Role of the null stream in the triangle-2L comparison

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Null Stream I

- The null stream is a signal-free linear combination of the interferometer strain data
- Particularly easy combination for the Δ configuration
  - The strain per detector can be written as

$$h^A(t) = d^A_{ij}\, h^{ij} = F^A_+\, h_+ + F^A_\times\, h_\times \tag{1}$$

  - where d^A_{ij} are the detector tensors

$$\begin{aligned}
\mathbf{d}^1 &= \frac{1}{2}(\mathbf{e}_1 \otimes \mathbf{e}_1 - \mathbf{e}_2 \otimes \mathbf{e}_2),\\
\mathbf{d}^2 &= \frac{1}{2}(\mathbf{e}_2 \otimes \mathbf{e}_2 - \mathbf{e}_3 \otimes \mathbf{e}_3),\\
\mathbf{d}^3 &= \frac{1}{2}(\mathbf{e}_3 \otimes \mathbf{e}_3 - \mathbf{e}_1 \otimes \mathbf{e}_1),
\end{aligned} \tag{2}$$

Null Stream II

- The sum of the individual responses is identically equal to zero

$$\begin{aligned}\sum_A h^A &= \sum_A d_{ij}^A \, h^{ij} \\ &= h^{ij} \sum_A d_{ij}^A \\ &= 0 \end{aligned} \tag{3}$$

- ▶ Two L-shaped detectors rotated relative to each other by an angle $\pi/4$ are completely equivalent to ET in terms of their response and resolvability of polarizations
  - However, their response cannot be used to construct a null stream
- ! Null stream assumes 1) co-located detectors, 2) all detectors are locked/online.

Null Space / Signal Space I

- Projection onto null space represented by a projection matrix

$$\boldsymbol{P}_{\mathsf{null}} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \tag{4}$$

- Projecting the strain signal $\boldsymbol{s}(t)$ onte the null space

$$\begin{aligned} \boldsymbol{P}_{\mathsf{null}}\boldsymbol{s}(t) &= \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix} \\ &= \frac{1}{3} \begin{bmatrix} s_1(t) + s_2(t) + s_3(t) \\ s_1(t) + s_2(t) + s_3(t) \\ s_1(t) + s_2(t) + s_3(t) \end{bmatrix} = \boldsymbol{0} \end{aligned} \tag{5}$$

Null Space / Signal Space II

- The orthogonal projection (signal projection) given by

$$\boldsymbol{P}_{\text{sig}} := \boldsymbol{I} - \boldsymbol{P}_{\text{null}} = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix} \tag{6}$$

  - $\boldsymbol{P}_{\text{sig}}$ projects the strain data onto the signal space
  - Removes data in the null space

Null Space / Signal Space III

- Since there is only one linearly independent row vector in $\boldsymbol{P}_{\text{null}}$
  - $\Rightarrow$ $\boldsymbol{P}_{\text{null}}$ is a rank one matrix
- The orthogonal projection matrix $\boldsymbol{P}_{\text{sig}}$ removes one dimension from the strain data $\boldsymbol{d}(t)$
  - $\Rightarrow$ only two dimensions in the 3-detector strain data that are relevant to GW data analysis

Key Point 1

The original 3-detector strain data with $3N$ data points where N is the number of data points in each time series could be compressed to a more compact representation with $2N$ data points without any loss of GW information

Inference with Null Stream I

- Likelihood in the detector spaces

$$p(\boldsymbol{d}|\boldsymbol{\theta}) = \prod_{i=1}^{3} \exp\left(-\frac{1}{2}(\boldsymbol{d}_i - \boldsymbol{s}_i(\boldsymbol{\theta}))^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{d}_i - \boldsymbol{s}_i(\boldsymbol{\theta}))\right) \tag{7}$$

