
Revealing the strength of three-nucleon interactions with the Einstein Telescope

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Common goal for NS study

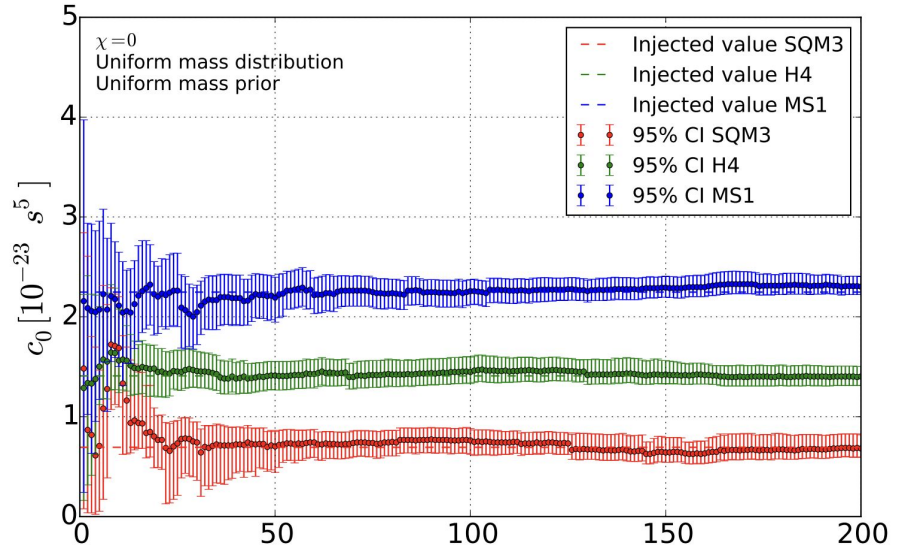
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 - Macroscopic
 - Mass, radius, ...

