Measuring the Dark Matter environments of black hole binaries with gravitational waves

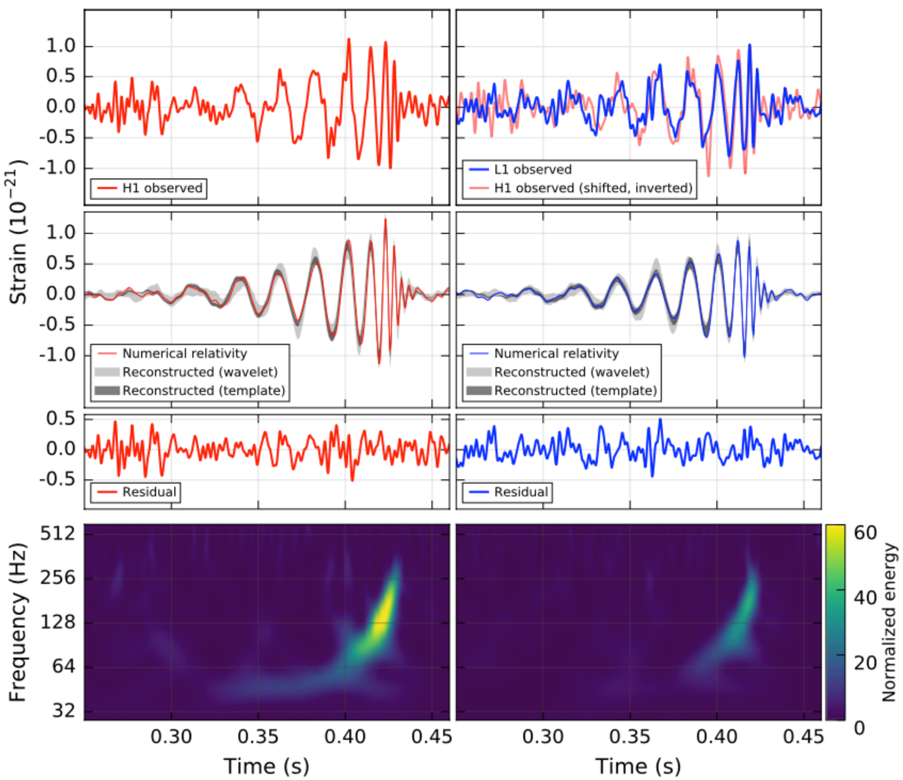
Alessandro Parisi
Scuola Normale Superiore di Pisa

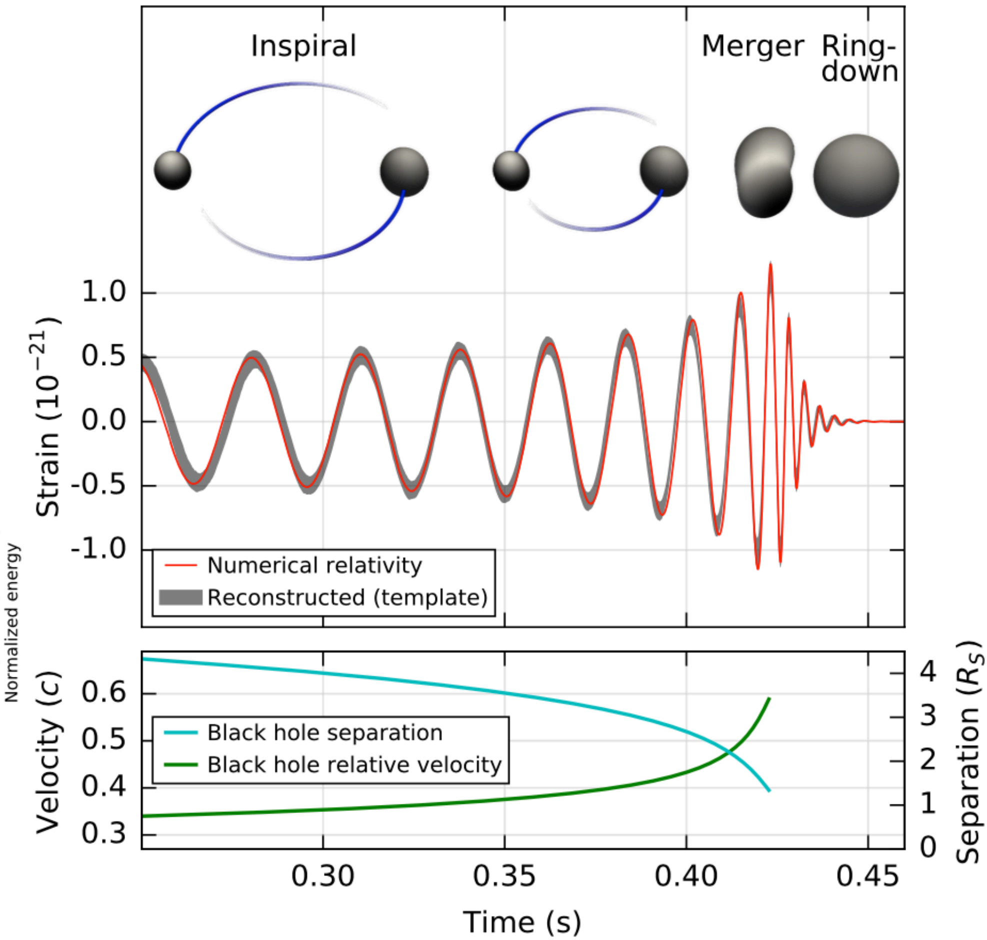
with Gianfranco Bertone, Philippa Cole, Adam Coogan, Bradley Kavanagh, Daniele Gaggero, Elena Cuoco, Alberto Iess

European Gravitational Observatory (EGO)
30 September 2022

First LIGO detection during O1: GW150914

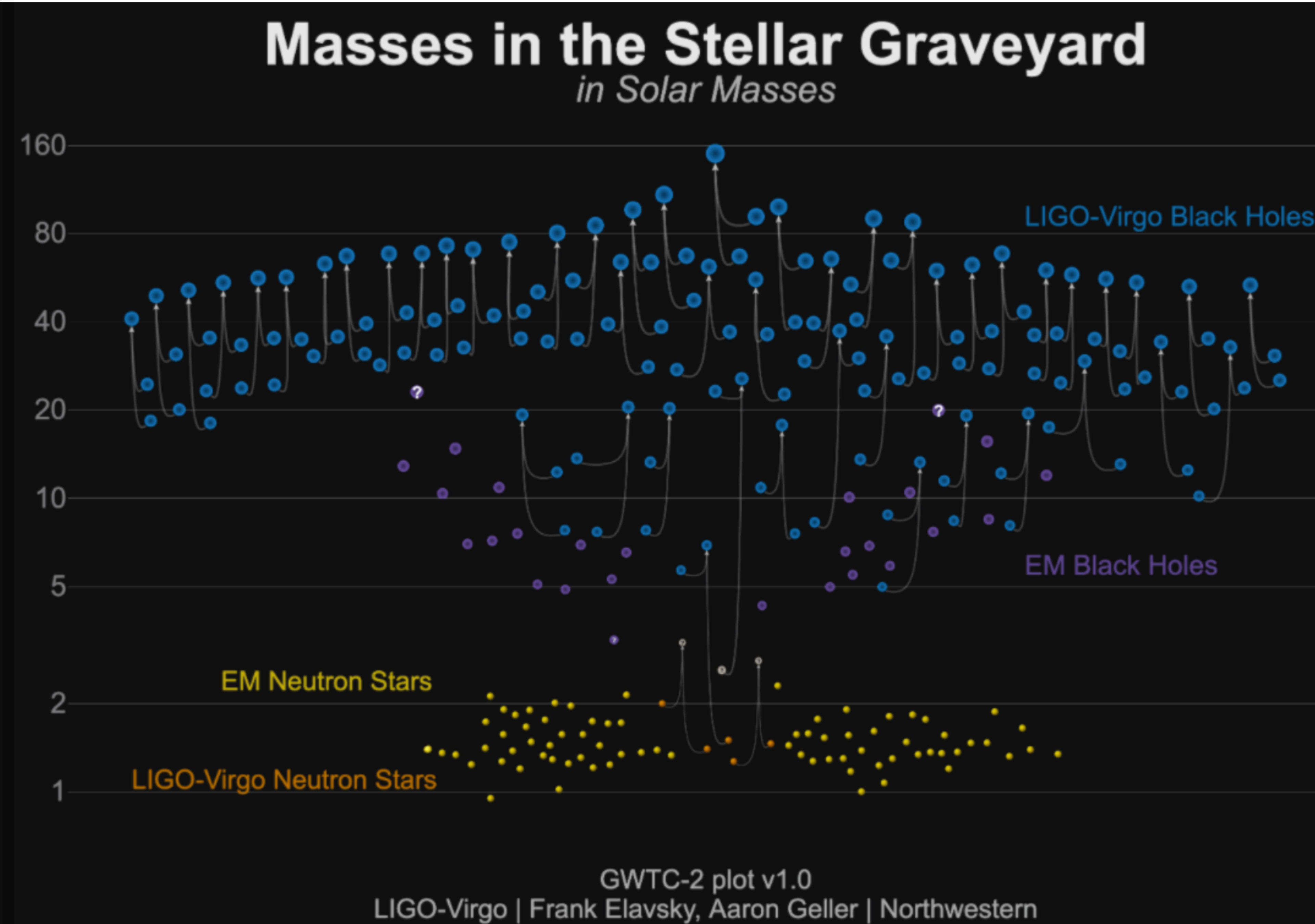
(Abbott et al. PRL 116 (2016) 061102)





- **Black holes** of radius of 90 km at separation of 350 km are **making 75 orbits** per second **before merging**.
- **Black holes** collide at (almost) speed of light, like **fundamental particles.**
- **Gravitational waves** carry **fingerprints** of **source**.

Abbott, R., Abbott, T. D., Abraham, S., Acernese, F., Ackley, K., Adams, A., ... & Agathos, M. (2020). GWTC-2: Compact Binary Coalescences Observed by LIG and Virgo During the First Half of the Third Observing Run. *arXiv preprint arXiv:2010.14527*.

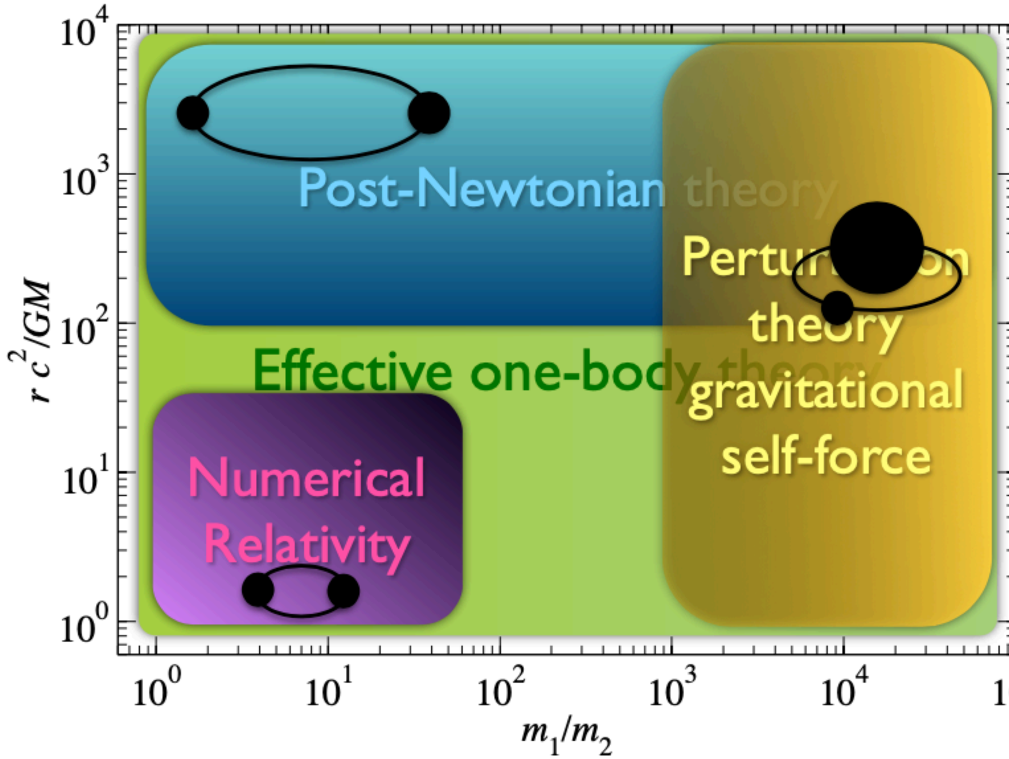


Solving two-body problem in General Relativity (including radiation)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

