Universitat de les Illes Balears
IAC3 Institute of Applied Computing & Community Code.

Modeling compact binary waveforms using machine learning methods.

G2Net WG1 meeting Valencia
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Waveform models I

- In order to identify the nature of the source (NS/BH) and its parameters (masses, spins, distance, …) we need accurate waveform models.
    - Synthesized from numerical relativity (NR) catalogs and perturbative results (post-Newtonian, …).
- 3 main approaches, standard code base: LAL, reviewed by LVC
    - “Surrogates” - interpolate waveform catalogs - limited in parameter space coverage
    - Effective one body: model Hamiltonian and radiative flux => integrate ODEs => expensive
    - “Phenomenological”: piecewise closed form expressions => fast
- Status: no generic inspiral-merger-ringdown waveforms (spin precession+eccentricity)
    - non-precessing non-eccentric models calibrated to NR - calibration of precessing models to NR still very limited

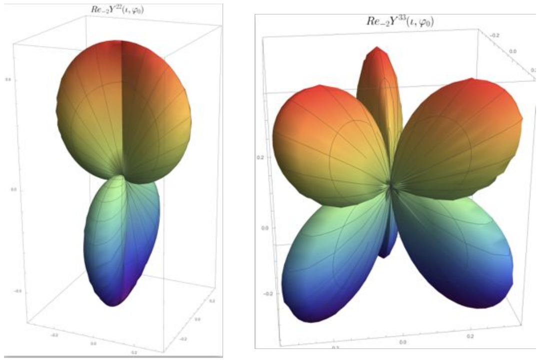
Third generation (IMRPhenomD-based): Simple, fast and accurate (for O1) models.
Standard tool for GW data analysis, still employed.

Fourth generation:
thorough improvement in accuracy + HM calibration + speed-up algorithms => IMRPhenomXPHM

New time-domain family: Similar techniques and data set as in IMRPhenomX* development.
Aimed to facilitate modelling generic waveforms, e.g. improve high mass precessing description, provide alternative handle on tests of GR.

Waveform models II

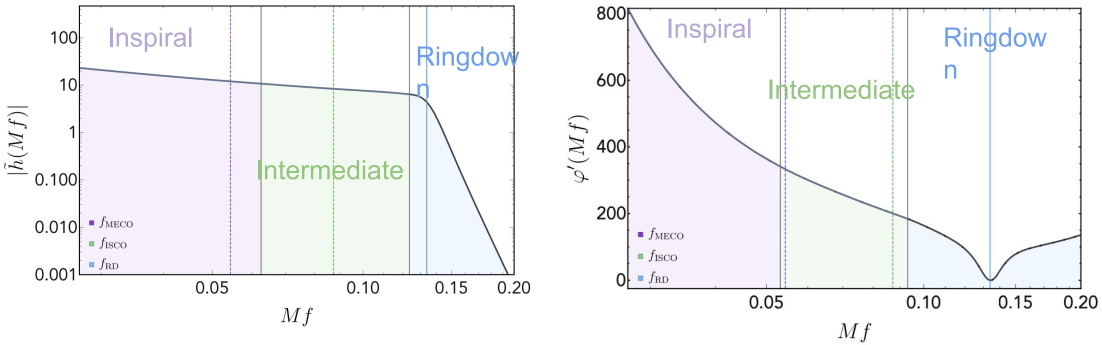
- Waveforms are modelled as sums of spherical harmonics
  - Quadrupole modes (2,2) are the dominant modes in GW signals.
  - Subdominant modes are important in the merger and ringdown, close to the edge-on and high unequal masses.



Spherical harmonics (2,2) and (3,3).

$$h(t;\lambda,\theta,\phi) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} Y_{l,m}^{-2}(\theta,\phi)h_{l,m}(t;\lambda) \qquad h_{l,m}(t) = A_{l,m}(t)e^{i\phi_{l,m}(t)}$$

- IMRPhenom waveform modelling use piecewise closed-form expressions for WF - key to simplicity and speed - compression of WF info.
  - Model the amplitudes and phases of the spherical harmonics using typically 3 different regions: inspiral/merger/RD



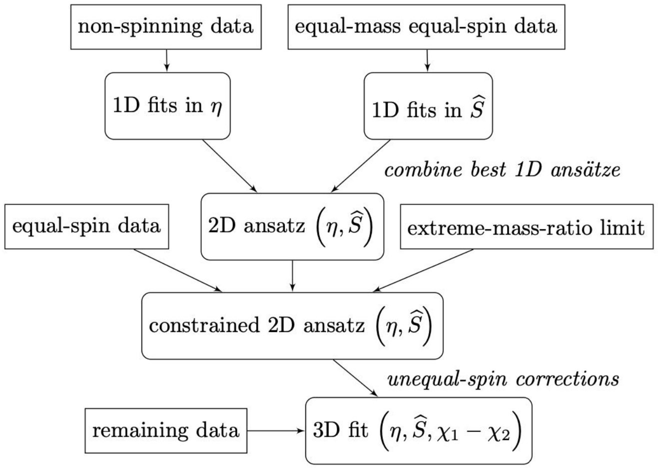
IMRPhenomXAS regimes, Pratten et al., 2020.

Phenomenological waveform modelling

Steps to construct a Phenom model:

1. Design appropriate WF ansatz across the parameter space.
    - Combine PN theory, BH perturbation theory and NR.
    - Flexibly use insight from analytical results and numerical studies of calibration.
    - Every point in parameter space is described by small number of coefficients.

2. "Direct" fit: Find the best coefficients for each waveform in the calibration data set.
3. Parameter space fit: model coefficients across parameter space.
4. Reconstruction: construct waveform from coefficients - more than one way!



Hierarchical fits, Jiménez-Forteza et al., 2017.

Ansätze used: Example of IMPhenomXHM

Non-precessing circular waveforms - model the amplitudes and phases of the (spherical or spheroidal) harmonics. In the frequency domain:

Inspiral	Intermediate	Ringdown
SPA transformation to PN waveform $A_{\ell m}^{\mathrm{SPA}}(f) = A_{\ell m}^{TD}(x)\sqrt{\frac{2\pi}{m\ddot{\phi}(x)}}$ $\mathcal{H}_{\ell m}(f) = \frac{\lvert A_{\ell m}^{PN}(f)\rvert}{A_{22}^{0}(f)} + \alpha\left(\frac{f}{f_{\ell m}^{\mathrm{Ins}}}\right)^{\frac{7}{3}} + \beta\left(\frac{f}{f_{\ell m}^{\mathrm{Ins}}}\right)^{\frac{8}{3}} + \gamma\left(\frac{f}{f_{\ell m}^{\mathrm{Ins}}}\right)^{\frac{9}{3}}$ 3 collocation points	Connect inspiral and RD fits 4 constraints at boundaries $\frac{A_{\ell m}^{\mathrm{Inter}}}{A_0} = \frac{1}{\delta_0+\delta_1 f+\delta_2 f^2+\delta_3 f^3+\delta_4 f^4+\delta_5 f^5}$ 2 collocation points	Phenomenological ansatz Use QNM natural frequencies $\frac{A_{RD}^{\ell m}}{A_0^{22}} = \frac{1}{f^{\frac{1}{12}}}\frac{\lvert a_\lambda\rvert\, f_{damp}^{\ell m}\,\sigma}{\left(f-f_{ring}^{\ell m}\right)^2+\left(f_{damp}^{\ell m}\,\sigma\right)^2} e^{-\frac{\left(f-f_{ring}^{\ell m}\right)\lambda}{f_{damp}^{\ell m}\sigma}}$ 3-2 coefficients
PN scaling $\phi_{\ell m}(f) = \frac{m}{2}\phi_{22}^{X}(2/mf) + \Lambda_{\ell m}(f) + d\phi_{\ell m}^{\mathrm{Ins}} f + \phi_{\ell m}^{\mathrm{Ins}}$ Smooth transition to intermediate	Fully calibrated $\frac{d\phi_{\ell m}^{\mathrm{Int}}}{df} = a_\lambda^{\ell m}\frac{f_{\mathrm{damp}}^{\ell m}}{(f_{\mathrm{damp}}^{\ell m})^2+(f-f_{\mathrm{ring}}^{\ell m})^2} + \sum_{k=0}^{4}\frac{a_k^{\ell m}}{f^k}$ 5 collocation points	Phenomenological scaling $\phi_{\ell m}^{\mathrm{RD}} = -\alpha_2^{\ell m}\frac{(f_{ring}^{\ell m})^2}{f} + \alpha_\lambda^{\ell m}\tan^{-1}\left(\frac{f-f_{ring}^{\ell m}}{f_{\mathrm{damp}}^{\ell m}}\right) + d\phi_{\mathrm{RD}}^{\ell m} f + \phi_{\mathrm{R}}^{\ell}$ Smooth transition to intermediate

Non-prececessing QC sector has been calibrated to NR => very accurate & fastest LAL models in the frequency and time domain.