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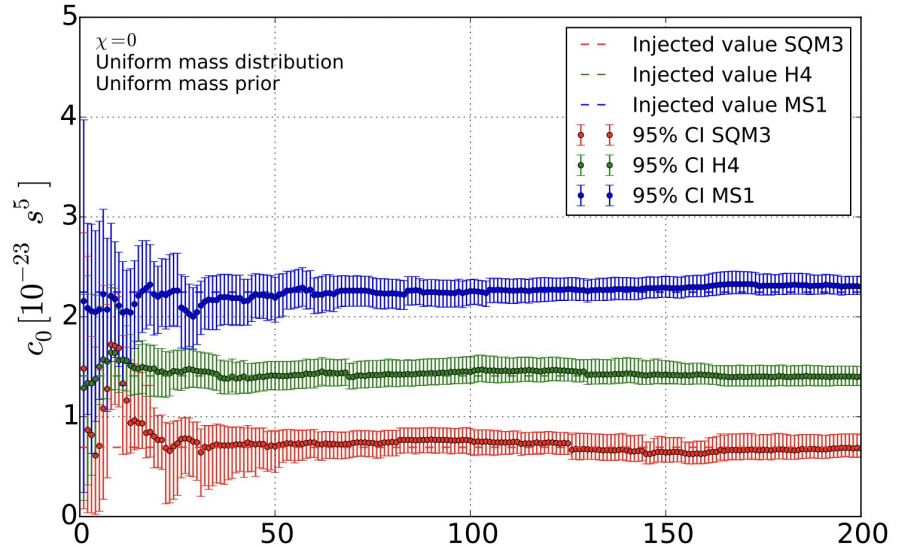
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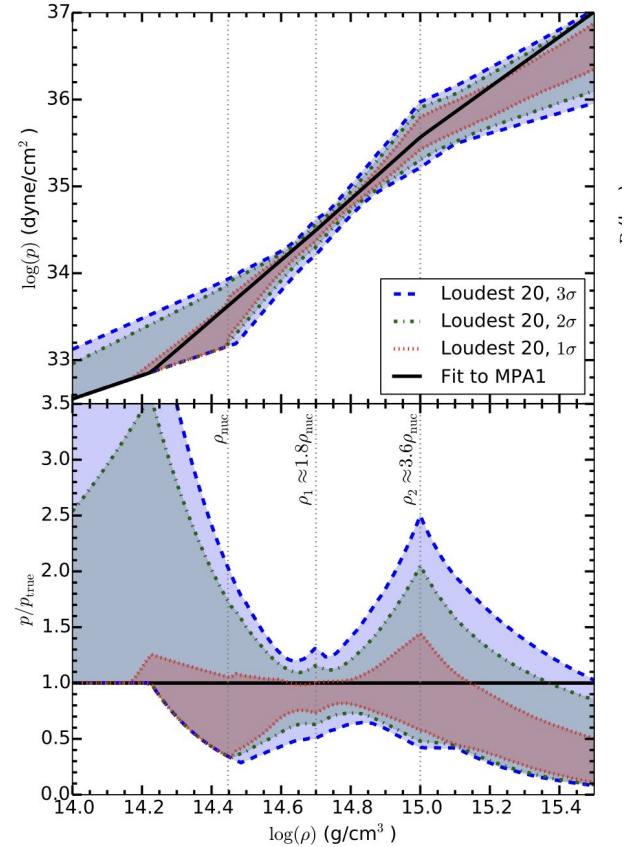
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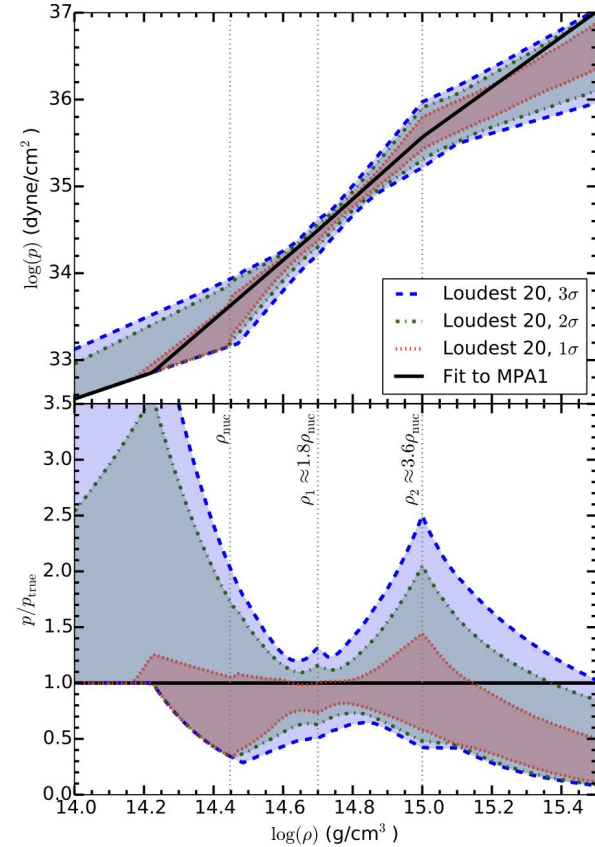
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Common goal for NS study

- Extract the equation-of-state
 - Macroscopic
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 - Pressure, density, ...
 - Microphysics?
 - **Nucleon interaction**



Effective hamiltonian

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$$\mathcal{H} = T + V_{\text{NN}} + V_{\text{3N}} + V_{\text{4N}} + \dots$$

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Two-nucleon interaction



Three-nucleon interaction



Four-nucleon interaction

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	NN	3N	4N
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$ (2 LECs)		—	—
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- Fundamental interaction
 - Quarks and gluons




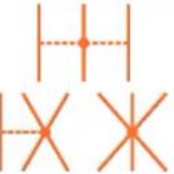
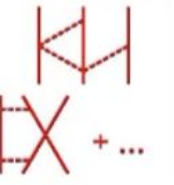
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How to model 3N interaction

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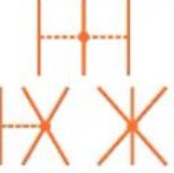

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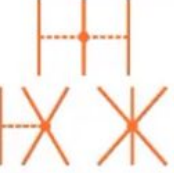

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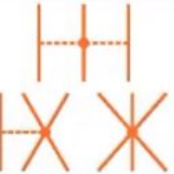

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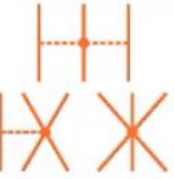

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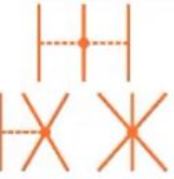

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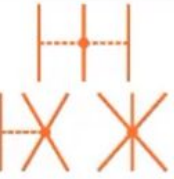
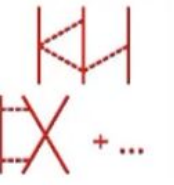
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What if we can study it directly with neutron star?

