0.6}$	$-0.0^{+1.2}_{-1.1}$	0.61	0.55	1.2 ± 0.1	

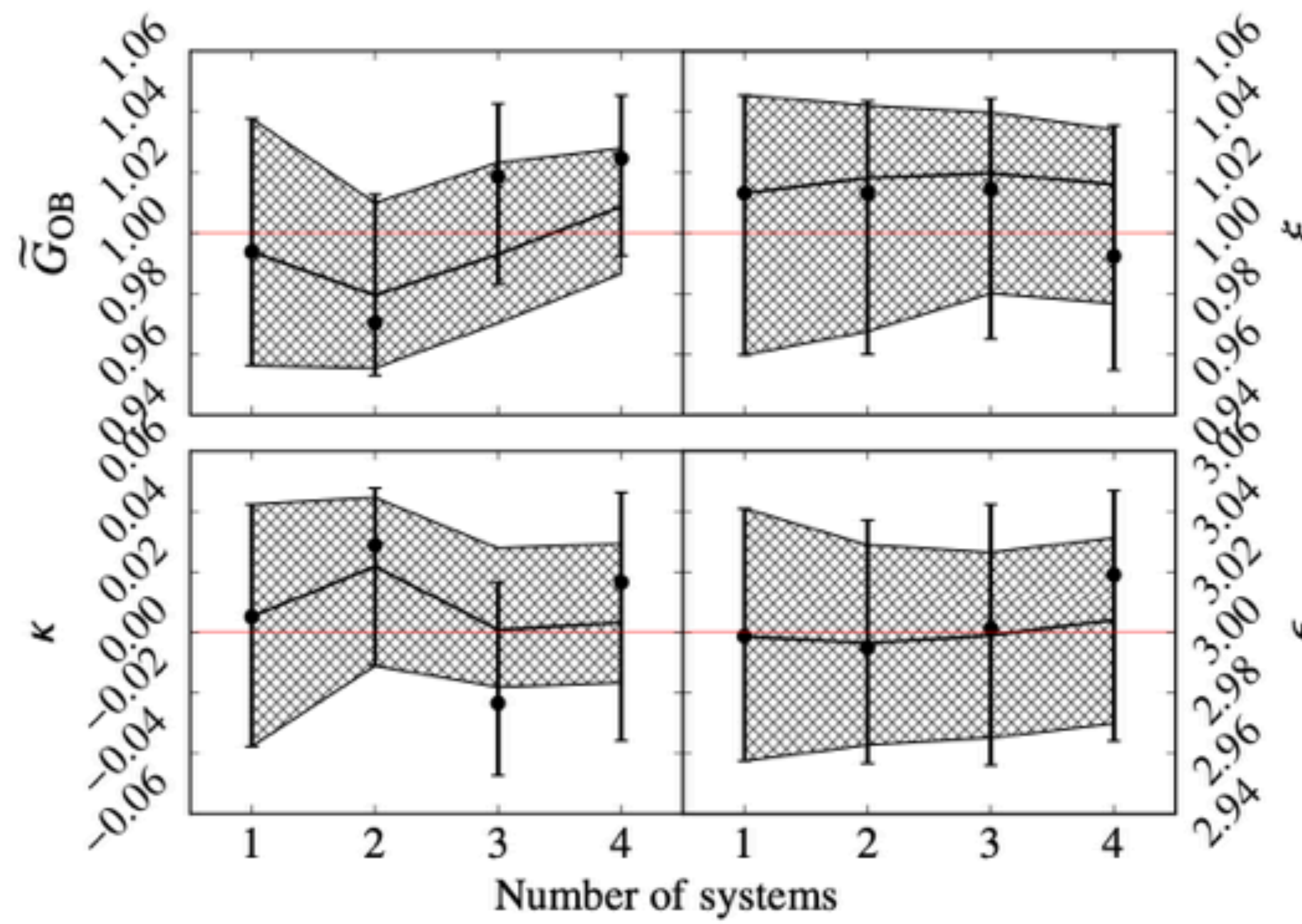
- Binary pulsar tests do not provide posteriors or Bayes factors
  - Kramer et al:

PK parameter	Observed	GR expectation	Ratio
$\dot{P}_b$	1.252(17)	1.24787(13)	1.003(14)
γ (ms)	0.3856(26)	0.38418(22)	1.0036(68)
s	0.99974(−39,+16)	0.99987(−48,+13)	0.99987(50)
$r(\mu s)$	6.21(33)	6.153(26)	1.009(55)

Bayesian PPK tests

- Binary pulsar tests can (and should) provide posteriors and Bayes factors (Del Pozzo & Vecchio)





A “one-size fits all” test of GR

- Null tests classically provide *p-values*
    - Summarise in a *single statement* the consistency of GR (Del Pozzo et al, in prep)
- Fisher combined statistics

