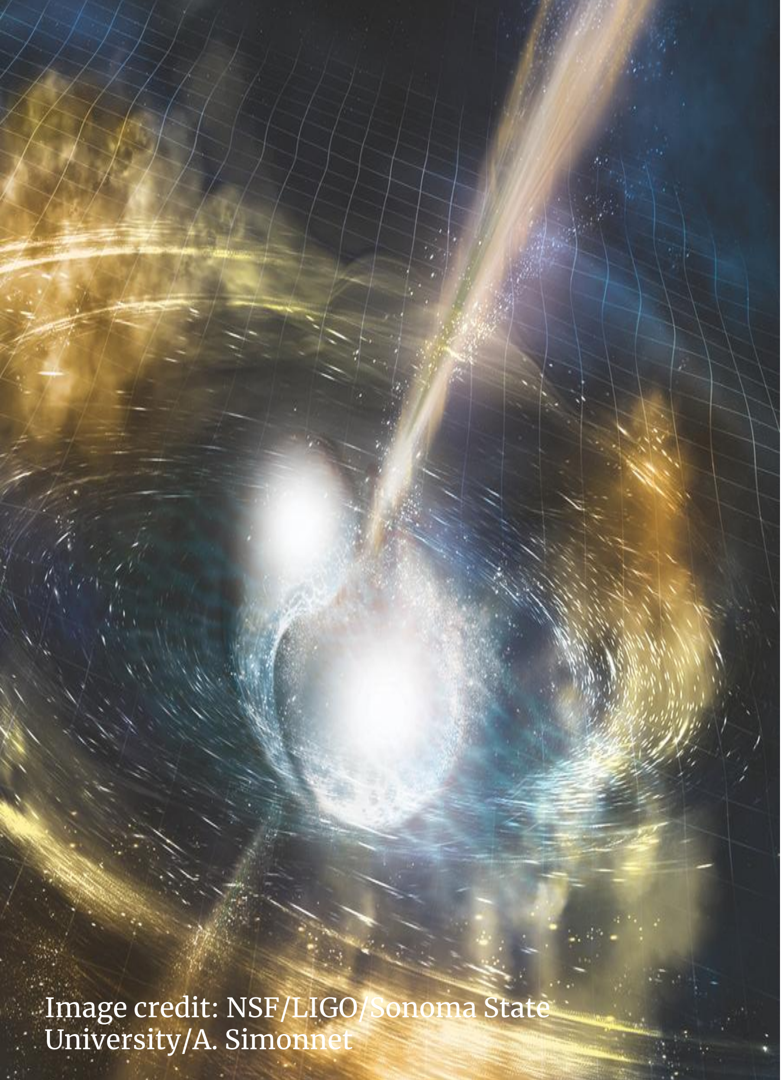
Image credit: NSF/LIGO/Sonoma State University/A. Simonnet

Searching for the phase transition in the dense matter equation of state using anomaly detection technique

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g2net WG1 meeting
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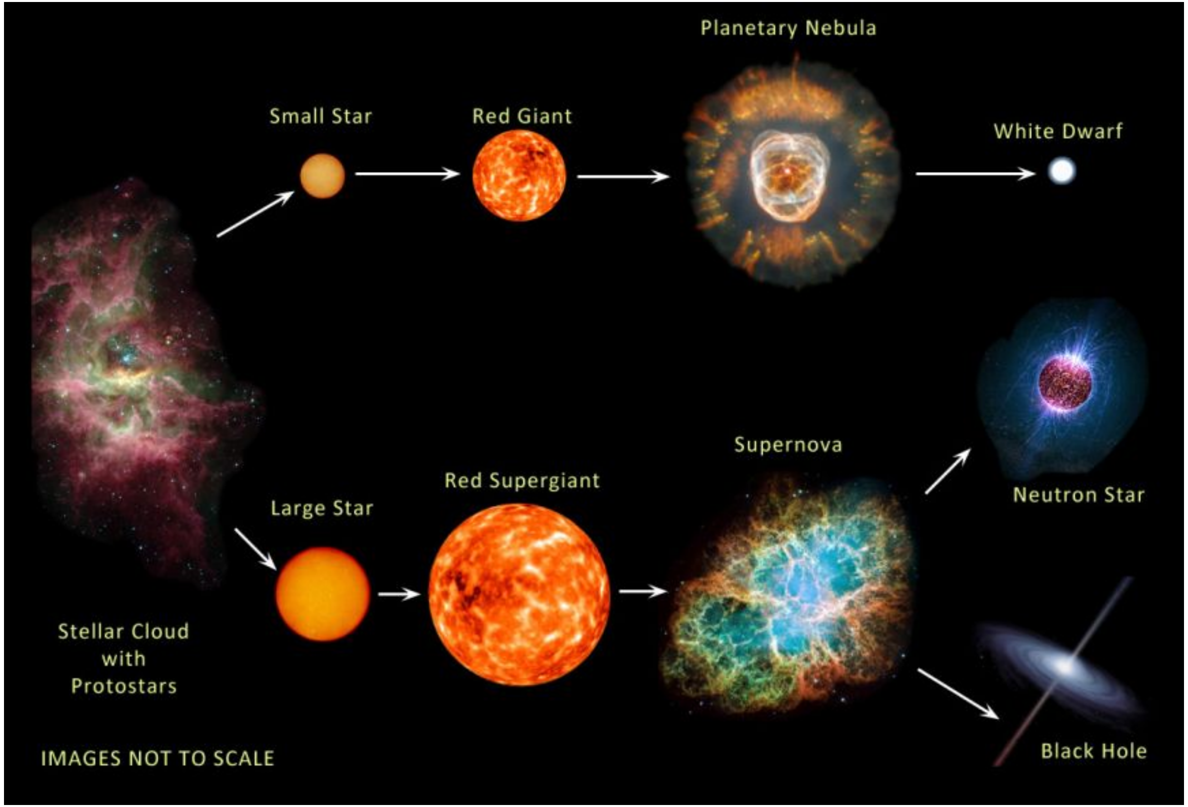
VIRGO G2NET cost EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY CAMK PAN