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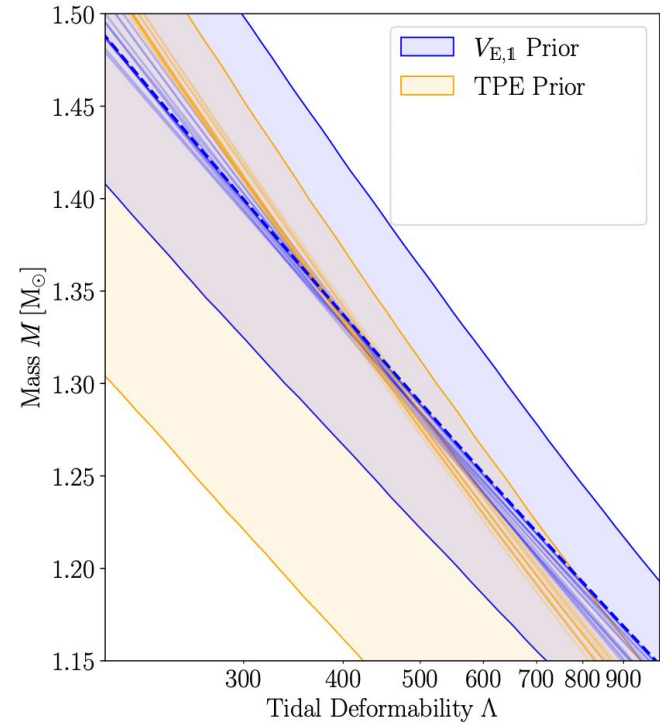
Effect of 3N interaction

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- TPE
 - Two-pion-exchange
- $V_{E,1}$
 - TPE + repulsive 3N

