

Forecasts for Λ CDM and Dark Energy models through Einstein Telescope mock data

Based on [Califano, De Martino, Vernieri, Capozziello, arxiv: 2205.11221 ET-0111A-22](#)

and [arxiv: 2208.13999 ET-0188A-22](#)

Gravitational Waves as Standard Sirens

GW Observables

- With GWs detectors we can measure the amplitude and the phase evolution of the signal

$$h_+ = \frac{2c}{D_L} \left(\frac{G\mathcal{M}^z}{c^3} \right)^{\frac{5}{3}} f^{\frac{2}{3}} (1 + \cos^2 i) \cos 2\Phi(t) \quad h_\times = \frac{2c}{D_L} \left(\frac{G\mathcal{M}^z}{c^3} \right)^{\frac{5}{3}} f^{\frac{2}{3}} \cos i \sin 2\Phi(t)$$

- From the observation, we can estimate

$$\tau_c = \frac{f}{\dot{f}}, \quad D_L \propto \frac{c}{(f^2 \tau_c h)}, \quad \mathcal{M}^z = \frac{(1+z)(m_1 m_2)^{\frac{3}{5}}}{(m_1 + m_2)^{\frac{1}{5}}} \propto f^{-\frac{11}{5}} \dot{f}^{\frac{3}{5}}$$

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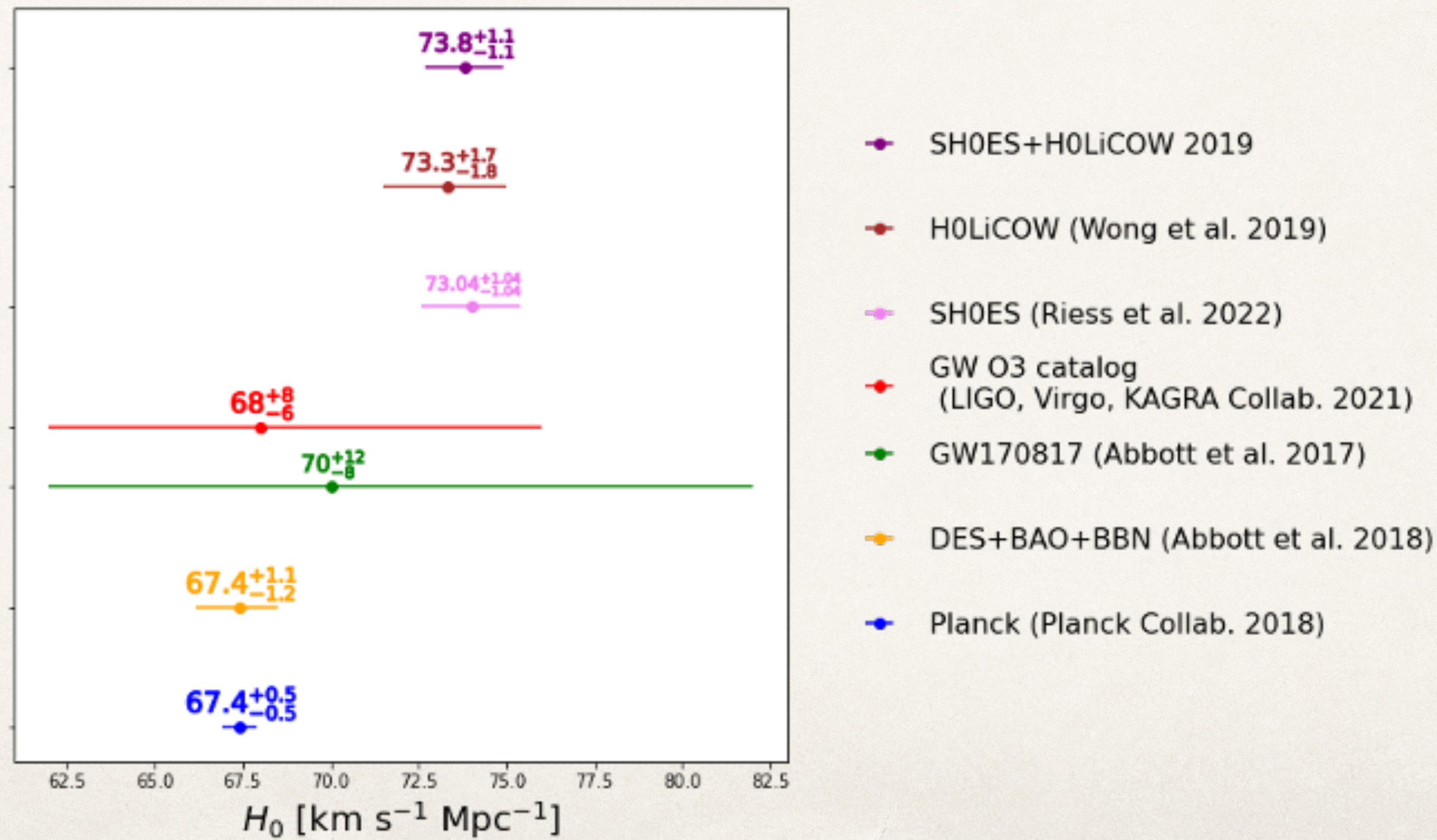
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- Binary inspiral allows for a determination of the distance to the source without any reference to the cosmic distance ladder.
- GWs suffer of mass-redshift degeneracy.

