Probing the Universe with binary black holes in the era of third-generation interferometers

Niccolò Muttoni[1]
in collaboration with Danny Laghi, Nicola Tamanini, Sylvain Marsat, David Izquierdo-Villalba

XIII ET Symposium, 8-12 May 2023, Cagliari

Laboratoire des 2 Infinis - L2IT Toulouse
[1]Université de Genève - Department of theoretical Physics

Motivations

- $\sim 5\sigma$ discrepancy between current electromagnetic (EM) wave-based H_0 estimates:

$$H_0 = 67.36 \pm 0.54 \mathrm{km\,s^{-1}\,Mpc^{-1}} \text{ (Early Universe)}^1$$
$$H_0 = 73.30 \pm 1.04 \mathrm{km\,s^{-1}\,Mpc^{-1}} \text{ (Late Universe)}^2$$

[1] Planck Collaboration, 2018 [arXiv:1807.06209]
[2] Riess et al., 2022 [arXiv:2112.04510]

Motivations

- $\sim 5\sigma$ discrepancy between current electromagnetic (EM) wave-based H_0 estimates:

$$H_0 = 67.36 \pm 0.54 \mathrm{km\,s^{-1}\,Mpc^{-1}} \text{ (Early Universe)}^1$$
$$H_0 = 73.30 \pm 1.04 \mathrm{km\,s^{-1}\,Mpc^{-1}} \text{ (Late Universe)}^2$$

Goal of gravitational wave (GW) cosmology

Provide an independent measurement for cosmological parameters

[1] Planck Collaboration, 2018 [arXiv:1807.06209]
[2] Riess et al., 2022 [arXiv:2112.04510]

GW cosmology: review

Quest for the two key quantities: distance and redshift

Standard sirens

- Merger of compact objects
- Absolute probes of luminosity distance:

$$h \propto d_L^{-1}$$

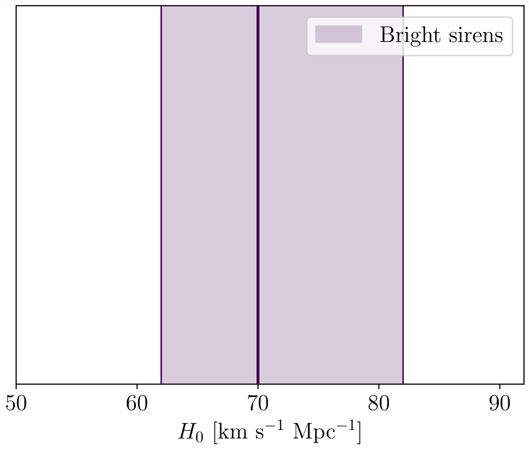
- No redshift information!

GW cosmology: where do we stand?

Standard sirens:

- Bright Sirens[a]

$$H_0 = 70^{+12}_{-8} \mathrm{km\,s^{-1}\,Mpc^{-1}}$$



[a] Abbott et al., 2017 [arXiv:1710.05835]

GW cosmology: where do we stand?

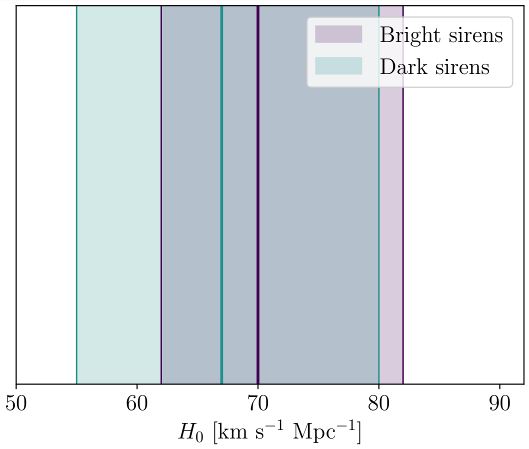
Standard sirens:

- Bright Sirens[a]

$$H_0 = 70^{+12}_{-8} \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

- Dark Sirens[b]

$$H_0 = 67^{+13}_{-12} \mathrm{km\,s^{-1}\,Mpc^{-1}}$$



[a] Abbott et al., 2017 [arXiv:1710.05835]
[b] Abbott et al., 2021 [arXiv:2111.03604]

GW cosmology: where do we stand?

Standard sirens:

- Bright Sirens[a]

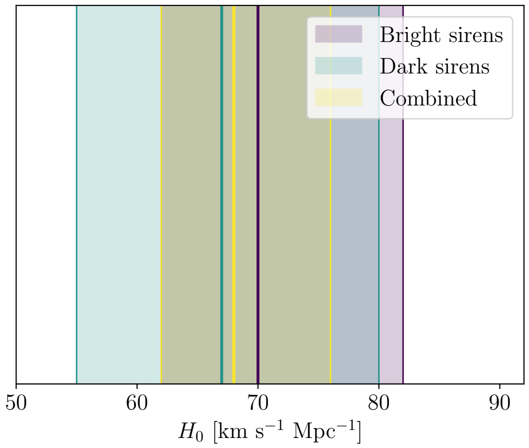
$$H_0 = 70^{+12}_{-8} \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

- Dark Sirens[b]

$$H_0 = 67^{+13}_{-12} \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

- Combined we obtain[b]

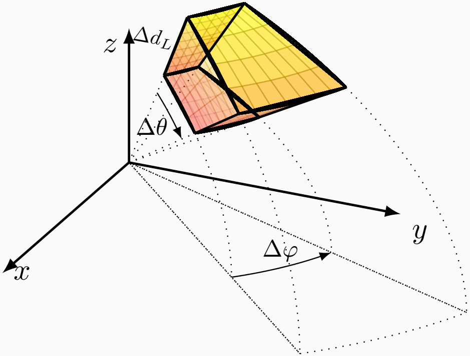
$$H_0 = 68^{+8}_{-6} \mathrm{km\,s^{-1}\,Mpc^{-1}}$$