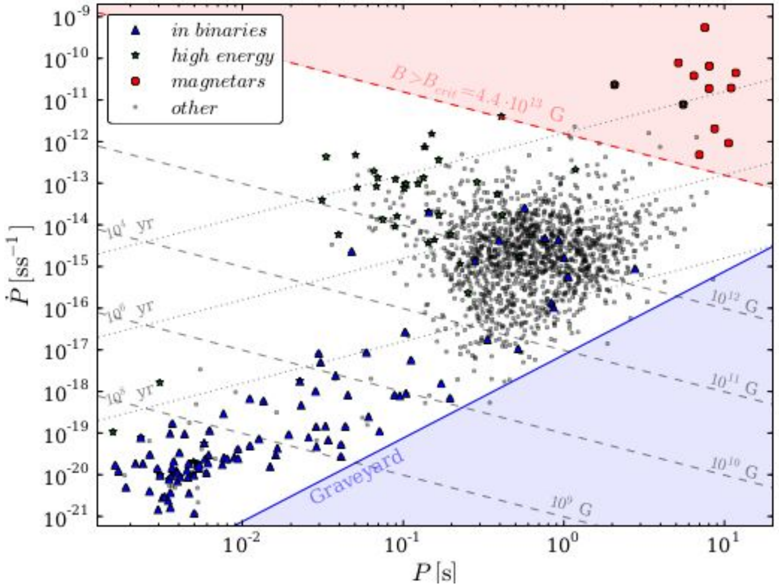
Origin



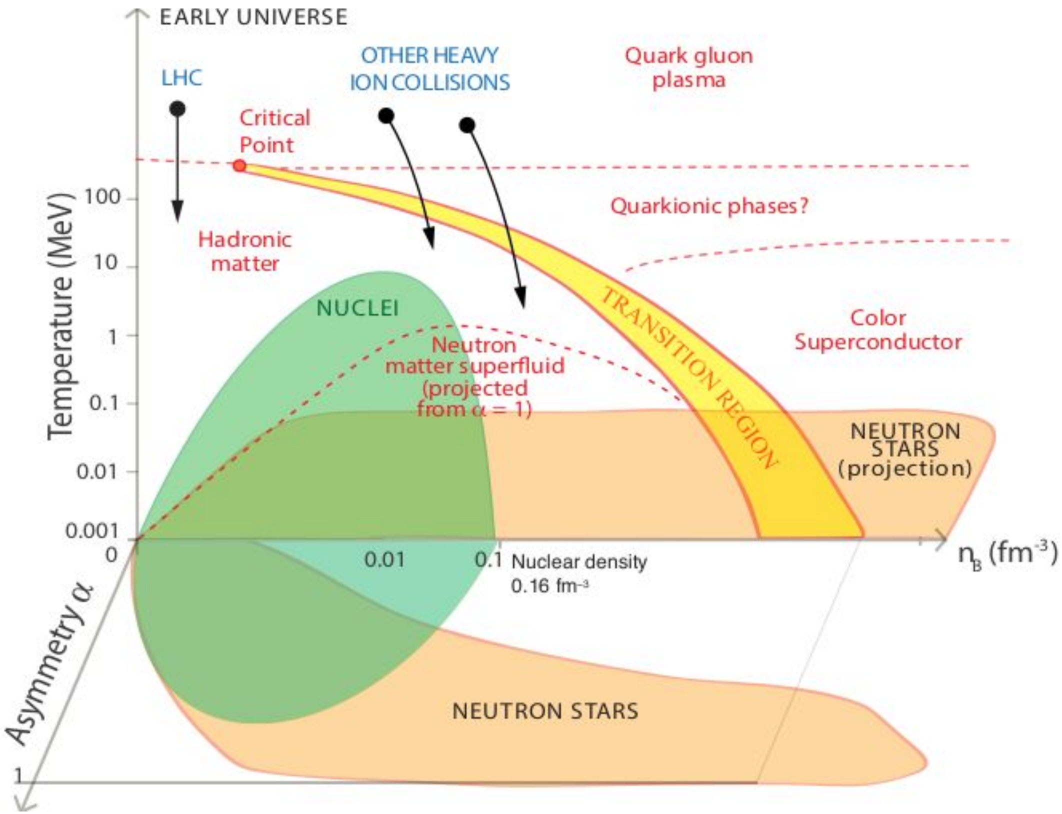
Planetary Nebula
Small Star
Red Giant
White Dwarf
Supernova
Red Supergiant
Large Star
Neutron Star
Stellar Cloud with Protostars
Black Hole
IMAGES NOT TO SCALE

Characteristics

- Compact
- Dense
- The most relativistic **material** objects in the Universe
- Laboratory for extreme physics allowing studies of dense matter equation of state (EOS)



