laser noise study for ET

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Beam path in the interferometer

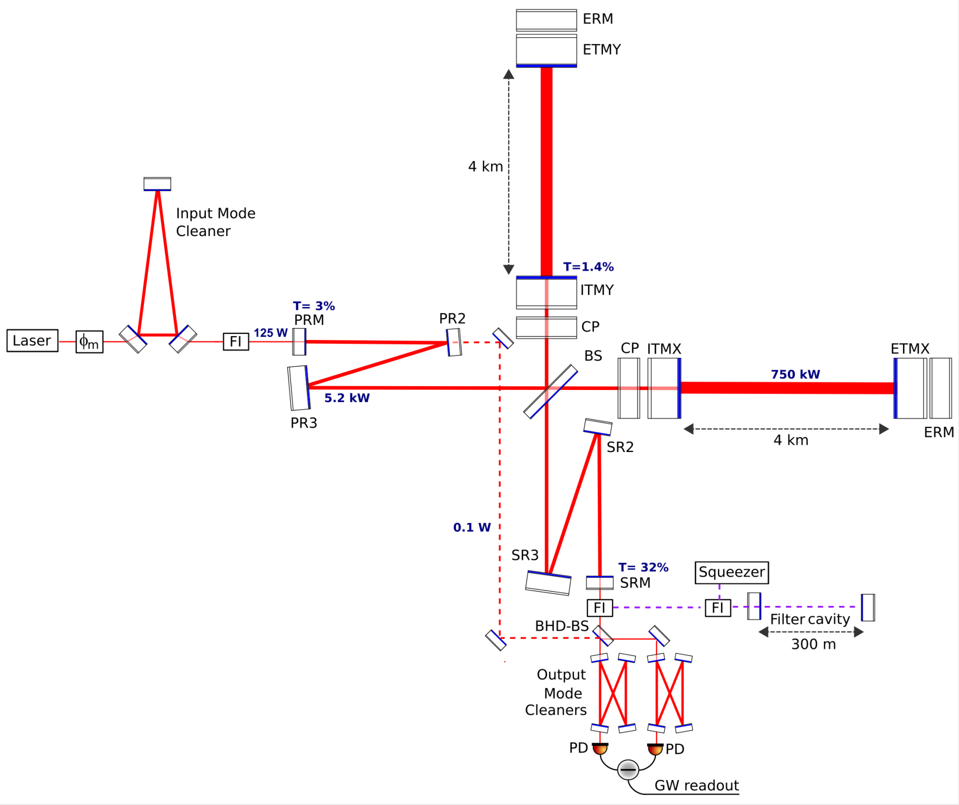
Laser noise in signal path:

1. Bright port ➡ PRC field , after circulating in coupled PRC-Arm.
2. PRC field ➡ SRC field, due to contrast defect, Schnupp asymmetry.
3. SRC field ➡ dark port, after circulating in coupled SRC-Arm.
4. dark port ➡ readout.

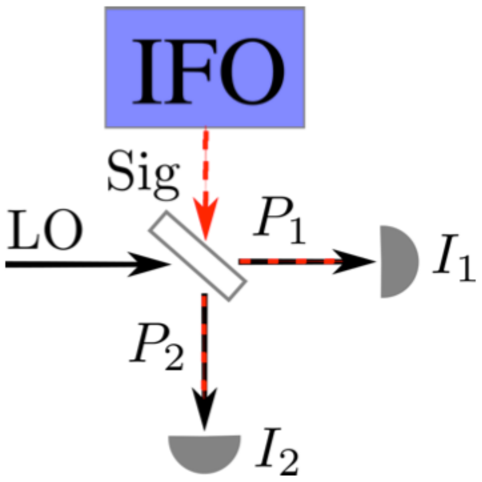
Laser noise in local oscillator path:

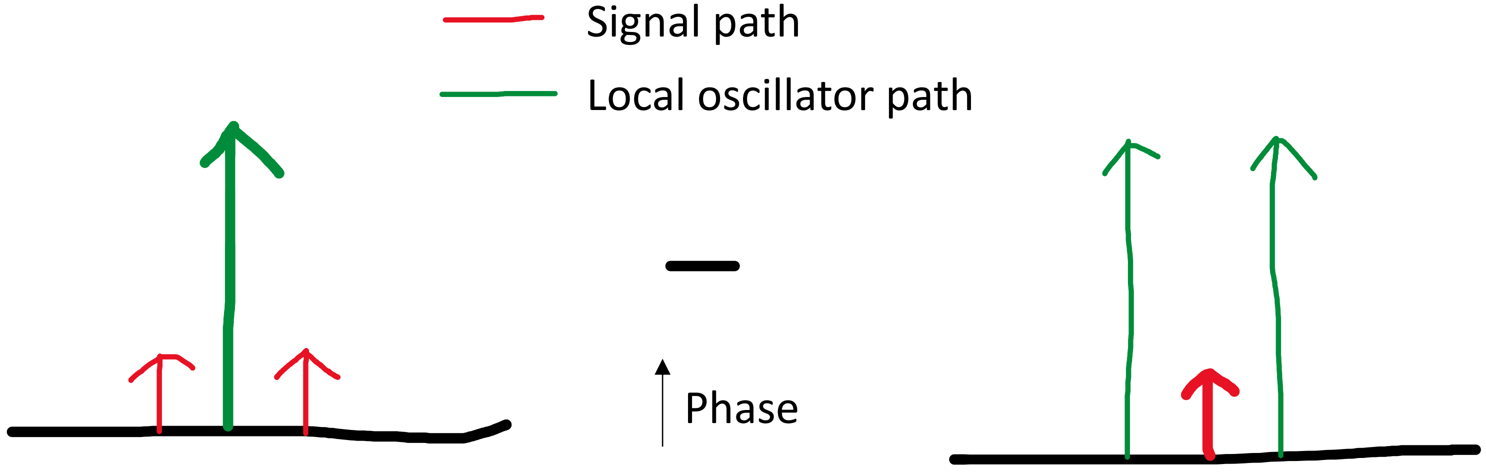
Balanced homodyne readout:

1. Bright port ➡ PRC field , after circulating in coupled PRC-Arm.
2. PRC field ➡ readout.



BHD readout scheme





The noise measurement mechanism is the same for balanced homodyne and DC readout:

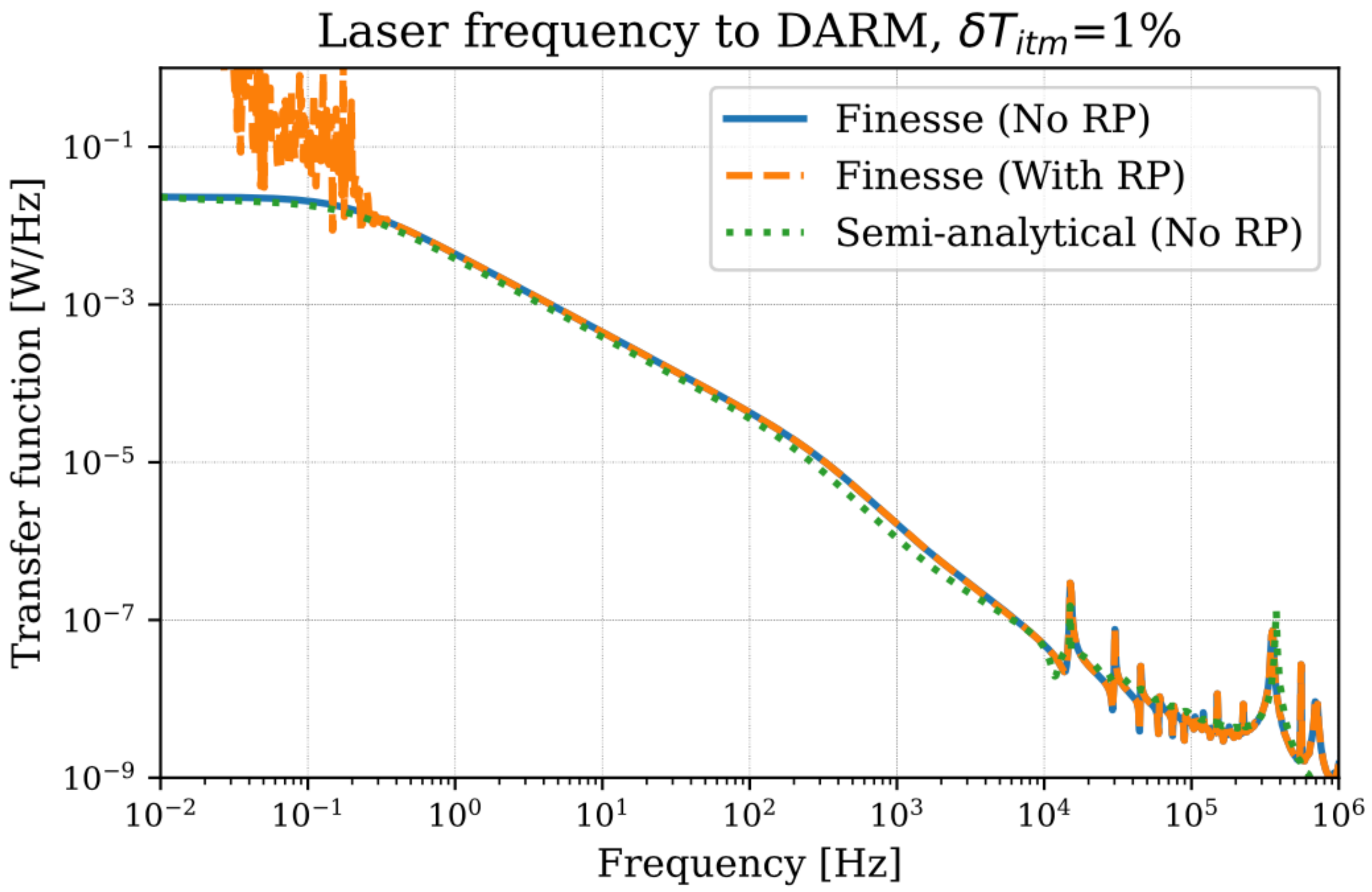
1. Static local oscillator beat against contrast defect noise — Static contrast defect beat against local oscillator noise. (on same phase quadrature)
2. At DC, the frequency noise is canceled.

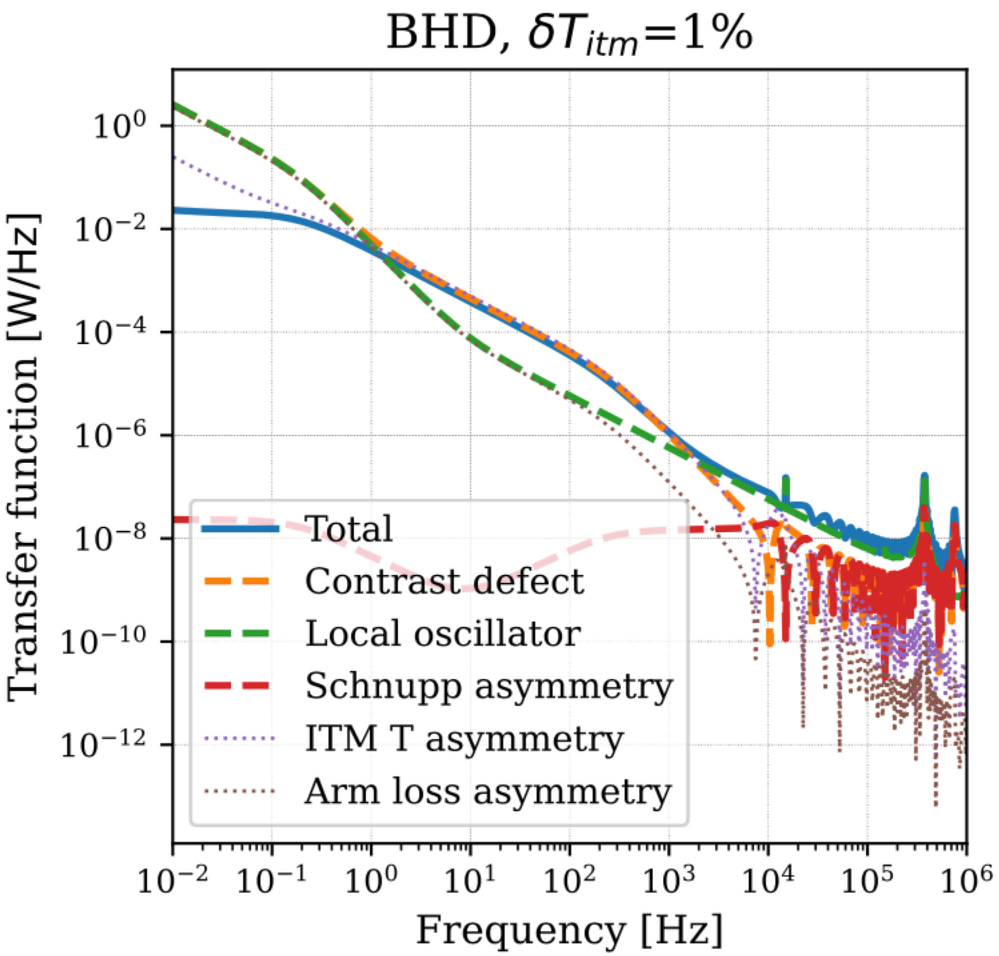
Parameters

Finesse	
ITM T	0.007
ITM T asymmetry	1%
Mirror loss	37.5e-6
Loss asymmetry	10%
LO power	100mW
Dark port power	1.41mW

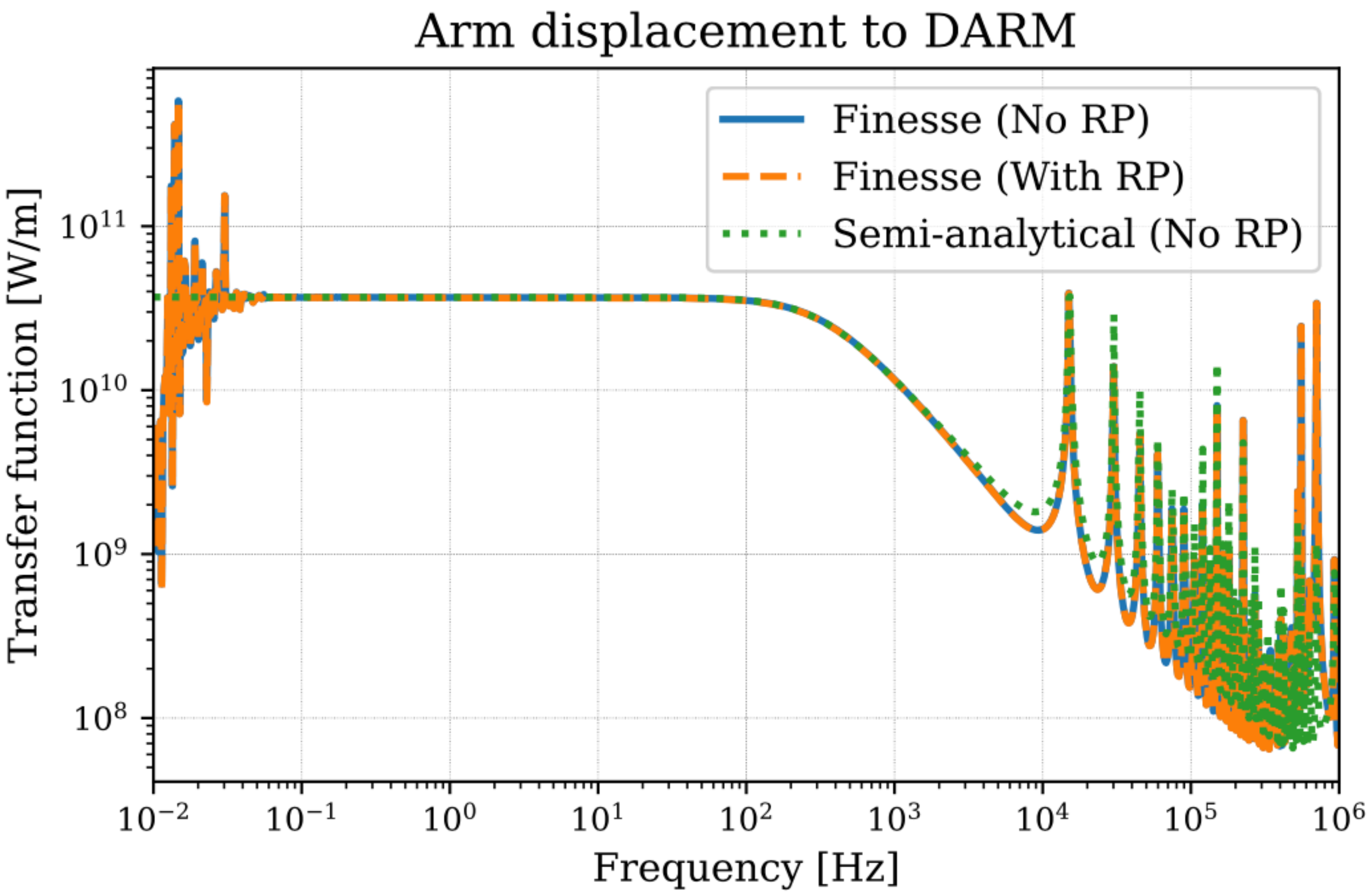
Semi-analytical	
ITM T	0.007
ITM T asymmetry	1%
Mirror loss	37.5e-6
Loss asymmetry	10%
LO power	100mW
Dark port power	1.56mW

