Response of the Einstein Telescope to Doppler anisotropies

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This talk is based on: D. Chowdhury, G. Tasinato, and I. Zavala, *Phys. Rev. D* **107**, *083516 (2023) (arXiv:2209.05770 [gr-qc])*.

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Gravitational wave perturbation

The two-point correlator of the gravitational wave modes is

$$\langle h^*_\lambda(f,\hat{n})\, h_{\lambda'}(f',\hat{n}')\rangle = \frac{\delta_{\lambda\lambda'}}{2}\,\delta(f-f')\,\frac{\delta^{(2)}(\hat{n}-\hat{n}')}{4\pi}\,\mathcal{I}(f)\,\underbrace{\mathbf{P}(f,\hat{n})}_{\substack{\text{Factorizable ansatz}\\ = H(f)\,\mathbf{P}(\hat{n}).}}.$$

First lack of assumption: We do not assume that $\mathbf{P}(f,\hat{n})$ is factorizable in this way.

The detector response function can be expressed as[1]

$$\mathcal{R}_{A_{BC},X_{YZ}}(f,t,t') = \sum_\lambda \int \frac{d^2\hat{n}}{4\pi}\,\mathbf{P}(f,\hat{n})\,F^{(\lambda)}_{A_{BC}}(\hat{n},t)F^{(\lambda)}_{X_{YZ}}(\hat{n},t').$$

[1] G. Mentasti and M. Peloso, JCAP **03**, 080 (2021).

Kinematic anisotropies

The GW energy density in a boosted frame B moving with relative velocity $\vec{v}$ with respect to frame A is given by[2]

$$\begin{aligned}\Omega_{\rm GW}^{(B)}(f,\hat{n}) &= \mathcal{D}^4\,\Omega_{\rm GW}^{(A)}\left(\mathcal{D}^{-1}\,f\right) \\ &= \frac{4\pi^2}{3\,H_0^2}\,f^3\,\mathcal{I}(f)\,\mathbf{P}_{\rm kin}(f,\,\hat{n}),\end{aligned}$$

where

$$\mathbf{P}_{\rm kin}(f,\,\hat{n}) = \frac{\mathcal{D}}{\mathcal{I}(f)}\,\mathcal{I}(\mathcal{D}^{-1}\,f),$$

and

$$\mathcal{D} = \frac{\sqrt{1-\beta^2}}{1-\beta\,\hat{n}\cdot\hat{v}}.$$

Second lack of assumption: We do not choose $\beta(=v/c)$ to be small. We consider $0 \leq \beta < 1$.

No anisotropies: $\mathbf{P}_{\rm kin}(f,\,\hat{n}) = 1$.

[2] G. Cusin and G. Tasinato, JCAP **08**, 036 (2022).

Computing the ET response function

The response function for the Einstein Telescope can be computed to be

$$\begin{aligned}\mathcal{R}_{\mathcal{O},\mathcal{O}'}(f,t,t') &= \frac{4}{5}\left[1+\frac{5}{2}c_1(f)\right] d^{ab}_{\mathcal{O}}(t)\, d_{\mathcal{O}'\,ab}(t') \\ &+4\,c_2(f)\,\left(\hat{v}_a\, d^{ab}_{\mathcal{O}}(t)\, d_{\mathcal{O}'\,bc}(t')\,\hat{v}^c\right) \\ &+c_3(f)\,\left(\hat{v}^a\hat{v}^b\, d_{ab\,\mathcal{O}}(t)\right)\,\left(\hat{v}^c\hat{v}^d\, d_{cd\,\mathcal{O}'}(t')\right),\end{aligned}$$

with the following coefficients:

$$c_1 = \frac{K_1}{8}+\frac{3\,K_2}{4}+\frac{K_3}{8}\;;\; c_2 = \frac{3\,K_1}{8}-\frac{3\,K_2}{4}-\frac{5\,K_3}{8}\;;\; c_3 = \frac{3\,K_1}{8}-\frac{15\,K_2}{4}+\frac{35\,K_3}{8},$$