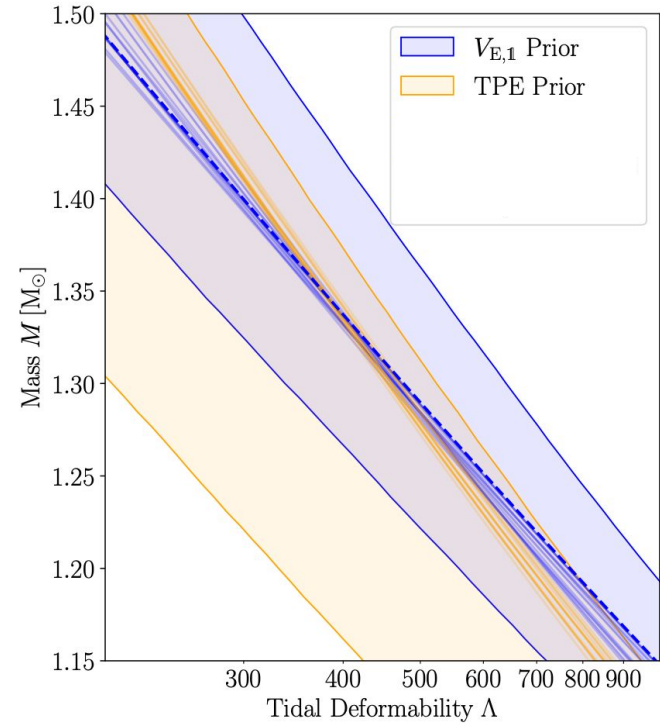
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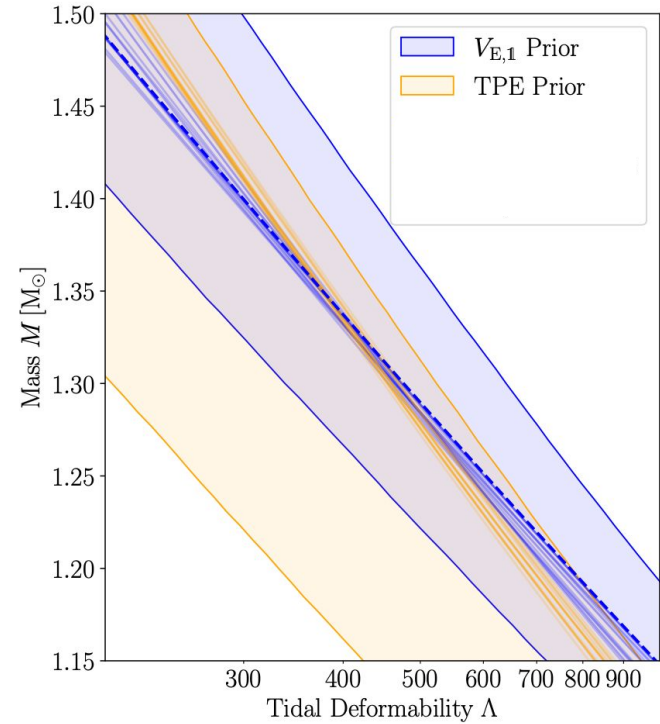
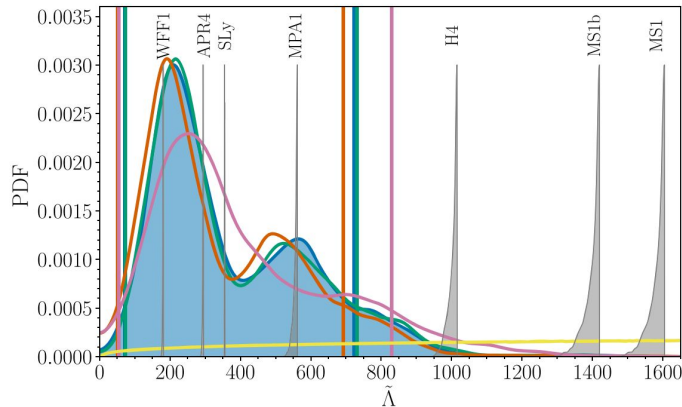
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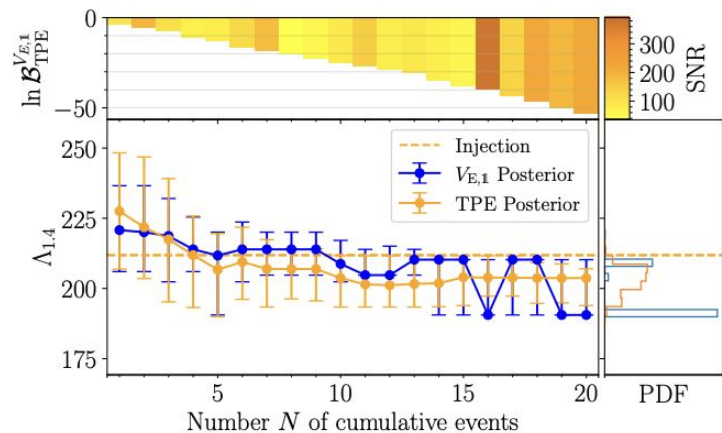
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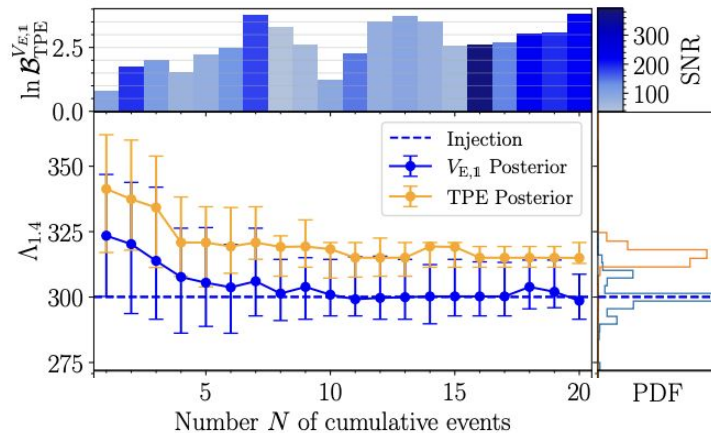
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 - Calculate the Bayes factor between the two models

