# Atmospheric Newtonian noise modeling for third-generation gravitational-wave detectors

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Mariano Cadoni
Piero Olla
Andrea P. Sanna

**Based on:** Phys. Rev. D (2022) **106**, 064040
**Datasets:** github.com/maurooi/AtmosphericNN
**More plots:** 10.5281/zenodo.6758920.svg

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# Motivations

Third-generation gravitational-wave (GW) detectors are expected to enhance the sensitivity of a factor ~10 and to push the frequency bandwidth down to 2-3 Hz

One of the major limitations in this region of the parameter space comes from Newtonian noise (NN)

NN is a fluctuation in the gravitational acceleration felt by the GW detector due to density perturbations in the nearby environment

# Motivations

Two main NN contributions

**Seismic:** extensively studied in the past [Saulson (1984)] [Harms (2019)]

**Atmospheric:** modeled only for surface-based detectors in a simplified picture (homogeneous, isotropic, and frozen density perturbations) [Creighton (2013)]

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**Atmospheric:** modeled only for surface-based detectors in a simplified picture (homogeneous, isotropic, and frozen density perturbations) [Creighton (2013)]

**Goal:** improve the estimation of atmospheric NN by building more accurate and realistic models

# Understanding atmospheric NN

Atmospheric NN is produced by temperature fluctuations $\delta T$ generated by the heat of the sun and mixed by convection

Density perturbations $\delta\rho \propto \delta T$ produce a fluctuation in the gravitational acceleration $\delta\boldsymbol{a}$

$$\delta\rho = -\frac{\langle\rho\rangle}{\langle T\rangle}\delta T$$

$$\delta\boldsymbol{a}(\boldsymbol{r},t) = \frac{G_N}{r^2}\delta\rho(\boldsymbol{r},t)\Delta V\,\hat{\boldsymbol{r}} = -\frac{\langle\rho\rangle}{\langle T\rangle}G(r)\delta T(\boldsymbol{r},t)\Delta V\,\hat{\boldsymbol{r}}$$

This has to be integrated over some volume of atmosphere and projected onto the detector arm, directed as $\hat{\boldsymbol{n}}_{arm}$, giving

$$\delta a(t) = \int_V d^3\boldsymbol{r}\,\delta\boldsymbol{a}(\boldsymbol{r},t)\cdot\hat{\boldsymbol{n}}_{arm}$$

# Understanding atmospheric NN

The magnitude of the noise can be estimated statistically through its spectral density

$$S_h(\omega) \propto \omega^{-4} \int dt\, e^{i\omega t} \langle \delta a(t) \delta a(0) \rangle$$
$$\propto \omega^{-4} \int dt\, d^3\boldsymbol{r}\, d^3\boldsymbol{r}' G(r) G(r') \langle \delta T(\boldsymbol{r}, t) \delta T(\boldsymbol{r}', 0) \rangle$$

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# Our models

We developed two distinct models to deal with atmospheric NN

**Homogeneous and isotropic (HI):** turbulence is homogeneously and isotropically distributed in the atmosphere

**Inhomogeneous:** we consider the inhomogeneities and anisotropies naturally present in atmospheric turbulence

In both models also include the effect of time correlations and decay of eddies

# HI turbulence

The key ingredient in our description is the choice of the correlation functions

We take the Fourier transform of $\langle \delta T(\boldsymbol{r}, t) \delta T(\boldsymbol{r}', t) \rangle$

$$\langle \delta T(\boldsymbol{r}, t) \delta T(\boldsymbol{r}', t') \rangle \propto \int d^3\boldsymbol{k}\, d\omega\, e^{-i\omega \mathrm{t} + i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \langle \delta T(k, \omega) \delta T(0,0) \rangle$$

We factorize the spatial and temporal part of $\langle \delta T(k, \omega) \delta T(0,0) \rangle$

$$\langle \delta T(k, \omega) \delta T(0,0) \rangle = f(k) h(\tau_k \omega)$$

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[Kolmogorov (1941)]

Spatial correlations
$f(k) \propto k^{-11/3}$

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[Kolmogorov (1941)]

**Frozen limit:**
$\tau_k \to \infty, h(\tau_k\omega) \to \delta(\omega - \boldsymbol{k}\cdot\boldsymbol{U})$

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# Inhomogeneous turbulence

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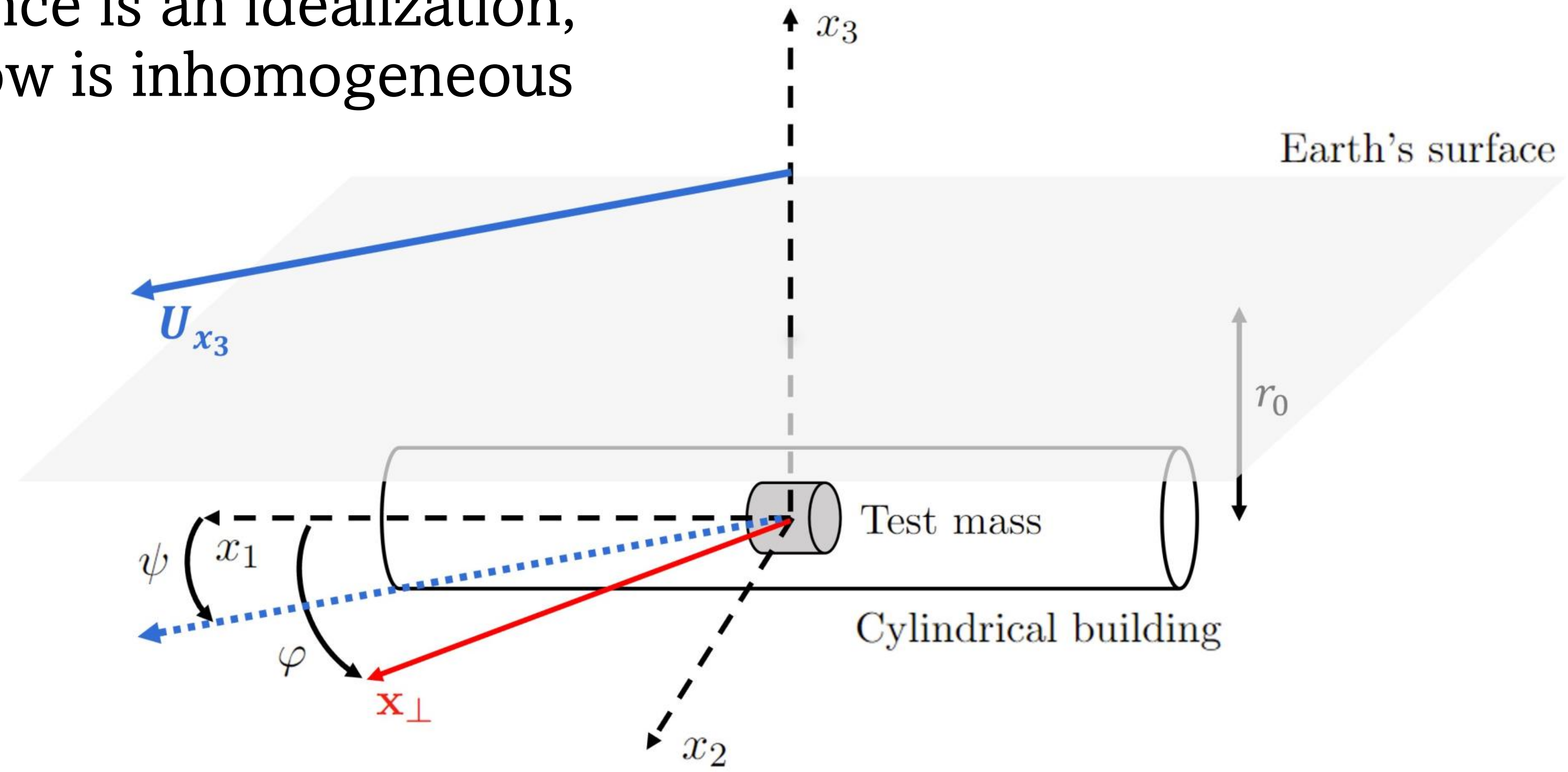
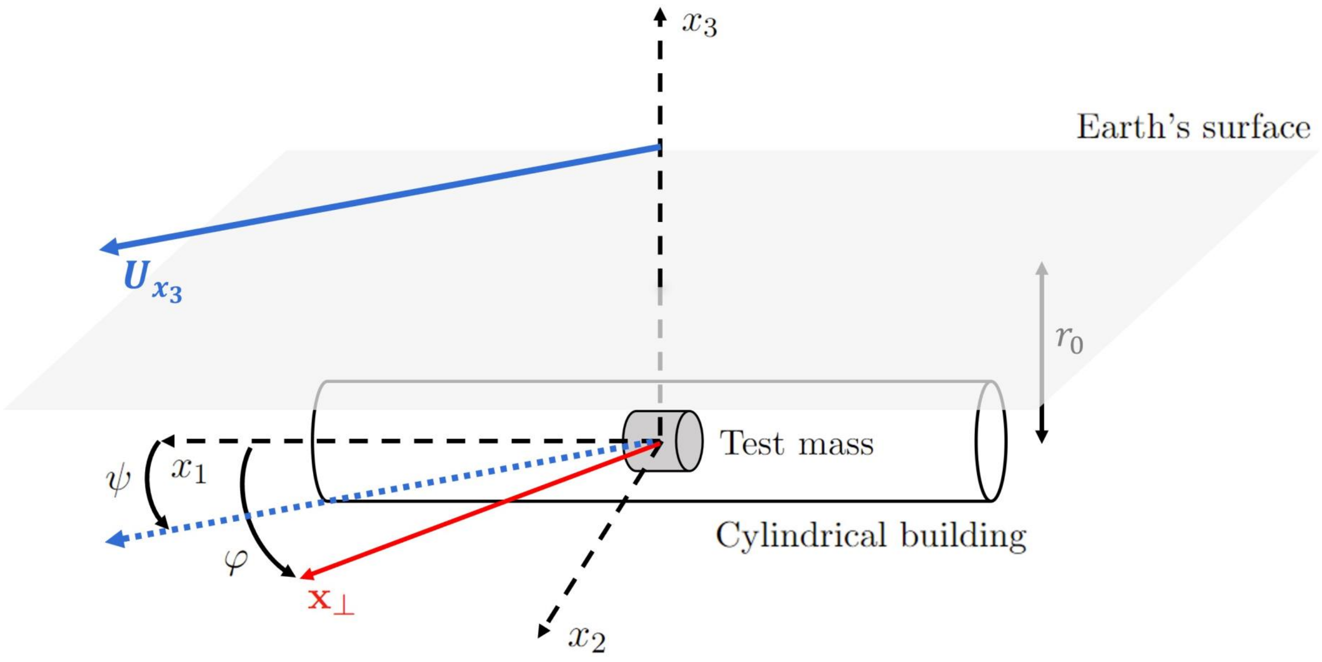


