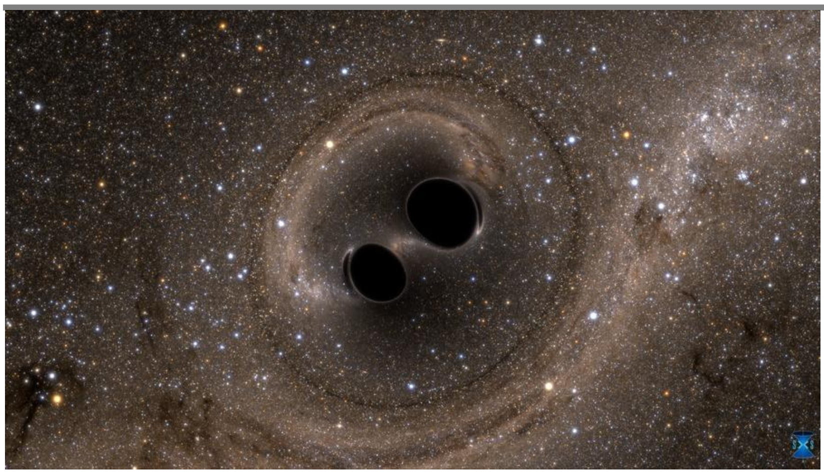
“Finding black-holes in a chirp”: how to understand the first gravitational-wave detection



PCCP demonstrator

Credit : The SXS (Simulating eXtreme Spacetimes) Project

Orienting and Asking Questions

Provide Contact with the content and/or provoke curiosity

In 1915 Albert Einstein formulated his famous General Relativity theory. "Space-time", a concept already introduced in the special relativity theory, becomes an "elastic object" with general relativity: it can be curved by matter and energy, and – as for any elastic object – waves can propagate through it. These space-time waves are the "gravitational waves": perturbations of space-time curvature propagating at the speed of light. The waves are generated when something that involves a rapid change of compact mass-energy distributions happens in the Universe. Then, the waves travel through the Universe and they arrive on Earth, often after a travel of hundreds of millions of years.

In 2015, a century after Einstein prediction, gravitational waves have been detected for the first time following the merger of two black holes, which we also call compact (massive and small) binary coalescence. With this detection, made by the US instrument LIGO, and with the following detections, made by LIGO and the European detector Virgo, a new kind of astronomy was born: the "gravitational-wave astronomy".

The first wave ever detected crossed the Earth on September 14th, at 11h50 Paris time.

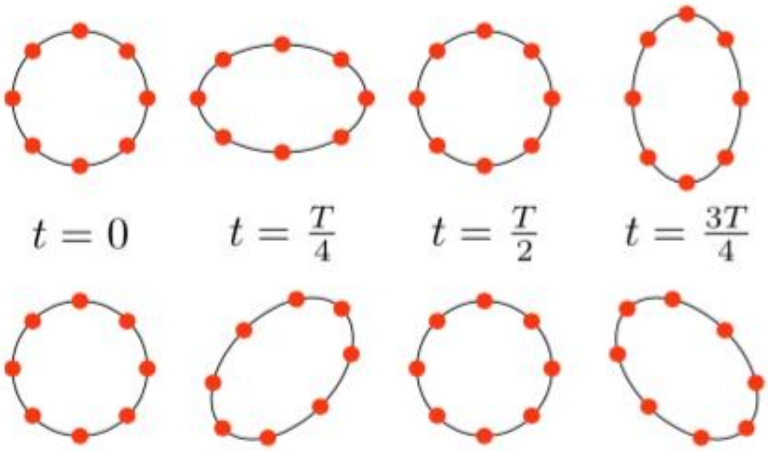
In this exercise: "finding black-holes in a chirp" we will try to take a closer look to this signal (called "a chirp") and understand how this is the exact signature of the merger of two black holes.

See the historic announcement of the first gravitational-wave detection: https://www.ligo.caltech.edu/video/ligo20160211v11

How do we detect gravitational waves? What is the first gravitational-wave signal telling us? How can we understand what the source is and, specifically, that the source is a pair of black holes?

Effect of gravitational-wave passing: gravitational waves are perturbations of the space-time. They stretch the space among "free-falling masses", masses subjected only to gravity.

