XIII ET Symposium

Cagliari, 8-12 May 2023

EINSTEIN TELESCOPE

Fisher Information Matrix for ET Forecasts:

How Informative is it?

OSB9 Division - Wednesday May 10th

GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

Ulyana Dupletsa

Different Fisher Matrix Codes

Various groups have developed their own Fisher matrix codes:

gwbench

S. Borhanian, 2021, *Class.Quant.Grav.*

GitLab link

GWFAST

F. Iacovelli,
M. Mancarella, et al.
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In OSB 9 division we compare and cross-check them all!

Quick Recap of Fisher Matrix Formalism

In the **high Signal to Noise Ratio (SNR) limit** and **linearized signal approximation**, the likelihood function can be approximated as a **multivariate Gaussian** with covariance matrix given by the inverse of the Fisher matrix:

$$p(d|\vec{\theta}) \approx \exp\left(-\frac{1}{2}\langle n - \left(\theta^i - \theta^i_0\right)\partial_{\theta^i} h(\vec{\theta}_0) | n - \left(\theta^i - \theta^i_0\right)\partial_{\theta^i} h(\vec{\theta}_0)\rangle\right)$$

$$\approx \exp\left(-\frac{1}{2}\langle n|n\rangle\right) + \exp\left(-\frac{1}{2}\Delta\theta^i \Gamma_{ij} \Delta\theta^j\right)$$

where:

$$d = n + h(\vec{\theta}_0)$$

data noise signal

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Fisher Matrix

$$\Gamma_{ij} = \langle \partial_i h(\vec{\theta}) | \partial_j h(\vec{\theta})\rangle\Big|_{\theta_0}$$

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$$\mathrm{Cov}_{ij} = \Gamma_{ij}^{-1}$$

forecasts!

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data noise signal

We use zero noise approximation!

forecasts!

$$\mathrm{Cov}_{ij} = \Gamma^{-1}_{ij}$$

Forecasts for 3G Detectors

- Fisher matrix codes are **particularly useful to study the performance of future GW observatories** (we focus here on ET and CE in particular)
- **Computationally convenient**: processing of entire populations (~1e5 events)
- Each event takes ~0.2-0.3 seconds to complete with GWFish (this is to compare with full PE softwares as **Bilby** that take some days to process a GW event)

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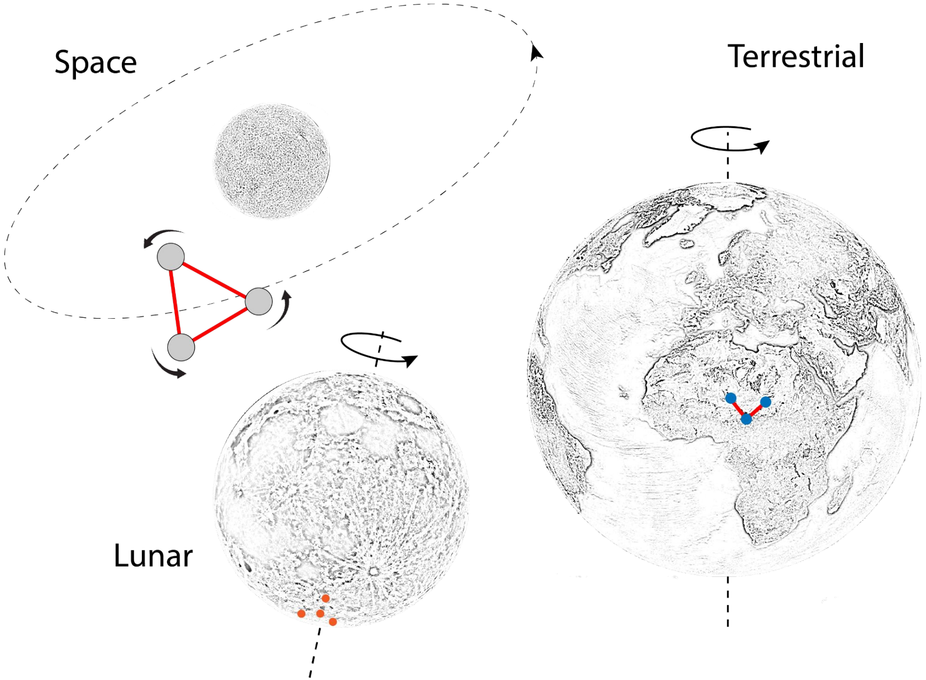
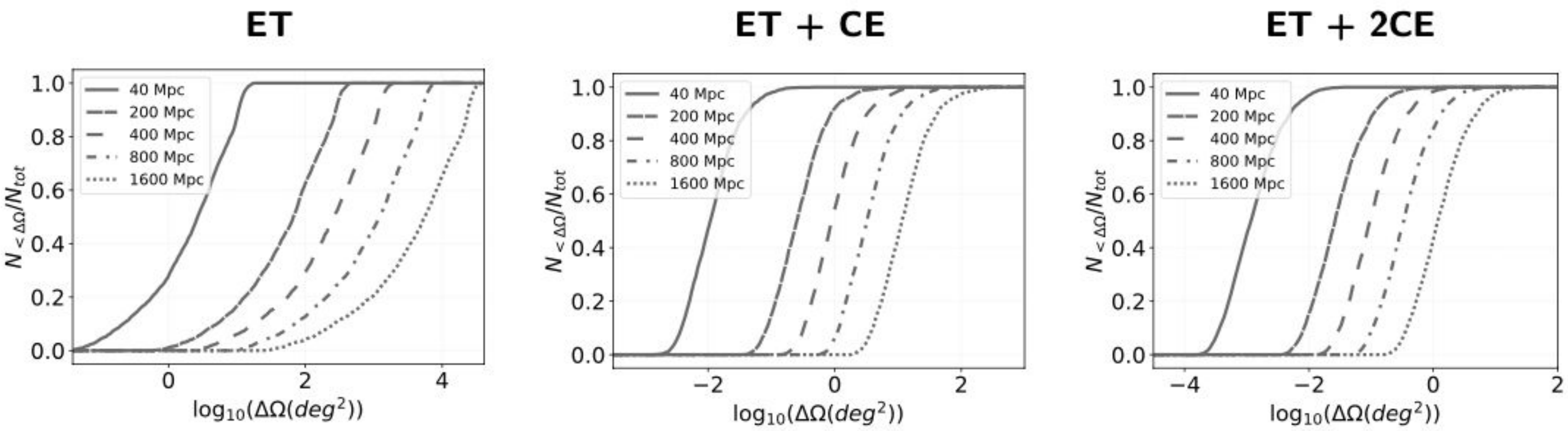
See also the CoBA study!
[Branchesi, Maggiore et al. 2023]

