Fast and accurate parameter estimation of high-redshift sources with the Einstein Telescope

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SCHOOL OF ADVANCED STUDIES
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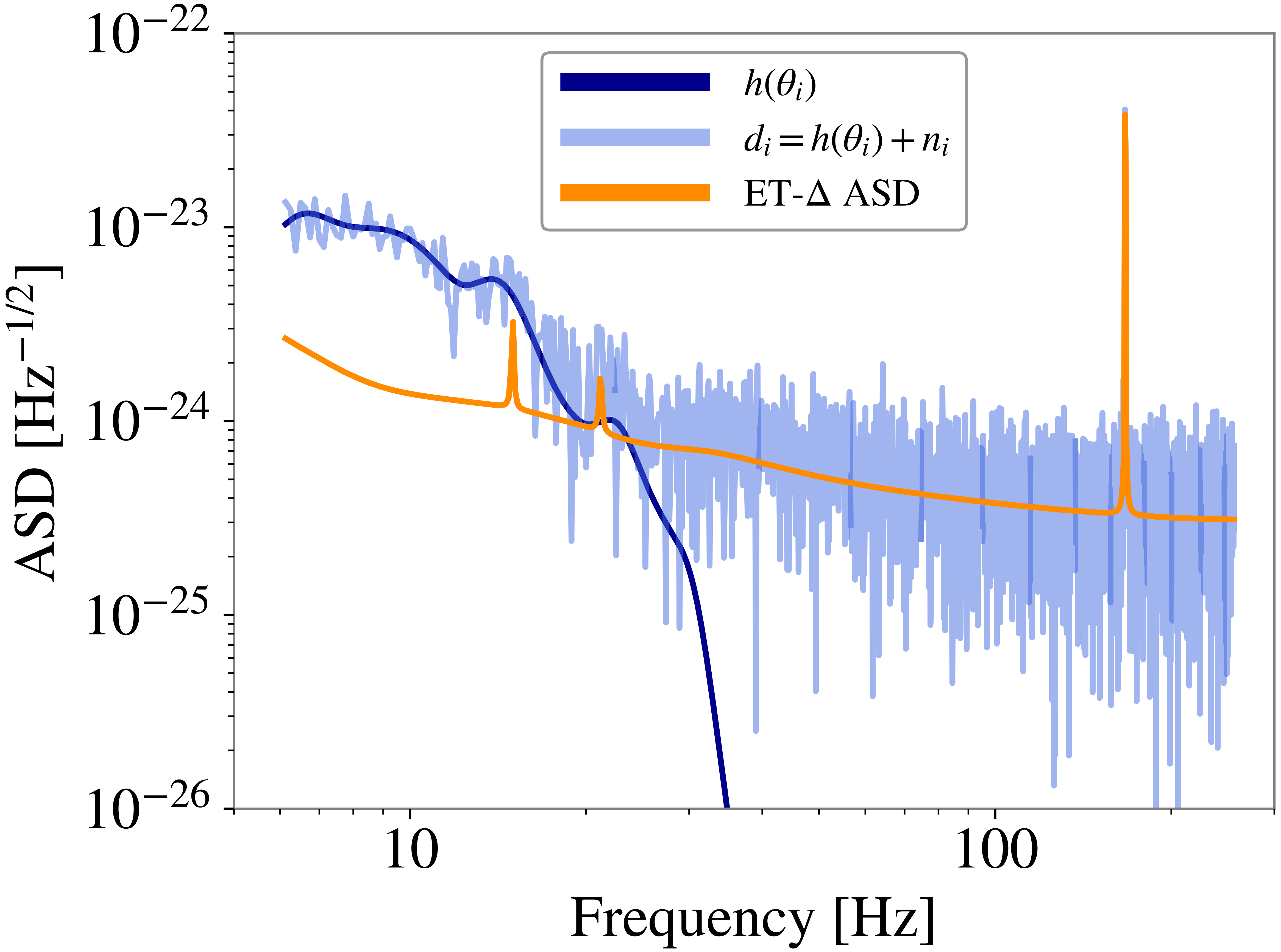
VIRGO

EINSTEIN TELESCOPE

Parameter estimation

$$p(\theta \,|\, d) = \frac{\mathscr{L}(d \,|\, \theta)\pi(\theta)}{\mathscr{Z}(d)}$$

$$\theta = \{\mathscr{M}_{\rm d},\ q,\ d_{\rm L},\ {\rm ra},\ {\rm dec},\ \theta_{\rm JN},\ \psi,\ \phi,\ t_{\rm geocent},\ \chi_1,\ \chi_2\}$$



Ref. Thrane et al. 2019, Christensen et al. 2022

Parameter estimation

θ through **stochastic sampling** (e.g. nested sampling).
This requires tens of millions of likelihood evaluations …

$$p(\theta | d) = \frac{\mathscr{L}(d | \theta)\pi(\theta)}{\mathscr{Z}(d)}$$

… and evaluating likelihood takes time

➡️ **3G detectors need new technology for PE**

Ref. Thrane et al. 2019, Speagle et al. 2019, Williams et al. 2021, Romero-Shaw et al. 2020, Wong et al. 2023, Papalini et al. 2025

Likelihood-free inference

Sampling parameters from the prior $\theta \sim \pi(\theta)$
and data from the likelihood $d \sim \mathscr{L}(d\,|\,\theta)$ is fast

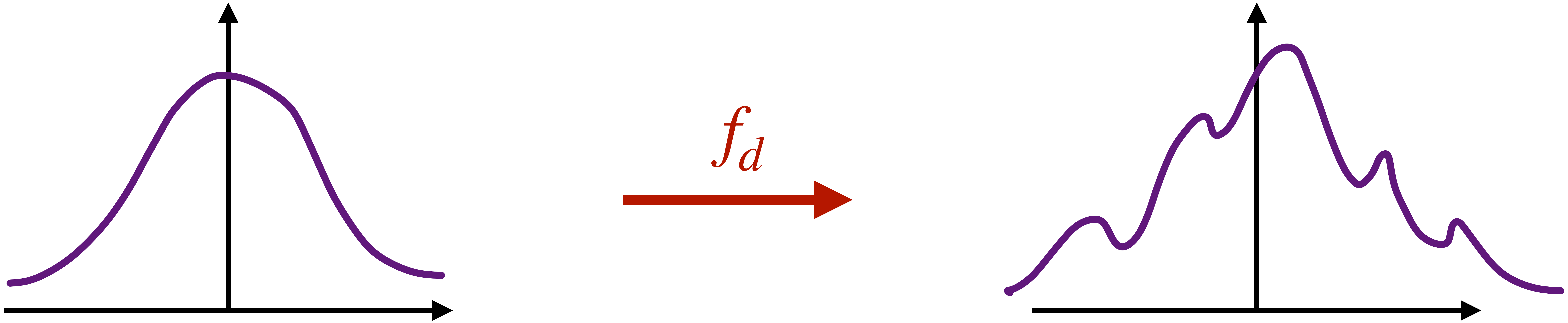
$$p(\theta\,|\,d) = \frac{\mathscr{L}(d\,|\,\theta)\pi(\theta)}{p(d)}$$

Using (θ, d) to construct with **deep learning** an estimator $q(\theta\,|\,d)$ of $p(\theta\,|\,d)$

Ref. Lueckmann et al. 2017, Greenberg et al. 2019, Cranmer et al. 2020, Chua et al. 2020, Dax et al. 2023

Normalising flows

Many advantages: they represent a complicated distribution q using a series of change of variables $f_d : u \rightarrow \theta$



$\mathcal{N}^D(0,1)$

$$q(\theta \mid d) = \mathcal{N}^D(0,1)(f_d^{-1}(\theta)) \mid \det J_{f_d}^{-1} \mid$$

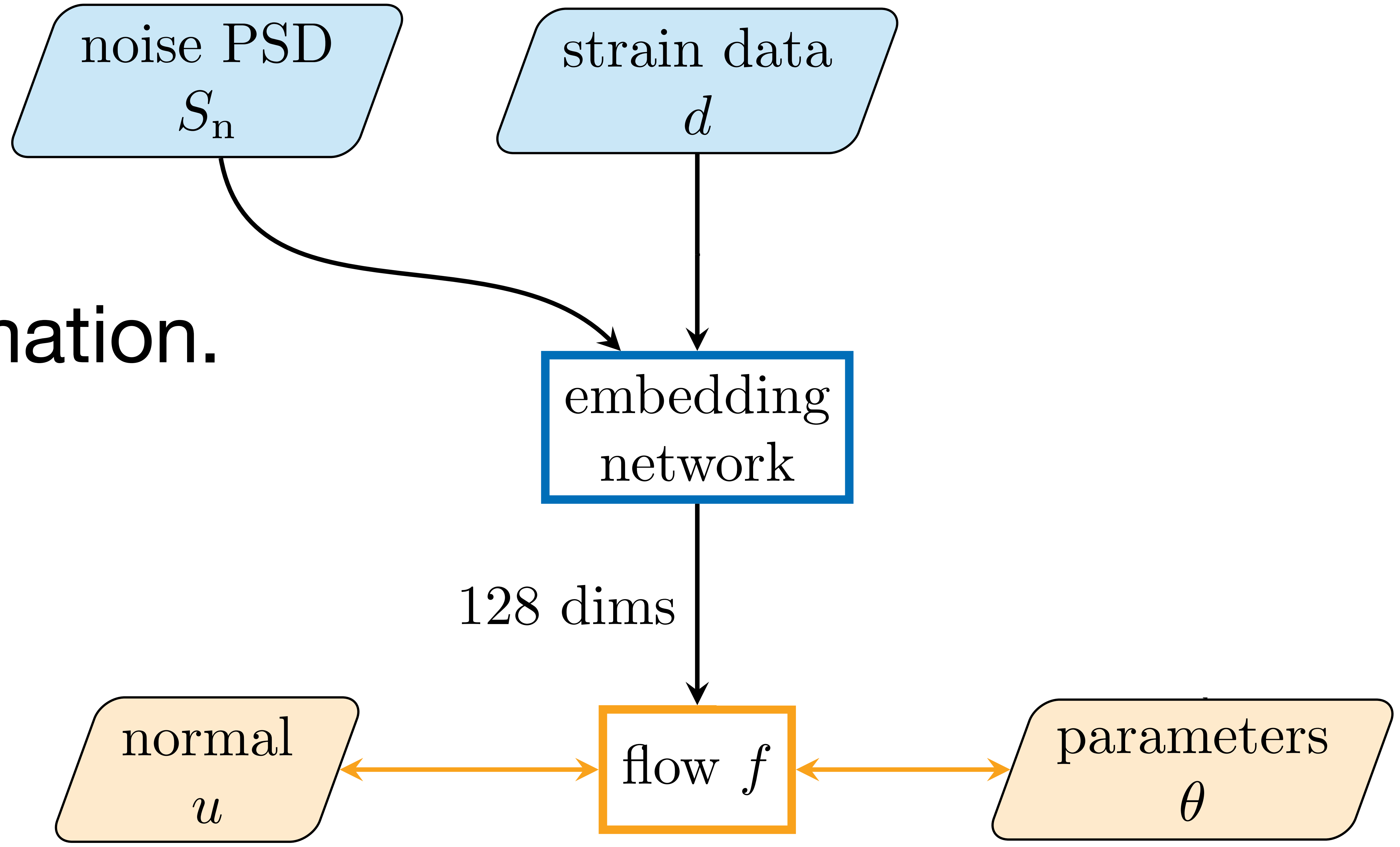
Rapidly evaluated and sampled from

Ref. Kobyzev et al. 2019, Durkan et al. 2019, Polanska et al. 2024

Dingo

Dingo implements neural posterior estimation.

