Challenges in Newtonian noise estimations: rheological effects

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Common work with:

Tamás FÜLÖP

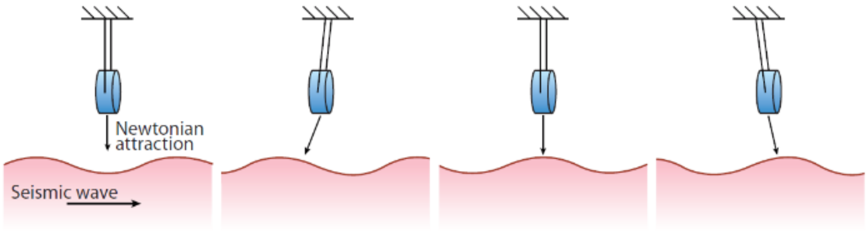
Róbert KOVÁCS

Péter VÁN

Detection of gravitational waves is based on displacement measurement $\Longrightarrow$ **Knowledge and separation of the motion of the surrounding environment and filtering and mitigation of noises are essential**

Several sources of noise (*e.g.*, standard quantum noise, thermal noise, scattering by residual gas), now we focus on:

- **Seismic noise:** persistent vibration of the ground, usually calculated via **elastic** models
- **Newtonian noise:** attraction of masses of nearby moving objects $\Longrightarrow$ direct coupling with seismic noises

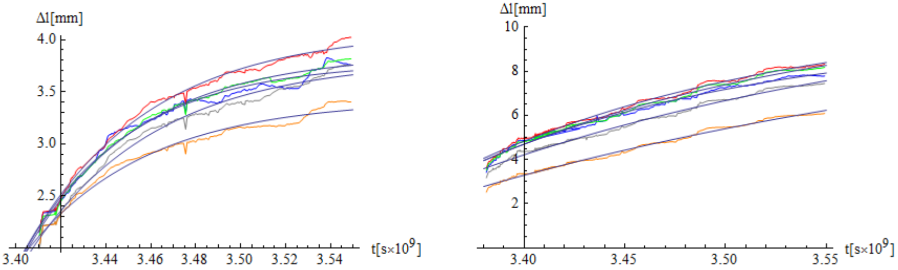


Soil and rocks are not so elastic as we expect

Tunnel squeezing: significant floor heave and wall closure

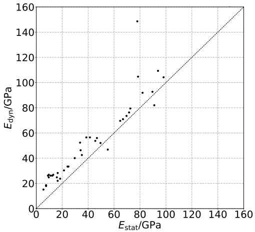
Basnet *et al.*: Analysis of squeezing phenomenon in the headrace tunnel of Chameliya Project, Nepal, *Hydro Nepal Journal of Water Energy and Environment* (2013)

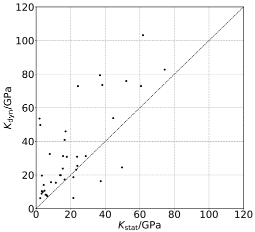
Measured (and fitted) exponential-like displacement history of tunnel walls at the National Radioactive Waste Repository, Bátaapáti, Hungary, with characteristic times of 3–10 years

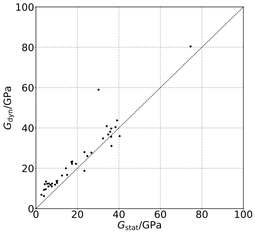


Kovács *et al.*: Revision and extension of the Geotechnical Interpretation Report (*in Hungarian*), Report RHK-K-032/12 by RockStudy Ltd. stored at the repository RHK Kft. Irattára, Pécs–Paks (2012)

Static vs. dynamic moduli of elasticity







Davarpanah *et al.*: Investigation of the relationship between dynamic and static deformation moduli of rocks. *Geomech. Geophys. Geo-energ. Geo-resour.* 6, 29 (2020)

Barnaföldi *et al.*: First report of long term measurements of the MGGL laboratory in the Mátra mountain range. *Class. Quantum Grav.* 34, 114001 (2017)

Ván *et al.*: Long term measurements from the Mátra Gravitational and Geophysical Laboratory. *Eur. Phys. J. Spec. Top.* 228, 1693—1743 (2019)

Rheological (a.k.a. viscoelastic) material behaviour: the response on the external effect is delayed and damped

Outline:

- Kluitenberg–Verhás rheological model family
- Wave propagation: numerical solutions
- Experimental investigations of rheological parameters