Frequency noise TF in W/Hz





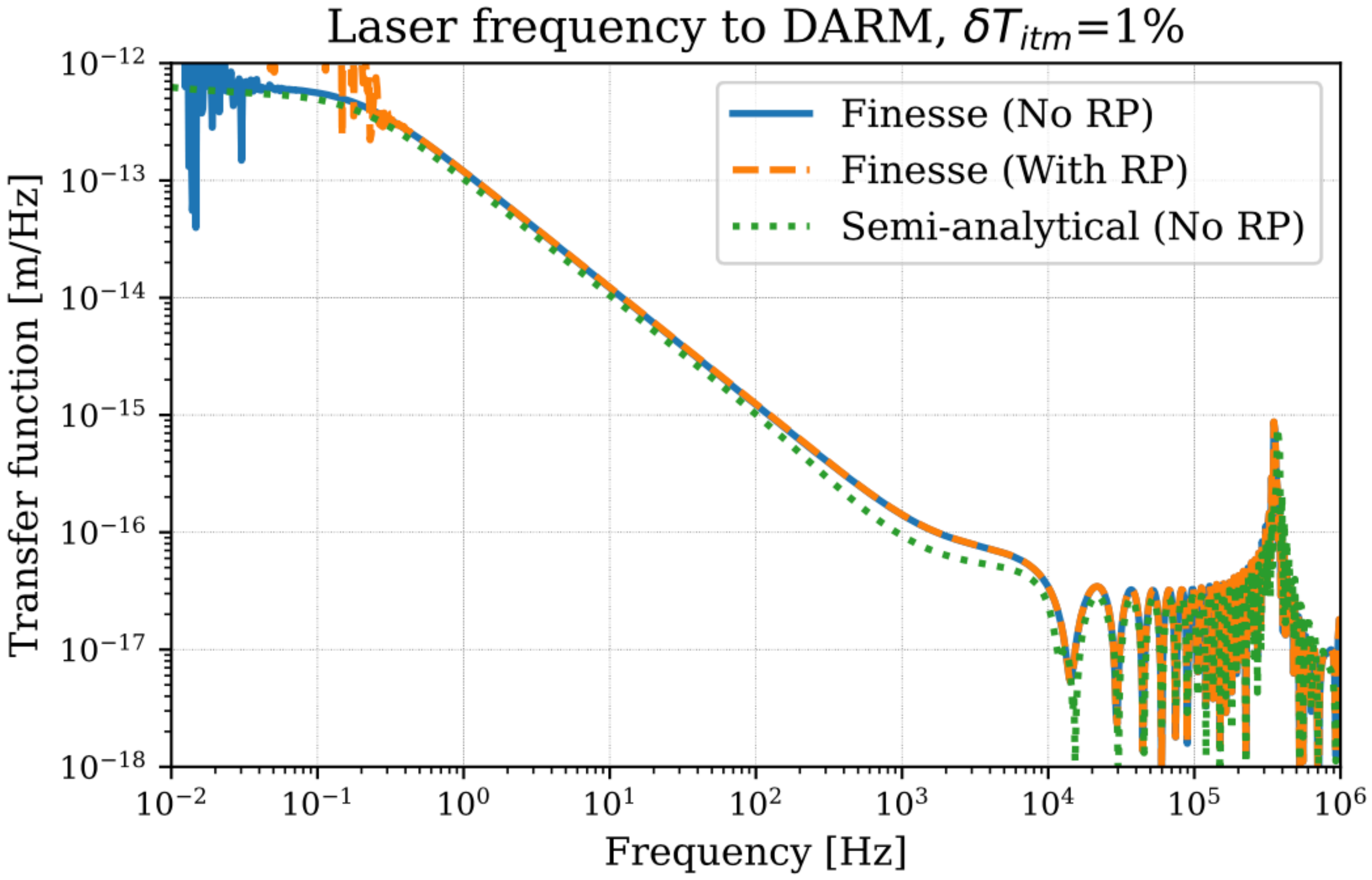
Darm motion TF in W/m

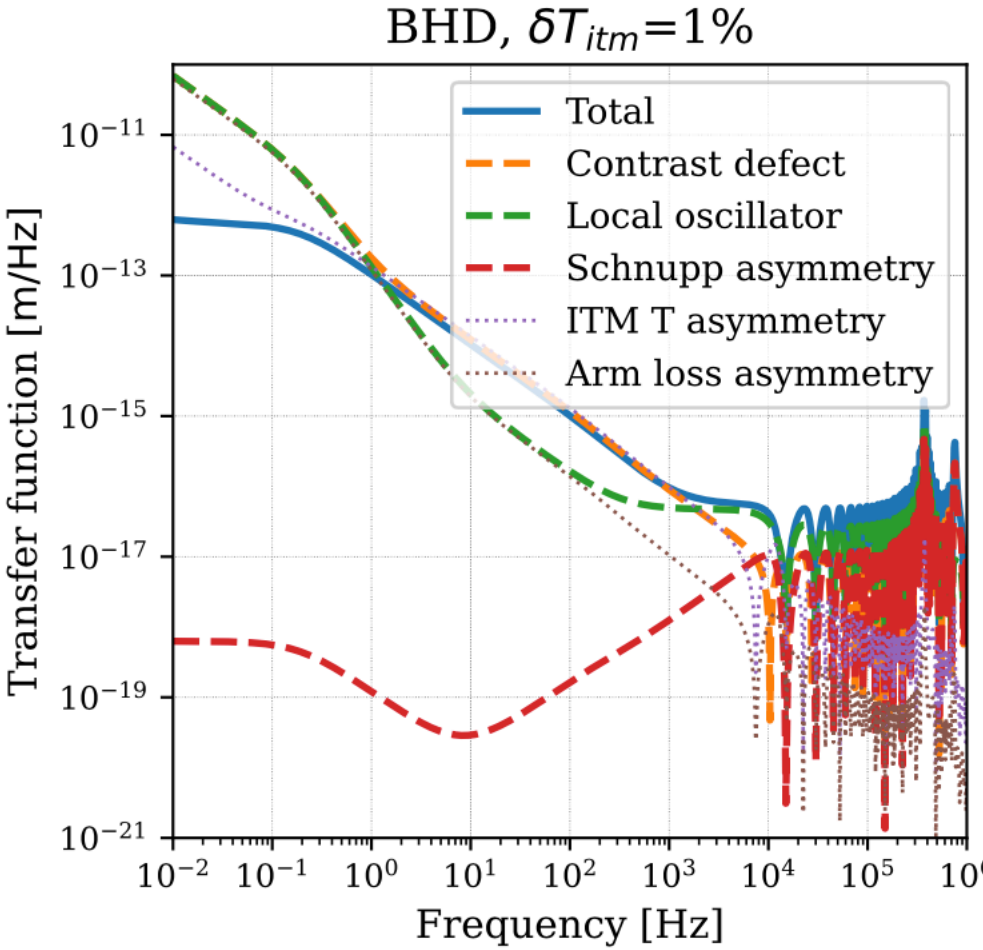


Frequency noise

HF

Calibrated frequency noise in $\frac{\mathrm{m}}{\mathrm{Hz}} = \frac{\mathrm{W}}{\mathrm{Hz}} / \frac{\mathrm{W}}{\mathrm{m}}$

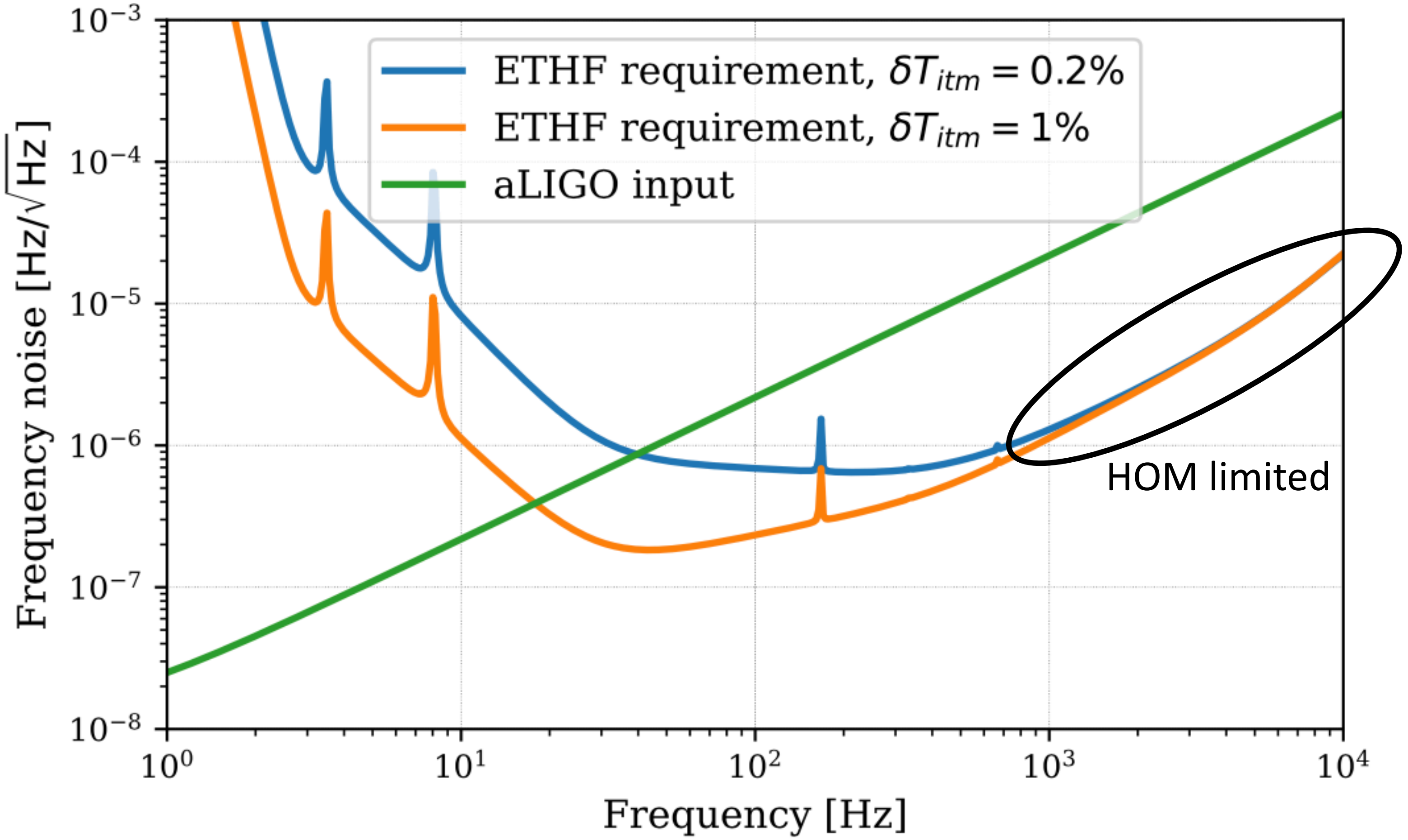




Frequency noise

HF

Requirement



Including a factor of 10 safe margin and 0.4e-15m/Hz constant noise from HOM.