"Twisting" up for precession

- Idea: precessing waveforms look simpler in a co-precessing frame => describe precessing waveform in terms of rotating a waveform in a co-precessing frame.
  - Can use Euler angle or quaternion description:

$$\tilde{h}^I_{lm}(f) = \mathcal{D}^l_{mm'}[\alpha(f), \beta(f), \gamma(f)]\tilde{h}^{\mathrm{coprec}}_{lm'}(f)$$

- Current precessing IMRPhenom models still use an approximation that allows to skip calibration to NR:
  - identify co-precessing WF with non-precessing WF.
- Future: calibrate to NR, e.g. single spin or complete double spin parameter space.

Machine Learning: Applications to waveform modeling

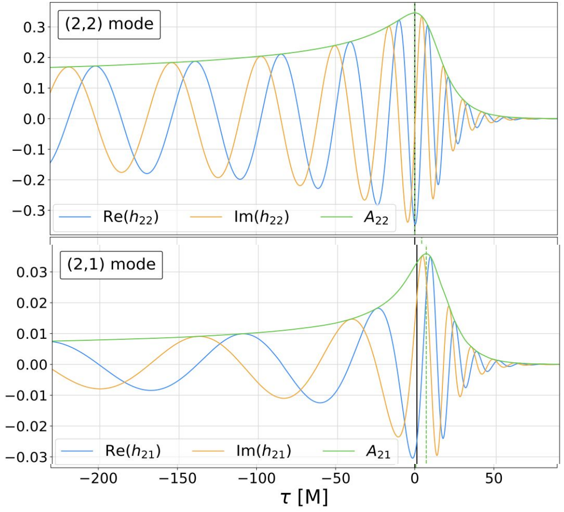
Fit a high dimensional parameter space with phenomenological techniques of waveform modeling is very difficult. The usage of ML could facilitate the procedure.

In this talk we present our current work applying ML techniques to predict the remnant properties of a BBH. As training data we use NR simulations from SXS catalog (2016 BBH NR simulations: 593 AS + 1423 Prec.)

- In the catalog there is information about the time evolution of the binary (waveform, spins or orbital evolution).
- We can extract this information and create a ML model which is able to predict the spin of the final object and the radiated energy.

Following the conventions:

- $m_1 > m_2$ where for simplicity the total mass of the binary is a scaling factor working in units of $m_1 + m_2 = 1$.
- $\chi_i \in [-1,1]$ and $m_i \in [0,1]$.



Waveform modes from the non-spinning simulation SXS:BBH:0169, Borhanian et al., 2020.

Aligned spin (AS) case

Individual spins // orbital angular momentum (⊥ to the orbital plane). Spin directions and the orbital plane itself is preserved in time.

3D parameter space fully described by:

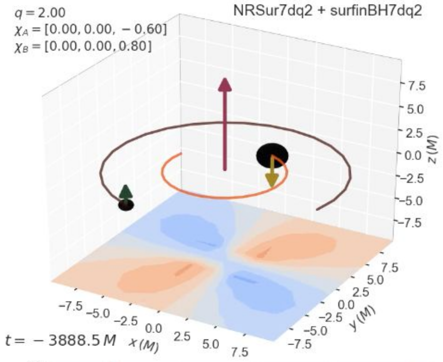
- The z-component of the individual spins (χ_{1z} and χ_{2z}).
- Either the asymmetric mass ratio (q) following the $m_1 > m_2$ convention, or the symmetric one (η).

$$\vec{\chi_i} = \frac{\vec{S_i}}{m_i^2} \qquad q = \frac{m_1}{m_2} \qquad \eta = \frac{m_1 m_2}{(m_1+m_2)^2}$$

Suggested by the post-Newtonian (PN) results and also studies of NR calibrated models we use a more convenient parametrization of the spins:

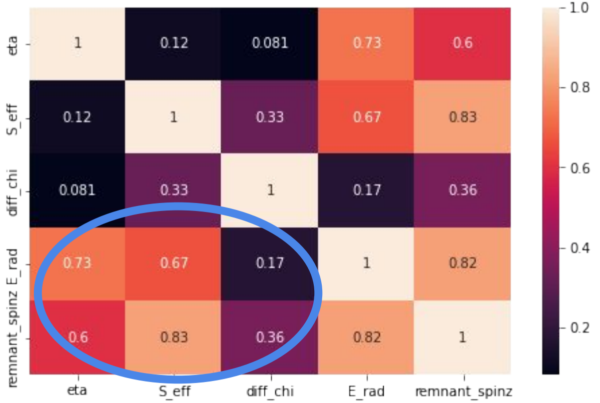
- The difference between spins ($\Delta\chi = \chi_1 - \chi_2$).
- The average spin ⊥ to the orbital plane (χ_{eff}).

$$\vec{\chi}_{eff} = \frac{m_1\vec{\chi_1} + m_2\vec{\chi_2}}{m_1 + m_2}$$



Motion of two aligned BBHs, Varma et al. 2018.

Correlation matrix between input and output quantities.



Input parameters
η, χ_{eff}, $\Delta\chi$

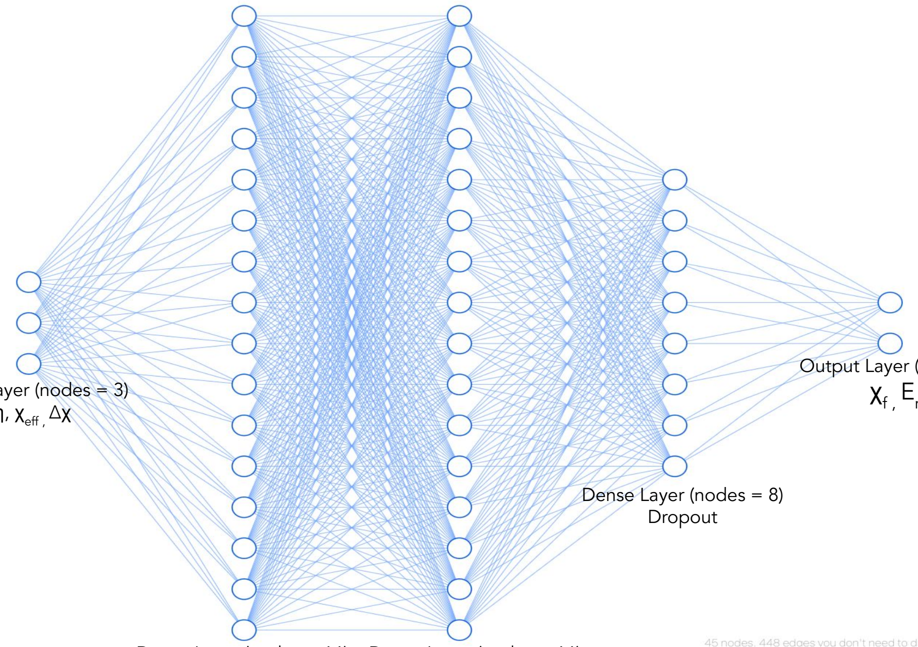
→

Output parameters
χ_f, E_{rad}

593 Numerical Relativity simulations (SXS catalog)

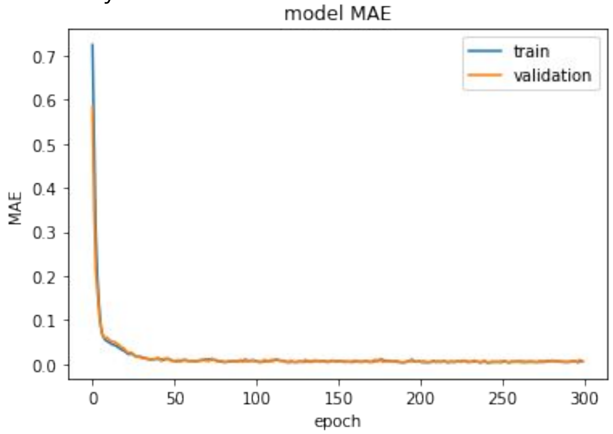
Aligned spin case - model structure

AS model structure



<u>*Radiated Energy*</u>
Mean abs. err.: 0.0029
Max abs. err.: 0.0126
Accuray: 99.7%

<u>*Final spin*</u>
Mean abs. err.: 0.0046
Max abs. err.: 0.0259
Accuray: 99.01%



Mean Absolute Error for the train and validation of AS model.