Kluitenberg–Verhás rheological model family

- Assuming:
  - A reversible model (here: Hooke elasticity/Duhamel–Neumann thermoelasticity)
  - Additional state variable: a second-order symmetric tensorial internal varibale
  - Additional stress contribution
- Second law of thermodynamics
- Linear Onsagerian equations $\Longrightarrow$ coupling between the strain and internal variable

$\Longrightarrow$ **Thermodynamical compatibility**

Asszonyi *et al.*: Distinguished rheological models for solids in the framework of a thermodynamical internal variable theory, *Contin. Mech. Thermodyn.*, 27, 971–986 (2015)

Elimination of the internal variable and assuming homogeneous and isotropic material $\Longrightarrow$ Kluitenberg–Verhás rheology:

$$\boldsymbol{\sigma}^{\rm dev} + \tau^{\rm dev}\dot{\boldsymbol{\sigma}}^{\rm dev} = E^{\rm dev}\boldsymbol{\varepsilon}^{\rm dev} + \hat{E}^{\rm dev}\dot{\boldsymbol{\varepsilon}}^{\rm dev} + \hat{\hat{E}}^{\rm dev}\ddot{\boldsymbol{\varepsilon}}^{\rm dev},$$

$$\boldsymbol{\sigma}^{\rm sph} + \tau^{\rm sph}\left(\dot{\boldsymbol{\sigma}}^{\rm sph} + \alpha E^{\rm sph}\dot{T}\right) =$$

$$= E^{\rm sph}\left[\boldsymbol{\varepsilon}^{\rm sph} - \alpha\left(T - T_{\rm aux}\right)\mathbf{1}\right] + \hat{E}^{\rm sph}\dot{\boldsymbol{\varepsilon}}^{\rm sph} + \hat{\hat{E}}^{\rm sph}\ddot{\boldsymbol{\varepsilon}}^{\rm sph}$$

where

$$\boldsymbol{\sigma}^{\rm sph} = \frac{1}{3}\left({\rm tr}\boldsymbol{\sigma}\right)\mathbf{1}, \qquad \boldsymbol{\sigma}^{\rm dev} = \boldsymbol{\sigma} - \boldsymbol{\sigma}^{\rm sph},$$

$$E^{\rm dev} = 2G, \qquad E^{\rm sph} = 3K$$

Special cases

Hooke's law of elasticity

$$\boldsymbol{\sigma}^{\text{dev}} = E^{\text{dev}} \boldsymbol{\varepsilon}^{\text{dev}}, \qquad \boldsymbol{\sigma}^{\text{sph}} = E^{\text{sph}} \boldsymbol{\varepsilon}^{\text{sph}}$$

Duhamel–Neumann thermoelasticity

$$\boldsymbol{\sigma}^{\text{dev}} = E^{\text{dev}} \boldsymbol{\varepsilon}^{\text{dev}}, \qquad \boldsymbol{\sigma}^{\text{sph}} = E^{\text{sph}} \left[\boldsymbol{\varepsilon}^{\text{sph}} - \alpha \left(T - T_{\text{aux}} \right) \mathbf{1} \right]$$

Kelvin–Voigt viscoelasticity

$$\boldsymbol{\sigma}^{\text{dev}} = E^{\text{dev}} \boldsymbol{\varepsilon}^{\text{dev}} + \hat{E}^{\text{dev}} \dot{\boldsymbol{\varepsilon}}^{\text{dev}}, \qquad \boldsymbol{\sigma}^{\text{sph}} = E^{\text{sph}} \boldsymbol{\varepsilon}^{\text{sph}} + \hat{E}^{\text{sph}} \dot{\boldsymbol{\varepsilon}}^{\text{sph}}$$

Poynting–Thomson–Zener rheology

$$\boldsymbol{\sigma}^{\text{dev}} + \tau^{\text{dev}} \dot{\boldsymbol{\sigma}}^{\text{dev}} = E^{\text{dev}} \boldsymbol{\varepsilon}^{\text{dev}} + \hat{E}^{\text{dev}} \dot{\boldsymbol{\varepsilon}}^{\text{dev}},$$
$$\boldsymbol{\sigma}^{\text{sph}} + \tau^{\text{sph}} \dot{\boldsymbol{\sigma}}^{\text{sph}} = E^{\text{sph}} \boldsymbol{\varepsilon}^{\text{sph}} + \hat{E}^{\text{sph}} \dot{\boldsymbol{\varepsilon}}^{\text{sph}}$$

