

# Advances in gravitational-wave predictions through quantum field theory methods



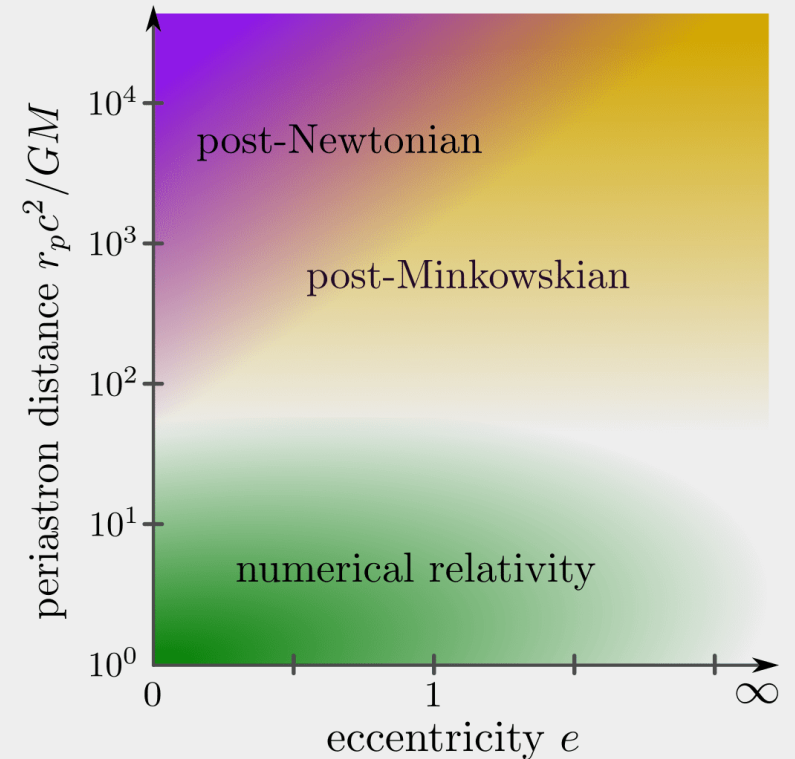
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# Post-Minkowskian (PM) expansion

- Natural pQFT approach  $\Rightarrow$  recycle tools  $G \ll 1$
- PM incorporates PN ( $v^2 \ll 1$ )  $\Rightarrow$  similar results for overlap
- PM velocity resummation increases accuracy for scattering-like events
  - large-eccentricity, hyperbolic
  - detection rate is expected to increase in 3G detectors!
  - [Khalil, Buananno, Steinhoff, Vines, '22; Damour, Retegno, '23]
  - higher accuracy needed
  - [Pürrer, Haster, '20]



[Khalil, Buananno, Steinhoff, Vines, '22]

# Post-Minkowskian expansion for non-spinning BH

		0PN	1PN	2PN	3PN	4PN	5PN	6PN	
→	1PM	1	$v^2$	$v^4$	$v^6$	$v^8$	$v^{10}$	$v^{12}$	$\times G^1$
→	2PM		1	$v^2$	$v^4$	$v^6$	$v^8$	$v^{10}$	$\times G^2$
→	3PM			1	$v^2$	$v^4$	$v^6$	$v^8$	$\times G^3$
→	4PM				1	$v^2$	$v^4$	$v^6$	$\times G^4$
	5PM					1	$v^2$	$v^4$	$\times G^5$
	6PM						1	$v^2$	$\times G^6$

## • State of the art for non-spinning BH:

- 4PM conservative [Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng, '21; CD, Kälin, Liu, Porto, '21]
- 4PM dissipative [CD, Kälin, Liu, Neef, Porto, '22]
- 4PN [Rothstein, Porto, Sturani, Foffa, Mastrolia, Marchand; Bernard, Blanchet, Faye; Damour, Jaranowski, Schäfer, ...]
- 5PN & 6PN (conservative): [Foffa, Mastrolia, Sturani, Sturm Bobadilla, '19; Blümlein, Maier, Marquard, '19; Bini, Damour, Geralico, Laporta, Mastrolia, '20]