Laboratories of extreme physics



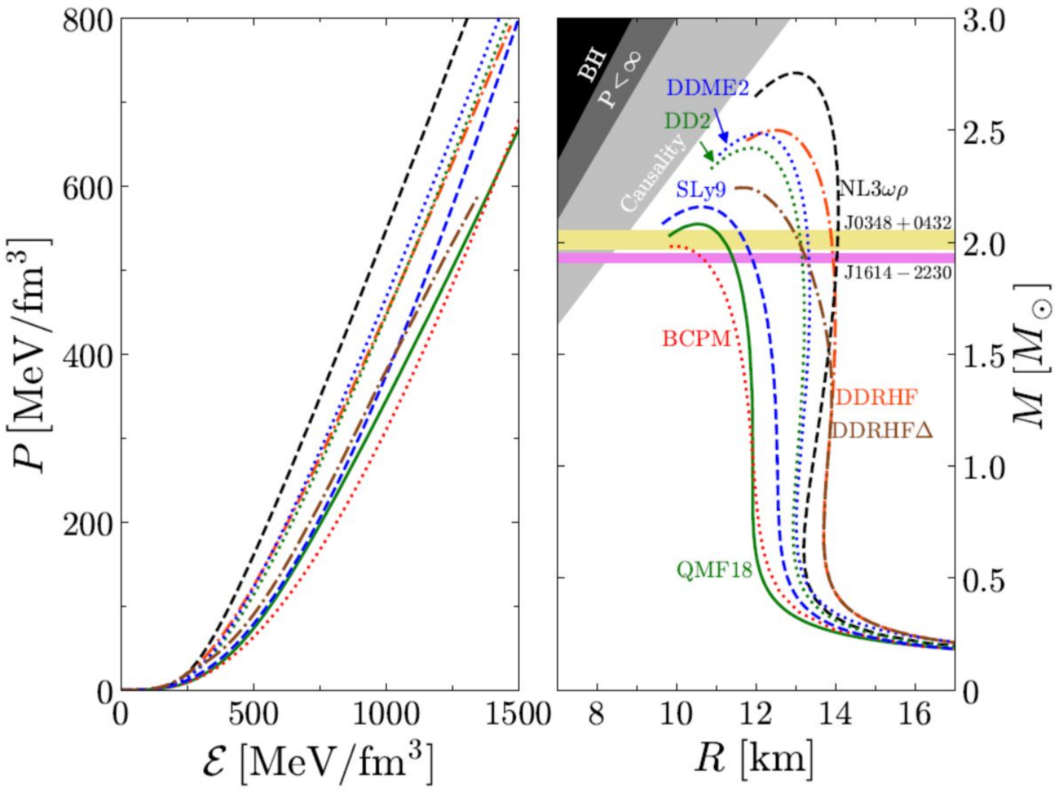
Source: Watts et al.
arxiv: 1507.00042

How to link microscopic to macroscopic properties of neutron stars?

Tolman-Oppenheimer-Volkoff equation:

$$\frac{dP}{dr} = \frac{-Gm}{r^2} \rho \left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi r^3 P}{mc^2}\right)\left(1 - \frac{2Gm}{rc^2}\right)^{-1}$$

Dense matter to neutron star properties



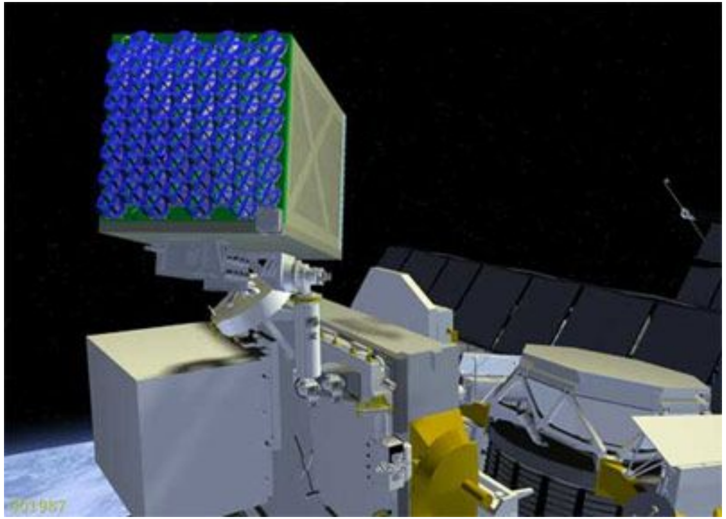
Source: Z.-Y. Zhu et al. (2018)

NICER: Neutron-star Interior Composition ExploreR

X-ray timing and spectroscopy instrument focusing on rotation-powered millisecond pulsars.