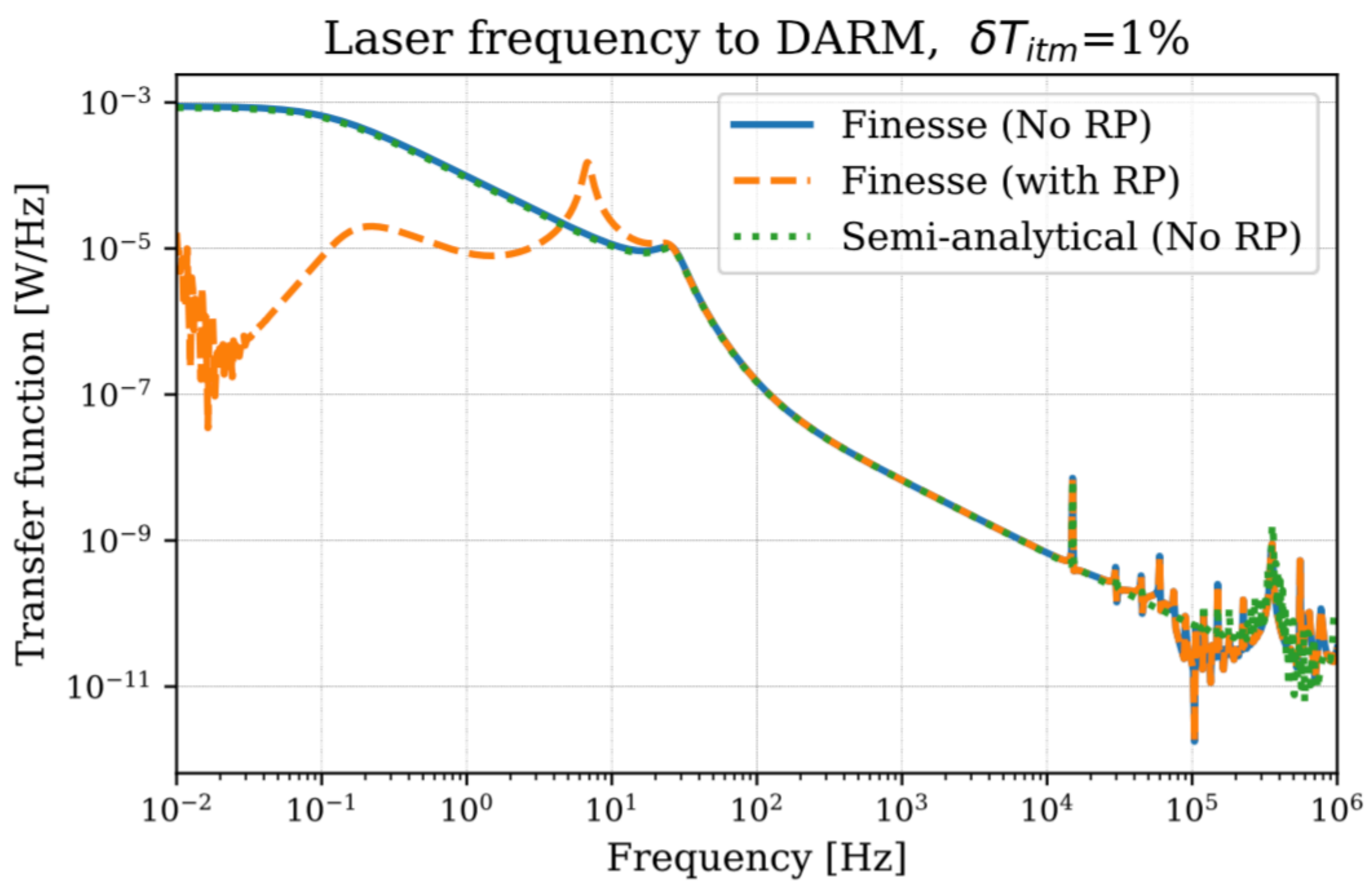
Parameters

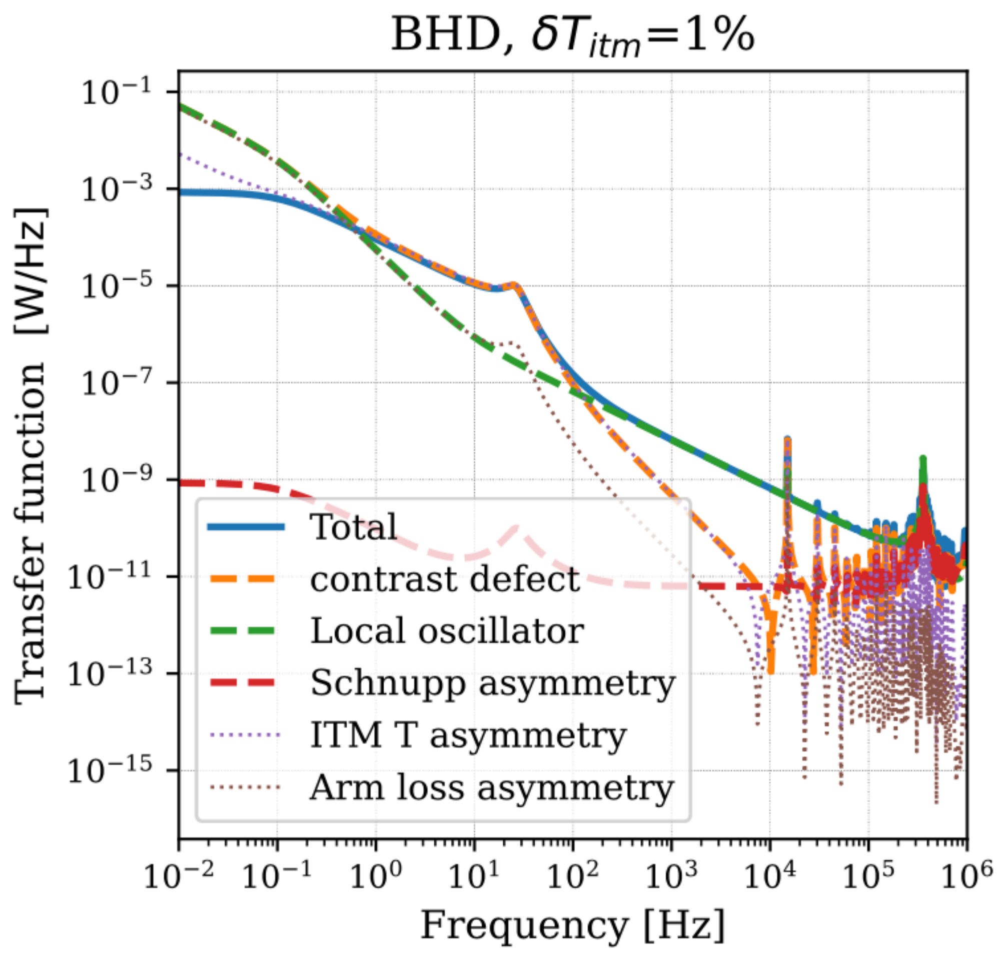
Finesse	
ITM T	0.007
ITM T asymmetry	1%
Mirror loss	20e-6
Loss asymmetry	10%
LO power	10mW
Dark port power	6.34uW

Semi-analytical	
ITM T	0.007
ITM T asymmetry	1%
Mirror loss	20e-6
Loss asymmetry	10%
LO power	10mW
Dark port power	6.37uW

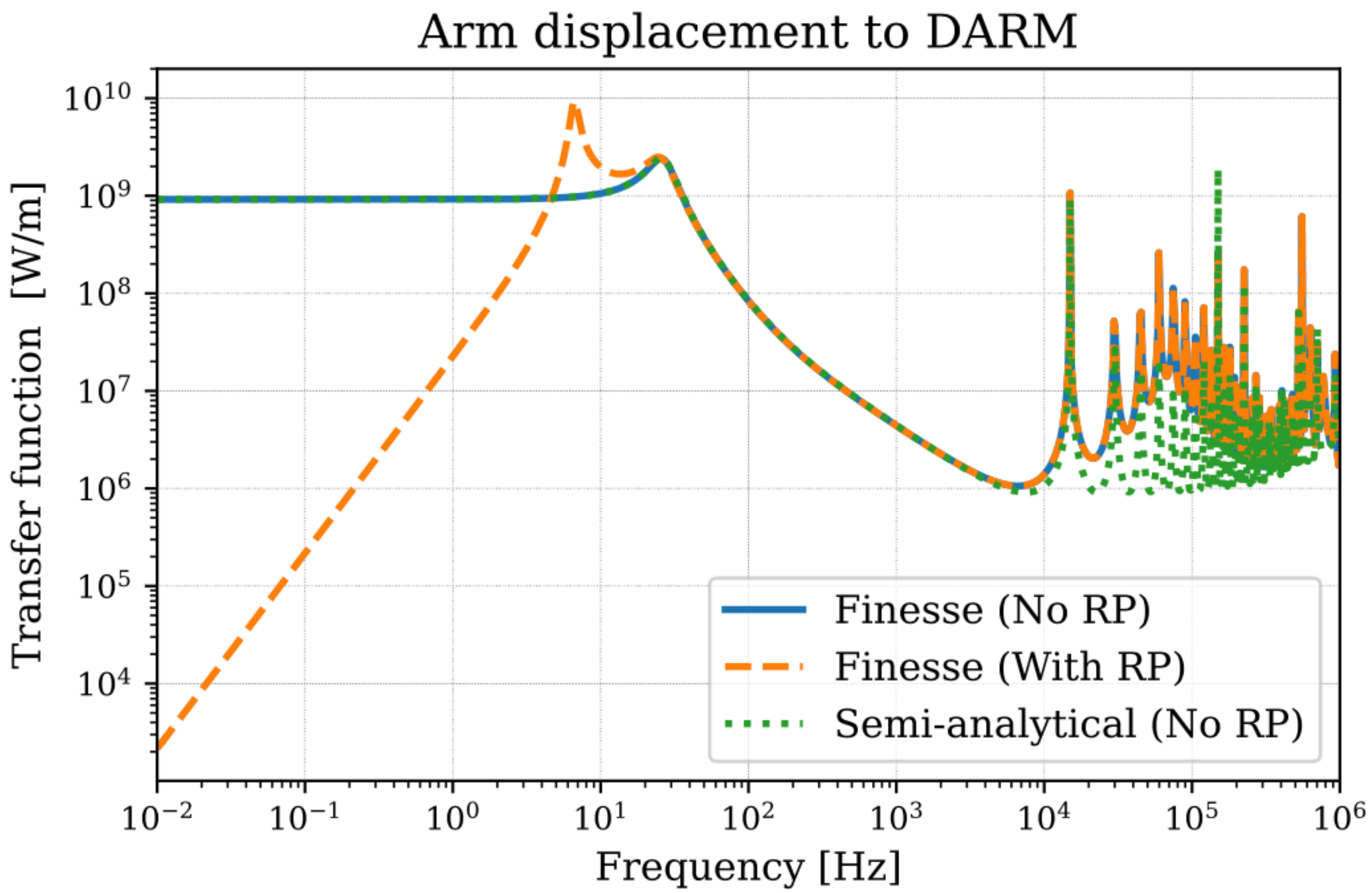
ITM T asymmetry ~0.2% is a promising scenario for ET.

Frequency noise TF in W/Hz





Darm motion TF in W/m

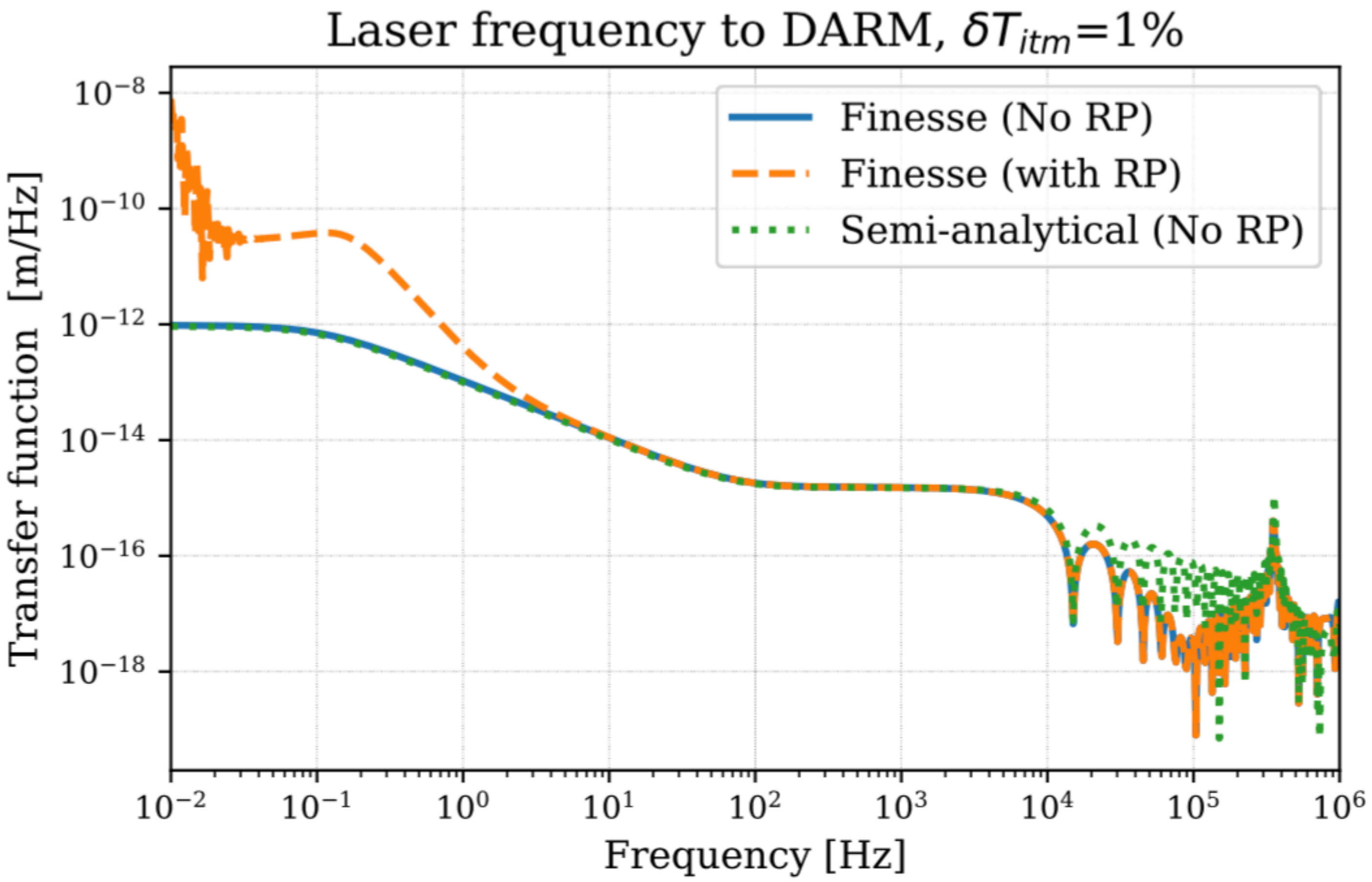


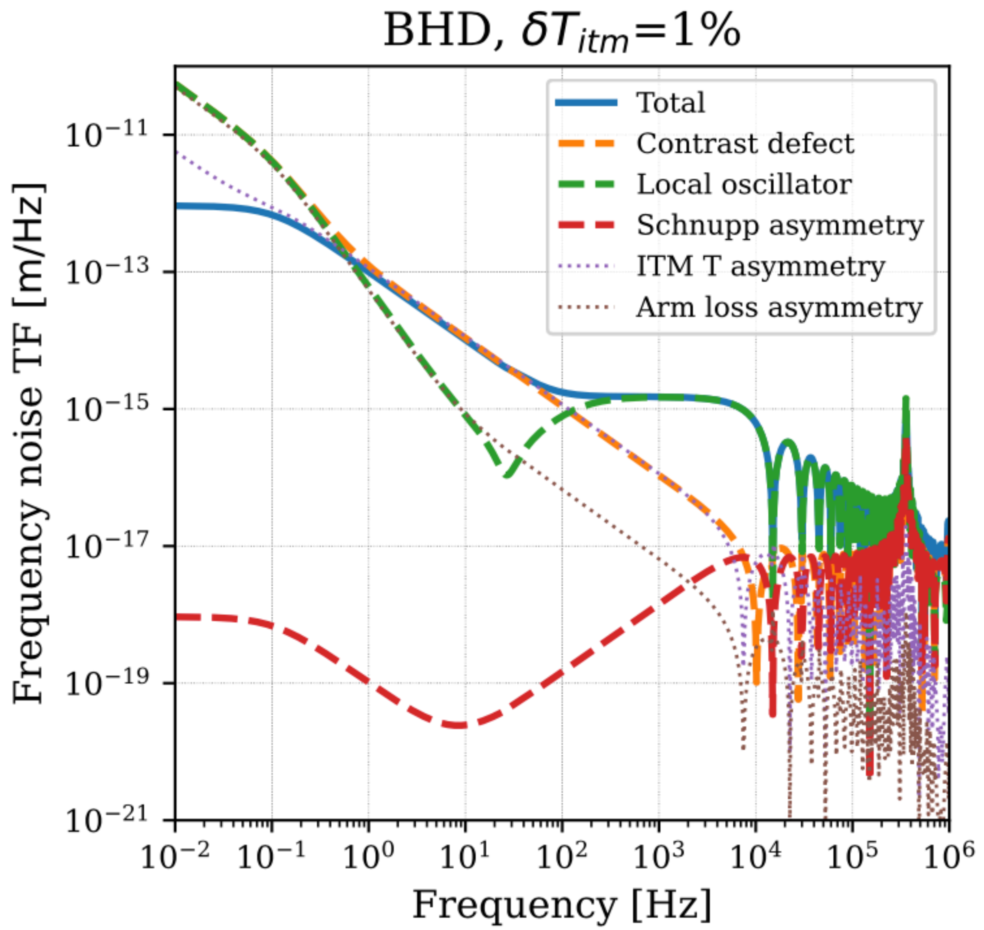
In Finesse, note that to measure the phase quadrature with SRM 34.2° detuned, the LO phase is set to −17.2°.