- EFT approach:

- Post-Minkowskian expansion:

- Total momentum change through EOM:

$$e^{iS_{\text{eff}}[x_a]} = \int \mathcal{D}h_{\mu\nu} e^{iS_{\text{EH}}[h] + iS_{\text{GF}}[h] + iS_{\text{pp}}[x_a, h]}$$
$$S_{\text{eff}} = \sum_{n=0}^{\infty} \int d\tau_1 G^n \mathcal{L}_n[x_1(\tau_1), x_2(\tau_2)]$$

$$\Delta p^\mu = \text{[diagram showing a series of Feynman diagrams representing the expansion of the total momentum change through EOM. The first diagram is a single wavy line with two orange dots. The second is a wavy line with a loop and two orange dots. The third is a wavy line with two loops and two orange dots. The series continues with an ellipsis.]}$$

Other approaches:

- Scattering amplitudes [Cheung, Rothstein, Solon; Bern, Parra-Martinez, Roiban, Ruf, Shen, Zeng, ...]
- WL-QFT [Jakobsen, Mogull, Plefka, Steinhoff, '21]
- Eikonal [Di Vecchia, Heissenberg, Russo, Veneziano]
- ...

# Loop integrals

- Integrand construction:
  - standard techniques in QFT
- Example integral family:
- Dimensional regularization:

$$D = 4 - 2\epsilon$$

- Kinematic dependence fixed by DE:

- solve by transforming to canonical form

- available algorithms:

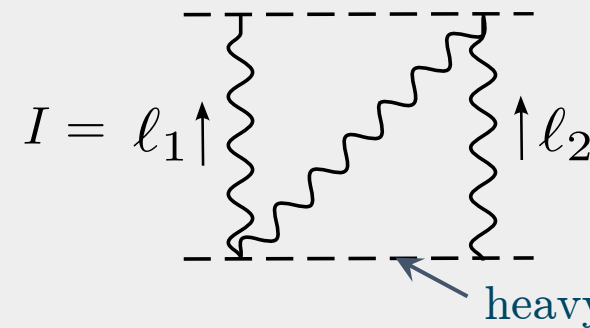
[INITIAL:  
CD, Henn, Wagner, '22;  
CD, Henn, Yan, '20]



used before for HQET integrals:  
[Brüser, CD, Henn, Yan, '20]

[Passarino, Veltman, '79]

[FORM, xAct, C++, GiNaC]



integrals very similar to  
Heavy Quark Effective Theory

[Goldberger, Rothstein, '06]

heavy particle

$$\partial_v \vec{f} = A(D, v) \vec{f}$$

[Henn, '13]

integrals to compute

# Boundary conditions

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- From PN-like limit

$$v^2 \ll 1$$

- method of regions:

[Beneke, Smirnov, '97]

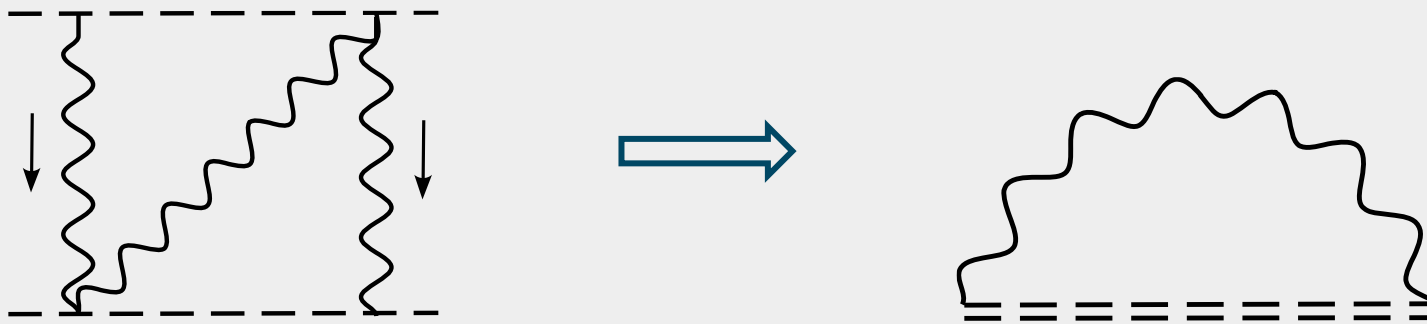
[asy2.m, Jantzen, Smirnov, Smirnov, '12; Pak, Smirnov, '10]