- Dropout = 10^{-4}
- Learning rate of the ADAM opt = 10^{-4}
- batch size = 32
- epochs = 300
- Using cross validation (cv=5) - (80% TR - 10% VL- 10% TS)

Aligned spin case - model behaviour

We evaluate the accuracy of the model comparing the results with existing models:

- NRHybSur3dq8 ($q \leq 8$ and $|\chi_i| \leq 0.8$).
- IMRPhenomX*
- SEOBNRv4

To understand better the behaviour of our model, we divide the parameter space in several subspaces reducing dimensionality:

- Equal mass - equal spin data (1D, χ_{eff} dependence) $\longrightarrow$ $\eta = 0.25$, $\Delta\chi = 0$
- Non-spinning data (1D, η dependence) $\longrightarrow$ $\chi_{eff} = \Delta\chi = 0$
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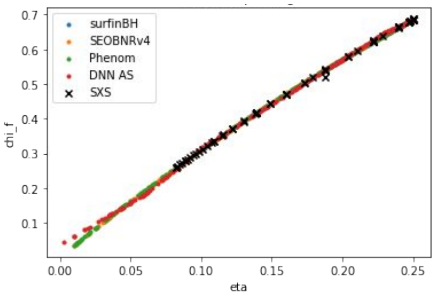
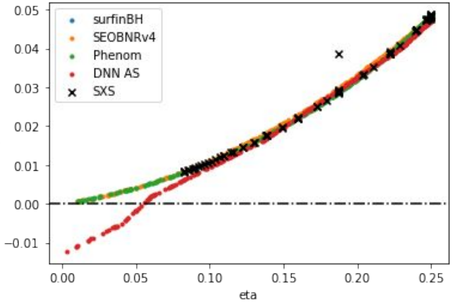
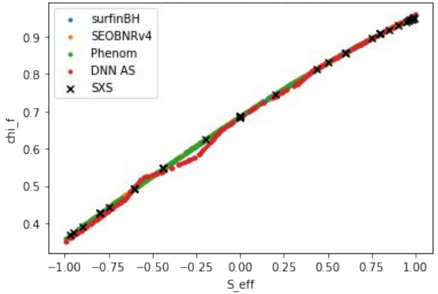
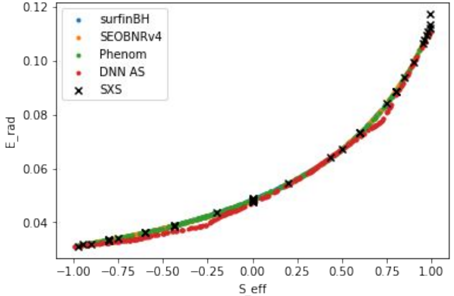


1D non-spinning

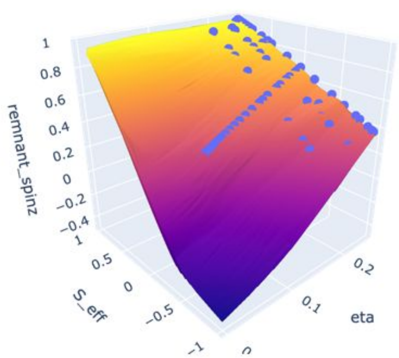


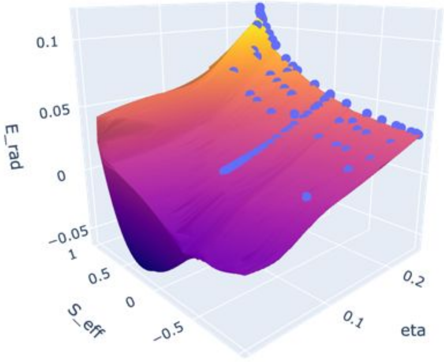
1D equal mass - equal spin





2D equal spin





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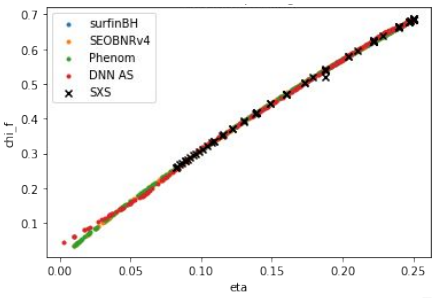

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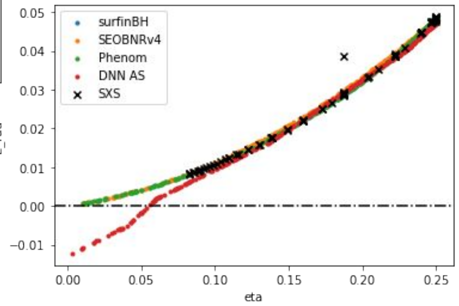

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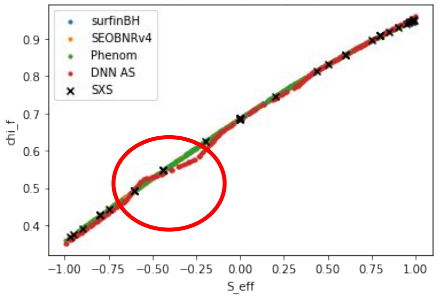
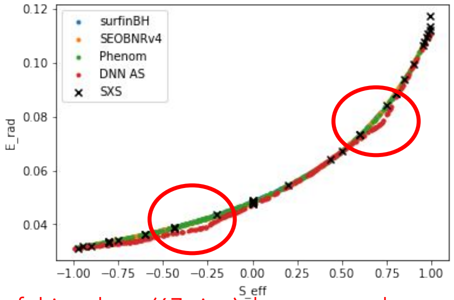
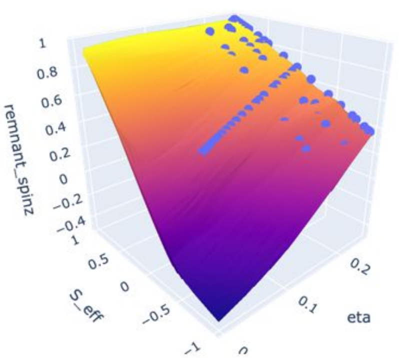
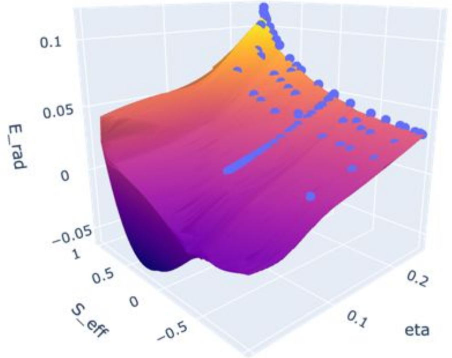




Our dataset has a good coverage of this subset (67 sim.), but we need better interpolation in large regimes.

2D equal spin





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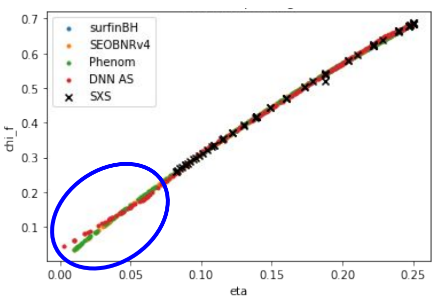
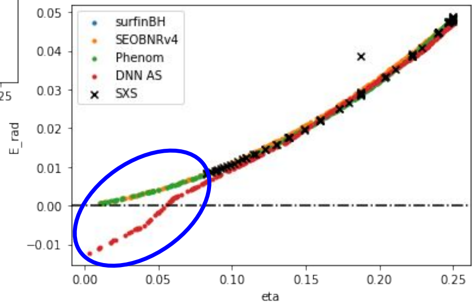
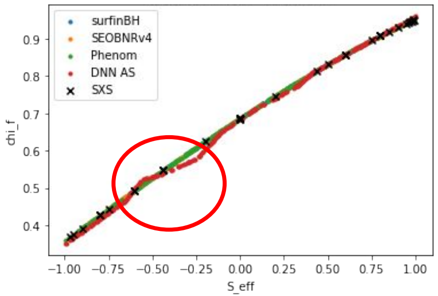
Our subset (116 sim.) does not have good coverage for large mass ratios. Extrapolation is really bad, we need more data (analytical solution).