# Inhomogeneous turbulence

HI turbulence is an idealization, any real flow is inhomogeneous

The presence of a terrain introduces inhomogeneities on the vertical direction

$$U = U(x_3) \sim \log(x_3/z_0)$$



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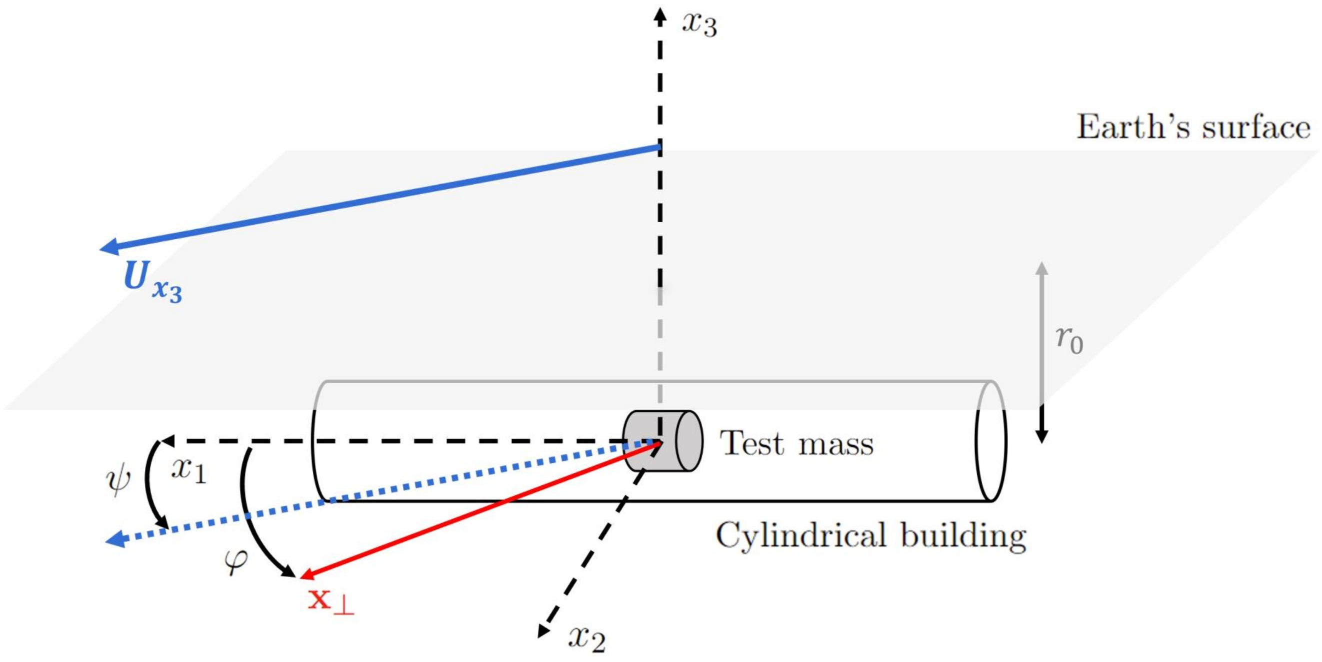

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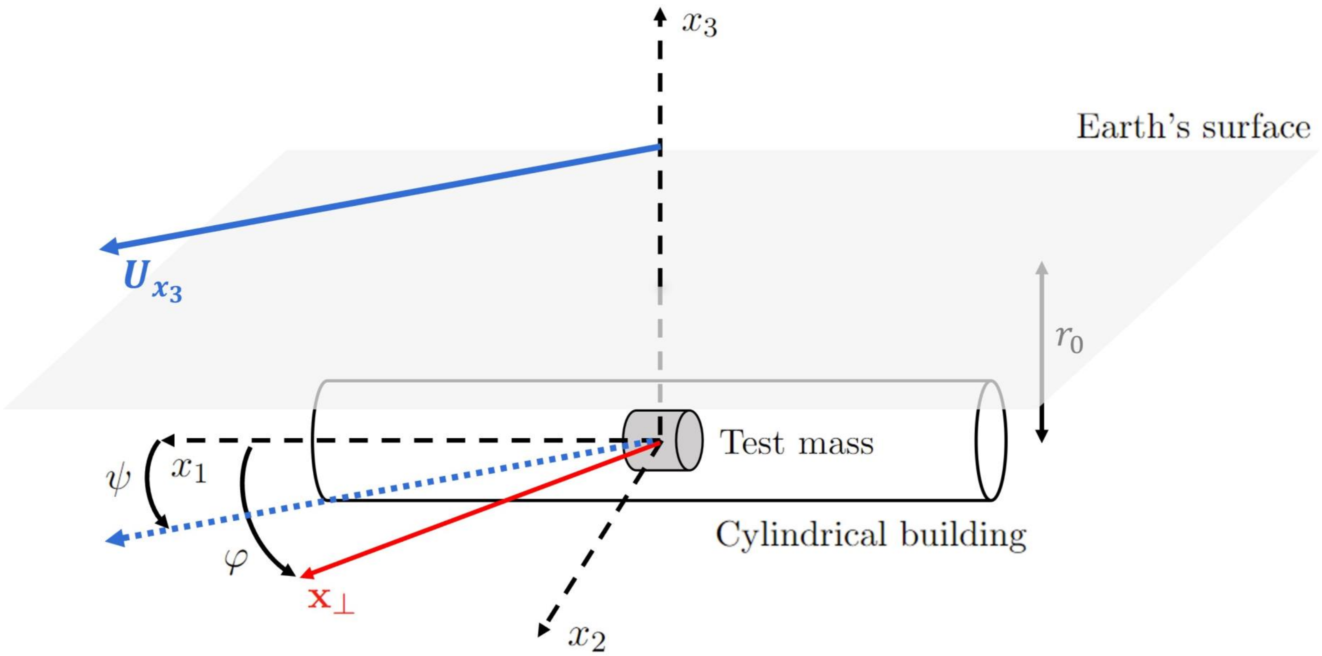