Training in **days**, inference in **minutes**

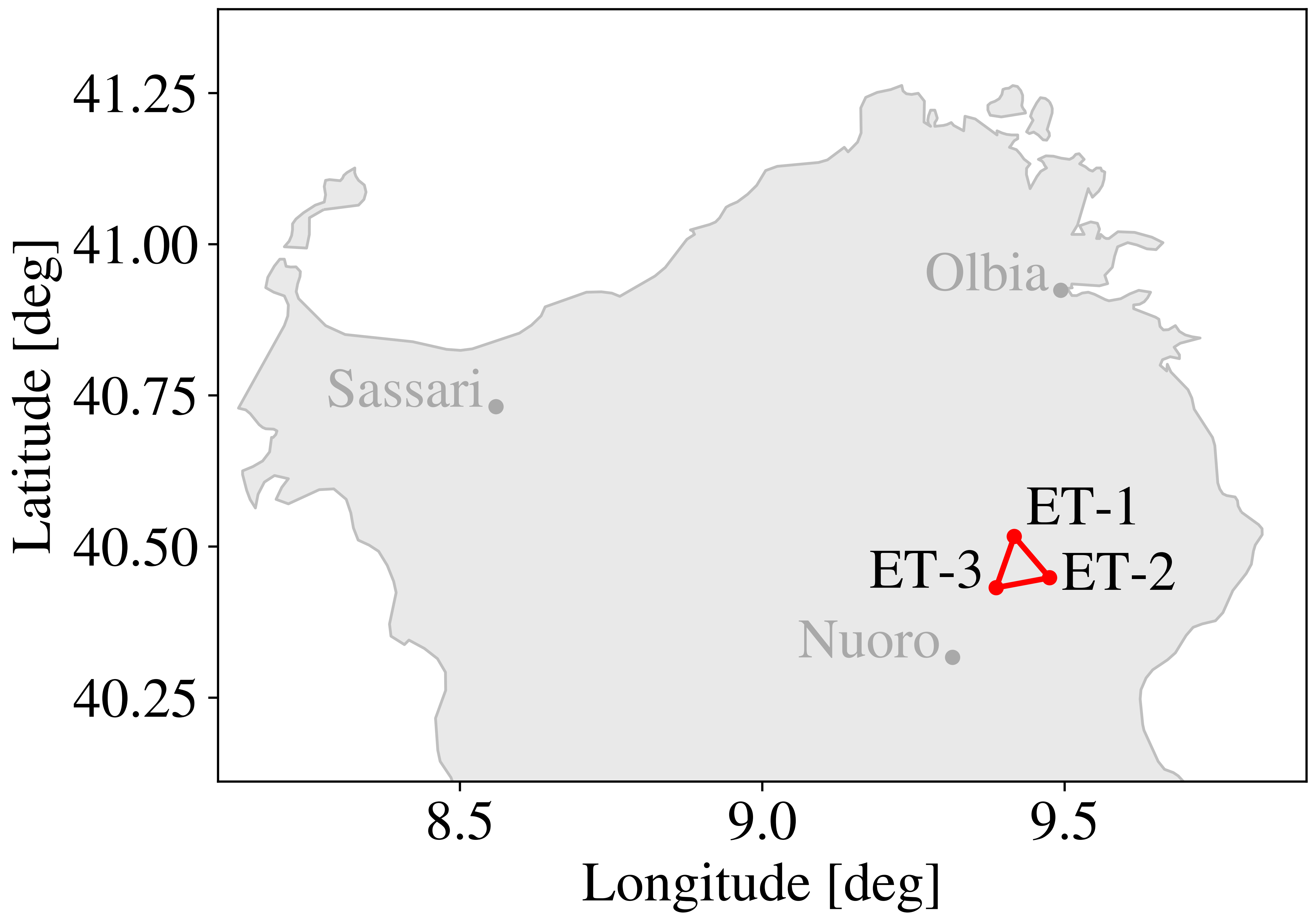


Adapted from Dax et al. 2023

Ref. Green et al. 2020, Green et al. 2021, Dax et al. 2021, Dax et al. 2022, Wildberger et al. 2022, Dax et al. 2024

Einstein Telescope

I trained Dingo using the **HFLF-cryo ASD** with **ET-Δ** configuration placed in Sardinia

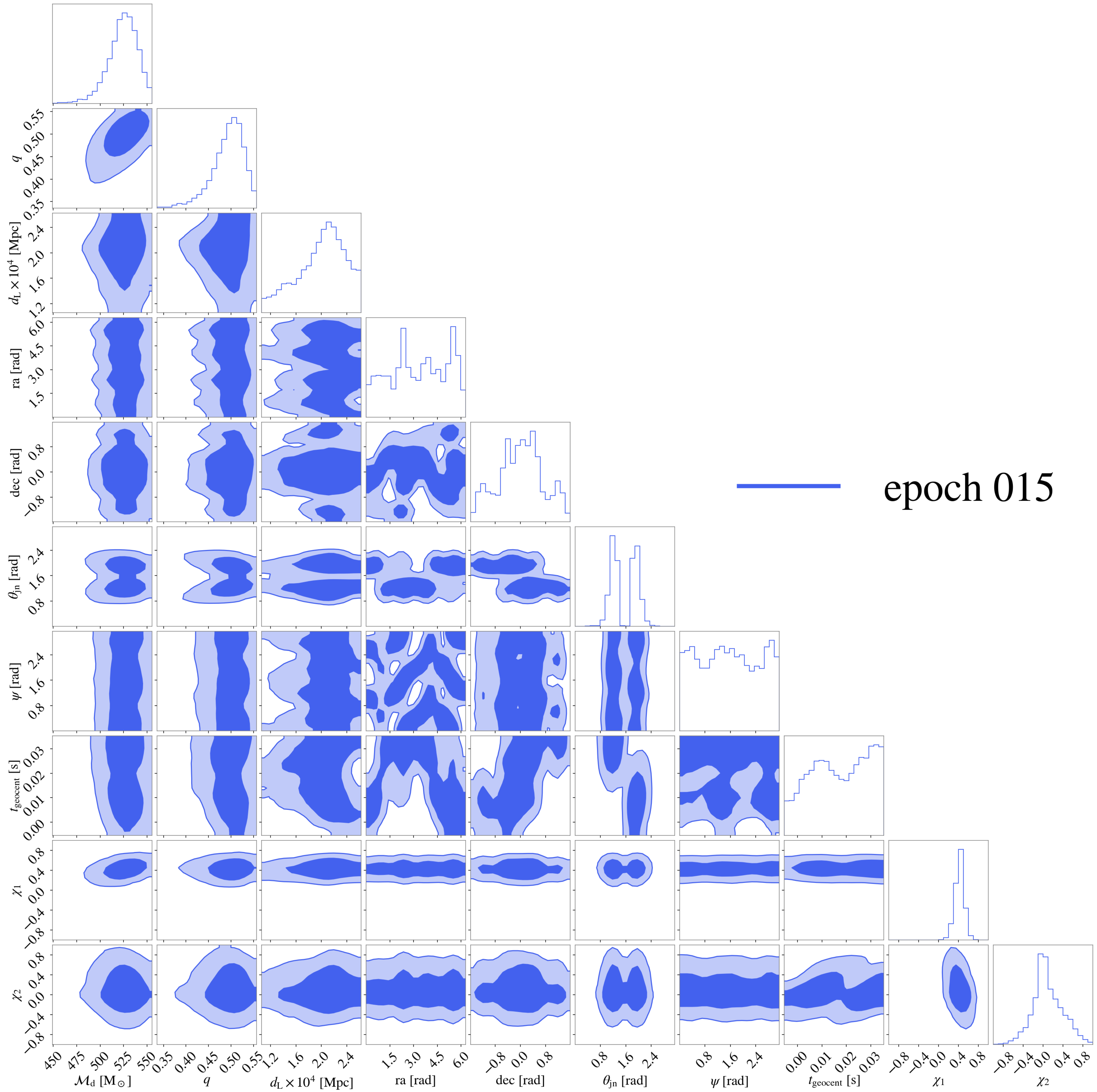


high-redshift sources

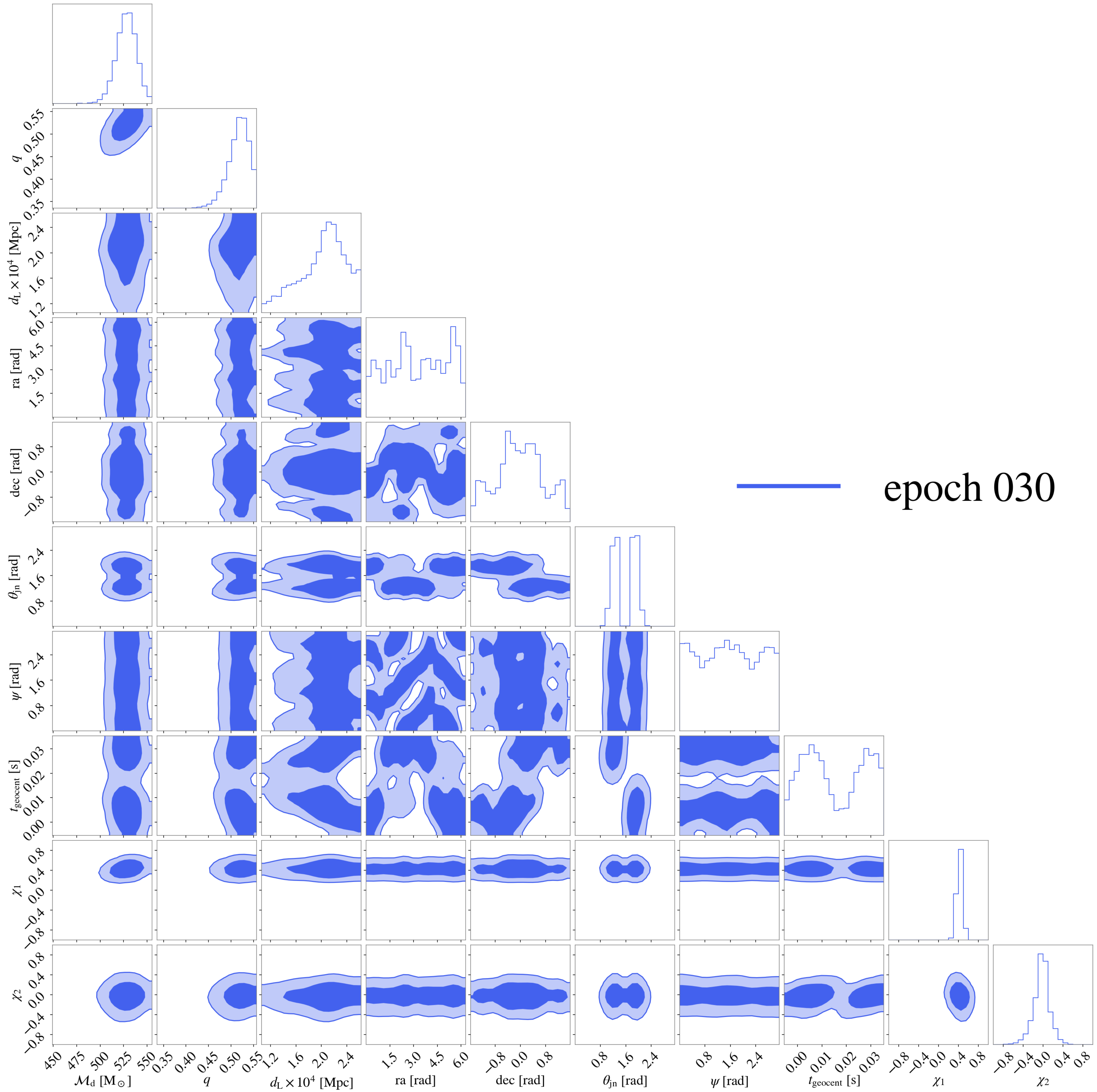
```
'chirp_mass': UniformInComponentsChirpMass(minimum=40, maximum=1100)

'luminosity_distance': UniformSourceFrame(minimum=5_000.0, maximum=500_000.0)
```