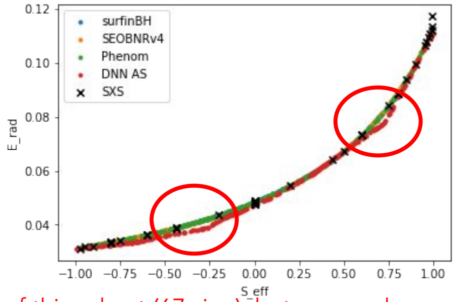


1D non-spinning



1D equal mass - equal spin





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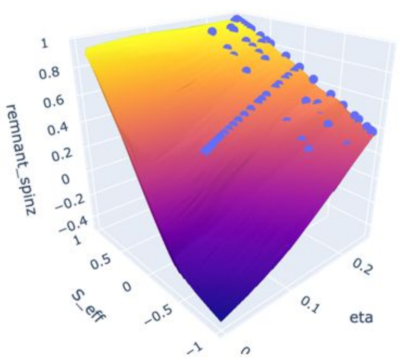
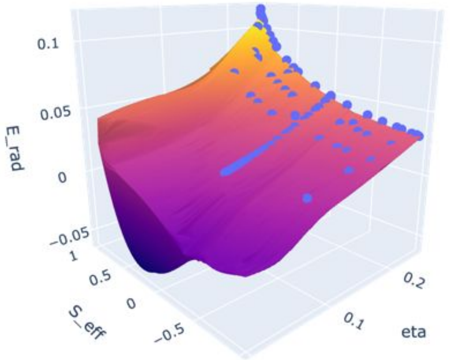
2D equal spin





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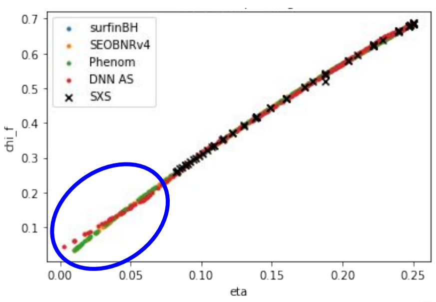
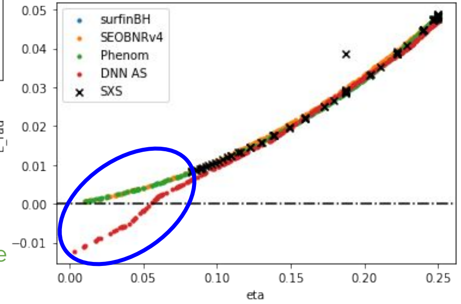
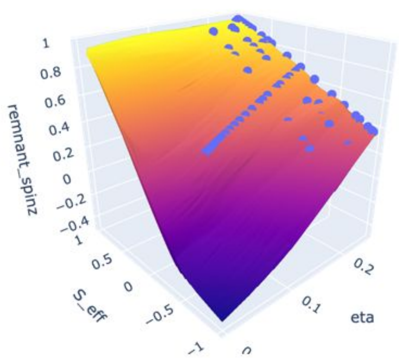
1D non-spinning

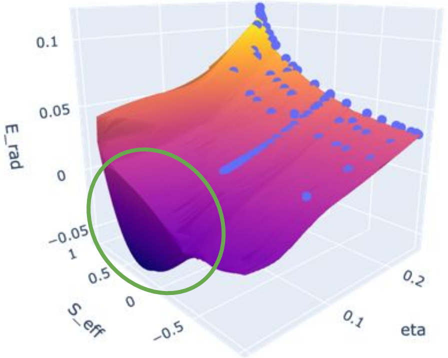




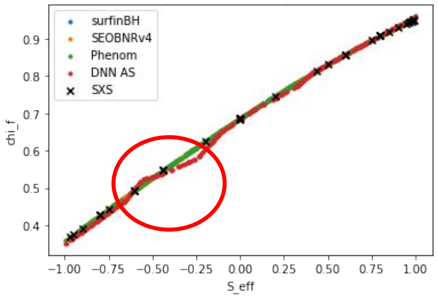
Also lack of information in the extreme mass ratio regime. Subset of 191 sim.

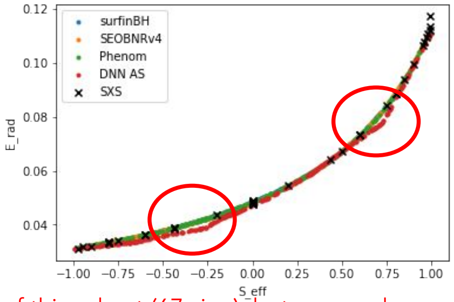
2D equal spin





1D equal mass - equal spin





Our dataset has a good coverage of this subset (67 sim.), but we need better interpolation in large regimes.

Extreme mass ratio approximation (EMRIs) - AS

- In the extreme mass ratio limit ($\eta \rightarrow 0$) we have a test particle orbiting a Kerr BH.
- The small BH plunges after reaching the ISCO and in linear order with the asymmetric mass ratio we have that $E_{rad} = E_{ISCO}$ and $M_f = 1 - E_{ISCO}$.
- Energy and orbital angular momentum are described as:

$$E_{ISCO}(\eta, \chi_f) = \eta\sqrt{1 - \frac{2}{3\rho_{ISCO}(\chi_f)}}$$

$$L^{orb}_{ISCO}(\eta, \chi_f) = \frac{2\eta(3\sqrt{\rho_{ISCO}(\chi_f)} - 2\chi_f)}{\sqrt{3\rho_{ISCO}(\chi_f)}}$$

$$\rho_{ISCO}(\chi) = 3 + Z_2 - sign(\chi)\sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}$$

$$Z_2 = \sqrt{3\chi^2 + Z_1^2}$$

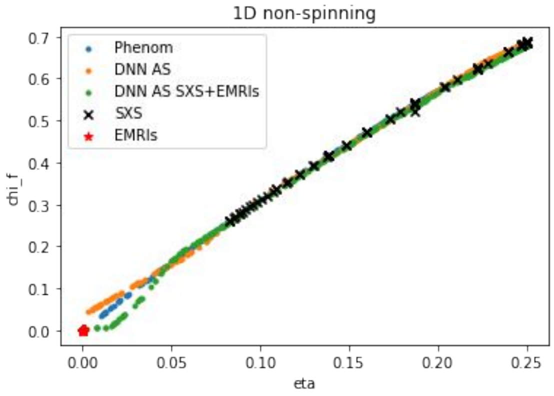
$$Z_1 = 1 + (1 - \chi^2)^{1/3})[(1 + \chi)^{1/3} + (1 - \chi)^{1/3}]$$

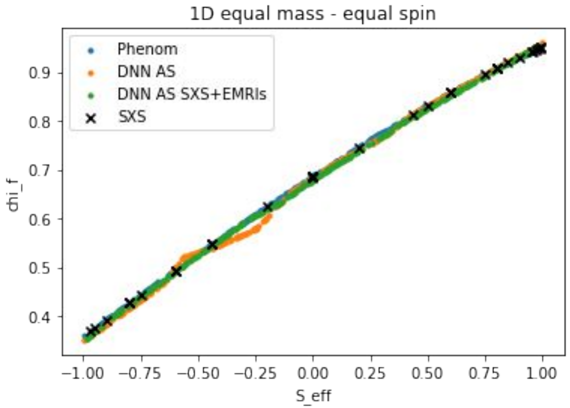
- We can obtain the final spin solving a numerical equation given by:

$$\chi_f M_f^2(\chi_f) = L_{orb}(\chi_f) + S_1 + \cancelto{0}{S_2}$$

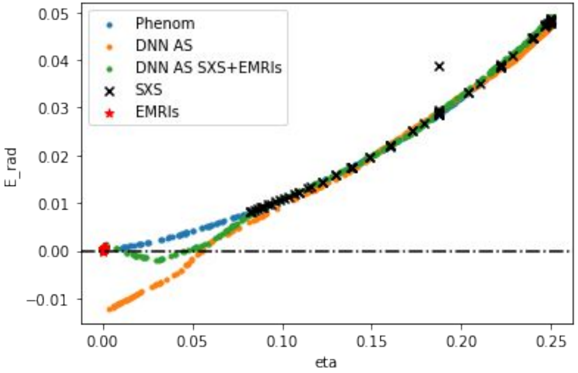
- In the boundary of this regime, we have a Kerr BH. It's final spin has to be equal or smaller than 1. And its derivative in the boundary has to be zero.

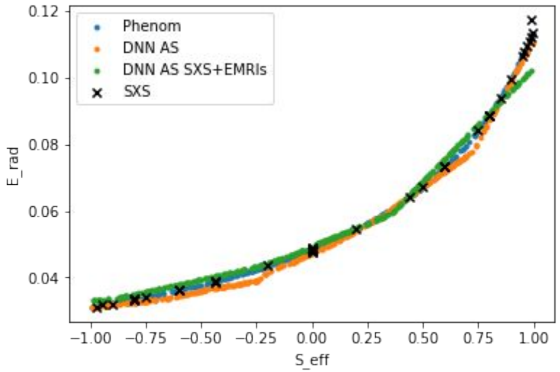
Aligned spin case - NR data + EMRI approx. (preliminar results)

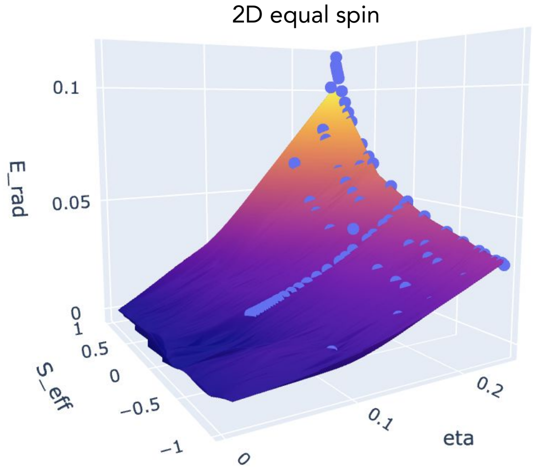




We add to our SXS dataset, 300 EMRI points between $\eta \in (10^{-5}, 10^{-3})$ and $\chi \in (-1,1)$.