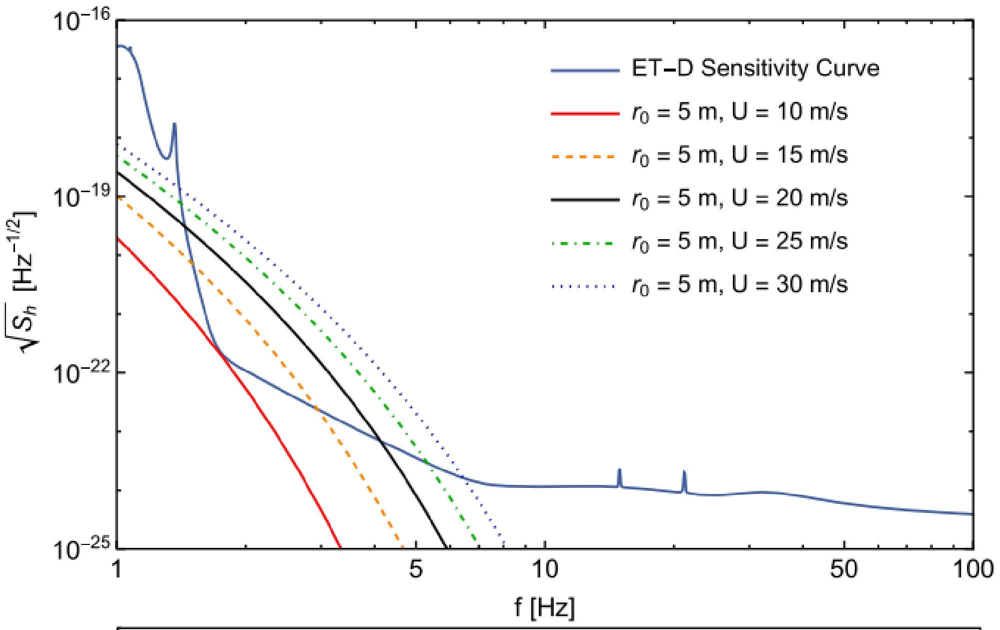

$z_0$ = roughness parameter

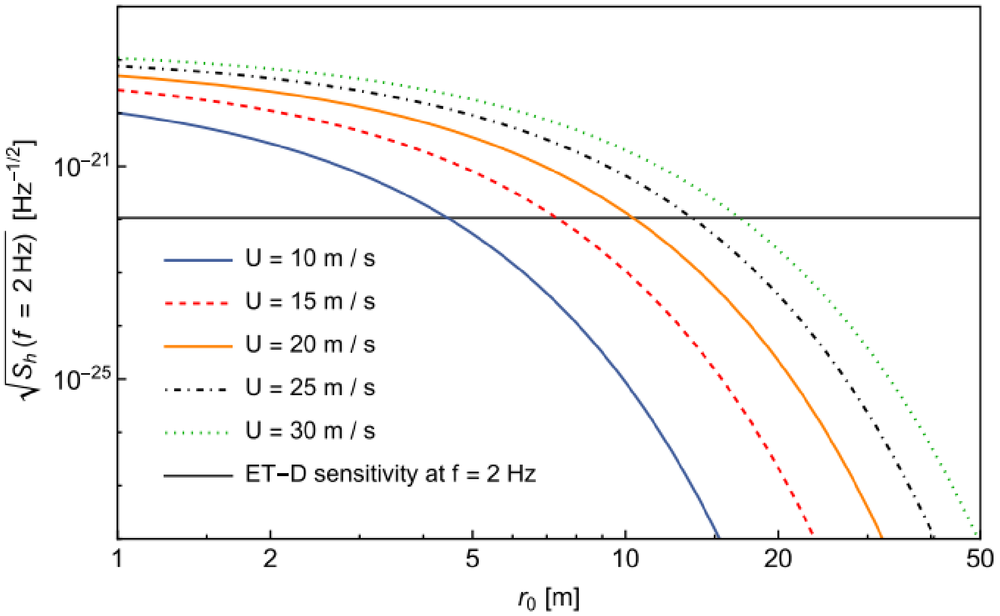
| $z_0$ [m] | Terrain | $z_0$ [m] | Terrain |
|---|---|---|---|
| 1 | City | 0.05 | Farmland (open) |
| 0.8 | Forest | 0.008 | Mown grass |
| 0.2 | Bushes | 0.005 | Bare soil (smooth) |

# Results — frozen HI

From HI turbulence in the frozen limit, we can understand the general features of the noise curves, which

- Decrease with $f$
- Decrease with $r_0$
- Increase with $U$



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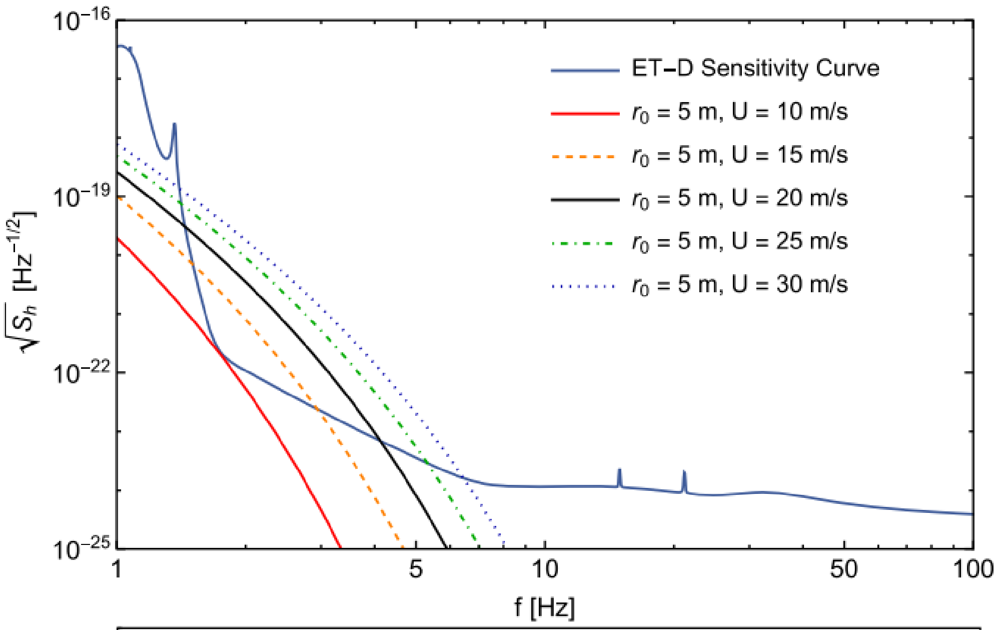
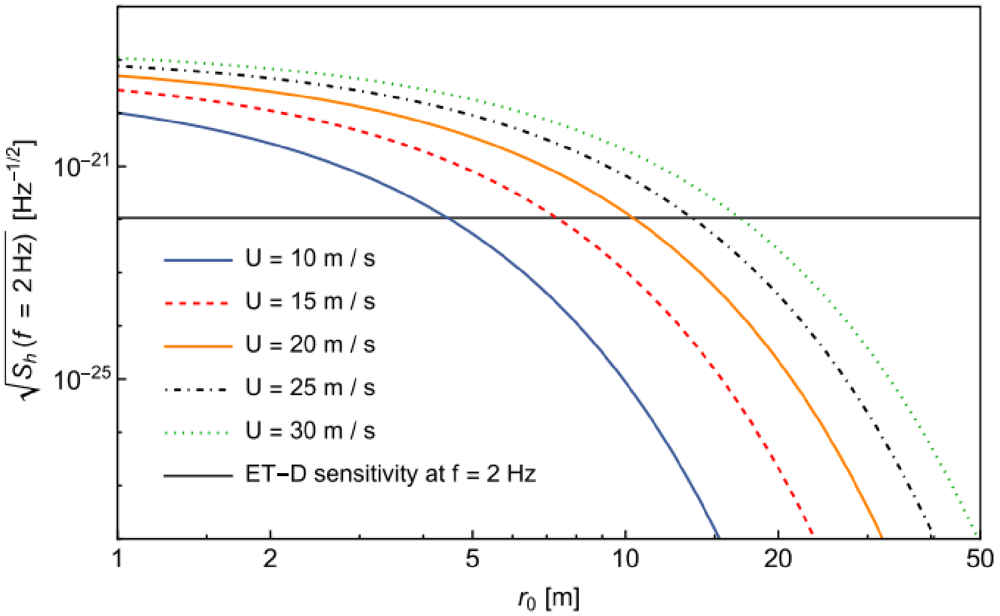
If the detector is located near the surface ($r_0 < 5 - 10$ m):
**noise larger than the sensitivity!**