Frequency noise

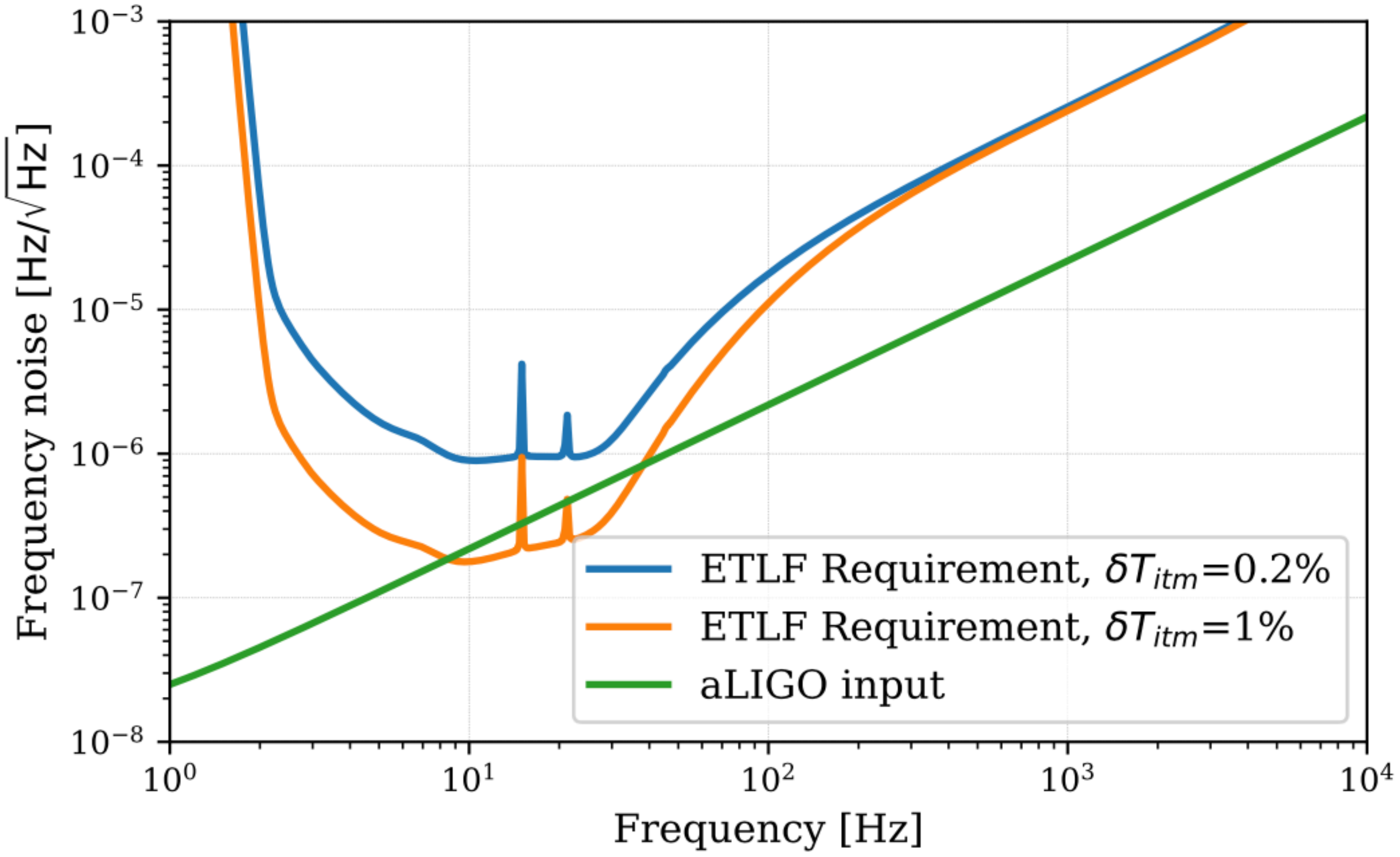
LF

Calibrated frequency noise in $\frac{\mathrm{m}}{\mathrm{Hz}} = \frac{\mathrm{W}}{\mathrm{Hz}} / \frac{\mathrm{W}}{\mathrm{m}}$





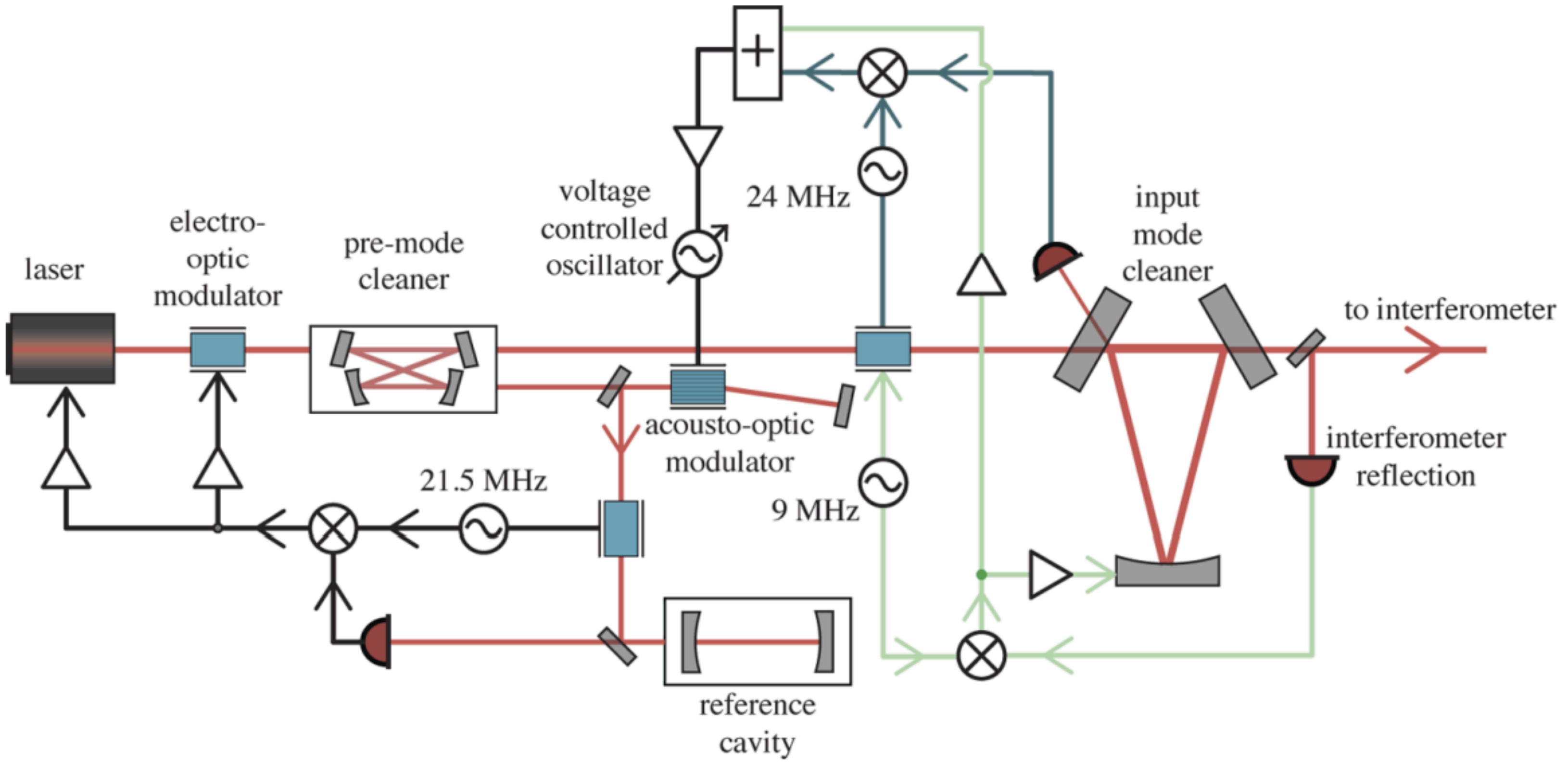
Requirement



Including a factor of 10 safe margin and 0.4e-15m/Hz constant noise from HOM.

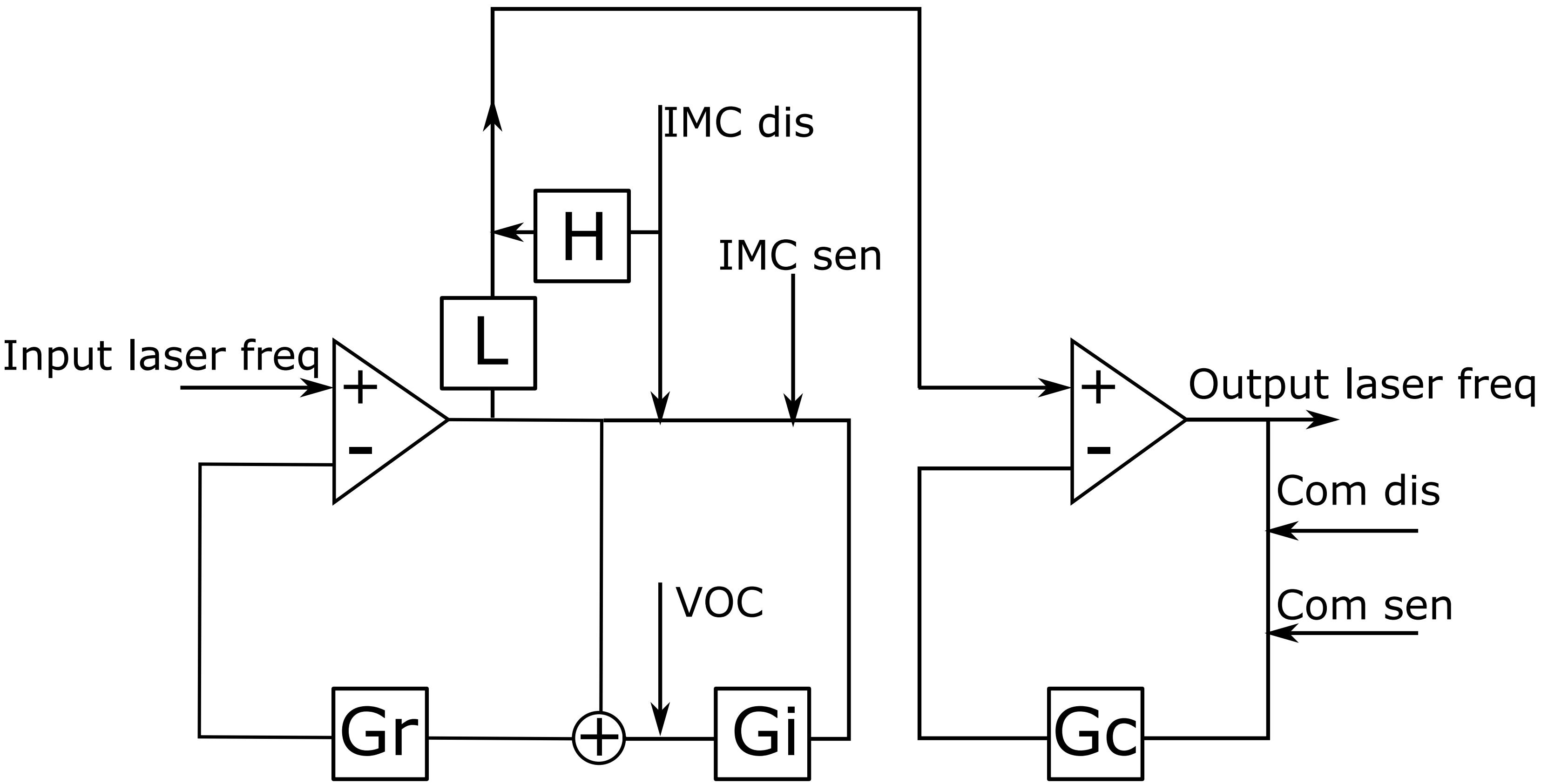
Laser stabilization

Setup in aLIGO

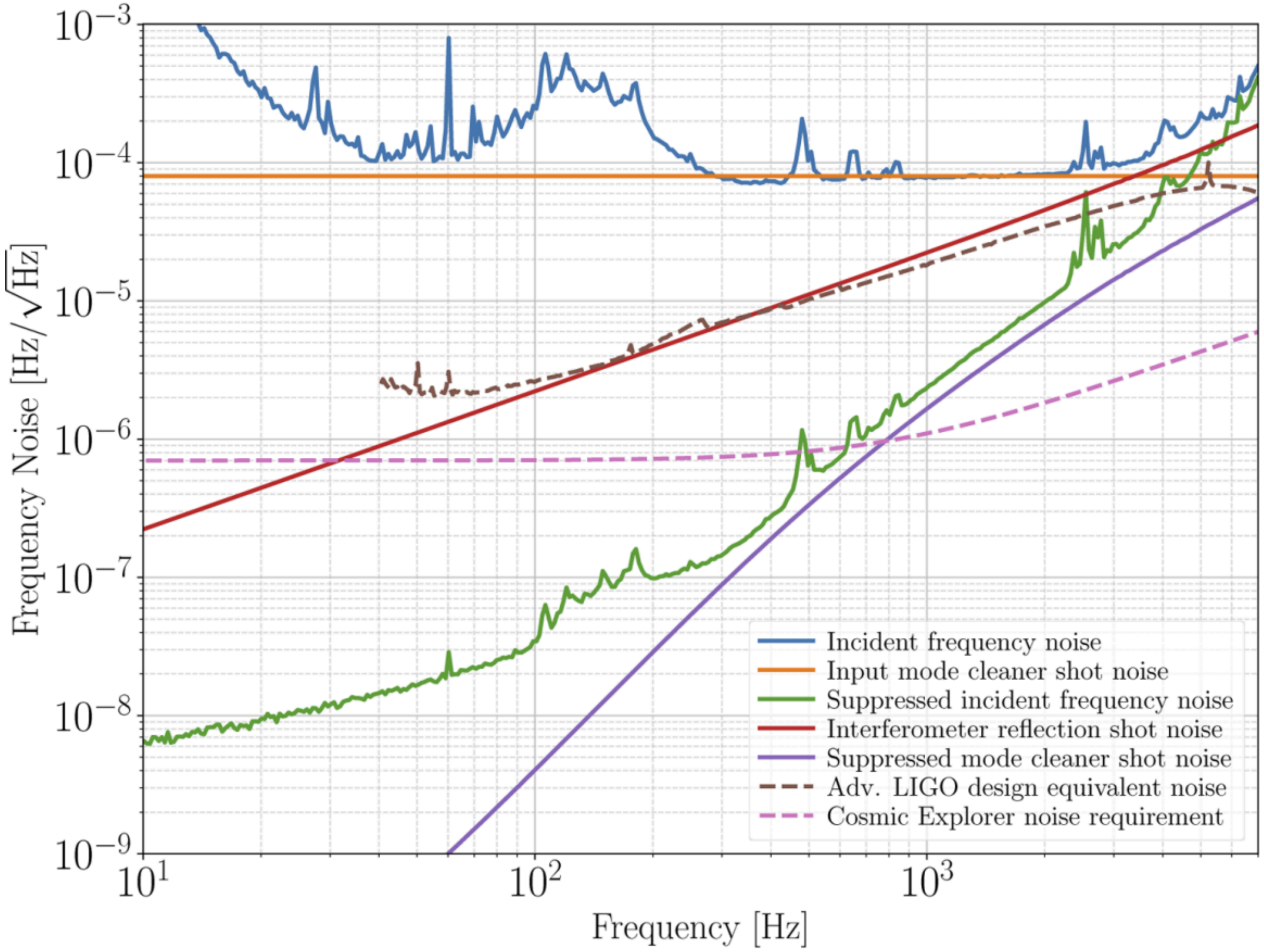


[1]Cahillane, Craig, Georgia L. Mansell, and Daniel Sigg. "Laser frequency noise in next generation gravitational-wave detectors." *Optics Express* 29.25 (2021): 42144-42161.

Laser stabilization



aLIGO performance



- The common mode control bandwidth is 15kHz.
- The coupled PRC and arm cavity pole is at 0.6Hz.
- The input frequency noise is limited by common mode sensing noise.
- The sensing noise is about $2\times10^{-8} f \frac{Hz}{\sqrt{Hz}}$, above 0.6 Hz.

The sensing noise of PDH

To evaluate aLIGO common mode control, I derive the sensing noise under some simplified assumption.

The reflection power from cavity is

$$P_{ref} = P_{in}\left(\frac{\mathrm{m}^2}{2} + \epsilon + \alpha_{\mathrm{HOM}}\right),$$

where m is the modulation index, ϵ is the 00 mode reflection coefficient, α_{HOM} represents the HOM contents at the input. The PD power is

$$P_{PD} = \frac{P_{ref}}{D},$$

Where D is the attenuation factor.
The shot noise is then

$$S_{shot} = \sqrt{\frac{P_{in}}{D}\left(\frac{3}{4}m^2 + \epsilon + \alpha_{\mathrm{HOM}}\right)4\hbar\omega}\ \frac{\mathrm{W}}{\sqrt{\mathrm{Hz}}}.$$

The PDH signal is

$$S_{PDH} = 2m\frac{F}{\mathrm{FSR}}\frac{P_{in}}{D}(1 - \alpha_{\mathrm{HOM}})/\left(1 + \frac{if}{f_{pole}}\right)\frac{\mathrm{W}}{\mathrm{Hz}}.$$

The sensing noise of PDH

The sensing noise in Hz is then

$$S_{sen} = \frac{\sqrt{\frac{m^2}{2} + \epsilon + \alpha_{\mathrm{HOM}}}}{1 - \alpha_{\mathrm{HOM}}} \frac{\mathrm{FSR}}{F} \sqrt{\frac{\left(\frac{3}{2} + \frac{2\epsilon}{m^2} + \frac{2\alpha_{\mathrm{HOM}}}{m^2}\right)\hbar\omega}{2\,P_{PD}}} \left(1 + \frac{if}{f_{pole}}\right) \frac{\mathrm{Hz}}{\sqrt{\mathrm{Hz}}}.$$

In aLIGO, there is $\mathrm{m} \approx 0.18$, $\epsilon = 1.6\%$, $P_{PD} = 25mW$, this gives $\alpha_{\mathrm{HOM}} \sim 0.39$.

