Mirror coating birefringence noise

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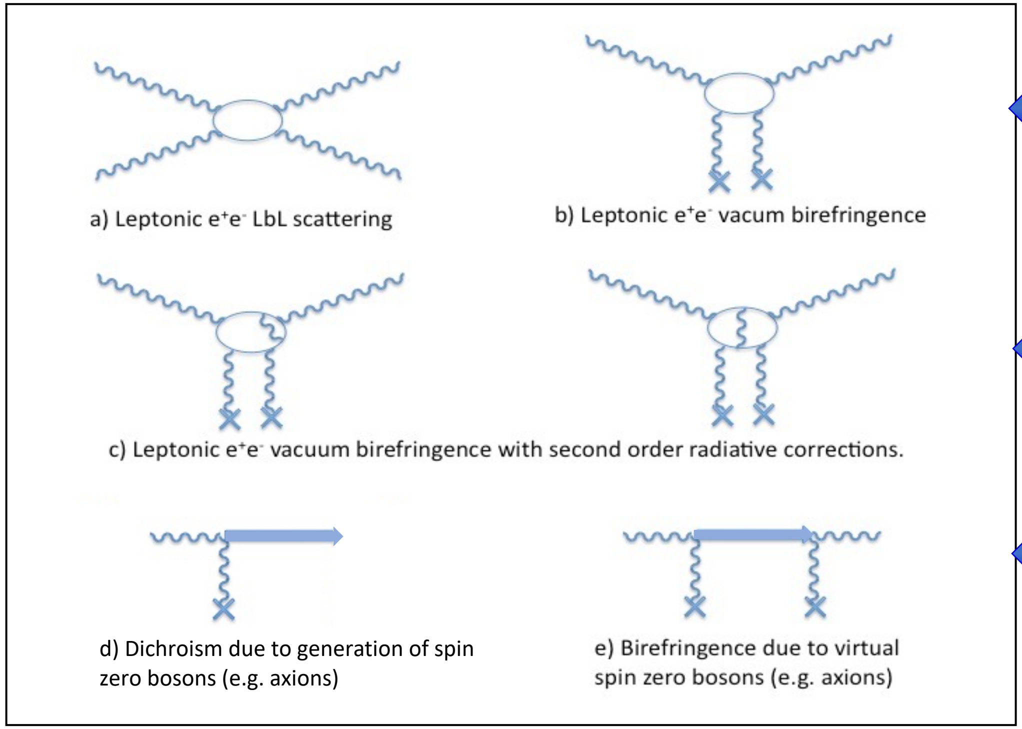
Background work in sensitive polarimetry

Experimental study of the induced birefringence by an external magnetic field in vacuum



Light-by-light interaction and vacuum magnetic birefringence.

Must be there: $\Delta n = 4 \mathrm{X} 10^{-24}$ B^2 with B in Tesla.

Includes MCPs

Radiative correction 1.45%

Contributions from hypothetical neutral light particles coupling to two photons: ALPs

Euler-Kockel-Heisenberg Lagrangian predicts VMB

$$\mathcal{L}_{\mathrm{EK}} = \frac{1}{2\mu_0}\left(\frac{E^2}{c^2} - B^2\right) + \frac{A_e}{\mu_0}\left[1\left(\frac{E^2}{c^2} - B^2\right)^2 + 7\left(\frac{\vec{E}}{c}\cdot\vec{B}\right)^2\right] + \ldots$$

$$A_e = \frac{2}{45\mu_0}\frac{\alpha^2 \lambda\!\!\!^{-}_e{}^3}{m_e c^2} = 1.32 \times 10^{-24}\ \mathrm{T}^{-2}$$

$$\Delta n = 3 A_e B_{\mathrm{ext}}^2$$

@ B_{ext} = 2.5 T

$\Delta n = 2.5 \cdot 10^{-23}$

Birefringence and ellipticity

The index of refraction is a complex number: $\tilde{n} = n + i\kappa$

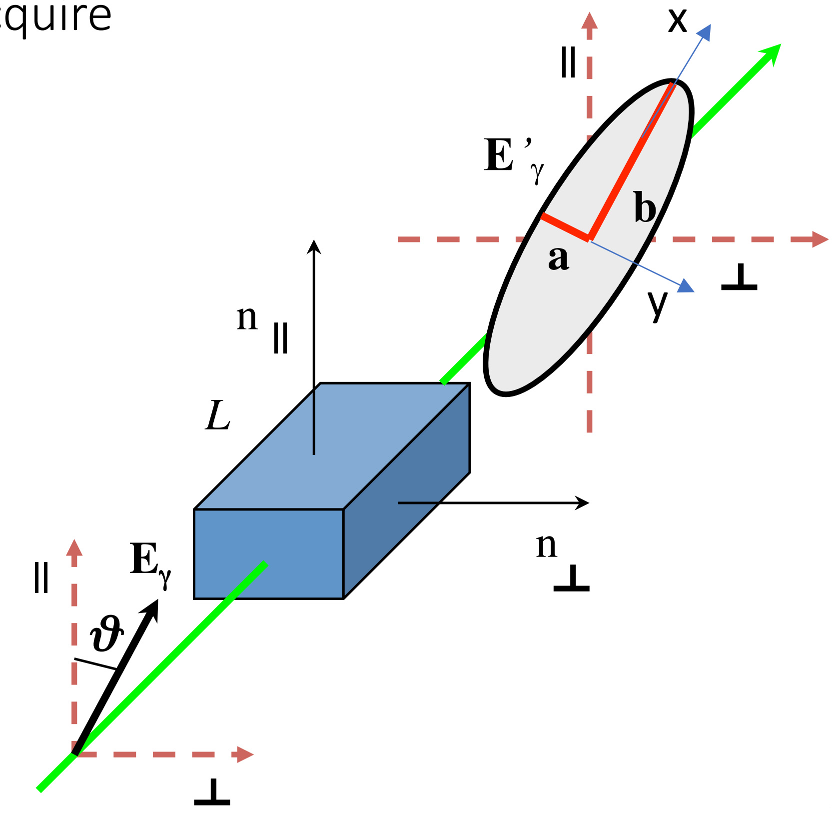
- In a birefringent medium $n_{\parallel} \neq n_{\perp}$
- A linearly polarized beam passing through a birefringent medium will acquire an ellipticity $\psi = \pm\, a/b$ (the sign determines the rotation direction of E_γ)

$$\mathbf{E}_\gamma = E_\gamma \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \Delta\phi = \frac{2\pi(n_\parallel - n_\perp)L}{\lambda}$$

$$\mathbf{E}'_\gamma = E_\gamma \begin{pmatrix} 1 + i\frac{\Delta\phi}{2}\cos 2\vartheta \\ i\frac{\Delta\phi}{2}\sin 2\vartheta \end{pmatrix}, \qquad \Delta\phi \ll 1$$

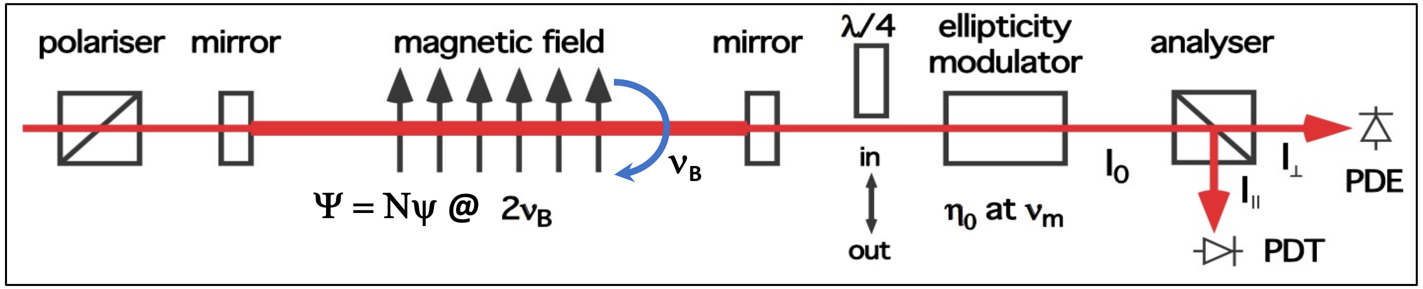
Immaginary

$$\psi = \pm\frac{a}{b} \approx \frac{\Delta\phi}{2}\sin 2\vartheta = \frac{\pi(n_\parallel - n_\perp)L}{\lambda}\sin 2\vartheta$$