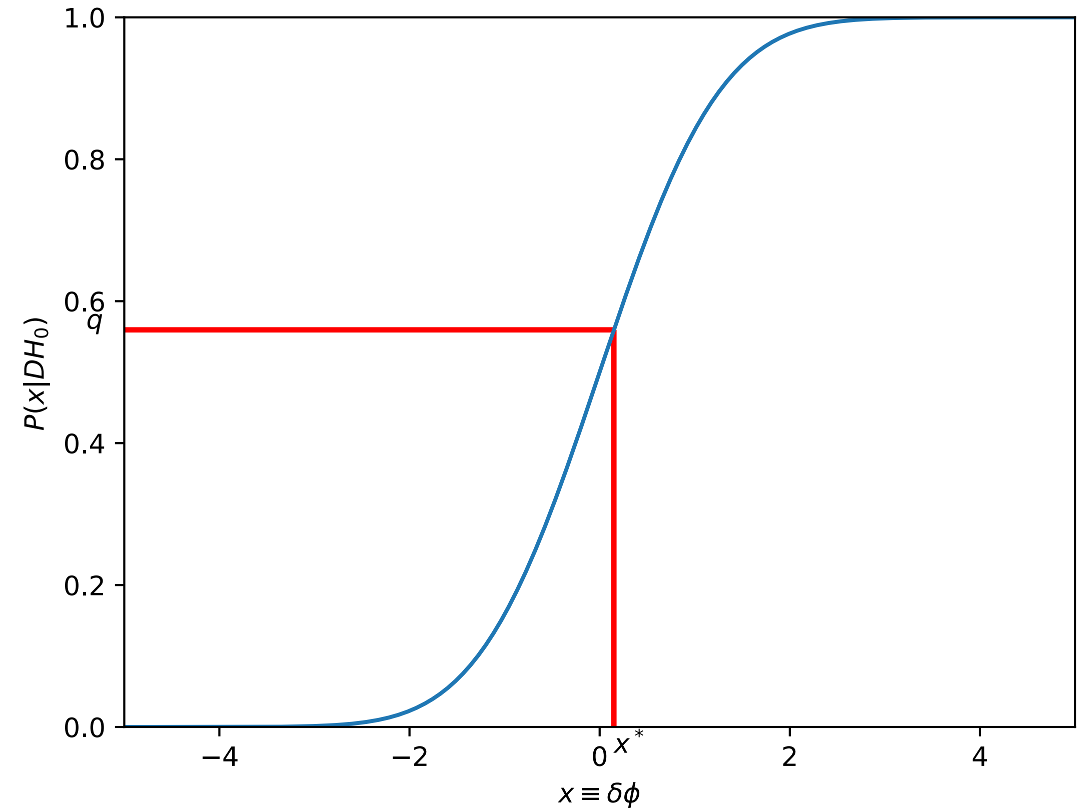
$$q = \int_{-\infty}^{x^\star} dx\, p(x|D,H_0)$$

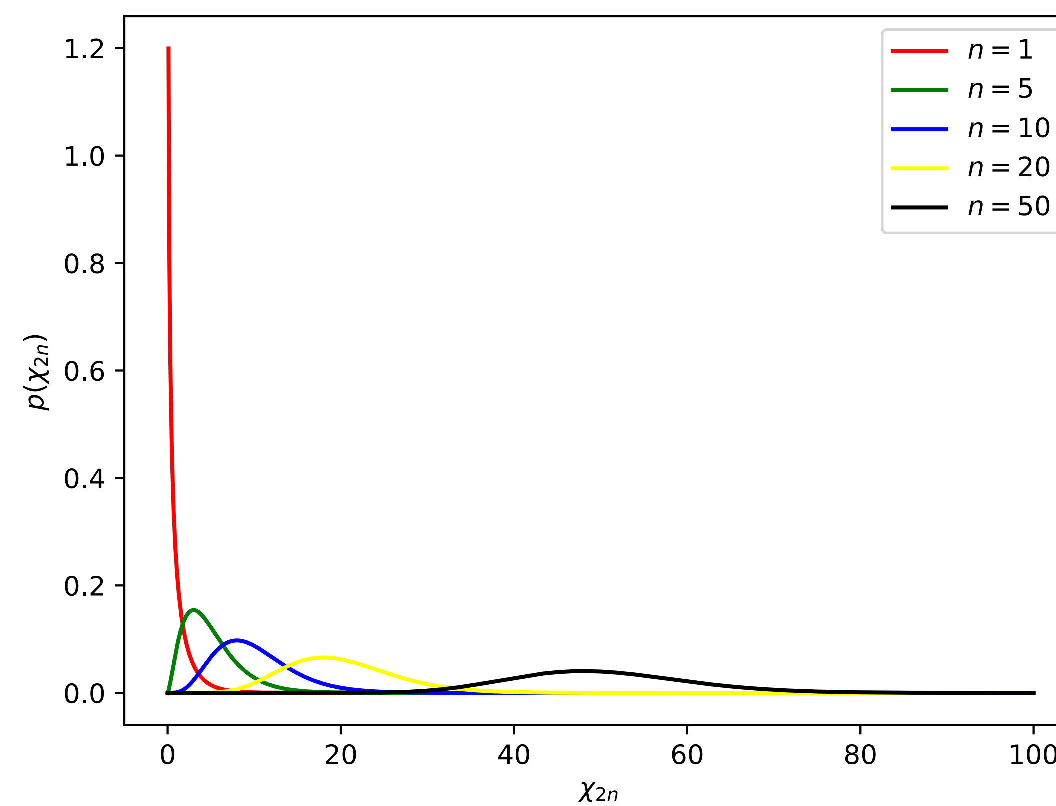
$$p = 1 - q$$

$$\chi^2_{2n} = -2\sum_{i=1}^{n}\log(p_i)$$

- GW150914+GW151226+Binary Pulsar +Cassini:

$$p = 0.7$$





- Dispersive nature of GWs have already been put to test using

$$E^2 = p^2 c^2 + A p^\alpha c^\alpha$$

Mirshekari, Yunes and Will, Phys. Rev. D85: 024041, 2012
LVC, Phys. Rev. Lett. 118, 221101 (2017)

- The two GW polarisations may in addition travel at different speeds, leading to birefringence.

Mewes, arxiv: 1905:00409

- Parity violation can in addition give rise to *amplitude-birefringence.*

Yagi and Yang, Phys. Rev. D 97, 104018 (2018)

- Tests for additional polarisations?

Isi and Weinstein, arxiv: 1710:03794

LVC, Phys. Rev. Lett. 119, 141101 (2017)

- Leading to further tests of Lorentz violation using GWs?