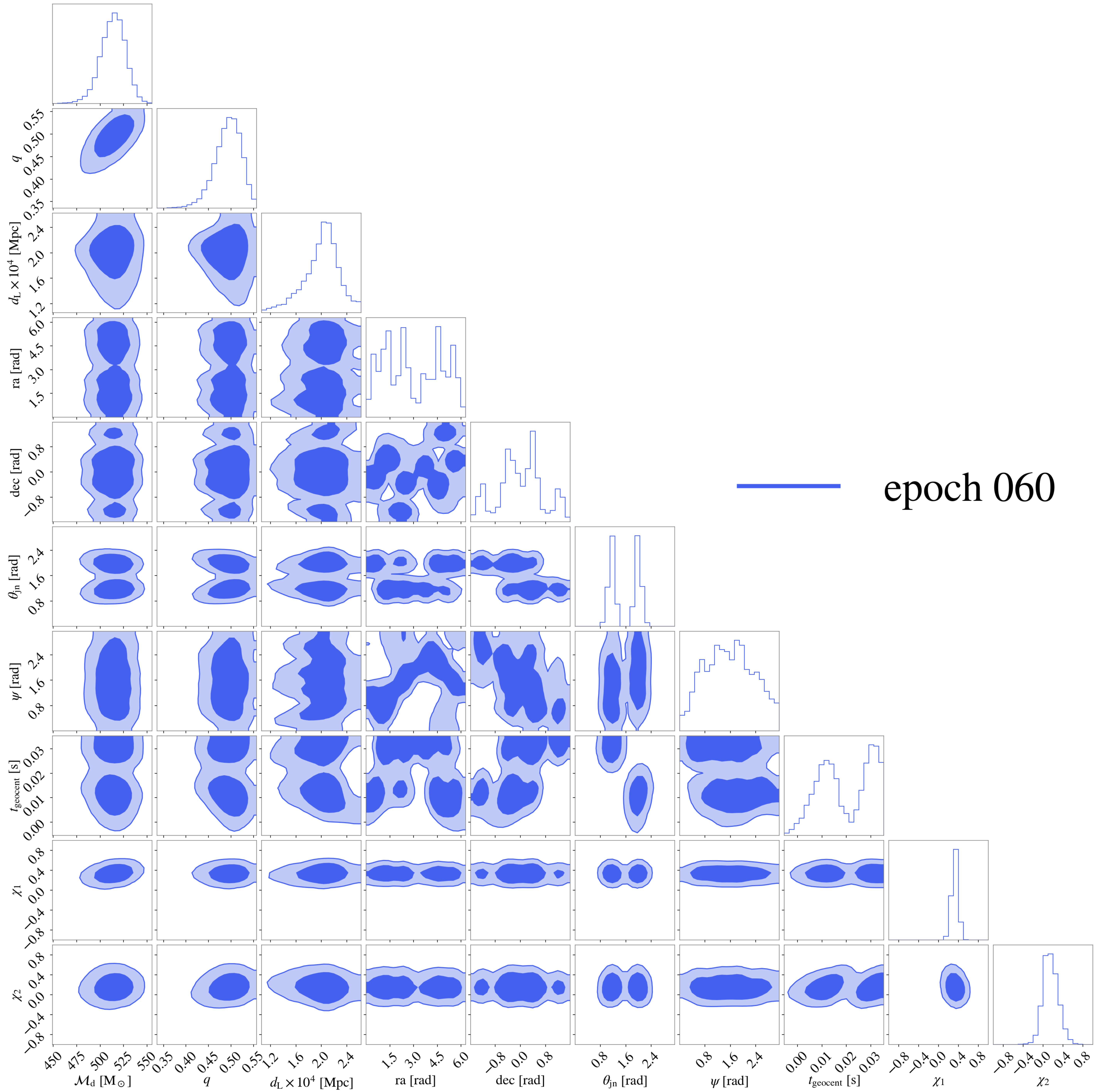
with $f \in [6, 256]$ Hz, `df = 1/8` Hz, waveform approximant = `IMRPhenomXPHM`



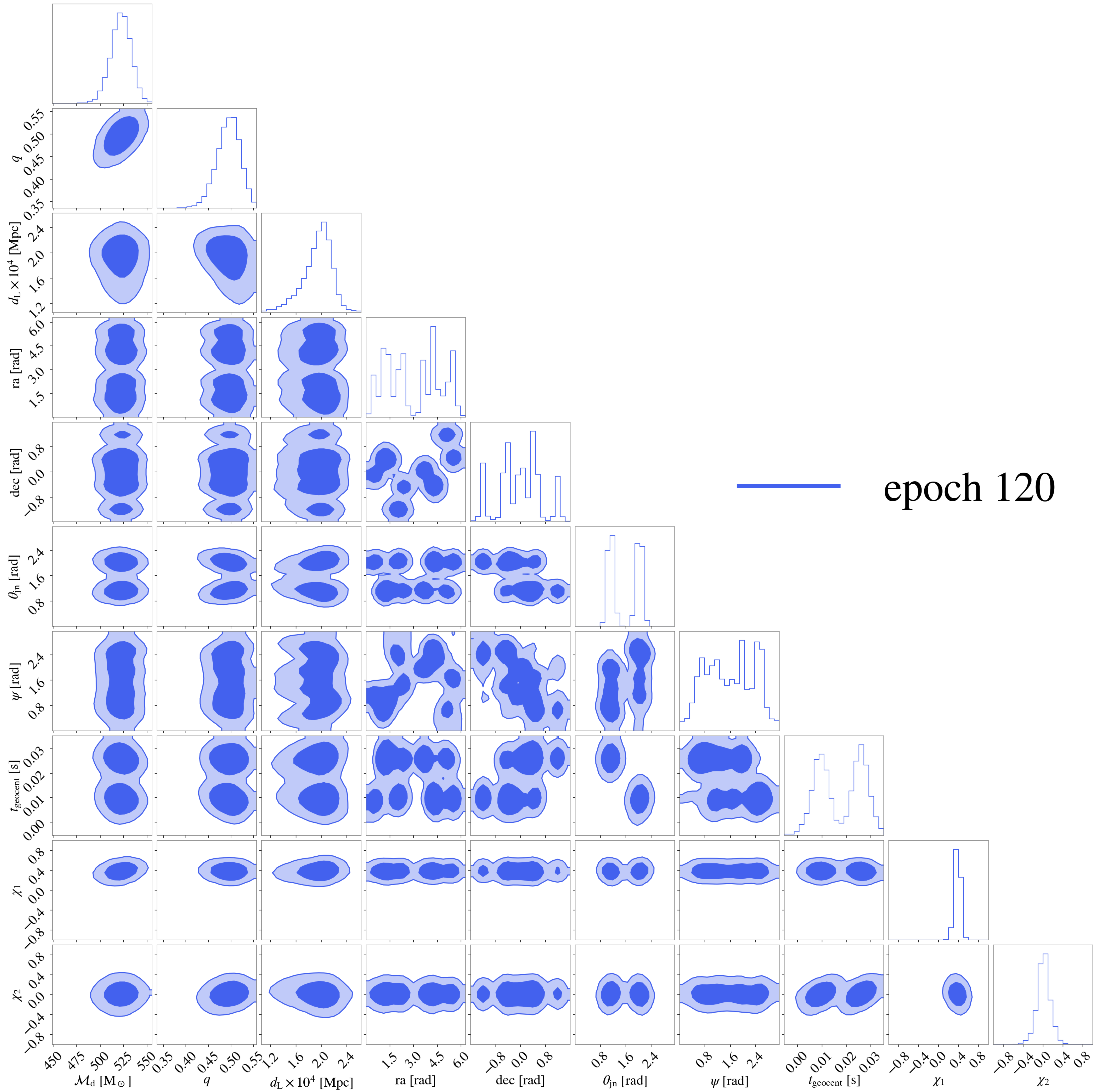
epoch 015
$\mathcal{M}_d$ [M$_\odot$]
q
$d_L \times 10^4$ [Mpc]
ra [rad]
dec [rad]
θ_{jn} [rad]
ψ [rad]
$t_{geocent}$ [s]
χ_1
χ_2