$$\Rightarrow P_{y,\mathrm{out}} = E_\gamma^2\,\psi^2$$

General scheme



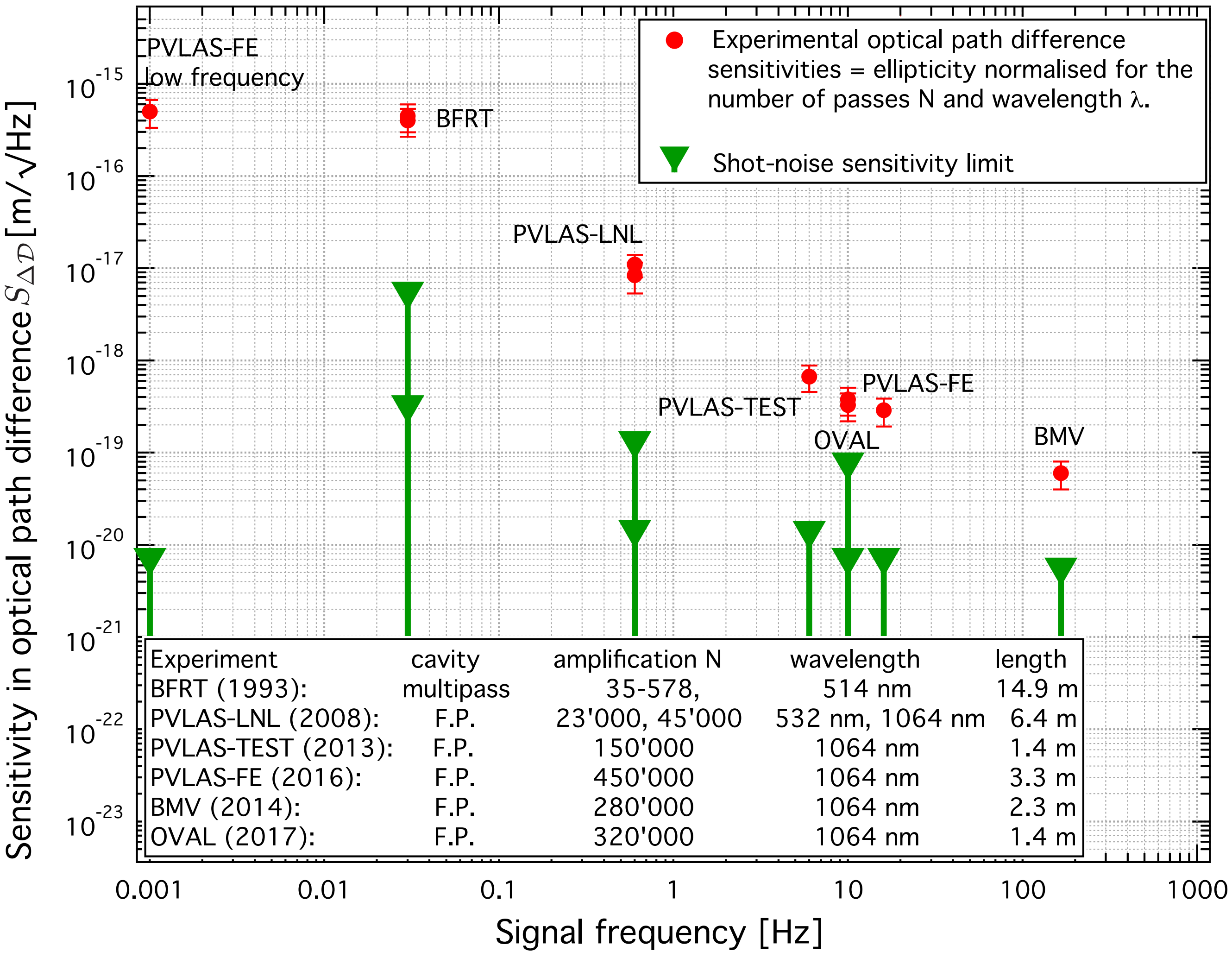
F. Della Valle et al. Eur. Phys. J. C (2016) 76:24
A. Ejlli et al. Physics Reports 871 (2020) 1–74

- Single pass ellipticity: $\psi = \dfrac{\pi \int \Delta n_{\rm B}\, dL}{\lambda} \sin 2\vartheta(t) = \psi_0 \sin 2\vartheta(t)$. Here $\vartheta(t)$ is the angle between the polarisation and the birefringence axis.
- The Fabry-Perot cavity amplifies ψ by a factor $N = 2\mathcal{F}/\pi$ We had $\mathcal{F} \approx 7 \times 10^5$.
- The ellipticity modulator allows heterodyne detection which linearizes the ellipticity ψ to be measured and allows the distinction between a rotation and an ellipticity. The insertion of the $\lambda/4$ wave plate allows measuring rotations.
- The rotating magnetic field modulates the desired signal $\Psi(t) = N\psi(t)$ due to VMB.
- Without the cavity or with a low finesse cavity, shot-noise is reached with ≈ 10 mW power: $S_\Psi \approx 5 \times 10^{-9}\, 1/\sqrt{\rm Hz}$

$$\Rightarrow I_{\rm out} = I_0 \left\{N\psi(t) + \eta(t)\right\}^2 \simeq I_0 \left\{\eta^2(t) + 2\eta(t)N\psi(t) + 2\eta(t)\Gamma(t) + \ldots\right\}$$

Intrinsic mirror birefringence noise

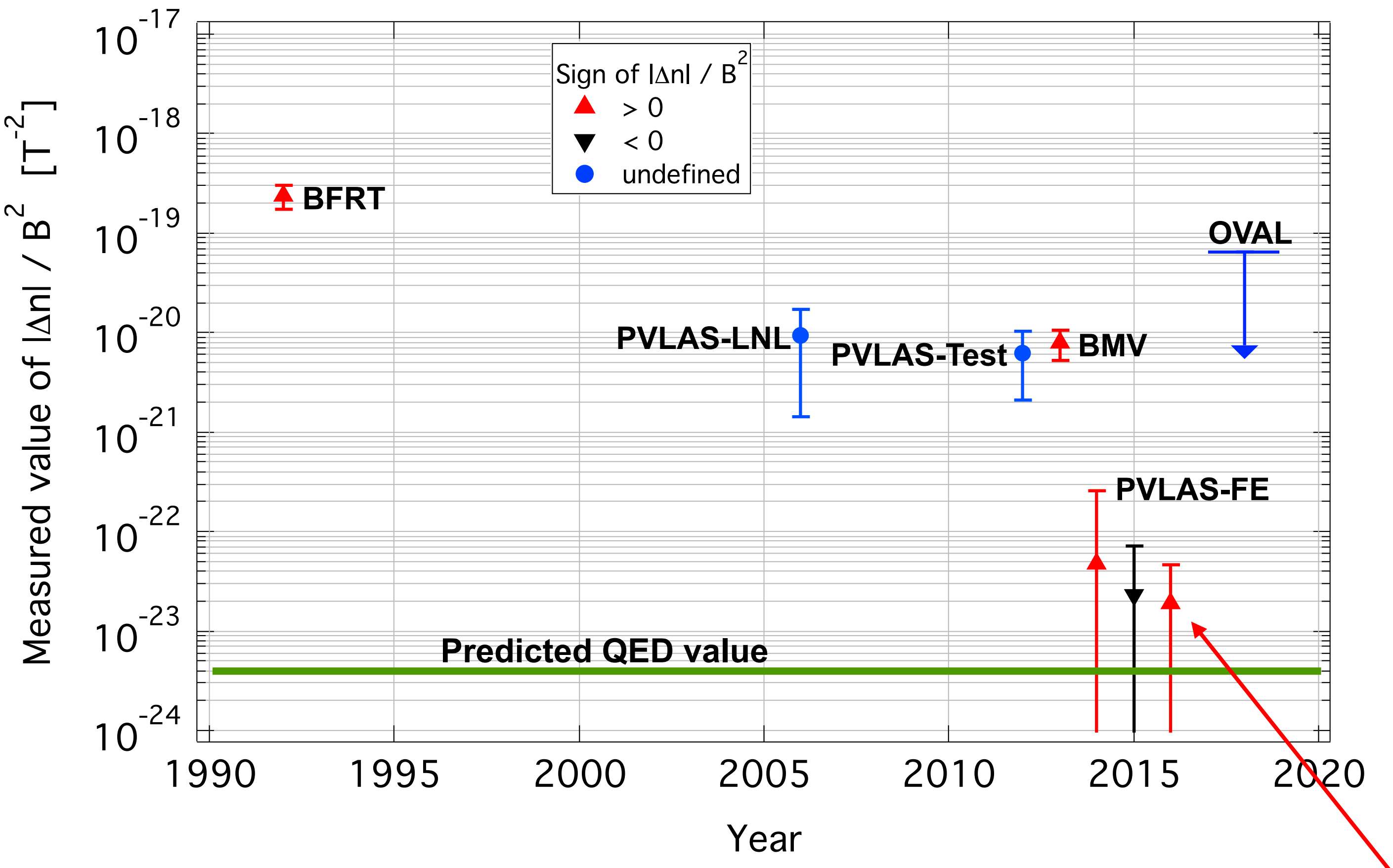
Optical path difference limits in the sensitivity of a polarimeter



- Optical path difference $\Delta\mathcal{D} = \int \Delta n \, dL$
- No experimental effort has reached shot-noise sensitivity (green) with a high finesse F.P.
- There seems to be a common problem afflicting all experiments
- Intrinsic noise from the mirrors limited the sensitivity and the SNR
- With a low finesse cavity one does reach shot-noise. The limit is not the method.