Detector Networks available in GWFish

- Potential upgrades of the current infrastructures: **Virgo**, **LIGO** (Hanford and Livingston), **LIGO India** and **KAGRA**
- The proposed **Einstein Telescope** and **Cosmic Explorer** (both second half of 2030s)
- The approved space-borne detector **LISA**, expected to begin observations in the second half of 2030s
- New detector concept on the lunar surface: **LGWA**



BNS Population [U. Dupletsa & J. Harms et al., *Astronomy and Computing, 2023*]



- BNSs at fixed luminosity distances (1e3 BNSs per curve) of 1.4+1.4Msol
- **Sky localization at 90% credible region**

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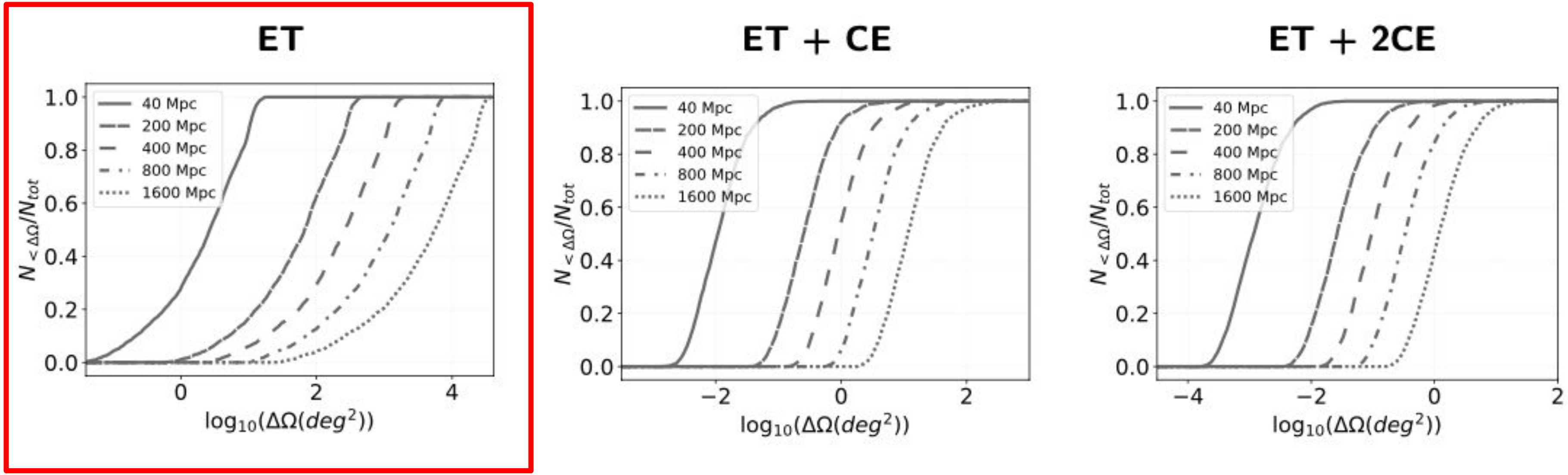