- Likelihood in the signal space

$$p(\boldsymbol{d}^p|\boldsymbol{\theta}) = \prod_{i=1}^{3} \mathcal{N} \exp\left(-\frac{1}{2}(\boldsymbol{d}_i^{\mathsf{p}} - \boldsymbol{s}_i^{\mathsf{p}}(\boldsymbol{\theta}))^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{d}_i^{\mathsf{p}} - \boldsymbol{s}_i^{\mathsf{p}}(\boldsymbol{\theta}))\right) \tag{8}$$

  - where the normalisation is given by

$$\mathcal{N} = \frac{1}{(2\pi)^{N/2}|\boldsymbol{\Sigma}|^{1/2}} \tag{9}$$

Inference with Null Stream II

- Signal-space likelihood and standard likelihood related by $\boldsymbol{\theta}$-independent factor

$$Cp(\bar{\boldsymbol{d}}^{\mathsf{p}}|\boldsymbol{\theta}) = p(\boldsymbol{d}|\boldsymbol{\theta}) \tag{10}$$

  - where $\boldsymbol{\theta}$-independent factor given by

$$C = \frac{1}{(2\pi)^{N/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(\boldsymbol{d}_3^{\mathsf{p}})^T\boldsymbol{\Sigma}^{-1}\boldsymbol{d}_3^{\mathsf{p}}\right) \tag{11}$$

- Posterior from the likelihood

$$p(\boldsymbol{\theta}|\boldsymbol{d}) = \frac{p(\boldsymbol{d}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{\int p(\boldsymbol{d}|\boldsymbol{\theta})p(\boldsymbol{\theta})d\boldsymbol{\theta}} \tag{12}$$

Inference with Null Stream III

- Equivalence between posteriors in signal space / detector space

$$p(\boldsymbol{\theta}|\boldsymbol{d}) = p(\boldsymbol{\theta}|\bar{\boldsymbol{d}}^{\mathsf{p}}) \tag{13}$$

- Equivalence between Bayes factor in signal space / detector space

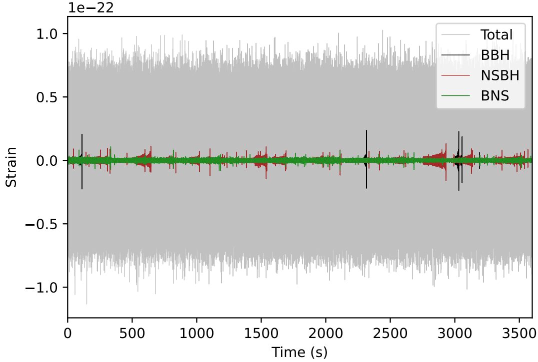
$$\mathcal{B}^{\mathcal{H}_2}_{\mathcal{H}_1}(\boldsymbol{d}) = \mathcal{B}^{\mathcal{H}_2}_{\mathcal{H}_1}(\bar{\boldsymbol{d}}^{\mathsf{p}}) \tag{14}$$

Key Point 2

The posterior distribution of the source parameters and the Bayes factor inferred from the signal-space data is identical to that inferred from the full set of data

Estimation of unbiased noise power spectrum I

- Many overlapping unresolvable GW signals
- Not trivial (without null stream) to estimate noise PSD without contamination



An hour of simulated data (Wu et al. [1])

Incoherent Homogeneous Noise I

- If the noise is homogeneous and incoherent among the detectors,

$$S_n^1(f) \simeq S_n^2(f) \simeq S_n^3(f)\,, \tag{15}$$

- Null stream only contains noise

$$x_{\text{null}}(t) = \sum_{A=1}^{3} n^A(t) + \sum_{A=1}^{3} d_{ij}^A h^{ij}(t) = \sum_{A=1}^{3} n^A(t) \tag{16}$$

- Noise PSD of each detector can be estimated by

$$S_n^i = \frac{1}{3} S_n^{\text{null}} \tag{17}$$

  - where S_n^{null} is the PSD of the null stream.