Intrinsic mirror birefringence noise

Results with the PVLAS polarimeter



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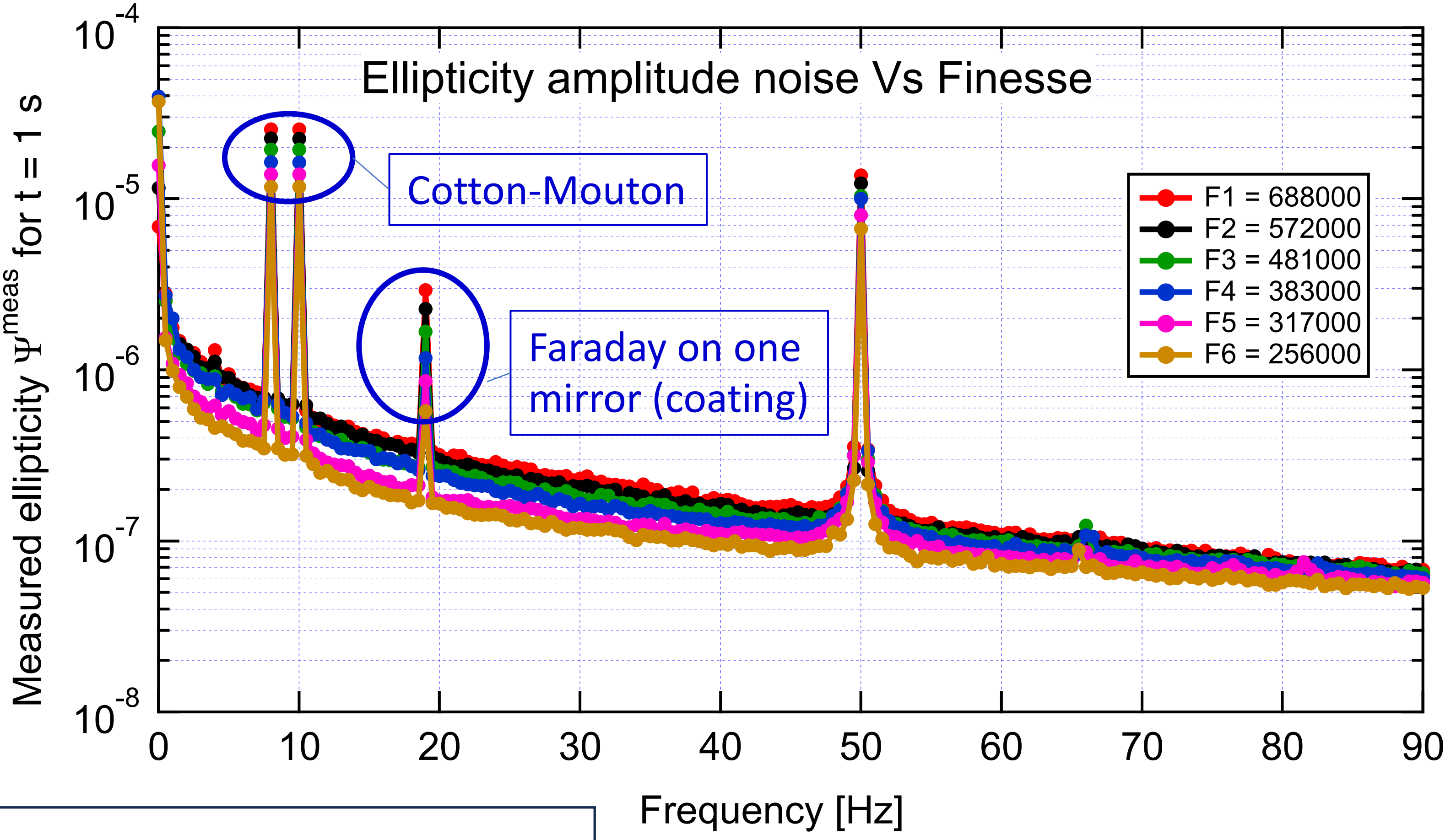
- There seems to be a common problem afflicting all experiments
- <u>Intrinsic noise from the mirrors limited the sensitivity and the SNR</u>
- With a low finesse cavity one does reach shot-noise. The limit is not the method.
- Nontheless we reached

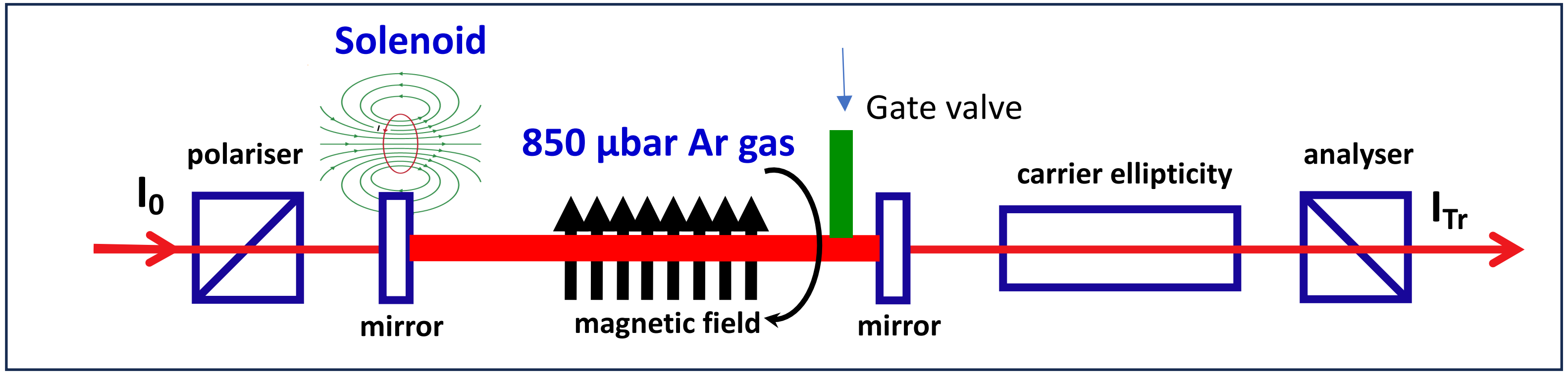
$\Delta n/B^2 = (1.9 \pm 2.7)\cdot 10^{-23}$ T^{-2} with 2.5 T

Physics Reports 871 (2020) 1–74

Ellipticity amplitude noise Vs Finesse

- The ellipticity noise depends on the finesse
- The ellipticity Cotton-Mouton peaks depend on the finesse, as expected
- With a birefringent cavity, rotations partially beat with the ellipticity modulator
- The Faraday peaks depend on the finesse and on the cavity birefringence