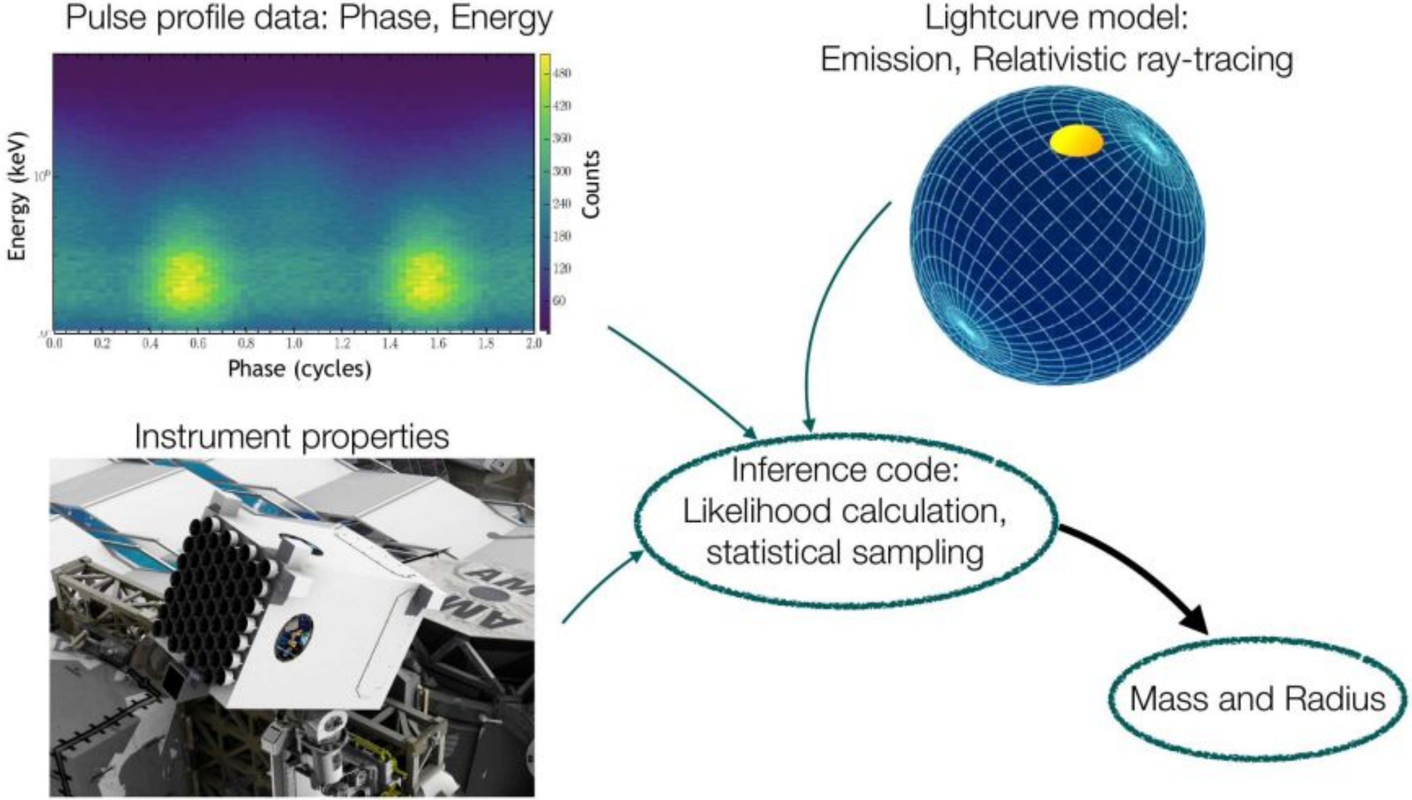
Masses and radii are computed via pulse-profile modeling.

Pulse-profile modeling is a technique that probes general relativistic effects on thermal emission from hot regions on the stellar surface.

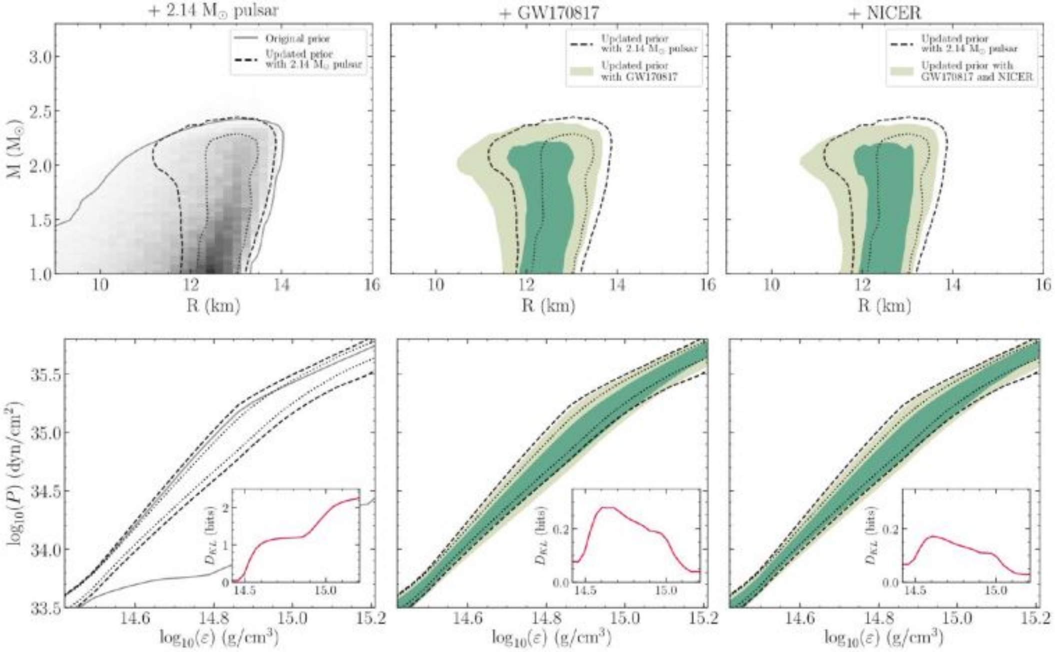


Source: NASA

NICER: from pulses to M(R)



Multi-messenger: EM (NICER) + GW (LIGO-Virgo)



Project aim

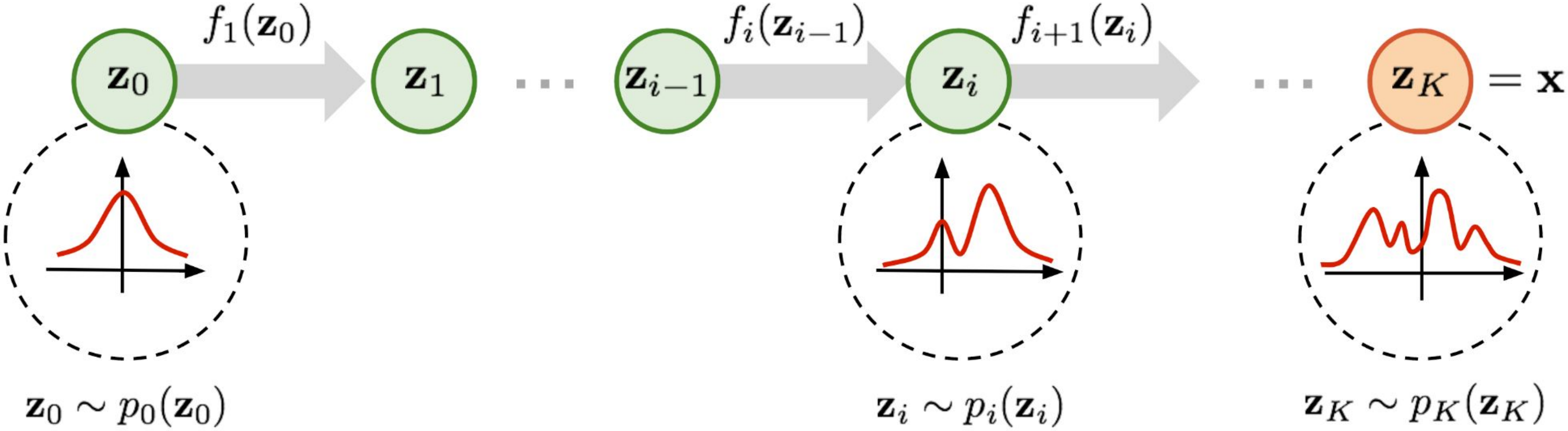
What?