[a] Abbott et al., 2017 [arXiv:1710.05835]
[b] Abbott et al., 2021 [arXiv:2111.03604]

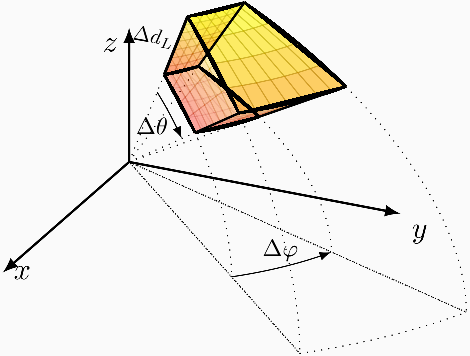
Dark sirens

1. Localise the source
2. Look for galaxies inside the localisation volume
3. Infer cosmological parameters

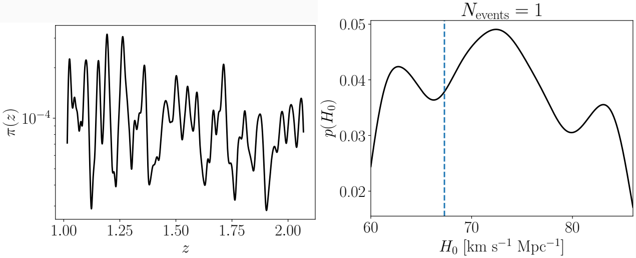


Dark sirens

1. Localise the source
2. Look for galaxies inside the localisation volume
3. Infer cosmological parameters $\Longrightarrow$

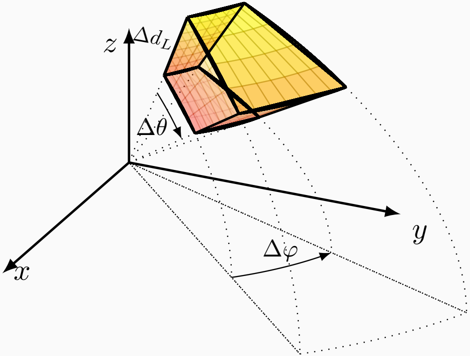


One event is usually not informative enough...

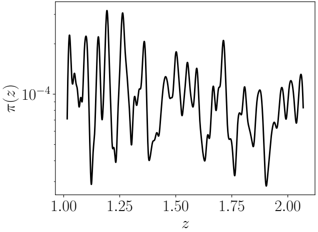


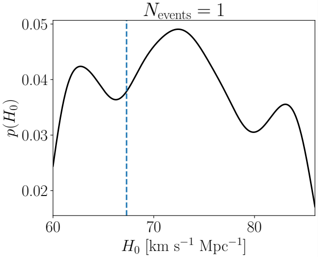
Dark sirens

1. Localise the source
2. Look for galaxies inside the localisation volume
3. Infer cosmological parameters

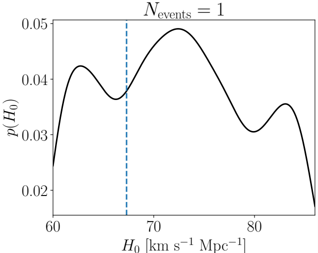


One event is usually not informative enough...



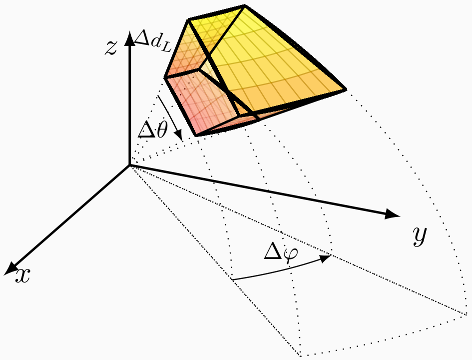


...but with lots of events:

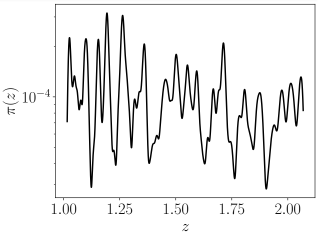


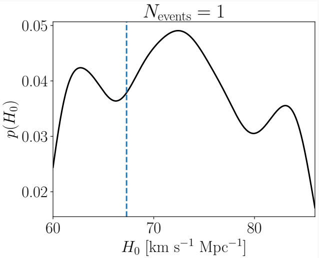
Dark sirens

1. Localise the source
2. Look for galaxies inside the localisation volume
3. Infer cosmological parameters

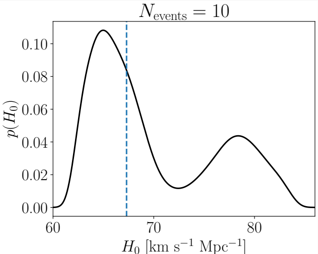


One event is usually not informative enough...