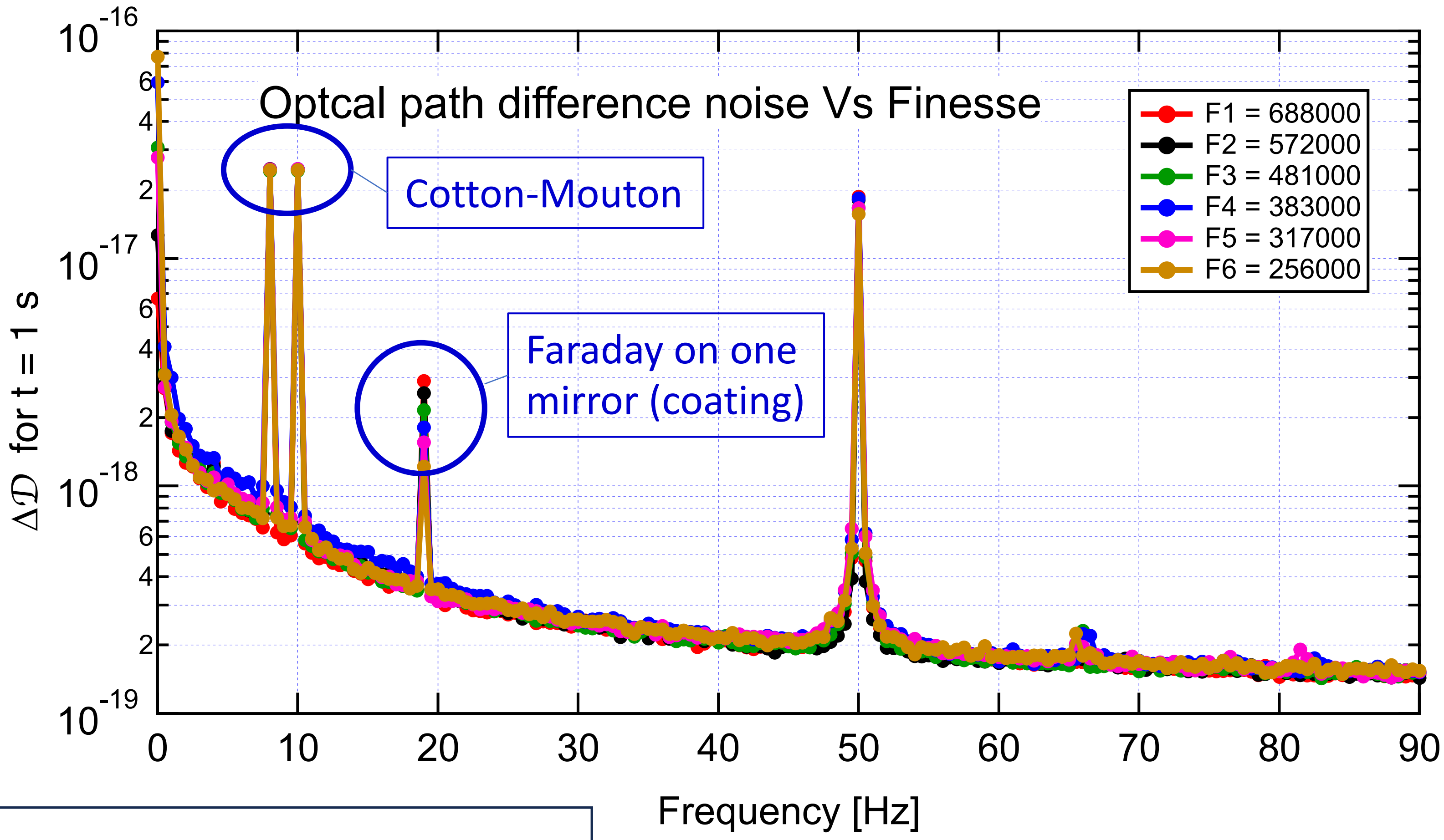


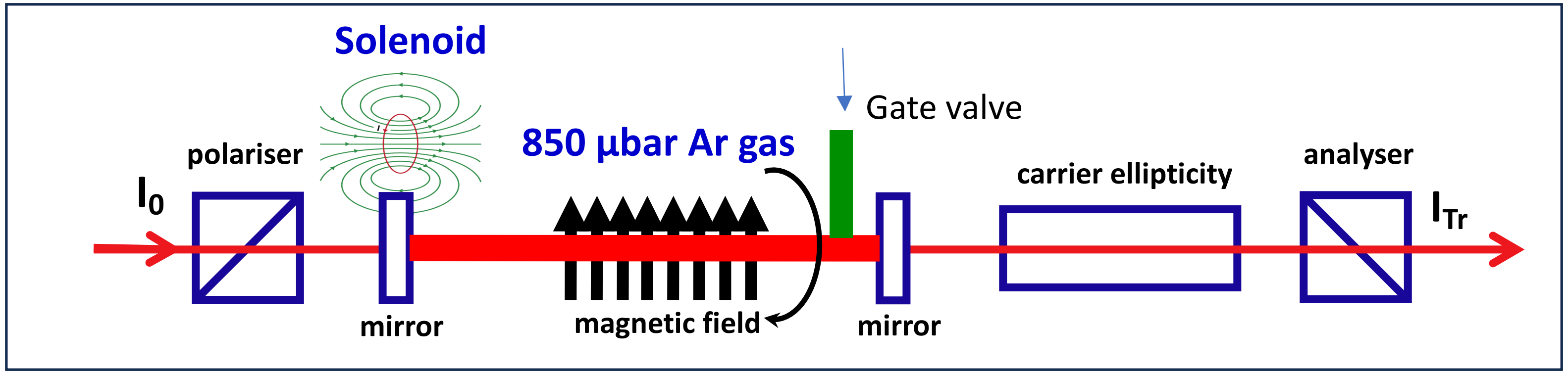


Optical path difference noise Vs Finesse

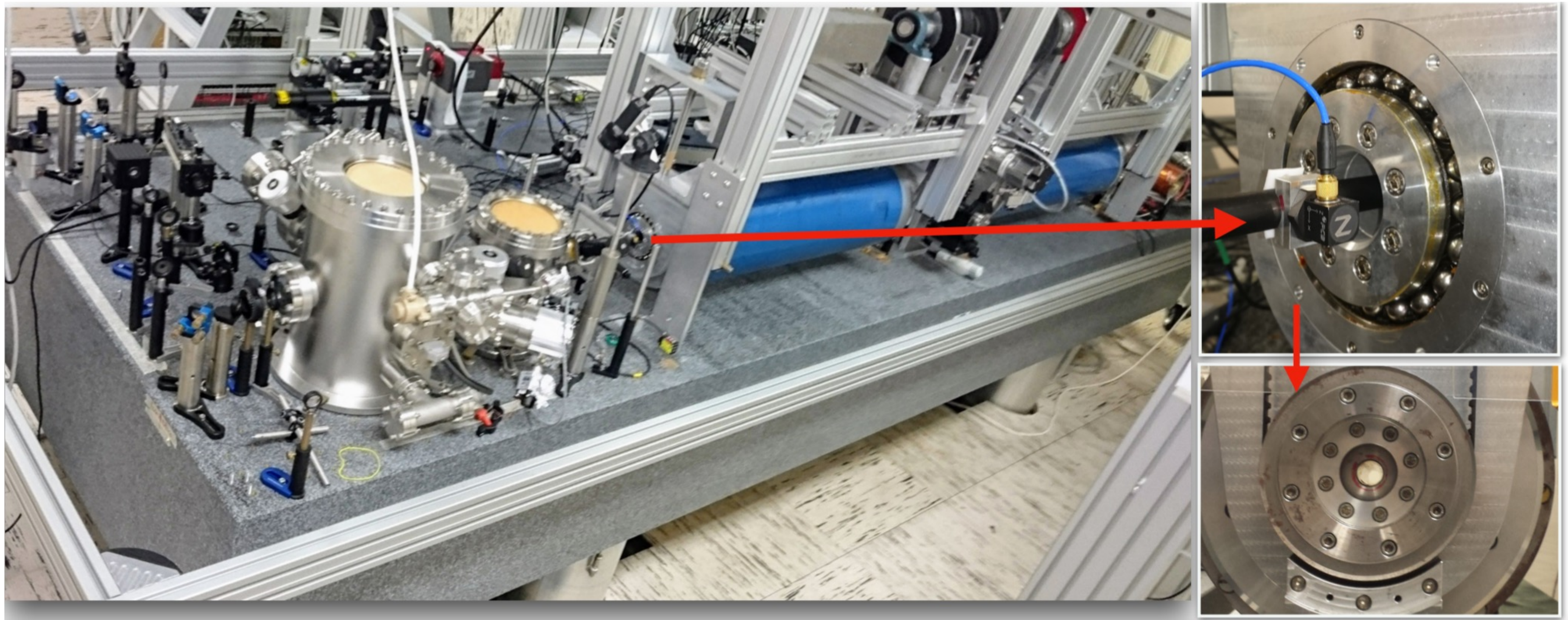
$$S_{\Delta\mathcal{D}} = S_{\Psi}\frac{\lambda}{2\mathcal{F}f(\nu)}$$

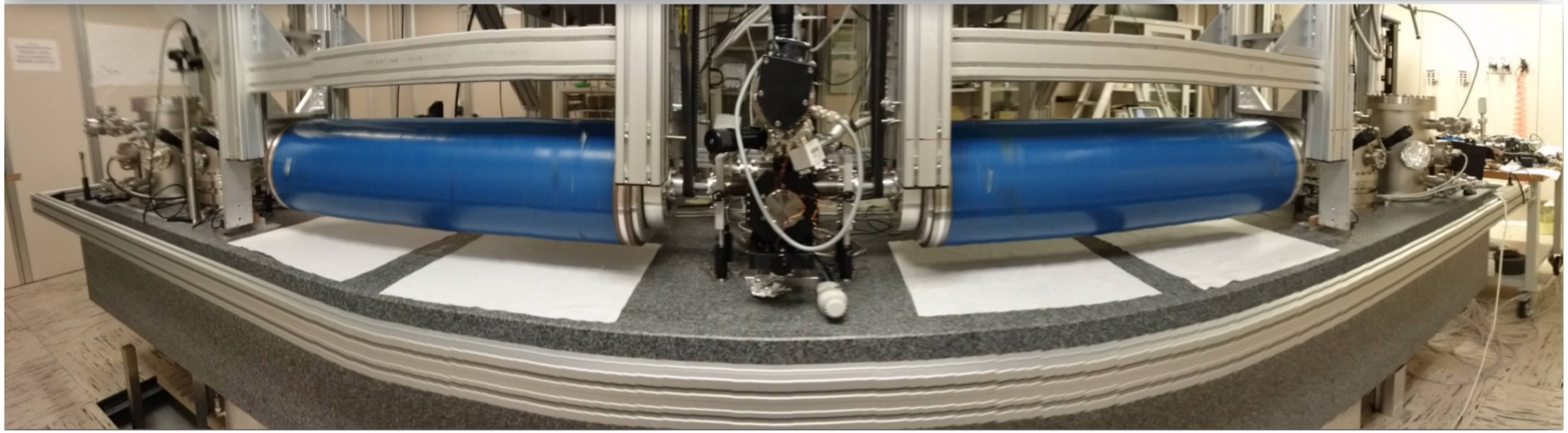
- The optical path difference noise does not depend on the finesse ➔ noise generated inside the cavity
- $f(\nu)$ = cavity transfer function
- The optical path difference due to the Cotton-Mouton effect doesn't depend on the finesse, as expected
- The Faraday peaks depend on the finesse and on the cavity birefringence