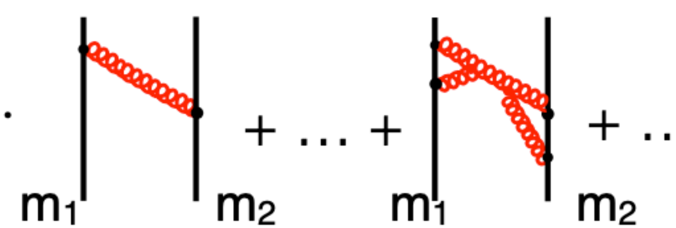
- **GR** is **non-linear theory**. Complexity similar to QCD.
- Einstein's field equations can be solved:
  - **approximately**, but **analytically** (**fast** way)
  - **exactly**, but **numerically** on supercomputers (**slow** way)

$v^2/c^2 \sim GM/rc^2$



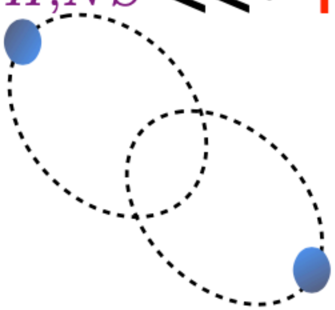
(AB & Sathyaprakash 14)

- **Analytical methods**: post-Newtonian/post-Minkowskian/post-Test-Body expansions effective-one-body theory
  - **effective field-theory**, **dimensional regularization**, etc.
  - **diagrammatic** approach to organize expansions



EFT for compact binary systems

r <<* $\lambda_{GW} \simeq \frac{r}{v}$



$l_{BH,NS}$ <<* r

in both cases
a multipole expansion is appropriate
to describe the interaction of the
source with the gravitational field:

$$S_m[x,h] = -\int \mathrm{d}\tau \left[m + \frac{1}{2}\mathcal{S}_{\mu\nu}\Omega^{\mu\nu} + I_{ij}E^{ij} + J_{ij}B^{ij} + c_E E^{ij}E_{ij} \ldots \right]$$

$$S_{EH} = -\frac{1}{64\pi G_N}\int dt d\mathbf{x} \left[\partial_\mu h_{\alpha\beta}\partial^\mu h^{\alpha\beta} - \partial_\mu h \partial^\mu h + 2\partial_\mu h^{\mu\nu}\partial_\nu h - 2\partial_\mu h^{\mu\nu}\partial_\rho h^\rho_\nu \right]$$

$$S_{GF\Gamma} = \frac{1}{32\pi G_N}\int dt d\mathbf{x} \left(\partial_\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h \right)^2$$

Useful ansatz:

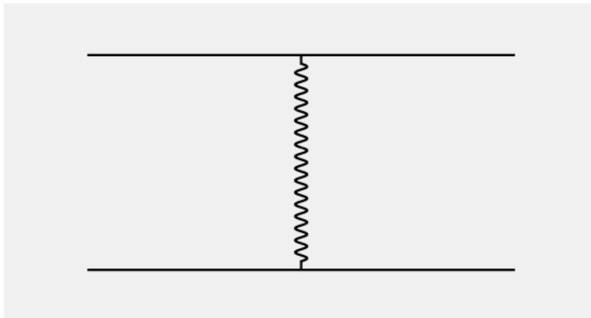
$$g_{\mu\nu} = e^{2\phi/m_{Pl}} \begin{pmatrix} -1 & A_j/m_{Pl} \\ A_i/m_{Pl} & e^{-c_d\phi}\left(\delta_{ij} + \sigma_{ij}/m_{Pl}\right) - A_iA_j/m_{Pl}^2 \end{pmatrix}$$

$$S_{pp} = \int dt\, e^{\phi/m_{Pl}} \sqrt{\left(1 - \frac{A_i v_i}{m_{Pl}}\right)^2 + e^{-c_d\phi/m_{Pl}}\left(v^2 + \frac{\sigma_{ij}}{m_{Pl}} v^i v^j\right)}$$

$$\begin{aligned} S_{EH} \;=\; & \int d^dx\sqrt{-\gamma}\left\{\frac{1}{4}\left[\left(\vec{\nabla}\sigma\right)^2 - 2\vec{\nabla}\sigma_{ij}{}^2\right] - c_d\left(\vec{\nabla}\phi\right)^2 + \right. \\ & + \frac{F_{ij}{}^2}{2} + \left(\vec{\nabla}\cdot\vec{A}\right)^2 \\ & \left. + \dot{\sigma}^2 + \dot{\phi}^2 + \dot{A}^2 + \text{interactions}\right\} \end{aligned}$$

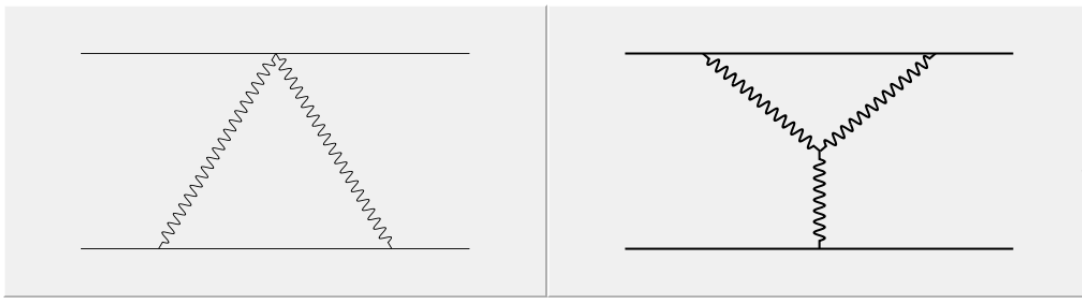
$$\begin{aligned} e^{iS_{eff}} \;=\; & Z[J, x_A]|_{J=0} = \int \mathcal{D}\Phi e^{iS_{quad}} \times \{1 \\ & -\frac{1}{2}\left[\sum_A m_A \int dt_A \Phi\left(t_A, \mathbf{x}_A(t_A)\right)\right]\left[\sum_B m_B \int dt_B \Phi\left(t_B, \mathbf{x}_B(t_B)\right)\right] + \ldots\} \end{aligned}$$

Newtonian potential:

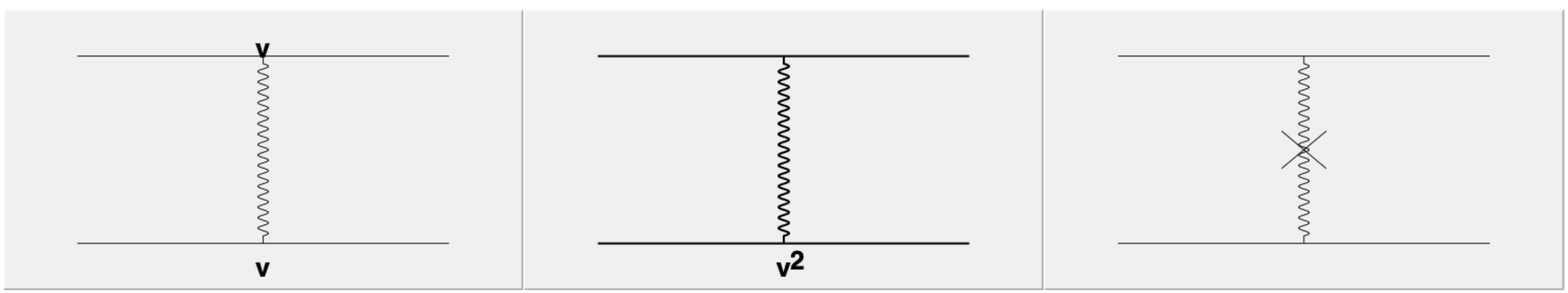


$$S_{eff} \sim \int dt \, \frac{G_N m_1 m_2}{r} \sim L$$

At 1PN:



$$S_{eff} = \int dt \frac{G_N^2 m_1^2 m_2}{r^2} \sim L v^2$$



$$V_{1PN} = -\frac{G_N m_1 m_2}{2r} \left[1 - \frac{G_N m_1}{2r} + \frac{3}{2}(v_1^2) - \frac{7}{2} v_1 v_2 - \frac{1}{2} v_1 \hat{r} v_2 \hat{r} \right] + 1 \leftrightarrow 2$$

Present status of 2 body problem

PM expansion parameter is $G_N M/r$, vs PN expansion

$$\mathcal{L} = -Mc^2 + \frac{\mu v^2}{2} + \frac{GM\mu}{r} + \frac{1}{c^2}[\ldots] + \frac{1}{c^4}[\ldots]$$

Terms known so far

		N	1PN	2PN	3PN	4PN	5PN	6PN	...
0PM	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	v^{14}	...
1PM		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	v^{12}/r	...
2PM			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	v^{10}/r^2	...
3PM				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	v^8/r^3	...
4PM					$1/r^4$	v^2/r^4	v^4/r^4	v^6/r^4	...
5PM						$1/r^5$	v^2/r^5	v^4/r^5	...
6PM							$1/r^6$	v^2/r^6	...
...								...	...

3PM recently computed by Z. Bern et al. PRL (2019)

5PN G^6 by S. Foffa, P.Mastrolia, RS, C. Sturm, W. Torres Bobadilla PRL (2019)

Short History of the PN Approximation

EQUATIONS OF MOTION

- 1PN equations of motion [Lorentz & Droste 1917; Einstein, Infeld & Hoffmann 1938]
- Radiation-reaction controvercy [Ehlers *et al* 1979; Walker & Will 1982]
- 2.5PN equations of motion and GR prediction for the binary pulsar [Damour & Deruelle 1982; Damour 1983]
- The "3mn" Caltech paper [Cutler, Flanagan, Poisson & Thorne 1993]
- 3.5PN equations of motion [Jaranowski & Schäfer 1999; BF 2001; ABF 2002; BI 2003; Itoh & Futamase 2003; Foffa & Sturani 2011]
- Ambiguity parameters resolved [DJS 2001; BDE 2003]
- 4PN [DJS, BBBFM]

RADIATION FIELD

- 1918 Einstein quadrupole formula
- 1940 Landau-Lifchitz formula
- 1960 Peters-Mathews formula
- EW multipole moments [Thorne 1980]
- BD moments and wave generation formalism [BD 1989; B 1995, 1998]
- 1PN orbital phasing [Wagoner & Will 1976; BS 1989]
- 2PN waveform [BDIWW 1995]
- 3.5PN phasing and 3PN waveform [BFIJ 2003; BFIS 2007]
- Ambiguity parameters resolved [BI 2004; BDEI 2004, 2005]
- 4.5PN (?)

Tidal effects in the gravitational wave signal emitted in NS-NS binary coalescence

$$h(f) = \mathcal{A}(f) e^{i\psi(f)} \qquad \psi(f) = \psi_{PP} + \psi_{\bar{Q}} + \psi_{\bar{\lambda}}$$

$x = (m\pi f)^{5/3}$ PN expansion parameter

point-particle contribution

$$\psi_{PP}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{1 + \left(\frac{3715}{756} + \frac{55}{9}\eta\right)x - (16\pi - 4\beta)x^{3/2} + \left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2 - 10\sigma\right)x^2 + \mathcal{O}(x^{5/2})\right\}$$

σ contains spin-spin and spin-orbit terms. Note that it appears in the 2-PN term (x^2)

Quadrupole contribution:

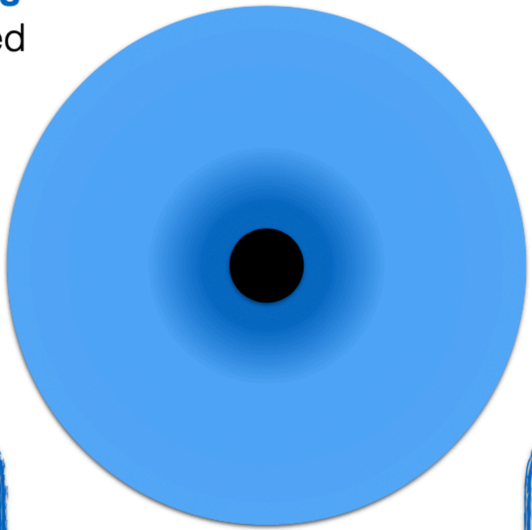
$$\psi_Q = \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{-50\left[\left(\frac{m_1^2}{m^2}\chi_1^2 + \frac{m_2^2}{m^2}\chi_2^2\right)(Q_S - 1) + \left(\frac{m_1^2}{m^2}\chi_1^2 - \frac{m_2^2}{m^2}\chi_2^2\right)Q_a\right]x^2\right\}$$

$$Q_S = \frac{\bar{Q}_1 + \bar{Q}_2}{2}, \quad Q_a = \frac{\bar{Q}_1 - \bar{Q}_2}{2}$$

both the quadrupole moments and the spin terms appear at the 2-PN order and cannot be measured independently : in this sense we say that there is complete degeracy

Dark Matter Spikes

Consider now a cold **DM** '**spike**' or '**dress**' around the central BH (not to be confused with ultralight boson clouds).