Results

TPE injection

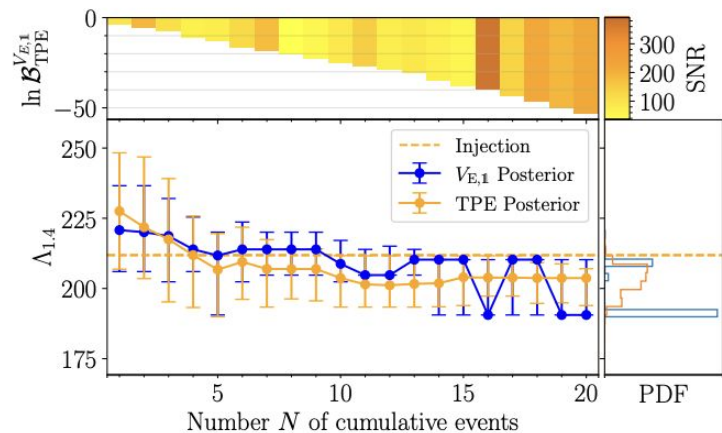


$V_{E,1}$ injection

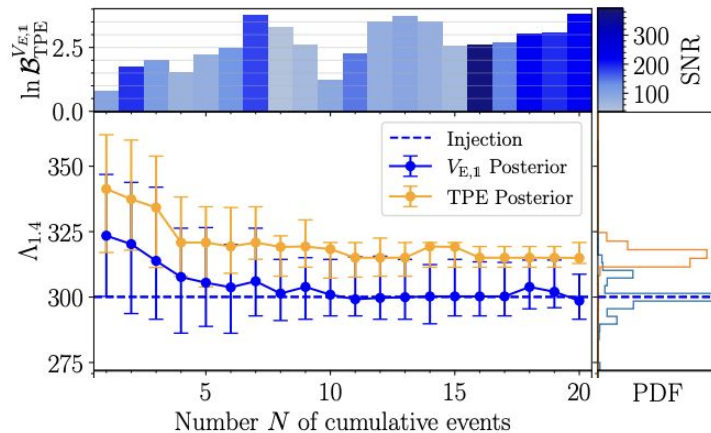


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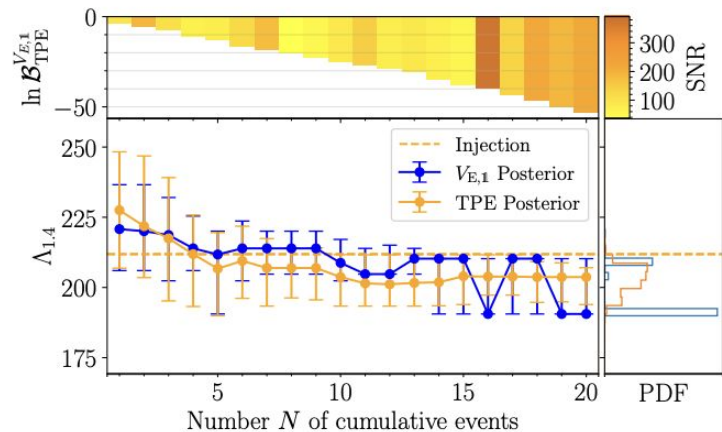
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ET is sensitive to microphysics

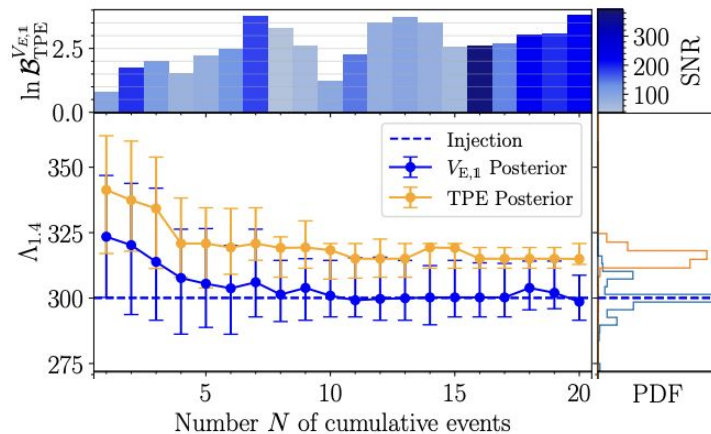
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ET is sensitive to microphysics systematics