For instance, if you have a circle of free-falling particles, the distances among the particles will vary when the gravitational wave crosses the circle (imagine the wave arriving perpendicular to the sheet). In the figure you see how the distance among the masses varies for two different "polarizations" of the waves (called + and X). An interesting feature of gravitational waves is that they stretch the space in an "asymmetric" way (when the horizontal coordinate becomes longer, at the same time the vertical coordinate becomes shorter). By observing the motion of test masses we can detect gravitational waves.

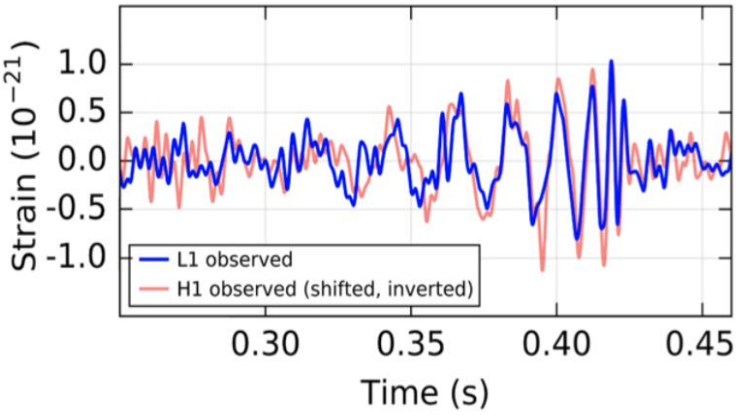


Sources of gravitational waves: Gravitational waves are emitted by mass distributions during acceleration of masses. With a caveat: spherical mass distributions don't emit gravitational waves. A spherical star, rotating on itself, does not emit gravitational waves. Two spherical stars, rotating one around the other, emit gravitational waves. In fact, a binary system can be described as a rotating bar with two heavy masses connected to its edges. The amplitude of the waves is higher if the bodies move at speed closer to the speed of light and if they are very compact: massive and small, like black holes and neutron stars.

Gravitational-wave detection: How to detect gravitational waves? We use laser interferometers: a tool very suitable to measure asymmetric changes in distances between points. A video clarifies well how a Michelson interferometer works.

https://www.ligo.caltech.edu/video/ligo20160211v6

LIGO and Virgo are giant interferometers, with kilometric arms. The instruments are very complex, and if you are interested you can try to understand the various aspects of their behaviour. Here we are interested in the signal produced by the first gravitational wave ever detected, in the figure below.



The wave holds for less than a second. We see a few cycles. The quantity on the Y-axis is the strain of the space: the variation of the distance over the arm of the instrument. It means that the arms (which are 4 km long) are stretched of the order of 10^{-18} m. An incredibly small quantity. However, the signal is pretty clear.

The journey of the wave detected on September 14th 2015 is told in the following video:

https://www.ligo.caltech.edu/video/ligo20160211v5

Teacher Guidelines:

- The sites of the LIGO and Virgo projects, www.ligo.org and www.virgo - gw.eu contain many other media resources.
- The video of the press conference of the first detection is 1 hour long. You can watch the first part, which is very simple, to discuss the basic concepts and it contains the famous sentence “we did it! We detected gravitational waves”.
- You can put the gravitational wave discovery in the context of other recent big discoveries in fundamental physics and astrophysics (1992 COBE’s spectrum of the cosmic microwave background, 1995 Top quark, 1998 Accelerated expansion of the Universe, 1998 Neutrino oscillations, 2012 Higgs Boson, 2019 Event horizon Telescope first direct observation of a supermassive black hole).
- You can discuss a little bit the special relativity theory, provoking curiosity with strange consequences and « paradoxes » of the theory: time dilation, twins ageing differently etc…

- You can also recall that the discovery of gravitational waves led to a Nobel Prize in physics in 2017 to three « fathers » of the gravitational-wave detection and of the LIGO project.

Define goals and/or questions from current knowledge

Take some minutes to discuss these questions:

- Why did it take so long to detect gravitational waves (from 1916 to 2015)? What happened between 1916 and 2015?

- What can the effect of a gravitational wave be on us? Can a gravitational wave be dangerous to us? Can we feel the passing of a gravitational wave?

- What do you know about black holes?

- Why are gravitational waves interesting?