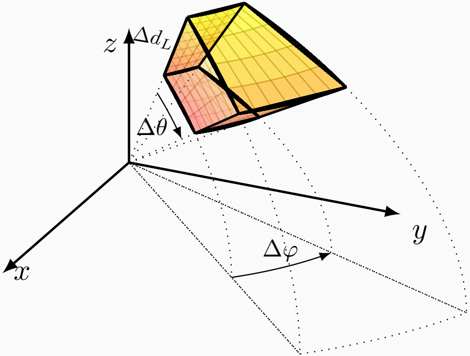


...but with lots of events:

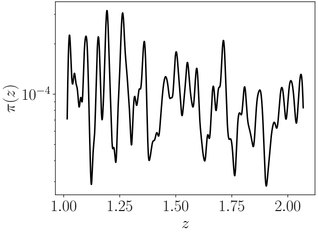


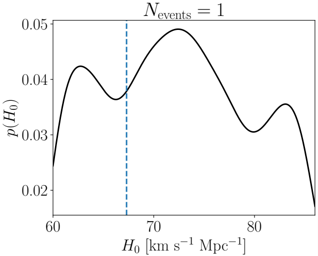
Dark sirens

1. Localise the source
2. Look for galaxies inside the localisation volume
3. Infer cosmological parameters

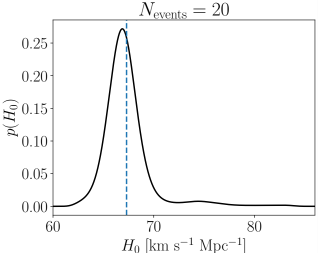


One event is usually not informative enough...



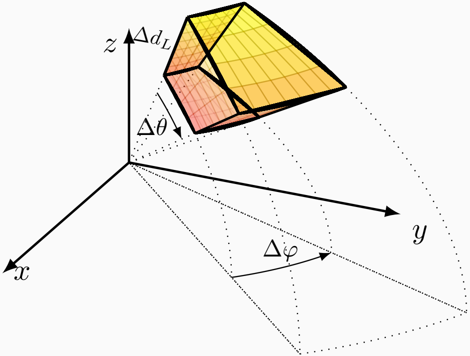


...but with lots of events:

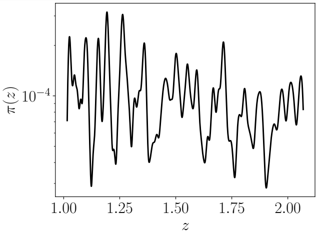


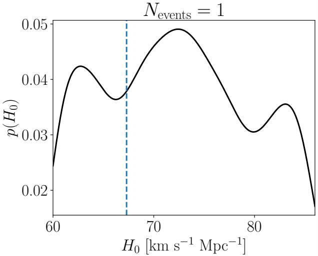
Dark sirens

1. Localise the source
2. Look for galaxies inside the localisation volume
3. Infer cosmological parameters

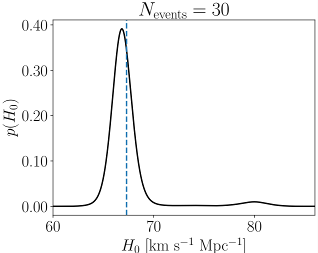


One event is usually not informative enough...



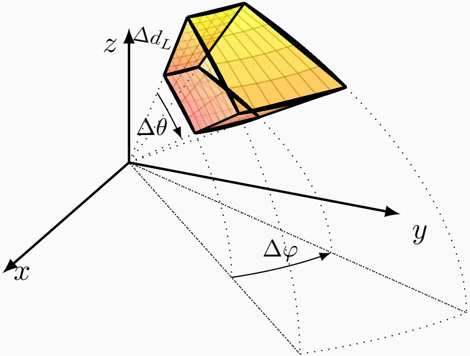


...but with lots of events:

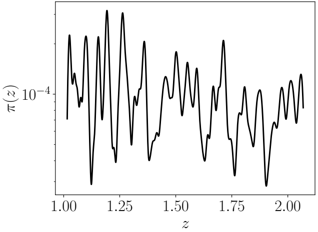


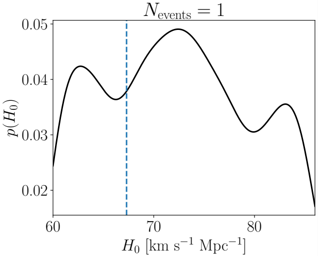
Dark sirens

1. Localise the source
2. Look for galaxies inside the localisation volume
3. Infer cosmological parameters

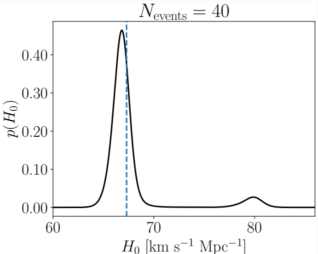


One event is usually not informative enough...



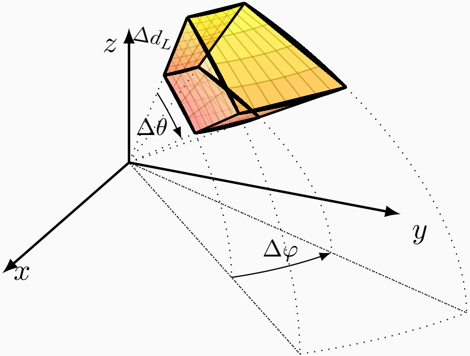


...but with lots of events:

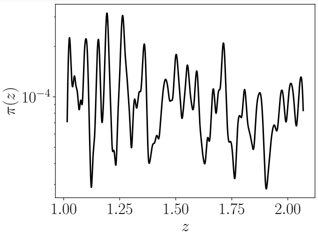


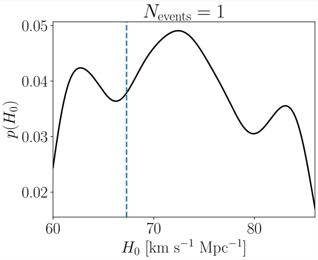
Dark sirens

1. Localise the source
2. Look for galaxies inside the localisation volume
3. Infer cosmological parameters

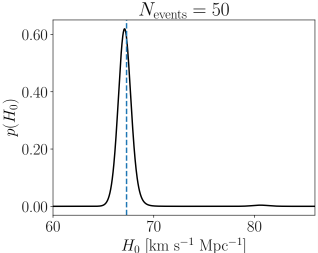


One event is usually not informative enough...