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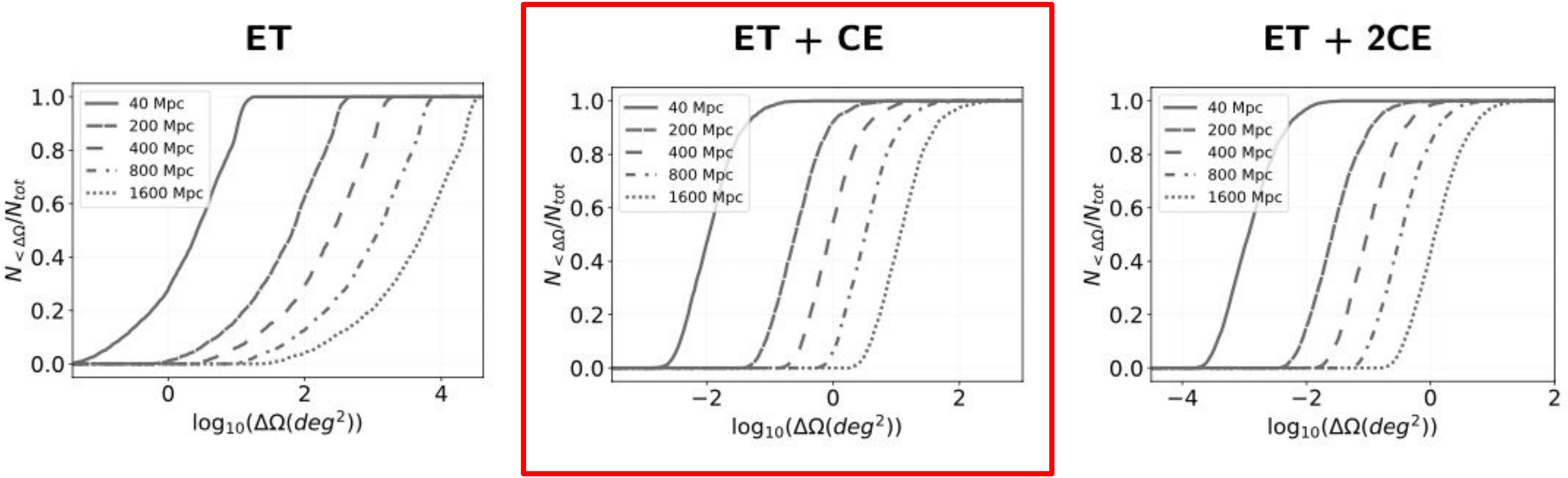

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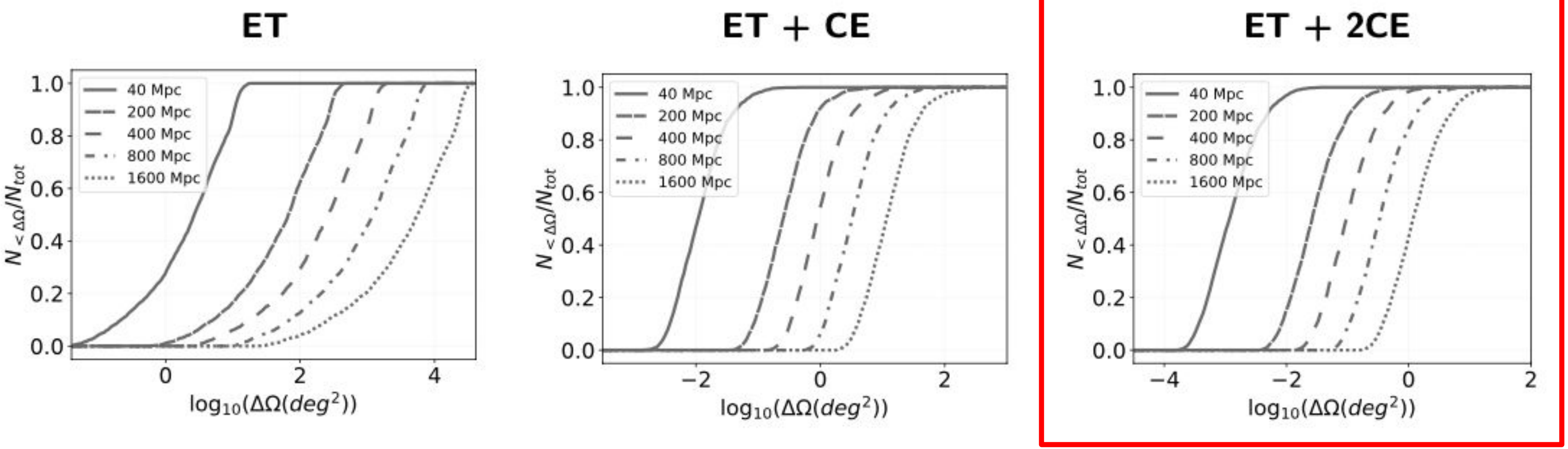