- How can we use the basic concepts of general relativity and the theory of gravitational waves to understand the source of the wave?

Teacher Guidelines:

- This exercise is inspired by the article "The basic physics of the binary black hole merger GW150914", that you can find in the Annex. The article uses simple general relativity equations to show that the source of the first waves detected is a black-hole binary system.

- Explain that, even if we can infer the type of source with simple equations, the complete analysis requires very complex numerical simulations and calculation. The results do not change, though.

- You can spend some time – depending on the interests of the students - in describing the instruments LIGO and Virgo, the fact that 10^{-18} is a very small number, and that the technology behind the instruments was developed during 40-50 years by hundreds of scientists.

Hypothesis Generation & Design

Generation of Hypotheses or Preliminary Explanations

Let us make the hypothesis that the signal observed by LIGO is a gravitational wave.

Which kind of sources can emit such an oscillatory wave?

We can quote here the article: "The basic physics of the binary black hole merger GW150914 ». While reading this text take again a look to the shape of the detected signal.

"*In general relativity, gravitational waves are produced by accelerating masses. Since the waveform clearly shows at least eight oscillations, we know that a mass or masses are oscillating*"

A mass or masses are oscillating. Something in the Universe is oscillating.

"*The increase in gravitational wave frequency and amplitude also indicate that during this time the oscillation frequency of the source system is increasing.*"

What kind of mass can oscillate in the Universe?

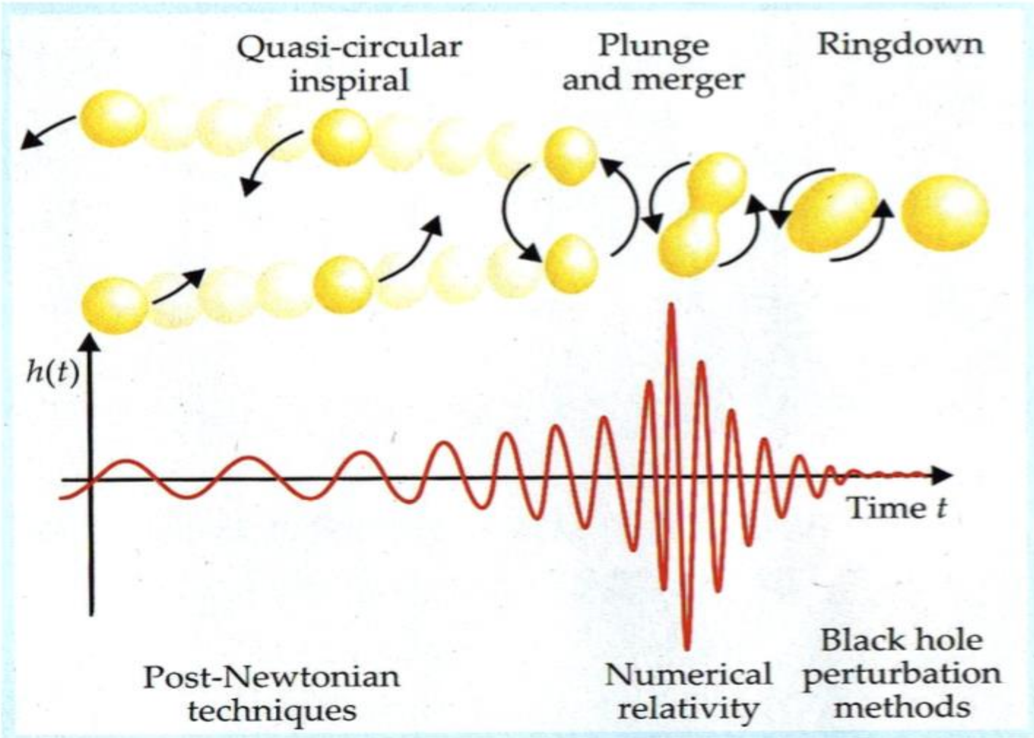
"*This initial behavior cannot be due to a perturbed system returning back to stable equilibrium, since oscillations around equilibrium are generically characterized by roughly constant frequencies and decaying amplitudes. For example, in the case of a fluid ball, the oscillations would be damped by viscous forces.*"

Did you notice that the frequency and amplitude are increasing?