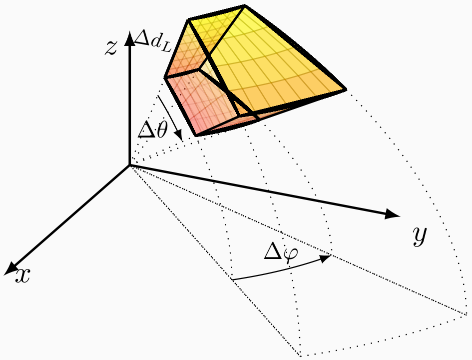


...but with lots of events:

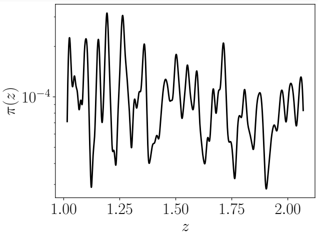


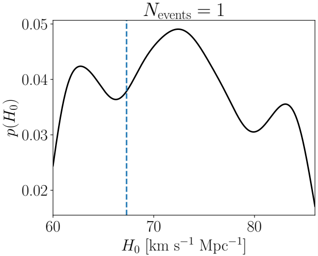
Dark sirens

1. Localise the source
2. Look for galaxies inside the localisation volume
3. Infer cosmological parameters

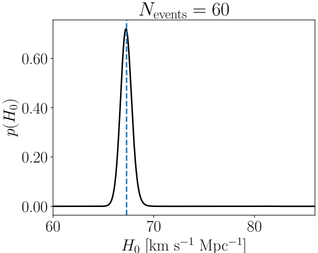


One event is usually not informative enough...



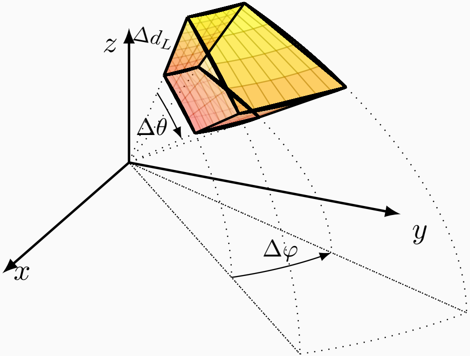


...but with lots of events:

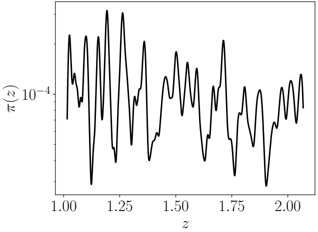


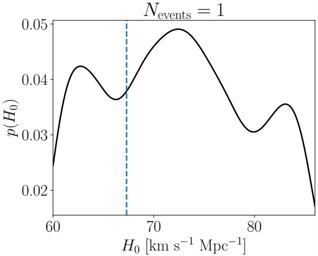
Dark sirens

1. Localise the source
2. Look for galaxies inside the localisation volume
3. Infer cosmological parameters

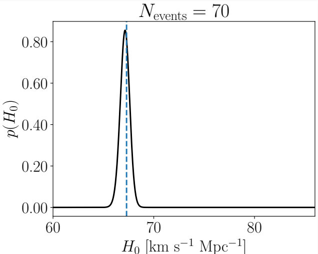


One event is usually not informative enough...



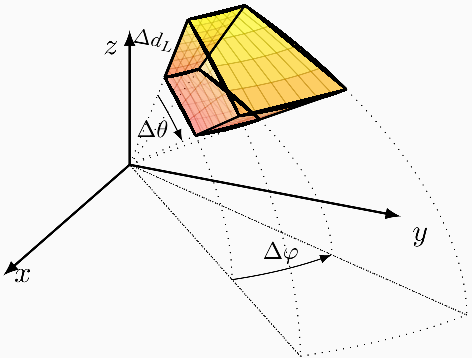


...but with lots of events:

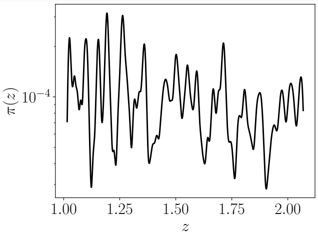


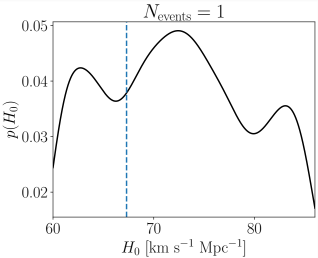
Dark sirens

1. Localise the source
2. Look for galaxies inside the localisation volume
3. Infer cosmological parameters

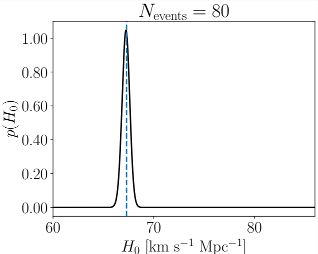


One event is usually not informative enough...