H_0 tension



Construction of mock sources catalog

- We assume a fiducial cosmological model Λ CDM
 $\Lambda_{cosmo} = \{H_0 = 67.66 \text{ km s}^{-1} \text{ Mpc}^{-1}, \Omega_{m,0} = 0.3111, \Omega_{k,0} = 0.00, \Omega_{\Lambda,0} = 0.6889\}$

- Given a cosmology the theoretical luminosity distance will be

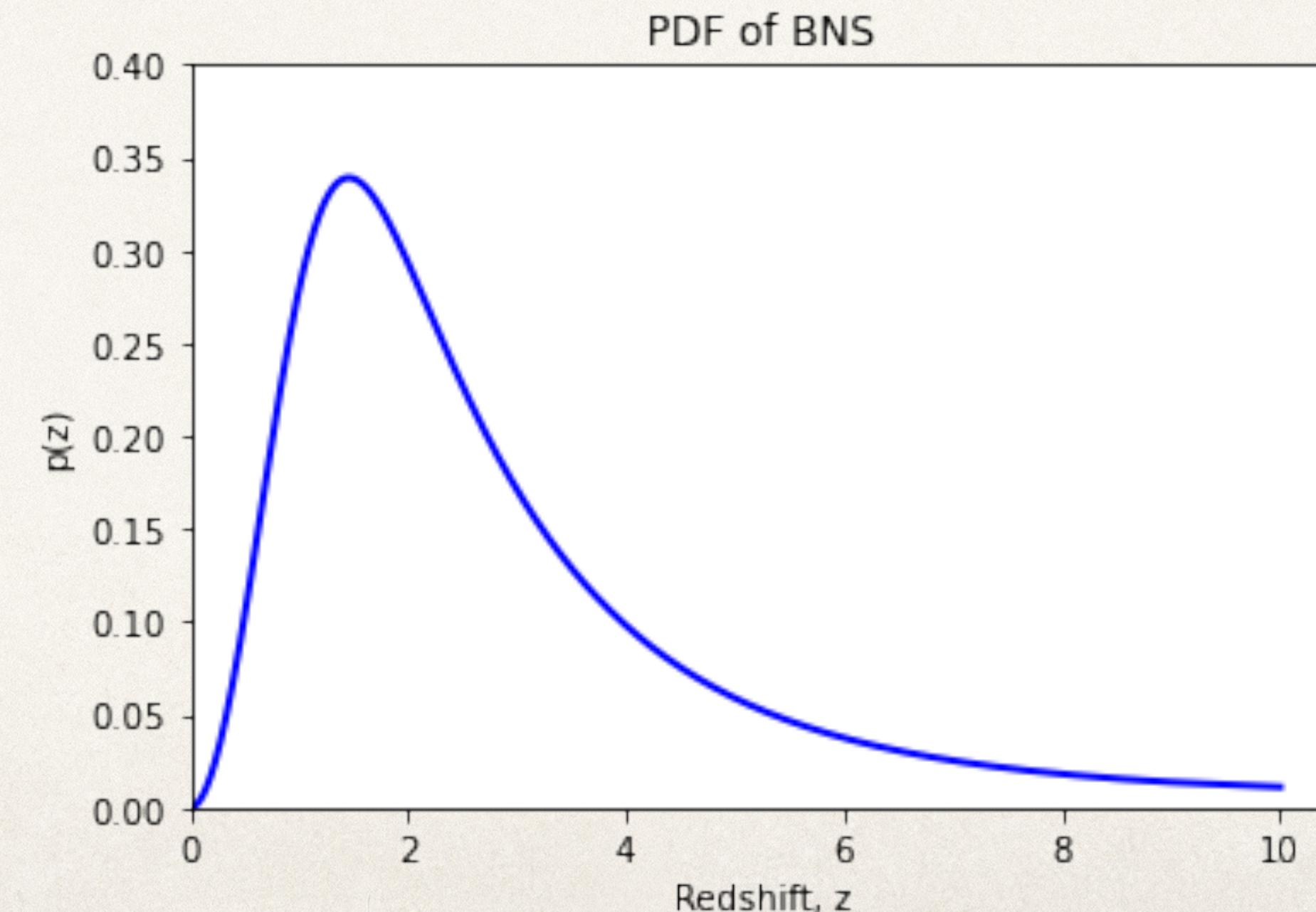
$$D_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z', \Lambda_{cosmo})}$$