Study the following benchmarks:

$$m_1 = 1 M_\odot$$
$$m_2 = 10^{-2} - 10^{-4} M_\odot$$
$$\rho_{\rm DM} = \rho_6 \left(\frac{10^{-6}\,{\rm pc}}{r}\right)^{\gamma_{\rm sp}}$$

Astrophysical scenario

$$\gamma_{\rm sp} = 7/3 \approx 2.3333\ldots$$
$$\rho_6 \approx 5.45 \times 10^{15}\, M_\odot\,{\rm pc}^{-3}$$

…depending on a number of environmental factors…

[astro-ph/9906391, astro-ph/0509565, 1305.2619, …]

PBH scenario

$$\gamma_{\rm sp} = 9/4 \approx 2.25$$
$$\rho_6 \approx 5.35 \times 10^{15}\, M_\odot\,{\rm pc}^{-3}$$

[Bertschinger (1985), astro-ph/0608642, 1901.08528, …]

Gondolo, P. & Silk, J. 1999, Phys. Rev. Lett., 83, 1719.

Bertone, G. & Merritt, D. 2005, Phys. Rev. D, 72, 103502.

Ullio, P., Zhao, H., & Kamionkowski, M. 2001, Phys. Rev. D, 64, 043504.

Feng, W.-X., Parisi, A., Chen, C.-S., et al. 2021, arXiv:2112.05160

Eroshenko, Y. N. 2016, Astronomy Letters, 42, 347.

Boucenna, S. M., Kühnel, F., Ohlsson, T., et al. 2018, J. Cosmology Astropart. Phys., 2018, 003.

Phase space distribution

Follow semi-analytically the phase space distribution of DM:

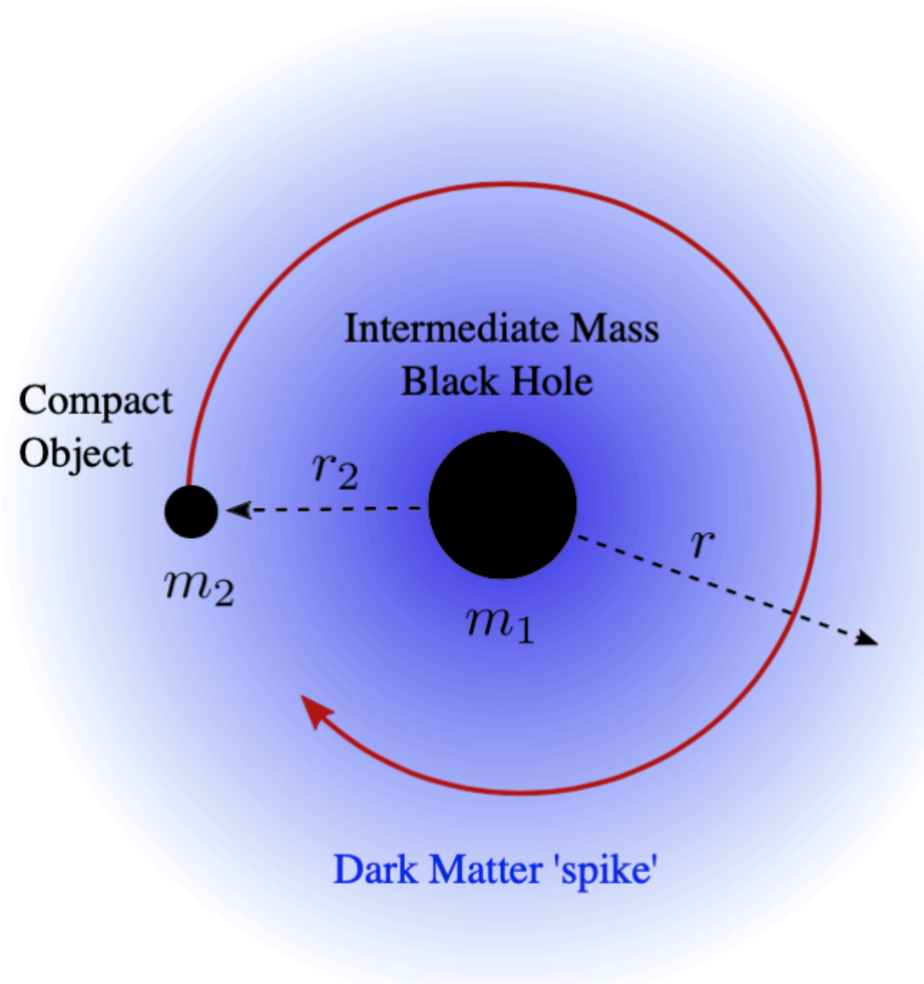
$$f = \frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\,\mathrm{d}^3\mathbf{v}} \equiv f(\mathcal{E})$$

$$\mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

Each particle receives a 'kick'

$$\mathcal{E} \to \mathcal{E} + \Delta\mathcal{E}$$

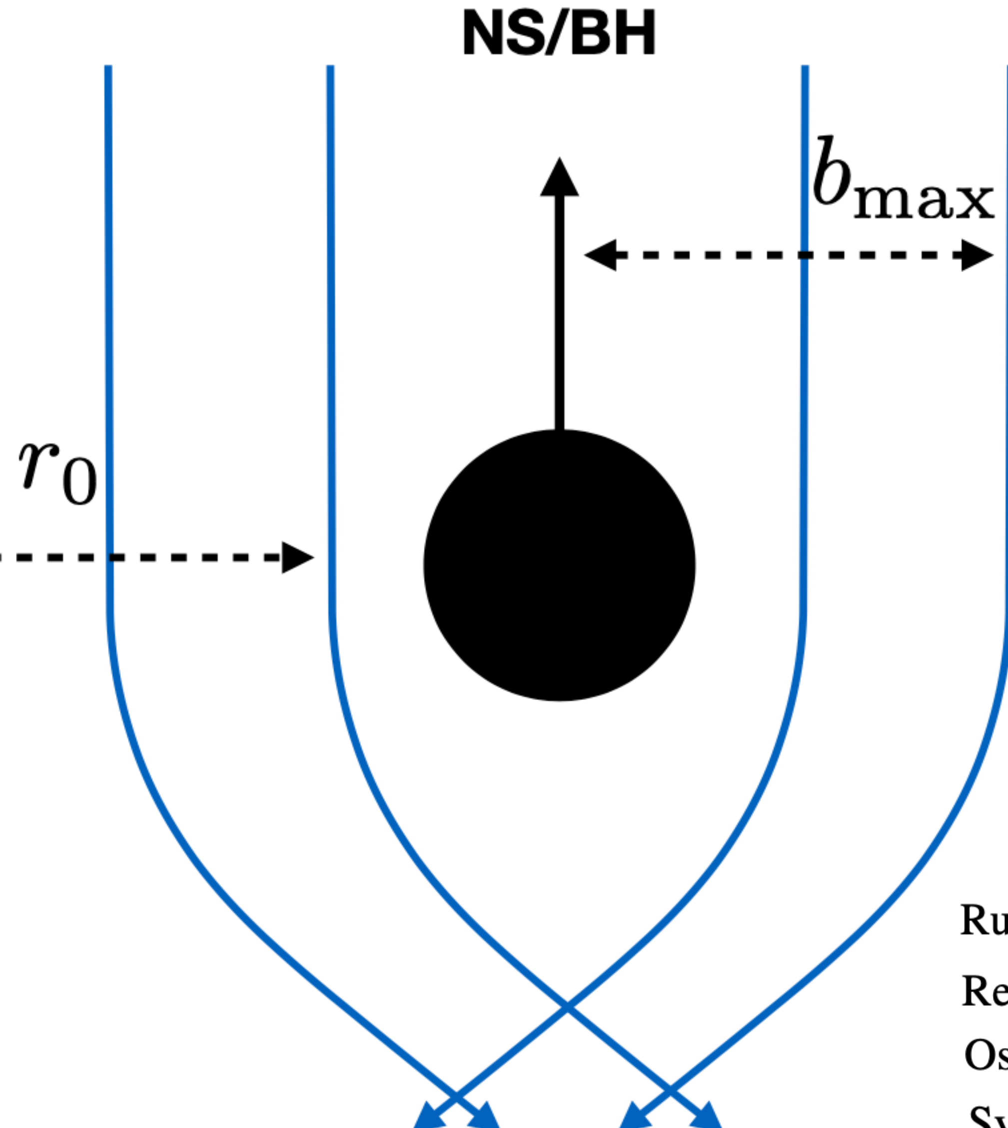
through gravitational scattering



Reconstruct density from distribution function:

$$\rho(r) = \int \mathrm{d}^3\mathbf{v}\, f(\mathcal{E})$$

Dynamical Friction



$$\frac{\mathrm{d}E_{\mathrm{DF}}}{\mathrm{d}t} = 4\pi (G m_2)^2 \rho_{\mathrm{DM}}(r_2) \xi(v) v^{-1} \log \Lambda$$

$$\Lambda = \sqrt{\frac{b_{\max}^2 + b_{90}^2}{b_{\min}^2 + b_{90}^2}},$$

Chandrasekhar, S. 1943, ApJ, 97, 255.
Lee, E. P. 1969, ApJ, 155, 687.
Ruderman, M. A. & Spiegel, E. A. 1971, ApJ, 165, 1.
Rephaeli, Y. & Salpeter, E. E. 1980, ApJ, 240, 20.
Ostriker, E. C. 1999, ApJ, 513, 252.
Syer, D. 1994, MNRAS, 270, 205.
Barausse, E. 2007, MNRAS, 382, 826.

Gravitational Wave

$$\frac{\mathrm{d}E_{\mathrm{orb}}}{\mathrm{d}t} = -\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}t} - \frac{\mathrm{d}E_{\mathrm{DF}}}{\mathrm{d}t}.$$

$$\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}t} = \frac{32G^4M(m_1m_2)^2}{5(cr_2)^5}.$$

$$\frac{\mathrm{d}E_{\mathrm{DF}}}{\mathrm{d}t} = 4\pi(Gm_2)^2\rho_{\mathrm{DM}}(r_2)\xi(v)v^{-1}\log\Lambda.$$

$$\dot{r}_2 = -\frac{64G^3Mm_1m_2}{5c^5(r_2)^3} - \frac{8\pi G^{1/2}m_2\rho_{\mathrm{sp}}\xi\log\Lambda r_{\mathrm{sp}}^{\gamma_{\mathrm{sp}}}}{\sqrt{M}m_1r_2^{\gamma_{\mathrm{sp}}-5/2}}$$

$$h_+(t) = \frac{4G_N\mu}{c^4D_L}\frac{1+\cos^2\iota}{2}(\omega r_2)^2\cos[2\Phi_{\mathrm{orb}}(t)+2\phi],$$

$$h_\times(t) = \frac{4G_N\mu}{c^4D_L}\cos\iota(\omega r_2)^2\sin[2\Phi_{\mathrm{orb}}(t)+2\phi],$$

$$E(v) = -\frac{1}{2}\eta Mv^2\left(1+\#(\eta)v^2+\#(\eta)v^4+\ldots\right)$$

$$P(v) \equiv -\frac{dE}{dt} = \frac{32}{5G_N}v^{10}\left(1+\#(\eta)v^2+\#(\eta)v^3+\ldots\right)$$

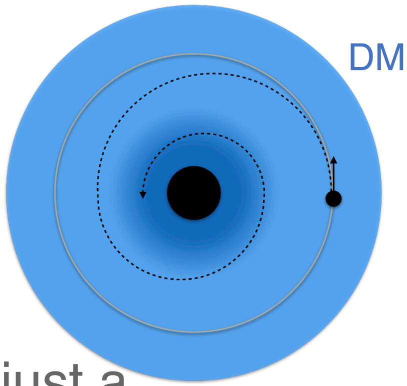
E(v)(P(v)) known up to 3(3.5)PN

$$\frac{1}{2\pi}\phi(T) = \frac{1}{2\pi}\int^T\omega(t)dt = -\int^{v(T)}\frac{\omega(v)dE/dv}{P(v)}dv$$

$$\sim \int\left(1+\#(\eta)v^2+\ldots+\#(\eta)v^6+\ldots\right)\frac{dv}{v^6}$$

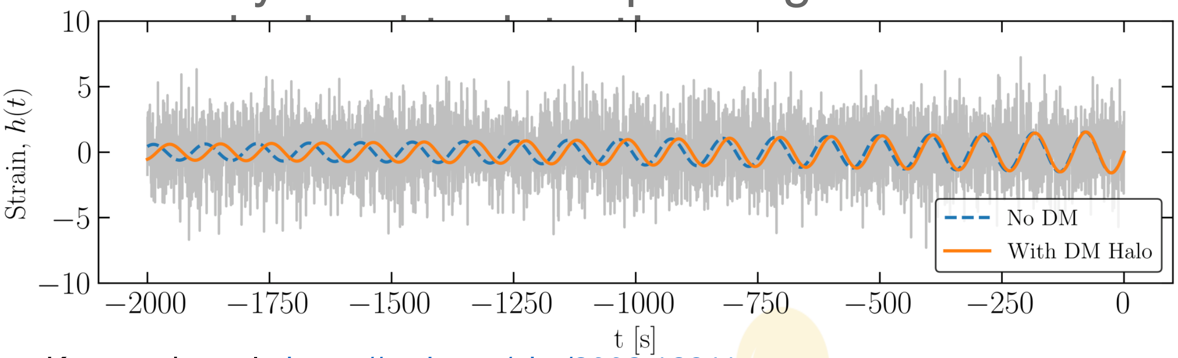
Detecting DM with Einstein Telescope

- Presence of DM ‘spikes’ around BHs can alter inspiral dynamics



- GW waveform gradually goes out of phase with the corresponding
  vacuum-only waveform