Incoherent Homogeneous Noise II

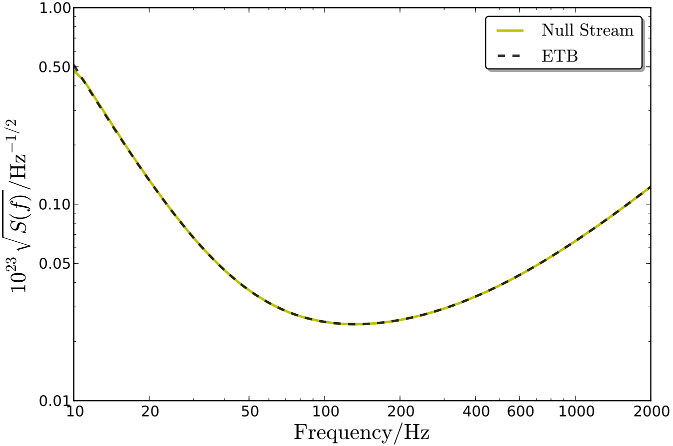


Figure: PSD estimate from null stream PSD (Regimbau et al. [2])

Incoherent Homogeneous Noise III

- Cross PSD (CPSD) of null stream with data streams

$$\left\langle \tilde{d}_{\text{null}}(f)\tilde{d}_i^*(f')\right\rangle = \frac{\delta(f-f')}{2\sqrt{3}}\left[\left(S_n^i(f) + \sum_{i\neq j} S_n^{ij}(f)\right)\right] \quad (18)$$

  - where S_n^{ij} is the CPSD between detectors i and j
- If noise is incoherent, then detector PSD can be directly estimated

Incoherent Homogeneous Noise IV

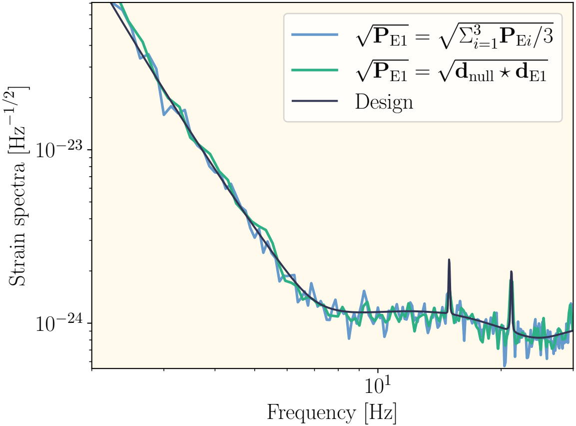


Figure: PSD estimate from null stream CPSD (Goncharov et al. [3])

Coherent Noise I

- Identical noise among detectors does not appear in null stream

$$x_{\text{null}}(t) = \sum_{A=1}^{3} n_{\text{incoh}}^{A}(t) + \sum_{A=1}^{3} d_{ij}^{A}\left(h^{ij}(t) + n_{\text{coh,id}}^{ij}(t)\right)$$
$$= \sum_{A=1}^{3} n_{\text{incoh}}^{A}(t) \quad (19)$$

- But is included in the total noise of each detector

$$n_{\text{tot}}^{A}(t) = n_{\text{incoh}}^{A}(t) + n_{\text{coh,id}}^{A}(t) \quad (20)$$

- NB: not all coherent noise sources are necessarily identical among detectors
  - See CoBA for more in-depth discussion of these noise sources

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Coherent Noise II

- Consider non-identical noise across detectors
- Cross PSD (CPSD) of null stream with data streams

$$\left\langle \tilde{d}_{\text{null}}(f)\tilde{d}_i^*(f')\right\rangle = \frac{\delta(f-f')}{2\sqrt{3}}\left[\left(S_n^i(f) + \sum_{i\neq j} S_n^{ij}(f)\right)\right] \quad (21)$$

  - where S_n^{ij} is the CPSD between detectors i and j
- If assume one knows CPSDs (e.g. witness sensors), then can recover estimate of detector PSD