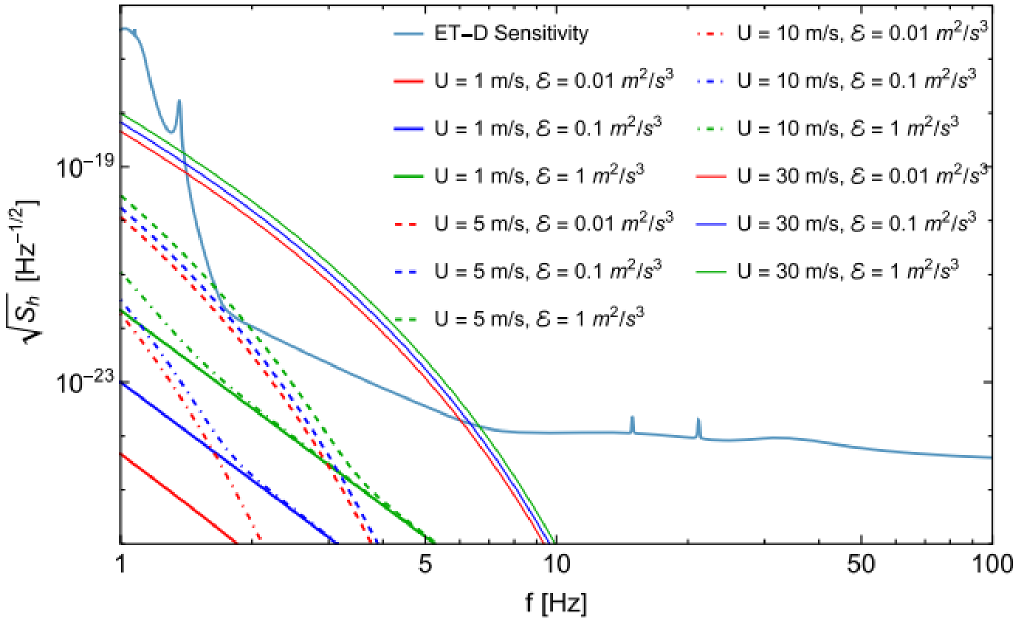



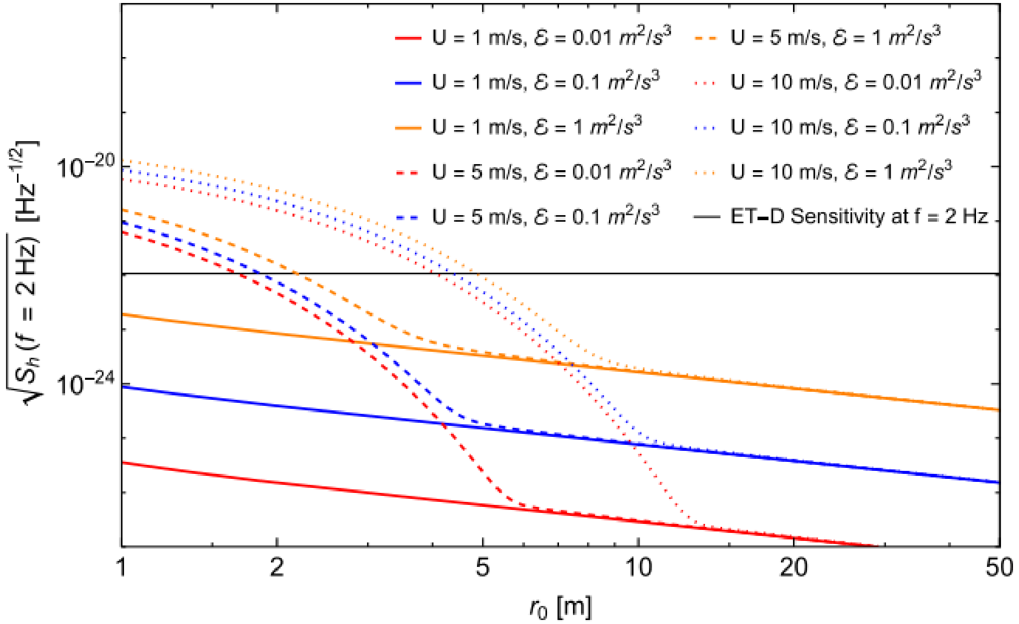
# Results — decaying HI

$\mathcal{E}$ = energy dissipation rate per unit mass

Frozen contribution is dominant at small frequencies and depths

The decay of eddies introduces a new power law behavior at large $f$ and at large $r_0$