Introducing

- **Index of damping:**

$$\mathcal{D}^{\text{dev}} = \hat{E}^{\text{dev}} - \tau^{\text{dev}} E^{\text{dev}} \geq 0, \quad \mathcal{D}^{\text{sph}} = \hat{E}^{\text{sph}} - \tau^{\text{sph}} E^{\text{sph}} \geq 0$$

based on thermodynamic criteria, hence

$$E^{\text{dev}}_{\text{dyn}} = \frac{\hat{E}^{\text{dev}}}{\tau^{\text{dev}}} \geq E^{\text{dev}} \equiv E^{\text{dev}}_{\text{stat}}, \quad E^{\text{sph}}_{\text{dyn}} = \frac{\hat{E}^{\text{sph}}}{\tau^{\text{sph}}} \geq E^{\text{sph}} \equiv E^{\text{sph}}_{\text{stat}}$$

- **Index of inertia:**

$$\mathcal{I}^{\text{dev}} = \hat{\hat{E}}^{\text{dev}} - \tau^{\text{dev}} \mathcal{D}^{\text{dev}}, \qquad \mathcal{I}^{\text{sph}} = \hat{\hat{E}}^{\text{sph}} - \tau^{\text{sph}} \mathcal{D}^{\text{sph}},$$

if $\mathcal{I} > 0$, possibility of material resonance, no known material yet

Dispersion of waves in a Poynting–Thomson–Zener medium

1 + 1D equations (equation of motion, kinematic equation and PTZ rheology)

$$\varrho\dot{v} = \sigma', \qquad \dot{\varepsilon} = v', \qquad \tau\dot{\sigma} + \sigma = E\varepsilon + \hat{E}\dot{\varepsilon}$$

Assuming plane wave solutions

$$\begin{pmatrix} v \\ \varepsilon \\ \sigma \end{pmatrix}(t,x) = \begin{pmatrix} A_v \\ A_\varepsilon \\ A_\sigma \end{pmatrix} \mathrm{e}^{-(-\mathrm{Im}\omega)t} \mathrm{e}^{\mathrm{i}k\left(x - \frac{\mathrm{Re}\omega}{k}t\right)}$$

the dispersion relation is

$$\omega^2 \frac{1 - \mathrm{i}\tau\omega}{1 - \mathrm{i}\frac{\hat{E}}{E}\omega} = \frac{E}{\varrho}k^2$$

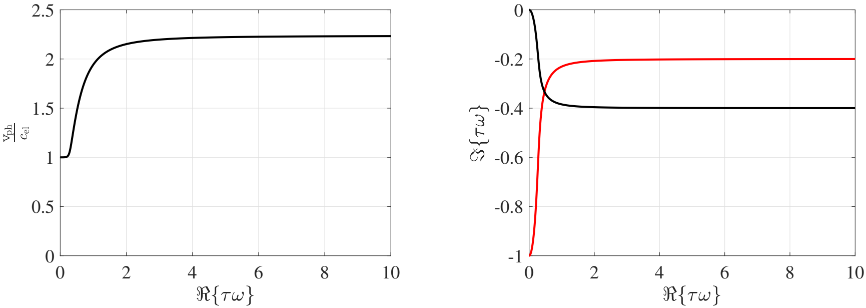
if $|\omega| \to 0$ (slow process):

$$\mathsf{v}_{\mathrm{ph}} = \sqrt{\frac{E}{\varrho}} = c_{\mathrm{el}}$$

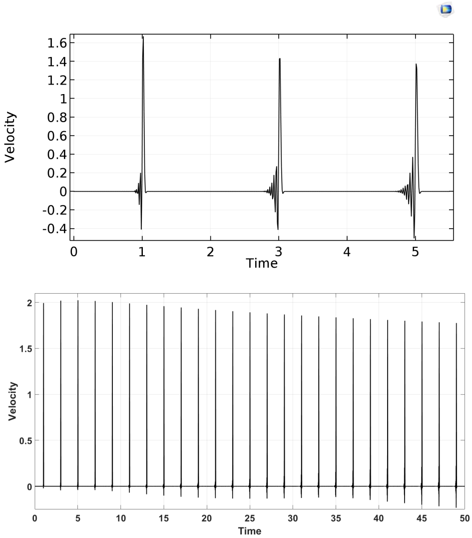
if $|\omega| \to \infty$ (fast process):

$$\mathsf{v}_{\mathrm{ph}} = \sqrt{\frac{\hat{E}_{\mathrm{dyn}}}{\varrho}} > c_{\mathrm{el}}$$

Phase velocity and temporal attenuation, here $\frac{E_{\text{dyn}}}{E_{\text{stat}}} = 5$



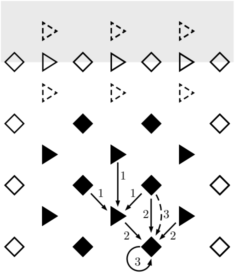
Numerical solution of elastic wave propagation with COMSOL $\Longrightarrow$ reliability?