- We extract the redshift from a normalized probability distribution

$$p(z) = \frac{R_z(z)}{\int_0^{10} R_z(z) dz}$$

$$R_z(z) = \frac{R_m(z)}{1+z} \frac{dV(z)}{dz}$$

$$R_m(z) = R_0 \int_{t_{min}}^{t_{max}} R_f[t(z) - t_d] P(t_d) dt_d$$



- To generate the synthetic signals of GWs, we assume the NS mass distribution be uniform in the interval $[1, 2.5] M_{\odot}$.
- We select sky angles θ and φ from an isotropic distribution, and orientation angle i and the polarization ψ from uniform distribution.
- We estimate the SNR ρ for the ET and we retain an event if $\rho > 9$.
- We extracted d_L from a Gaussian distribution $\mathcal{N} \left(d_L^{fid}, \sigma_{d_L} \right)$

$$\sigma_{d_L} = \sqrt{\sigma_{inst}^2 + \sigma_{lens}^2 + \sigma_{pec}^2}$$

Selection of electromagnetic counterpart

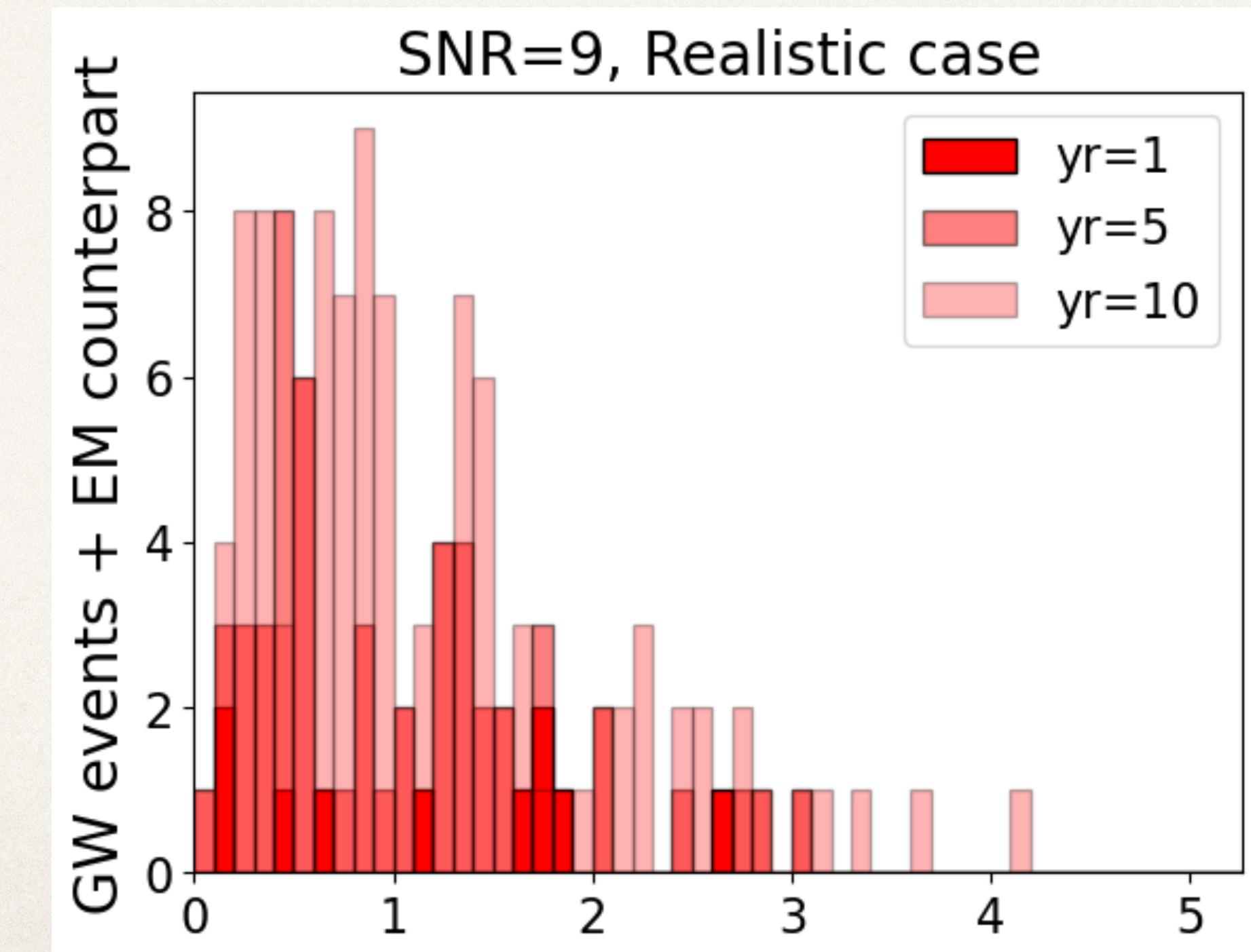
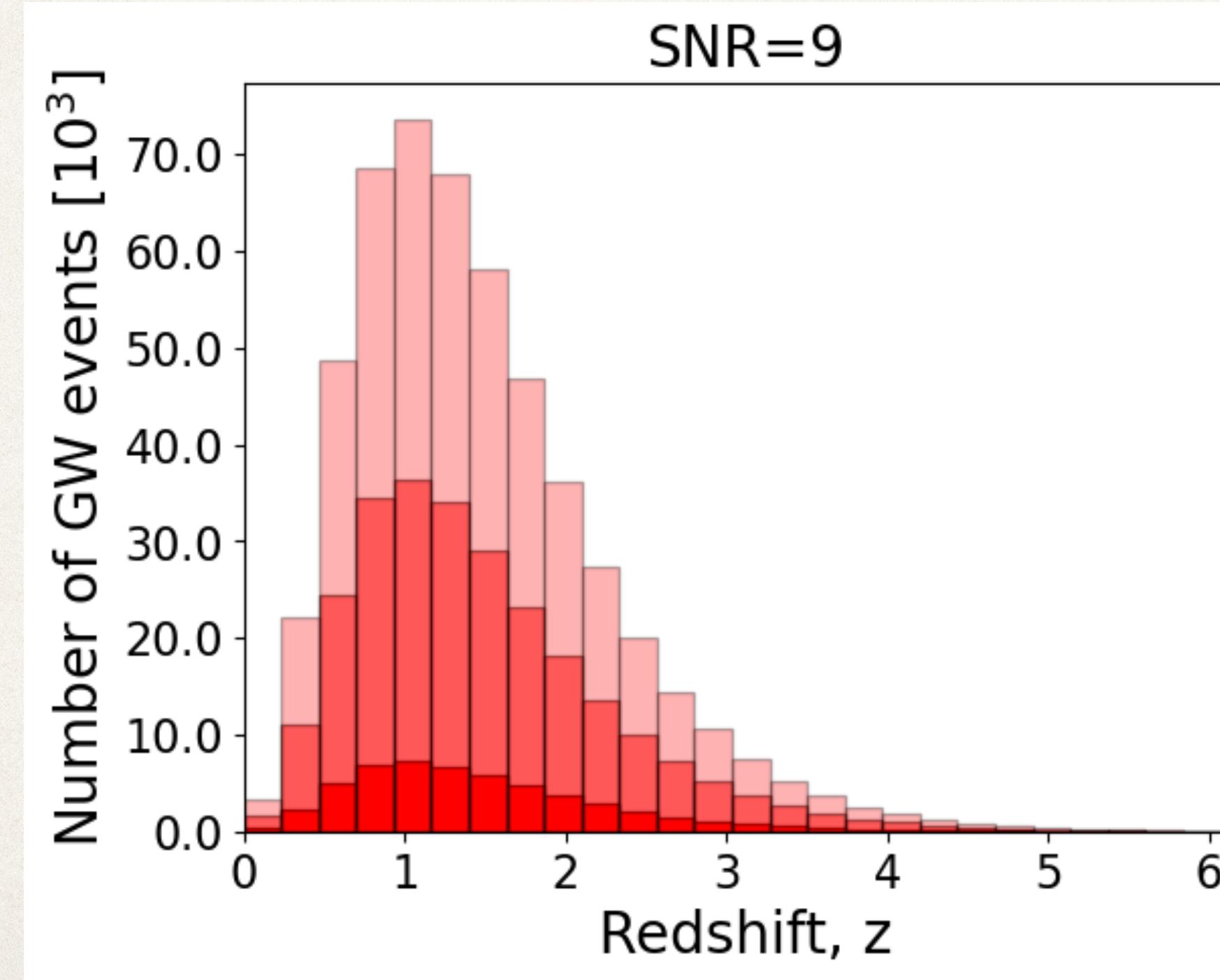
- We estimate the flux for the coincident short-GRB