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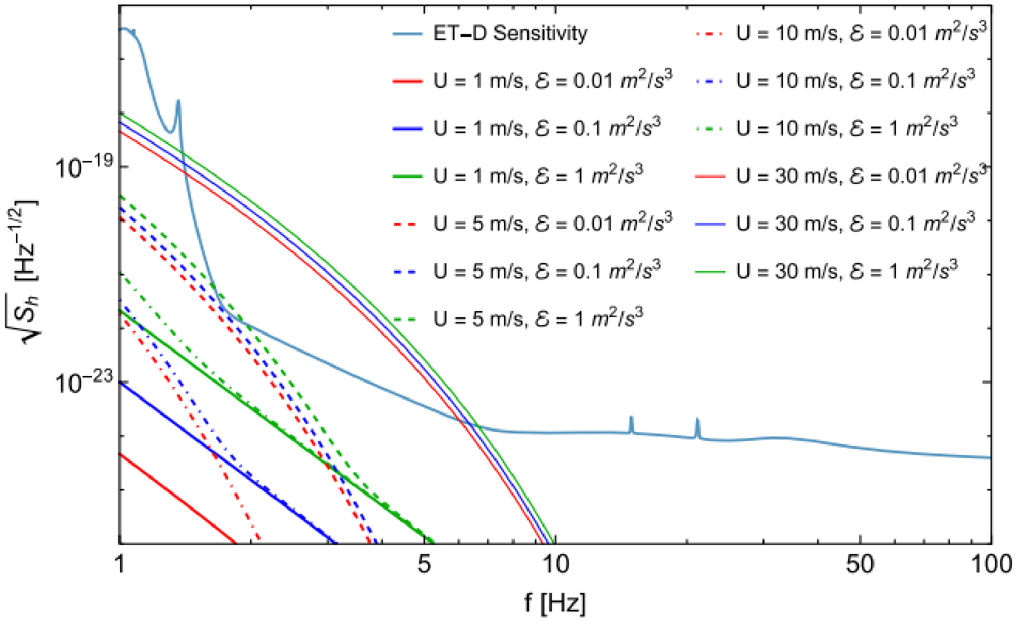
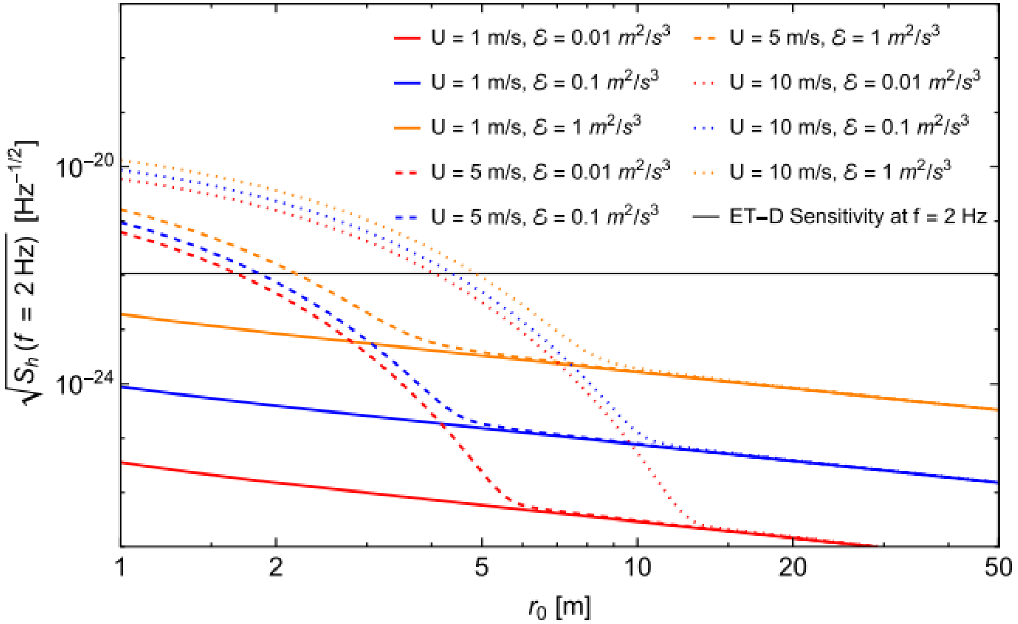
**Power spectrum scaling as $1/r_0^2$**
**Noise amplitude scaling as $1/r_0$**



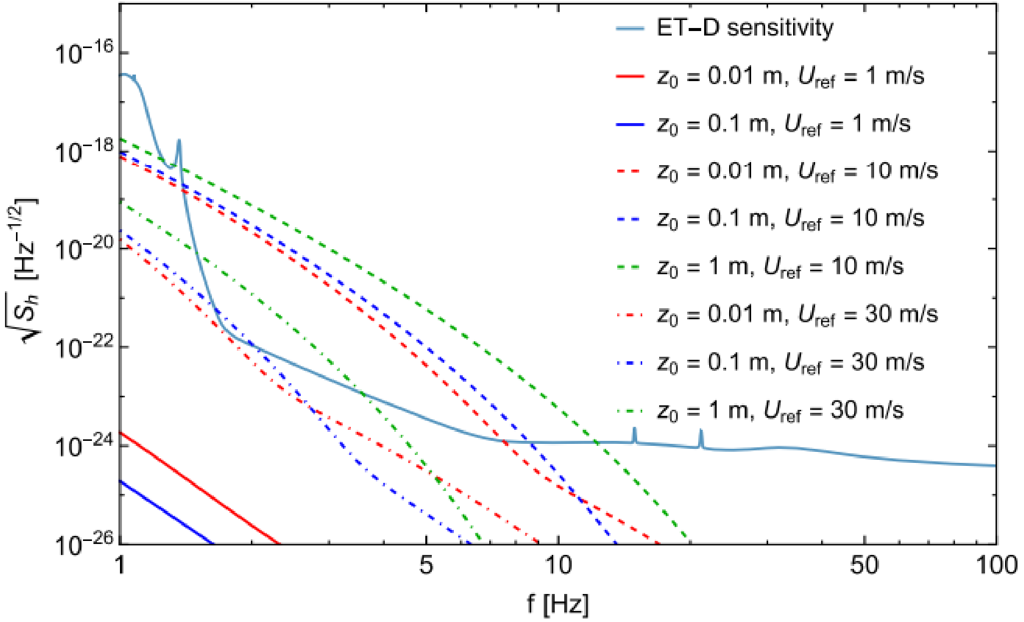
# Results - Inhomogeneous

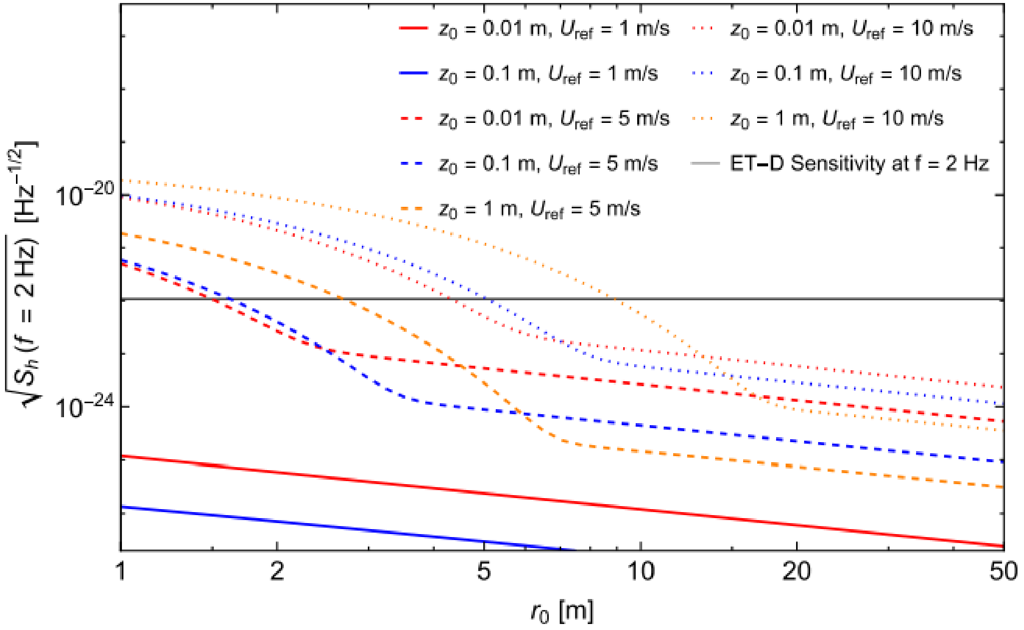
Again, frozen contribution is dominant at small frequencies and depths

Power law behavior at large $f$ and $r_0$

Weak dependency on $z_0$ at small $f$ and $r_0$

Power spectrum scaling as $\mathbf{1/r_0^2}$
Noise amplitude scaling as $\mathbf{1/r_0}$



# Conclusions

We improved previous models for the atmospheric contribution to NN by studying

- Time decay of turbulent structures
- Inhomogeneities along the vertical direction
- Dependency of NN from the detector depth

General features

- Noise is always suppressed at large frequencies
- Noise is partially mitigated placing the detector underground