...but with lots of events:



Mock binary black hole (BBH) population

We built a mock population of BBH starting from the LIGO, Virgo, Kagra collaboration (LVK) analysis on GWTC-3[3]:

- Masses: `POWER LAW+PEAK`, mass ratio
- Spins: uniformly in $[-0.75, 0.75]$
- Sky location angles: uniformly on a sphere
- Polarisation and coalescence phase: uniformly in $[0, 2\pi]$
- Inclination: uniformly in $\cos \iota$
- Merger time: uniformly in a 1 yr time window
- Redshift: Star formation rate density[a] + time delays

[a] Madau & Fragos, 2017 [arXiv:1606.07887]

[3] LVK, 2022 [arXiv:2111.03634]

Setting up the simulation

Waveform

We modelled each signal with the `IMRPhenomXHM` waveform:

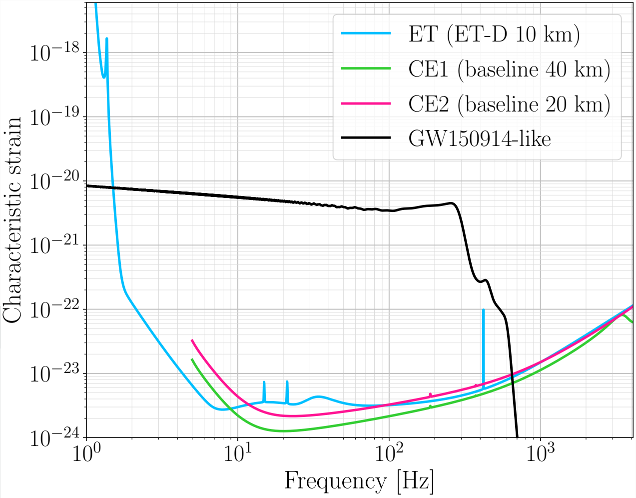
- Fast
- Non-precessing systems
- Higher modes break degeneracies

Detectors

- ET (IT)
- CE1 (US)
- CE2 (AU)

Networks

- ET
- ET+CE1
- ET+CE1+CE2



Detection and Fisher Matrix

Signal-to-noise ratio (SNR)

$$(A|B) = 4\mathrm{Re}\int_0^\infty df\,\frac{\tilde{A}(f)\tilde{B}^*(f)}{S_n(f)}$$

- $\mathrm{SNR} = (h|h)^{1/2}$
- $\mathrm{SNR}_{\mathrm{net}}^2 = \sum_{k=1}^{M} \mathrm{SNR}_k^2$
- Detection if $\mathrm{SNR}_{\mathrm{net}} \geq 12$
- Dark siren if $\mathrm{SNR}_{\mathrm{net}} \geq 300$

Detection and Fisher Matrix

Signal-to-noise ratio (SNR)

$$(A|B) = 4\mathrm{Re}\int_0^\infty df\, \frac{\tilde{A}(f)\tilde{B}^*(f)}{S_n(f)}$$

- $\mathrm{SNR} = (h|h)^{1/2}$
- $\mathrm{SNR}^2_{\mathrm{net}} = \sum_{k=1}^{M} \mathrm{SNR}^2_k$
- Detection if $\mathrm{SNR}_{\mathrm{net}} \geq 12$
- Dark siren if $\mathrm{SNR}_{\mathrm{net}} \geq 300$

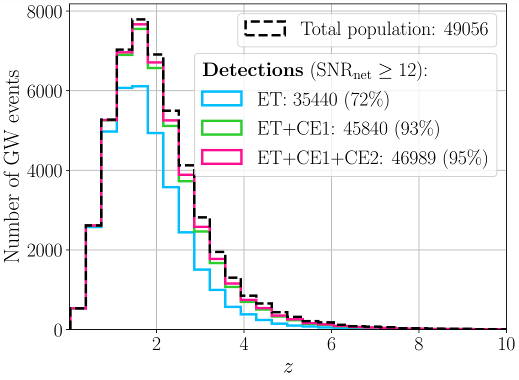
Fisher Matrix

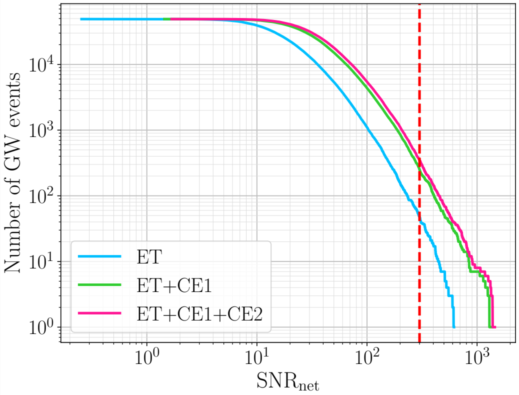
In the limit of strong signals (i.e. high SNR):

$$\mathcal{L}(d|\theta) = \mathcal{N}(\theta^t, \Gamma^{-1})$$

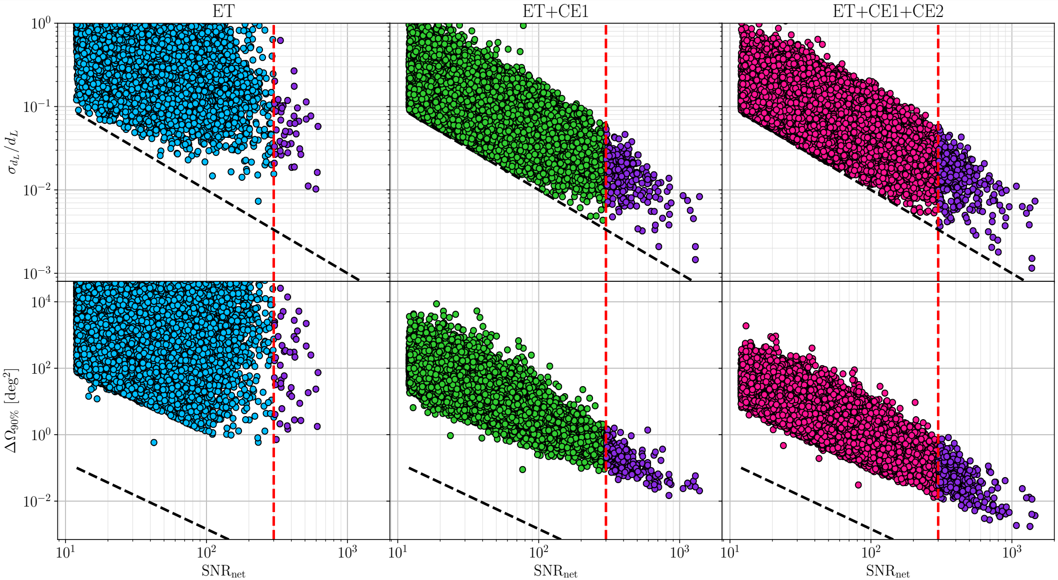
- $\Gamma_{ij} = (\partial_i h|\partial_j h)$ is the Fisher Matrix
- $\Gamma_{\mathrm{net}} = \sum_{k=1}^{M} \Gamma_k$
- $\Sigma_{\mathrm{net}} = \Gamma^{-1}_{\mathrm{net}}$ is the Covariance Matrix