$$F(\theta_V) = \frac{L(\theta_V)(1+z)}{4\pi d_L^2 k(z) b}$$
$$b = \frac{\int_{1\text{ keV}}^{10^4\text{ keV}} E N(E) dE}{\int_{E_1}^{E_2} N(E) dE}$$
$$k(z) = \frac{\int_{E_1}^{E_2} N(E) dE}{\int_{E_1(1+z)}^{E_2(1+z)} N(E) dE}$$

- We record the combined event if $F(\theta_V) > F^t$ $\left(= 0.2 \frac{\text{photon}}{\text{cm}^2 \text{ s}} \right)$ for the THESEUS-XGIS satellite.

GW events distribution

- ❖ Imposing a SNR equal 9, we estimate a rate of GW signals of $\sim 10^4$ events/year.
- ❖ We estimate a rate of combined detection with the THESEUS-XGIS satellite of ~ 11 events/year.



Analysis

CASE I: Bright Sirens

- We include in the single-event likelihood the selection effects $\rho > \rho_t$, $F(\theta_V) > F_t$

$$p(d_i | \Lambda_{cosmo}) = \frac{\int p(d_i | D_L, \Lambda_{cosmo}) p_{pop}(D_L | \Lambda_{cosmo}) dD_L}{\int p_{det}(D_L, \Lambda_{cosmo}) p_{pop}(D_L | \Lambda_{cosmo}) dD_L}$$

[Mandel et al. MNRAS (2019)]



$$p_{pop}(D_L | \Lambda_{cosmo}) = \delta(D_L^{th}(\Lambda_{cosmo}) - D_L)$$



$$p(d_i | D_L, \Lambda_{cosmo}) \propto \exp -\frac{1}{2} \frac{(d_i - D_L)^2}{\sigma_{d_i}^2}$$



$$p_{det}(D_L, \Lambda_{cosmo}) = \int_{\rho > \rho_t, F > F_t} p(d_i | D_L, \Lambda_{cosmo}) dd_i$$

CASE II: Dark Sirens

- ❖ When we cannot extract the redshift information from electromagnetic signal, we have to marginalise over the redshift

$$p(d_i | \Lambda_{cosmo}) = \int p(d_i | D_L, \Lambda_{cosmo}) p_{pop}(D_L | \Lambda_{cosmo}) p(z) dz dD_L$$

[Ding et al. JCAP (2019)]

Results

$K\Lambda CDM$

$$E^2(z) = \Omega_{m,,0}(1+z)^3 + \Omega_{K,0}(1+z)^2 + \Omega_{\Lambda,0}$$

	σ_{H_0}	$\sigma_{\Omega_{K,0}}$	$\sigma_{\Omega_{\Lambda,0}}$	
Bright Sirens	0.79	0.16	0.13	
Dark Sirens	0.04	0.01	0.01	

$K\omega CDM$

$$E^2(z) = \Omega_{m,,0}(1+z)^3 + \Omega_{K,0}(1+z)^2 + \Omega_{\Lambda,0}(1+z)^{3(1+\omega)}$$

	σ_{H_0}	$\sigma_{\Omega_{K,0}}$	$\sigma_{\Omega_{\Lambda,0}}$	σ_{ω}
Bright Sirens	0.80	0.18	0.18	0.93
Dark Sirens	0.06	0.02	0.03	0.10

Interacting DE

$$E^2(z) = \Omega_{m,,0}(1+z)^3 + \Omega_{\Lambda,0} \left[(1+z)^{3(1+\omega^{eff})} + \frac{\xi}{3\omega^{eff}}(1 - (1+z)^{3\omega^{eff}})(1+z)^3 \right], \quad \omega^{eff} = \omega + \frac{\xi}{3}$$

	σ_{H_0}	$\sigma_{\Omega_{m,0}}$	σ_{ξ}	
Bright Sirens	1.03	0.14	0.88	
Dark Sirens	0.05	0.01	0.06	