"*Here, the data demonstrate very different behavior. During the period when the gravitational wave frequency and amplitude are increasing, orbital motion of two bodies is the only plausible explanation: there, the only "damping forces" are provided by gravitational wave emission, which brings the orbiting bodies closer (an "in-spiral"), increasing the orbital frequency and amplifying the gravitational wave energy output from the system.*"

It cannot be a « perturbed system coming back to the equilibrium ». It should be something that diverges : the system loses energy through gravitational-wave emission and spirals. The speed increases and the gravitational-wave emission becomes more important. There is a maximum: the frequency and the amplitude increase up to the merger of two bodies.

The event can be summarized with the following scheme:



And a video explaining the process is:

https://www.ligo.caltech.edu/video/ligo20160615v1

The signal is called «a chirp » (a signal increasing in frequency and amplitude). A very interesting software called « black-hole hunter » allows to find chirps in the instrument noise using your ears. In real life, chirps are found by very complex computer algorithms scanning the data.

INSERT IFRAME: https://blackholehunter.org

Teacher Guidelines:

- It's important to spend a few minutes to make sure that the students have understood the following point: the chirp signal is closely related to the motion of the two inspiralling masses. Can the source be something else than two bodies orbiting one another? Can one imagine another source? For instance, a supernova explosion? How could the shape of that signal be?
- Another interesting question is: how did the LIGO and Virgo scientists understand that the signal is "real", meaning that it comes from the sky

and it is not an instrumental artefact? It can be interesting to read the discovery paper: **Observation of Gravitational Waves from a Binary Black Hole Merger,** B. P. Abbott et al. (LIGO and Virgo Collaborations), Phys. Rev. Lett., Vol. 116, 061102, 2016.

Planning & Investigation

Plan Investigation

Now that we are convinced that the source is the in-spiral and merger of two bodies, we want to understand which kind of bodies are merging.

Are they stars similar to the Sun? Are they more compact stars, like neutron stars, or are they extreme objects like black holes?

Black holes: a black hole is another consequence of the general theory of relativity. It is an object so compact that nothing can escape from it, even if it travels at the largest possible speed: the speed of light. The region inside which nothing can escape is defined by the "black-hole horizon". The horizon is a mathematical surface (not a hard surface, like the one of the Earth or the surface of a metal ball): entering the horizon of the black-hole means to be lost forever. For the same reason it is impossible to receive information from the interior of the black hole. The information we have on an object is given by "messengers" coming from the interior to the exterior. If nothing can escape, no information is possible about the interior of the black hole.

The radius of the horizon is called the Schwarzschild radius and depends on the mass of the black hole.

$$r_s = \frac{2GM}{c^2} \approx 2.95 \, \frac{M}{M_{\mathrm{Sun}}} \, \mathrm{km},$$

Therefore, for a black hole having the mass of the Sun, the Schwarzschild radius is approximately 3 km, and it changes proportionally to the mass of the black hole.

We immediately see how compact black holes are: the Sun has a radius of 700 000 km, meaning 200 000 times larger than that of a black hole of the same mass. For comparison, a neutron star, a very compact object, would have a radius of about 10 km.

You can take a look to the Wikipedia page for black holes.

In order to understand if an astronomical object is a black hole, we should compute its compactness, and compare it to the one of the corresponding black hole of the same mass.

A very recent highlight about black holes: A first "image" of a black hole has been taken by the Event Horizon Telescope, in April 2019.

Perform Investigation

In order to understand the compactness of the objects producing the gravitational waves we should have a measurement of their mass and their radius.

First, let's discuss how to determine the mass.

1. Determination of the mass of the two objects

In order to determine the mass, we will use a property of the gravitational-wave emission: the larger the mass of the objects, the larger is the gravitational-wave emission. The larger is the gravitational-wave emission, the larger is the gravitational-wave emission during the inspiral of the system (and thus the increase of the frequency of the gravitational waves). This translates into the following equation:

$$\mathcal{M} = \frac{c^3}{G}\left(\left(\frac{5}{96}\right)^3 \pi^{-8} (f_{GW})^{-11} \left(\dot{f}_{GW}\right)^3\right)^{1/5} \quad \text{Equation 1}$$

.

where M is the "chirp mass", a combination of the mass of the 2 objects.

