Gravitational Waves from Domain Wall Dynamics

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Based on ET blue book contribution with
Alberto Mariotti, Oriol Pujolàs and
Simone Blasi

ET symposium

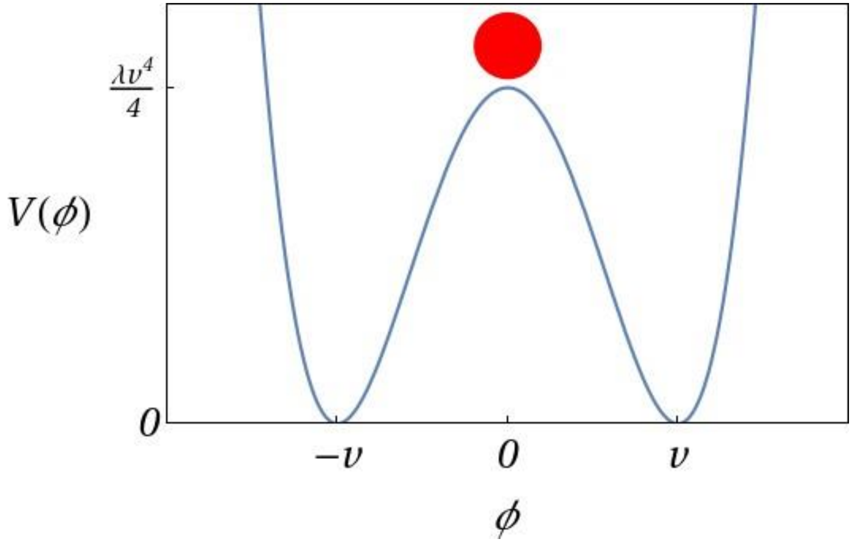
May 6, 2024

Domain walls

Def.: **Topological defects** from spontaneously broken **discrete symmetry**

$\mathbb{Z}_2 : \phi \to -\phi$

$$V = \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2$$



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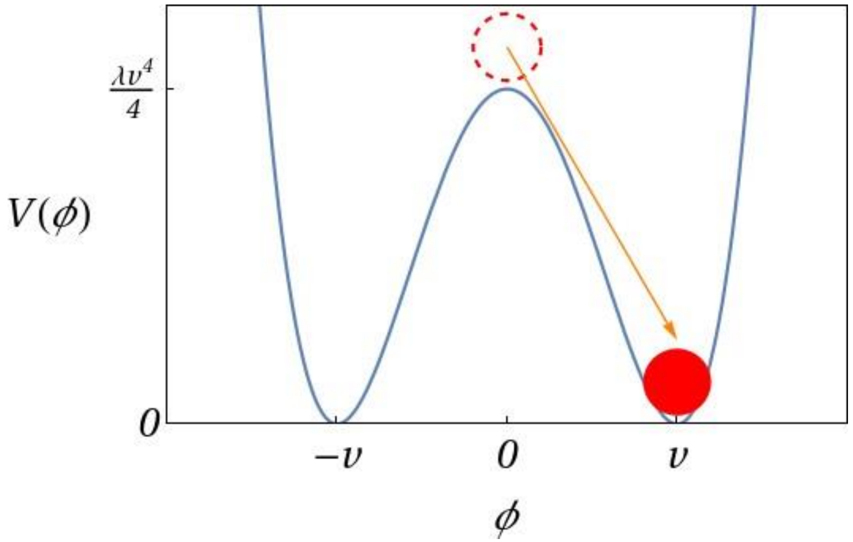
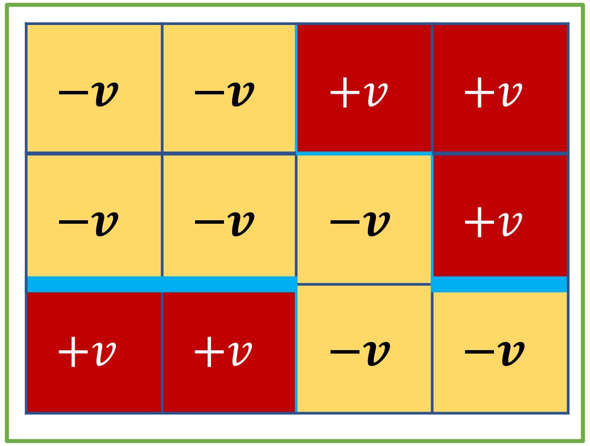


Universe



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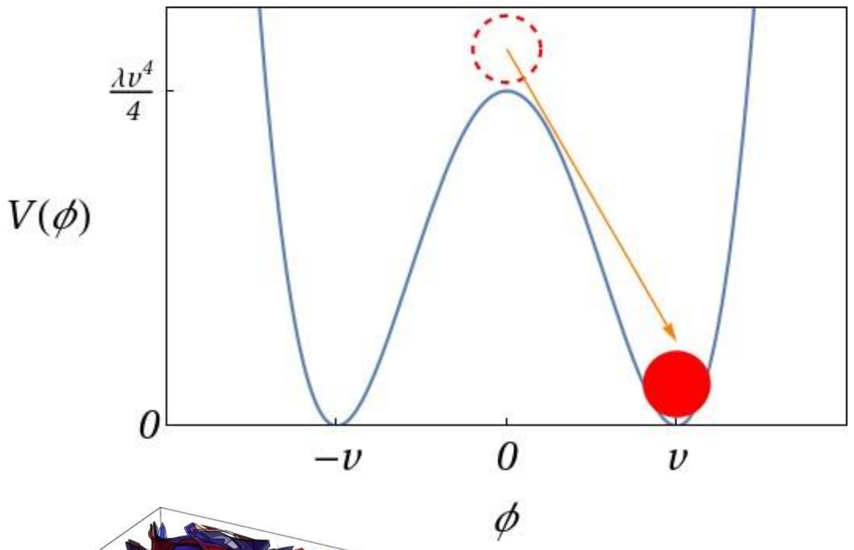

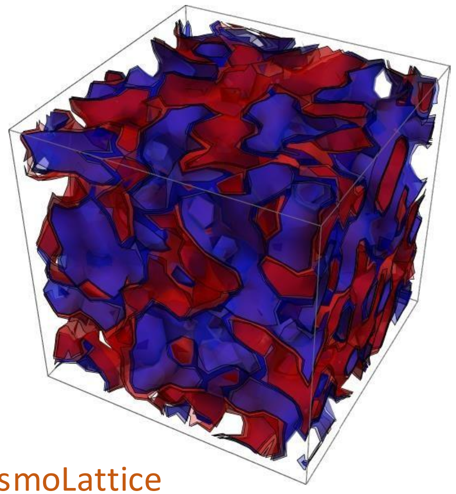

Blue: $-v$
Red: $+v$

credits: CosmoLattice

Creation of uncorrelated **domains** with **different** vacuum expectation values

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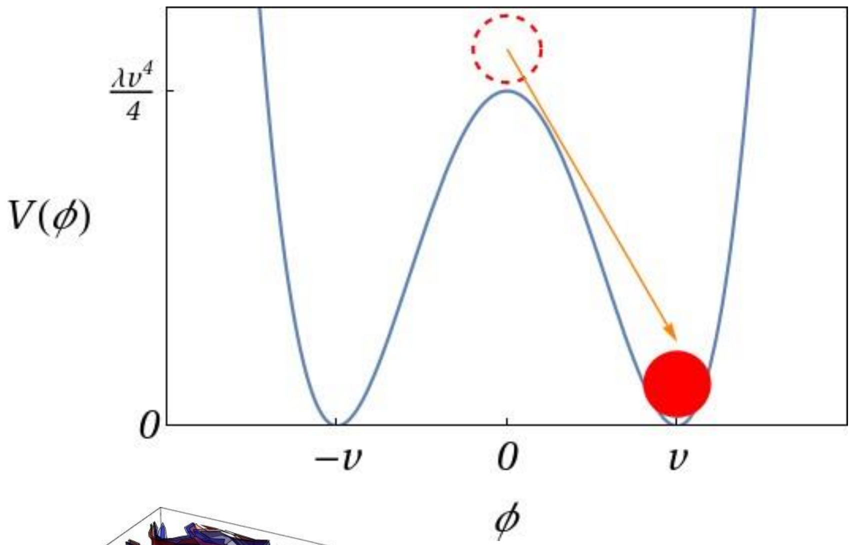