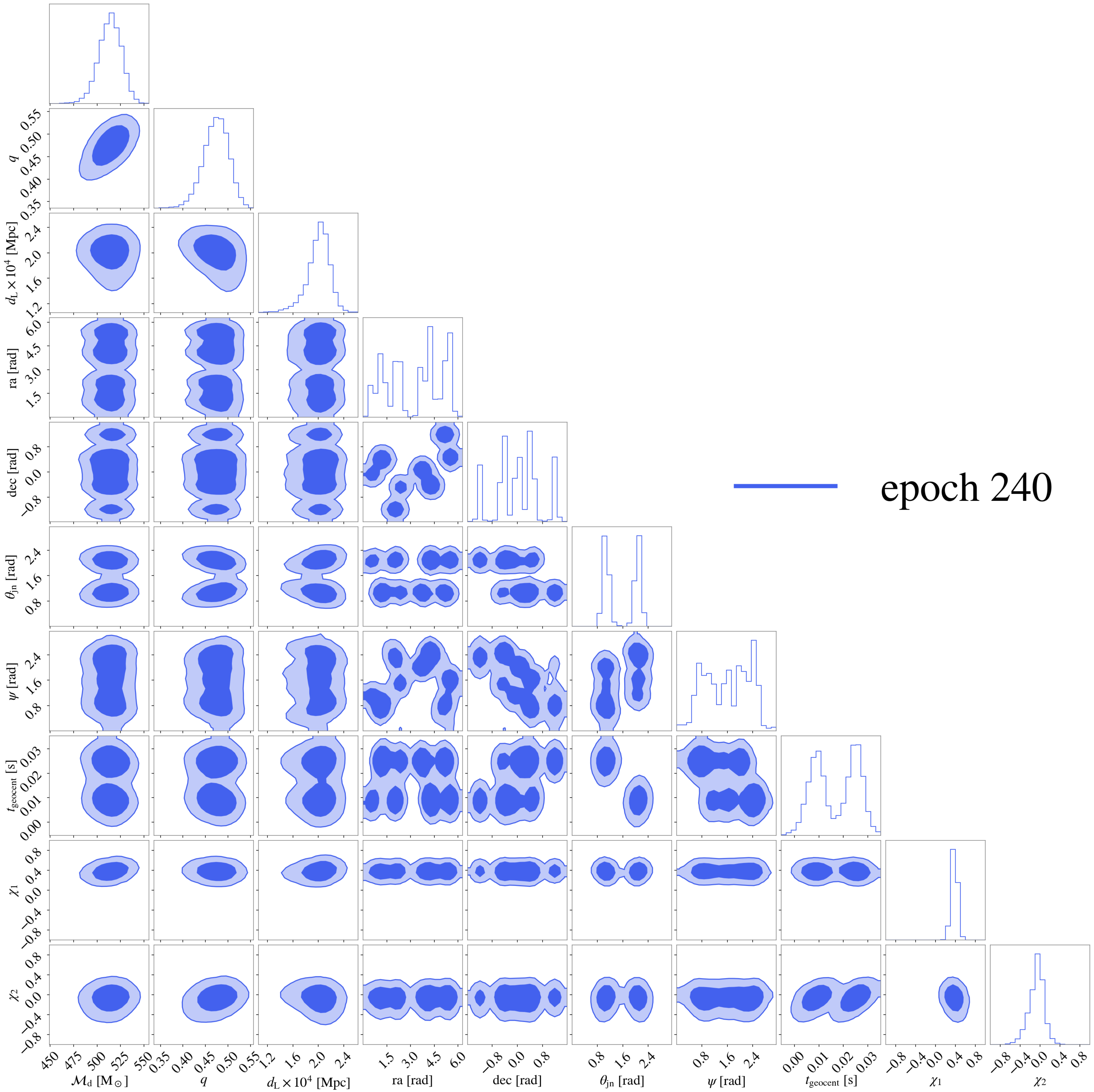


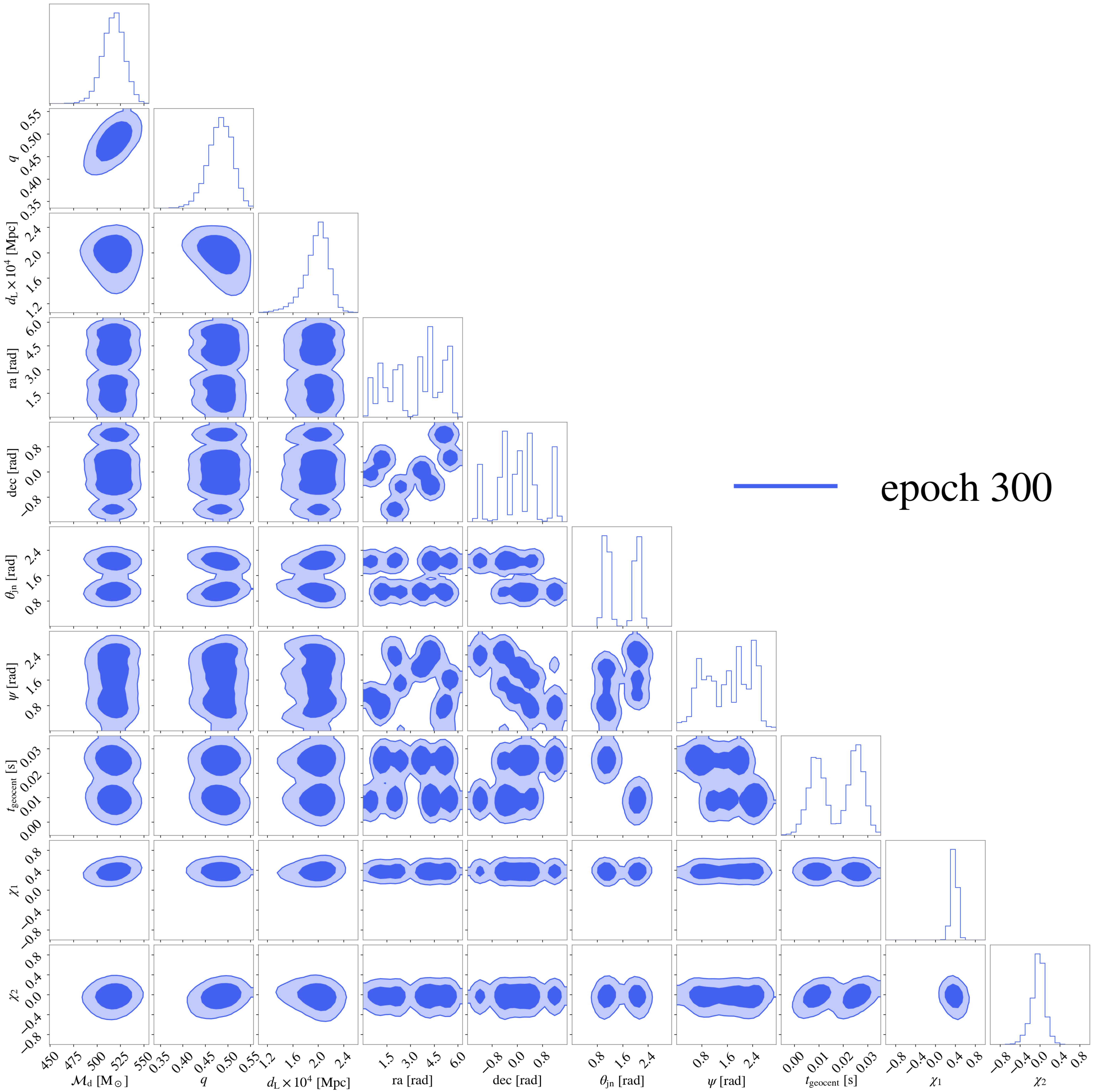
epoch 030
$\mathcal{M}_{\rm d}$ [M$_\odot$]
q
$d_{\rm L} \times 10^4$ [Mpc]
ra [rad]
dec [rad]
$\theta_{\rm jn}$ [rad]
ψ [rad]
$t_{\rm geocent}$ [s]
χ_1
χ_2



epoch 060
$\mathcal{M}_{\rm d}$ [M$_\odot$]
q
$d_{\rm L} \times 10^4$ [Mpc]
ra [rad]
dec [rad]
$\theta_{\rm jn}$ [rad]
ψ [rad]
$t_{\rm geocent}$ [s]
χ_1
χ_2


epoch 120
$\mathcal{M}_d$ [$M_\odot$]
q
$d_L \times 10^4$ [Mpc]
ra [rad]
dec [rad]
θ_{jn} [rad]
ψ [rad]
$t_{geocent}$ [s]
χ_1
χ_2


epoch 240
$\mathcal{M}_d$ [M$_\odot$]
q
$d_L \times 10^4$ [Mpc]
ra [rad]
dec [rad]
θ_{jn} [rad]
ψ [rad]
$t_{geocent}$ [s]
χ_1
χ_2



epoch 300
$\mathcal{M}_{\rm d}$ [M$_\odot$]
q
$d_{\rm L}\times10^4$ [Mpc]
ra [rad]
dec [rad]
$\theta_{\rm jn}$ [rad]
ψ [rad]
$t_{\rm geocent}$ [s]
χ_1
χ_2

Importance sampling

We can correct inaccuracies

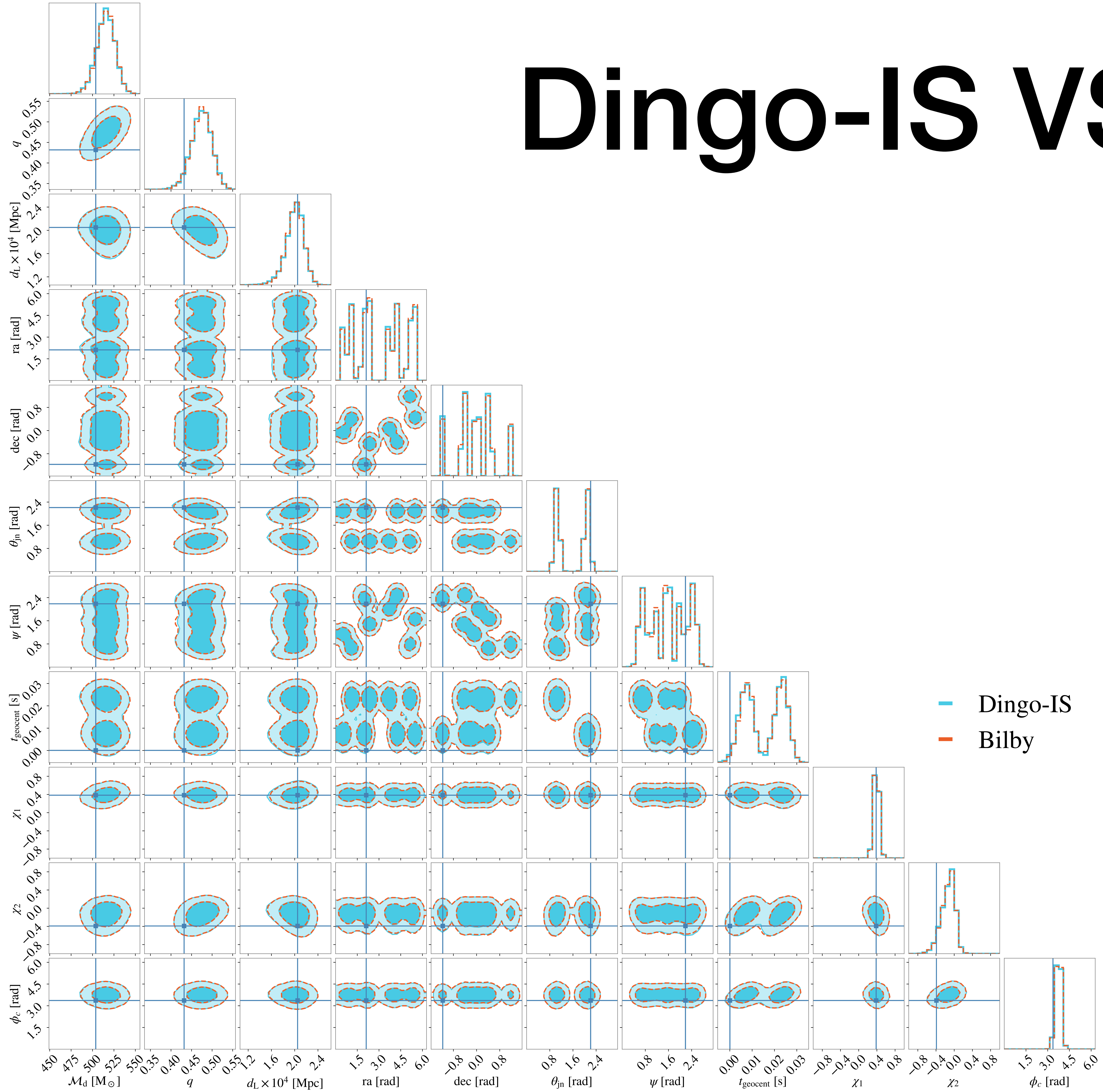
$$w_i = \frac{\mathscr{L}(d\,|\,\theta)\pi(\theta)}{q(\theta\,|\,d)}$$

Target (likelihood x prior)