We can do better by filtering the HOM contents!

The sensing noise of PDH

In ET, I assume $\epsilon = 1\%$, and $m = 0.14$ to give to RF power equals to the DC reflection.
Thus the sensing noise simplified to

$$S_{sen} = \left(1 + \frac{if}{f_{pole}}\right) \frac{\sqrt{2\% + \alpha_{\mathrm{HOM}} * R}}{1 - \alpha_{\mathrm{HOM}}} \frac{\mathrm{FSR}}{F} \sqrt{\frac{\left(\frac{5}{2} + 100\alpha_{\mathrm{HOM}} * R\right)\hbar\omega}{2\, P_{PD}}},$$

Where R represents the HOMs filtering at the reflection. Assuming $\alpha_{\mathrm{HOM}} = 0.1$, depending on the filtering, there is

$$S_{sen} = \left(1 + \frac{if}{f_{pole}}\right) \frac{10}{9} \frac{\mathrm{FSR}}{F} \sqrt{\frac{5\hbar\omega}{4P_{PD}}} \times (0.14 \sim 0.76)$$

I take the number **0.15** meaning filtering 98.5% HOMs, equivalent to 1W effective power.

The sensing noise of PDH

At the reflection of IMC, there must be more HOM content. For instance, assuming $\alpha_{\mathrm{HOM}} = 0.8$, depending on the filtering, there is

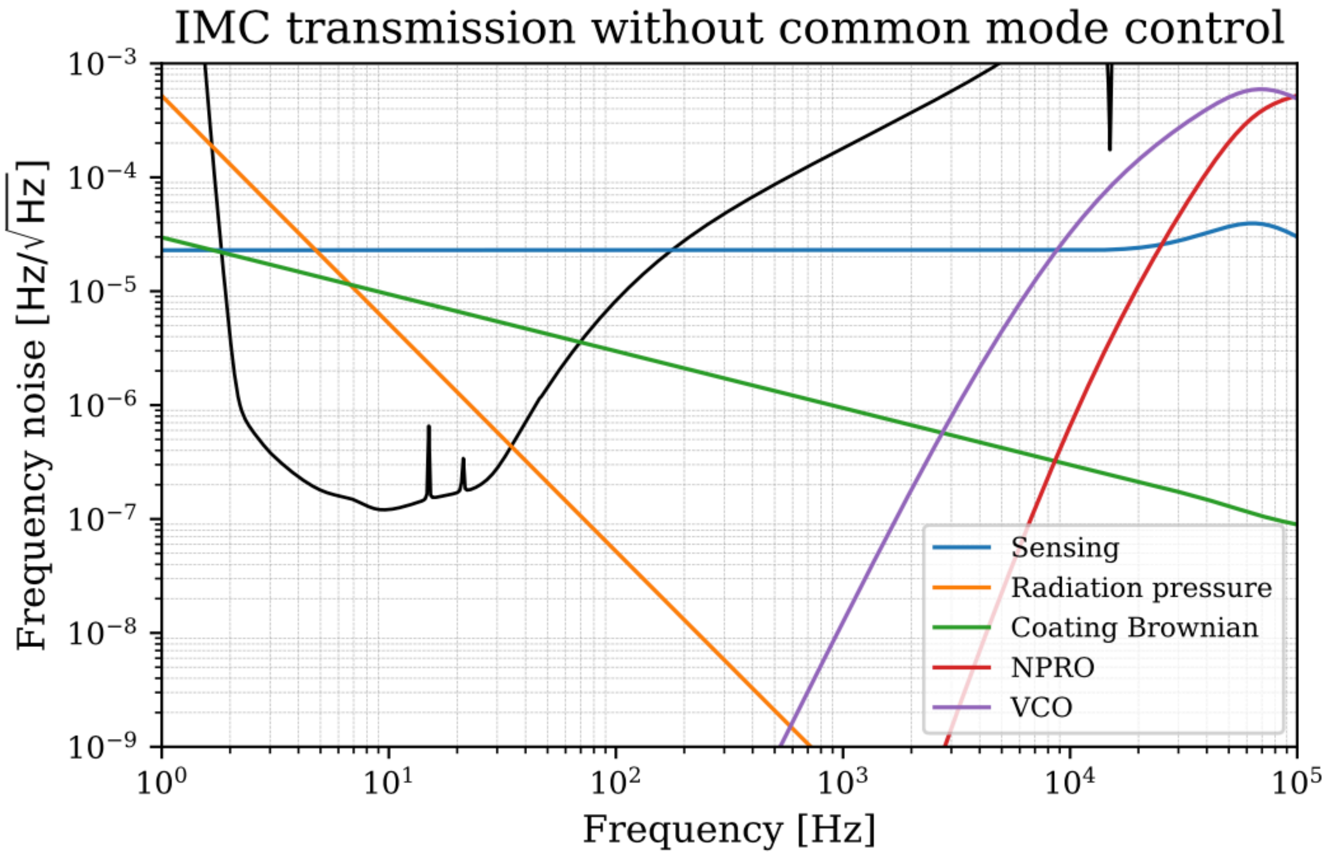
$$S_{sen} = \left(1 + \frac{if}{f_{pole}}\right)\frac{10}{9}\frac{\mathrm{FSR}}{F}\sqrt{\frac{5\hbar\omega}{4P_{PD}}} \times (0.126 \sim 5.8)$$

I take the number 0.2**,** meaning filtering 98.5% HOMs, equivalent to 0.62W effective power.

Laser stabilization of LF

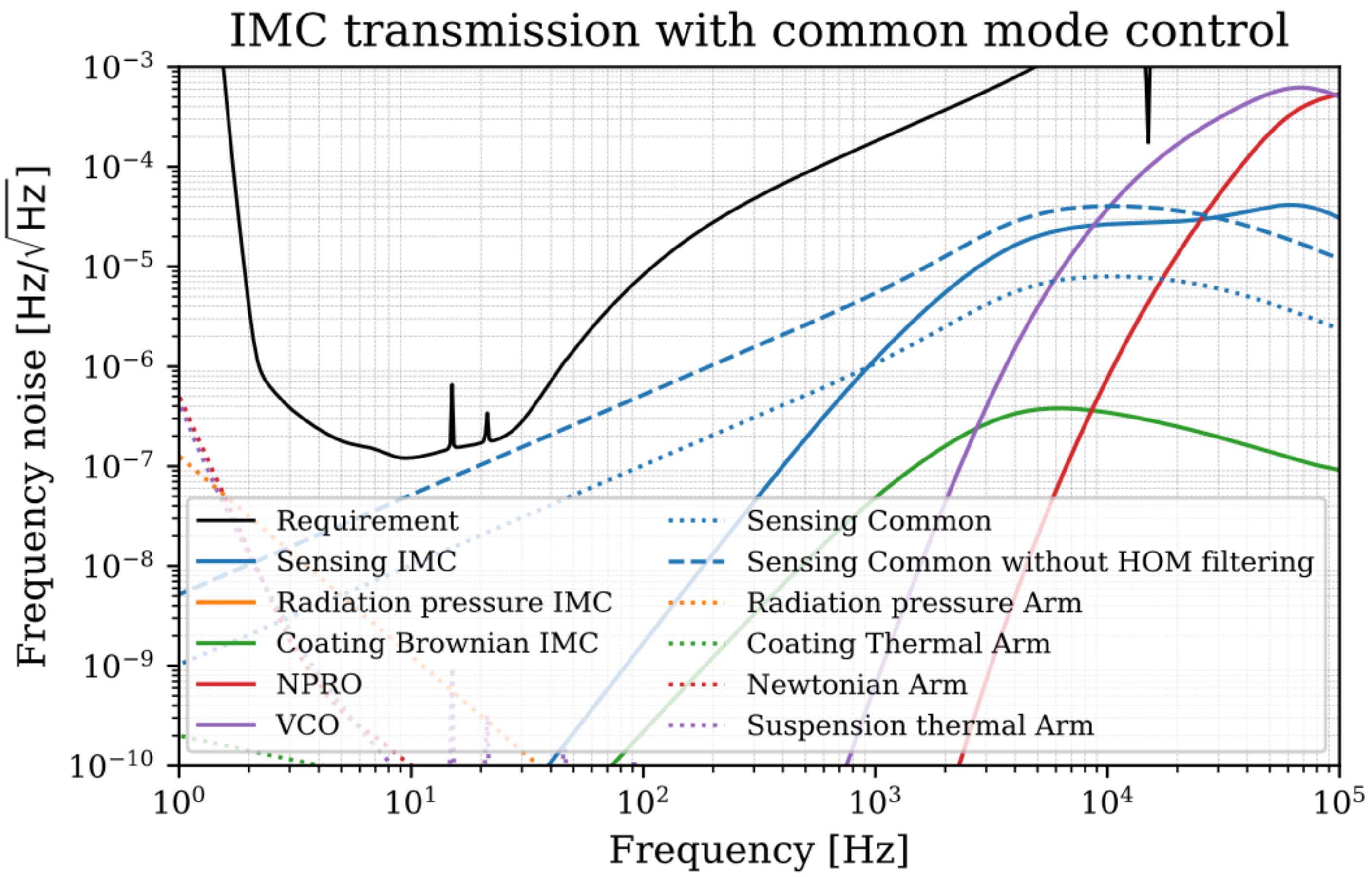
For LF, the requirement for IMC noise is not stringent.

IMC length: 20m IMC gain bandwidth: 100kHz. IMC Finesse: 1000 Reference cavity gain bandwidth: 500kHz



Laser stabilization

The noise can be suppressed with common mode control. The common gain bandwidth is chosen as 7kHz



Laser stabilization

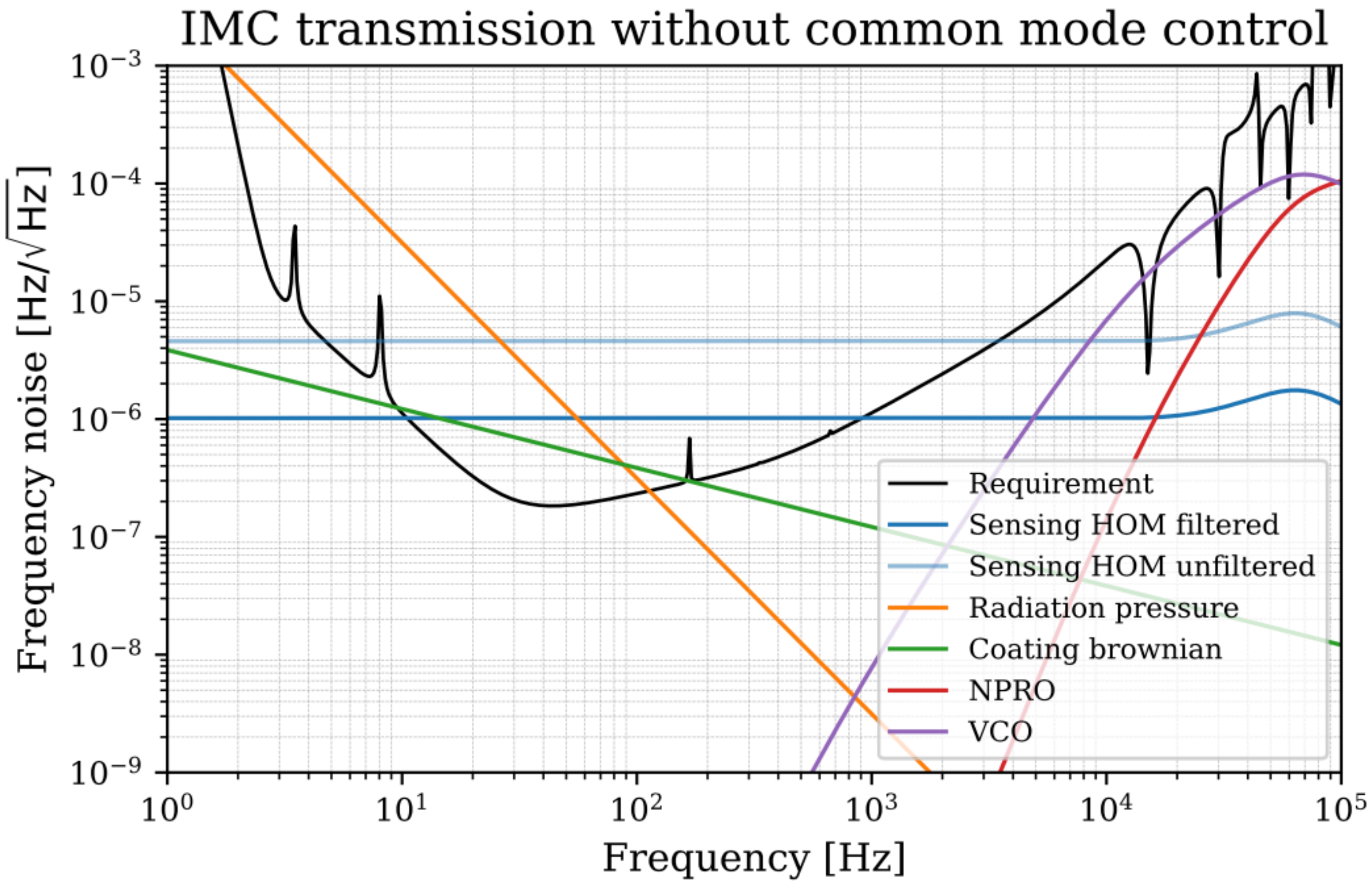
The HF has stringent requirement at high frequencies.

Here two adjustments are made:

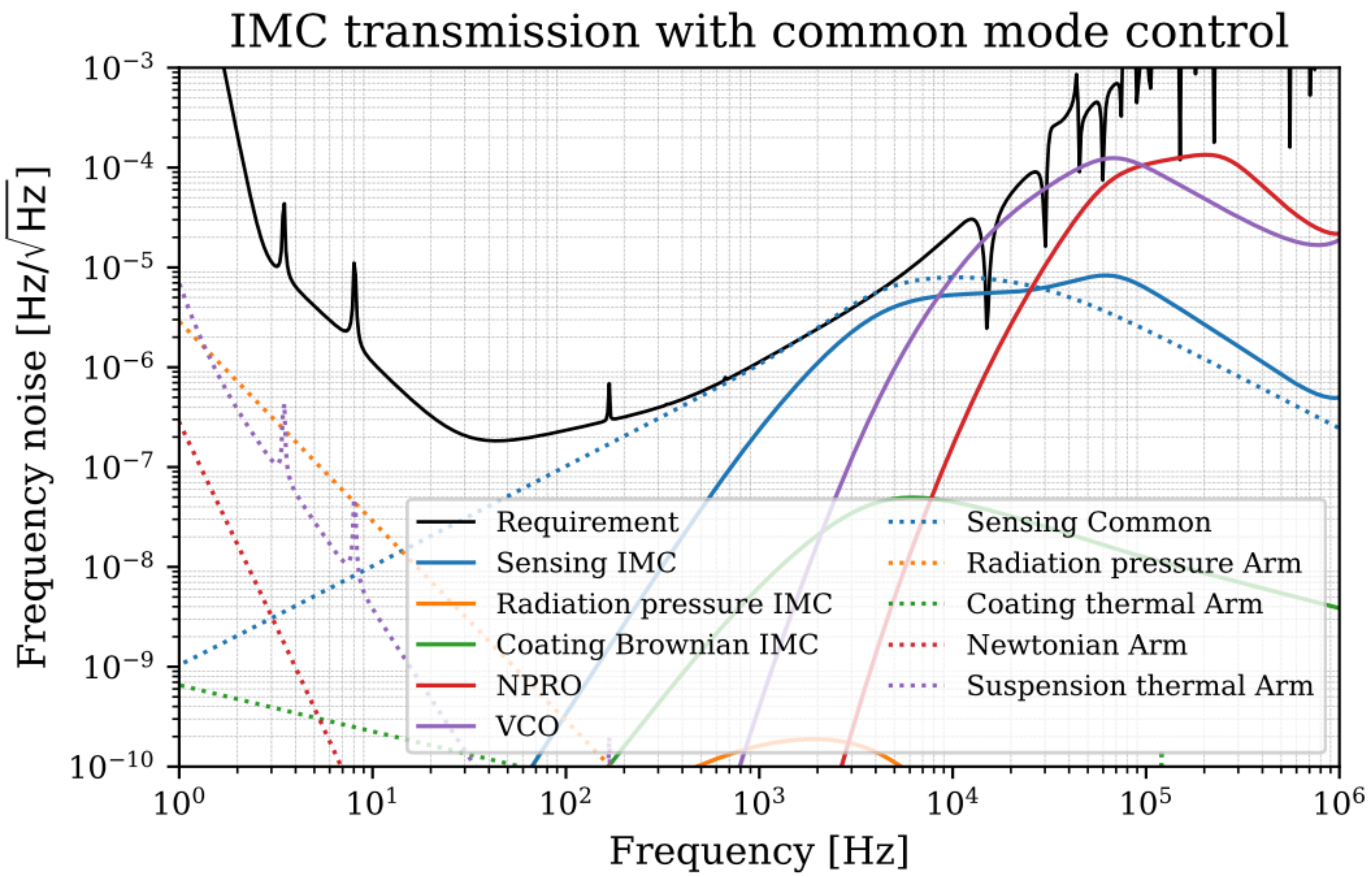
1. Increase the IMC length to reduce the IMC noise.
2. Filter the HOM at IMC reflection, assuming the sensing noise suppression factor 0.2 .