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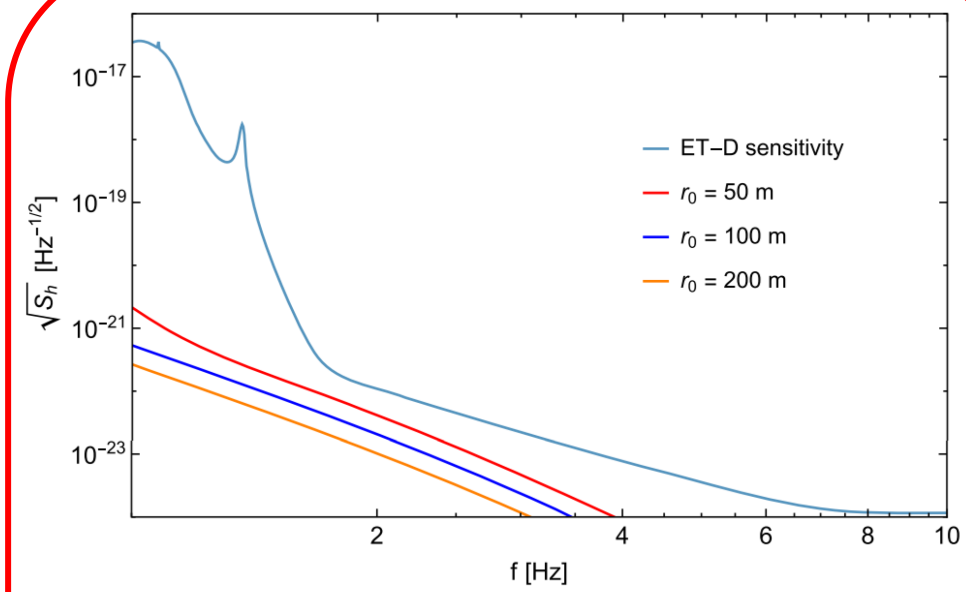


**Passive mitigation can be insufficient!**

# Thanks for your attention!

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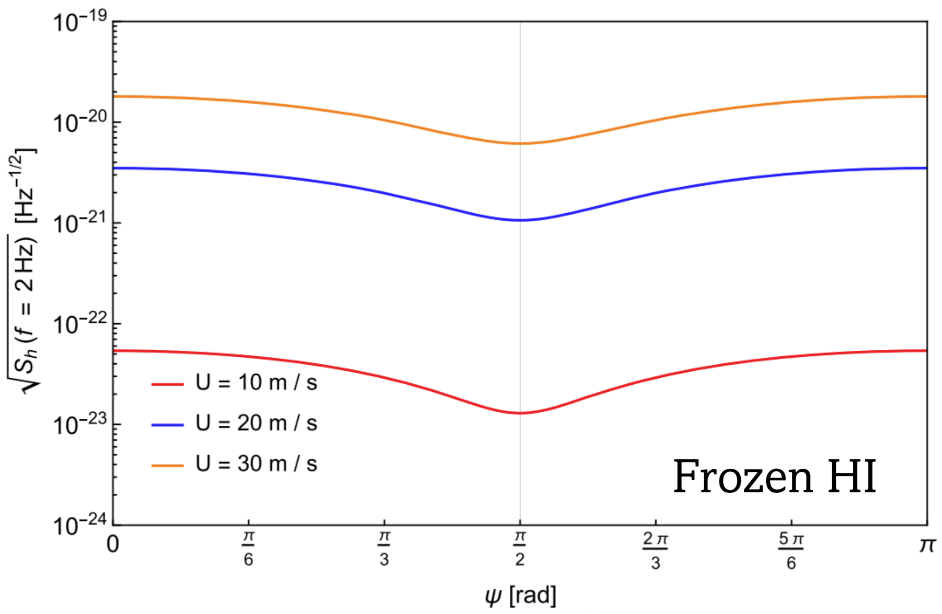
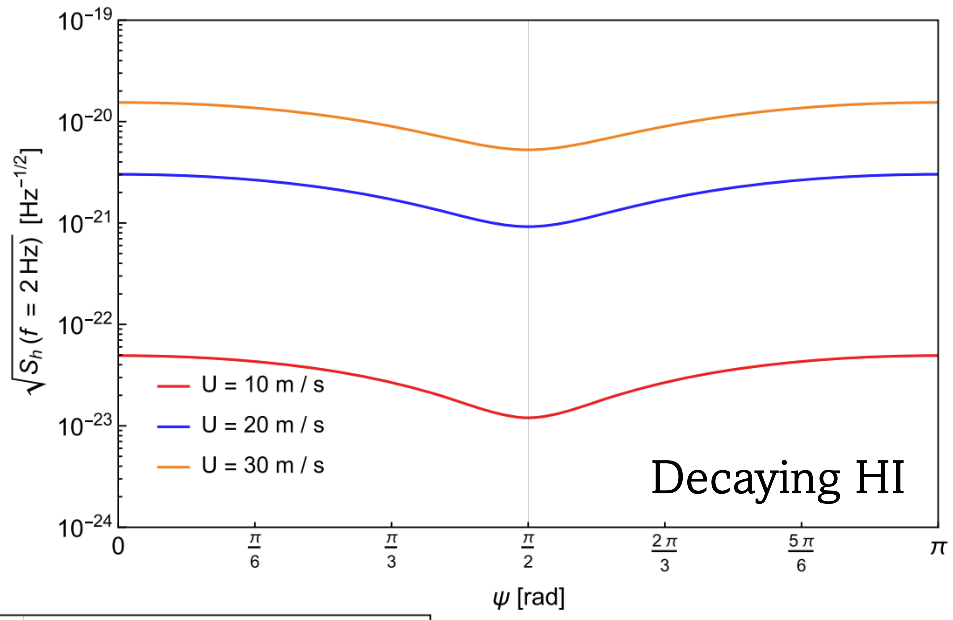
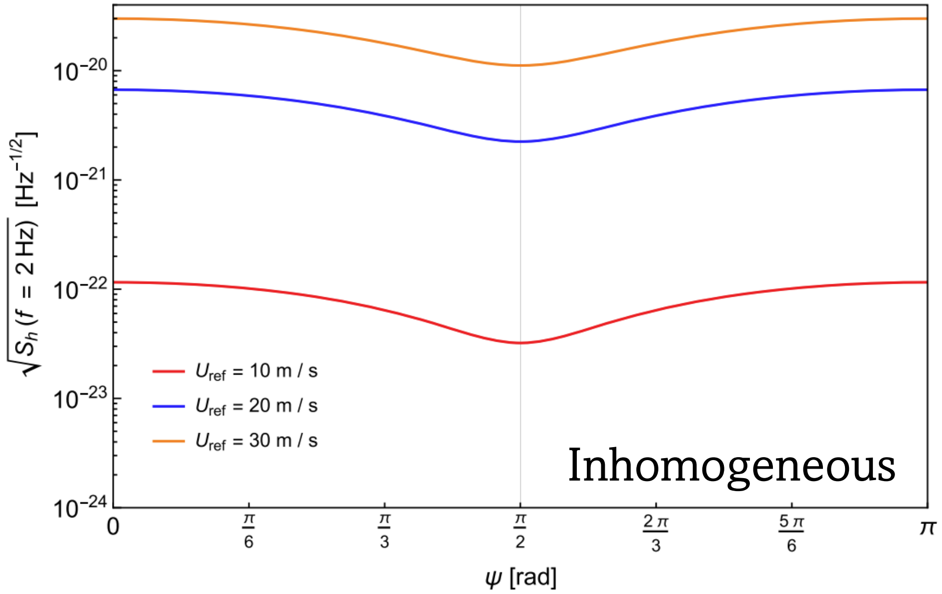
# Backup slides