Dingo proposal

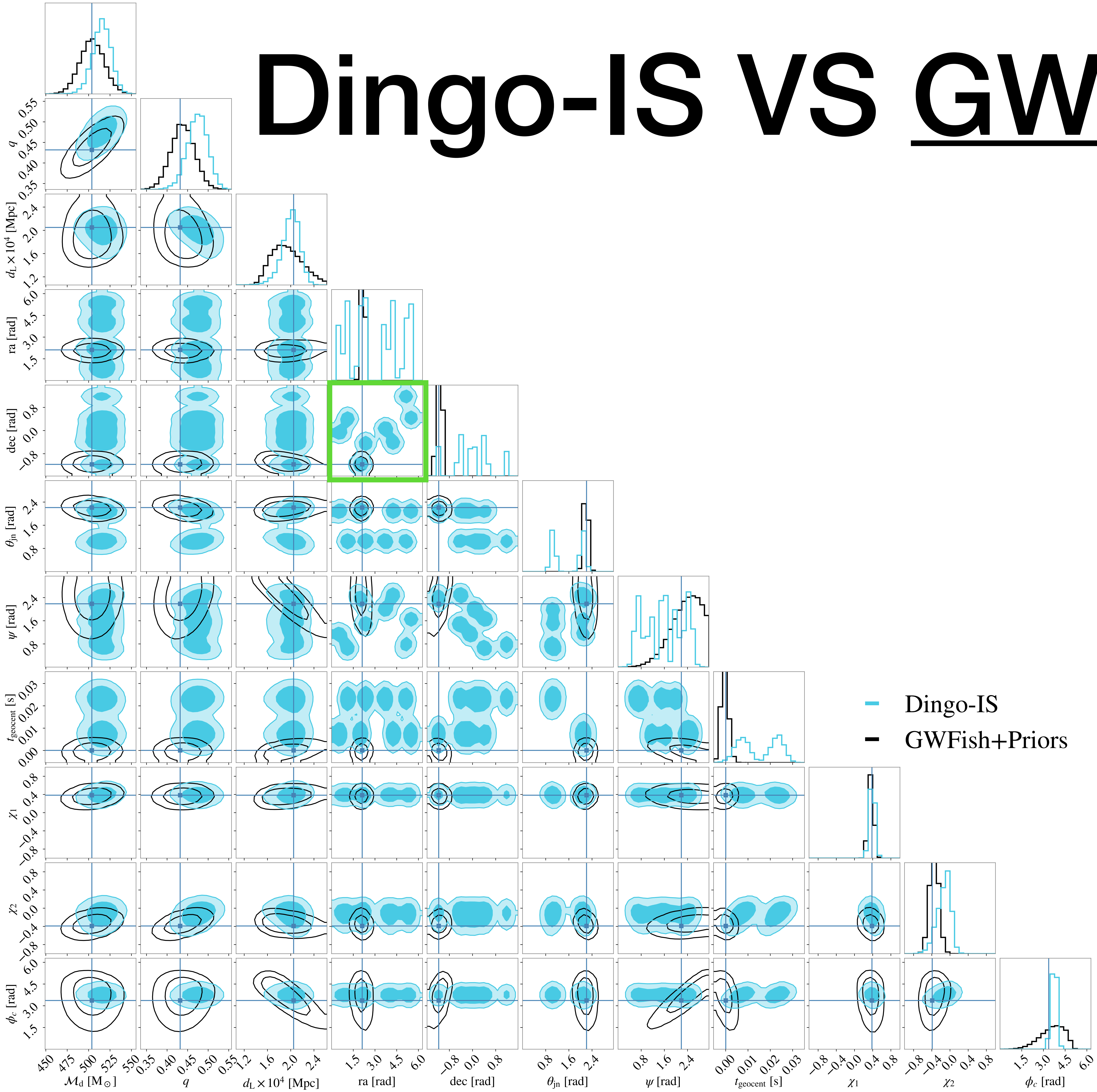
Ref. Todkar et al. 2010, Owen 2013

Dingo-IS VS Bilby



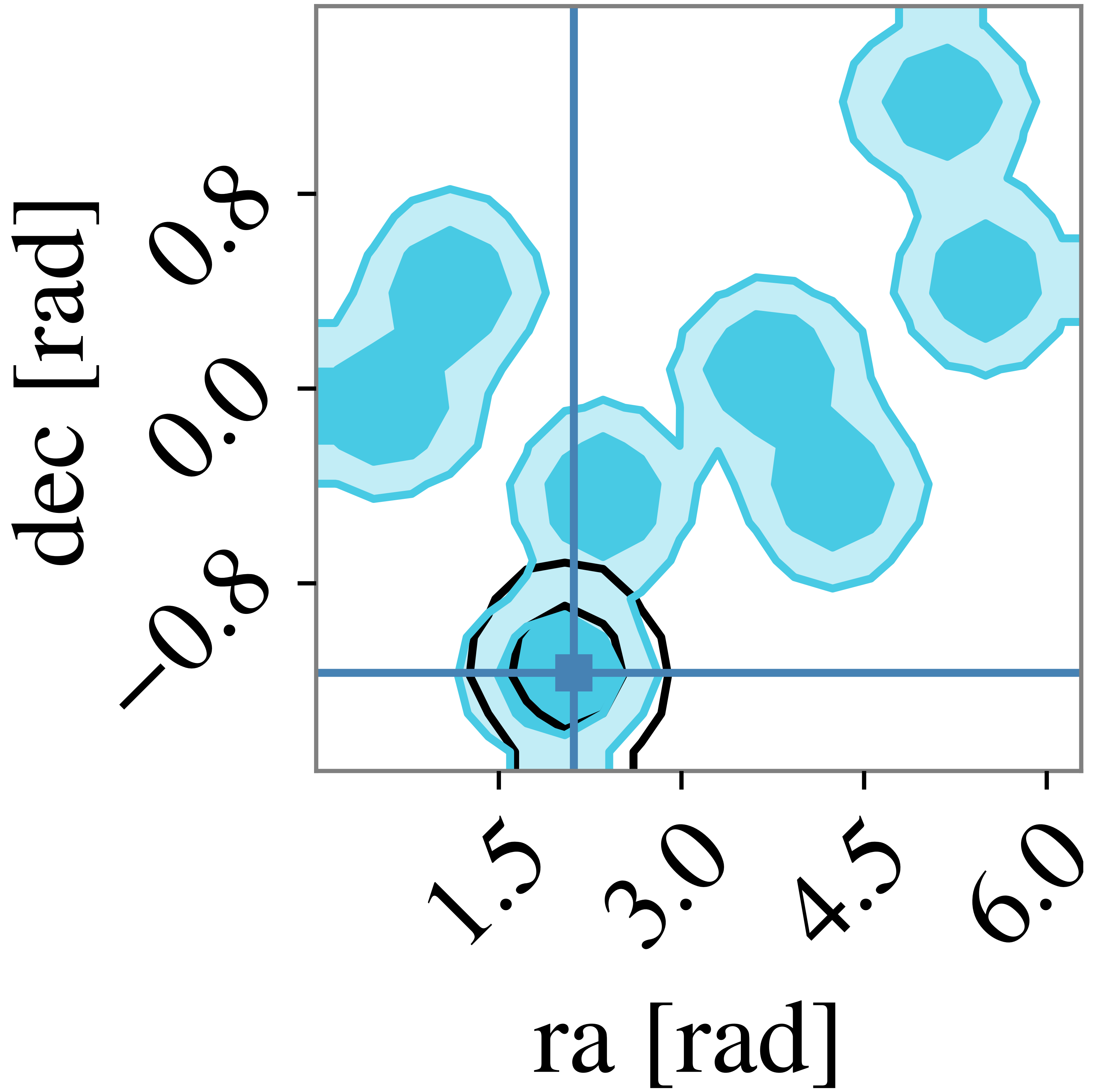
Ref. [Romero-Shaw et al. 2020]()

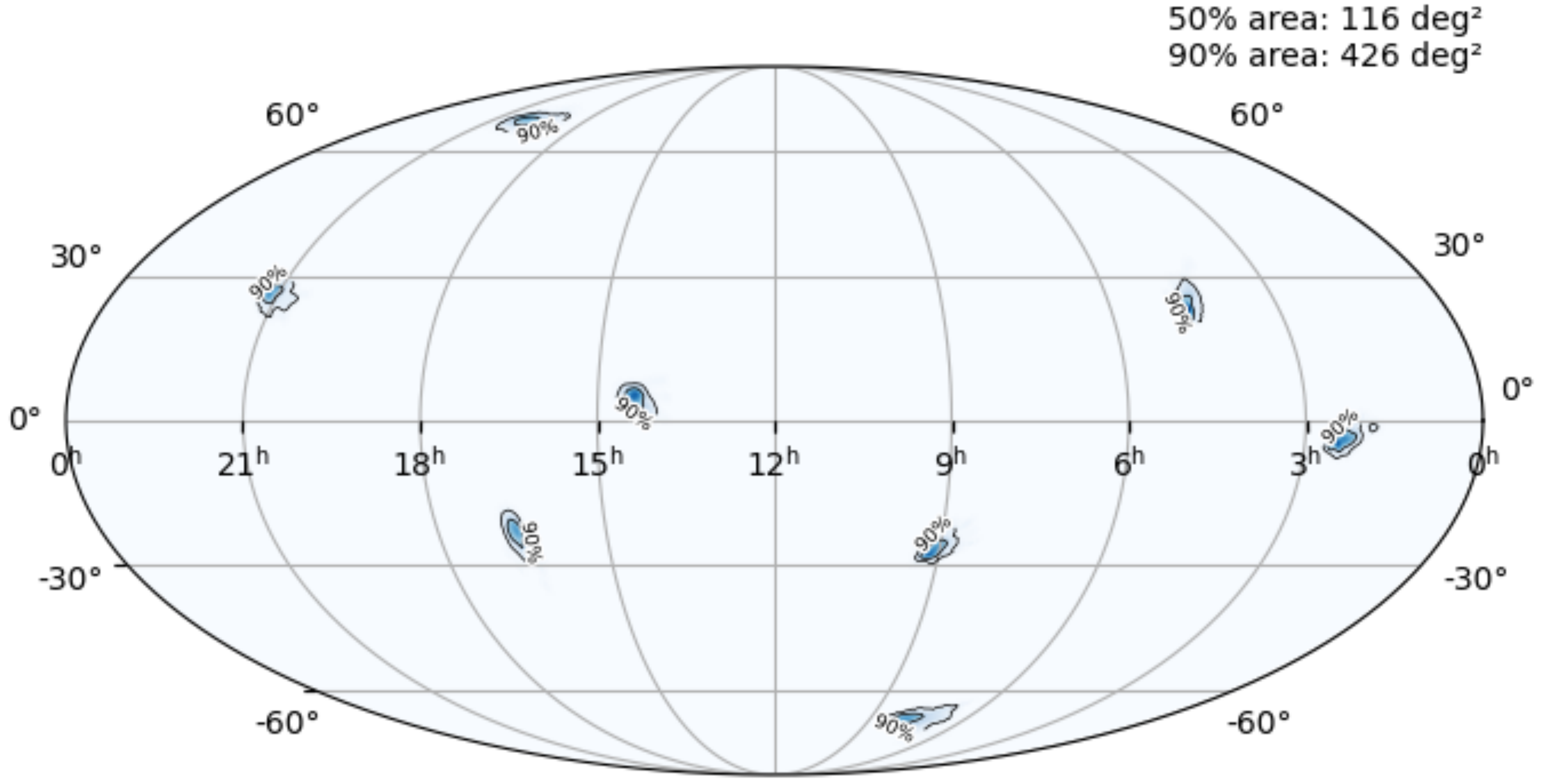
Dingo-IS VS GWFish+Priors



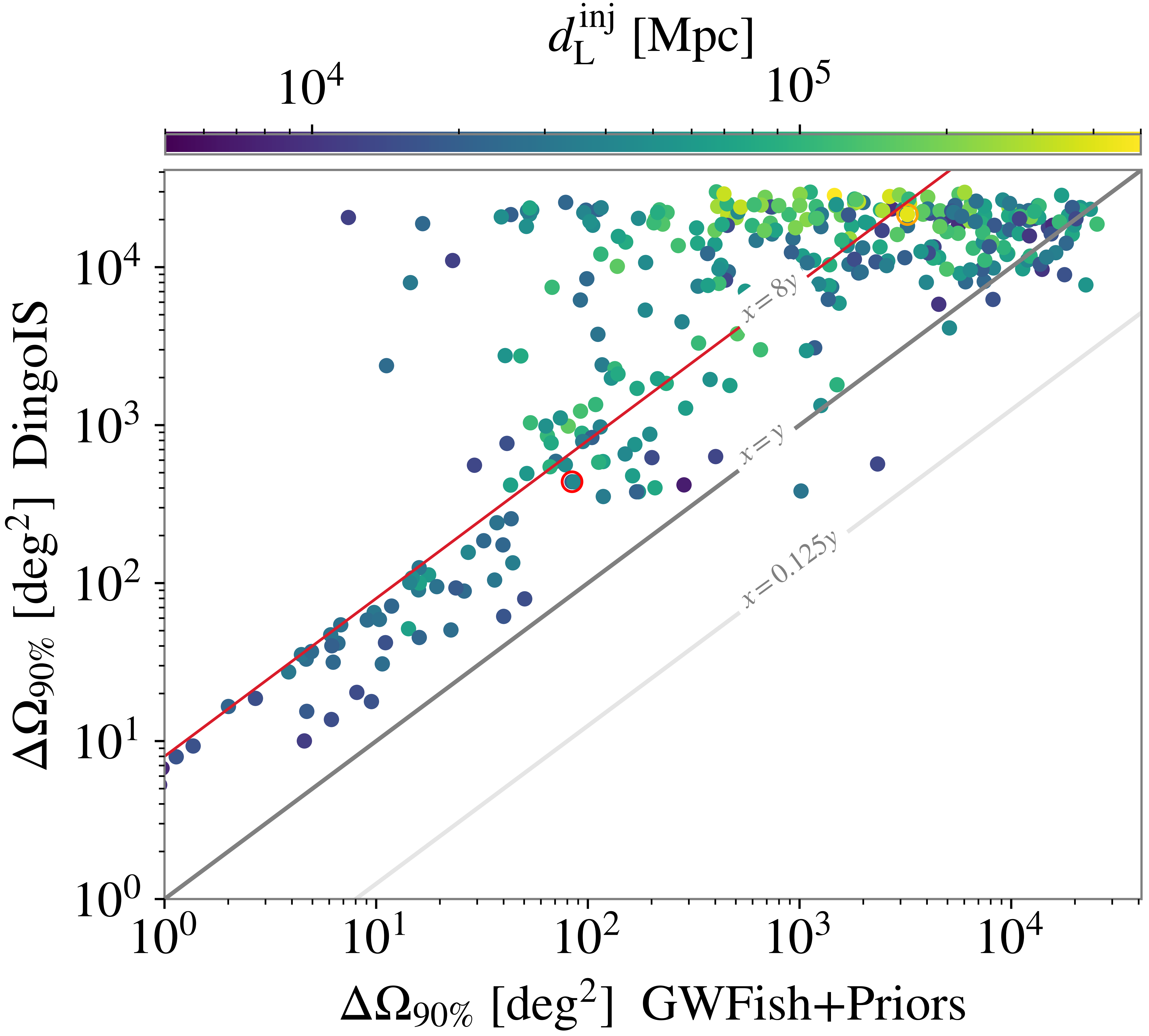
Ref. Dupletsa et al. 2025

Sky modes


dec [rad]
0.8
0.0
−0.8
1.5
3.0
4.5
6.0
ra [rad]