IMC length: 100m   IMC gain bandwidth: 100kHz. Common gain bandwidth: 7kHz
IMC Finesse: 1000   Reference cavity gain bandwidth: 500kHz

Laser stabilization

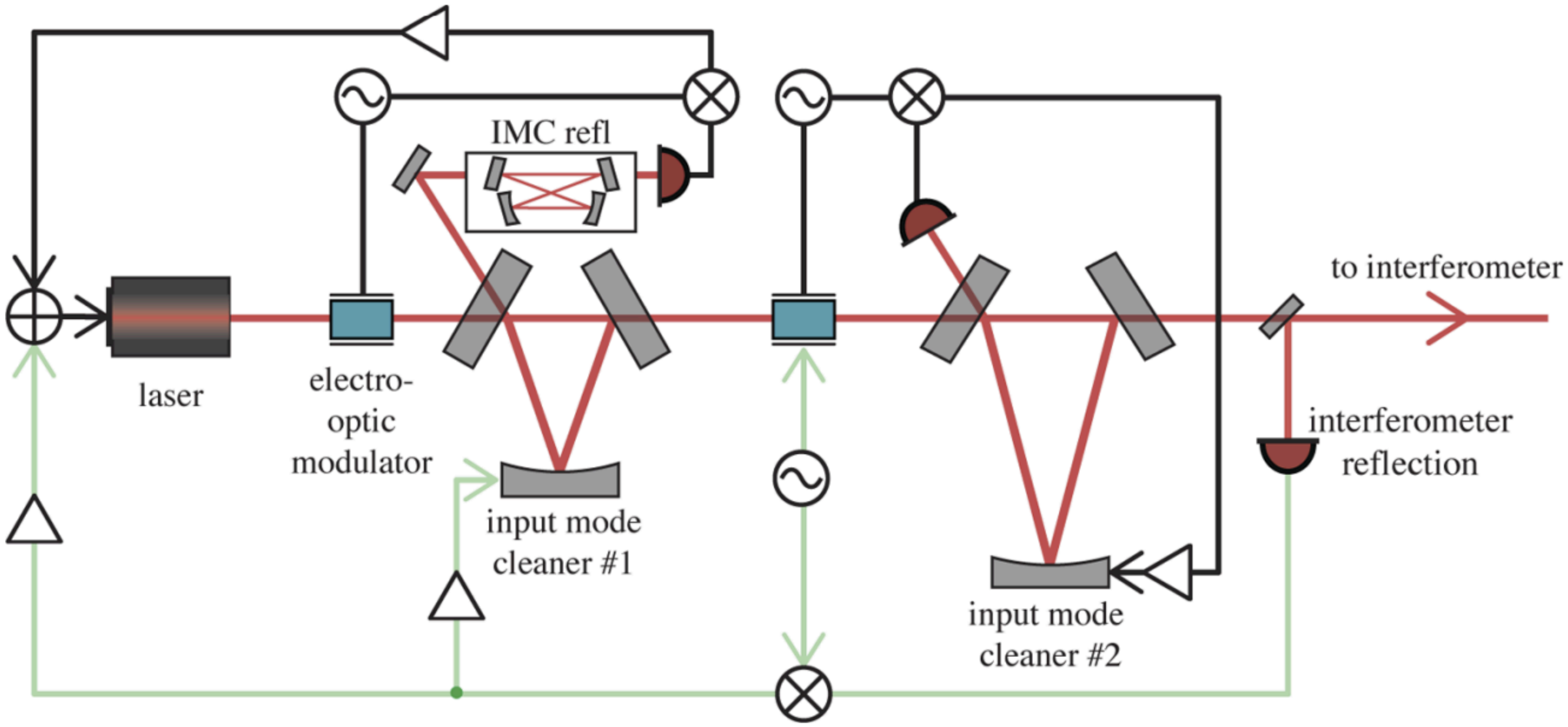


Laser stabilization

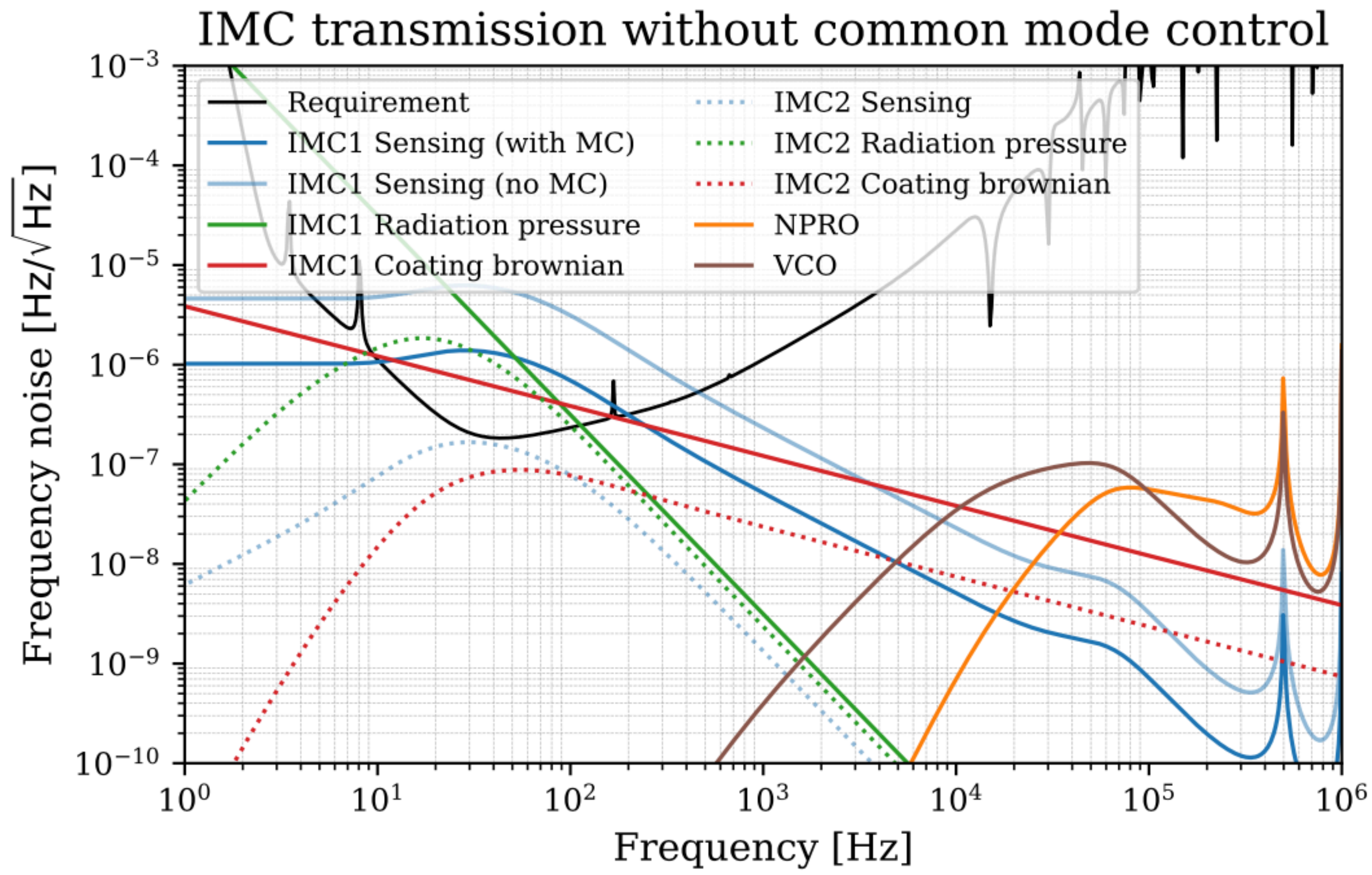


Laser stabilization

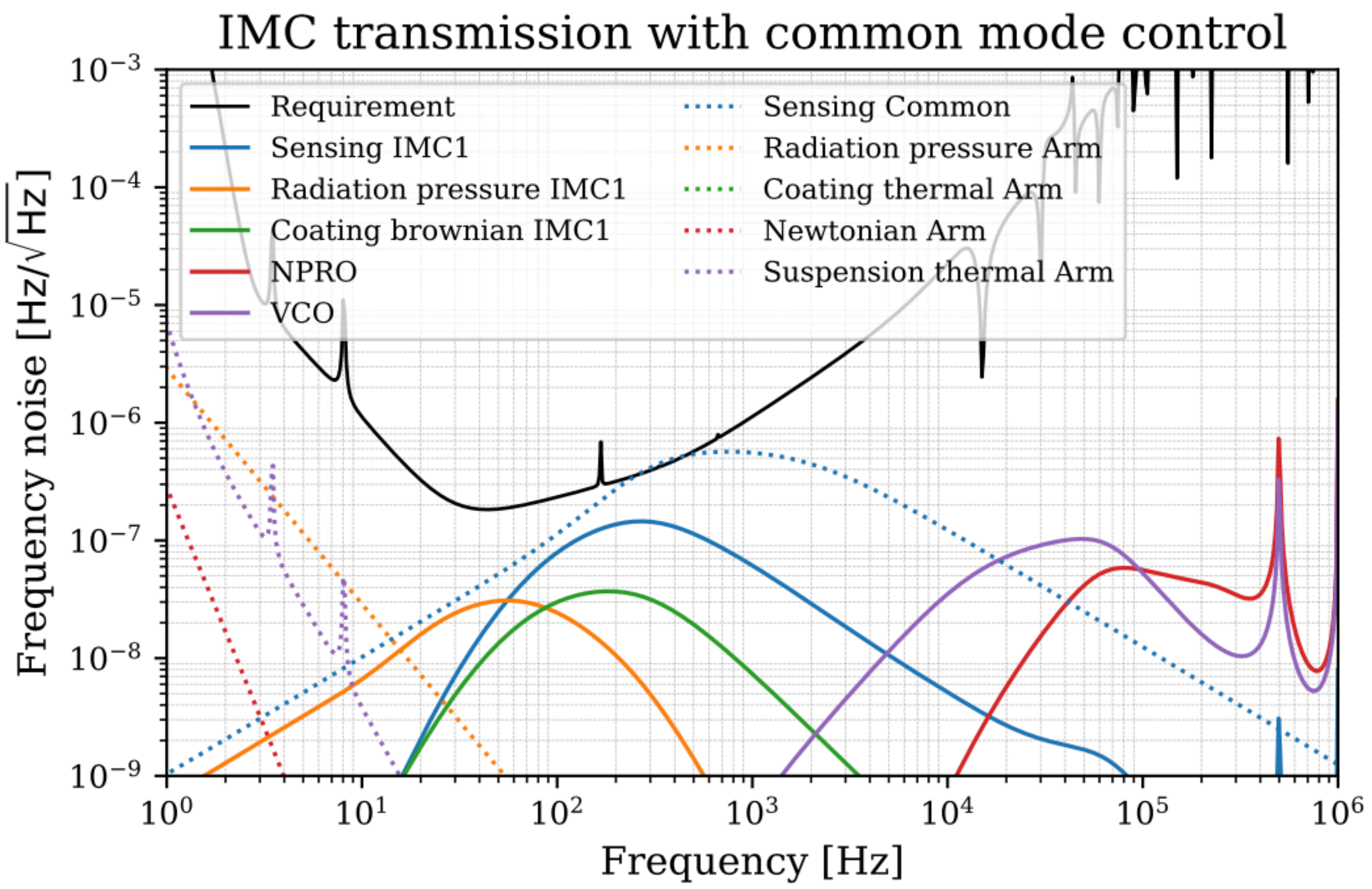
The requirement can be marginally satisfied with before assumptions. There is still risk in terms of the HOM content and filter effectiveness. Here I also consider a second mode cleaner as a passive filter for high frequency noise.



Laser stabilization



Laser stabilization



Summary

- The HF detetcor has a more strignet requirement on the frequency noise pre-stabilisation.
- >=100m IMC,
- Enhancing the IMC sensing noise is desired (filtering HOMs).
- Enhancing the common mode sensing noise is desired. (filtering HOMs)
- A second mode cleaner can help to filter the high frequency IMC noise and satisfy more strigent requirement if there is stronger HOM coupling.