- Possibility to detect and constrain dense DM ‘spikes’ with just a few cycles of GW ‘dephasing’ → but these subtle differences



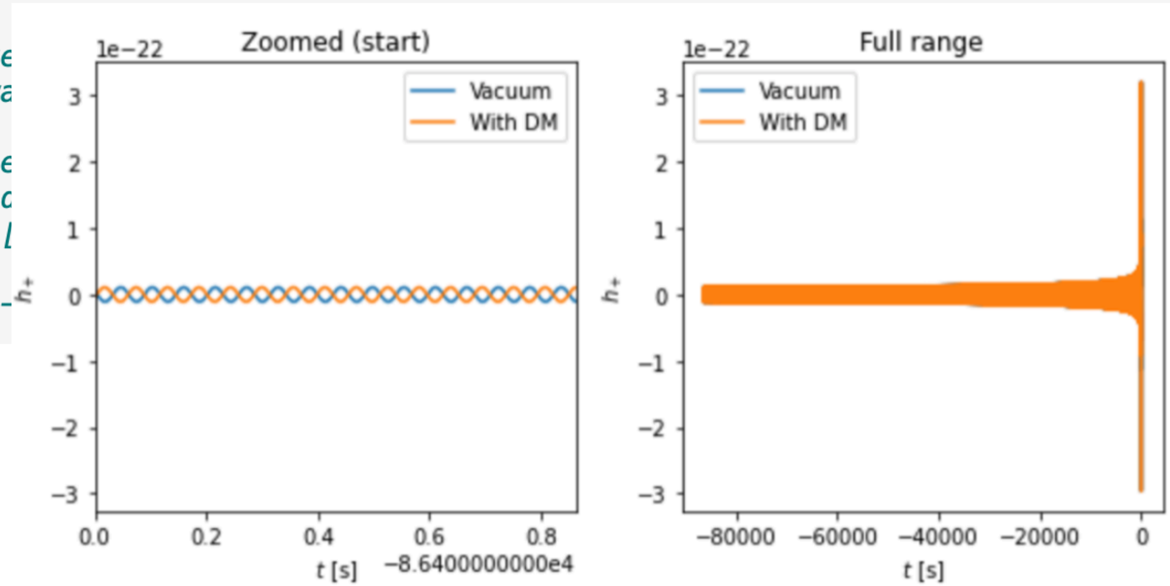
Ideal case for
Machine learning!

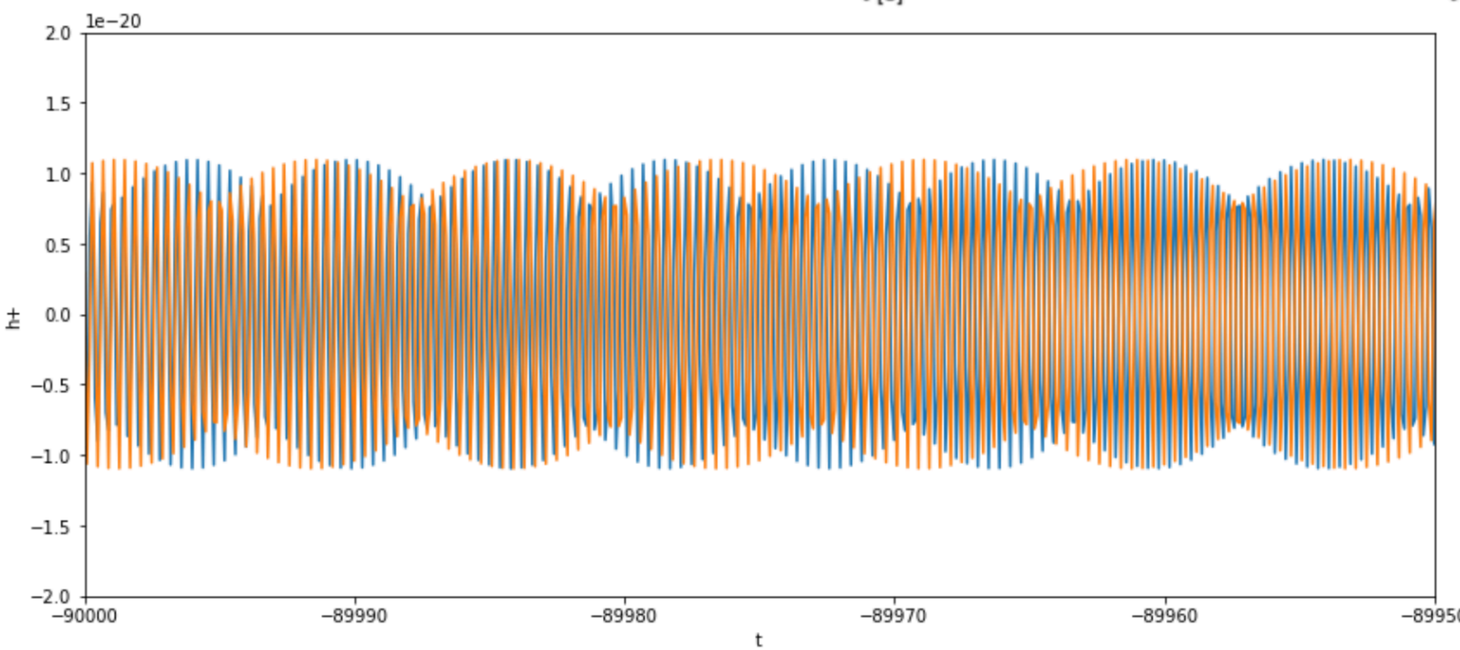
Kavanagh et al., https://arxiv.org/abs/2002.12811
Coogan et al., https://arxiv.org/abs/2002.12811

Funded by the European Union’s
Horizon 2020 - Grant N° 824064

```
#EDIT WAVEFORM PARAMETERS BELOW:
#---------------------------------------------------
m_1      = 1.0     # [MSun]
m_2      = 0.001   # [MSun]
t_obs    = 1*DAY   # Duration of the wave
t_end    = -0.0*YR   # End time of the wa
f_samp   = 200 # Sampling frequency [Hz]
d_l      = 100000    # Luminosity distance
iota     = 0.0   # Inclination angle [rad
phi_c    = 0.0   # Phase at coalescence [

#---------------------------------------------
```

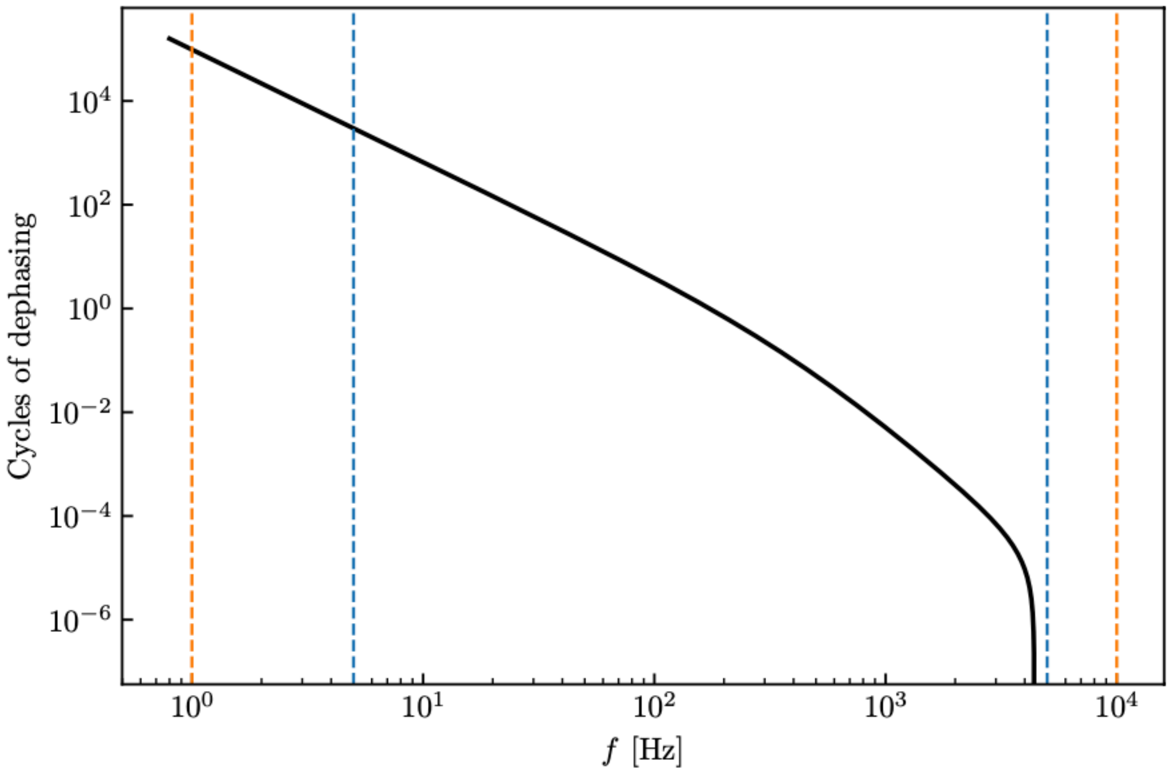




Dephasing

$$N_{\text{cycles}}(t_{\max}, t_{\min}) = \int_{t_{\min}}^{t_{\max}} f_{\text{gw}}(t)dt = \int_{f_{\min}}^{f_{\max}} df_{\text{gw}} \frac{f_{\text{gw}}}{\dot{f}_{\text{gw}}}$$

$$\Delta N_{\text{cycles}} = N_{\text{cycles}}^{\text{vac}}(f_{\max}, f_{\min}) - N_{\text{cycles}}^{\text{DM}}(f_{\max}, f_{\min})$$