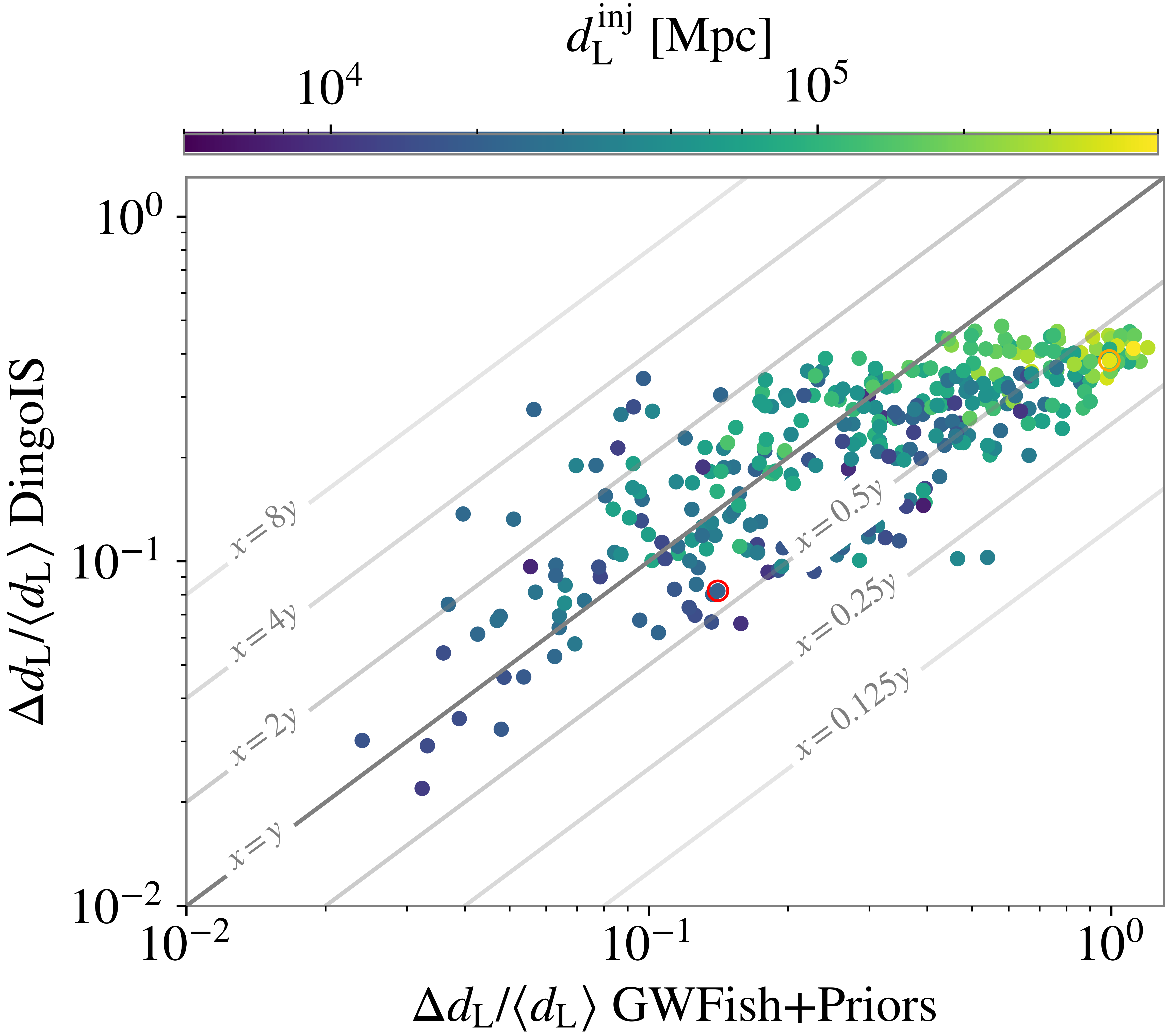

50% area: 116 deg²
90% area: 426 deg²
60°
30°
0°
-30°
-60°
0h
21h
18h
15h
12h
9h
6h
3h
0h
90%

Sky localisation



Ref. [Santoliquido et al. 2025]

Luminosity distance



Ref. [Santoliquido et al. 2025]

Contributions

Fast and accurate parameter estimation for high-redshift sources

Look at multimodalities in sky localisation

This approach thrives where Fisher matrix approximation is less reliable

arXiv: [2504.21087]

Backup slides

Other priors

```
‘mass_ratio’: UniformInComponentsMassRatio(minimum=0.125, maximum=1.0)

'dec': Cosine(minimum=−π/2, maximum=π/2)

'ra': Uniform(minimum=0, maximum=2π)

'theta_jn': Sine(minimum=0.0, maximum=π)

'psi': Uniform(minimum=0, maximum=π)

'chi_1': AlignedSpin(a_prior=Uniform(minimum=0, maximum=0.9))

'chi_2': AlignedSpin(a_prior=Uniform(minimum=0, maximum=0.9))

'phase': Uniform(minimum=0, maximum=2π)
```

The loss function

Kullback–Leibler divergence

$$D_{\mathrm{KL}}[p(\theta, s) \,||\, q(\theta, s)] = \int \mathrm{d}s \; p(s) \left[\int \mathrm{d}\theta \; p(\theta \,|\, s) \log \frac{p(\theta \,|\, s)}{q(\theta \,|\, s)} \right]$$

Bayes' theorem

$$= \int \mathrm{d}s \; p(s) \left[\int \mathrm{d}\theta \; \frac{p(s \,|\, \theta) p(\theta)}{p(s)} \log \frac{p(\theta \,|\, s)}{q(\theta \,|\, s)} \right]$$

This term is not affected by the neural network

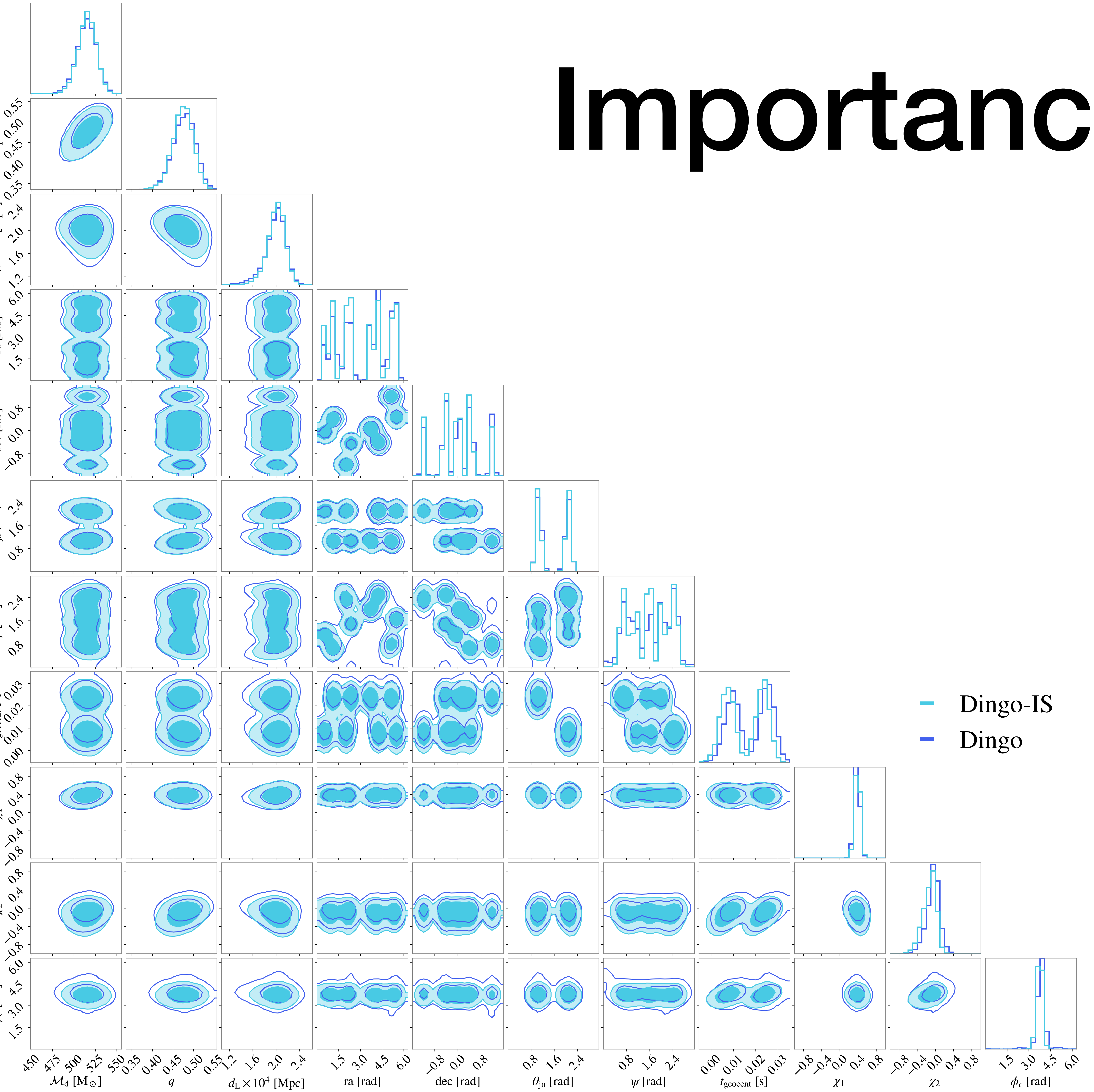
$$= \int \mathrm{d}s \; p(s) \left[\int \mathrm{d}\theta \; \frac{1}{p(s)} \left[p(\theta) p(s \,|\, \theta) \log p(\theta \,|\, s) - p(\theta) p(s \,|\, \theta) \log q(\theta \,|\, s) \right] \right]$$

Neglecting constant values

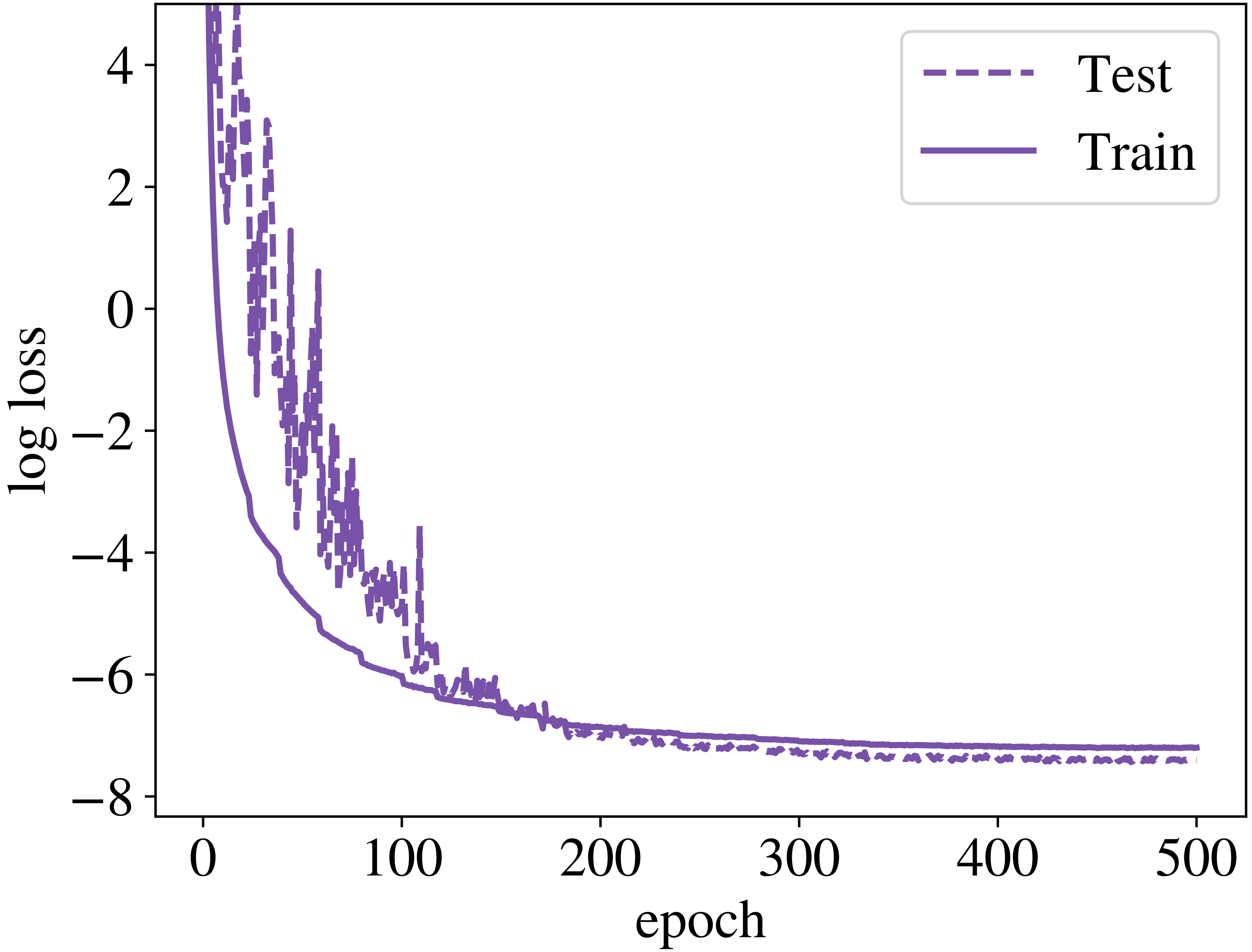
$$\propto - \int \mathrm{d}s \; \mathrm{d}\theta \; p(\theta) p(s \,|\, \theta) \log q(\theta \,|\, s) = - \frac{1}{N_S} \sum_{i=1}^{N_S} \log q(\theta_i, s_i) = \mathbb{E}_{p(\theta)} \mathbb{E}_{p(s|\theta)} [-\log q(\theta \,|\, d)]$$