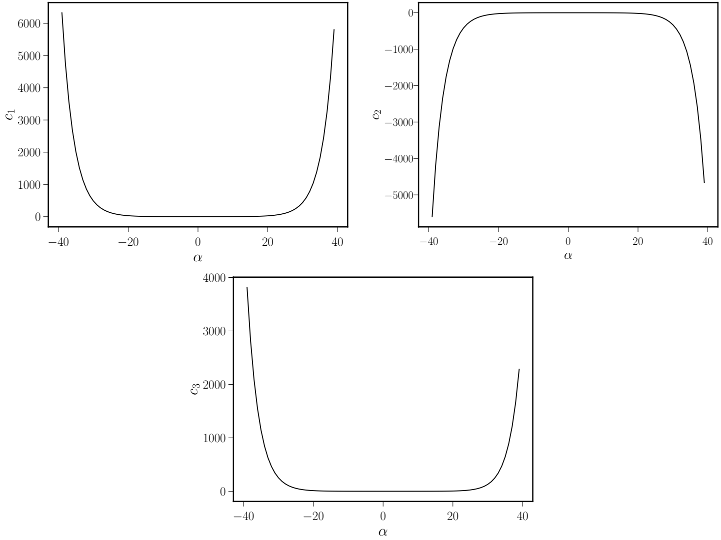
where

$$K_1 = \int\frac{d^2\hat{n}}{4\pi}\,(\mathbf{P}_{\rm kin}-1)\;;\; K_2 = \int\frac{d^2\hat{n}}{4\pi}\,(\mathbf{P}_{\rm kin}-1)\,(\hat{n}\cdot\hat{v})^2\;;\; K_3 = \int\frac{d^2\hat{n}}{4\pi}\,(\mathbf{P}_{\rm kin}-1)\,(\hat{n}\cdot\hat{v})^4.$$