- Stability (numerical originated artificial exponential-like blow up)
- Dispersion error (numerical originated artificial oscillations)
- Dissipation error (numerical originated artificial increase/decrease of the amplitudes)

Our numerical method

- Thermodynamical extension of a symplectic numerical scheme on a staggered space-time grid
- The discretized equations:



$$v_{n+1/2}^{j+1/2} = v_{n+1/2}^{j-1/2} + \frac{E}{\varrho} \frac{\Delta t}{\Delta x} \left(\varepsilon_{n+1}^{j} - \varepsilon_{n}^{j} \right),$$

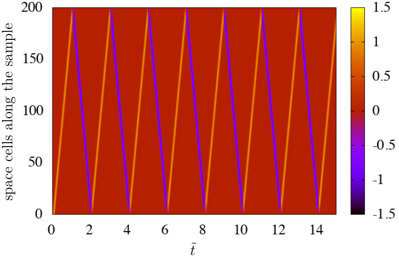
$$\varepsilon_{n}^{j+1} = \varepsilon_{n}^{j} + \frac{\Delta t}{\Delta x} \left(v_{n+1/2}^{j+1/2} - v_{n-1/2}^{j+1/2} \right),$$

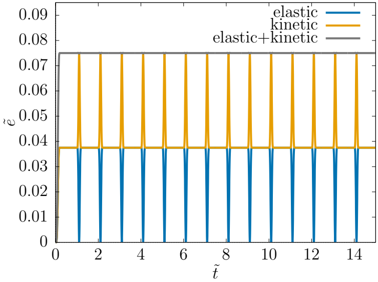
$$\sigma_{n}^{j+1} = \frac{1}{1 - \alpha + \frac{\tau}{\Delta t}} \left\{ \left(\frac{\tau}{\Delta t} - \alpha \right) \sigma_{n}^{j} + E \left[\alpha \varepsilon_{n}^{j} + (1-\alpha) \varepsilon_{n}^{j+1} \right] + \hat{E} \frac{\varepsilon_{n}^{j+1} - \varepsilon_{n}^{j}}{\Delta t} \right\}$$

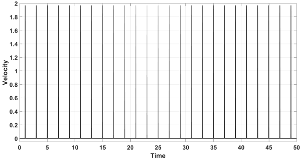
if $\alpha = \frac{1}{2}$ the scheme is second order accurate

Fülöp *et al.*: Thermodynamical extension of a symplectic numerical scheme with half space and time shifts demonstrated on rheological waves in solids. *Entropy* 22, 155 (2020)

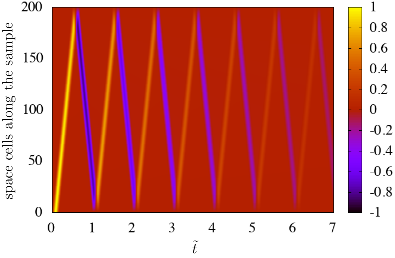
Numerical results: elastic wave propagation