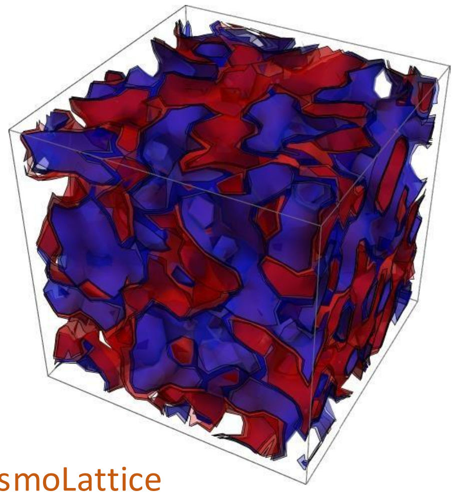

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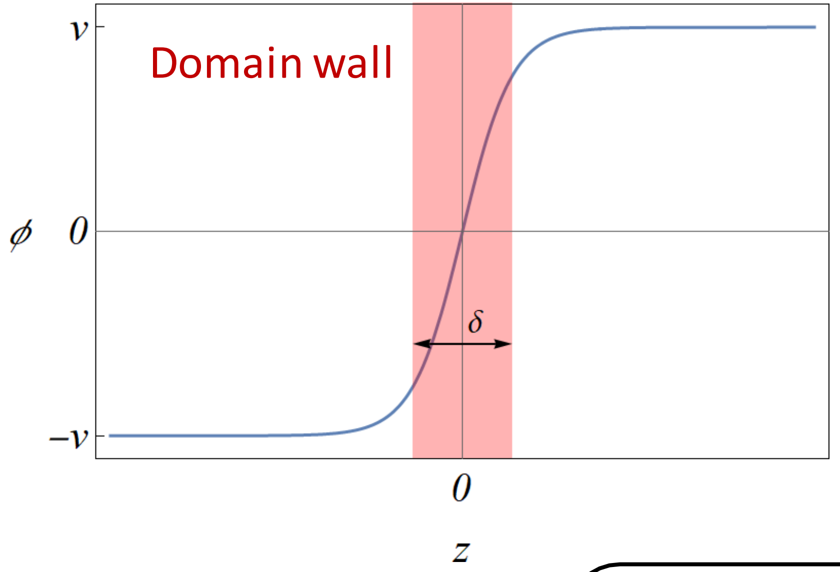
Creation of uncorrelated **domains** with **different** vacuum expectation values

But what is a domain wall?



- Width $\delta \approx \left(\sqrt{\frac{\lambda}{2}}\, v\right)^{-1} \sim m_\phi^{-1}$
- Tension $\sigma \sim m_\phi v^2$

Domain wall:

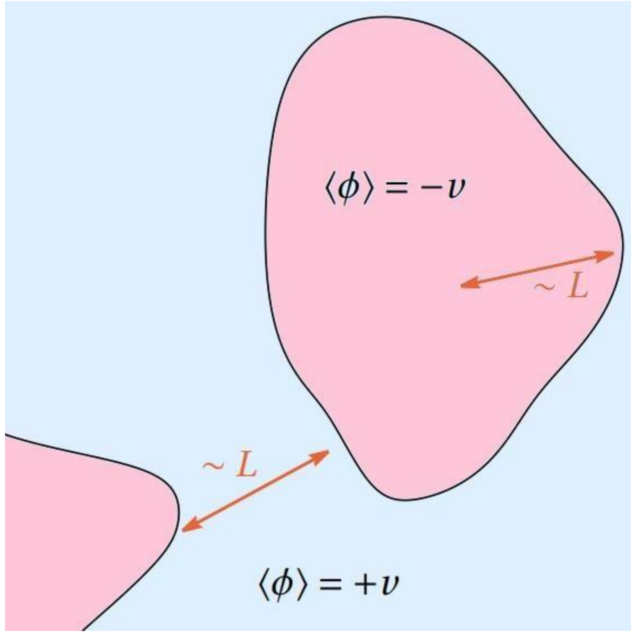
Large energy density **localized** in 2D **surface**

Domain wall dynamics: the scaling regime

- **Tension force** stretches the walls up to horizon sizes

$$p_T \sim \frac{\sigma}{L}$$

- Characteristic **length scale** L
  - Curvature radius
  - Average distance



Numerical

- Press et al., Astrophys. J., 1989
- Hindmarsh et al., PRD, 2003
- Avelino et al., PRD, 2005
- Avelino et al., PLB, 2005
- Martins et al., [1110.3486], PRD
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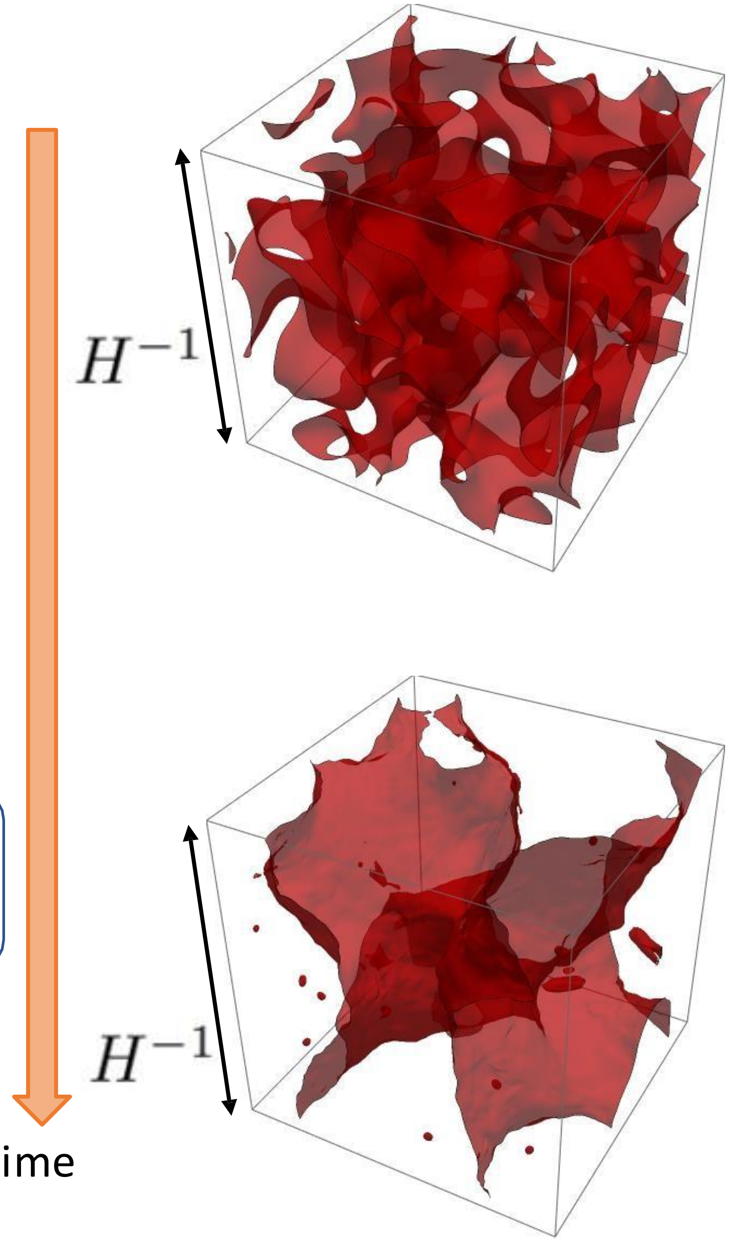
Scaling regime

$$L \sim H^{-1} \sim t$$



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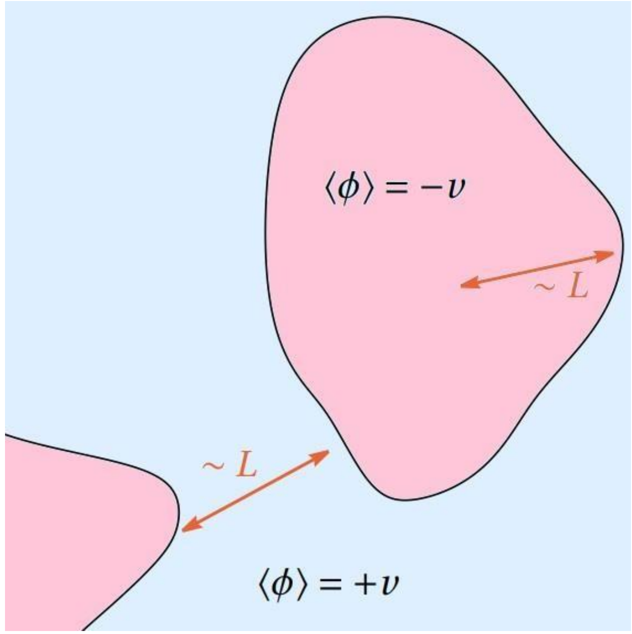

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Scaling regime

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But wait… problem???

- Domain wall energy density:

$$\rho_{\rm dw} \sim \sigma L^2/L^3 \sim \sigma/t$$

- In radiation dominated Universe: $\rho_{\rm rad} \sim 1/t^2$

Domain walls will dominate!