Detection results

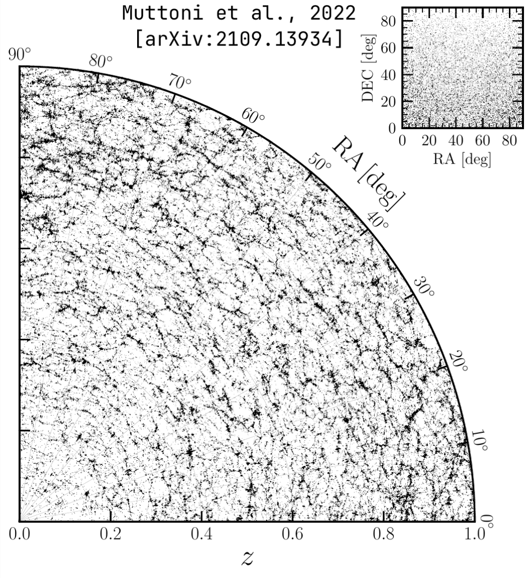




Fisher Matrix results (preliminary)



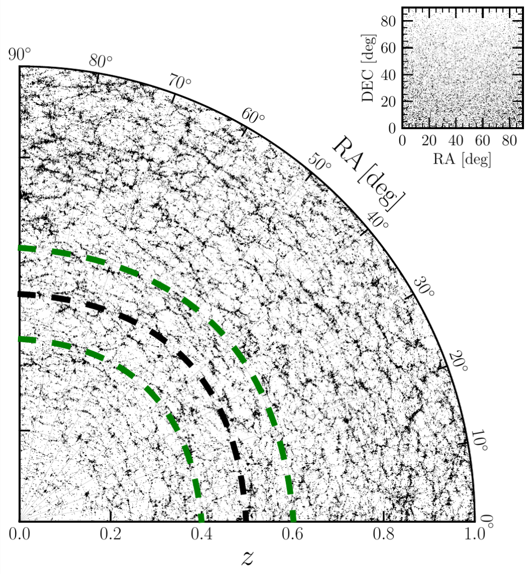
Localisation error-volumes: generation



1. Assume the cosmology of the catalog[a].

[a] Izquierdo-Villalba et al., 2019 [arXiv:1907.02111]

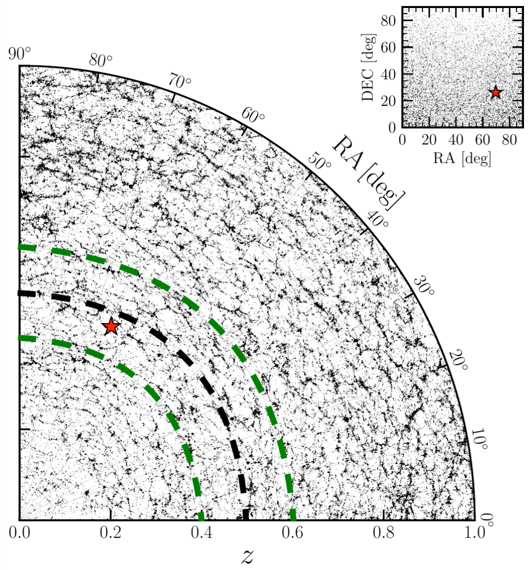
Localisation error-volumes: generation



1. Assume the cosmology of the catalog[a].
2. Convert $d_L^t \pm 3\sigma_{d_L}$ into a redshift interval and consider the galaxies within.

[a] Izquierdo-Villalba et al., 2019 [arXiv:1907.02111]

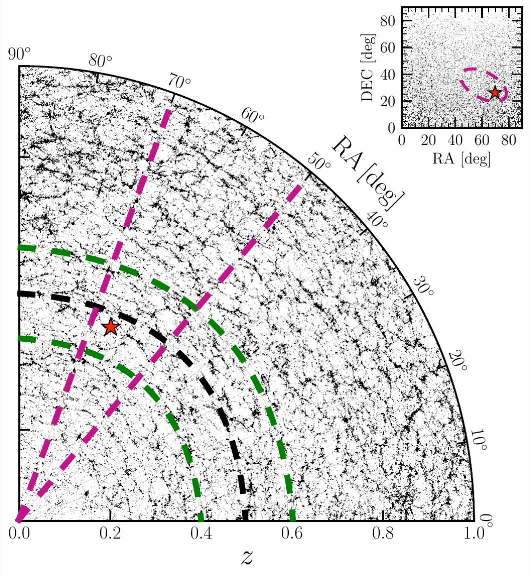
Localisation error-volumes: generation



1. Assume the cosmology of the catalog[a].
2. Convert $d_L^t \pm 3\sigma_{d_L}$ into a redshift interval and consider the galaxies within.
3. Select one of them as the true host according to $\mathcal{N}(d_L^t, \sigma_{d_L}^2)$.