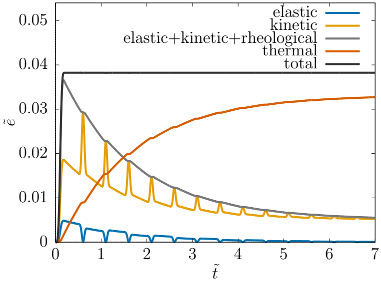




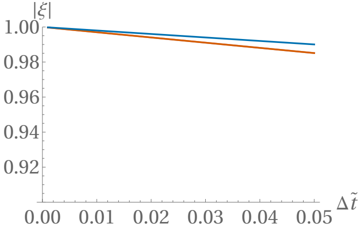


Numerical results: rheological wave propagation

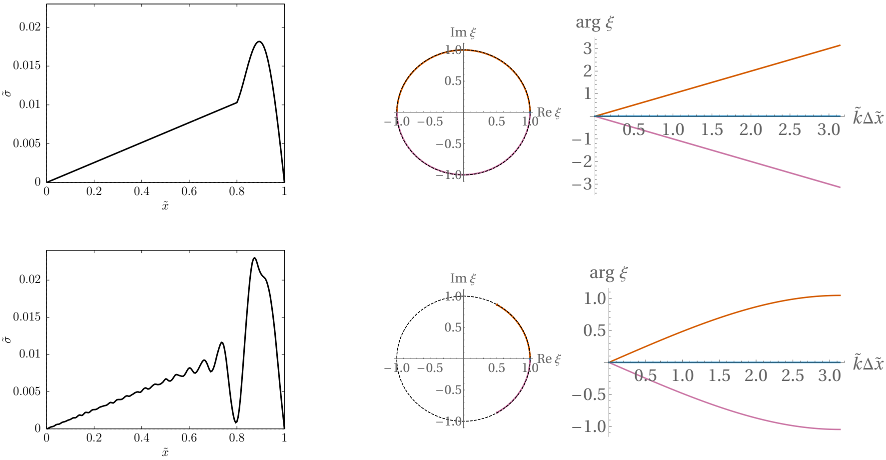




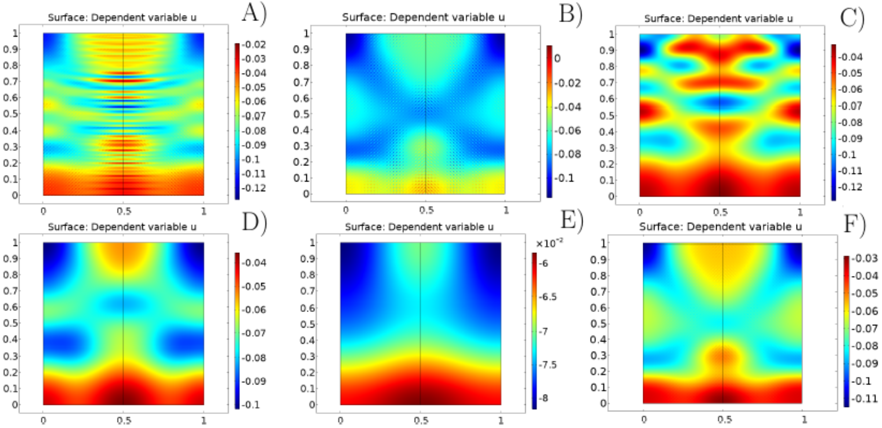
Dissipation error



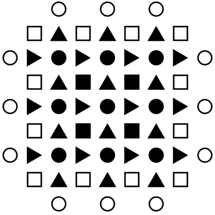
Dispersion error

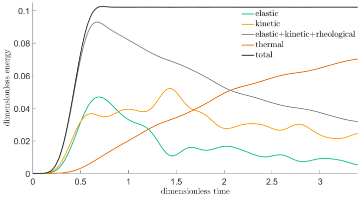


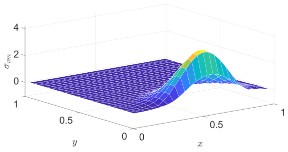
COMSOL 2D scalar wave equation $\Longrightarrow$ reliability?

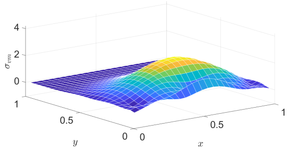


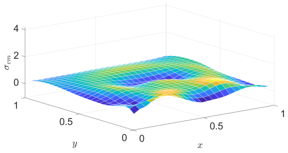
Generalization the self-developed scheme for 3 spatial dimensions

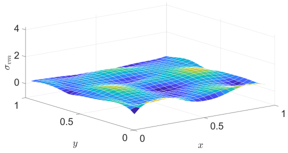












Pozsár *et al.*: Four spacetime dimensional simulation of rheological waves in solids and the merits of thermodynamics. *Entropy* 22, 1376 (2020)

Experimental investigations

László KOVÁCS

ROCKSTUDY

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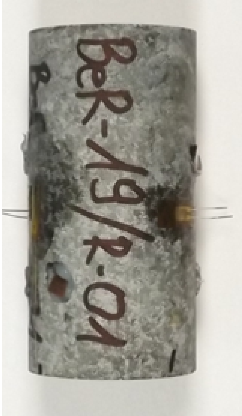
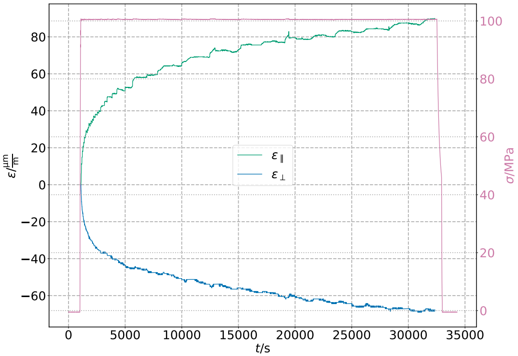
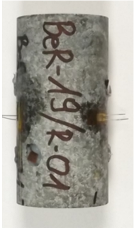
Measurement of creep: viscosities