Time-Varying G

$$E^2(z) = \Omega_{m,0}(1+z)^{3-\delta_G} + \Omega_{\Lambda,0}(1+z)^{\delta_G \frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}}$$

	σ_{H_0}	$\sigma_{\Omega_{m,0}}$	σ_{δ_G}
Bright Sirens	0.95	0.09	0.44
Dark Sirens	0.04	0.01	0.02
Emergent DE	$E^2(z) = \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} \left[\frac{1 + \tanh(\Delta \log(\frac{1+z}{1+z_t}))}{1 + \tanh(\Delta \log(1+z_t))} \right]$		
	σ_{H_0}	$\sigma_{\Omega_{m,0}}$	σ_{Δ}
Bright Sirens	0.86	0.06	0.86
Dark Sirens	0.03	0.02	0.01

Conclusions

- ✿ In the analysis, we distinguish the catalogs depending on whether the redshift information comes from the GRB (Bright Sirens) or the BNS merger rate (Dark Sirens). We assume the rate is a priori known.
- ✿ We show the huge capability of ET to solve the Hubble tension independently by the theoretical framework chosen, achieving an accuracy on the Hubble constant less than 1%.
- ✿ The ET standard sirens will represent an alternative approach to constrain the cosmological parameters and the DE models.

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THANK YOU!
QUESTIONS?

Backup slides

Open Issue: Redshift information

- ❖ Host galaxy identification

[Schutz, Nature, 1986]

- ❖ Source sky localization error.

- ❖ σ_{d_L} error.

- ❖ Cross-Correlation

[Mukherjee and Wandelt, 2018]

- ❖ Overlapping sky area between GW sources and galaxy surveys.
- ❖ The accurate redshift estimation of galaxies.

- ★ Coincident short GRB

- ★ Only 0.1% of GW events could have a detected counterpart.

- Prior information on redshift

- We need to assume astrophysical model for the p(z).

- Tidal deformation

[Messenger and Read, PRL, 2012]

- We need high precision in the signal analysis.

- It depends on neutron star equation of state.

Impact of different assumptions

- ❖ Star Formation Rate:

- ◆ Vangioni model, $R_f(z) = \frac{\nu a \exp(b(z - z_m))}{a - b + b \exp(a(z - z_m))}$

[Vangioni et al., MNRAS, 2015]