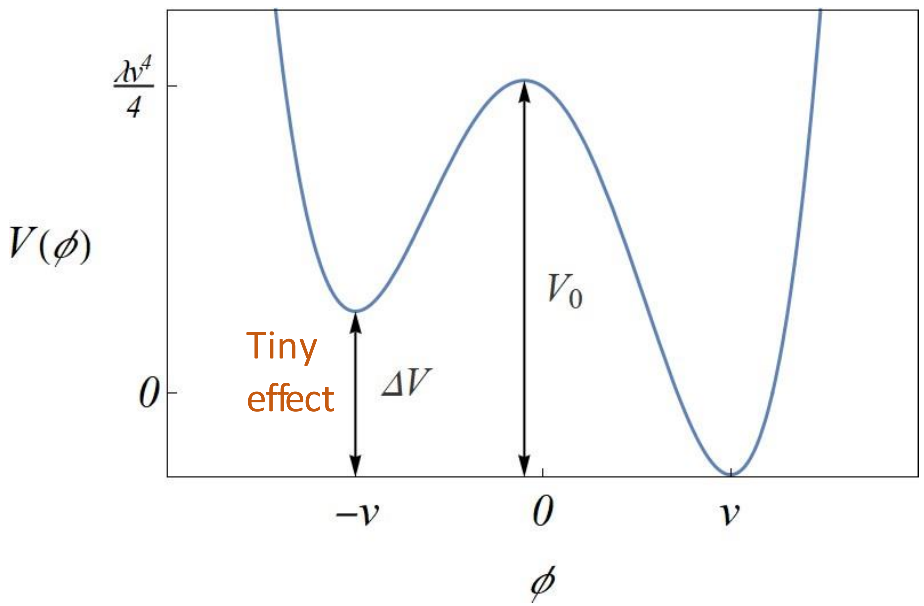
- Strong bound on tension

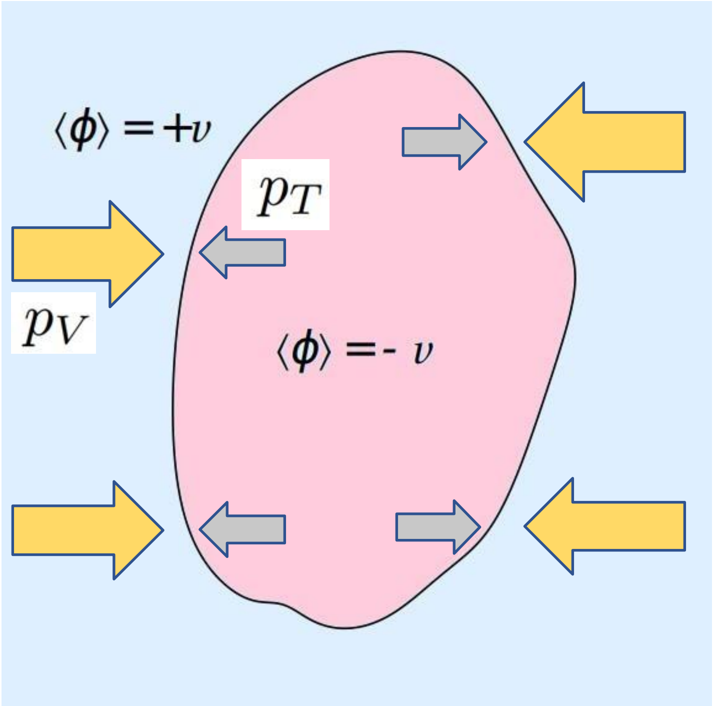
$$\sigma < \mathcal{O}({\rm MeV}^3)$$

Zel'dovich et al., 1974

Domain wall solution: introducing a bias

- Make symmetry slightly approximate (**energy bias**)





- Creates **volume pressure force**

$$p_V \sim \Delta V$$

Annihilation when

$$p_T \lesssim p_V$$

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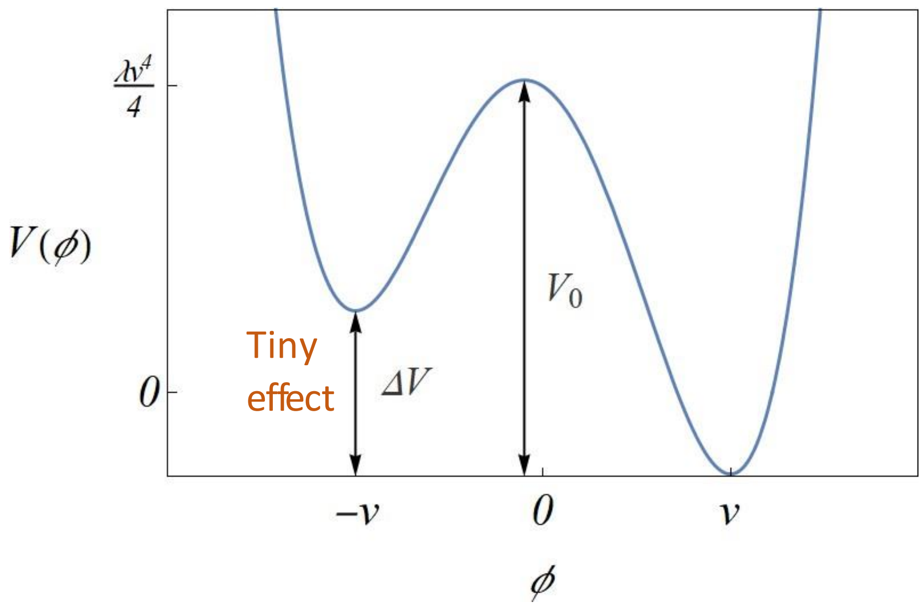

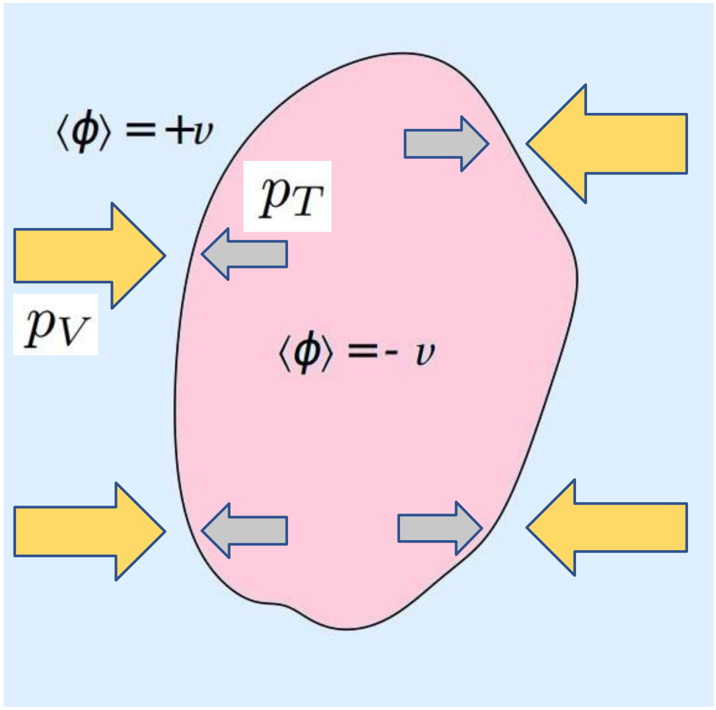

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Important time scales

Domination

$$\rho_{\rm dw} = \rho_{\rm rad}$$

$$t_{\rm dom} = \frac{3}{4}\frac{M_p^2}{\sigma}$$

Planck mass

Annihilation

$$p_T \lesssim p_V$$

$$t_{\rm ann} = \frac{\sigma}{\Delta V}$$

Gravitational wave spectrum from domain walls

- Production of gravitational waves until domain walls annihilate

From **dimensional arguments** using quadrupole formula:

domain wall mass

$$P_{\rm gw} \sim G\dddot{Q}_{ij}\dddot{Q}_{ij} \qquad Q_{ij} \sim m_{\rm dw} L^2$$

Scaling

$$\rho_{\rm gw} \sim \frac{P_{\rm gw} t}{t^3} \sim G\sigma^2$$

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Simulations on the lattice (expanding universe):

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2 \phi}{a(t)^2} = -\frac{\partial V}{\partial \phi}$$

$$\cdot \equiv \partial / \partial t$$

Scale factor

Scalar field **sources** the gravitational waves

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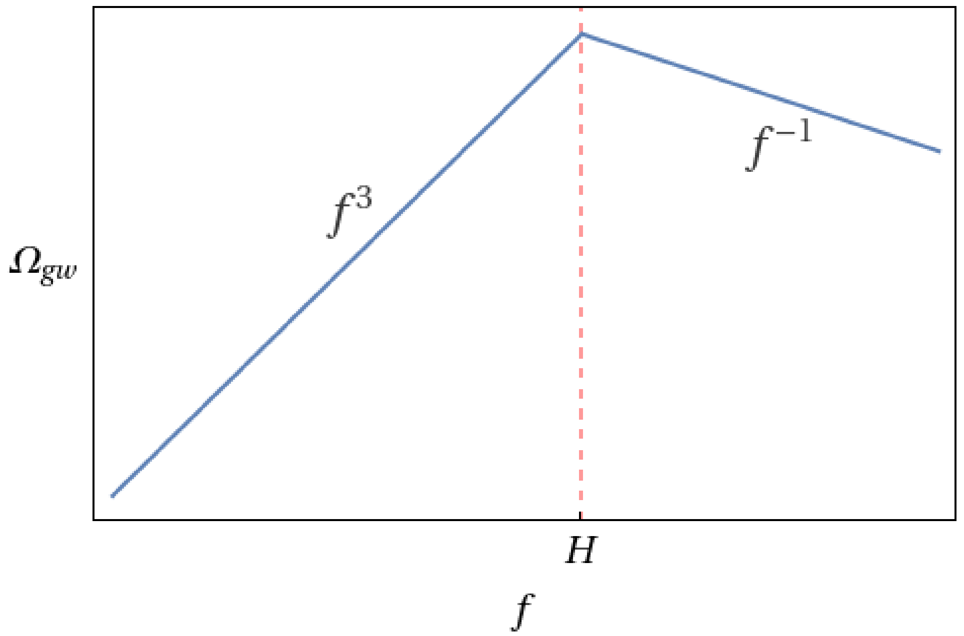
From **simulations**:

Saikawa et al., JCAP, 2014

$$\Omega_{\rm gw}(f,t) = \Omega_{\rm gw}^{\rm peak}(t) \begin{cases} \left(\frac{f}{f_{\rm peak}}\right)^3 & f \le f_{\rm peak} \\ \left(\frac{f}{f_{\rm peak}}\right)^{-1} & f > f_{\rm peak} \end{cases}$$

$$\Omega_{\rm gw}^{\rm peak}(t) = \frac{\tilde{\epsilon}_{\rm gw} \mathcal{A}^2 G\sigma^2}{\rho_c(t)} \qquad f_{\rm peak}(t) = H(t)$$



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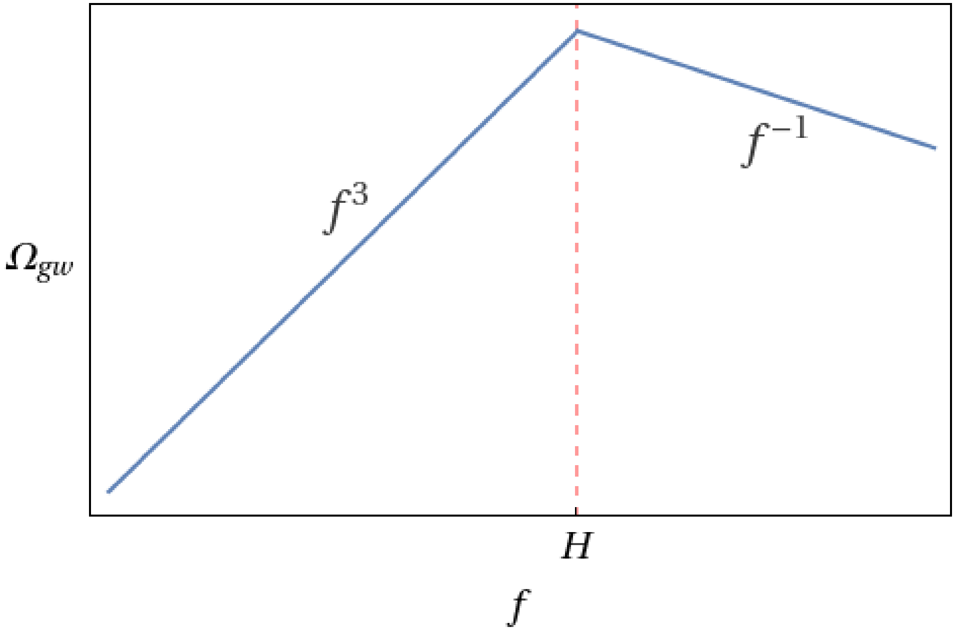
With a little bit of algebra:

$$\Omega_{\rm gw}^{\rm peak}(t) = \frac{\tilde{\epsilon}_{\rm gw} \mathcal{A}^2 G \sigma^2}{\rho_c(t)} \sim \left(\frac{t}{t_{\rm dom}}\right)^2$$

time of domination

Late time emissions contribute the most!



Gravitational wave spectrum from domain walls

Biggest contribution from the time of annihilation

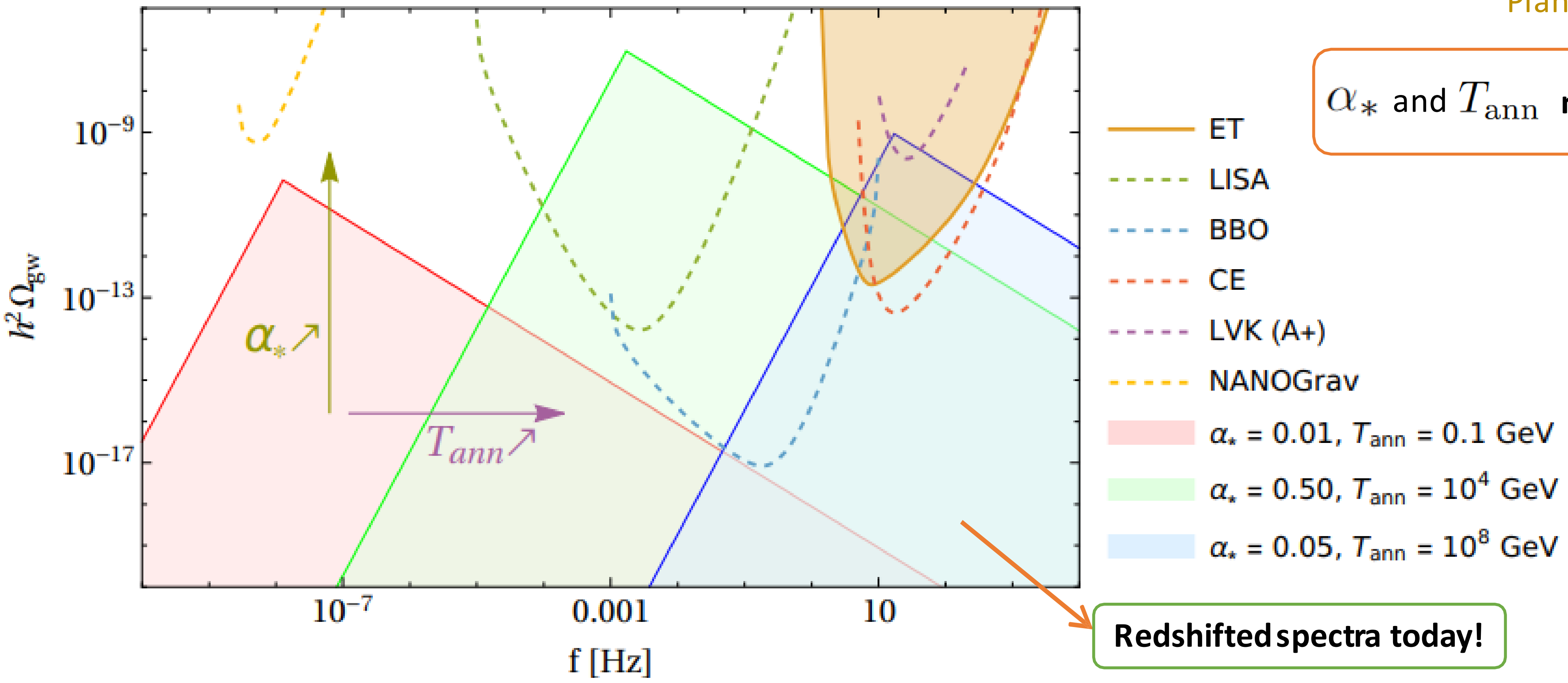
$$\Omega_{\rm gw}^{\rm peak}(t) = \frac{3}{32\pi}\tilde{\epsilon}_{\rm gw}\alpha_*^2$$