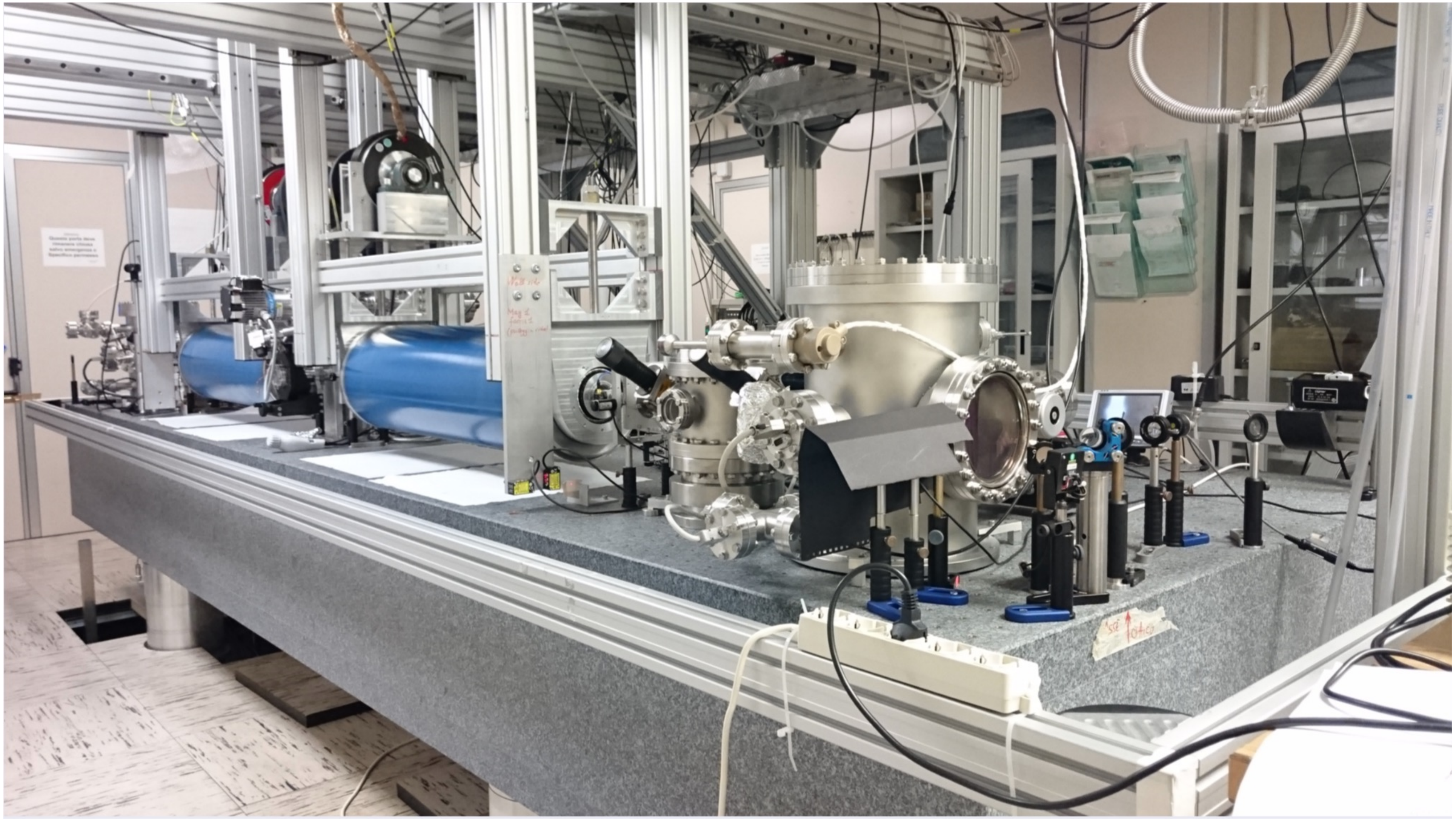


PVLAS input end

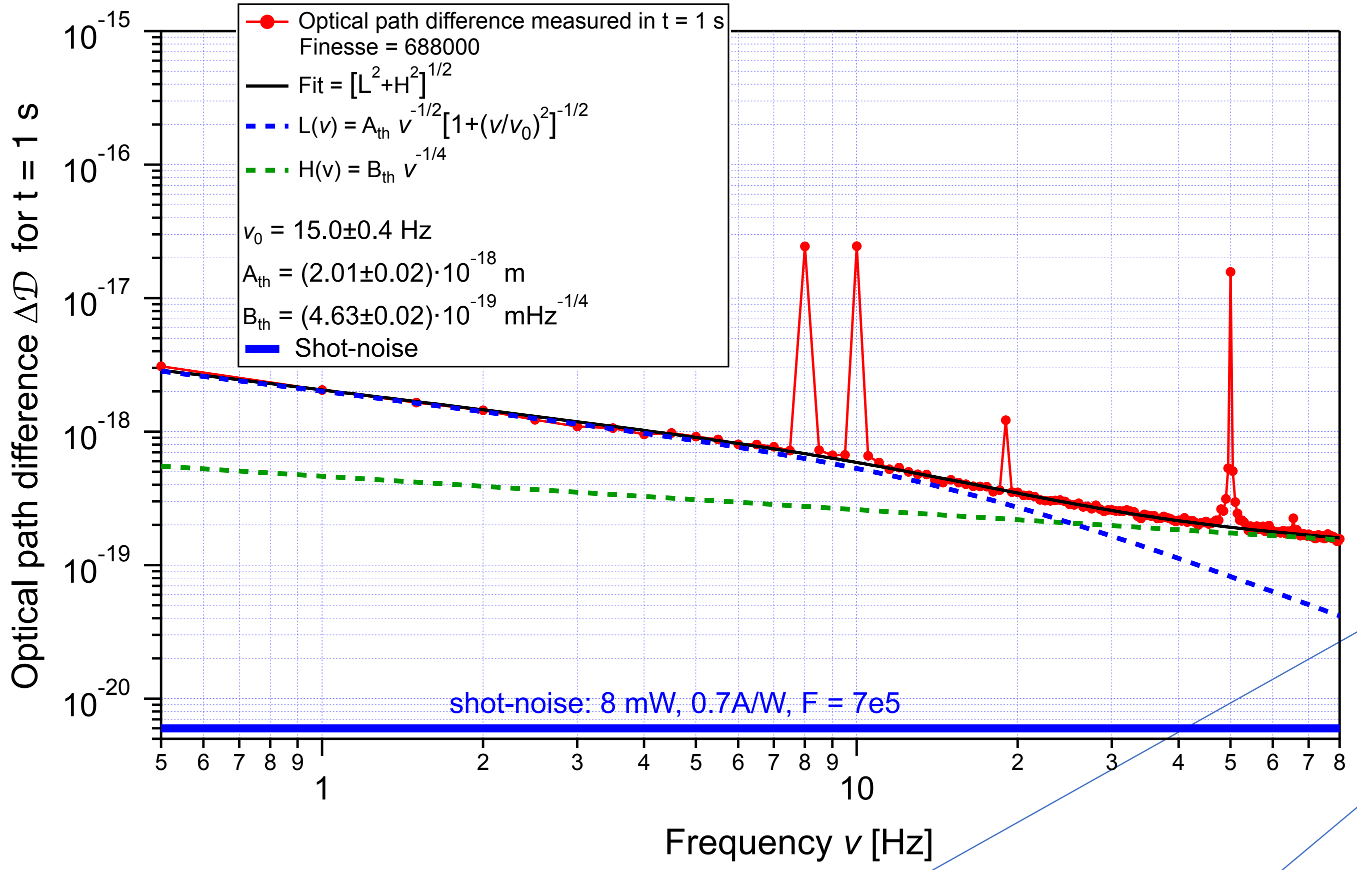




PVLAS output end



Fitting the intrinsic $\Delta\mathcal{D}$ noise



- Interesting to measure new coatings. Finesse must be F ≥ 5e4 (R ≥ 99.995%): the amplified mirror noise must be greater than shot-noise.
- Will be testing crystaline GaAs/AlGaAs mirrors.
- Brownian? Why the cut-off? Beam radius?
- Thermo-elastic model points to tantala.

$$S_{\mathrm{OPD}}(\nu) = \sqrt{\left(\frac{A_{\mathrm{th}}\nu^{-1/2}}{\sqrt{1+(\nu/\nu_0)^2}}\right)^2 + \left(B_{\mathrm{th}}\nu^{-1/4}\right)^2}$$

$$A_{\mathrm{th}} = (2.01 \pm 0.02) \times 10^{-18}\ \mathrm{m}, \quad \nu_0 = (15.0 \pm 0.4)\ \mathrm{Hz}, \quad B_{\mathrm{th}} = (4.63 \pm 0.02) \times 10^{-19}\ \mathrm{m/Hz}^{1/4}$$