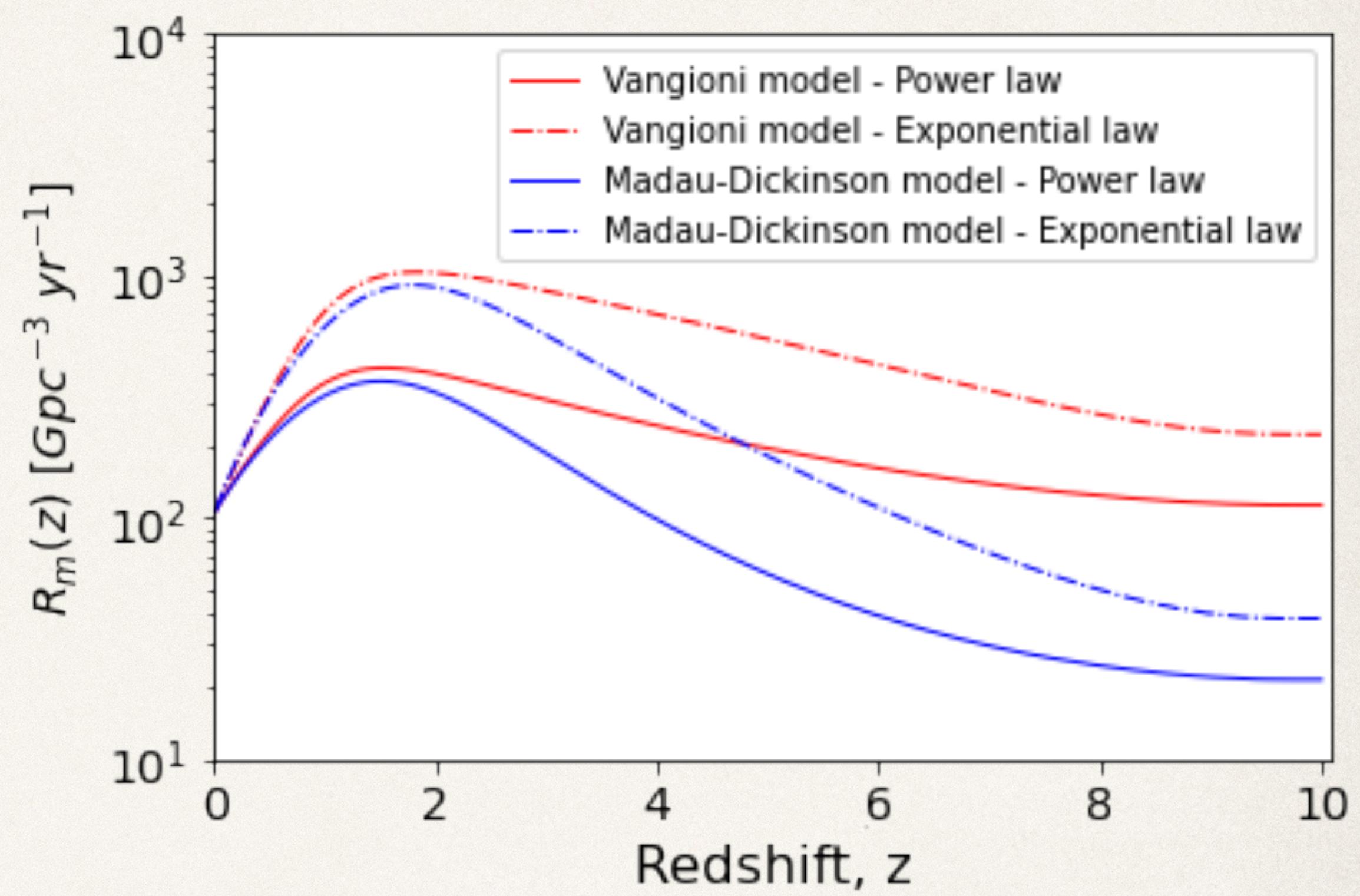
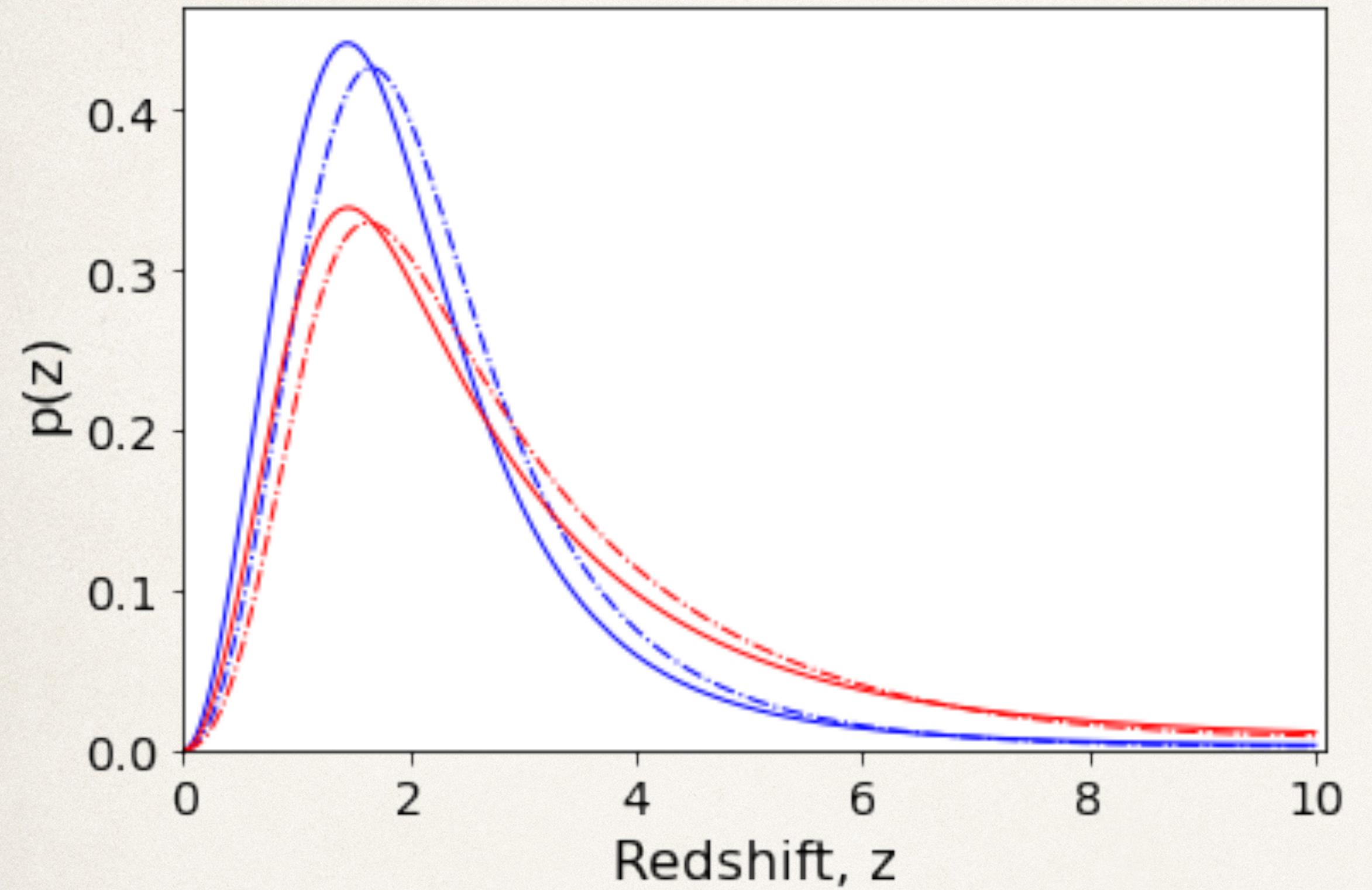
- ◆ Madau-Dickinson model, $R_f(z) = \frac{1 + z)^\alpha}{1 + \left[\frac{1 + z}{C}\right]^\beta}$