$$\alpha_* = \left.\frac{\rho_{\rm dw}}{3H^2M_{\rm pl}^2}\right|_{\rm ann}$$

$$H \sim \frac{1}{t} \sim \frac{T^2}{M_p}$$

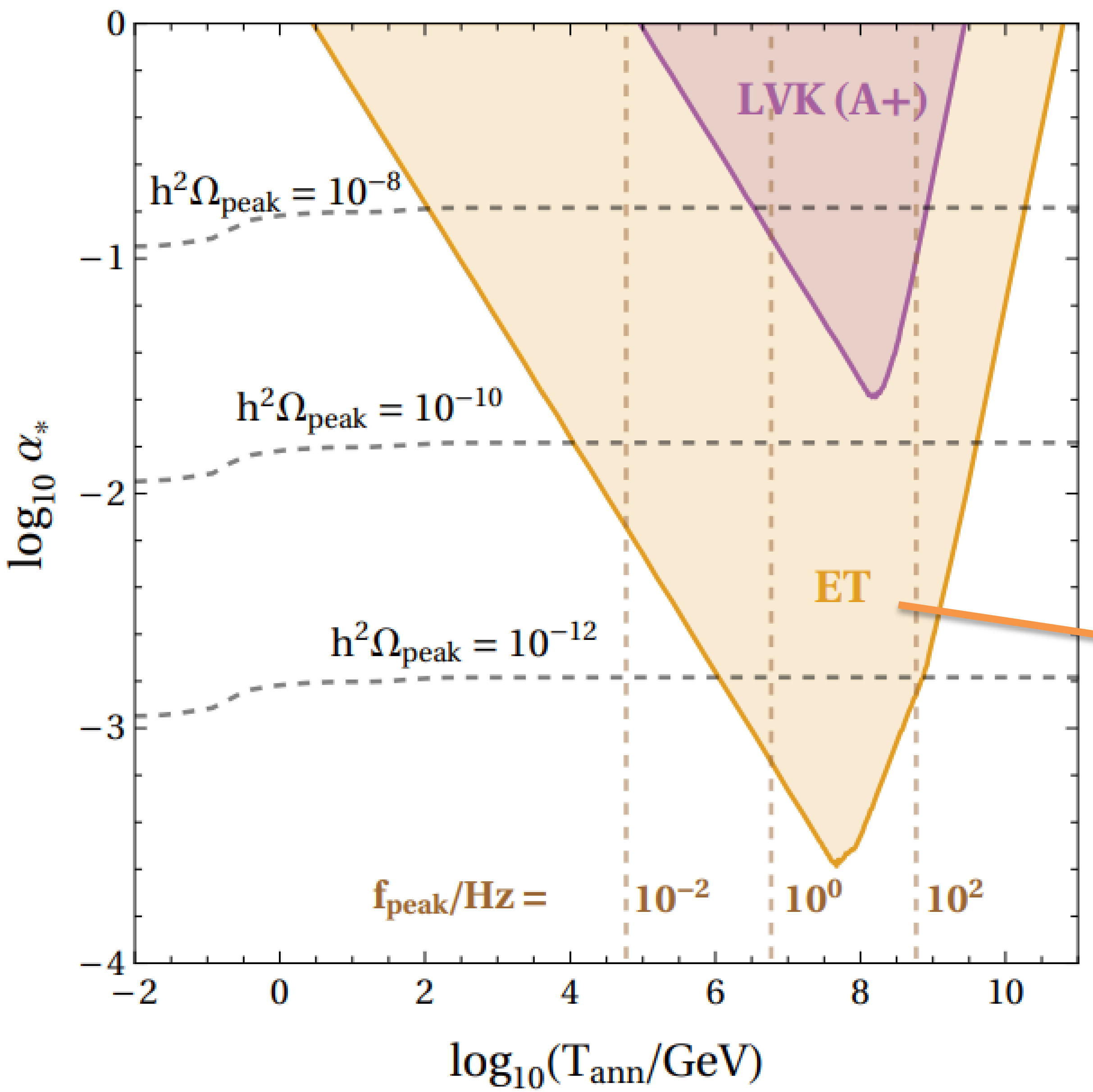
Planck mass

α_* and $T_{\rm ann}$ **model dependent**



Redshifted spectra today!

ET probes a significant part of the parameter space



$$\alpha_* = \frac{\rho_{dw}}{3H^2 M_{pl}^2}\Big|_{ann}$$

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Planck mass

α_* and T_{ann} **model dependent**

Several orders of magnitude gained

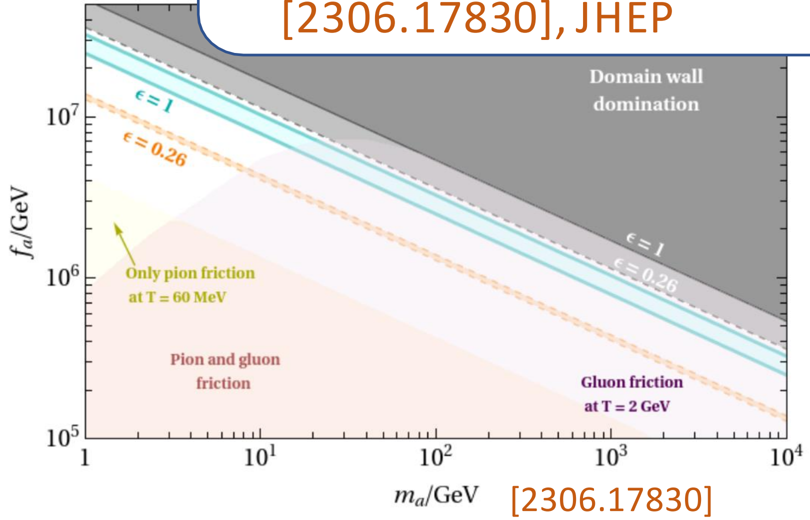
Domain walls as an ongoing field of research

Axion models

- Craig et al., [2012.13416], JHEP
- Sikivie et al., PRD, 1999
- Pujolàs et al., [2107.07542], PRL
- Gelmini et al., [2103.07625], PRD
- …

Friction effects

- Blasi, Mariotti, **Rase**, Sevrin, Turbang, [2210.14246], JCAP
- Blasi, Mariotti, **Rase**, Sevrin, [2306.17830], JHEP



[2306.17830]

Production of Primordial Black Holes

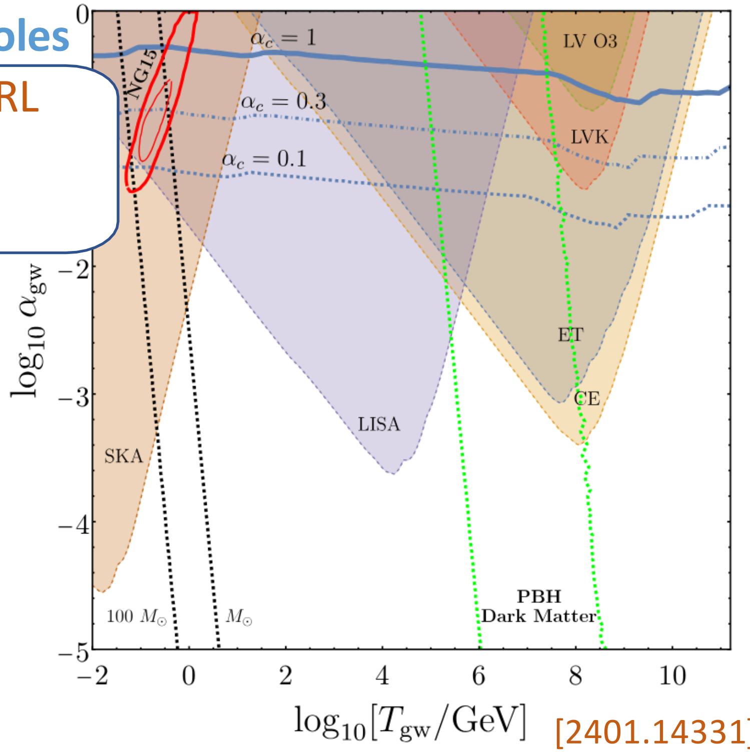
- Pujolàs et al., [1807.01707], PRL
- Pujolàs et al., [2401.14331]
- …

Baryogenesis

- Takahashi et al., [1504.07917], JCAP
- …

Dark Matter production

- Saikawa et al., [1412.0789], PRD
- …



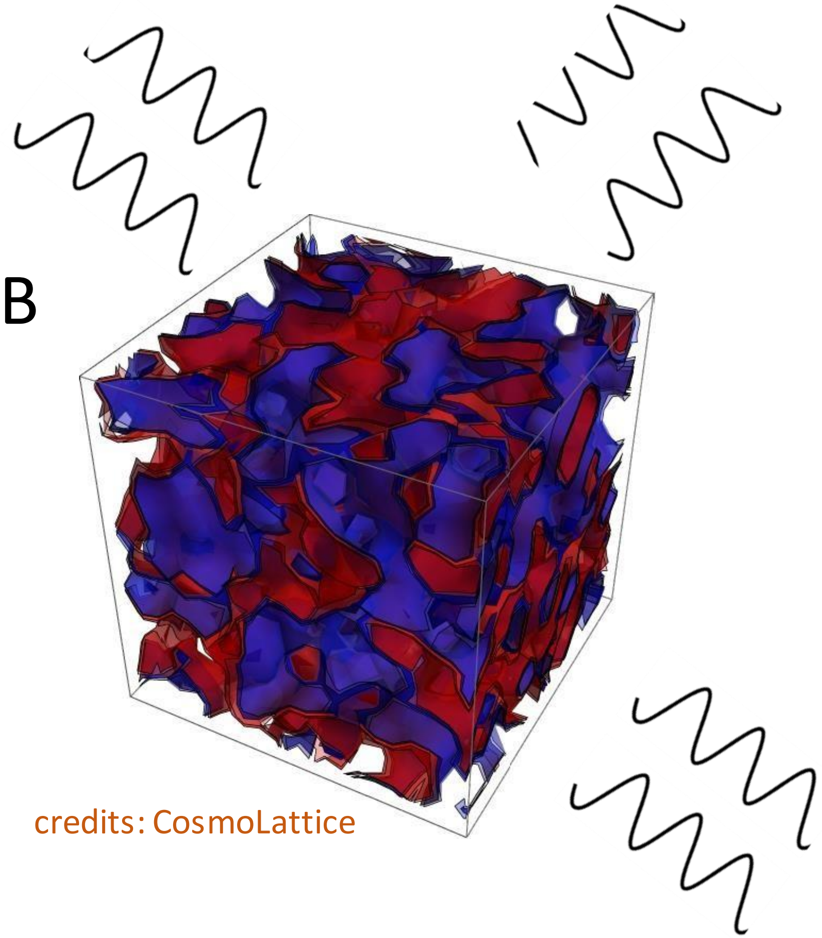
[2401.14331]