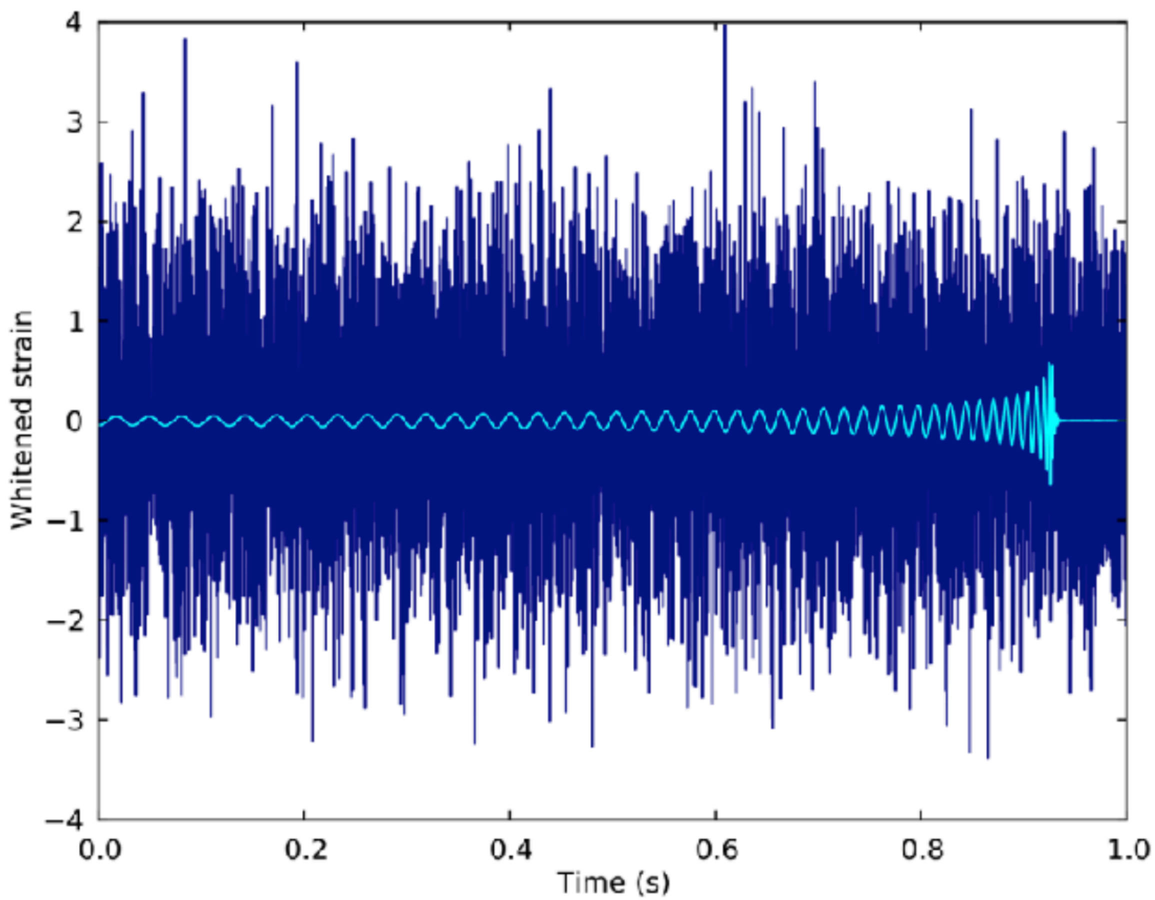
$q = 10^{-3}$, $m_1 = 1 M_\odot$

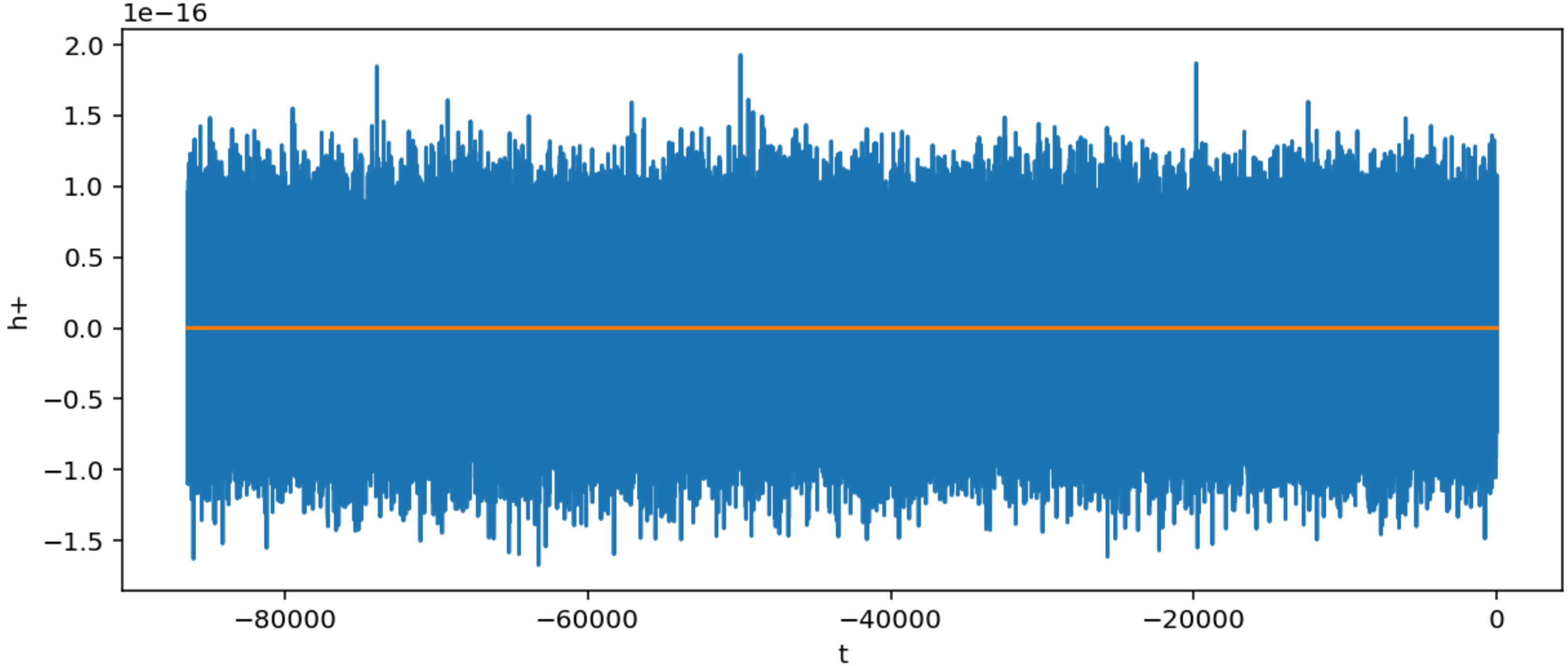
$\rho_{\text{sp}} = 835 M_\odot/\text{pc}^3$

$\gamma_{\text{sp}} = 9/4.$

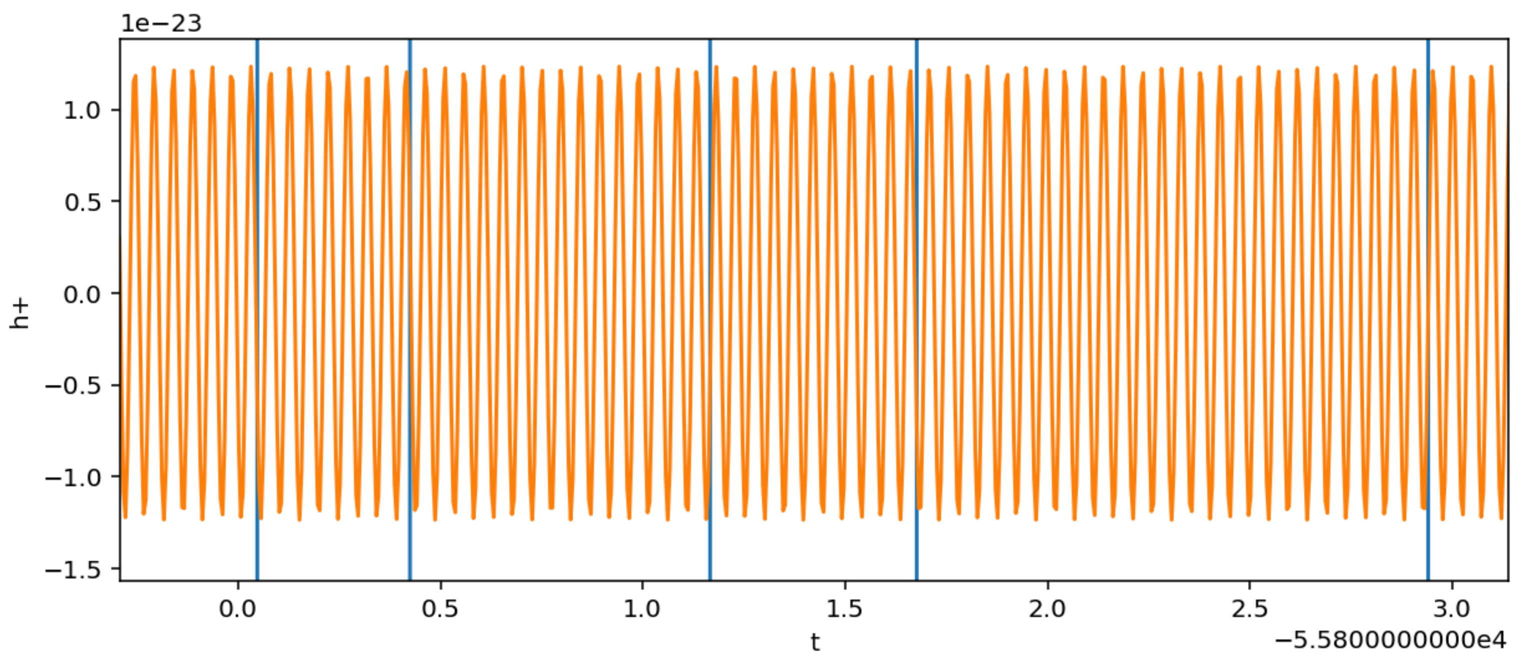
Matched Filtering

Naively, one might think that we can only make confident detections when $|h(t)| > |n(t)|$
However, the **majority of signals are expected to be $|h(t)| \ll |n(t)|$**
Therefore, we need a method to detect signals from noise-dominated data
If we know the possible forms of h(t), we can "filter" out things that are non-signal-like





1e−16
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
h+
−80000
−60000
−40000
−20000
0
t



1e−23
1.0
0.5
0.0
−0.5
−1.0
−1.5
h+
0.0
0.5
1.0
1.5
2.0
2.5
3.0
t
−5.5800000000e4

Detector Antenna Sensitivity

- **Antenna patterns**

$$F = \begin{bmatrix} \cos(2\psi) & \sin(2\psi) \\ -\sin(2\psi) & \cos(2\psi) \end{bmatrix} \begin{bmatrix} F_+[\theta,\phi] \\ F_\times[\theta,\phi] \end{bmatrix}$$

$$F_+[\theta,\phi] = \tfrac{1}{2}(1+\cos^2\theta)\cos$$

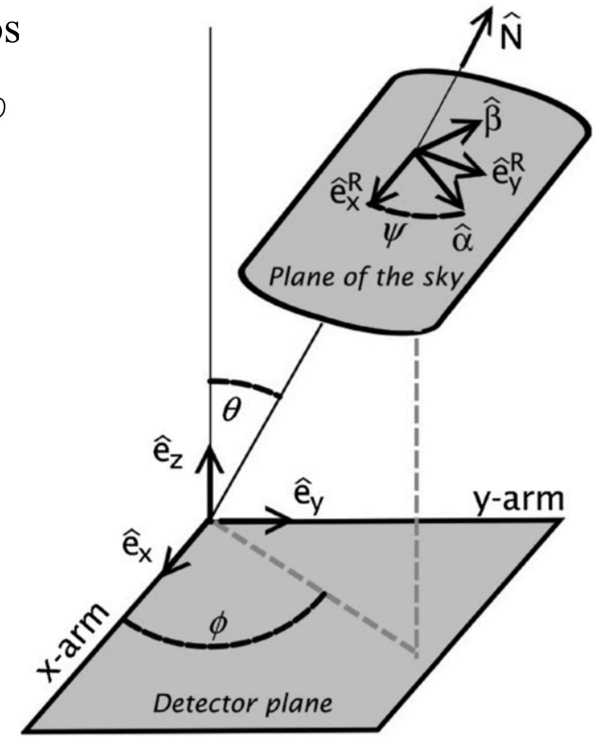
$$F_\times[\theta,\phi] = \cos\theta \sin 2\varphi$$

- **Sampled GW signal**

$$h[i] = \begin{bmatrix} \cos(2\psi) & \sin(2\psi) \\ -\sin(2\psi) & \cos(2\psi) \end{bmatrix} \begin{bmatrix} h_+[i] \\ h_\times[i] \end{bmatrix}$$

- **Sampled detector response**

$$\xi[i] = F_+ h_+[i] + F_\times h_\times[i] = F^T \cdot h[i]$$



- **Direction to the source θ, φ and polarization angle Ψ define relative orientation of the detector and wave frames.**
- **Rotation of the wave frame $R_z(2\Psi)$ induces transformations both for F and h, but ξ is INVARIANT**