[Madau and Dickinson, ARAA, 2014]

- ❖ t_d probability distribution:

- ★ Power law, $P(t_d) = t_d^{-1}$

- ★ Exponential, $P(t_d) = \tau^{-1} \exp\left(-\frac{t}{\tau}\right)$



GW + EM events				
MODEL	# events	H_0	$\Omega_{k,0}$	$\Omega_{\Lambda,0}$
Baseline model	332	$67.47^{+0.39}_{-0.40}$	$-0.08^{+0.08}_{-0.09}$	$0.72^{+0.07}_{-0.07}$
Model 1	603	$67.18^{+0.34}_{-0.32}$	$0.01^{+0.07}_{-0.07}$	$0.65^{+0.06}_{-0.06}$
Model 2	271	$67.48^{+0.30}_{-0.30}$	$-0.09^{+0.09}_{-0.10}$	$0.71^{+0.07}_{-0.07}$
Model 3	536	$67.20^{+0.27}_{-0.28}$	$0.01^{+0.08}_{-0.07}$	$0.65^{+0.05}_{-0.06}$

Dark Sirens				
MODEL	# events	H_0	$\Omega_{k,0}$	$\Omega_{\Lambda,0}$
Baseline model	521552	$67.68^{+0.04}_{-0.04}$	$0.00^{+0.01}_{-0.01}$	$0.69^{+0.01}_{-0.01}$
Model 1	1143212	$67.64^{+0.04}_{-0.04}$	$0.00^{+0.01}_{-0.01}$	$0.69^{+0.01}_{-0.01}$
Model 2	443560	$67.62^{+0.05}_{-0.05}$	$0.01^{+0.01}_{-0.01}$	$0.68^{+0.01}_{-0.01}$
Model 3	966659	$67.68^{+0.04}_{-0.04}$	$-0.01^{+0.01}_{-0.01}$	$0.68^{+0.01}_{-0.01}$

Table 6: The *baseline* model adopts the *Vangioni model* for the SFR and the *power law* form of the time delay distribution; **Model 1** is based on the *Vangioni model* for the SFR and the *exponential distribution* of the time delay distribution; **Model 2** is based on the *Madau - Dickison model* for the SFR and the *power law* form of the time delay distribution; **Model 3** is based on the *Madau - Dickison model* for the SFR and the *exponential distribution* of the time delay distribution.

- We extracted d_L from a Gaussian distribution $\mathcal{N}\left(d_L^{fid}, \sigma_{d_L}\right)$:

$$\sigma_{d_L} = \sqrt{\sigma_{inst}^2 + \sigma_{lens}^2 + \sigma_{pec}^2}$$

$$\sigma_{inst} = \frac{2}{\rho} d_L(z)$$

[Cutler and Flanagan, PRD (1994)]

[Dalal et al., PRD (2006)]

$$\sigma_{lens} = F_{delens}(z) \left[0.066 \left(\frac{1 - (1+z)^{-0.25}}{0.25} \right)^{1.8} d_L(z) \right]$$

[Speri et al., PRD (2021)]

$$\sigma_{pec} = \left[1 + \frac{c(1+z)^2}{H(z)d_L(z)} \right] \frac{\sqrt{\langle v^2 \rangle}}{c} d_L(z)$$

[Kocsis et al., Astrop. J. (2006)]