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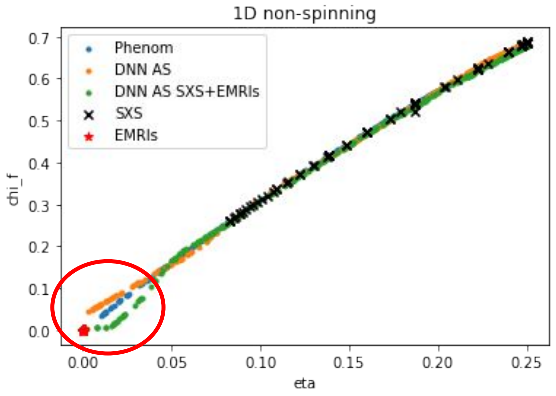

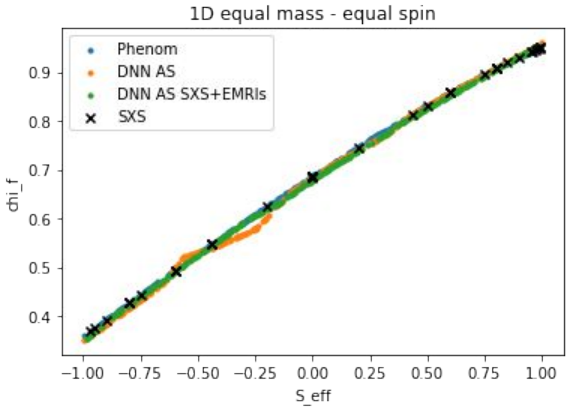

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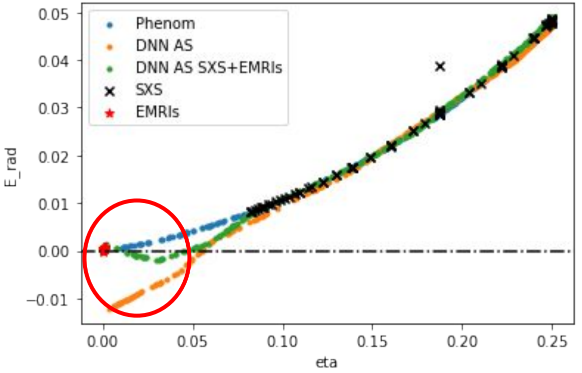
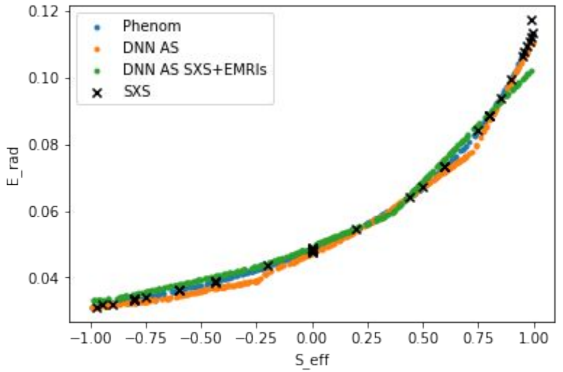
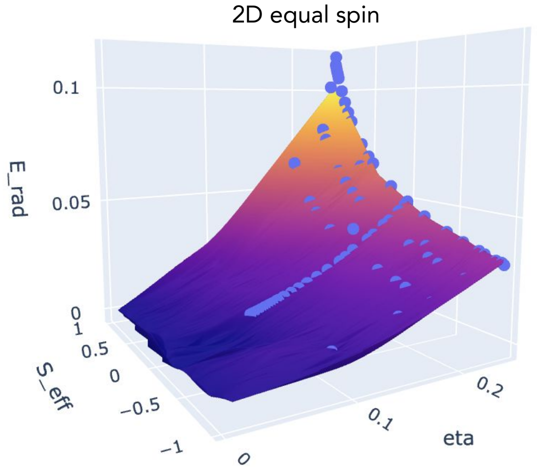
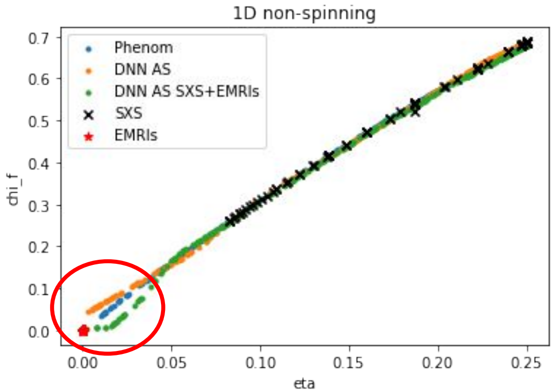
Adding EMRI data corrects a bit the behaviour in large q regime. It's a toy model - need further exploration.

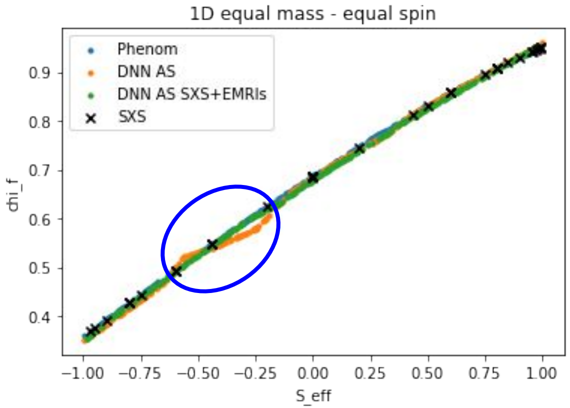






Aligned spin case - NR data + EMRI approx. (preliminar results)





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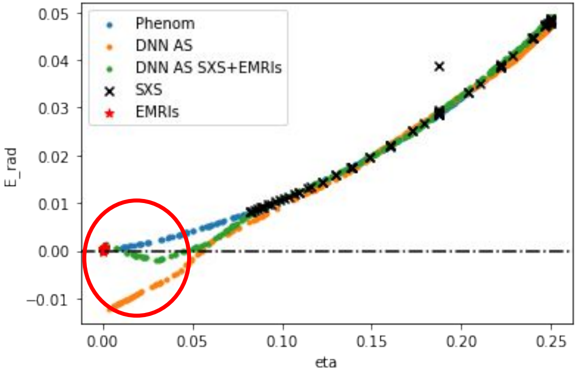

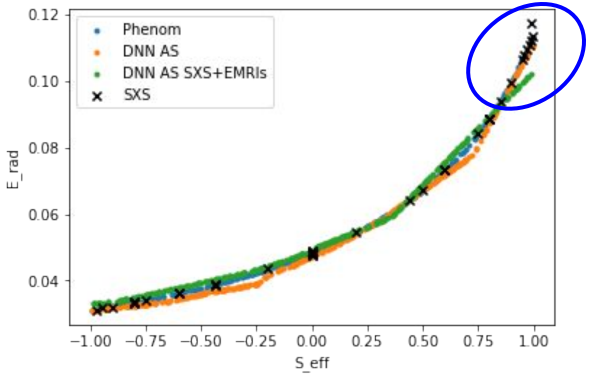

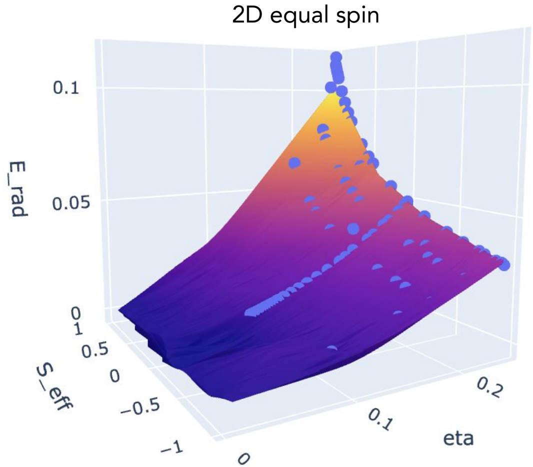

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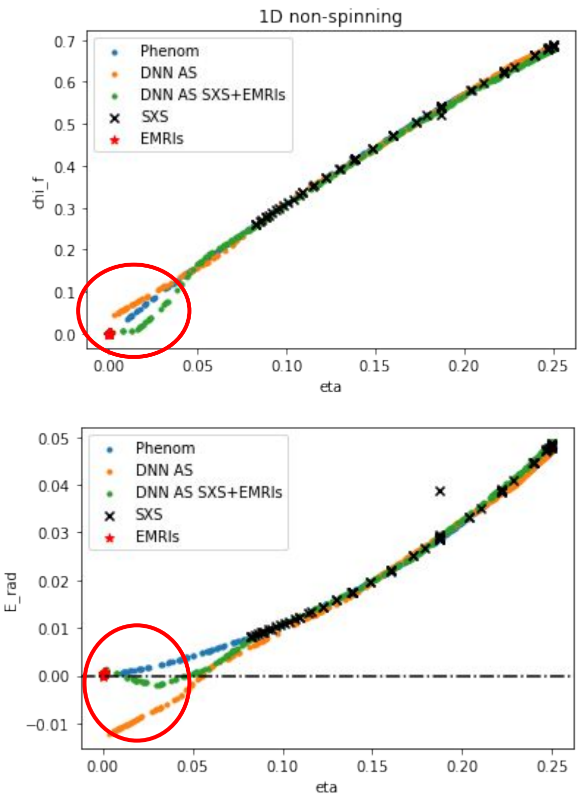