Coherent Noise III

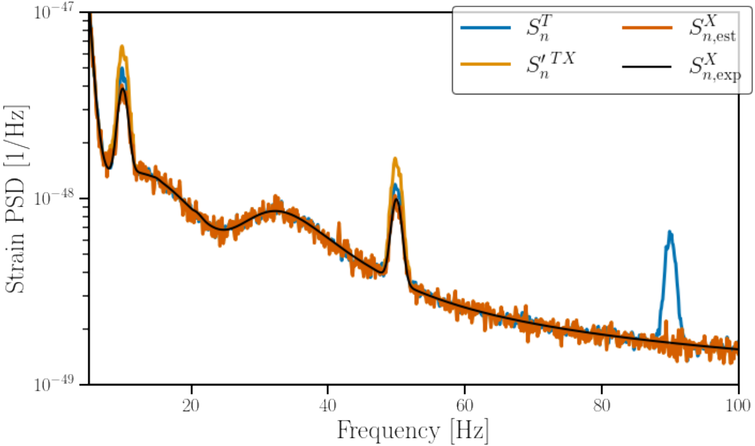

Figure: Estimate detector PSD using null stream CPSD (Janssens et al. [4])

Estimation of unbiased noise power spectrum

Key Point 3

The null stream can be used to obtain an unbiased estimation of the noise power spectrum (of each detector)

Unbiased noise power spectrum: Science Impact I

- Recover PSD of the GW signals S_h present in the data

$$S_h^i \simeq S_{\rm tot}^i - S_n^i, \tag{22}$$

- S_h can be directly related to the SGWB if the data contains no resolvable signals
- Otherwise, one has to subtract loud/resolvable sources (see e.g. Wu et al. [1] and Sachdev et al. [5])

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Unbiased noise power spectrum: Science Impact II

- Inability to disentangle detector noise from confusion noise has the effect of raising the overall perceived noise level
- In a templated GW search, the effect manifests as a loss of matched filtering SNR

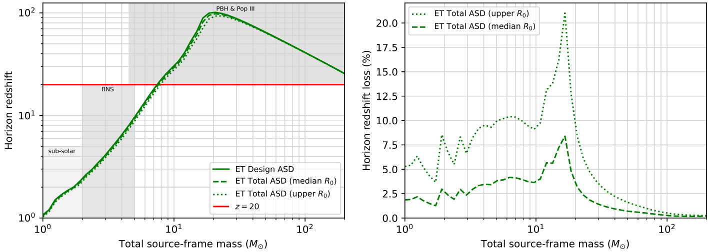
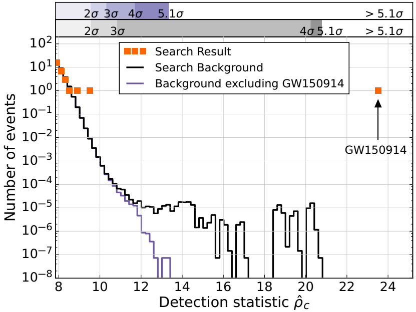


Figure: Loss in ET detection horizon (Wu et al. [1])

Unbiased noise power spectrum: Science Impact III

- Confusion noise also impacts calculation FAR
- FAR is estimated from noise-induced (background) distribution of detection statistic (e.g. SNR)
- E.g. perform matched filtering on time shifted data (e.g. Was et al. [6])
- FAR esimate assumes number of genuine GW detectable signals is low



Background for GW150914 (Abbott et al. [7])

⇒ Estimate background distribution directly from null stream (e.g. time shifting null stream)

Glitches I

- GW observatories suffer from instrumental noise artifacts
- Non-stationary sources of noise (glitches) affect all searches