Other

- Gelmini et al., [2009.01903], JCAP
- Fornal and Pierre, [2209.04788], PRD
- Eto et al., [1805.07015], JHEP
- …

Conclusion

- Domain walls are interesting physics case for SGWB
  - **BSM motivated**
  - Overclosure problem can be avoided
- SGWB signal is **broken power law**
- ET paves the way to further exploration



credits: CosmoLattice

Back up

Specific case: axion domain walls

$$\mathcal{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - \lambda \left(\Phi^\dagger \Phi - \frac{v_a^2}{2} \right)^2 - \Lambda^4 \left[1 - \cos \left(\frac{a N_{DW}}{v_a} \right) \right]$$

Anomalous U(1) in strong gauge theory

e.g. Peccei-Quinn symmetry in QCD
(Peccei and Quinn, PRL, 1977)

$$\Phi = \frac{\rho}{\sqrt{2}} e^{ia/v_a}$$

Pseudo-Nambu Goldstone boson = **Axion**

$$f_a \equiv \frac{v_a}{N_{DW}}$$

Axion decay constant

Discrete symmetry

$H \sim m_a$

Axion domain walls

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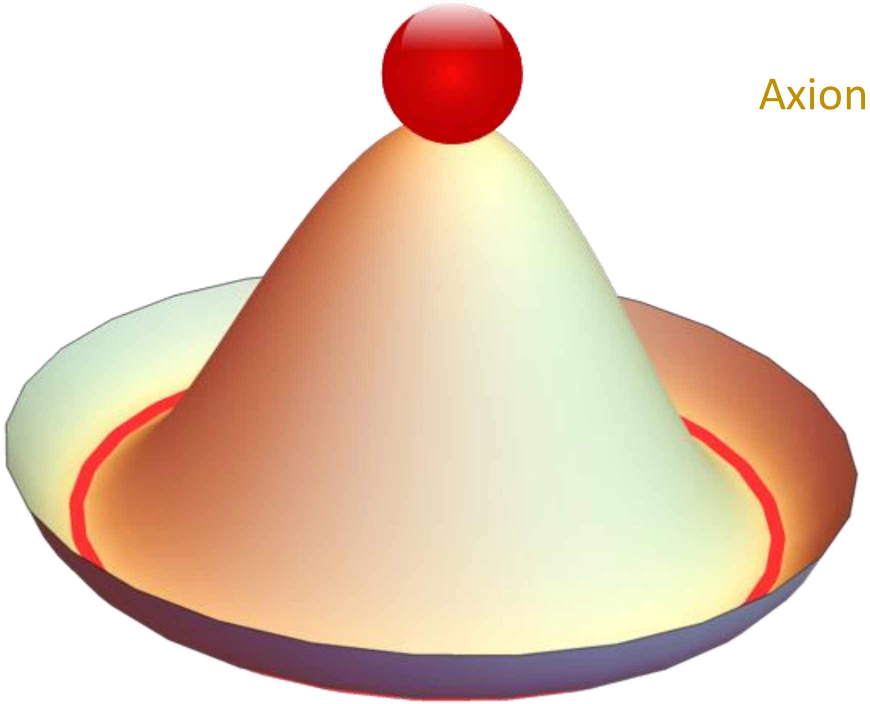

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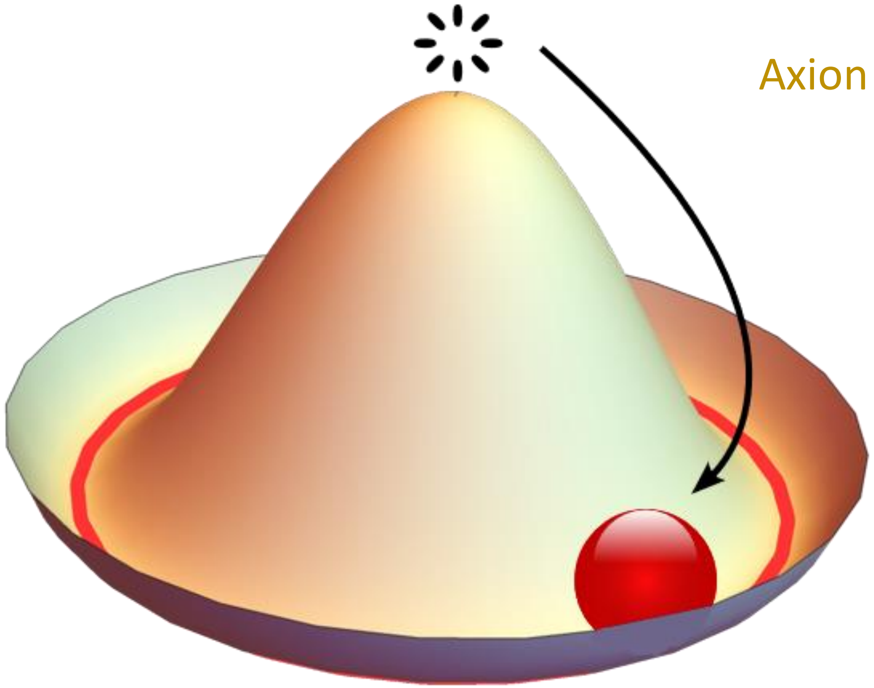

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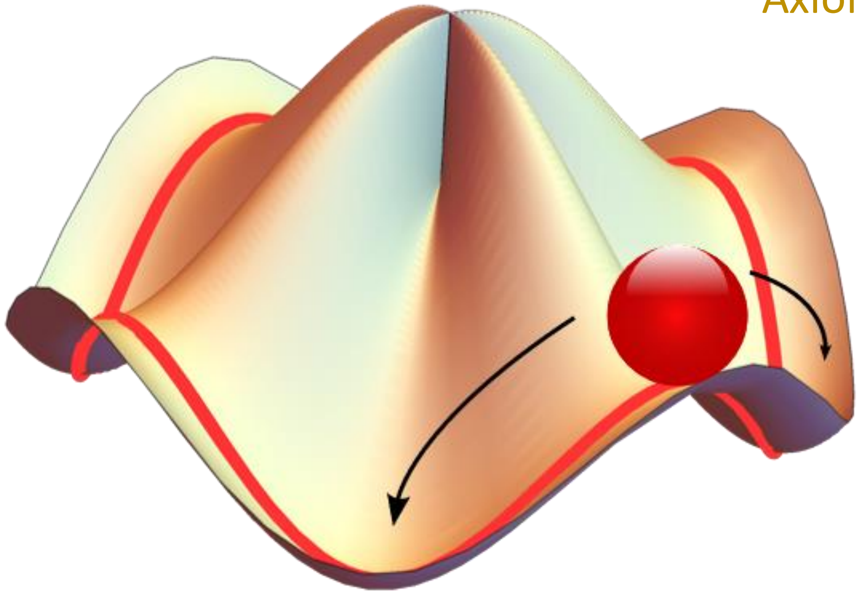

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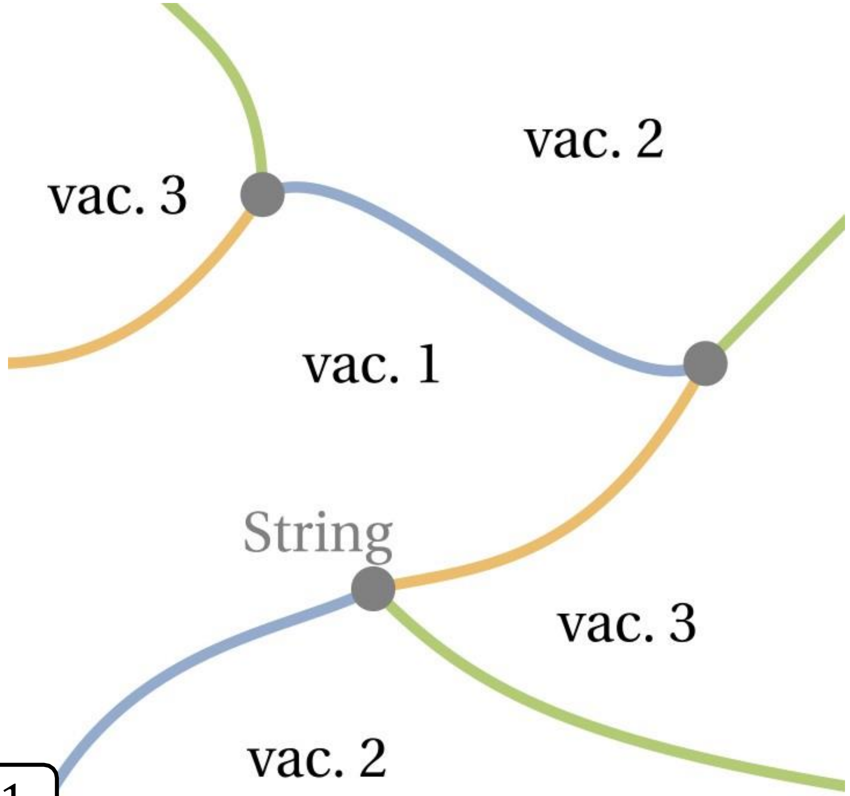
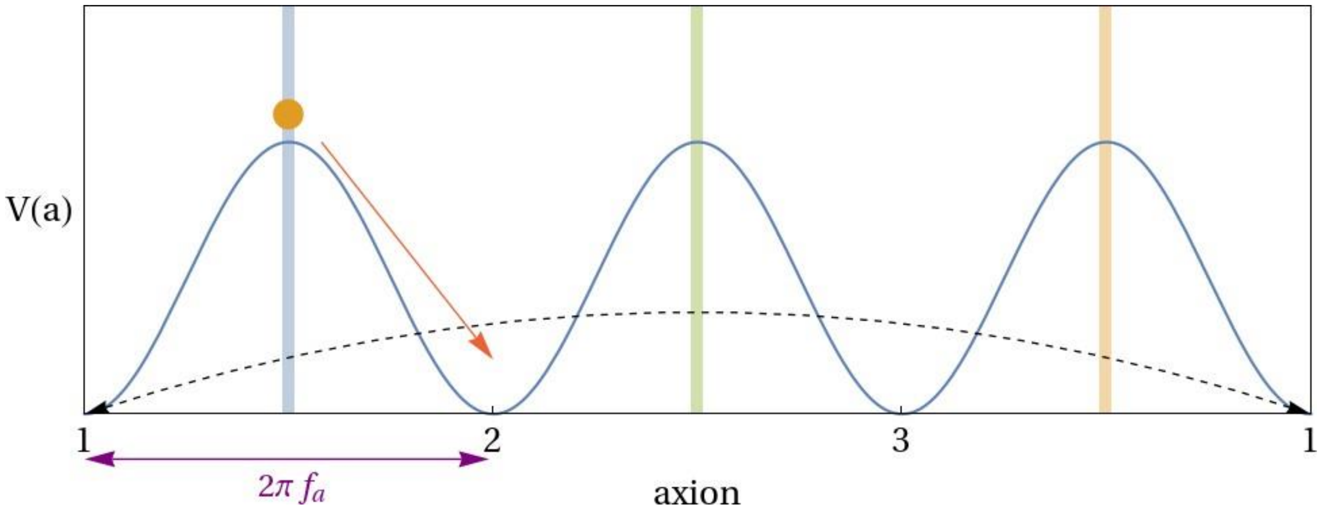