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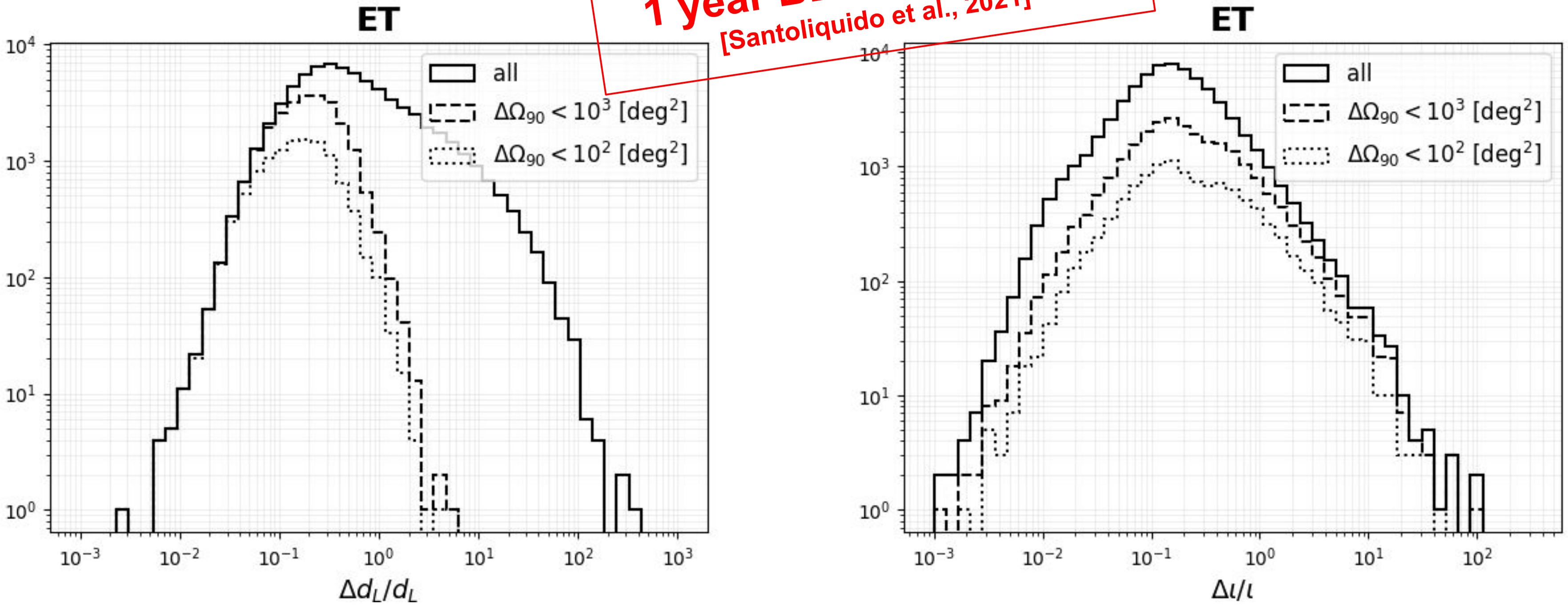
BBH Population

1 year BBH population
[Santoliquido et al., 2021]



Multi-Band Example [U. Dupletsa & J. Harms et al., *Astronomy and Computing, 2023*]

Multi-band observation with ET+LISA. We simulated an IMBH binary system of $4000M_\odot$ and $2000M_\odot$ at a distance of 1.2 Gpc. The errors are given at 1σ and the sky localization is at 90% C.L.. We used a frequency resolution of 1/16 Hz for ET and 10^{-4} Hz for LISA.

	inj_value	err_LISA	err_ET	err_LISA_ET
SNR		70.5	250	260
$m_{1,\mathrm{src}}$ [$M_\odot$]	4000	± 0.036	± 19	± 0.032
$m_{2,\mathrm{src}}$ [$M_\odot$]	2000	± 0.016	± 21	± 0.015
d_L [Mpc]	1200	± 21	± 218	± 19
RA [rad]	0.37	± 0.022	± 0.029	± 0.0072
DEC [rad]	-0.36	± 0.0082	± 0.046	± 0.0054
ι [rad]	1.6	± 0.010	± 0.040	± 0.0081
Ψ [rad]	1.7	± 0.015	± 0.040	± 0.0054
phase [rad]	3.5	± 0.72	± 0.11	± 0.028
t_c [s]	1120381489.6	± 0.59	± 0.0050	± 0.0020
sky-loc [deg^2]		7.2	56	1.1