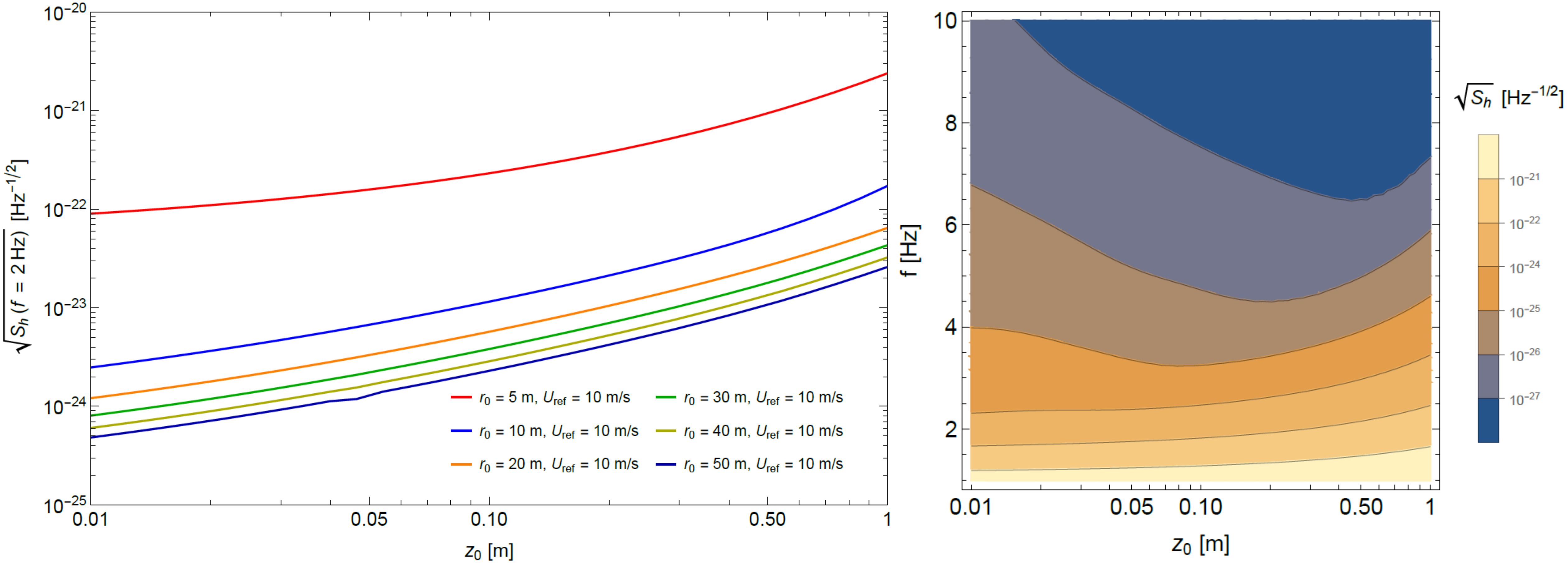
# Dependence on the wind angle



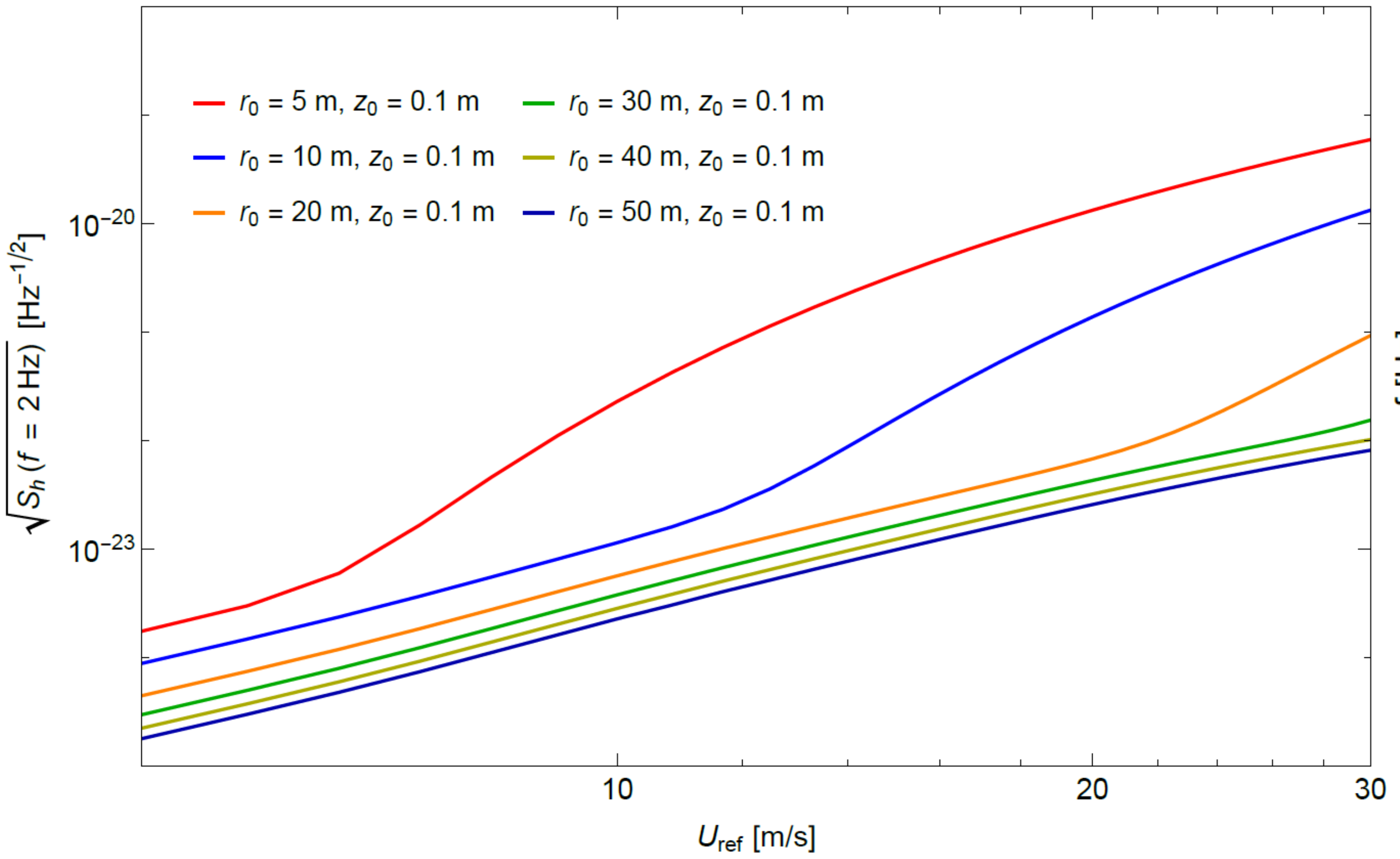



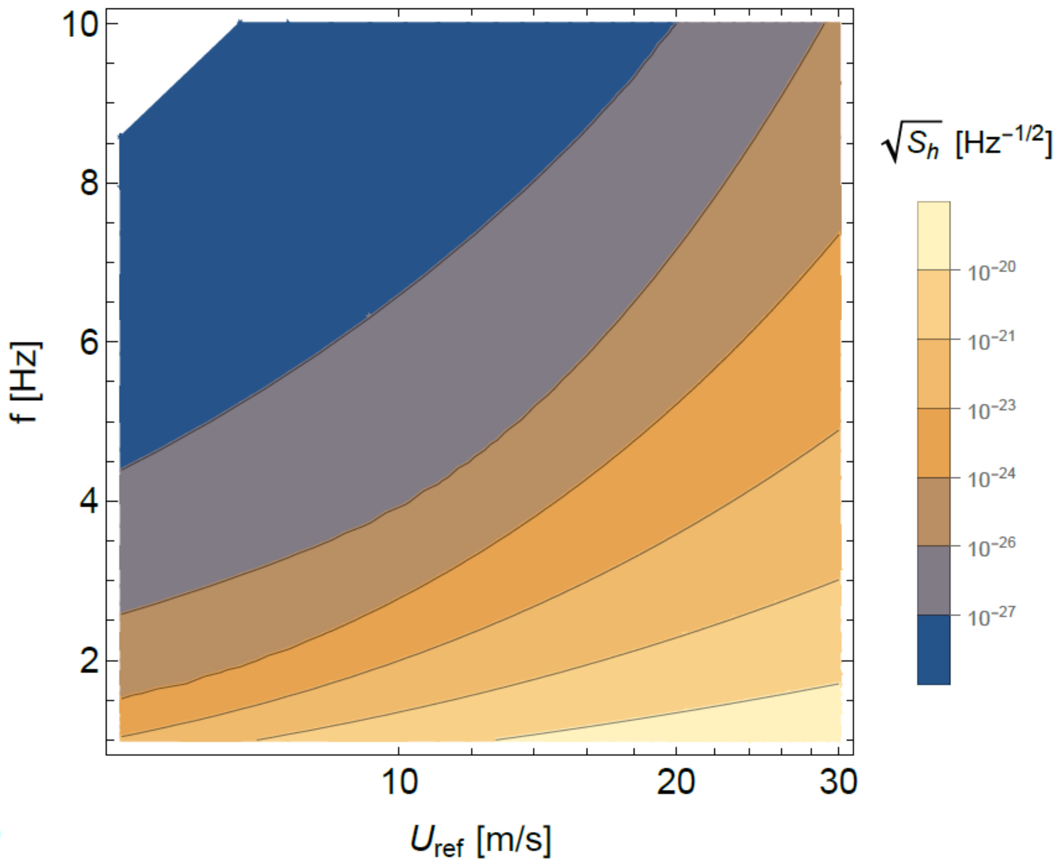



# Dependence on roughness

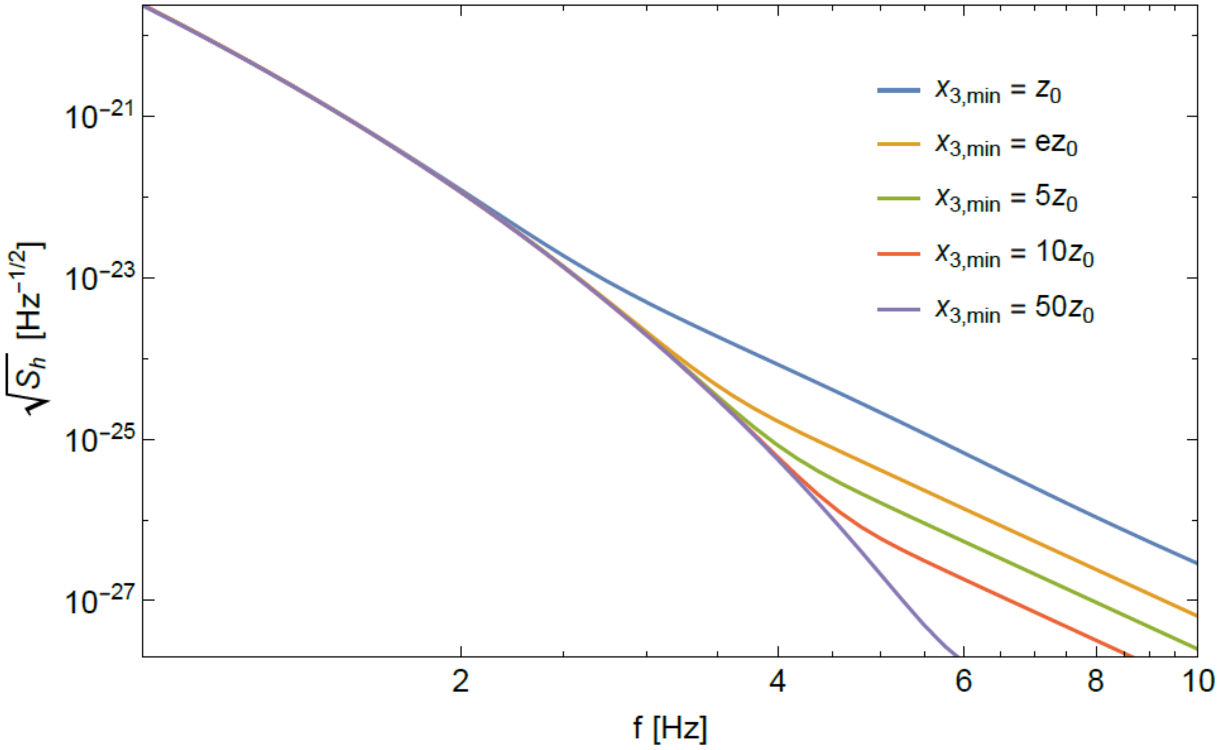


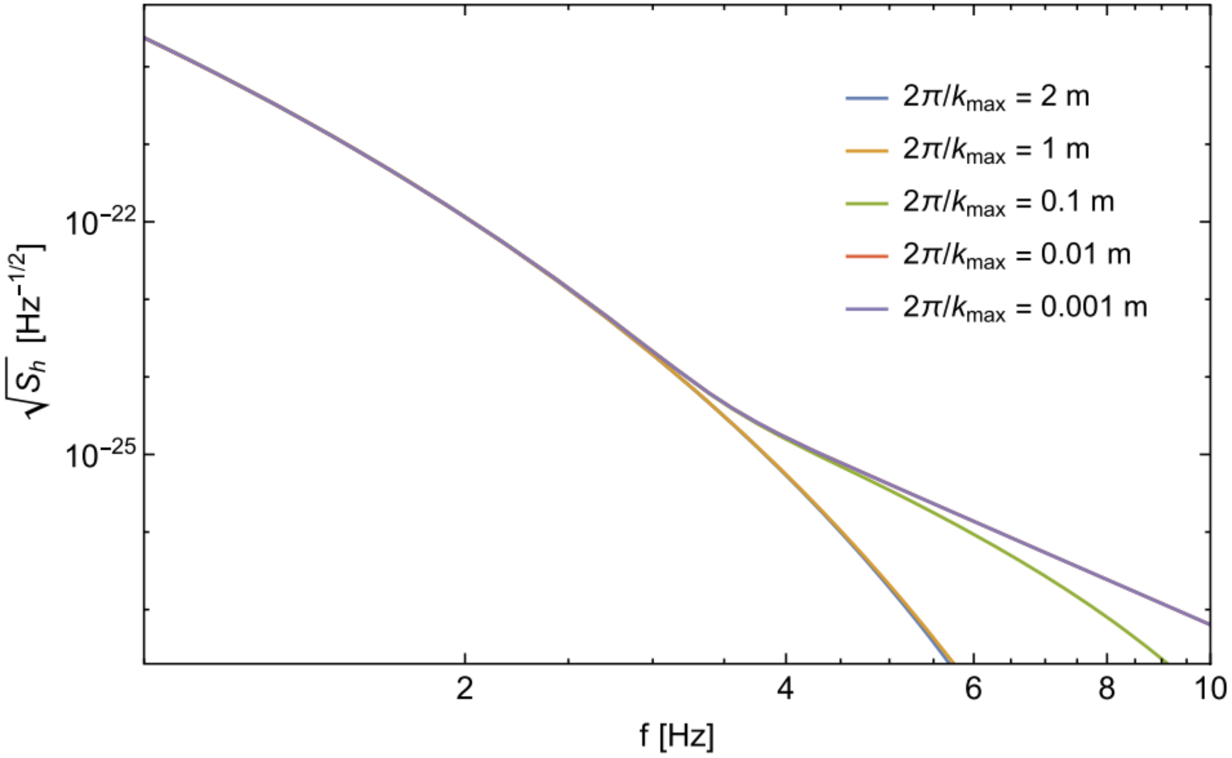
# Dependence on wind speed, Inhomogeneous



# Dependence on cutoffs, Inhomogeneous



# Explicit form of correlation functions, frozen HI

$\mathcal{E}$ = energy dissipation rate per unit mass
$\mathcal{E}_T$ = energy dissipation rate per unit mass per Kelvin

In the wind reference frame:

$$\langle \delta T(\boldsymbol{r}, t)\delta T(\boldsymbol{r}', 0)\rangle_U = \frac{\mathcal{C}_K \mathcal{E}_T}{(2\pi)^3 \mathcal{E}^{\frac{1}{3}}} \int d^3\boldsymbol{k}\, d\omega\, k^{-\frac{11}{3}} \delta(\omega - \boldsymbol{k}\cdot\boldsymbol{U}) e^{-i\omega t + i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}$$

Therefore:

$$f(k) = \mathcal{C}_K \mathcal{E}_T \mathcal{E}^{-2/3} k^{-11/3}$$
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