[a]Izquierdo-Villalba et al., 2019 [arXiv:1907.02111]

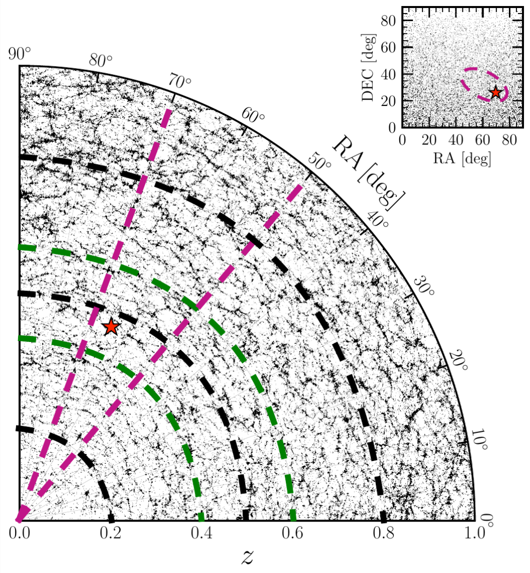
Localisation error-volumes: generation



1. Assume the cosmology of the catalog[a].
2. Convert $d_L^t \pm 3\sigma_{d_L}$ into a redshift interval and consider the galaxies within.
3. Select one of them as the true host according to $\mathcal{N}(d_L^t, \sigma_{d_L}^2)$.
4. Scatter the center of the error-box by extracting a new sky position $(\mathrm{RA_c}, \mathrm{DEC_c})$ from $\mathcal{N}((\mathrm{RA_{th}}, \mathrm{DEC_{th}}), \Sigma_{\Delta\Omega})$.

[a] Izquierdo-Villalba et al., 2019 [arXiv:1907.02111]

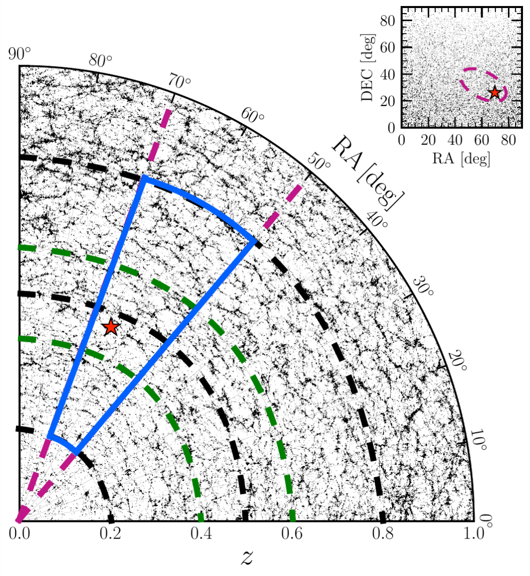
Localisation error-volumes: generation



1. Assume the cosmology of the catalog[a].
2. Convert $d_L^t \pm 3\sigma_{d_L}$ into a redshift interval and consider the galaxies within.
3. Select one of them as the true host according to $\mathcal{N}(d_L^t, \sigma_{d_L}^2)$.
4. Scatter the center of the error-box by extracting a new sky position $(\mathrm{RA_c}, \mathrm{DEC_c})$ from $\mathcal{N}((\mathrm{RA_{th}}, \mathrm{DEC_{th}}), \Sigma_{\Delta\Omega})$.
5. Extend the redshift interval to include all the cosmologies in the prior.

[a] Izquierdo-Villalba et al., 2019 [arXiv:1907.02111]

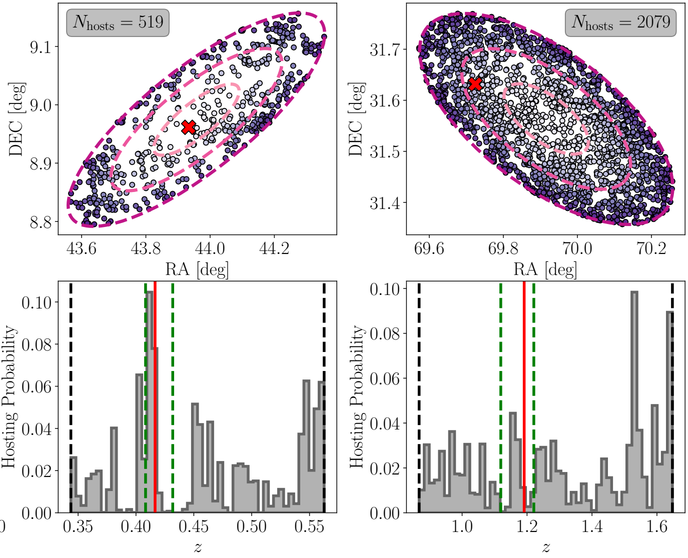
Localisation error-volumes: generation



1. Assume the cosmology of the catalog[a].
2. Convert $d_L^t \pm 3\sigma_{d_L}$ into a redshift interval and consider the galaxies within.
3. Select one of them as the true host according to $\mathcal{N}(d_L^t, \sigma_{d_L}^2)$.
4. Scatter the center of the error-box by extracting a new sky position $(\mathrm{RA_c}, \mathrm{DEC_c})$ from $\mathcal{N}((\mathrm{RA_{th}}, \mathrm{DEC_{th}}), \Sigma_{\Delta\Omega})$.
5. Extend the redshift interval to include all the cosmologies in the prior.
6. Select all the galaxies that fall inside.