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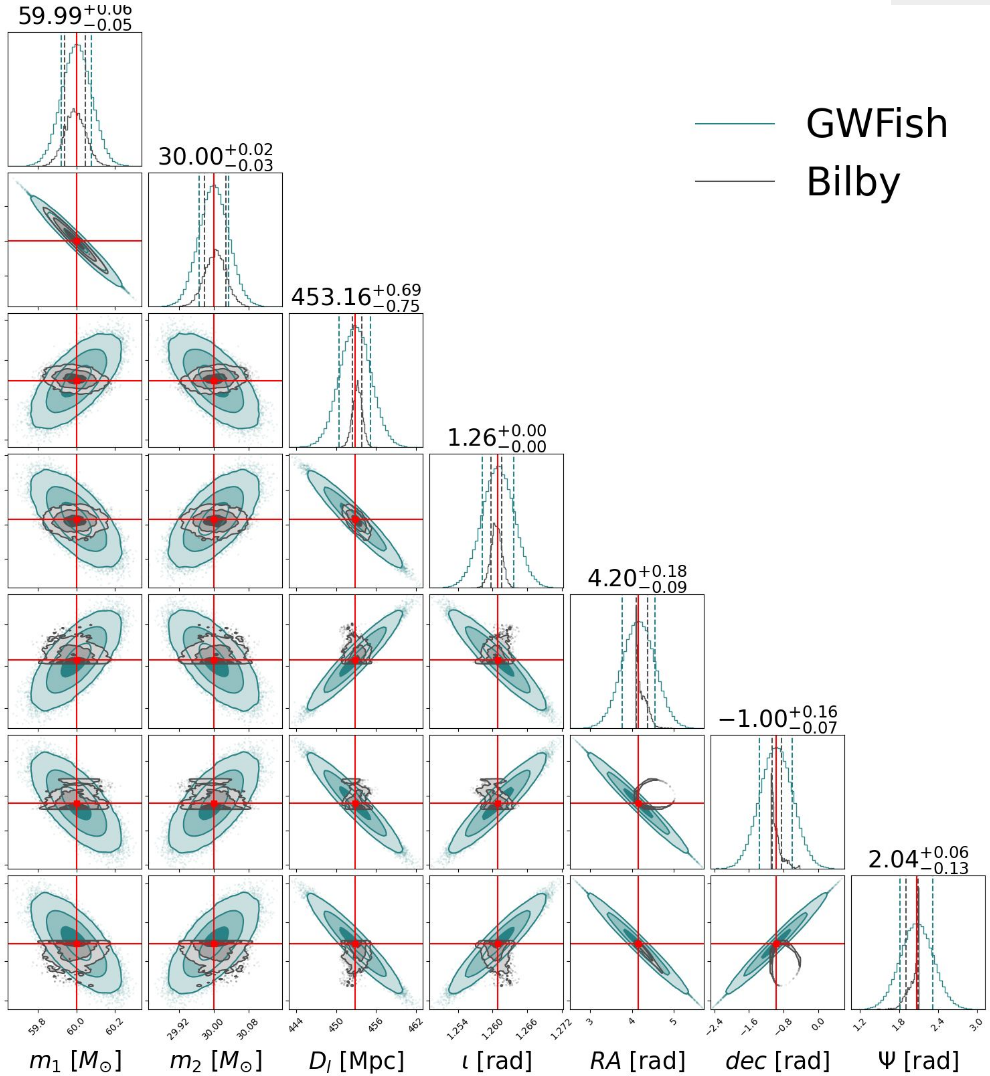

GWFish vs Bilby

Results Comparison for one BBH signal

[U. Dupletsa & J. Harms et al., *Astronomy and Computing, 2023*]

Low SNR Case [U. Dupletsa & J. Harms et al., *Astronomy and Computing, 2023*]

	BILBY	GWFISH
RA [rad]	$4.5^{+0.5}_{-0.5}$	4.1 ± 83
DEC [rad]	$-0.66^{+0.32}_{-0.39}$	-0.97 ± 81
Ψ [rad]	$1.8^{+0.35}_{-0.43}$	2.1 ± 54
ι [rad]	$1.2^{+0.19}_{-0.31}$	1.3 ± 0.6
d_L [Mpc]	68508^{+26173}_{-18288}	58115 ± 64040
$m_{1,det}$ $[M_\odot]$	$61.3^{+7.6}_{-7.2}$	65.9 ± 18
$m_{2,det}$ $[M_\odot]$	$35.2^{+4.3}_{-3.6}$	32.9 ± 8

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Huge errors!

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Beyond Fisher matrix: GWFish + Priors

In some cases with a Fisher matrix analysis we estimate an uncertainty on a parameter which goes **out of the range** the parameter can be in

This happens especially for **angles** which have a limited range:

$$\iota \in [0, \pi]$$
$$\mathrm{DEC} \in [-\pi, \pi]$$
$$\mathrm{RA} \in [0, 2\pi]$$
$$\Psi \in [0, 2\pi]$$
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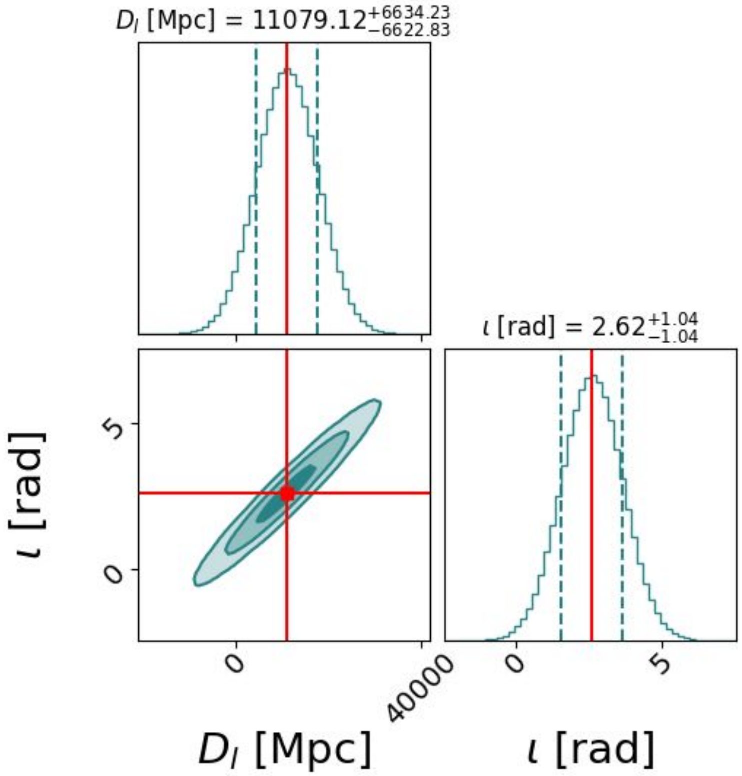
or parameters like **luminosity distance** which can extend to negative values



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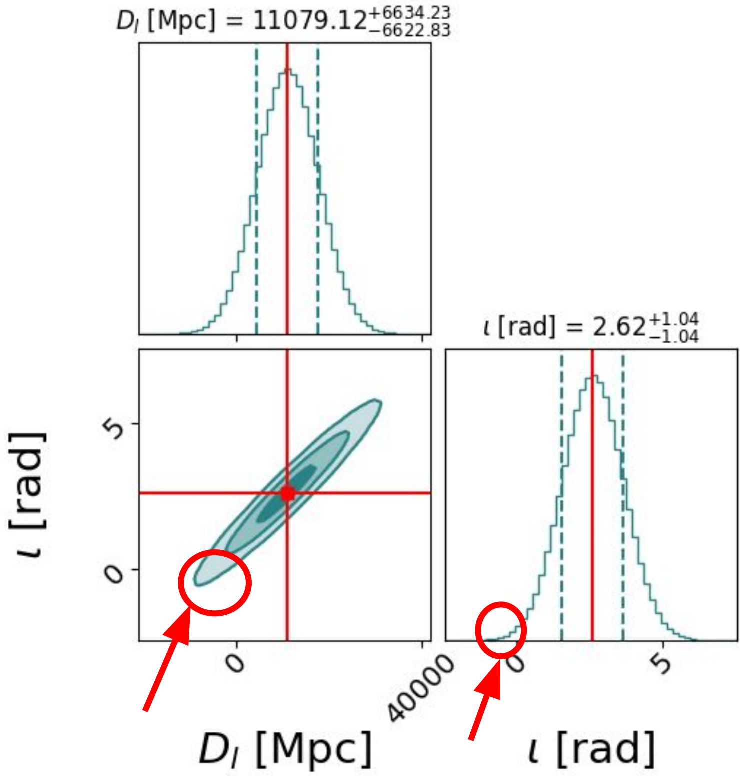
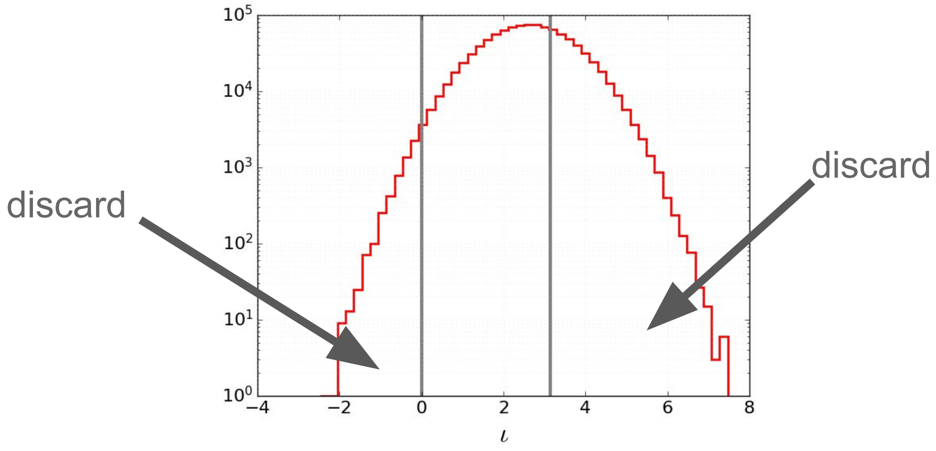
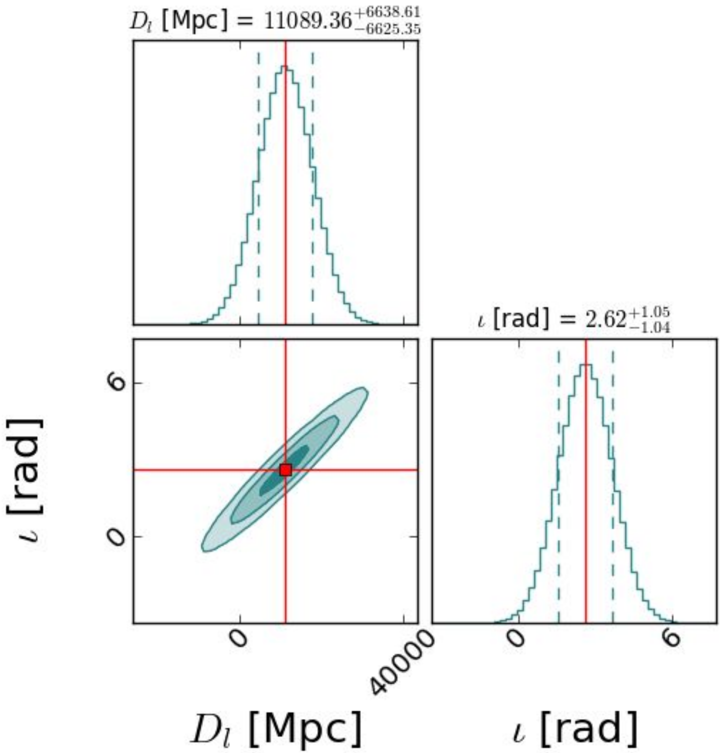