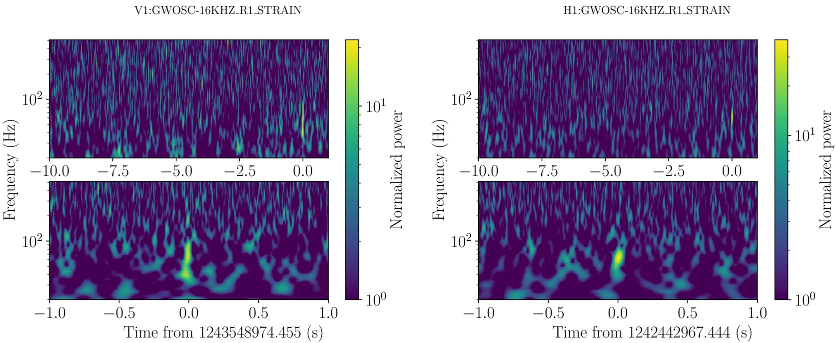


Figure: Visual similarity between a merger signal and instrumental artifact (Goncharov et al. [3])

Glitches II

- Use null SNR as discriminator

$$\rho^2_{\text{null}} = \rho^2_{\text{coinc}} - \rho^2_{\text{coh}} \tag{23}$$

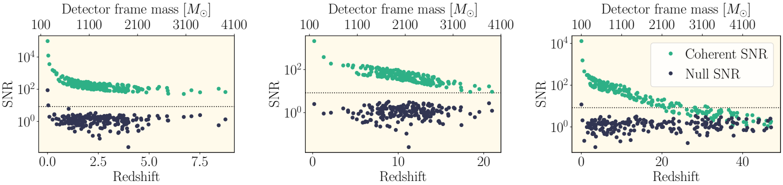
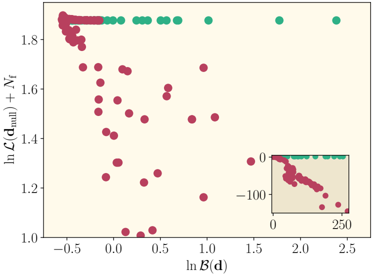


Figure: Null SNR as veto for glitches (Goncharov et al. [3])

Glitches III

- Use null likelihood as discriminator

$$\log \mathcal{L}(\tilde{\boldsymbol{d}}_{\text{null}}) \sim -\frac{1}{2}\langle\tilde{\boldsymbol{d}}_{\text{null}}|\tilde{\boldsymbol{d}}_{\text{null}}\rangle \quad (24)$$



Null likelihood as veto for glichtes (Goncharov et al. [3])

Mitigation of transient detector glitches

Key Point 4

The null stream can be used to mitigate the effects of transient detector glitches

Control of Systematic Errors I

- Any errors in detector calibration can propagate into null stream to cause incomplete cancellations of GW signals [8]
- If signal waveform is a-priori well understood, and if its parameters are well-measured by network of detectors, then residual signal in null stream will be product of the calibration error and known weighted amounts of signal [9]
- Detect residual by performing matched filtering on null stream
- Calibration error can be obtained by fitting with a family of specific functions supplemented by the SNR output of the matched filters over a number of detected events
- Calibration error can be inferred at the percent level if supplemented with $O(100)$ relatively loud (SNR=20) events (Schutz et al. [9])

Control of known and unknown systematic errors

Key Point 4

The null stream can be used to control known and unknown systematic errors.

Concluding Remarks

- Null stream is the Swiss army knife of noise handling
- Allows one to straightforwardly optimise science extraction
- While the individual improvement of having a null stream seems modest, it is not immediately obvious how well one can optimise science extraction in the absence of the null stream

References I

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References II

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[8] S. Chatterji et al. "Coherent network analysis technique for discriminating gravitational-wave bursts from instrumental noise". *Phys. Rev. D* 74 (2006), p. 082005. arXiv: gr-qc/0605002.

[9] B. F. Schutz et al. "Self-calibration of Networks of Gravitational Wave Detectors". (Sept. 2020). arXiv: 2009.10212 [gr-qc].