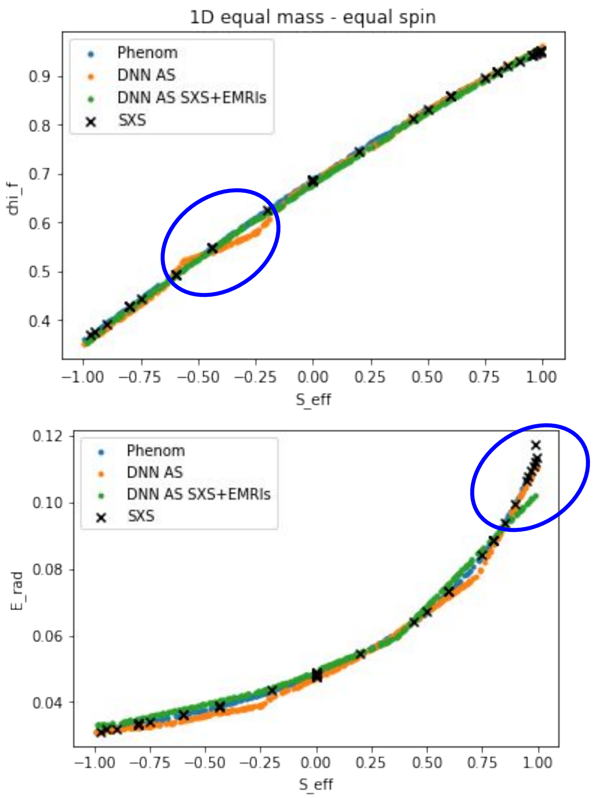

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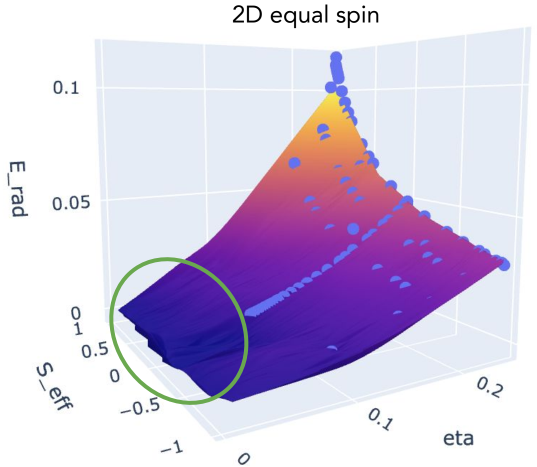
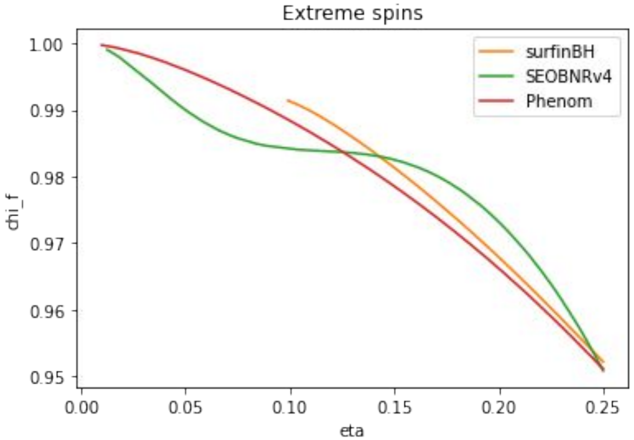


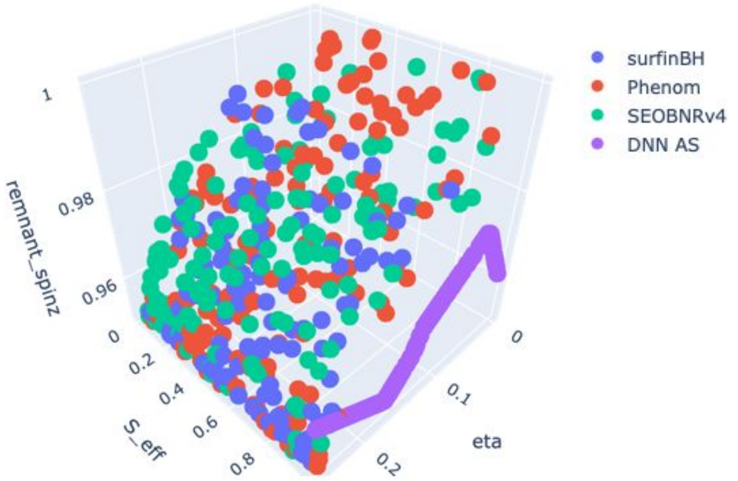
Aligned spin case - Extreme spins

The extremal aligned spins regime ($\chi_1 = \chi_2 = 1$), is very challenging. Any NR catalog covers it.

Different ansatz constructions, especially in the EMRI regime, produce discrepancies. One has to take into account that the final state is a Kerr BH ($\chi_f \leq 1$). We expect that in $\chi_{eff} = 1$, the final spin decreases monotonically with increasing η: $\chi_f'(\eta \to 0, \chi_{eff} \to 1) = 0$



Comparison of different models in the limit of extremal aligned spins.



Representation of the final spin for the extreme spins subset.

The IMRPhenomX family fits do not violate the Kerr bound in the extreme-spin limit at low η. The NRSur family does not cover the region $q > 8$.

Having models which cover the whole parameter space (PS) is needed. High SNR events need very accurate models covering all the PS and extrapolation is not a good choice.

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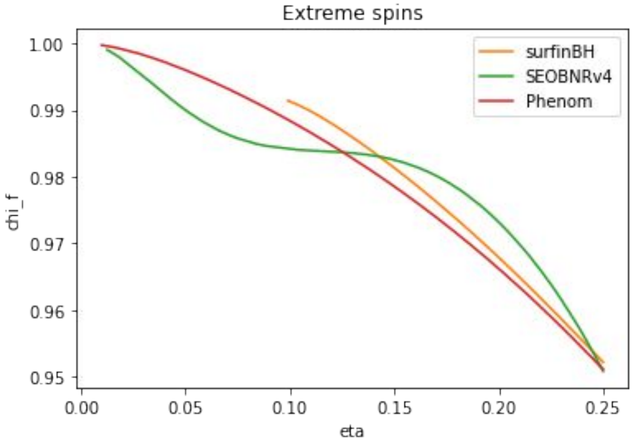

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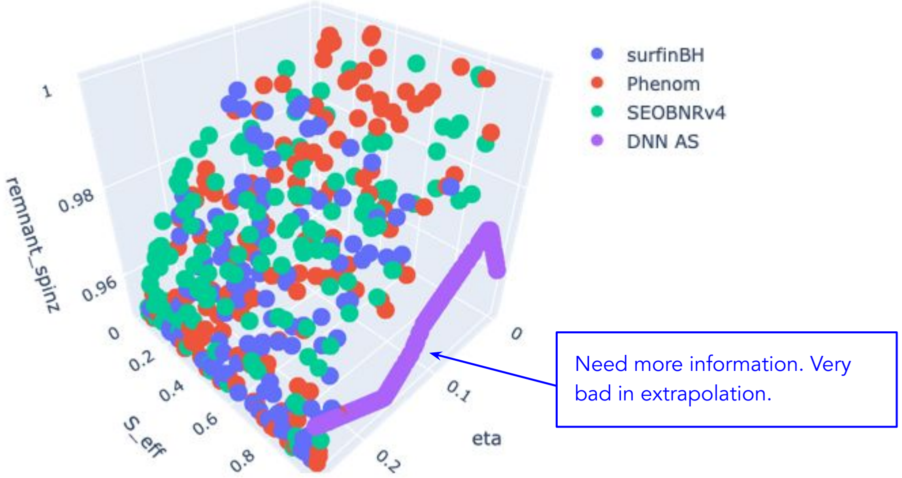

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Precessing spin case

One or both spins not // to the orbital angular momentum. The orbital plane is not preserved - temporal dependence.

7D parameter space fully described by:

- The components of the individual spins (χ_{1x},χ_{1y},χ_{1z} and χ_{2x}, χ_{2y}, χ_{2z}).
- Either the asymmetric mass ratio (q) following the $m_1 > m_2$ convention, or the symmetric one (η).