$$\mathbb{Z}_{N_{DW}}: a \to a + 2\pi k f_a, \quad 0 \le k \le N_{DW} - 1$$



Specific case: axion domain walls

Anomalous U(1) in strong gauge theory

↓ Pseudo-Nambu Goldstone boson = **Axion**

Discrete symmetry

↓ $H \sim m_a$

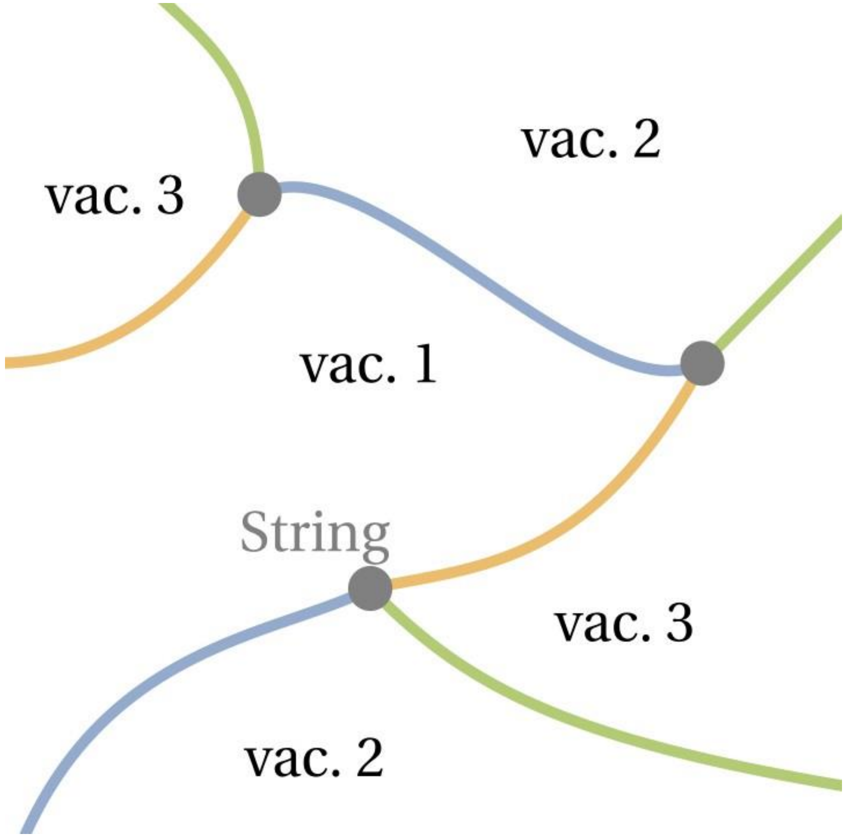
Axion domain walls

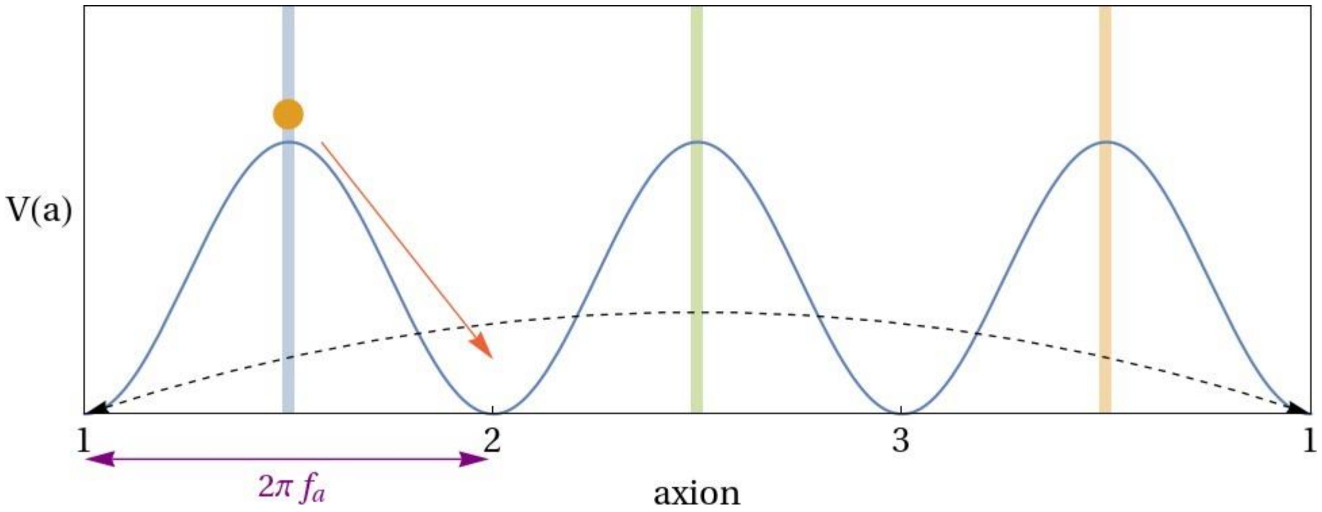
Domain Wall System

$$\rho_s \sim \mu/t^2 \sim v_a^2/t^2$$

$$\rho_{\rm dw} \sim \sigma/t \sim m_a f_a^2/t$$

$$\rho_s/\rho_{\rm dw} \sim \frac{1}{m_a t}$$





Naturalness of the bias

Small explicit breaking... Is it natural?

Discrete symmetry descending from anomalous U(1)

- Not expected to be exact
  (e.g. Peccei-Quinn quality problem)
  Barr and Seckel, PRD, 1992
  Kolb et al., PLB, 1992

- Explicitly broken by **higher dimensional operators**

$$V_{M_{\mathrm{Pl}}} = C_{n,m} \frac{\left(\Phi^\dagger \Phi\right)^m \Phi^n}{M_{\mathrm{Pl}}^{2m+n-4}} + \mathrm{h.c.}$$