Ref. Dax et al. 2022, Dax et al. 2023

Importance sampling



Training



Ref. [Santoliquido et al. 2025]

Antenna amplitude pattern functions

$$F_+^i(\beta, \lambda, \psi) = -\frac{\sqrt{3}}{4}[(1 + \cos^2\beta)\sin 2\lambda \cos 2\psi + 2\cos\beta \cos 2\lambda \sin 2\psi],$$

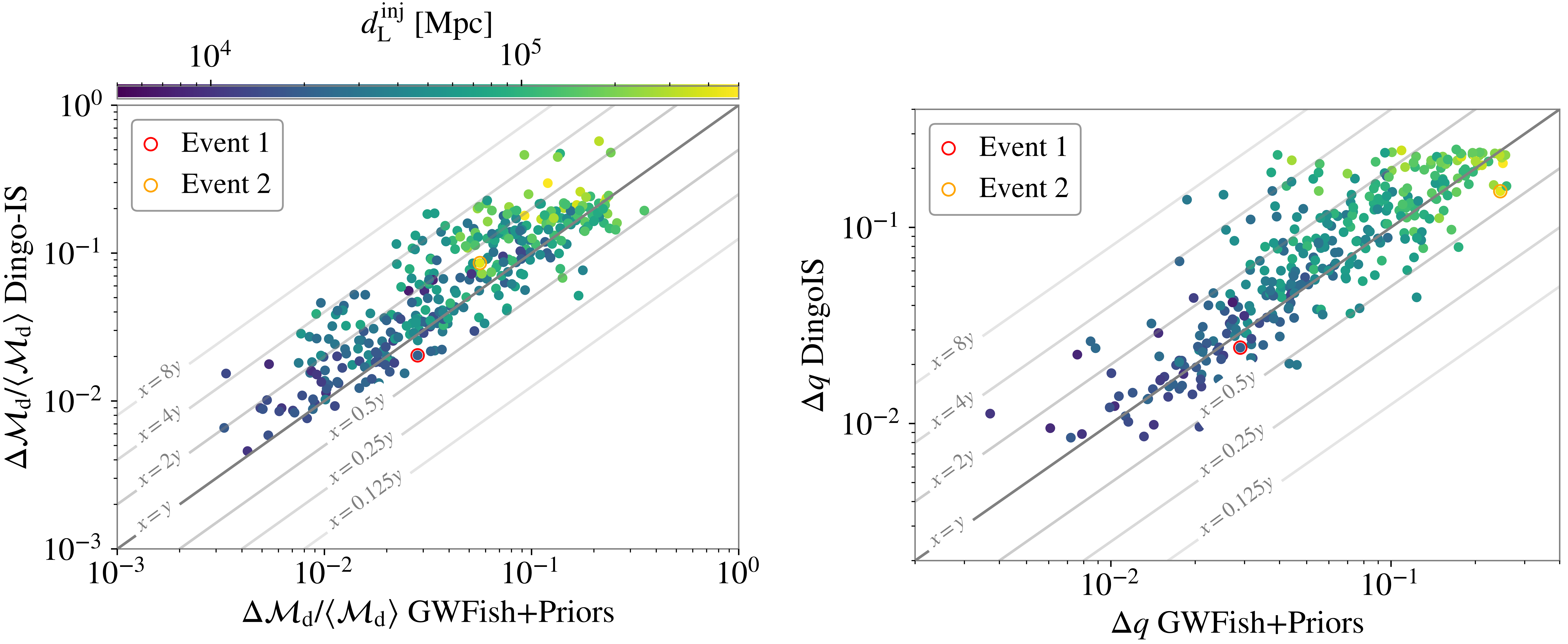
$$F_\times^i(\beta, \lambda, \psi) = +\frac{\sqrt{3}}{4}[(1 + \cos^2\beta)\sin 2\lambda \sin 2\psi - 2\cos\beta \cos 2\lambda \cos 2\psi],$$

Ref. Regimbau et al. 2012, Vishal et al. 2020, Sylvain et al. 2021

Antenna power pattern function

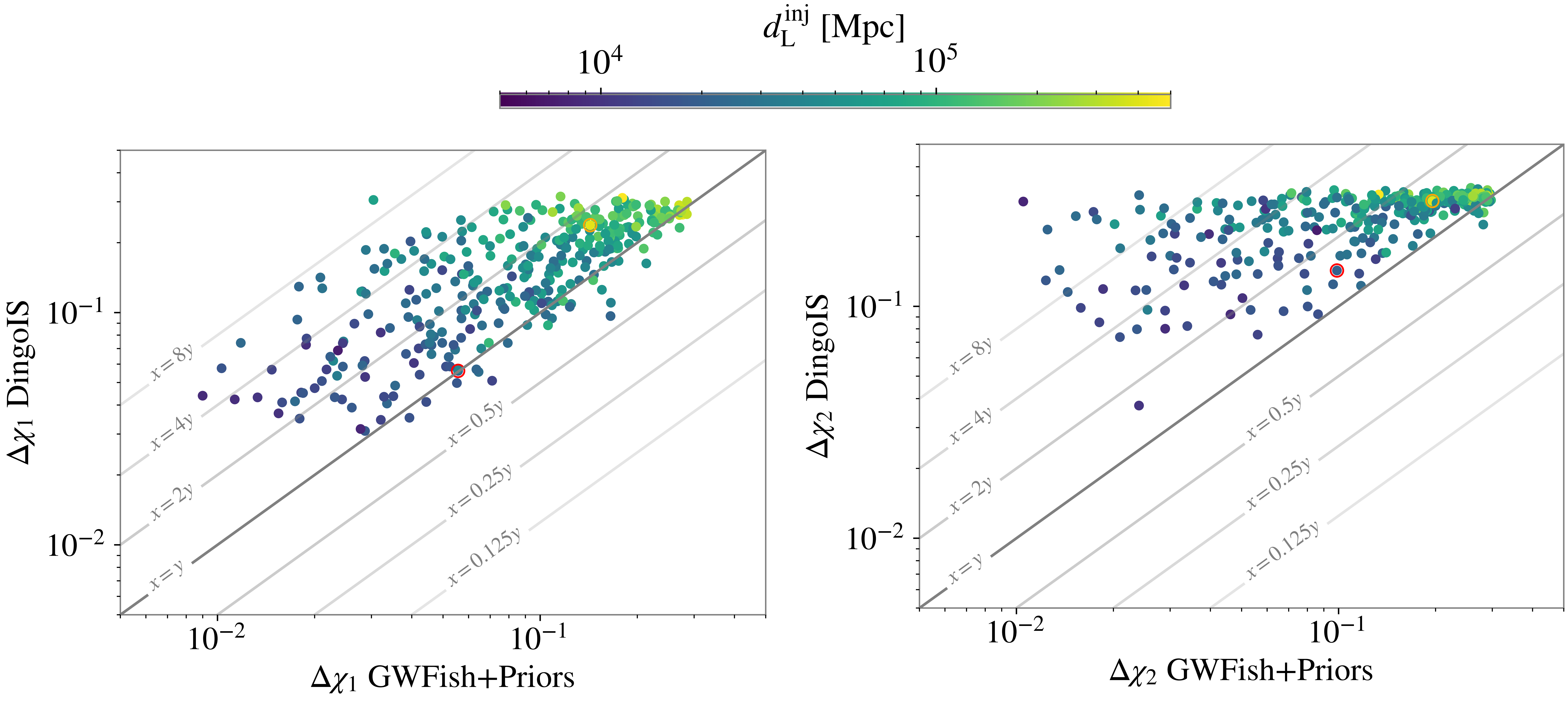
$$\sqrt{(F^i_+)^2 + (F^i_\times)^2} = \sqrt{\frac{3}{12}\left[(1+\cos^2\beta)^2\sin^2 2\lambda + \cos^2\beta\cos^2 2\lambda\right]}$$