One can add extra parameters to the training giving information about the spin angles (zenith angle between the spin and orbital momenta, $\Theta_{1,2}$ and the planar spin projection angle difference, Φ_{12}).

$$cos\theta_{1,2} = \frac{\vec{\chi}_{1,2} \cdot \vec{L}}{|\vec{\chi}_{1,2}| \cdot |\vec{L}|} \qquad cos\phi_{12} = \frac{\vec{\chi}_1^{plane} \cdot \vec{\chi}_2^{plane}}{|\vec{\chi}_1^{plane}| \cdot |\vec{\chi}_2^{plane}|}$$

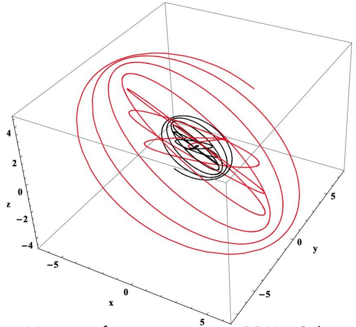
$$\vec{\chi}_i^{plane} = \vec{\chi}_i - (\vec{\chi}_i \cdot \hat{L}) \cdot \hat{L}$$

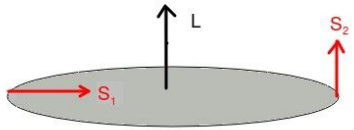
Input parameters
η, χ_{1x},χ_{1y},χ_{1z}, χ_{2x}, χ_{2y}, χ_{2z}, Φ_{12},Θ_1,Θ_2

→

Output parameters
χ_{fx},χ_{fy},χ_{fz}, E_{rad}

1423 Numerical Relativity simulations (SXS catalog)

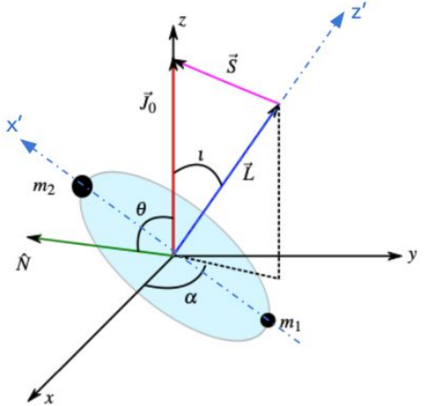




Motion of two precessing BBHs, Schmidt et al., 2010.

* PROBLEM: The frame used for AS is not convenient any more.

Rotation of all the simulations to the co-orbital frame at a $t_{ref} = t_{peak}$- 100M (non-oscillatory).



Source frame of a precessing binary. Schmidt et al., 2014.

Preparing training data: Precessing

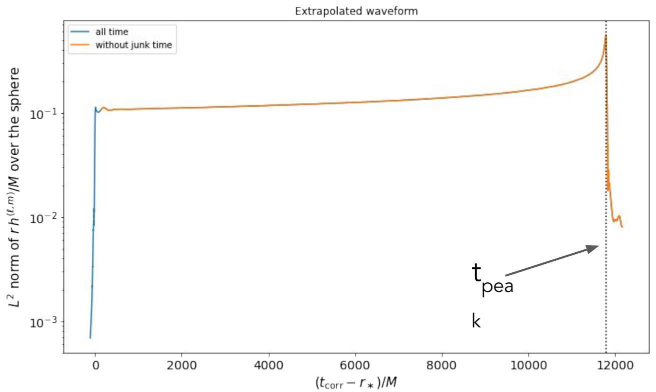
1. Get the spin evolution for each simulation.
2. Remove the junk radiation due to NR (first 500 points).
3. Compute the maximum of the total WF amplitude - peak of the WF

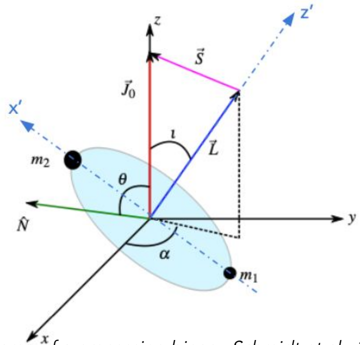
$$A(t) = \sqrt{\sum_{\ell m} |h_{\ell m}(t)|^2}.$$

4. Get the time reference: 100M time before the peak.
5. Coprecessing rotation:
   a. Get the position evolution for each simulation.
   b. Derive it in order to get the velocity evolution.
   c. Compute the angular velocity and normalize it.

$$\boldsymbol{\omega} = \frac{\mathbf{r} \times \mathbf{v}}{r^2}$$

   d. Compute the rotation matrix in order to have $\boldsymbol{\omega}(t_{ref})$ in the z axis.
6. Coorbital rotation:
   a. Compute the rotation matrix in order to have $\mathbf{r}(t_{ref})$ in the x axis.



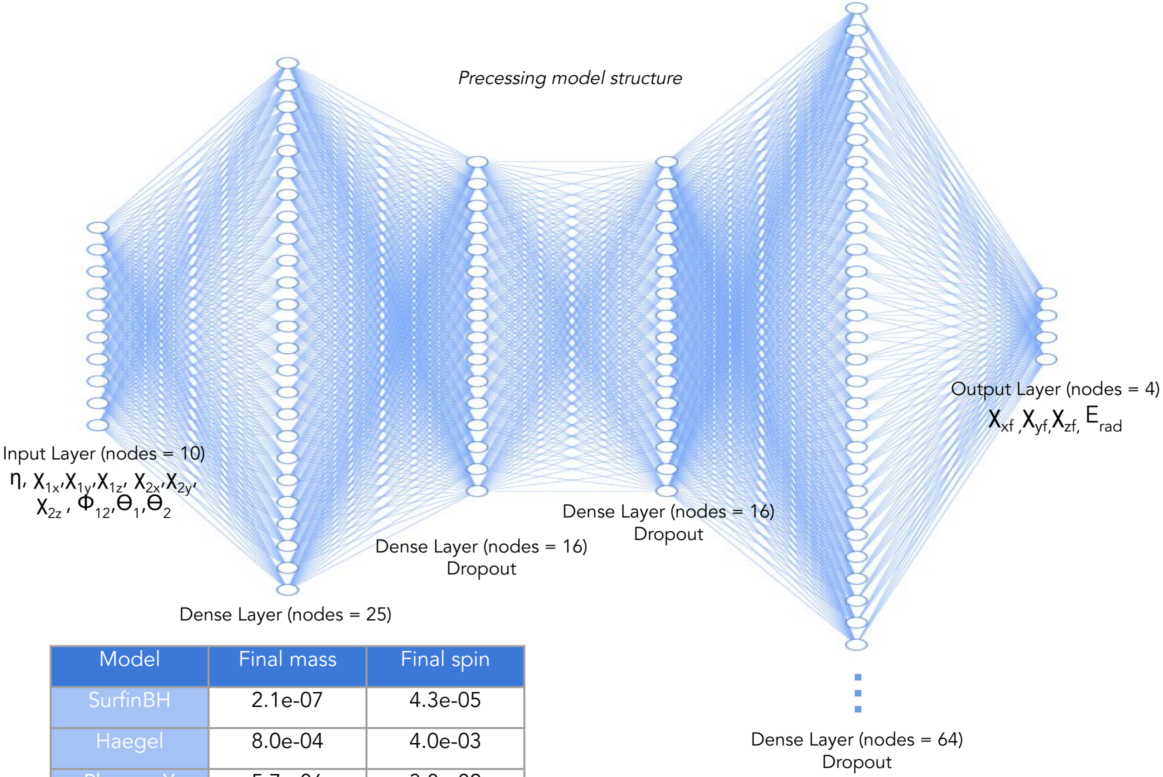


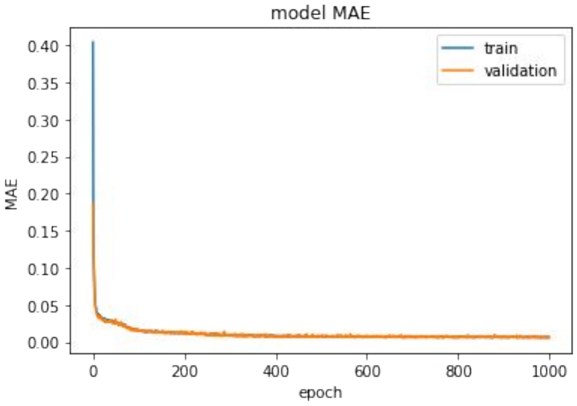
Source frame of a precessing binary. Schmidt et al., 2014.

* Now we can rotate the initial and the final spins for all the simulations and we will have all of them in the same frame.

Precessing case - Toy model structure

Precessing model structure





Mean Absolute Error for the train and validation of precessing model.