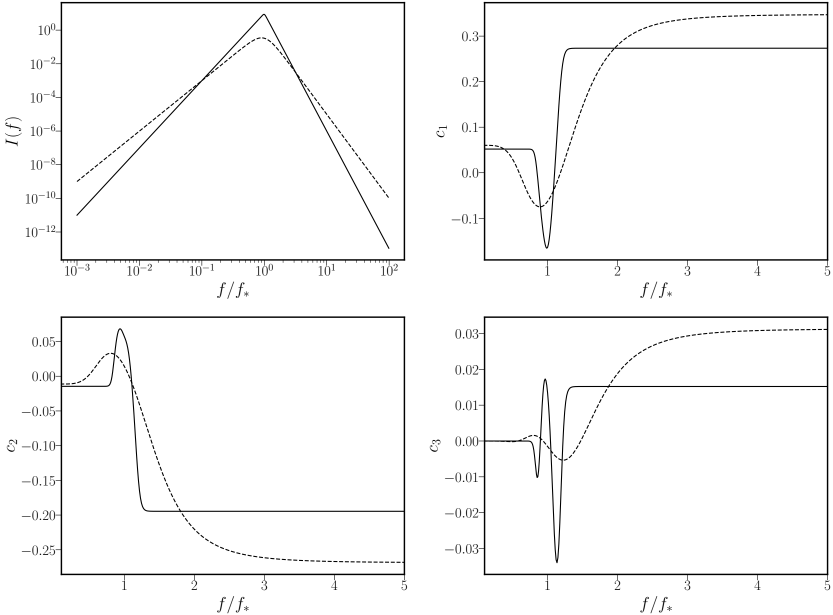
Power-law SGWB profile

$$\mathbf{P}_{\text{kin}} = \mathcal{D}^{1-\alpha}.$$

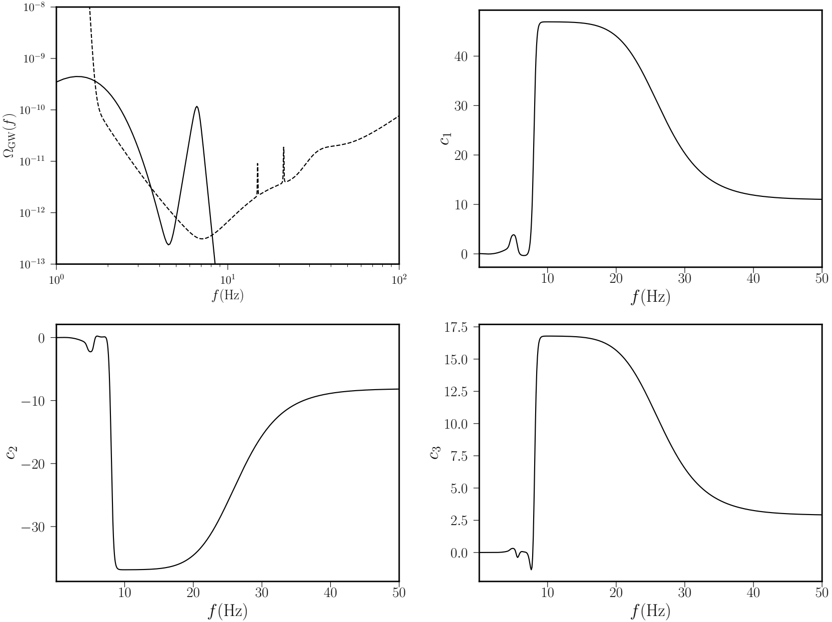
- When $\beta = 0$, we get $c_i = 0 \; \forall \; i$ and $\forall \; \alpha$, as expected.
- Curiously, we also find that $c_i = 0 \; \forall \; i$ when $\alpha = 1$, irrespective of the value of β.



Broken power-law SGWB profile

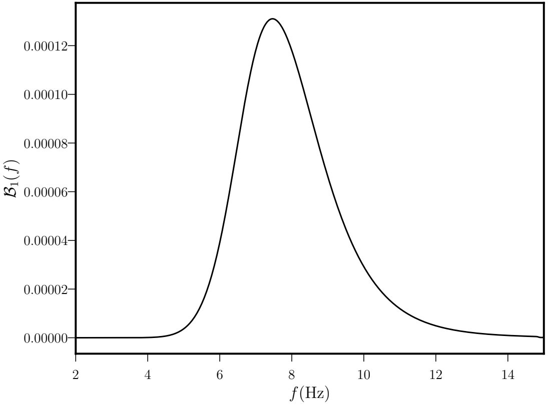


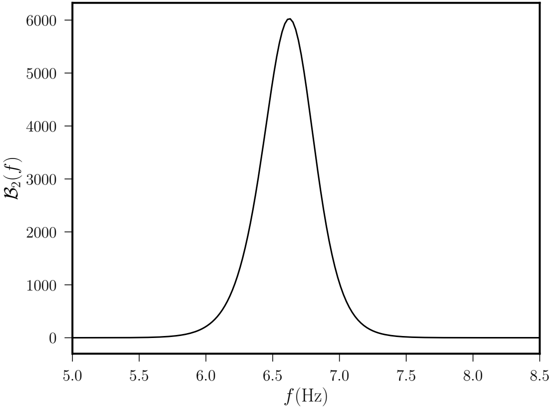
Double power-law SGWB profile



Signal-to-noise ratio

$$\mathrm{SNR}_4 = \left(\sqrt{\frac{T}{1\mathrm{year}}} \times \frac{\Omega_{\mathrm{GW}}^{(0)}}{10^{-12}}\right) \left(\frac{\Sigma_{\mathcal{O}\mathcal{O}'}\, d_{\mathcal{O}\,ab}\, d_{\mathcal{O}'\,cd}\, v_\perp^a\, v_\perp^b\, v_\perp^c\, v_\perp^d}{\frac{4}{5}\,\Sigma_{\mathcal{O}\mathcal{O}'}\, d_{\mathcal{O}\,ab}\, d_{\mathcal{O}'}^{ab}}\right) \left(\int_{f_{\min}}^{f_{\max}} \mathrm{d}f\, \mathcal{B}_{1,2}(f)\right)^{1/2} .$$





Summary

- We have considered the effect of kinematic anisotropies on the SGWB power spectra.
- We have computed the response function of the Einstein Telescope to these anisotropies.
- We have considered three types of profiles for the SGWB power spectra and investigated each case.
- We have estimated the signal-to-noise ratio that could be detected by the Einstein Telescope for each of these spectra.

Thank you!