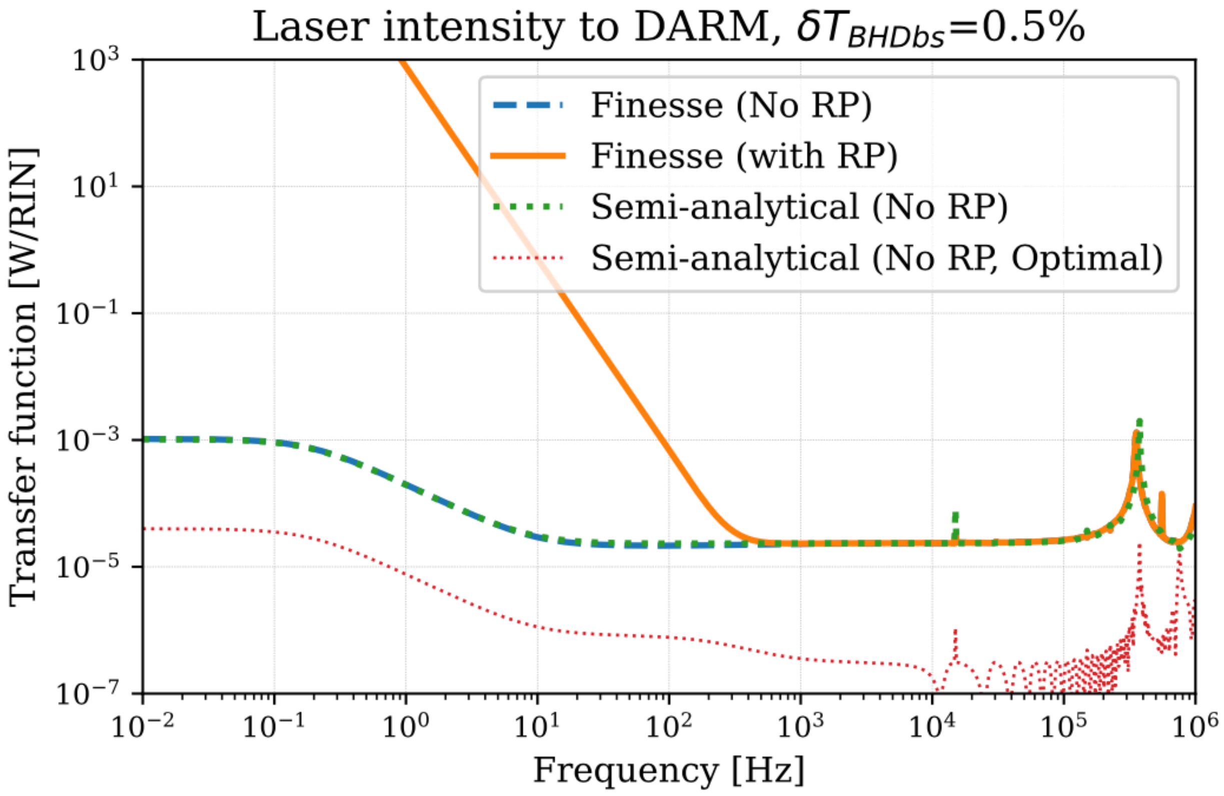
Parameters

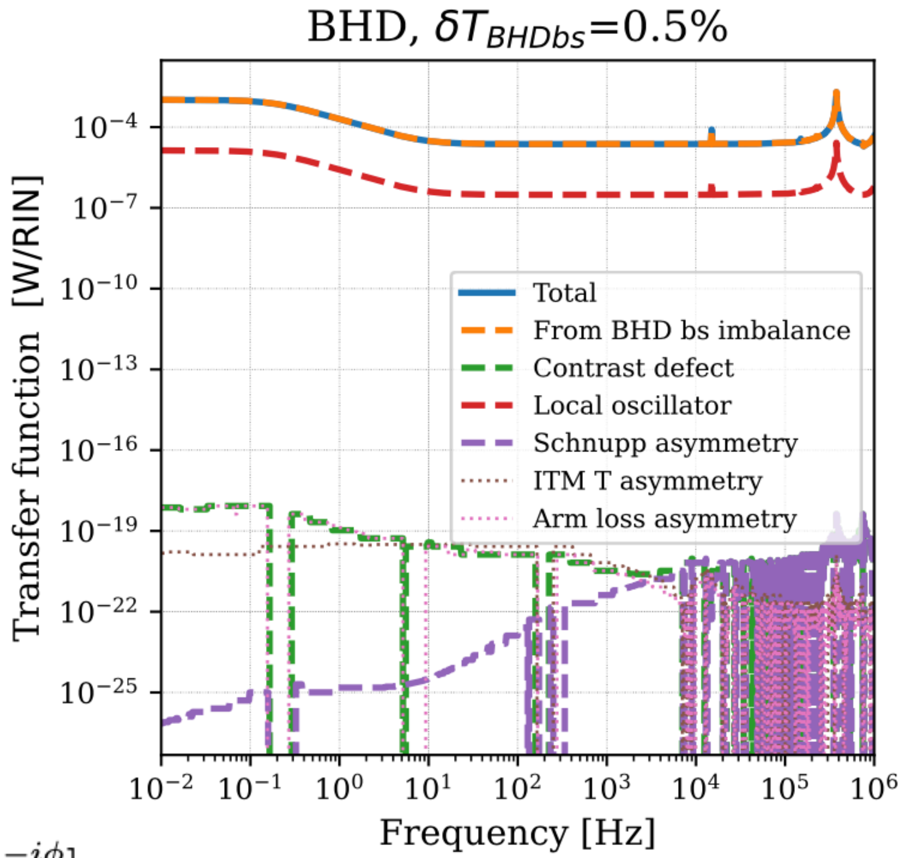
Finesse	
ITM T	0.007
ITM T asymmetry	1%
Mirror loss	20e-6
Loss asymmetry	10%
BHD BS asymmetry	0.5%
LO power	100mW
Dark port power	1.41mW

Semi-analytical	
ITM T	0.007
ITM T asymmetry	1%
Mirror loss	37.5e-6
Loss asymmetry	10%
BHD BS asymmetry	0.5%
LO power	100mW
Dark port power	1.56mW

The LO static field (on phase quadrature) is orthogonal to the amplitude noise in signal beam. Here a imperfection on BHD beamsplitter can introduce the noise from LO itself. : LO static field x the local noise.

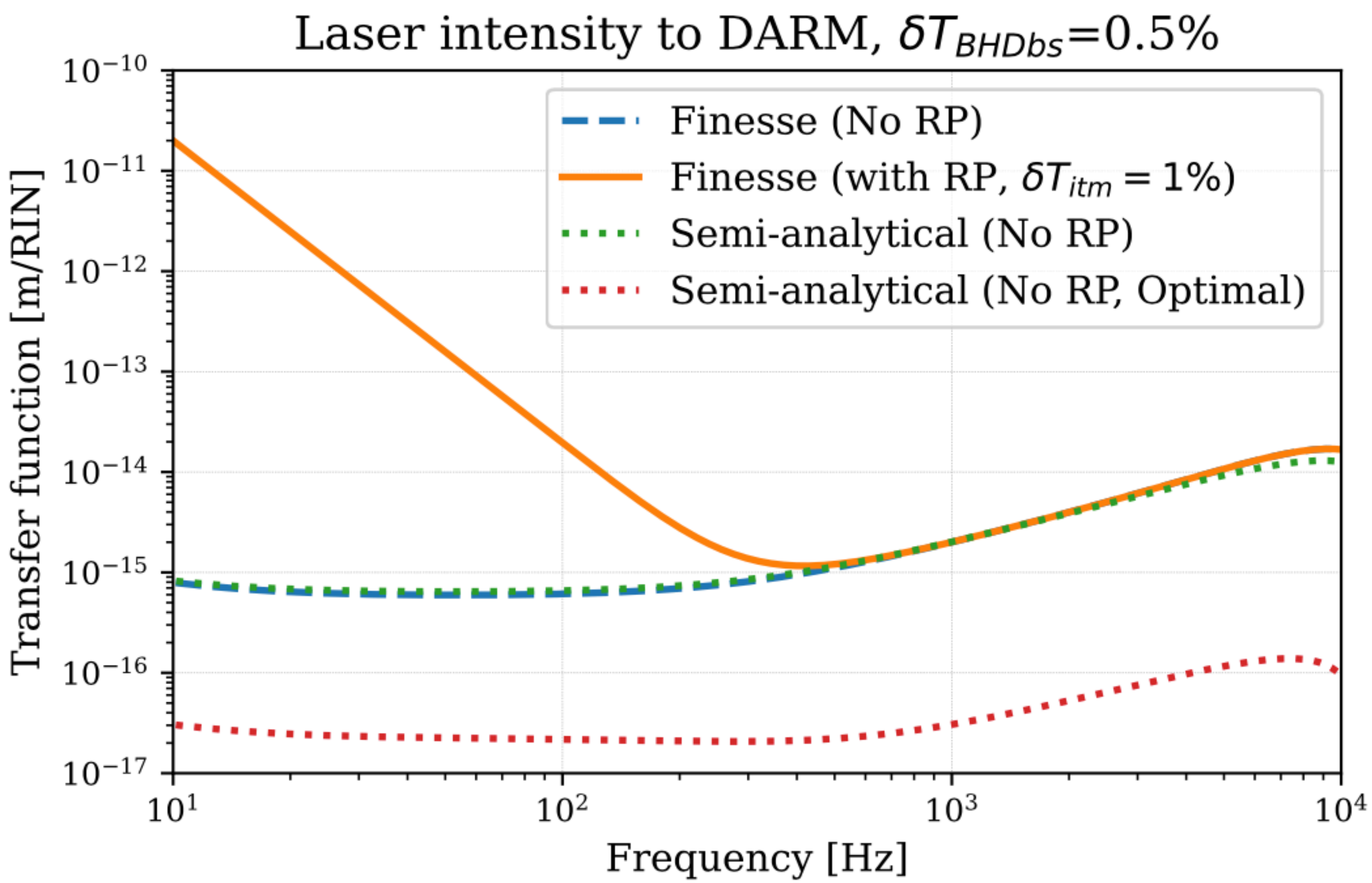
Intensity noise TF in W/RIN

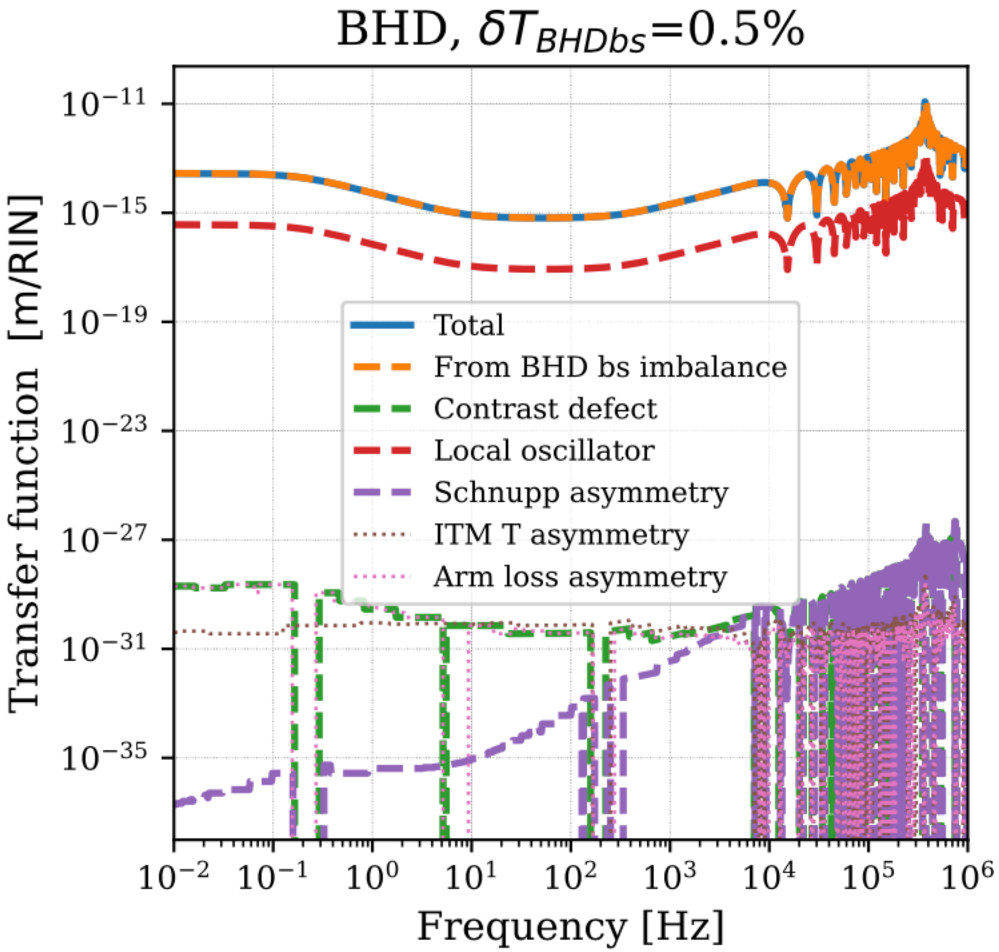




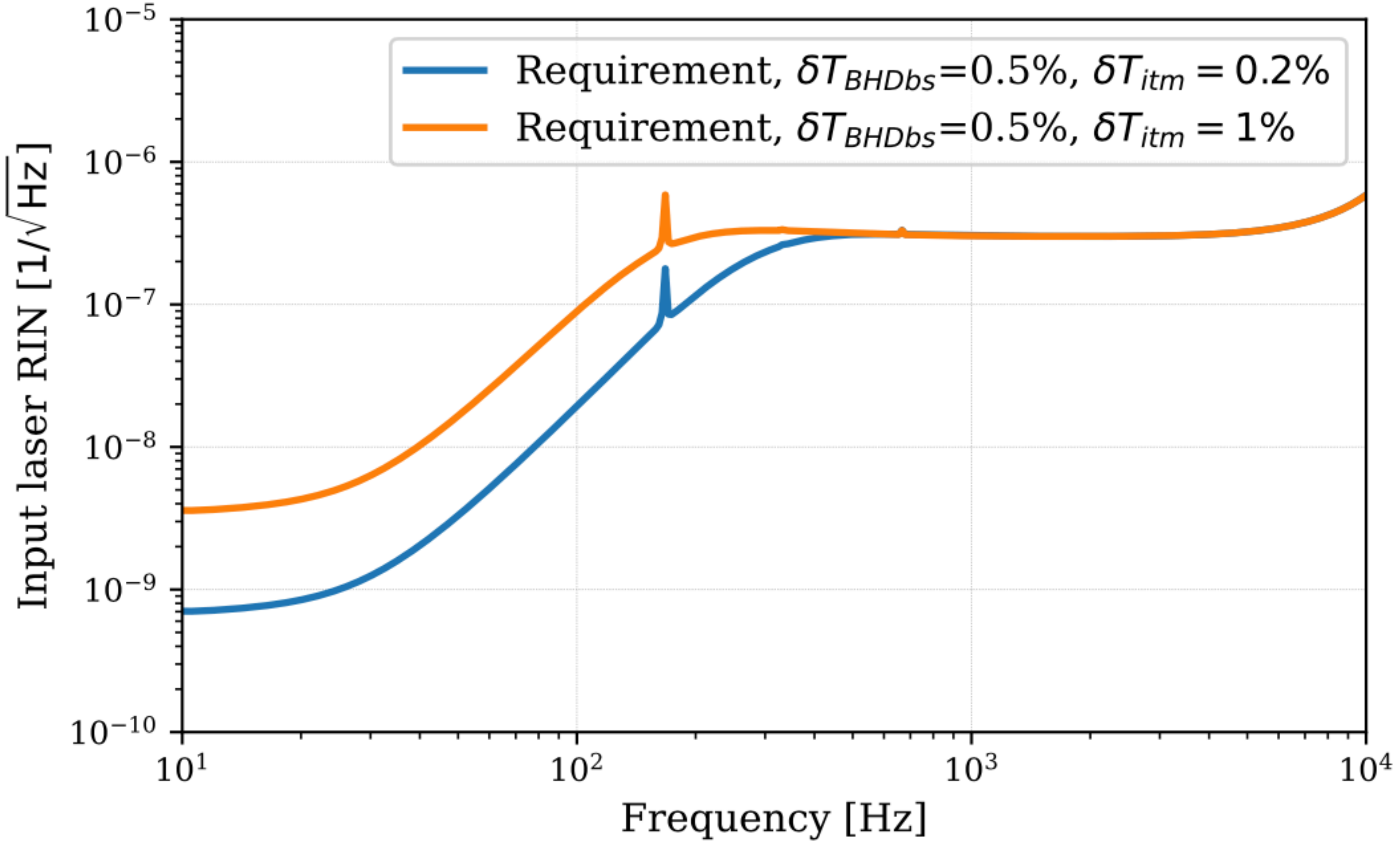
$$I(t) = (2-\epsilon)t_b r_b[L_0^* s(t)e^{-i\phi} + L_0 s(t)^\dagger e^{i\phi} + S_0^* l(t)e^{i\phi} + S_0 l(t)^\dagger e^{-i\phi}]$$
$$+\cancel{(t_b^2 - r_b^2 + \epsilon r_b^2)(L_0^* l(t) + L_0 l(t)^\dagger)} + (r_b^2 - t_b^2 + \epsilon t_b^2)(S_0^* s(t) + S_0 s(t)^\dagger)$$

Intensity noise TF in m/RIN





Requirement



Including a factor of 10 safe margin.