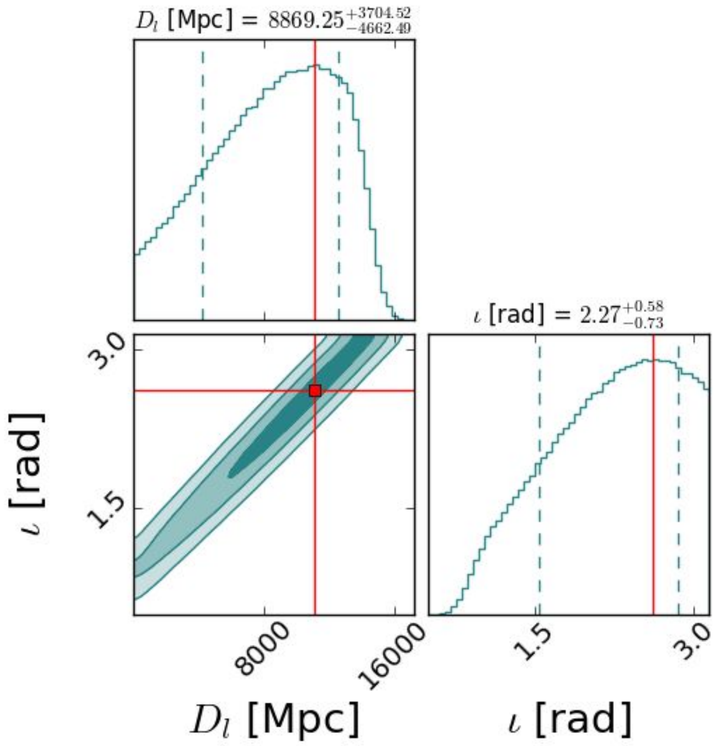
Adding a Flat Prior: Sample Rejection

1. Generate samples from the multivariate Gaussian distribution obtained from the Fisher matrix analysis
2. Discard samples that go out of the prior range for a given variable (**uniform prior**)
3. Re-calculate the standard deviation on that variable using the filtered sample