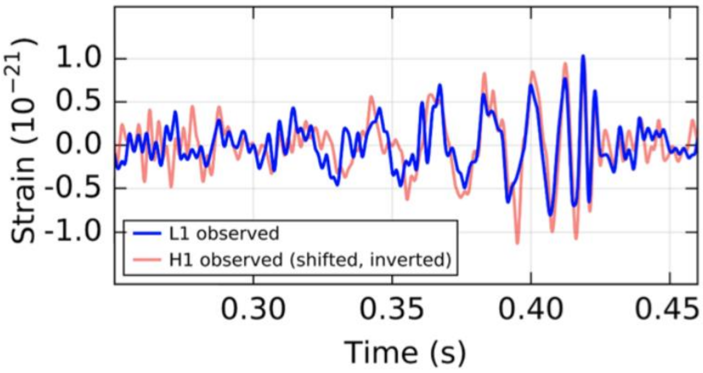
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \quad \text{Equation 2}$$

For this exercise we can consider that the two masses are equal.

A little exercise: Find the mathematical expression of the chirp mass when $m_1 = m_2 = m$.

f_dot is the frequency rate of change with time, also called the "derivative" of the frequency.

From the data

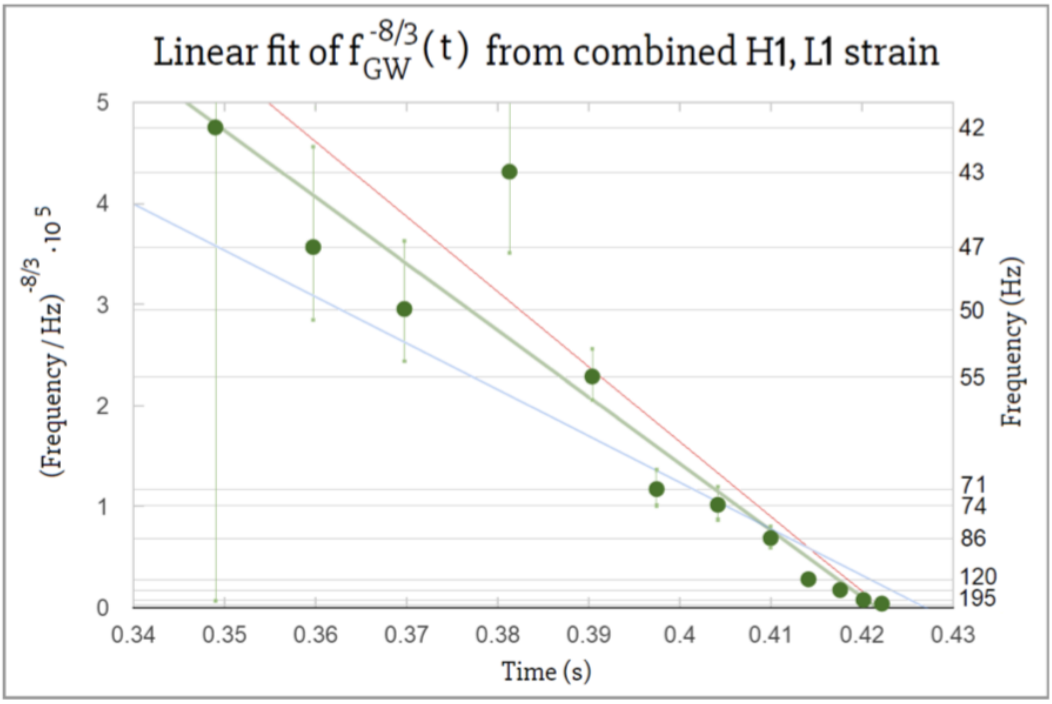


We can compute f and f_dot and then compute the chirp mass. We can also compute the chirp mass of the system making the hypothesis that the two stars have the same mass.