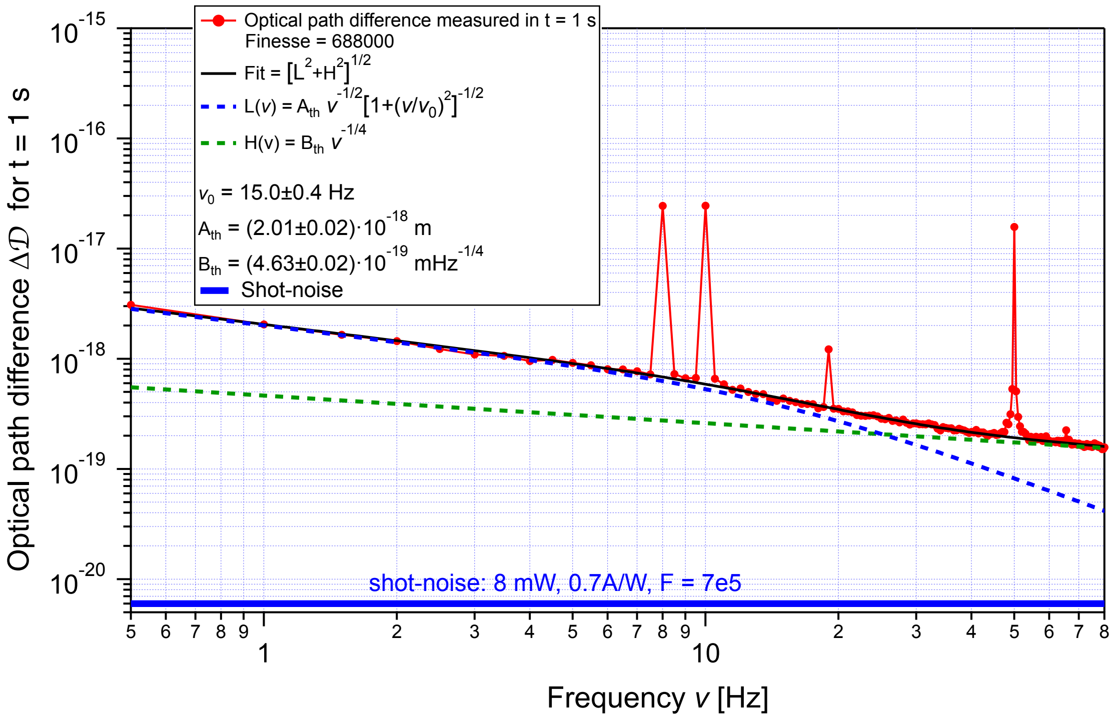
Model

- Thermoelastic effect
- Followed Braginsky's line of thought
- Ingredients:
  - diffuse heat transfer length r_T
  - intrinsic temperature fluctuations in a volume V: $\langle \delta T^2 \rangle = \dfrac{\kappa_B T^2}{\rho C V}$
  - linear thermal expansion coefficient α_T
  - stress optic coefficient C_{SO}
  - ellipticity accumulated in first few layers d_e
  - beam radius r_0

➔ Find $S_{\Delta\mathcal{D}} \propto \nu^{-1/4}$

A. Ejlli, F. Della Valle, U. Gastaldi et al. Physics Reports 871 (2020) 1–74

Intrinsic mirror birefringence noise



- Estimated the thermoelastic birefringence noise in reflection*
- C_{SO} = stress optic coefficient
- Y = Young's modulus
- α_T = thermal expansion coefficient
- r_0 = beam radius on mirror
- C_T = specific heat capacity
- ρ = density
- λ_T = themal conductivity

Temperature spectral density

$$S_T(\nu) = \sqrt{\frac{8k_B T^2}{\pi r_0^2\sqrt{\pi\rho C_T\lambda_T\nu}}} \propto \nu^{-1/4}$$

Optical path difference spectrum

$$S_{\Delta\mathcal{D}} = 2d_e\sqrt{2}C_{SO}Y\alpha_T S_T(\nu)$$

Fused silica

$$S_{\Delta\mathcal{D}}^{(\mathrm{FS})} \sim 4\times10^{-21}\ \mathrm{m}/\sqrt{\mathrm{Hz}} \quad @ \quad 1\ \mathrm{Hz}$$

Tantala

$$S_{\Delta\mathcal{D}}^{(\mathrm{Ta})} \sim (1\div5)\times10^{-19}\ \mathrm{m}/\sqrt{\mathrm{Hz}} \quad @ \quad 1\ \mathrm{Hz}$$

Compatible with $B_{\mathrm{th}} = (4.63\pm0.02)\times10^{-19}\ \mathrm{m/Hz}^{1/4}$ from the fit

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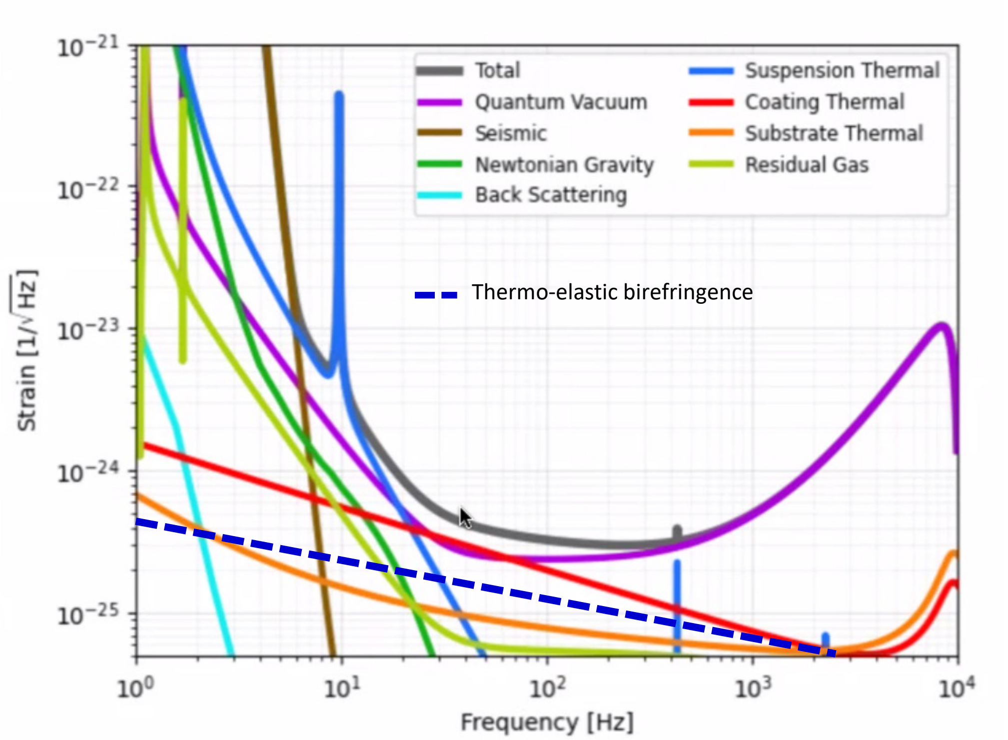
Scaled to ET-HF, 10 km



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Optical path difference spectrum

$$S_{\Delta\mathcal{D}} = 2d_e \sqrt{2} C_{SO} Y \alpha_T S_T(\nu)$$

Fused silica

$$S_{\Delta\mathcal{D}}^{(FS)} \sim 4 \times 10^{-21}\ \mathrm{m}/\sqrt{\mathrm{Hz}} \quad @ \quad 1\ \mathrm{Hz}$$

Tantala

$$S_{\Delta\mathcal{D}}^{(Ta)} \sim (1 \div 5) \times 10^{-19}\ \mathrm{m}/\sqrt{\mathrm{Hz}} \quad @ \quad 1\ \mathrm{Hz}$$

Compatible with $B_{th} = (4.63 \pm 0.02) \times 10^{-19}\ \mathrm{m}/\mathrm{Hz}^{1/4}$ from the fit