Search for the phase transition presence using neutron star observables such as masses and radii.

How?

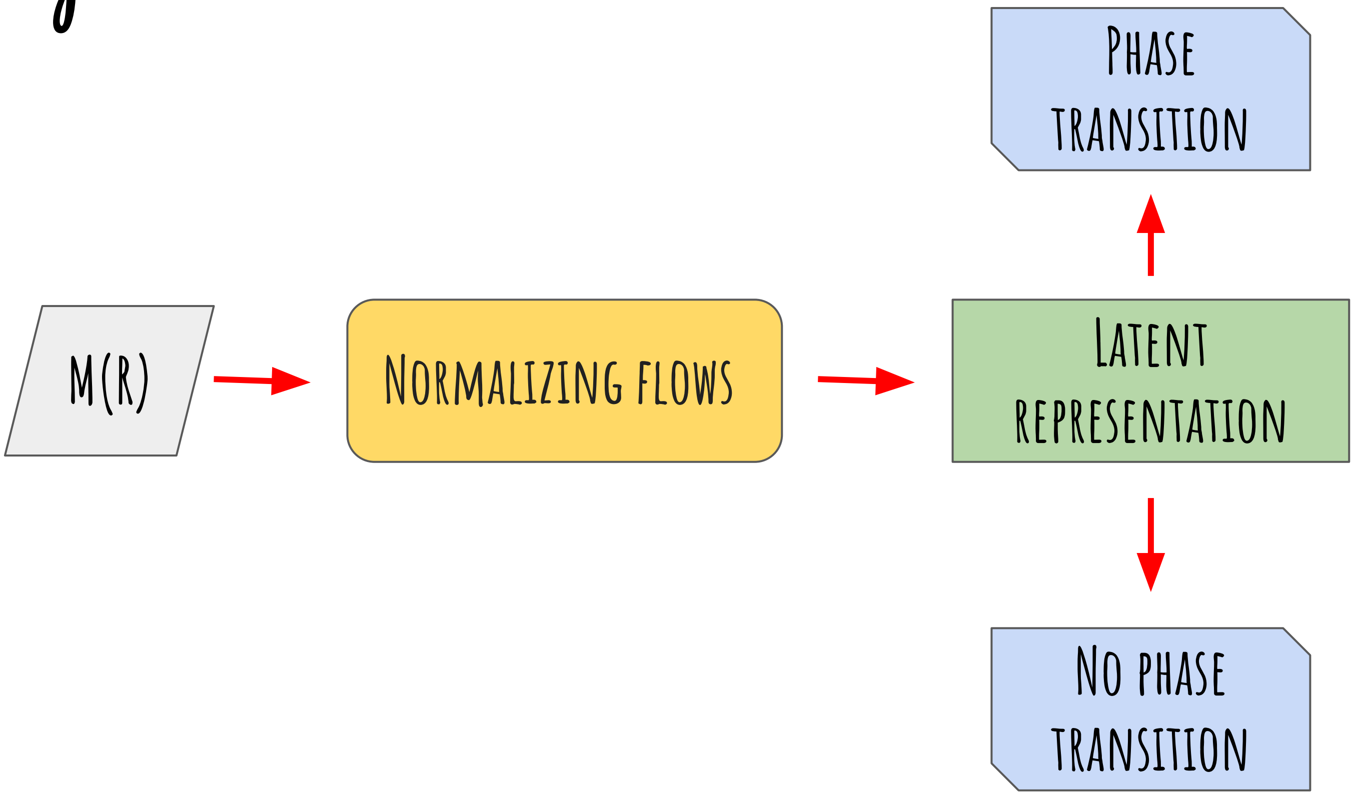
Use anomaly detection approach based on normalizing flows algorithm to train the model against data without phase transition and test it against the data that contains it.

Normalizing flows



Source: Lilian Weng tutorial on normalizing flows

Concept


M(R)
Normalizing flows
Latent representation
Phase transition
No phase transition

Data

Training:

Low-density part of EOS is adopted from existing astrophysical model (Sly4) up to particular baryon density n_0. This part is combined with piecewise relativistic polytrope:

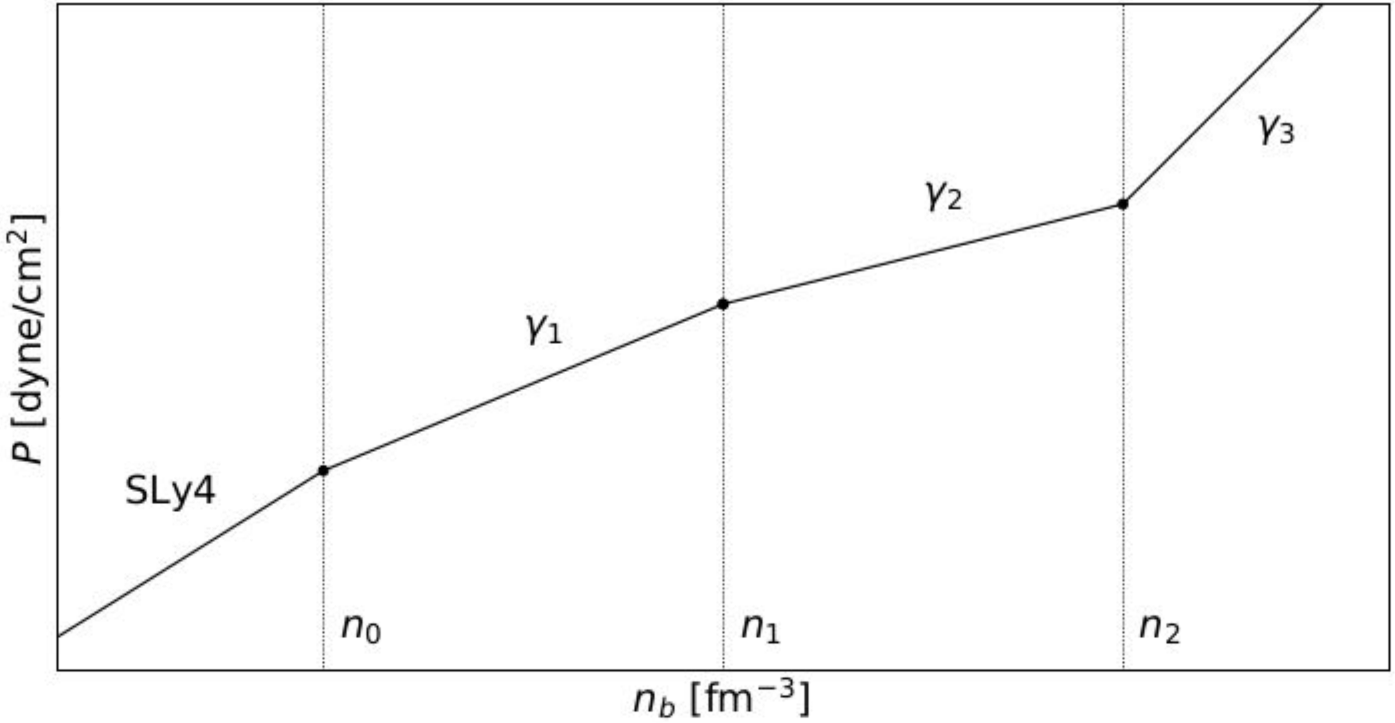
$$P(n) = \kappa n^{\gamma}$$

$$\rho c^2 = \frac{P(n)}{\gamma - 1} + n m_b c^2$$

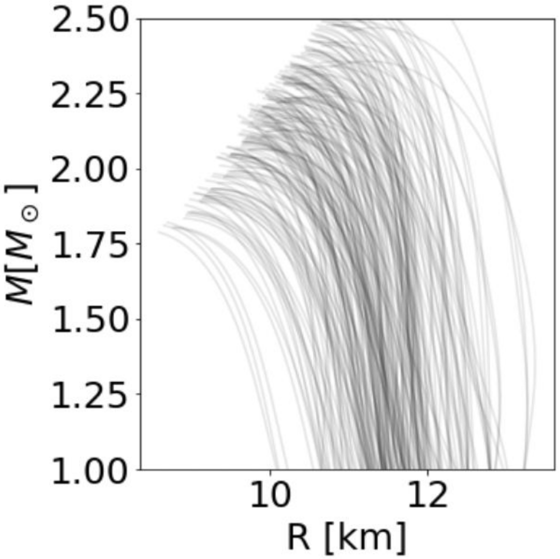
Testing:

Polytropes including phase transition as well as astrophysical model (Drago et al. 2014)