From the equation above, with some mathematical manipulation, we can obtain the equivalent formula:

$$f_{\mathrm{GW}}^{-8/3}(t) = \frac{(8\pi)^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(t_c - t) \qquad \text{Equation 3}$$

that tells us that the frequency (to the power -8/3) is directly proportional to time. In fact, the blue term in Eq. 3 is the angular coefficient K of a straight-line equation. Indeed, from the frequency of GW150914 measured with respect to the time we obtain the following plot:



Where the green line represents our best fit of the frequency using two detectors. You can now use the above plot to estimate the angular coefficient "K" of the green line (notice that you have to read the ordinate value on the left axis) and you can recover the chirp mass as:

$$\mathcal{M} = \left[\frac{5K}{(8\pi)^{8/3}}\right]^{3/5} \frac{c^3}{G}$$

Teacher guidelines:

- In a more advanced version of the exercise, we can use the REAL DATA, which can be downloaded at the GWOSC (Gravitational-wave open data science:

https://www.gw-openscience.org/about/

The GWOSC contains the data and also python notebook to extract and plot the data.

As an alternative, an ascii file is given and the students can plot the signal with a very basic plotting tool.

- Let us stress again the fact that the phase of the gravitational wave ALONE can be used to determine the mass of the objects. We don't need the amplitude of the gravitational wave. This will become very important in the following, when we will introduce the cosmological applications of gravitational waves.

- The hypothesis of equal mass should be of course justified, but this goes beyond the scope of this exercises. Please take a look to the publication for further details.

2. Determination of the radius of the two objects

In order to determine the radius of the two objects we use another property of the gravitational wave: the frequency of the wave is the double of the orbital frequency of the source. If the frequency of the wave is 100 Hz (100 cycles per second), the frequency of the orbit is 50 Hz (50 cycle per second). Then, knowing the frequency of the waves, we can determine the frequency of the orbit.

The third Kepler law allows, knowing the frequency of the orbit and the mass of the orbiting objects, to determine their distance.

$$R = \left(\frac{GM}{\omega^2_{\text{Kep}}\big|_{\text{max}}} \right)^{1/3}$$

Where M is the total mass of the system and omega is the maximal radial frequency before the merger of the two bodies.

A little exercise:

1) Find the expression above starting from the Newton's law of gravitation.

2) How is the angular frequency related to the frequency?

Teacher Guidelines:

- We underline the fact that here we are using Keplerian orbit. This is an approximation (in general relativity the orbits are not Keplerian, since the bodies are spiralling). But, in this case, the approximation is justified. See further details in the publication.

Analysis & Interpretation

Analysis and Interpretation: Gather result from data

Let us analyse now the LIGO data.

- Compute the frequency and the rate of change of the frequency.
- Compute the distance between the two bodies.

Which is the radius of the objects, compared to the Schwarzschild radius?

Analyse and plot the data using more than one oscillation, and the two formulas described above and involving the chirp mass. Try to estimate the errors in the determination of the masses.

Which are the error bars?

What are your conclusions, based on your analysis?

Teacher Guidelines:

If Eq. 3 is used, we can perform a linear fit, as in the publication.

Conclusion & Evaluation

Conclude and communicate result/explanation

1. Write a 15 lines report on what you have done addressing the following points (context, methodology, results, discussions).

2. Record a 1 minute video or audio on what you have done addressing the following points (context, methodology, results, discussions). If you use a video, you can show your results (in the form of plots and schemes).

Teachers Guidelines:

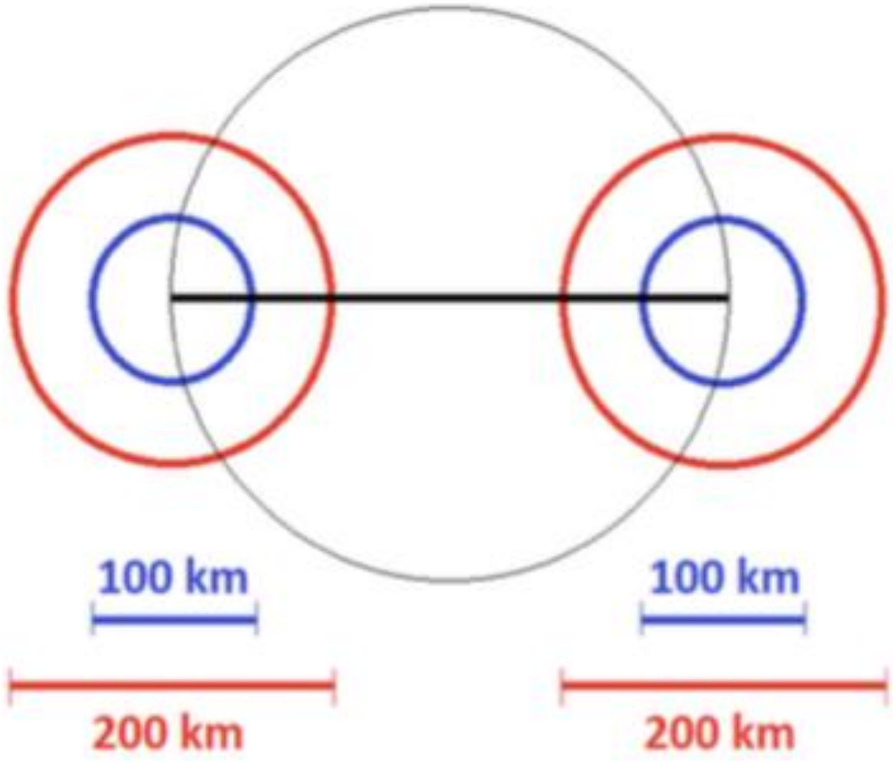
Here we can try to “justify” the working hypothesis (for example the fact that we are taking equal mass objects), as it is done in the publication.