Model	Final mass	Final spin
SurfinBH	2.1e-07	4.3e-05
Haegel	8.0e-04	4.0e-03
PhenomX	5.7e-06	3.8e-02
SEOBNRv4	5.0e-06	5.6e-03
Our model	3.3e-06	1.0e-04

Comparison table of the Root Mean Absolute Errors using different models

- Dropout = 10^{-4}
- Learning rate of the ADAM opt = 10^{-3}
- batch size = 32
- epochs = 1000
- Using cross validation (cv=5) - (80% TR - 10% VL- 10% TS)

Precessing case - model accuracy

Our precessing model has very good accuracy in both remnant quantities.

- Both SEOB and Phenom models are not very accurate. Our model predicts much better the remnant quantities. For the final spin, they only predict the norm of the vector, and in our case we predict the 3 components.
- SurfinBH model has one order of magnitude more accuracy. This model used 1528 NR simulations, and we used a few less (1423).

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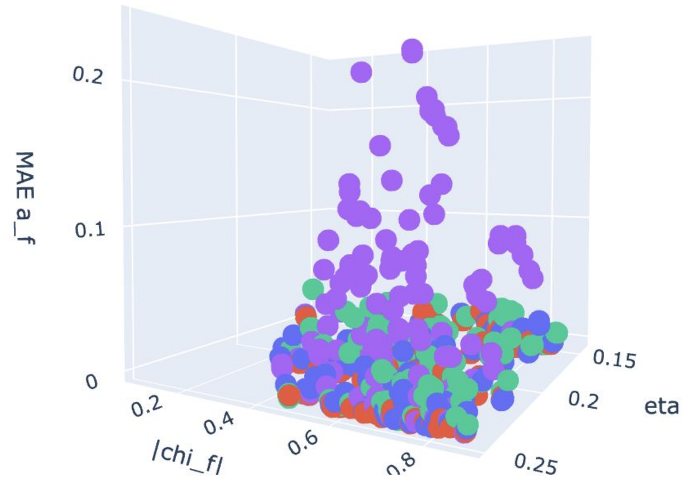

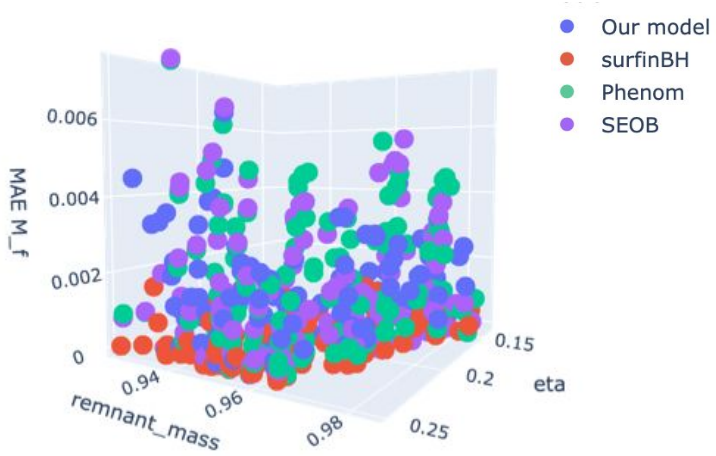

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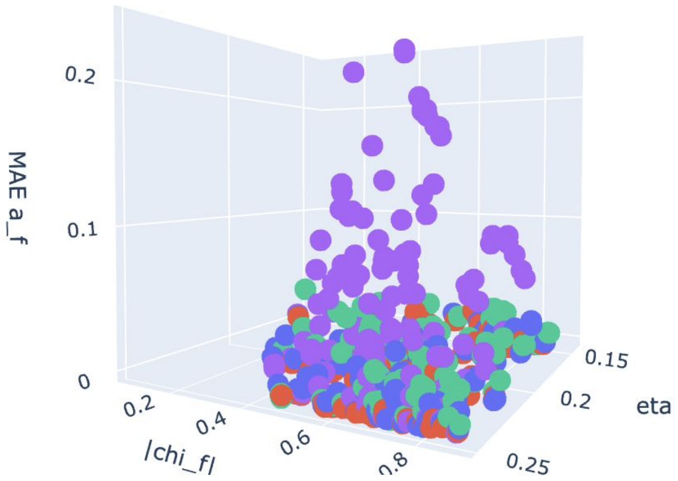

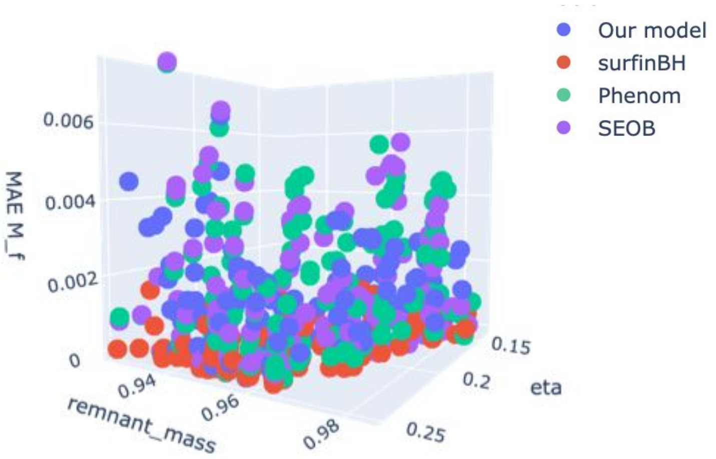

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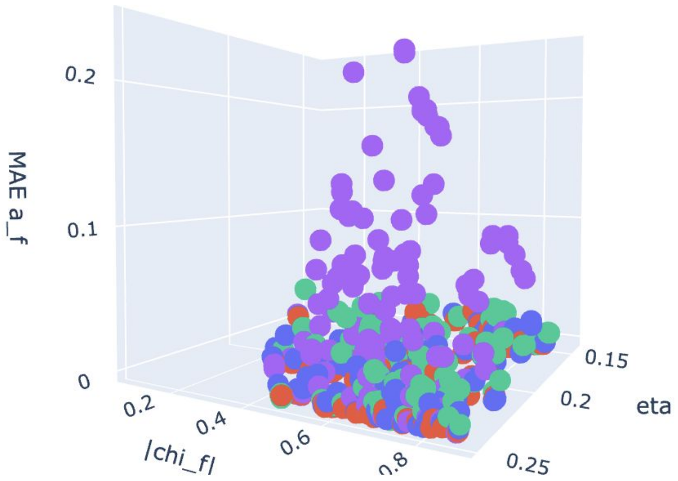

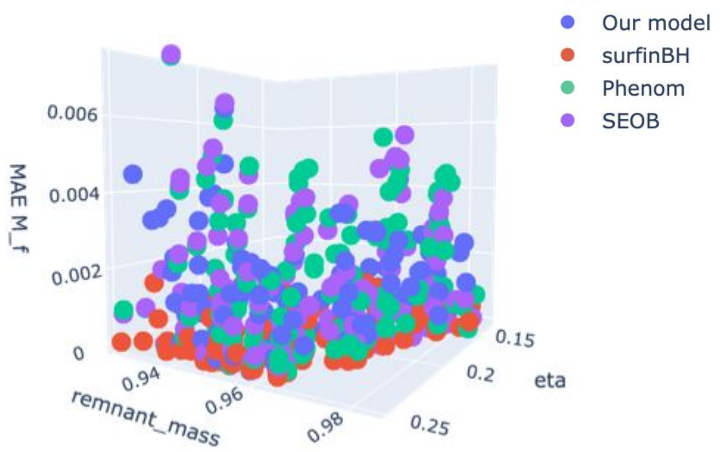

Conclusions

- Constant development and improvement of waveform models is necessary with the increasing sensitivity of the detectors.
- Current models does not cover the whole parameter space (PS) and extrapolation is not accurate.
- For a high dimensional PS (precessing or eccentric), cover it is extremely important. Events with high SNR need very accurate models in all the PS.

Future work

- We need to add more data points in our dataset - RIT catalog and UIB NR simulations.
  - Extreme spins and extreme mass ratio
- Gain accuracy in the interpolation of two very separate regions.
- Weight data taking into account the resolution of the simulations and their errors.
- Figure out how to interpret results of the precessing model.

Acknowledgments

This work is supported by the Universitat de les Illes Balears; European Union FEDER funds; the Spanish Ministry of Science and Innovation and the Spanish Agencia Estatal de Investigación grants PID2019-106416GB-I00/ MCIN/ AEI/10.13039/501100011033, RED2018-102661-T, RED2018-102573-E; the European Union NextGenerationEU (PRTR-C17.I1); the Comunitat Autònoma de les Illes Balears through the Direcció General de Política Universitària i Recerca with funds from the Tourist Stay Tax Law ITS 2017-006 (PRD2018/24, PRD2020/11); the Conselleria de Fons Europeus, Universitat i Cultura del Govern de les Illes Balears; the Generalitat Valenciana (PROMETEO/2019/071); and EU COST Actions CA18108, CA17137, CA16214, and CA16104. Maite Mateu-Lucena is supported by the Conselleria de Fons Europeus, Universitat i Cultura del Govern de les Illes Balears (ref. FPI /2215/2019).

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