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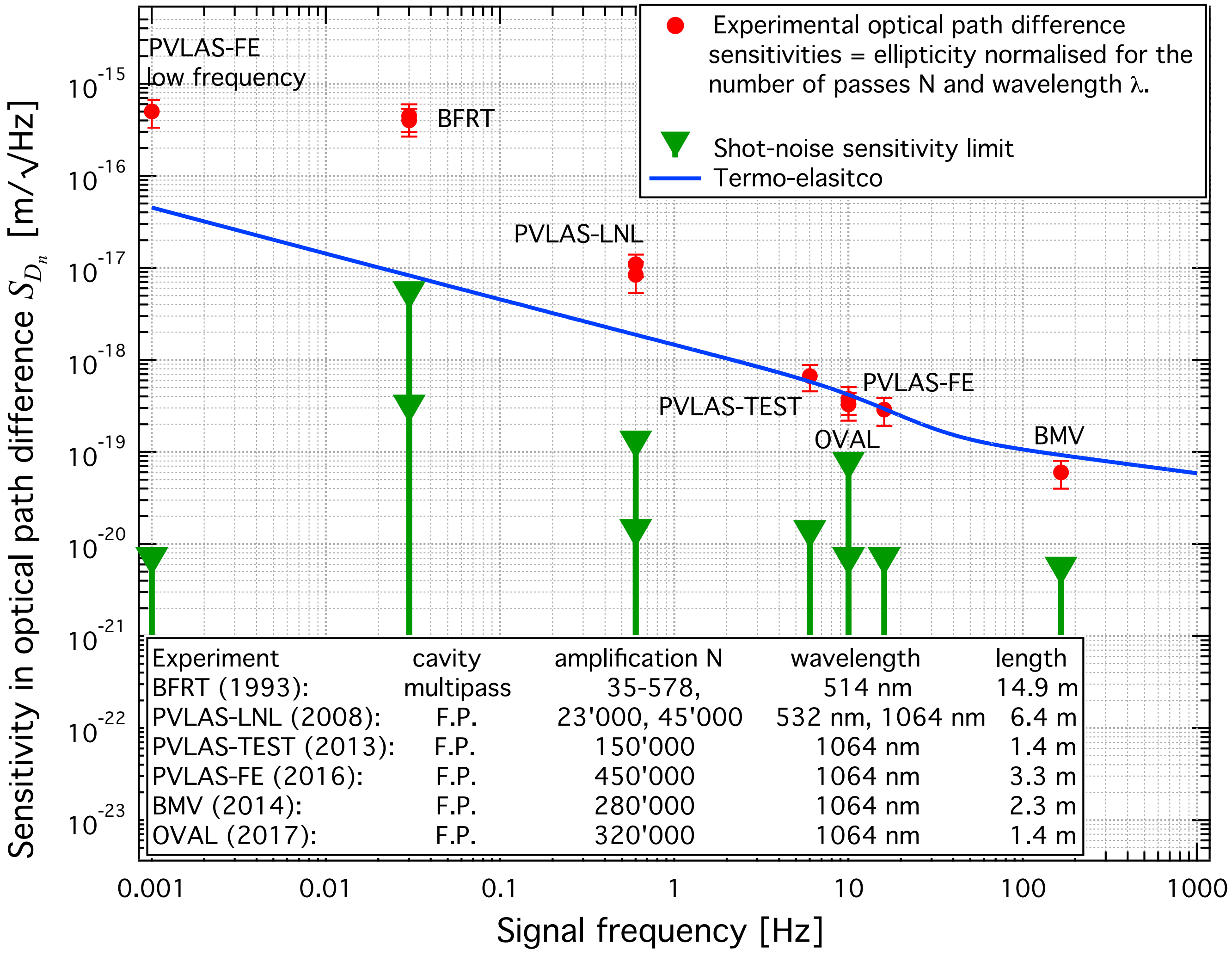
Conclusions

- Measured an intrinsic birefringence noise deriving from mirror coatings (AtFilms)
- We found two components: the first proportional to $\boldsymbol{\nu}^{-1/2}$ with a frequency cut-off; the second proportional to $\boldsymbol{\nu}^{-1/4}$.
- A thermo-elastic model points to the $\boldsymbol{\nu}^{-1/4}$ component dominated by the tantala layer
- New measurements with crystalline mirrors will be performed

Thank you

Intrinsic mirror birefringence noise

Limits in the sensitivity of a polarimeter

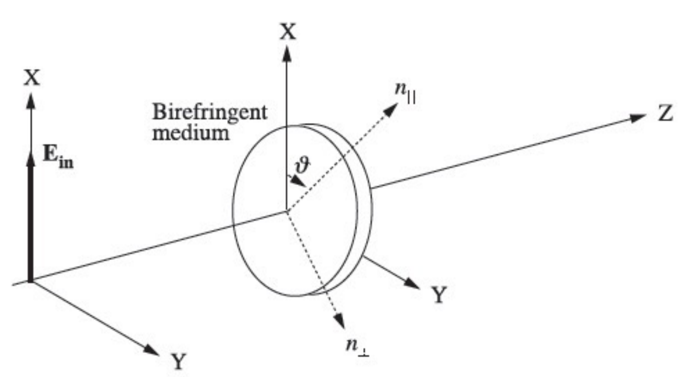


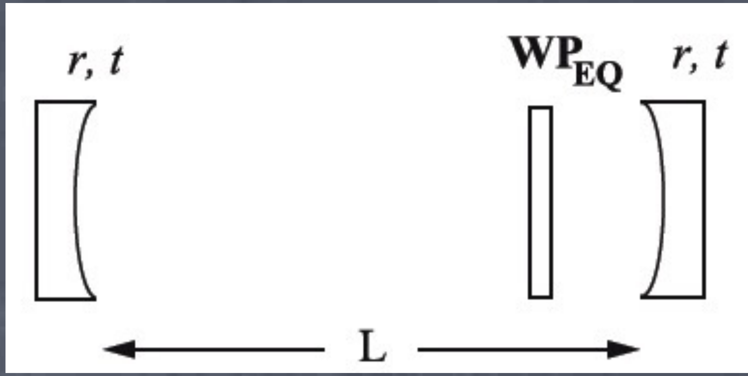
Experiment	cavity	amplification N	wavelength	length
BFRT (1993):	multipass	35-578,	514 nm	14.9 m
PVLAS-LNL (2008):	F.P.	23'000, 45'000	532 nm, 1064 nm	6.4 m
PVLAS-TEST (2013):	F.P.	150'000	1064 nm	1.4 m
PVLAS-FE (2016):	F.P.	450'000	1064 nm	3.3 m
BMV (2014):	F.P.	280'000	1064 nm	2.3 m
OVAL (2017):	F.P.	320'000	1064 nm	1.4 m

- No experimental effort has reached shot-noise sensitivity (green) with a high finesse F.P.
- There seems to be a common problem afflicting all experiments
- This noise seems to be an intrinsic property of the cavity mirrors
- With a low finesse cavity one does reach shot-noise. The limit is not the method.

Mirror birefringence

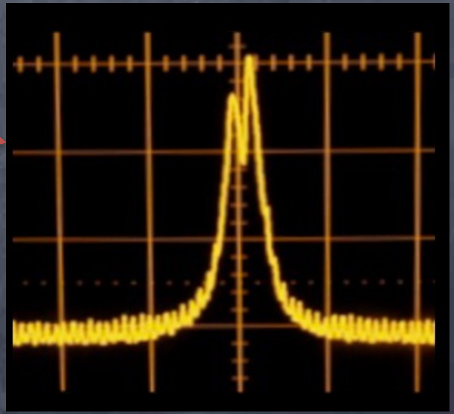
Fabry Perot cavity mirrors have **intrinsic static birefringence**





The resulting cavity behaves like a **waveplate.** This results in:

- cavity mode splitting



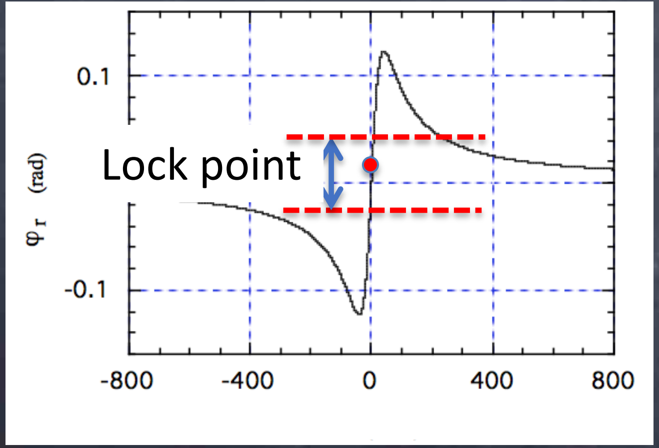
- Cavity mirrors must be rotated to reduce total birefringence
- Polarization must be aligned with one of the equivalent waveplate axes.