Evaluation/reflection

You can compare your results with the LIGO results.

- The total mass of the system is ~70 solar masses.
- The distance between the two objects is ~ 350 km.

See the figure below:



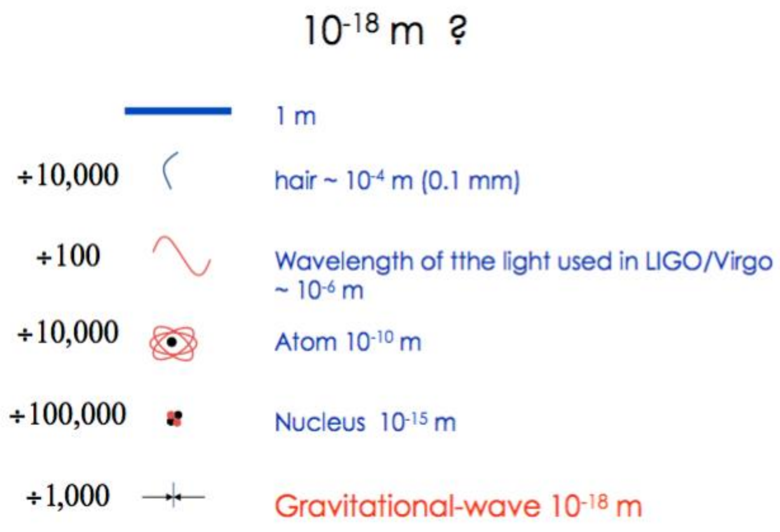
The two objects are still orbiting one another, they are not in contact and their radius is smaller than 350 km / 2 = 175 km.

A black hole of mass 35 solar masses has a Swarztschild radius of 100 km if it is not spinning and up to 200 km if it is spinning. Since the radius of the object detected is less than 175 km, the two objects must be black holes.

In the previous explanation we have used the "phase" of the gravitational waves to determine the mass of the system. Is it possible to use the phase of an electromagnetic wave? Explore the difference between the detection of an electromagnetic waves (with a telescope) and that of gravitational waves

You can start by recalling how the electromagnetic radiation (light, UV, IR, ...) is detected. Which is the observed quantity?

- In order to measure a gravitational wave, we had to measure a variation of distance of 10^{-18} m over 4 km. Try to connect this distance (10^{-18} m) with the scales of distances you know (atoms, molecules).



Compare the compactness of a black hole (M/r) with the compactness of the Earth or the compactness of an atom.

Start from the radius and mass of those objects. Before immediately looking on the internet, try to make some reasonable guess.

- If you have used the two methods (Eq. 1 and Eq. 2) to determine the mass of the objects, are the results compatible? If not, why?

If you need the actual data (instead of the figure) to better analyse the event, you can take a look to the site: https://www.gw-openscience.org/about/

Teacher Guidelines:

The 4 questions addressed try to connect the knowledge developed with 4 important points: • The difference between EM radiation and gravitational-wave radiation. • The difficulty of the measurement (10-18m). • The extreme compactness of the black hole. • The importance of indicating and discussing the measurement errors.