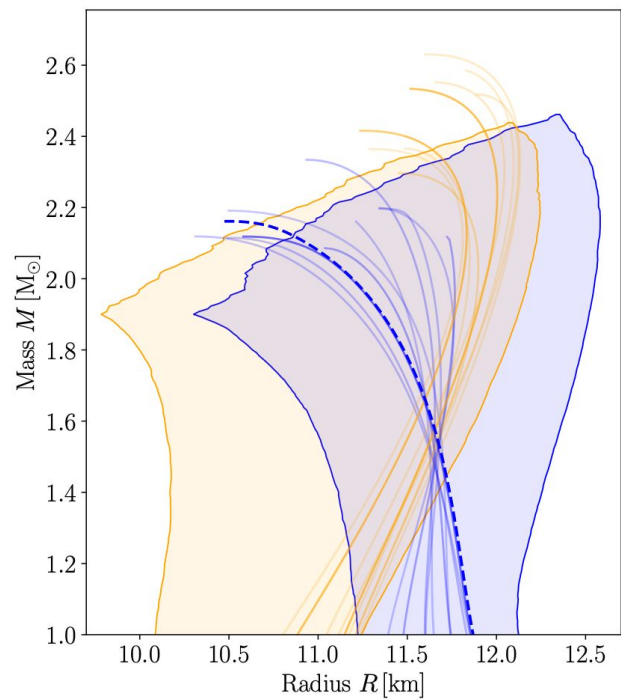
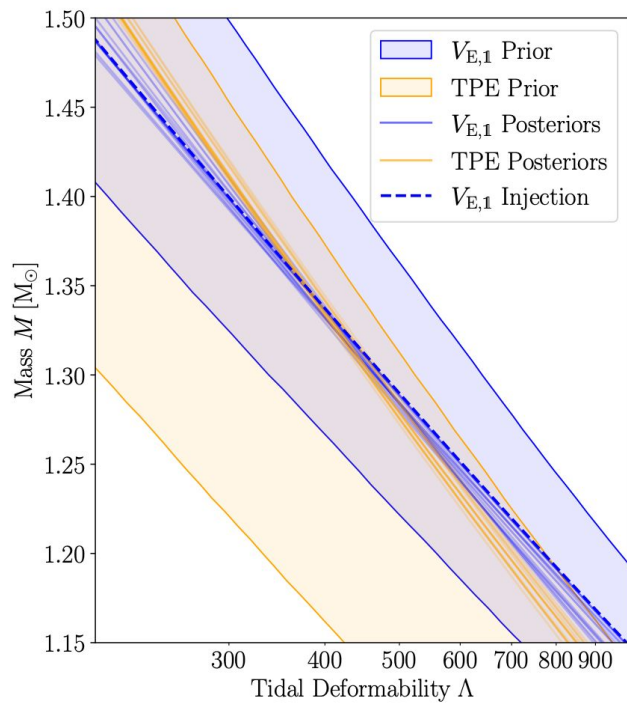
Conclusion

- ET is sensitive to microphysics
- 3N nucleon interaction
 - Distinguishable with ~ 20 events
- ET also suffer from microphysics systematics
 - Significant bias
 - Crucial for extracting nuclear physics

Injection setup

- Masses following galactic binary neutron star
 - Gaussian with mean = 1.33, std = 0.09
- Uniform in co-moving volume
 - Within 200 Mpc
 - A SNR 30 cutoff
- 50-th percentile EOS in TOV mass
- 30Hz - 2048Hz

Results



Prior

	parameter	symbol	prior bounds
observational	luminosity distance [Mpc]	d_L	5 – 500
	inclination	$\cos \theta_{JN}$	-1 – 1
	phase [rad]	ϕ	0 – 2π
	polarisation [rad]	ψ	0 – π
	right ascension [rad]	α	0 – 2π
	declination [rad]	δ	$-\pi$ – π
orbital	chirp mass [M_\odot]	\mathcal{M}	1.20 – 1.30*
	source chirp mass [M_\odot]	\mathcal{M}_s	1.15 – 1.30*
	mass ratio	q	0.125 – 1
	source comp. mass [M_\odot]	$M_{i,s}$	> 0.5
	aligned component spin	χ_i	-0.15 – 0.15
hyper	Equation of State	EOS	1 – 3000