[a] Izquierdo-Villalba et al., 2019 [arXiv:1907.02111]

Localisation error-volumes: examples



For the j-th galaxy in the error-box:

Hosting Probability

$$w_j \propto \mathcal{N}((\mathrm{RA_c}, \mathrm{DEC_c}), \Sigma_{\Delta\Omega})\big|_{(\mathrm{RA}_j, \mathrm{DEC}_j)}$$

Redshift uncertainty

$$\sigma_{v_p}(z_j) = (1 + z_j)\frac{v_p}{c}$$

$$v_p = 700\,\mathrm{km\,s^{-1}}$$

Inference of cosmological parameters

- Set of dark sirens: $\mathcal{D} = \{\mathcal{D}_i\}_{i=1}^{N}$
- Set of cosmological parameters: $\Omega = \{H_0, \Omega_m\}$
- Cosmological model: $\mathcal{H} \equiv \Lambda\text{CDM}$
- Redshift information: I

- Flat prior on the set Ω.
- We obtain posterior samples with `cosmoLISA`[a].

[a] Del Pozzo & Laghi

Bayesian formalism

Bayes' theorem:

$$p(\Omega|\mathcal{D},\mathcal{H},I) \propto \mathcal{L}(\mathcal{D}|\Omega,\mathcal{H},I)\pi(\Omega|\mathcal{H},I)$$

Likelihood:

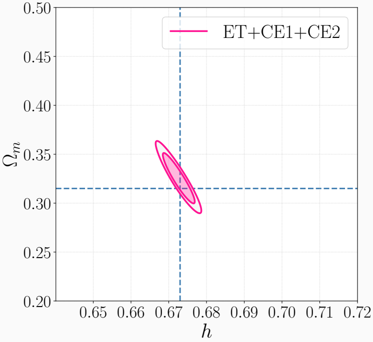
$$\mathcal{L}(\mathcal{D}|\Omega,\mathcal{H},I) = \prod_{i=1}^{N} \mathcal{L}(\mathcal{D}_i|\Omega,\mathcal{H},I)$$

$$\mathcal{L}(\mathcal{D}_i|\Omega,\mathcal{H},I) = \int_{z_{\min}}^{z_{\max}} dz_{\rm GW}\, \mathcal{N}(d_L^t, \sigma_{d_L}^2)\big|_{d_L(z_{\rm GW},\Omega)} \sum_{j=1}^{N_{\rm hosts}} w_j \mathcal{N}(z_j, \sigma_{v_p}^2)$$

Cosmology: preliminary results

We provide results assuming different scenarios:

1. Fiducial: galaxy catalog complete up to $z = 1$
2. Optimistic: galaxy catalog complete up to $z = 3$
3. Only H_0 (Ω_m fixed to its fiducial value)



Network	N		$\Delta H_0/H_0$ (%)			$\Delta\Omega_m/\Omega_m$ (%)	
	Fiducial	Optimistic	Fiducial	only H_0	Optimistic	Fiducial	Optimistic
ET+CE1+CE2	278	335	0.4 (0.7)	0.2 (0.3)	0.4 (0.6)	5.4 (9.0)	4.5 (7.5)

Future improvements and conclusions

Future improvements

- Precessing waveform model
- Realistic spin distribution
- Realistic duty cycle
- $w_j = w_j(\mathrm{RA}_j, \mathrm{DEC}_j, L_j)$
- Selection effects

Future improvements and conclusions

Future improvements

- Precessing waveform model
- Realistic spin distribution
- Realistic duty cycle
- $w_j = w_j(\mathrm{RA}_j, \mathrm{DEC}_j, L_j)$
- Selection effects

Take-home messages

- Dark sirens show great potential in the context of GW cosmology with a network of 3G detectors
- Strong constraints on H_0, sub % in the case of ET+CE1+CE2
- Weaker constraints on Ω_m, of the order of few % in ET+CE1+CE2

Future improvements and conclusions

Future improvements

- Precessing waveform model
- Realistic spin distribution
- Realistic duty cycle
- $w_j = w_j(\mathrm{RA}_j, \mathrm{DEC}_j, L_j)$
- Selection effects

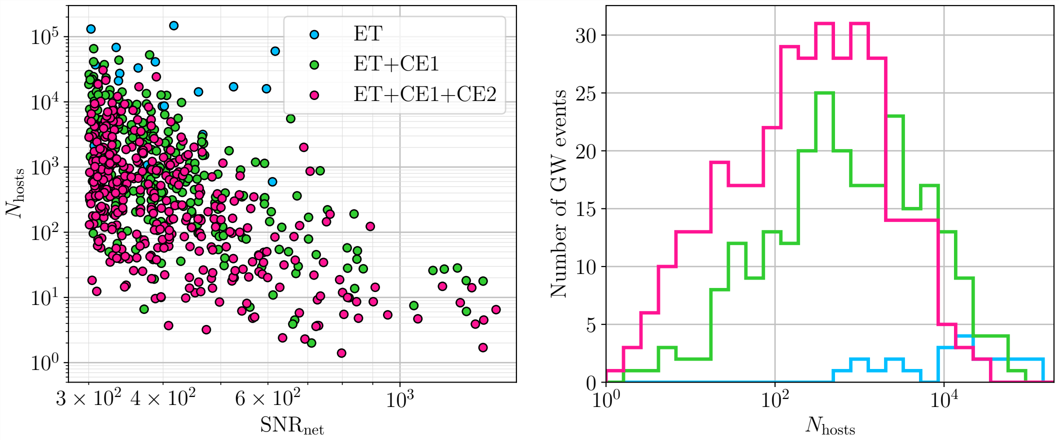
Take-home messages

- Dark sirens show great potential in the context of GW cosmology with a network of 3G detectors
- Strong constraints on H_0, sub % in the case of ET+CE1+CE2
- Weaker constraints on Ω_m, of the order of few % in ET+CE1+CE2

Thank you for your attention.

More information in (a soon to be v2 of) Muttoni et al., 2023 [arXiv:2303.10693]

Number of hosts per error-box (preliminary)



Redshift distribution of dark sirens (preliminary)