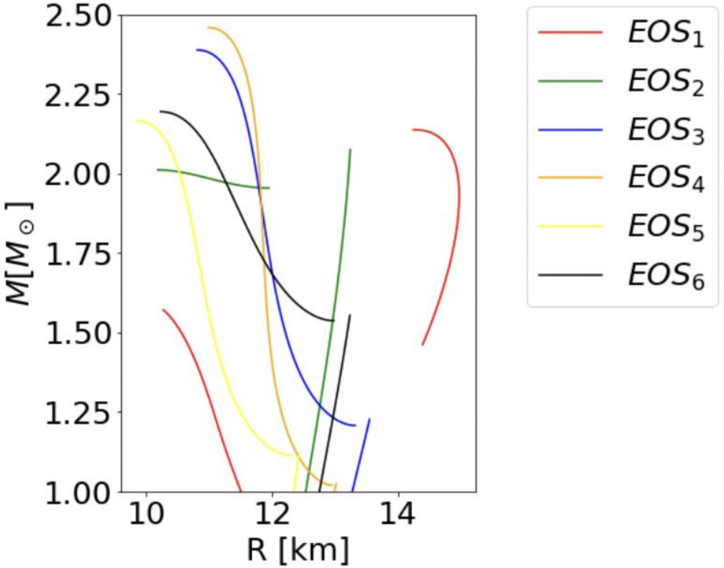
Relativistic polytrope



Types of EOS



No phase transition

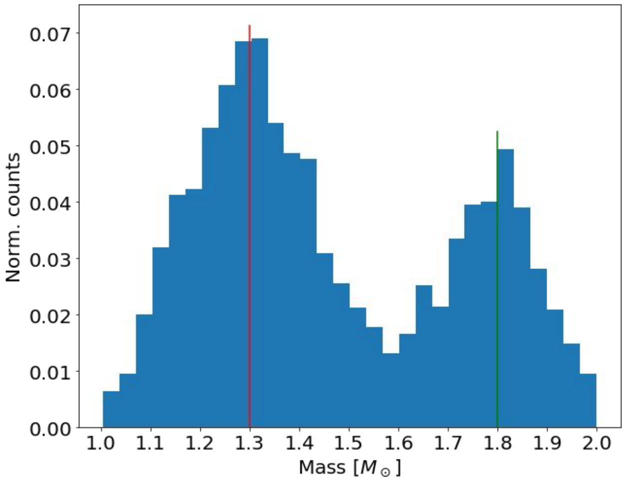


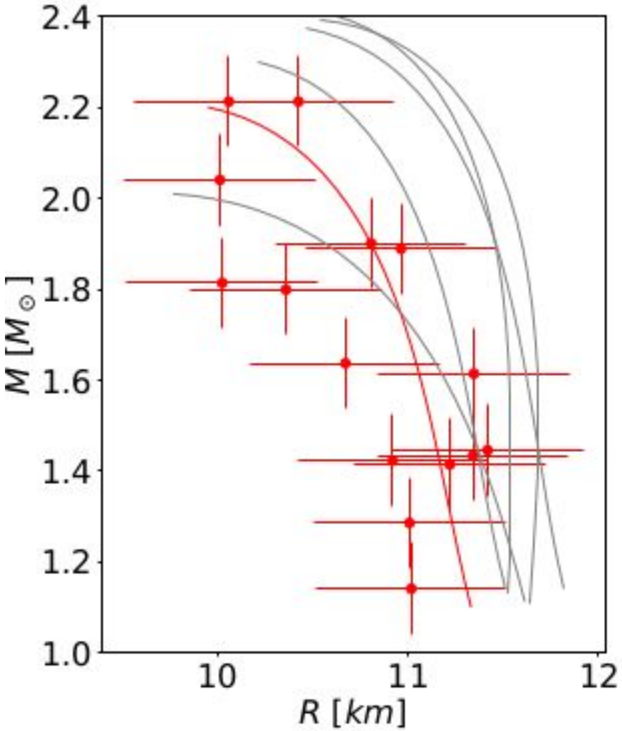
Phase transition

Neutron star mass distribution

Mass range restricted to the astrophysically-realistic range: [1, 2,2] $M_\odot$.

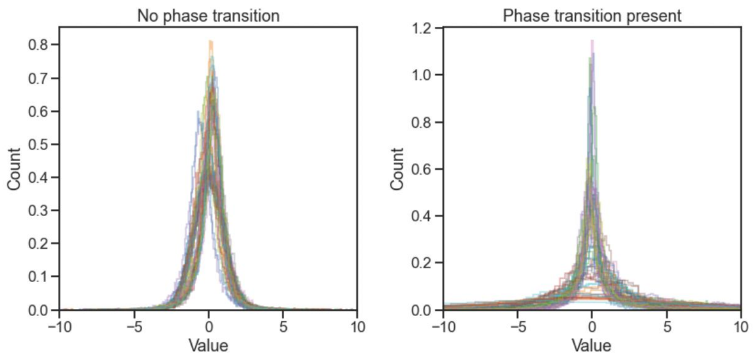
It corresponds to the observed NS masses.