Generated dynamically

- Induced by strong dynamics effect, e.g. **Standard Model QCD**

- ALP couples anomalous to QCD

↓

QCD induced potential acts as bias at **QCD scale**

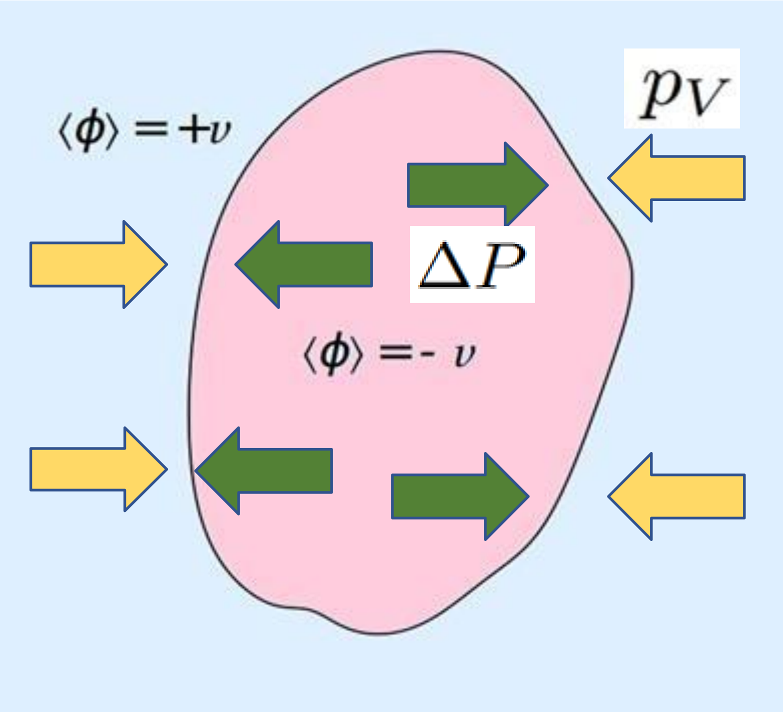
Friction effects

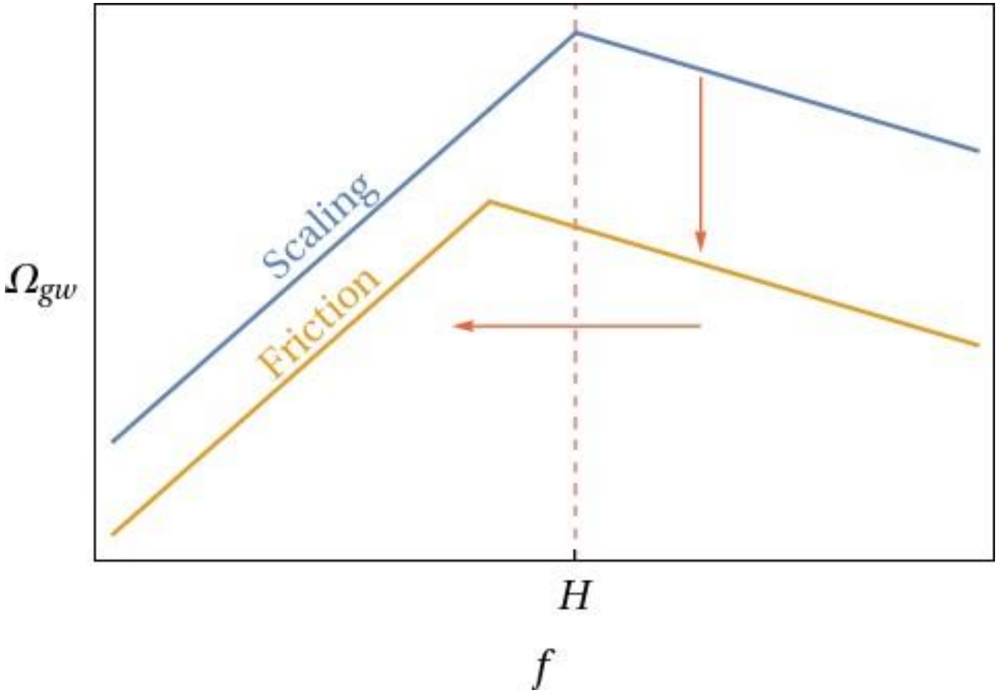
Blasi, Mariotti, **Rase**, Sevrin, Turbang, [2210.14246], JCAP

- Slow down average wall velocity of the network

General **effects**:

- No scaling regime
- Lower peak frequency (annihilation at later time)
- Lower peak amplitude (energy loss)





The Stochastic Gravitational Wave Background

- Superposition of GW signals produced by a large number of **independent** and **unresolved** sources.

SGWB

- Astrophysical
  - BBH
  - BNS
  - Pulsars
  - Supernovae
  - …
- **Cosmological**
  - Inflation
  - Phase transitions
  - Topological defects
  - …

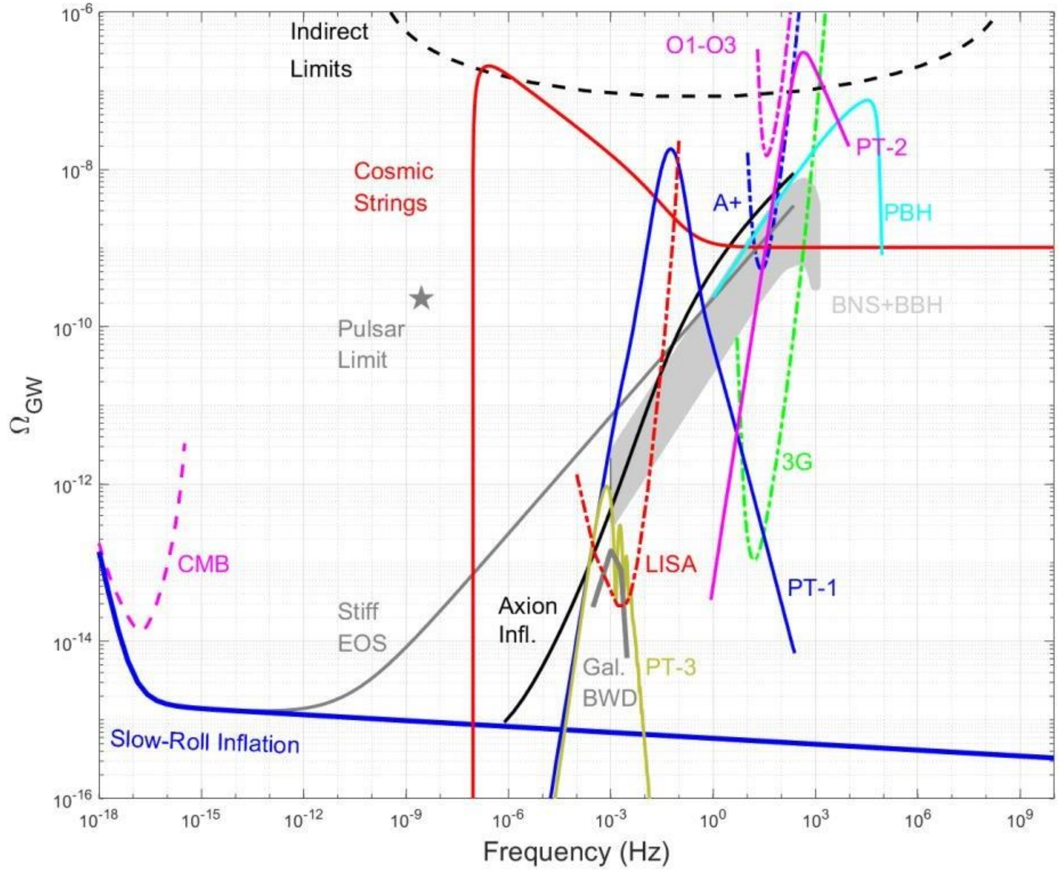
Probing High Energy Physics

Spectrum of GW energy density per logarithmic frequency interval

$$\Omega_{\rm gw}(f) = \frac{1}{\rho_c}\frac{{\rm d}\rho_{\rm gw}}{{\rm d}\ln f} \qquad \rho_c = \frac{3H_0^2}{8\pi G}$$

Critical density

Fig. from Caldwell, R. et al., [2203.07972], Contribution to the 2022 Snowmass Summer Study



The Stochastic Gravitational Wave Background

Stochastic nature

Gravitational wave signal today is superposition of **many independent horizon volumes**

➡ $h_{ij}(t, \mathbf{x})$ random variable, characterized statistically by **ensemble average**

$$\langle \tilde{h}_A^*(f, \hat{\mathbf{n}}) \tilde{h}_{A'}(f', \hat{\mathbf{n}}') \rangle = \delta(f - f') \frac{\delta^2(\hat{\mathbf{n}}, \hat{\mathbf{n}}')}{4\pi} \delta_{AA'} \frac{1}{2} S_h(f)$$

- $\delta(f - f')$: stationary
- $\frac{\delta^2(\hat{\mathbf{n}}, \hat{\mathbf{n}}')}{4\pi}$: homogeneous + isotropic
- $\delta_{AA'}$: unpolarized
- $S_h(f)$: Spectral density

$$\rho_{\rm gw} = \frac{c^2}{32\pi G} \langle \dot{h}_{ij} \dot{h}^{ij} \rangle \longrightarrow \rho_{\rm gw} = \int_{f=0}^{f=\infty} \mathrm{d}\ln f \, \frac{\mathrm{d}\rho_{\rm gw}}{\mathrm{d}\ln f} \longrightarrow \Omega_{\rm gw}(f) = \frac{1}{\rho_c} \frac{\mathrm{d}\rho_{\rm gw}}{\mathrm{d}\ln f} = \frac{4\pi^2}{3H_0^2} f^3 S_h(f)$$

Cross correlation $s_1 = h_1 + n_1 \qquad s_2 = h_2 + n_2$

$$\langle \hat{C}_{12} \rangle = \langle s_1 s_2 \rangle = \langle h_1 h_2 \rangle + \langle h_1 n_2 \rangle + \langle n_1 h_2 \rangle + \langle n_1 n_2 \rangle \simeq \langle h_1 h_2 \rangle$$

Assume noise **uncorrelated**