potential graviton:  $(\ell^0, \ell) \sim (1, v)$

radiation graviton:  $(\ell^0, \ell) \sim (v, v)$

[Goldberger, Rothstein, '06]

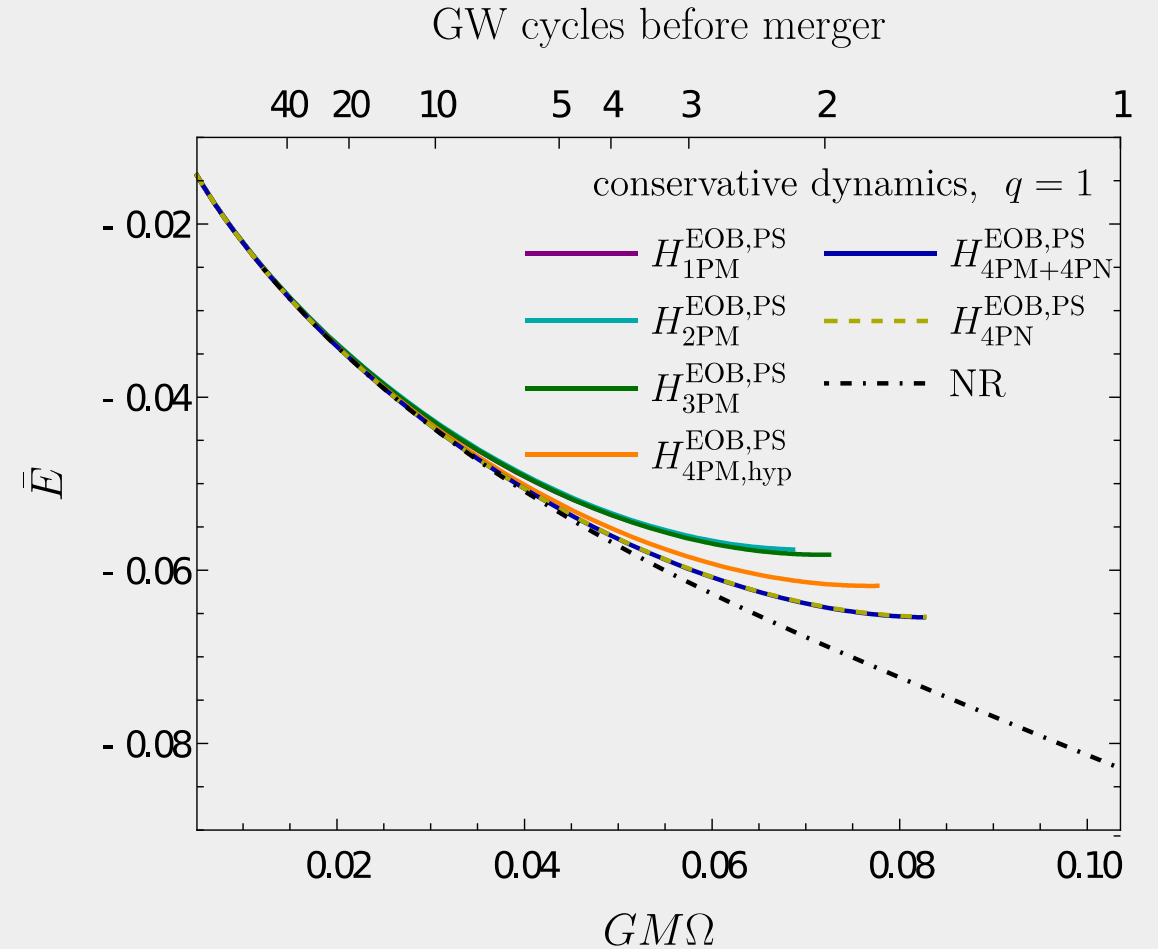
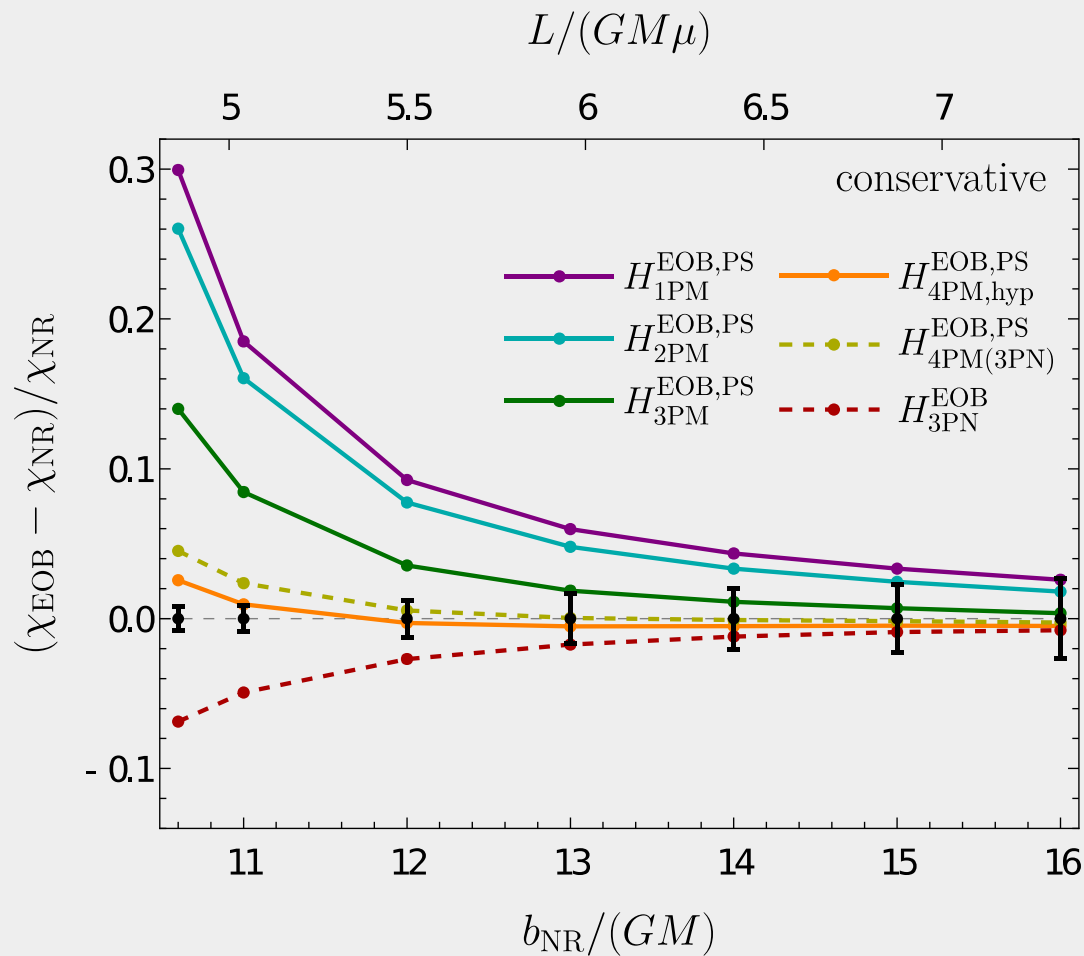
- correspond nicely to PN regions and (3d) integrals