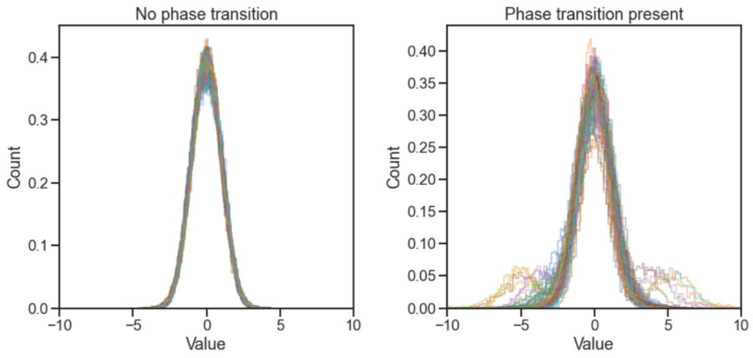
Results - latent representation

Without measurement uncertainties



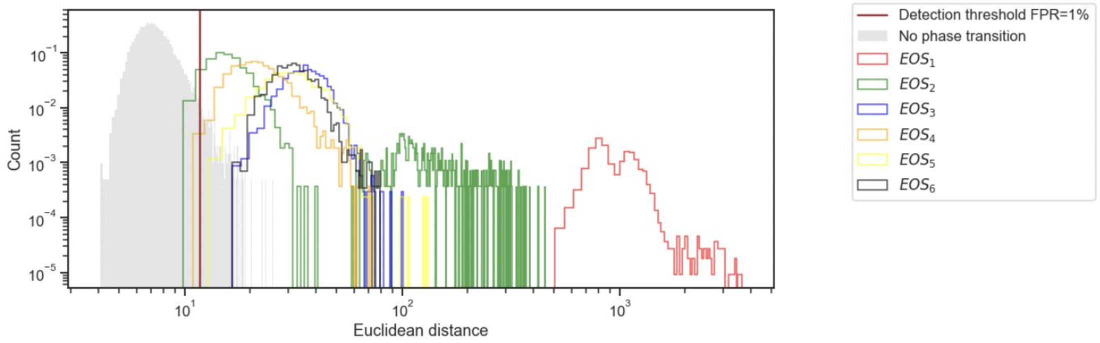
$U(R) = (-1, 1)$ km

$U(M) = (-0.1, 0.1)\ M_{\odot}$



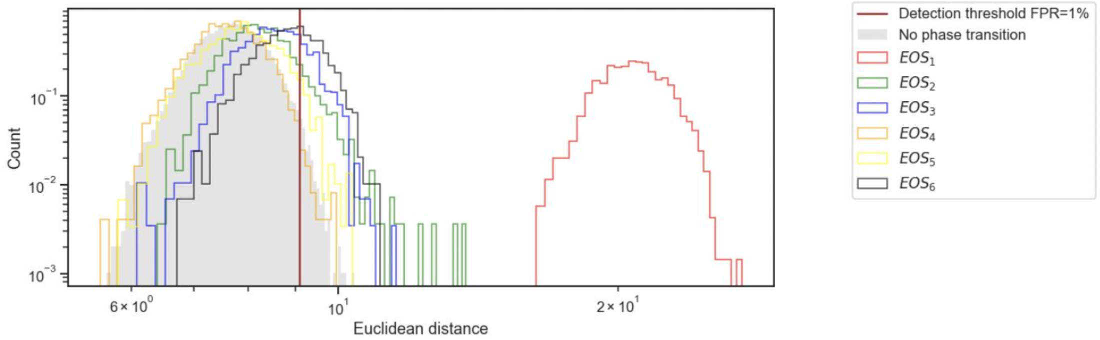
Results - Euclidean distance

Without measurement uncertainties

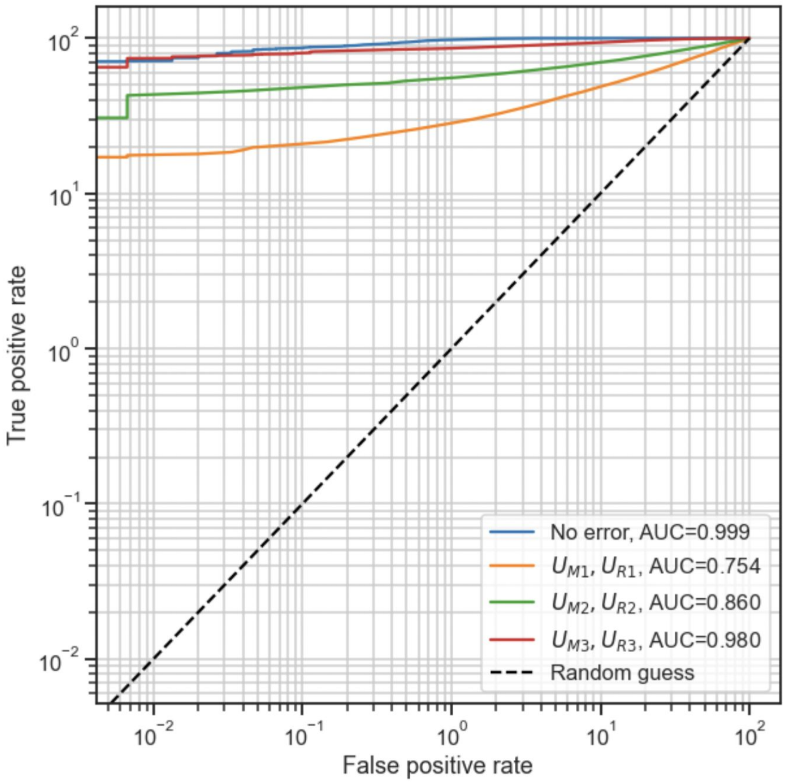


$U(R) = (-1, 1)$ km

$U(M) = (-0.1, 0.1)\ M_\odot$



Results - ROC



No error, AUC=0.999
U_{M1}, U_{R1}, AUC=0.754
U_{M2}, U_{R2}, AUC=0.860
U_{M3}, U_{R3}, AUC=0.980
Random guess
True positive rate
False positive rate
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}

Results - AUC of ROC

$M(R)$	$N = 10$	$N = 30$	$N = 50$
No error	99.72%	99.91%	99.98%
U_{M1}, U_{R1}	95.92%	98.02%	98.82 %
U_{M2}, U_{R2}	81.56%	86.01%	94.43%
U_{M3}, U_{R3}	72.97%	75.41%	88.49%

Results - detection efficiency for FPR=1%

$M(R)$ input data	EOS_1	EOS_2	EOS_3	EOS_4	EOS_5	EOS_6
No error, $N = 10$	100%	86.95%	100%	93.75%	99.4%	100%
U_{M1}, U_{R1}, $N = 10$	100%	46.45%	98.3%	48.25%	85.65%	100%
U_{M2}, U_{R2}, $N = 10$	100%	25.15%	57.4%	2.65%	18.35%	75.55%
U_{M3}, U_{R3}, $N = 10$	100%	11.80%	24.25%	1.65%	5.5%	35.8%
No error, $N = 30$	100%	95.85%	100%	99.8%	100%	100%
U_{M1}, U_{R1}, $N = 30$	100%	35.25%	100%	90.85%	100%	100%
U_{M2}, U_{R2}, $N = 30$	100%	24.4%	84.35%	4.1%	32.70%	99.15%
U_{M3}, U_{R3}, $N = 30$	100%	12.0%	22.75%	0.85%	5.05%	39.65%
No error, $N = 50$	100%	100%	100%	90.95%	100%	100%
U_{M1}, U_{R1}, $N = 50$	100%	47.3%	100%	98.8%	100%	100%
U_{M2}, U_{R2}, $N = 50$	100%	50.35%	97.45%	23.20%	73.15%	99.60%
U_{M3}, U_{R3}, $N = 50$	100%	32.35%	57.35%	8.75%	21.10%	72.30%

Summary

- We provided overview of anomaly detection based searches of phase transition using masses and radii observations

- Efficiency of methods mostly depends on the measurement uncertainties; it can be improved by increasing number of observations

- Extreme cases of EOS (like EOS_1 - Drago et al. model) are easily detected regardless of measurement uncertainties or number of observations