Parameters

Finesse	
ITM T	0.007
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LO power	10mW
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ITM T	0.007
ITM T asymmetry	1%
Mirror loss	20e-6
Loss asymmetry	10%
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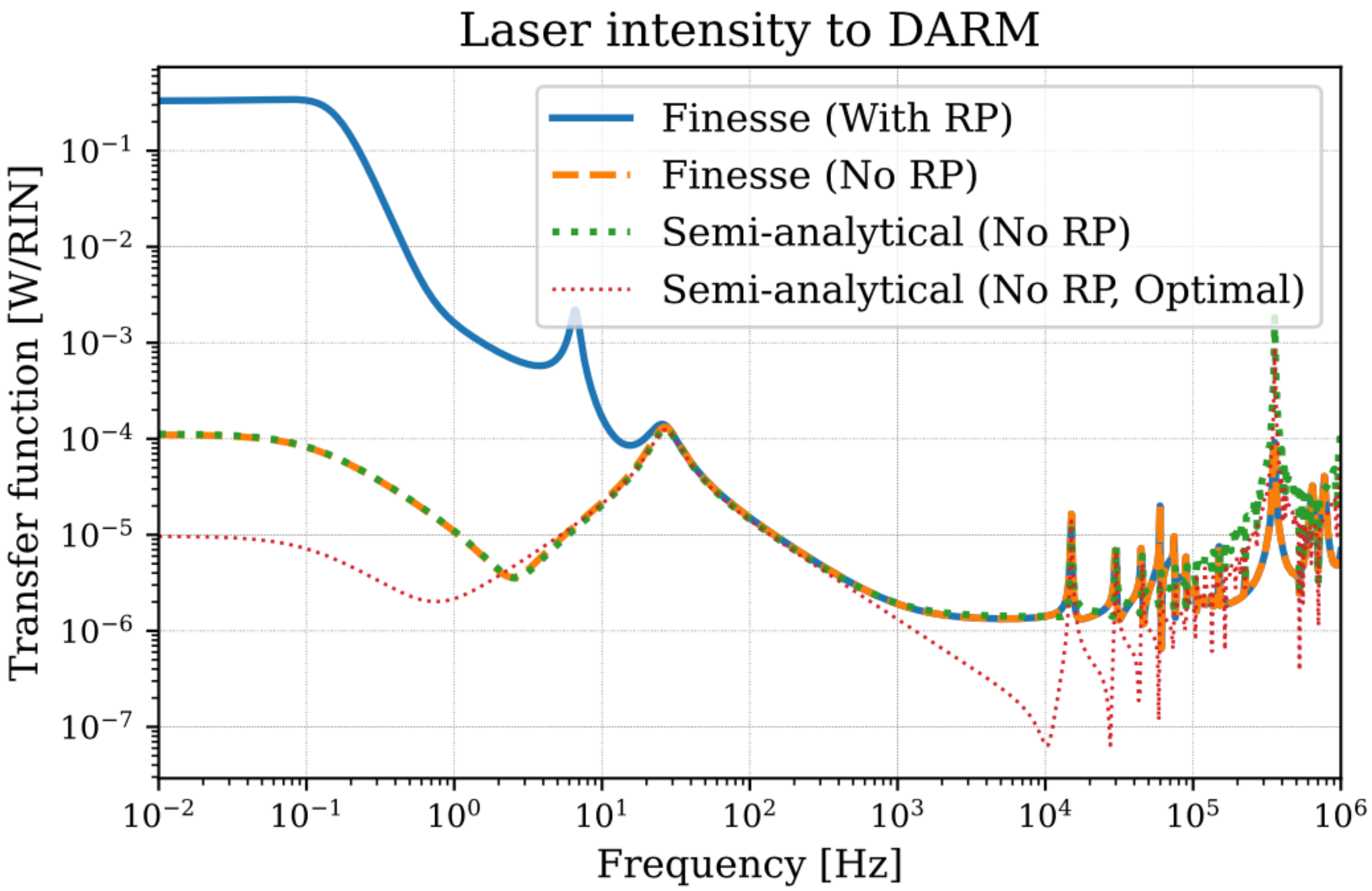
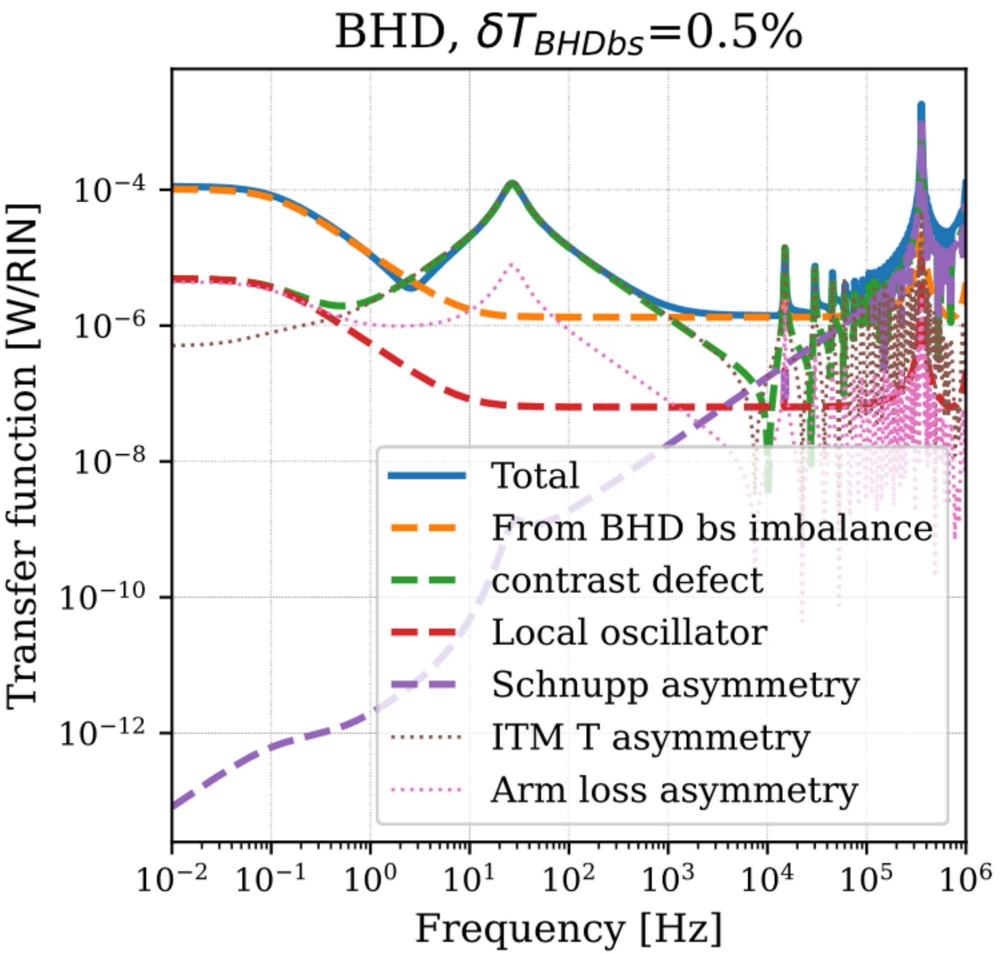
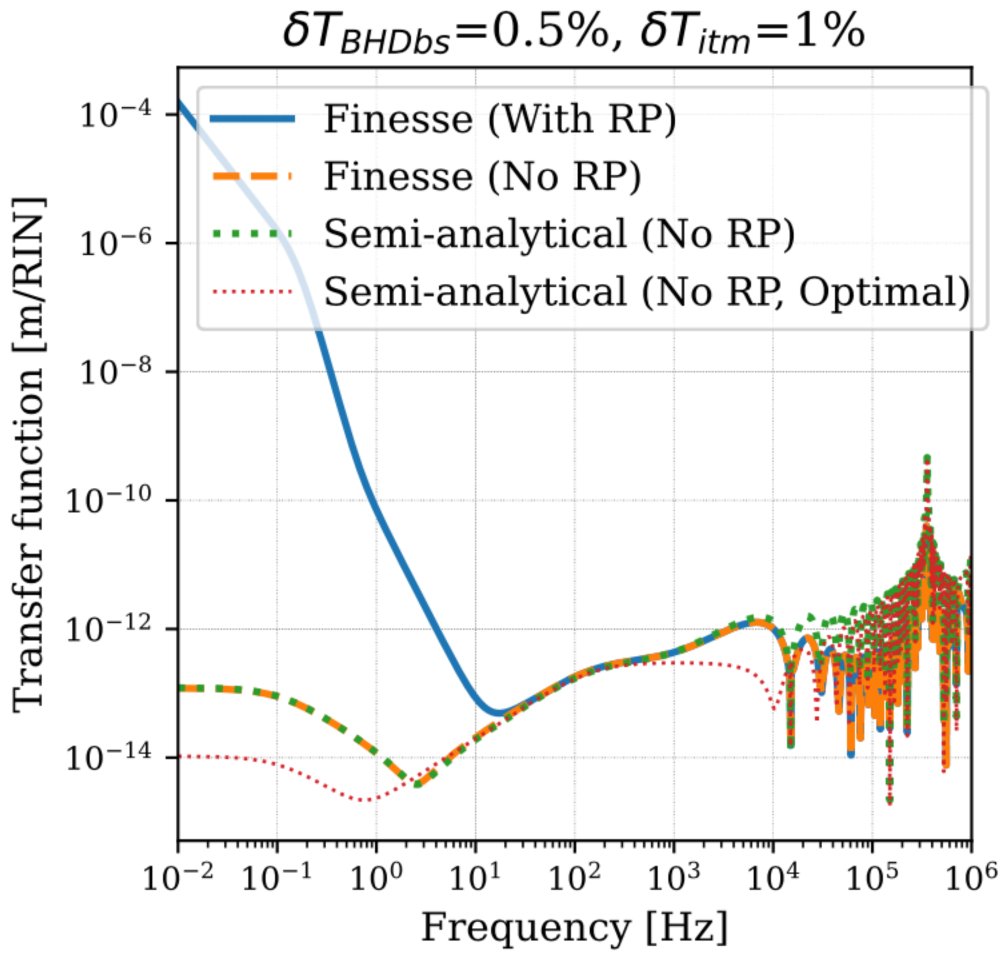
LO power	10mW
Dark port power	6.37uW

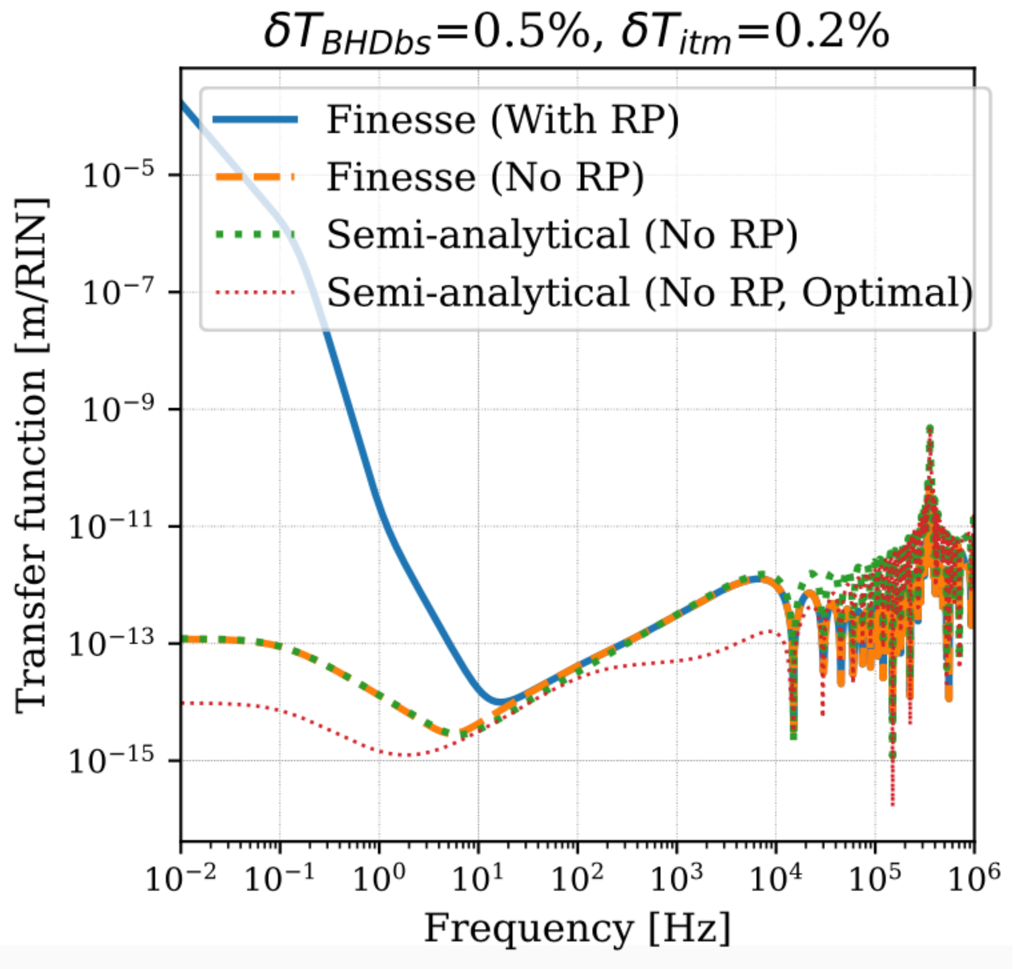
Intensity noise TF in W/Hz



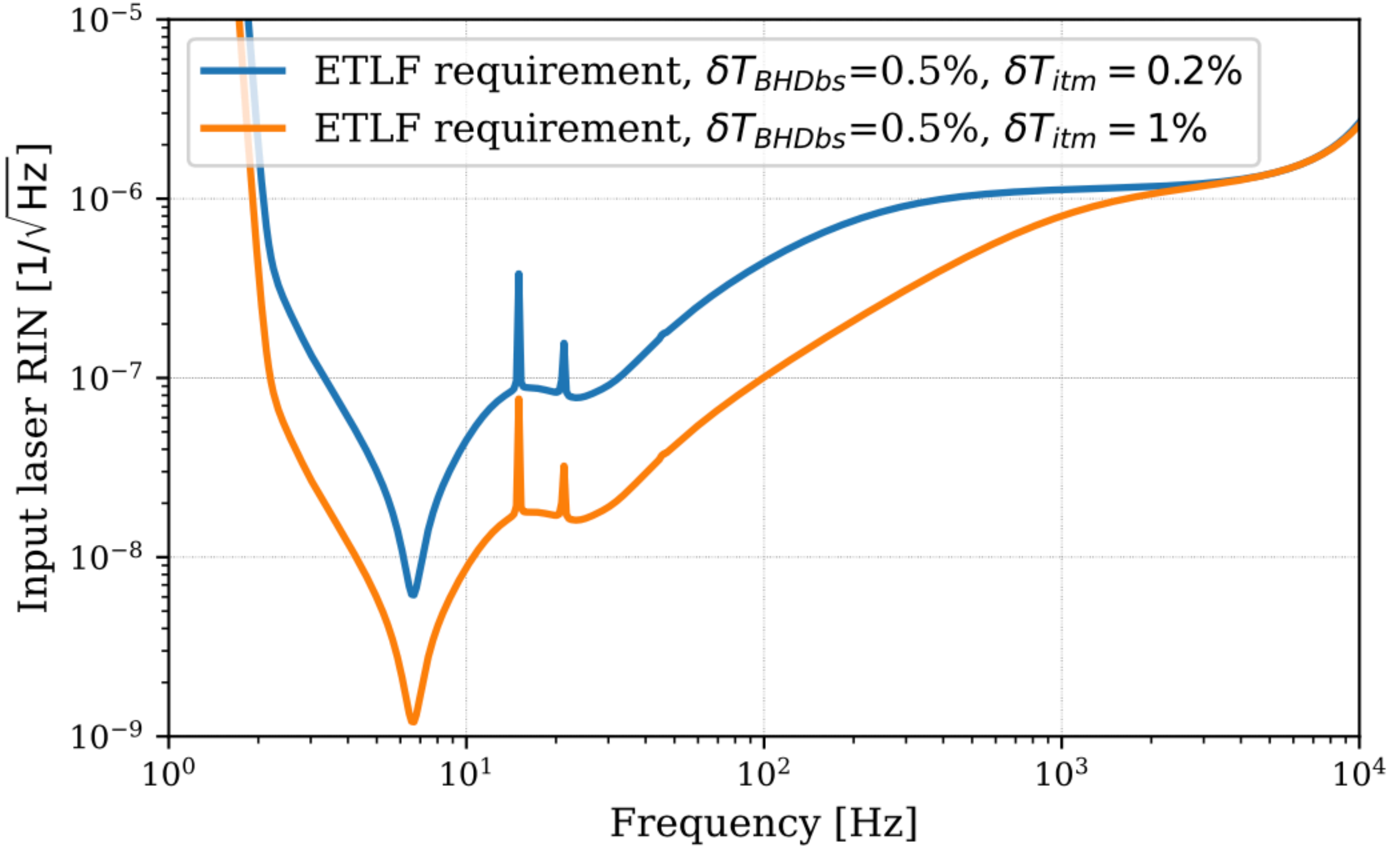


Intensity noise TF in m/RIN





Intensity noise requirement



Including a factor of 10 safe margin.