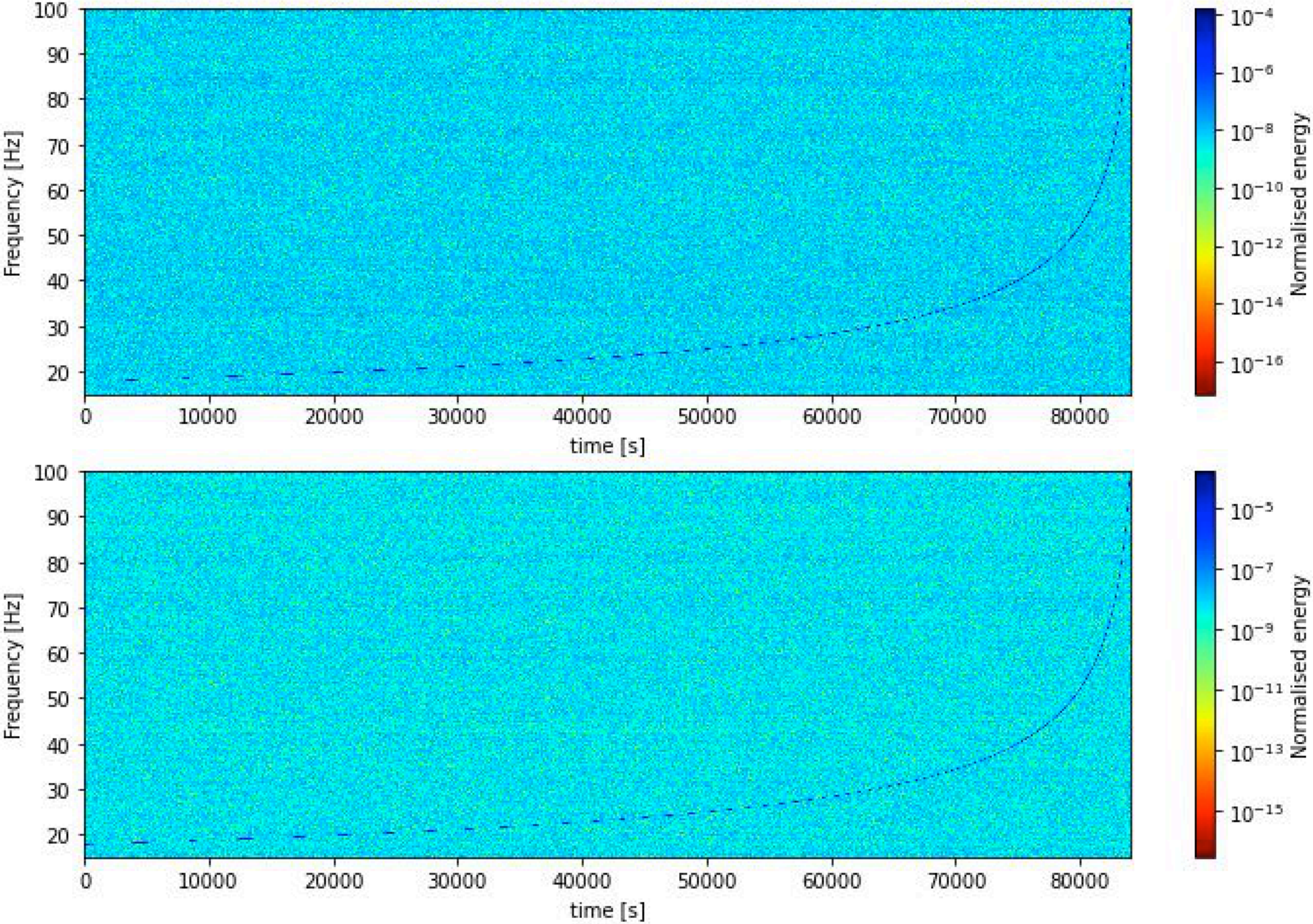
Waveform Dataset

- Develop a catalog of waveforms for different luminosity distances and masses

- **Luminosity distance d=10kpc, 20kpc, 30kpc, 40kpc, 50kpc , 60kpc,100kpc**
- **Mass** $m_1 = 1M_\odot$ $m_2 = 10^{-2} - 10^{-4}M_\odot$ $\Delta m_2 = 0.001M_\odot$
- **Antenna Sensitivity 100 different directions**

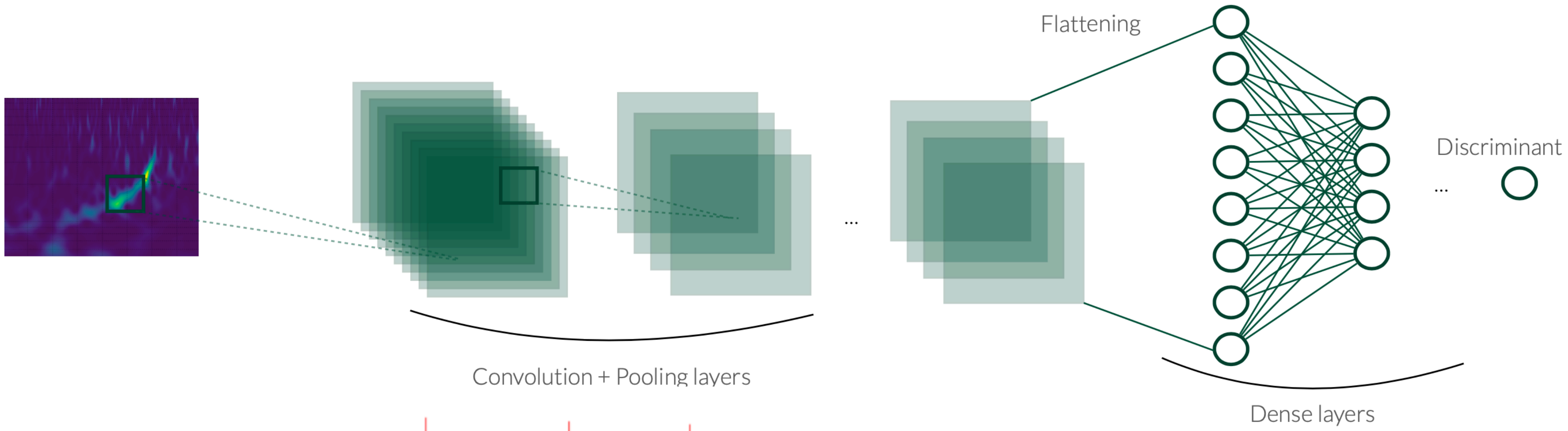
We have 11400 GW for the vacuum and 11400 GW with dark matter +802 GW at 100kpc

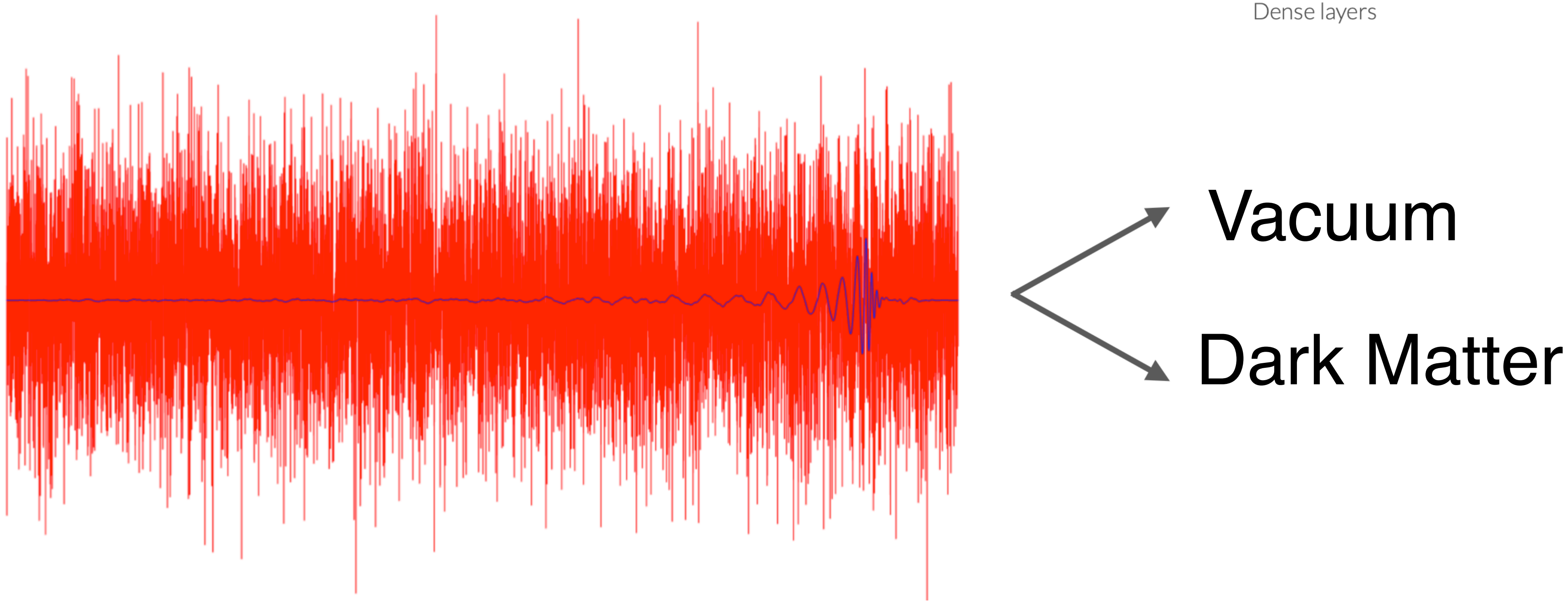
Total: 22800 waveform +802



Frequency [Hz]
time [s]
Normalised energy
0
10000
20000
30000
40000
50000
60000
70000
80000
100
90
80
70
60
50
40
30
20
10^-4
10^-6
10^-8
10^-10
10^-12
10^-14
10^-16
Frequency [Hz]
time [s]
Normalised energy
10^-5
10^-7
10^-9
10^-11
10^-13
10^-15

Machine Learning for GW Classification





Pipeline Structure

Input GW data

- **Basic GW wavedorm**
- **Add a noise**
- **Antenna Sensitivity 100 different directions**
- **Whitened strain**

Classification

- **Basic GW Image creation from time frequency (spectrograms)**
- **Tested various networks, including a 4-block layers**

High Performance Computing Center

Scuola Normale Superiore

Conclusions

- We can measure the properties of dark matter spike around binaries with Einstein Telescope
- We can distinguish between vacuum and dark matter for distance up to 100kpc

Futuro work:

- Eccentric waveforms
- Post-Newtonian corrections

Thank you for your attention

Self-consistent evolution

Assuming everything evolves slowly compared to the orbital period:

$$T_{\mathrm{orb}}\frac{\mathrm{d}f(\mathcal{E})}{\mathrm{d}t}=\boxed{-p_{\mathcal{E}}f(\mathcal{E})+}$$

Particles scattering from
$\mathcal{E}\to\mathcal{E}+\Delta\mathcal{E}$

$$\boxed{\int\left(\frac{\mathcal{E}}{\mathcal{E}-\Delta\mathcal{E}}\right)^{5/2}f(\mathcal{E}-\Delta\mathcal{E})P_{\mathcal{E}-\Delta\mathcal{E}}(\Delta\mathcal{E})\,\mathrm{d}\Delta\mathcal{E}}$$

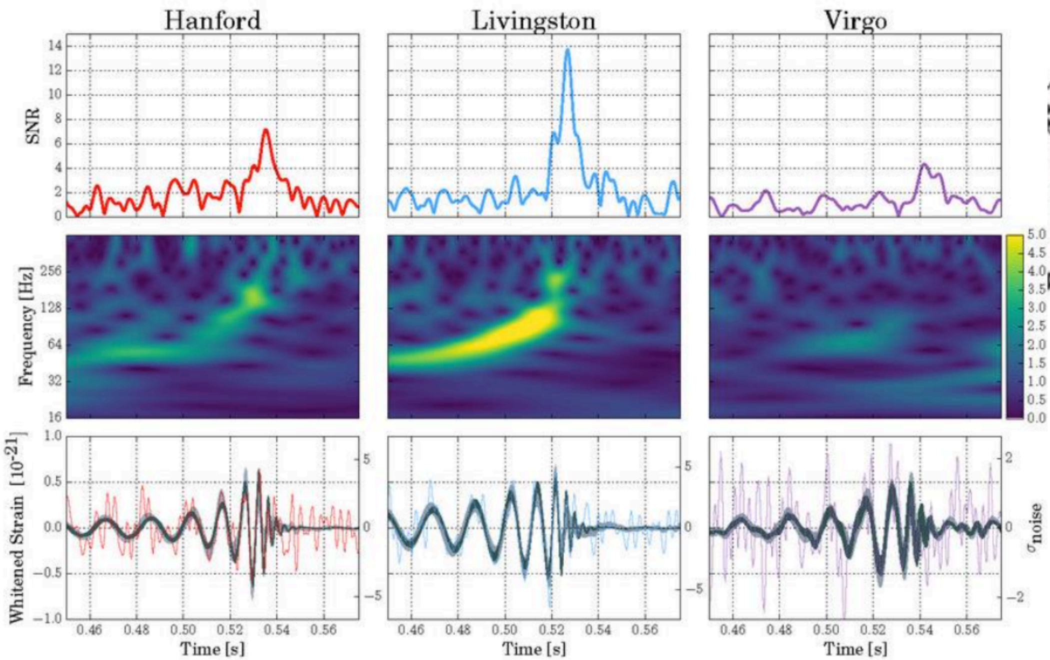
Particles scattering from
$\mathcal{E}-\Delta\mathcal{E}\to\mathcal{E}$