GWFish + Flat Prior: Results Comparison





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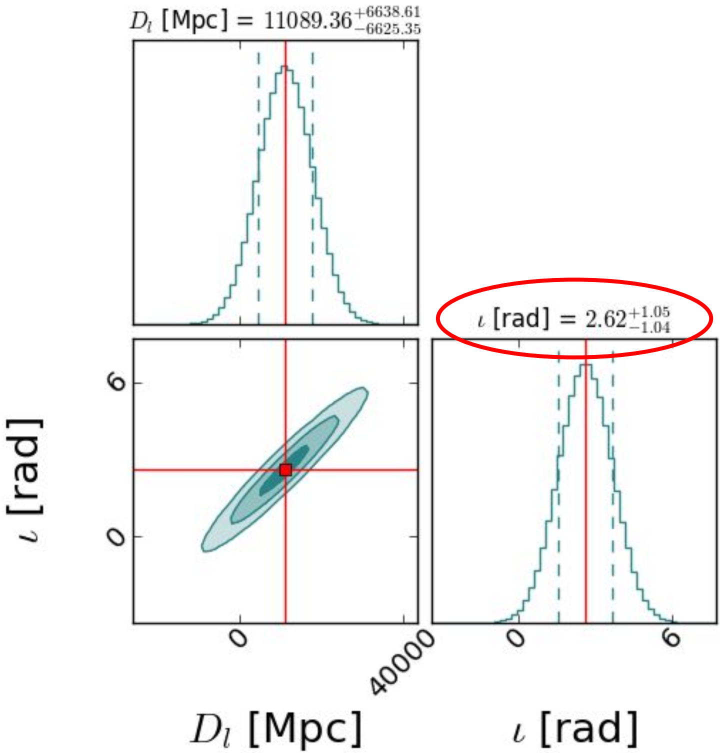
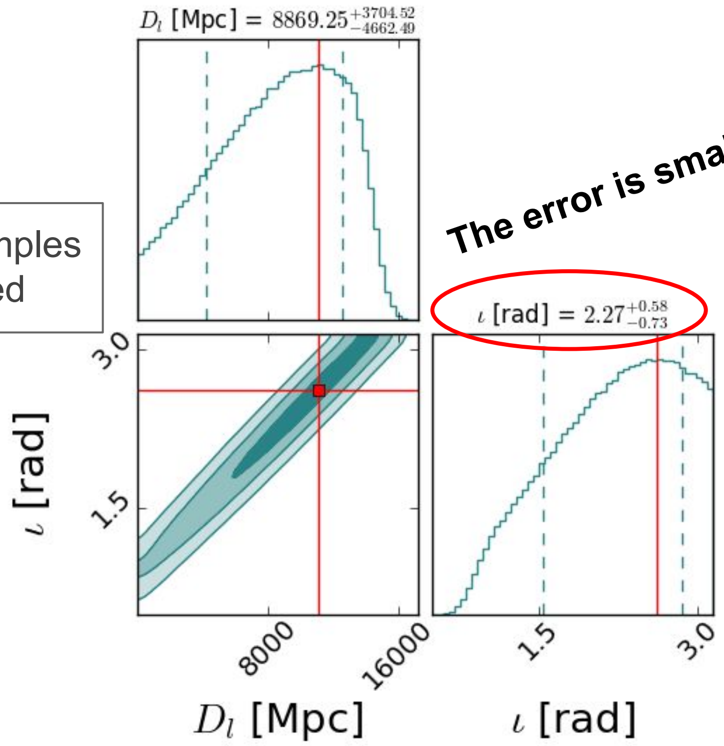


36% of samples was rejected



The error is smaller!

Adding a non Uniform Prior

- Accept/reject samples according to a distribution of inclination angle which is **uniform in cosine**

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def iota_distr(x):
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- A bigger sample is rejected (~67%) and the **final error is smaller**

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random_vec = np.random.uniform(0,0.5,int(N_points))
mask_iota_prior = iota_distr(data['iota']) > random_vec
data = data.loc[mask_iota_prior]
data_per_cov = (data.to_numpy()).T

return np.cov(data_per_cov)
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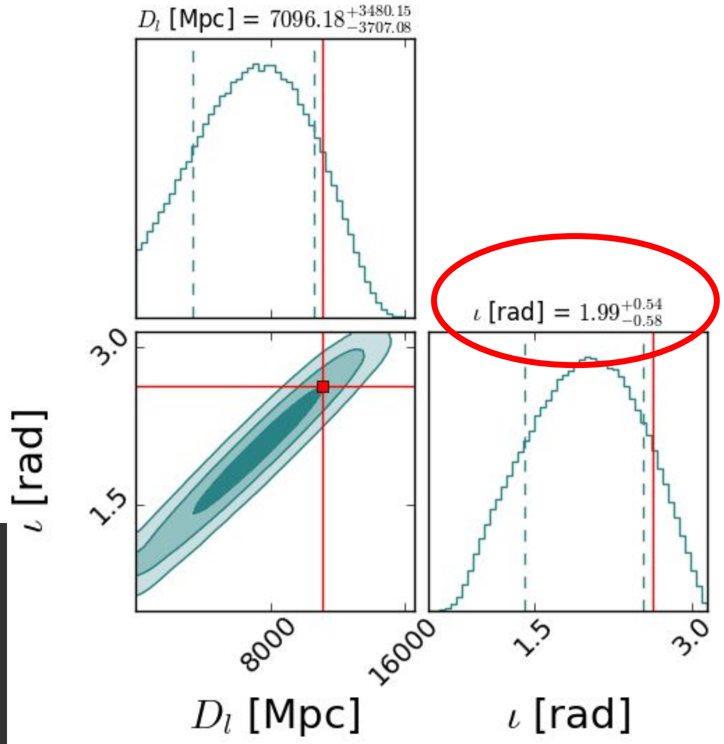

Conclusions

- A **great wealth of data** is expected in the near future in the GW field posing a new **computational challenge**
- Certain analyses are currently not possible with state-of-the-art detector simulation and **Bayesian analysis** software like bilby, due to practical constraints on available computational resource
- Fisher-matrix codes like **GWFish** are the best option to exploit in preparation of a new Bayesian parameter-estimation software
- We can go beyond Fisher matrix analysis **adding priors** on Fisher matrix parameters and still remaining computationally competitive

Thank you all!