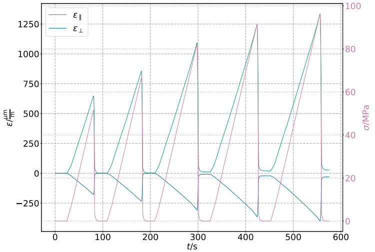


Contaminated
monzonite

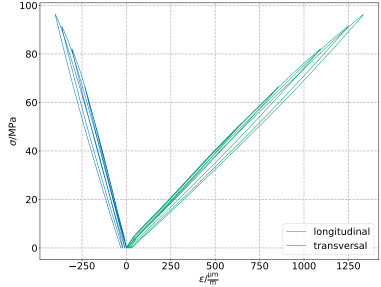


Measurement of hysteresis: time constants and beyond

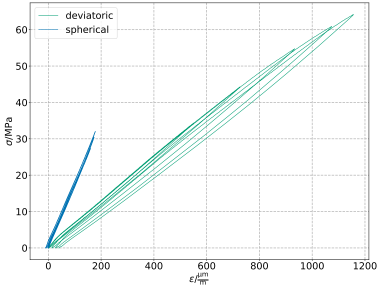




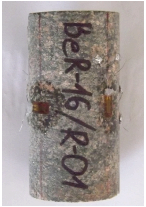
longitudinal–transversal



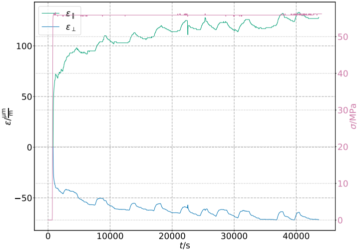
deviatoric–spherical



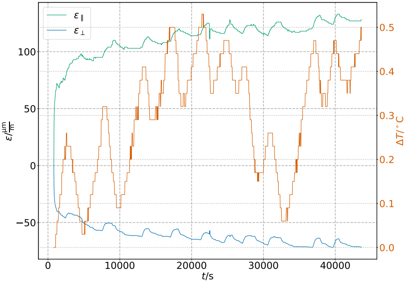
The effect of thermal expansion



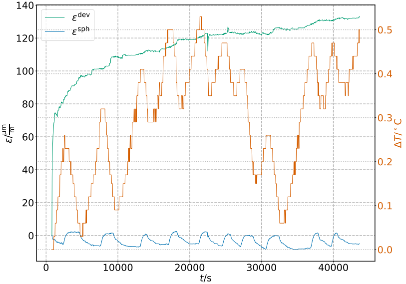
Reddish-gray hybrid monzonite



longitudinal–transversal



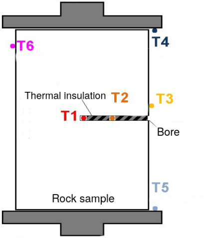
deviatoric–spherical

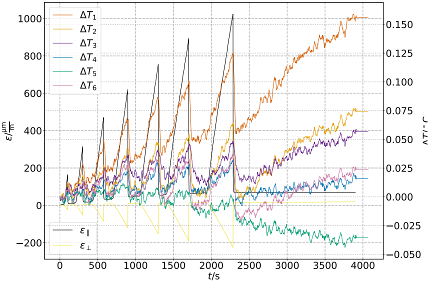


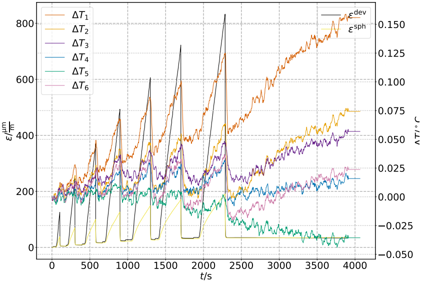
Temperature is space dependent

Gray andesite

T1 . . . T6: temperature sensors







- Rheology of the surrounding rocks may notably affect the filtering and mitigation of Newtonian noise
- Advantage or disadvantage?
- Measurements of the rocks in the planned locations are required!

THANK YOU FOR YOUR KIND ATTENTION!