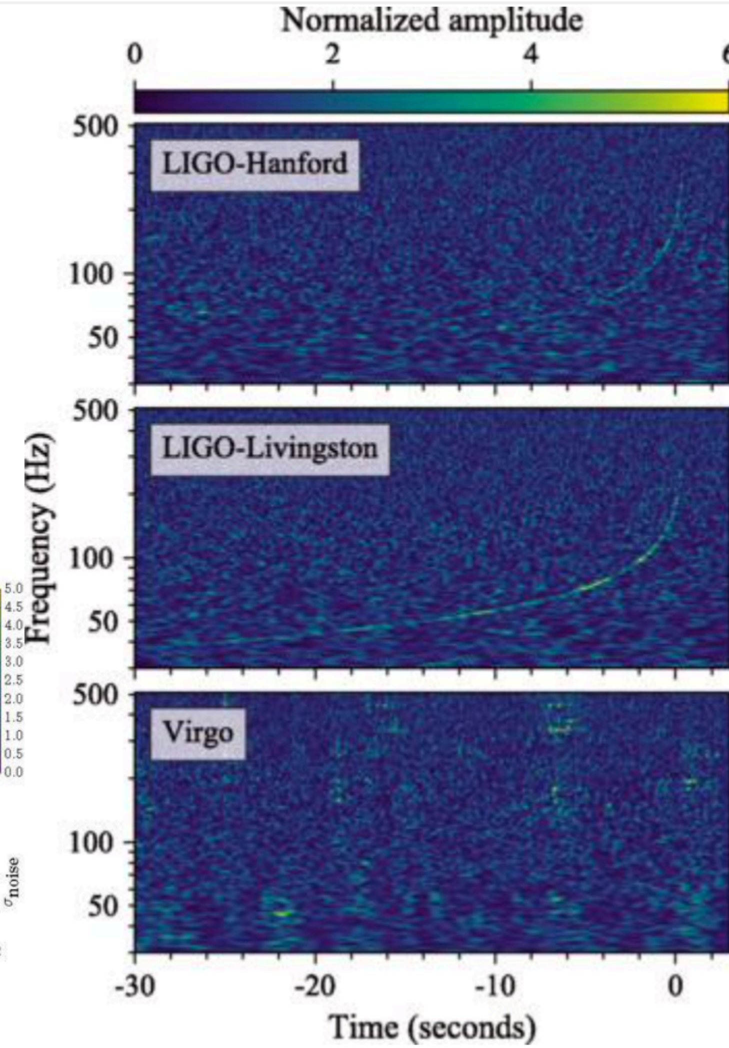
$P_{\mathcal{E}}(\Delta\mathcal{E})$ - probability for a particle with energy $\mathcal{E}$ to scatter and receive a ‘kick’ $\Delta\mathcal{E}$

$$p_{\mathcal{E}}=\int P_{\mathcal{E}}(\Delta\mathcal{E})\,\mathrm{d}\Delta\mathcal{E}$$

- total probability for a particle with energy $\mathcal{E}$ to scatter

Example of GW signals in Time-Frequency plots





Tidal effects in the gravitational wave signal emitted in NS-NS binary coalescence

$$h(f) = \mathcal{A}(f)e^{i\psi(f)} \qquad \psi(f) = \psi_{PP} + \psi_{\bar{Q}} + \psi_{\bar{\lambda}}$$

$x = (m\pi f)^{5/3}$ PN expansion parameter

point-particle contribution

$$\psi_{PP}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{1 + \left(\frac{3715}{756} + \frac{55}{9}\eta\right)x - (16\pi - 4\beta)x^{3/2} + \left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2 - 10\sigma\right)x^2 + \mathcal{O}(x^{5/2})\right\}$$

σ contains spin-spin and spin-orbit terms. Note that it appears in the 2-PN term (x^2)

Quadrupole contribution:

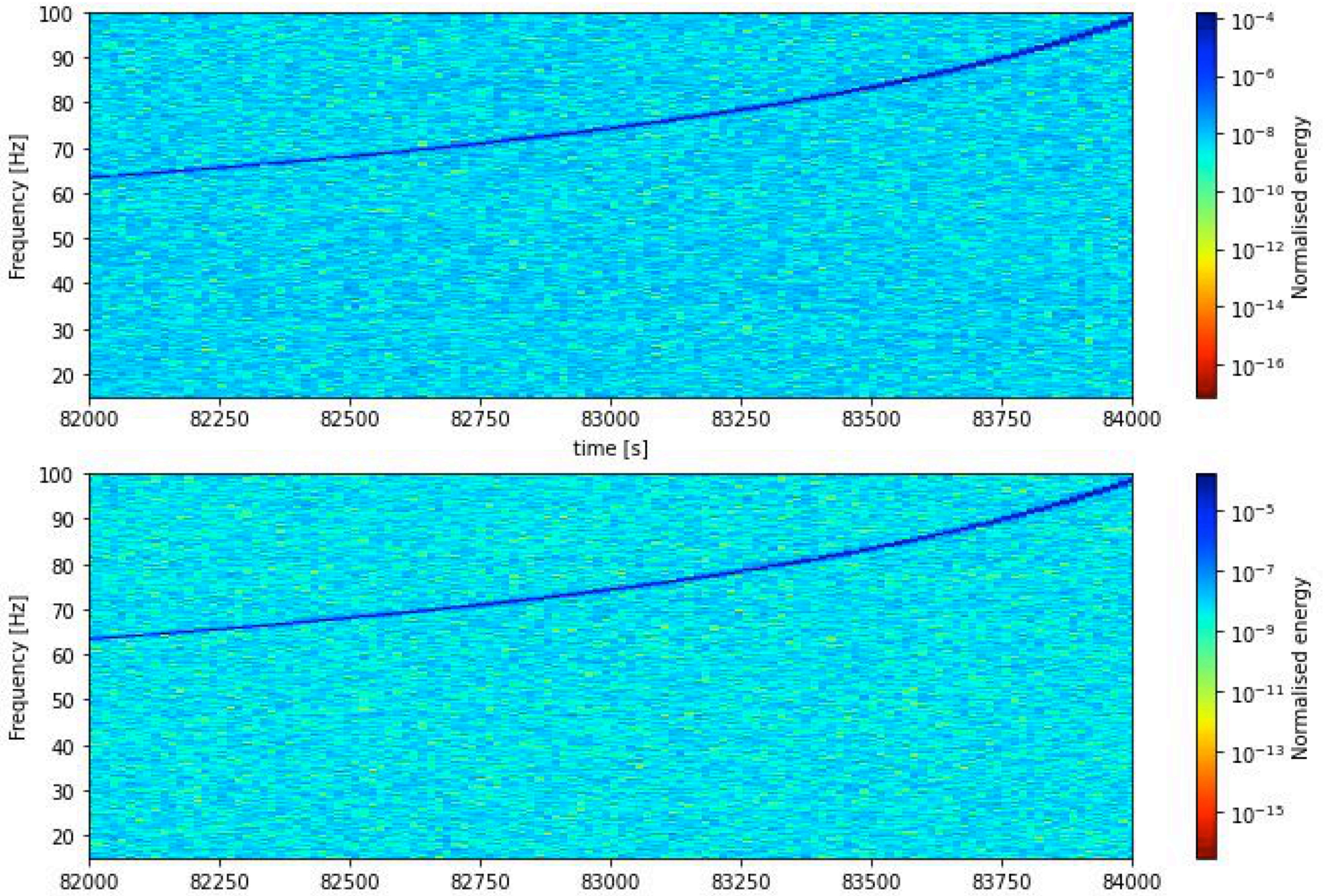
$$\psi_Q = \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{-50\left[\left(\frac{m_1^2}{m^2}\chi_1^2 + \frac{m_2^2}{m^2}\chi_2^2\right)(Q_S - 1) + \left(\frac{m_1^2}{m^2}\chi_1^2 - \frac{m_2^2}{m^2}\chi_2^2\right)Q_a\right]x^2\right\}$$

$$Q_S = \frac{\bar{Q}_1 + \bar{Q}_2}{2}, \quad Q_a = \frac{\bar{Q}_1 - \bar{Q}_2}{2}$$

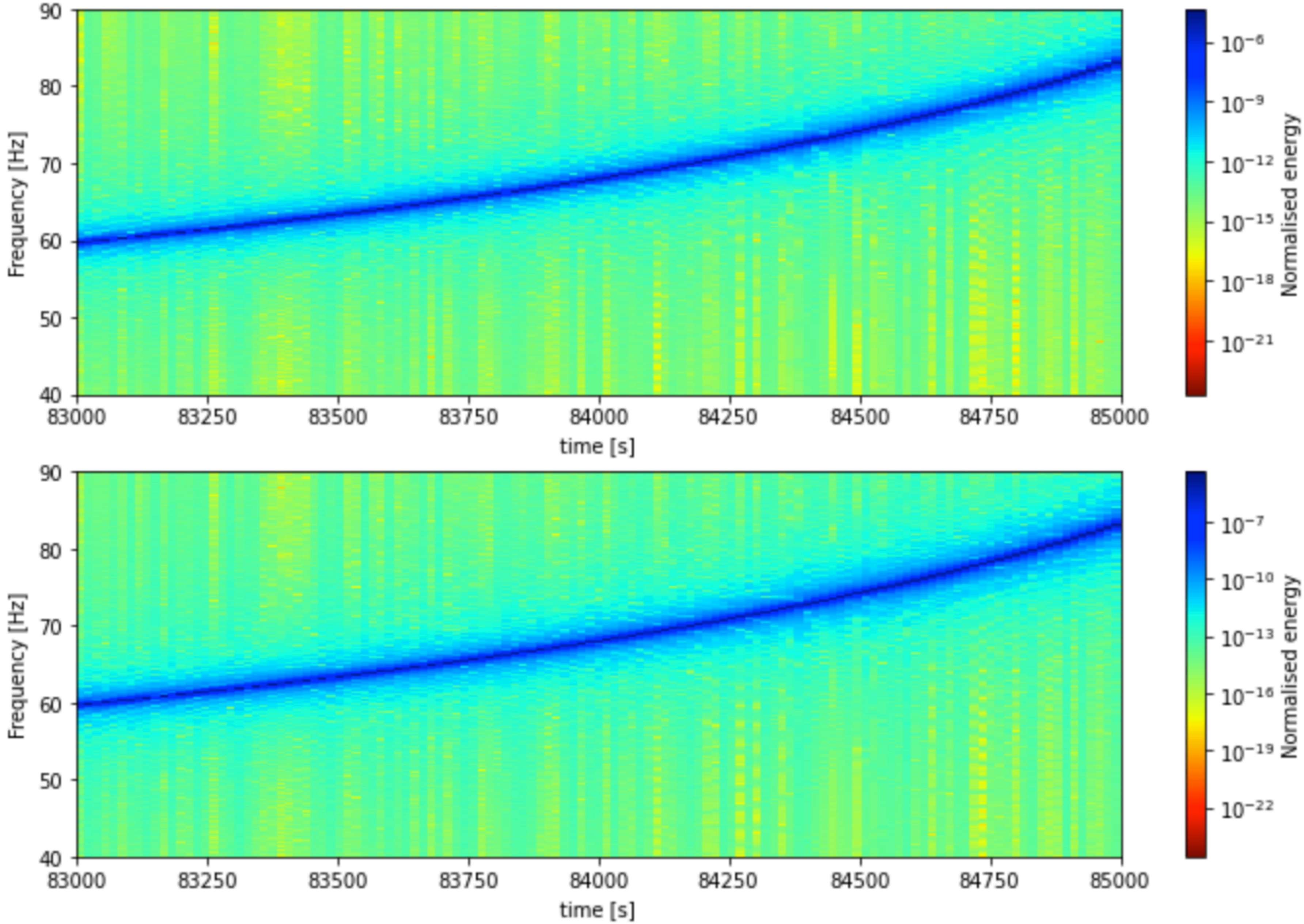
Tidal contribution: $\psi_{\bar{\lambda}} = -\frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{24[(1 + 7\eta - 31\eta^2)\lambda_S + (1 + 9\eta - 11\eta^2)\lambda_a\delta m]x^5 + \right\} +$

$$\lambda_S = \frac{\bar{\lambda}_1 + \bar{\lambda}_2}{2}, \quad \lambda_a = \frac{\bar{\lambda}_1 - \bar{\lambda}_2}{2} \qquad \delta m = \frac{m_1 - m_2}{m}$$

degeracy can be removed by expressing the Q's in terms of λ using the universal relations
NOTE THAT: λ is independent of the spins



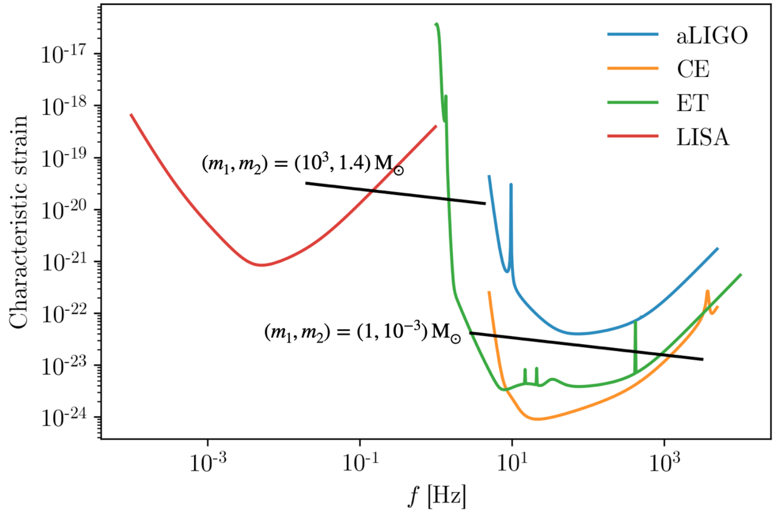
Frequency [Hz]
time [s]
Normalised energy
100
90
80
70
60
50
40
30
20
82000
82250
82500
82750
83000
83250
83500
83750
84000
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}