[Galley, Leibovich, Porto, Ross, '15]

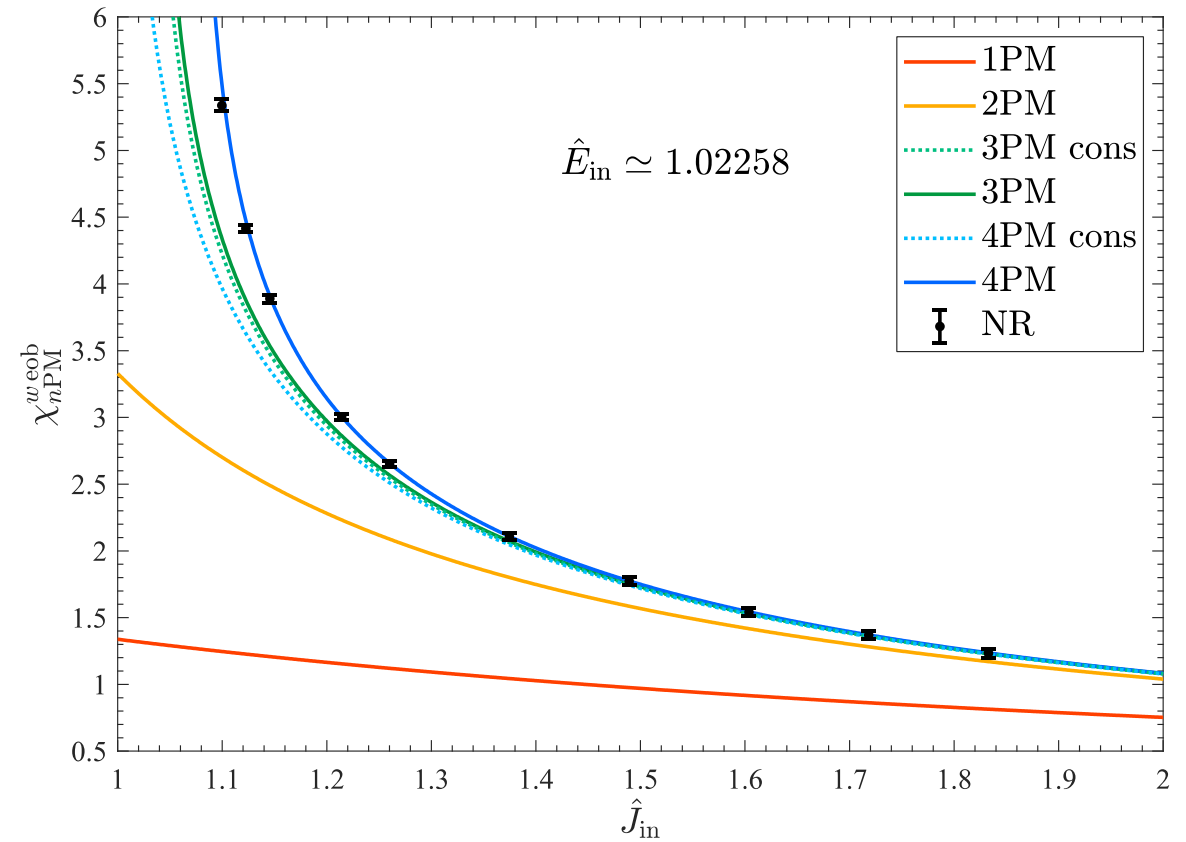
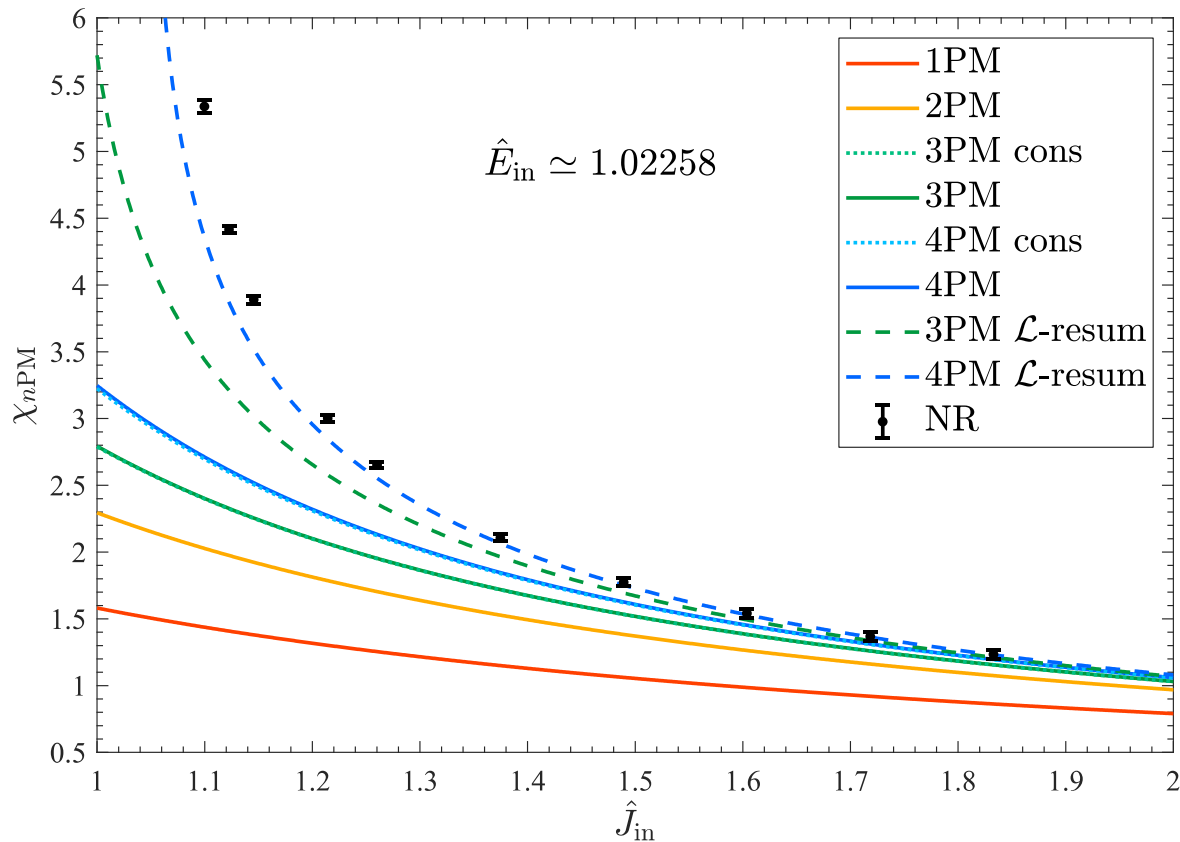
# PM conservative accuracy

[Khalil, Buananno, Steinhoff, Vines, '22]



# PM radiative state-of-the-art accuracy

[Damour, Retegno, '22]





# Challenges

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- Detector accuracy
  - Higher PM, spin, internal structure, ...
  - Adjust QFT tools more to our needs
  - Continue scattering to bound state data  
[B2B: Kälin, Porto, '19; Cho, Kälin, Porto, '21]
  - Incorporate PM more directly into waveform generation (EOB)  
[Damour, '16; Damgaard, Vanhove, '21; Khalil, Buananno, Steinhoff, Vines, '22; Damour, Rettegno, '23; ...]