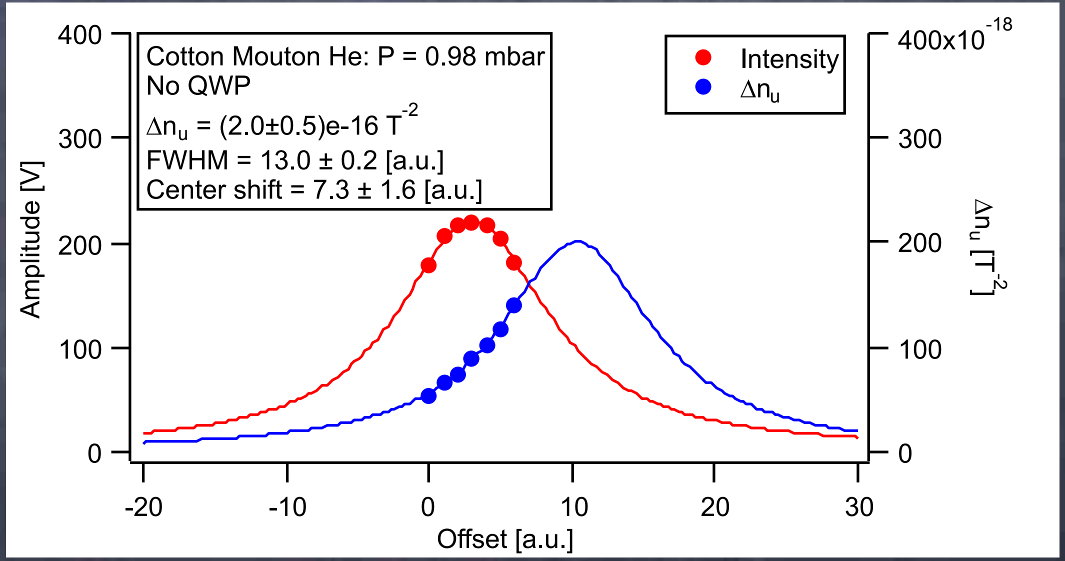
Cavity birefringence

- With He gas at ≈ 1mbar pressure we measured the ellipticity as a function of feedback lock point (δ)
- The imaginary part of $E(t)$ will beat with the ellipticity of the modulator

$$E(t) = E_0 \left(\frac{2\mathcal{F}}{\pi}\right) i\psi \sin 2\theta \left(1 - i\left(\frac{\alpha_{\rm EQ}}{2} - \delta\right)\frac{2\mathcal{F}}{\pi}\right)\left(\frac{1}{1+\left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2\left(\frac{\alpha_{\rm EQ}}{2} - \delta\right)}\right)$$

Error signal





Example with P = 0.98 mbar He

Cavity birefringence

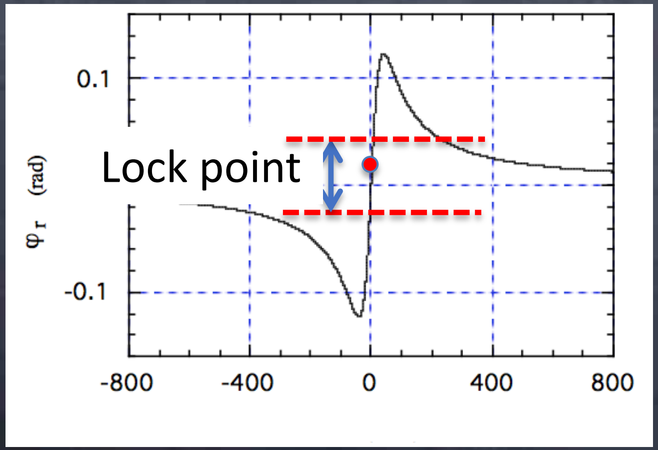
The laser is locked with its polarization along one of the cavity's axis.

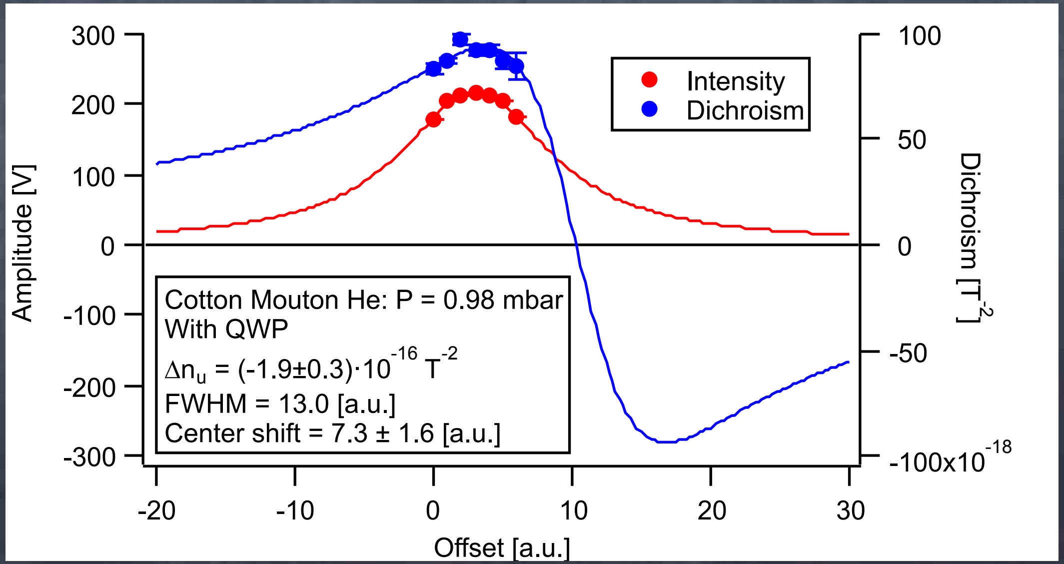
- the perpendicular polarization acquires an extra phase due to the cavity birefringence
- there is also an induced rotation (real component) [Appl. Phys. B 83, 571-577 (2006)]

$$E(t) = E_0 \left(\frac{2\mathcal{F}}{\pi}\right) i\psi \sin 2\theta \left(1 - i\left(\frac{\alpha_{\rm EQ}}{2} - \delta\right)\frac{2\mathcal{F}}{\pi}\right)\left(\frac{1}{1 + \left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2\left(\frac{\alpha_{\rm EQ}}{2} - \delta\right)}\right)$$

With a QWP and the ellipticity modulator one can measure the induced rotation.

Error signal



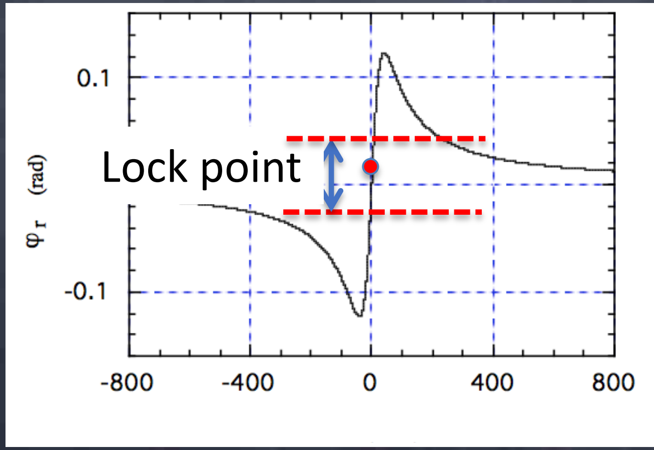


Cavity birefringence

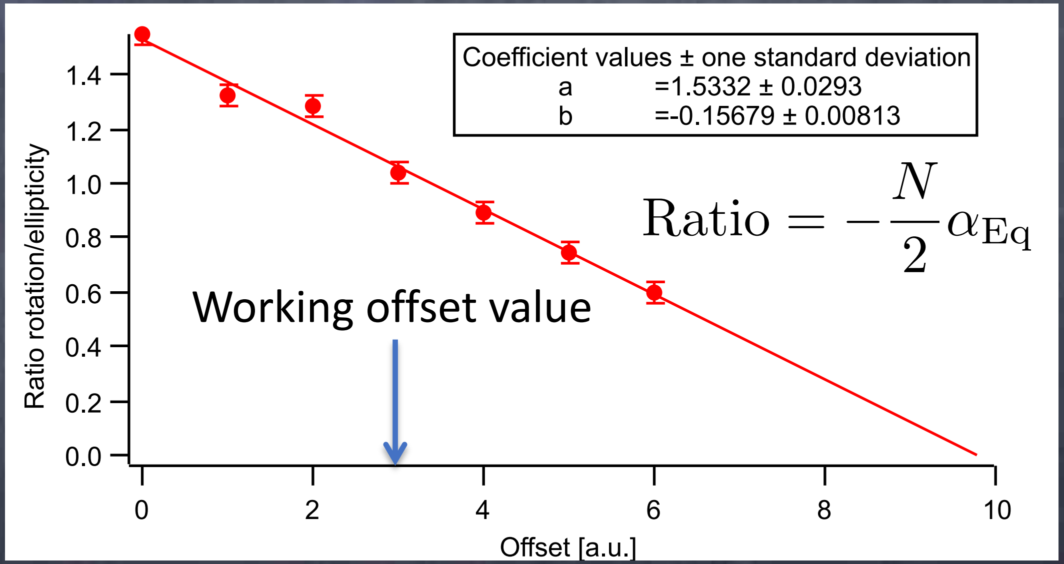
- The ratio of the rotation to the ellipticity allows the determination of the cavity birefringence

$$\frac{\text{rotation}}{\text{ellipticity}} = \frac{2\mathcal{F}}{\pi}\left(\delta - \frac{\alpha_{\mathrm{EQ}}}{2}\right)$$

Error signal



Ratio rotation/ellipticity



Working offset value = 3.1 → Rotation/ellipticity = 1