Frequency [Hz]
time [s]
Normalised energy
90
80
70
60
50
40
83000
83250
83500
83750
84000
84250
84500
84750
85000
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-18}
10^{-21}
10^{-7}
10^{-10}
10^{-13}
10^{-16}
10^{-19}
10^{-22}

Detecting DM with Einstein Telescope

- Frequency band of ET means that most promising target would be solar and sub-solar mass binaries



- **Primordial black holes** (PBHs) could form such binaries, and must be surrounded by dense spike of particle DM

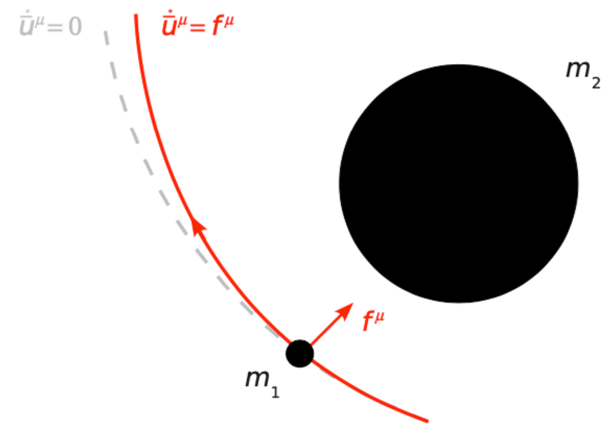
- Currently developing an ET pipeline to search for 'de-phased' GW waveforms due to DM around PBHs, using machine learning

- Waveform generation & search pipeline will all be public → implementation in virtual research environment will allow easy **access** and **re-use**

Funded by the European Union's
Horizon 2020 - Grant N° 824064

Self force causes deviation from background geodesic

- A particle is moving on a background space-time
- Its own stress-energy tensor modifies the background gravitational field
- Because of the "back-reaction" the motion of the particle deviates from a background geodesic hence the appearance of a gravitational self force (GSF)



The self acceleration of the particle is proportional to its mass

$$\boxed{\frac{\mathrm{D}\bar{u}^\mu}{\mathrm{d}\tau} = f^\mu = \mathcal{O}\left(\frac{m_1}{m_2}\right)}$$

Assume space-time slightly differs from Minkowski space-time $\eta_{\alpha\beta}$

$$\mathfrak{g}^{\alpha\beta} = \eta^{\alpha\beta} + h^{\alpha\beta} \qquad \text{with} \quad |h| \ll 1$$

$$\Box h^{\alpha\beta} = \frac{16\pi G}{c^4}|g|T^{\alpha\beta} + \overbrace{\Lambda^{\alpha\beta}[h, \partial h, \partial^2 h]}^{\text{non-linear source term}} \equiv \frac{16\pi G}{c^4} \underbrace{\tau^{\alpha\beta}}_{\text{stress-energy pseudo-tensor}}$$

$$\underbrace{\partial_\mu h^{\alpha\mu} = 0}_{\text{harmonic-gauge condition}}$$

where $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$ is the flat d'Alembertian operator

$$\mathfrak{g}^{\alpha\beta} = \eta^{\alpha\beta} + \underbrace{\sum_{n=1}^{+\infty} G^n \, h^{\alpha\beta}_{(n)}}_{G \text{ labels the PM expansion}}$$

Inspiral $h = A\cos(\phi(t))$ $\quad \frac{\dot{A}}{A} \ll \dot{\phi}$

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$\begin{aligned} E(v) &= -\frac{1}{2}\eta M v^2 \left(1 + \#(\eta)v^2 + \#(\eta)v^4 + \dots\right) \\ P(v) \equiv -\frac{dE}{dt} &= \frac{32}{5G_N} v^{10} \left(1 + \#(\eta)v^2 + \#(\eta)v^3 + \dots\right) \end{aligned}$$

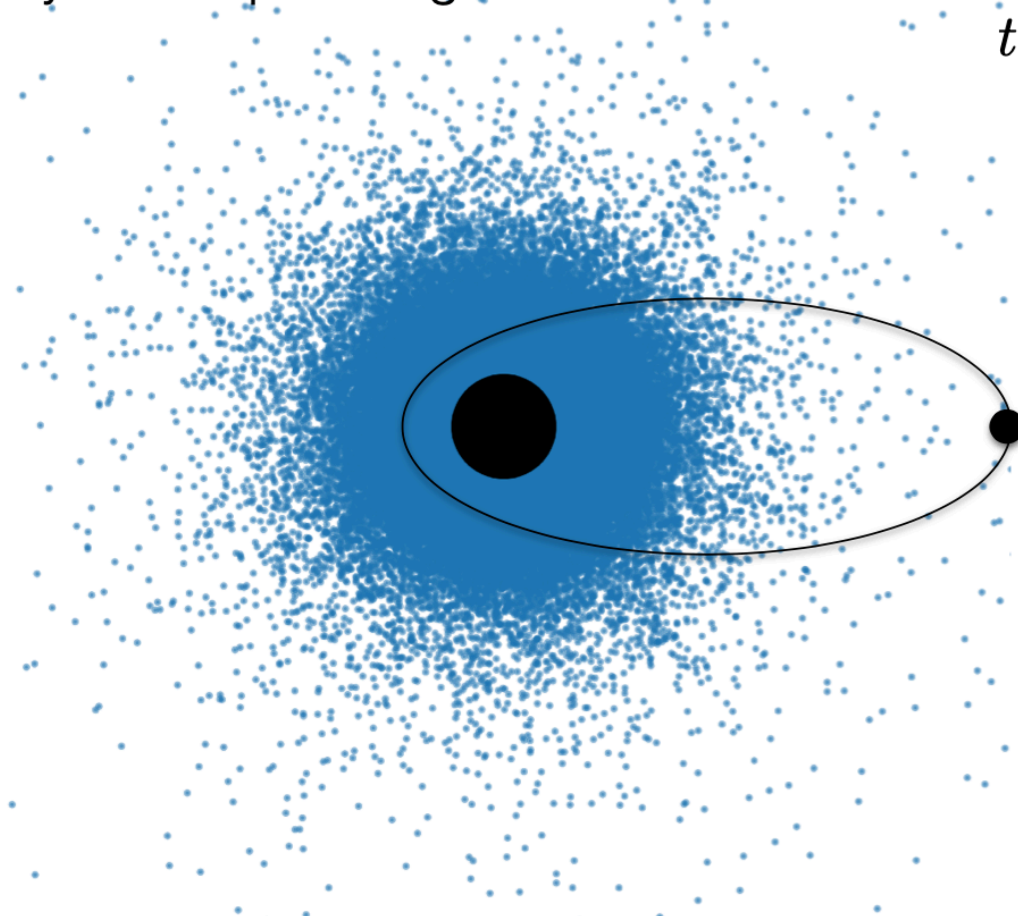
E(v)(P(v)) known up to 3(3.5)PN

$$\begin{aligned} \frac{1}{2\pi}\phi(T) &= \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)} dv \\ &\sim \int \left(1 + \#(\eta)v^2 + \dots + \#(\eta)v^6 + \dots\right) \frac{dv}{v^6} \end{aligned}$$

Next Steps

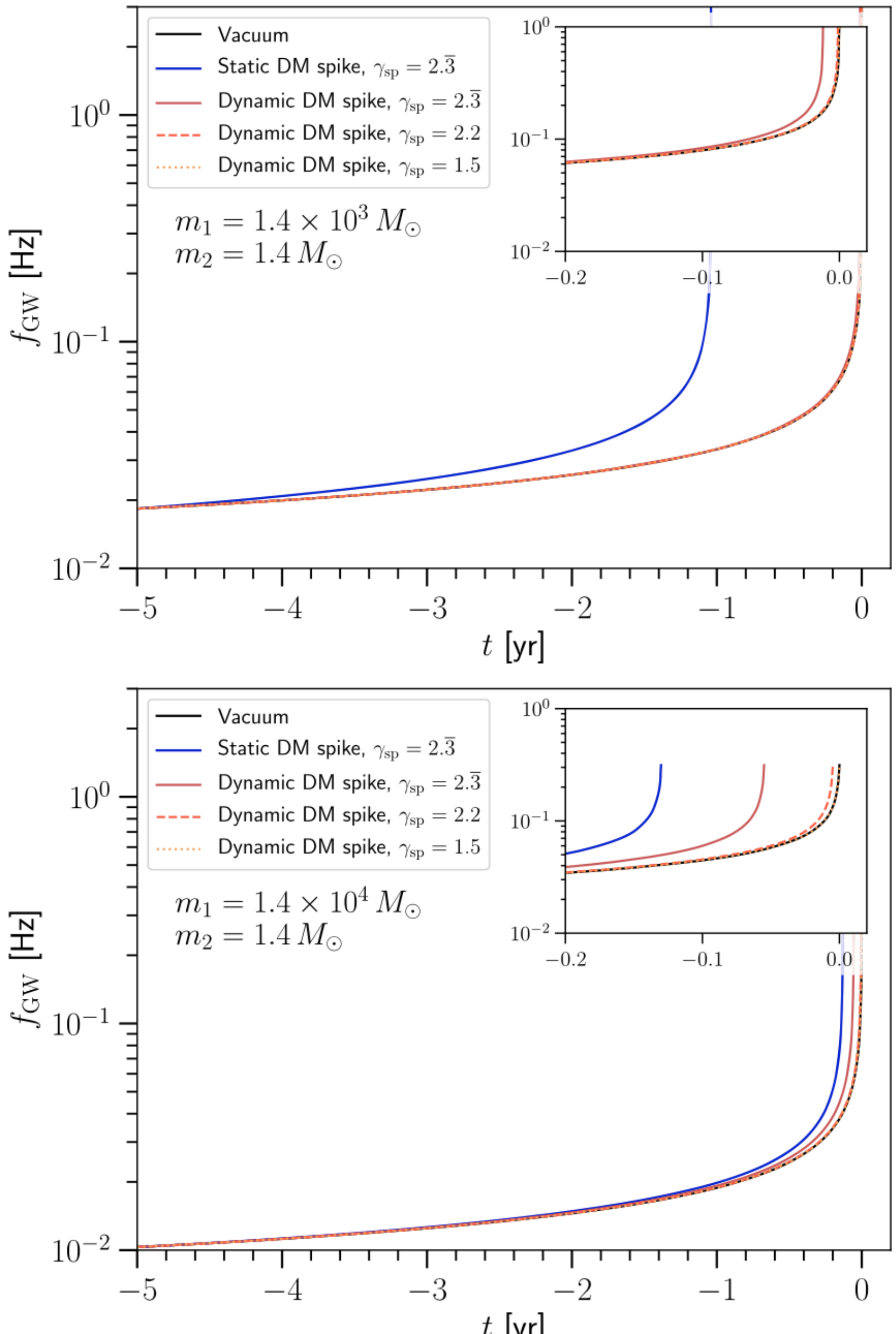
PBH binaries typically formed (in the early Universe) with very high eccentricity —> Rapid merger

$$t_{\rm merge} \sim a_i^4 (1 - e_i^2)^{7/2}$$



Dark Dresses around PBH IMRIs are likely to accelerate merger…

Do dressed PBH IMRIs merge slowly enough to be detected at low redshift?



Vacuum
Static DM spike, $\gamma_{\rm sp} = 2.\overline{3}$
Dynamic DM spike, $\gamma_{\rm sp} = 2.\overline{3}$
Dynamic DM spike, $\gamma_{\rm sp} = 2.2$
Dynamic DM spike, $\gamma_{\rm sp} = 1.5$
$m_1 = 1.4 \times 10^3\, M_\odot$
$m_2 = 1.4\, M_\odot$
$f_{\rm GW}$ [Hz]
t [yr]
Vacuum
Static DM spike, $\gamma_{\rm sp} = 2.\overline{3}$
Dynamic DM spike, $\gamma_{\rm sp} = 2.\overline{3}$
Dynamic DM spike, $\gamma_{\rm sp} = 2.2$
Dynamic DM spike, $\gamma_{\rm sp} = 1.5$
$m_1 = 1.4 \times 10^4\, M_\odot$
$m_2 = 1.4\, M_\odot$
$f_{\rm GW}$ [Hz]
t [yr]