Masses



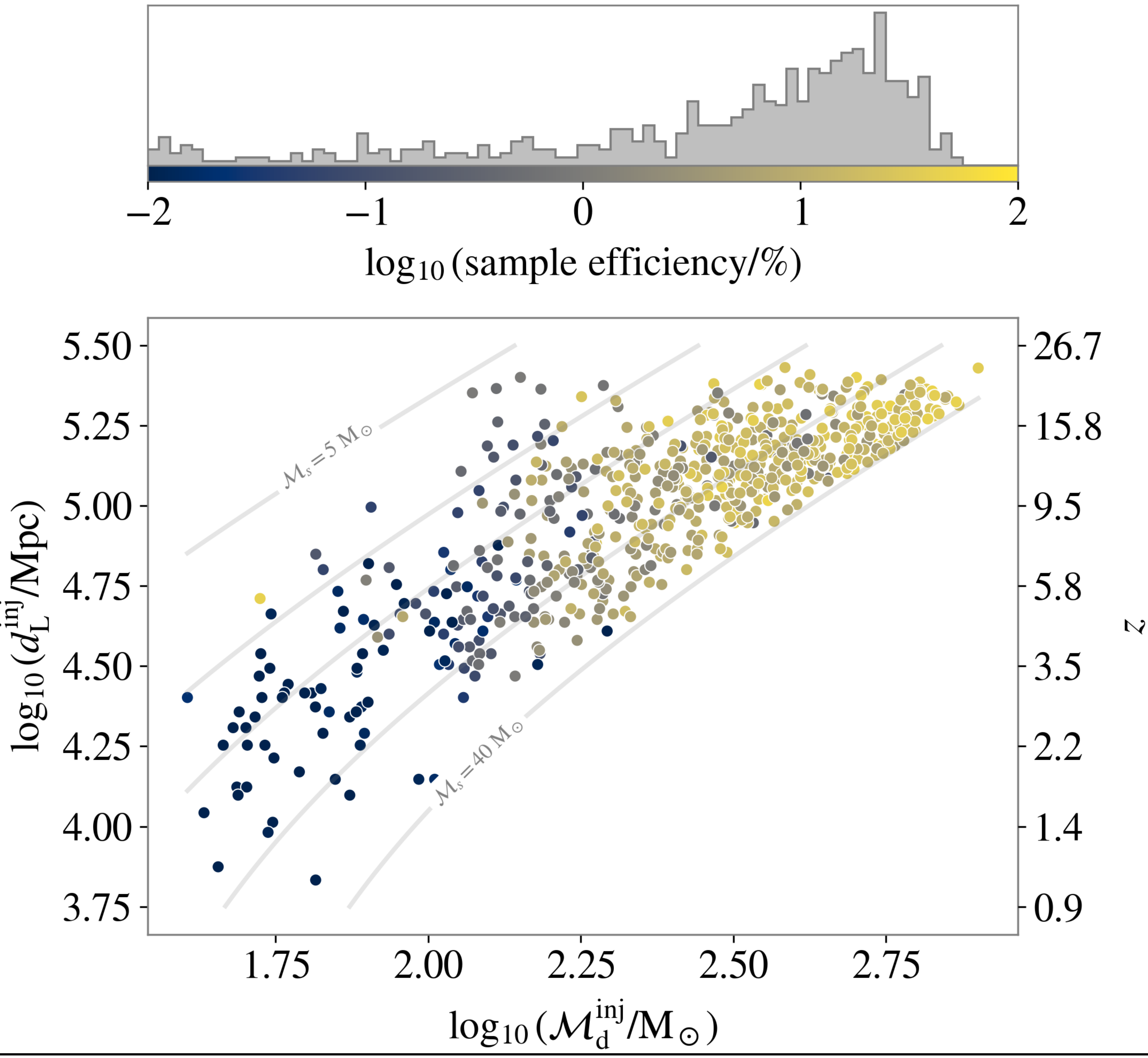
Ref. Santoliquido et al. 2025

Aligned spins



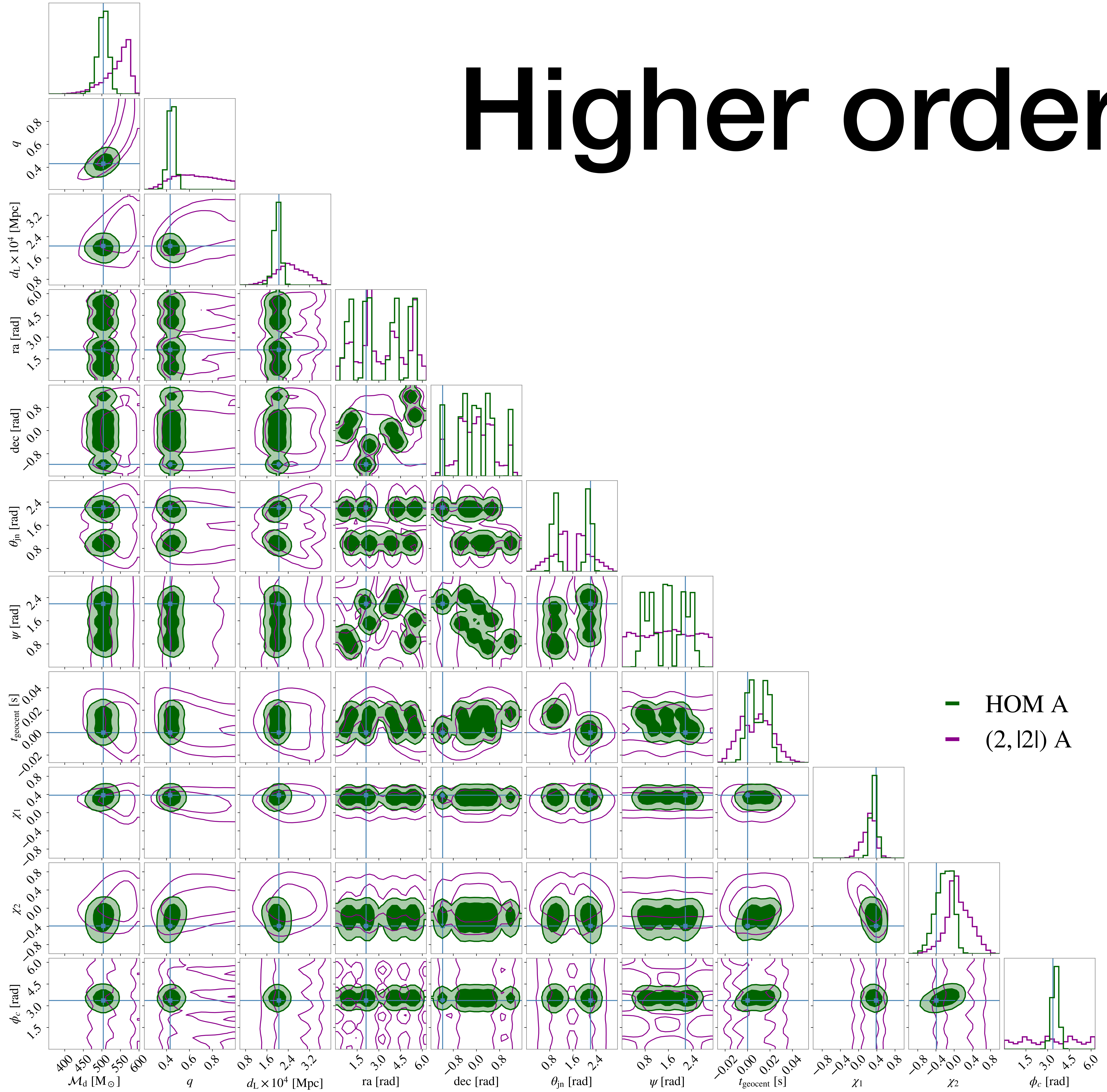
Astrophysical population

Binary black holes formed from Population III stars



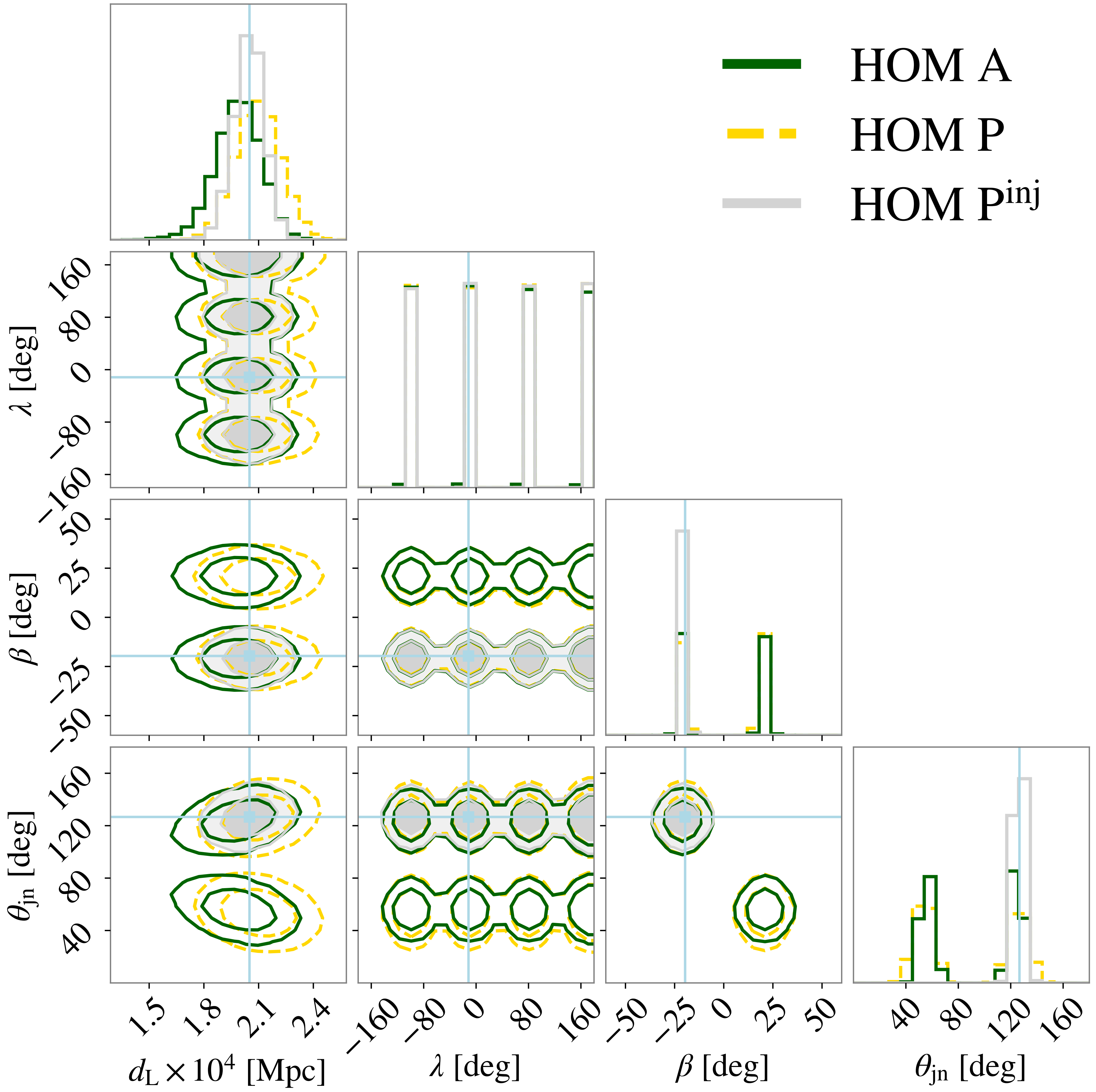
Ref. Costa et al. 2023, Santoliquido et al. 2023, Santoliquido et al. 2024, Santoliquido et al. 2025

Higher order modes



Ref. [Santoliquido et al. 2025]

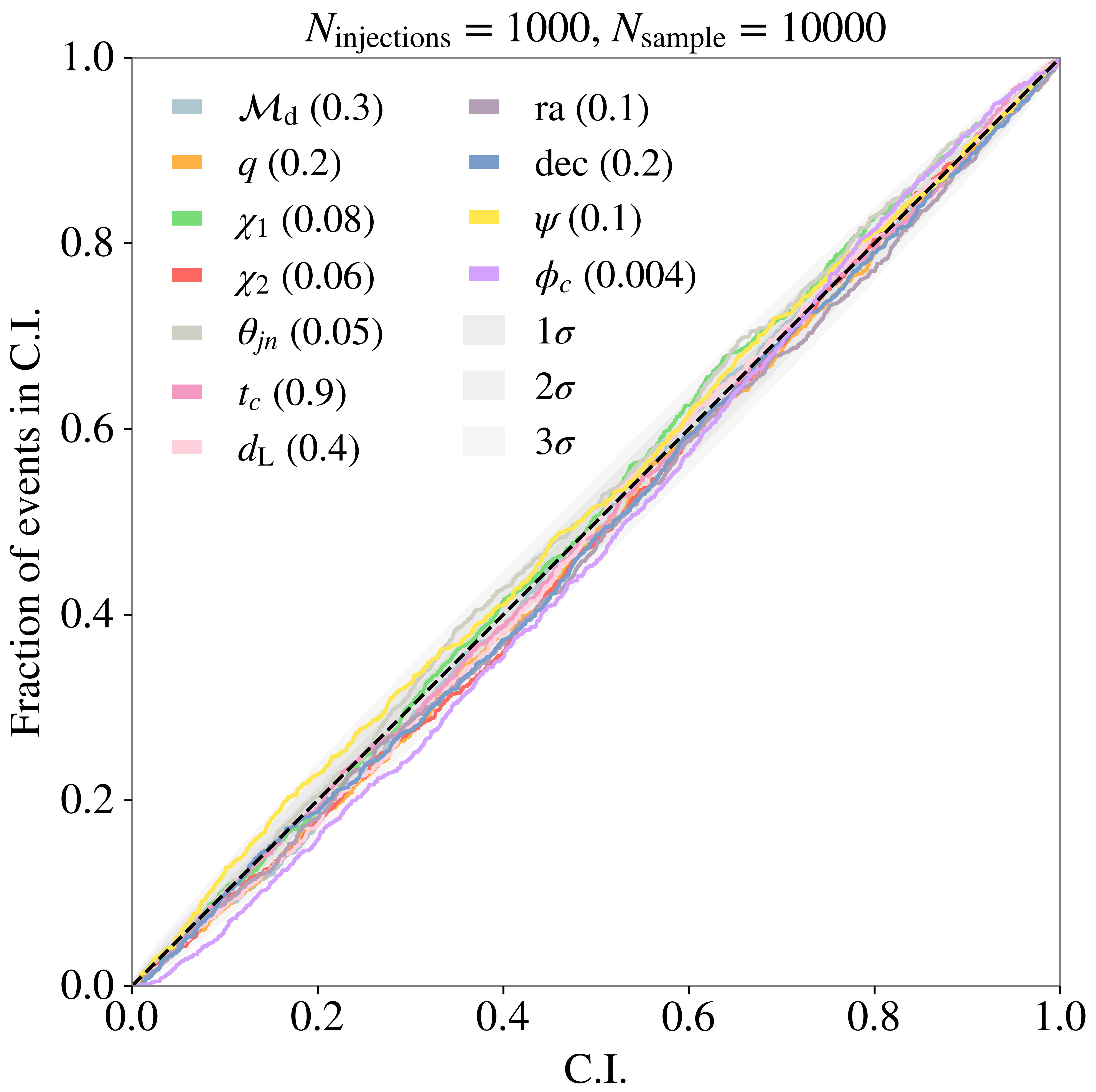
Precessing spins



Ref. [Santoliquido et al. 2025]

Validating results

1000 random injections
sampled from the priors ➡️



Importance sampling

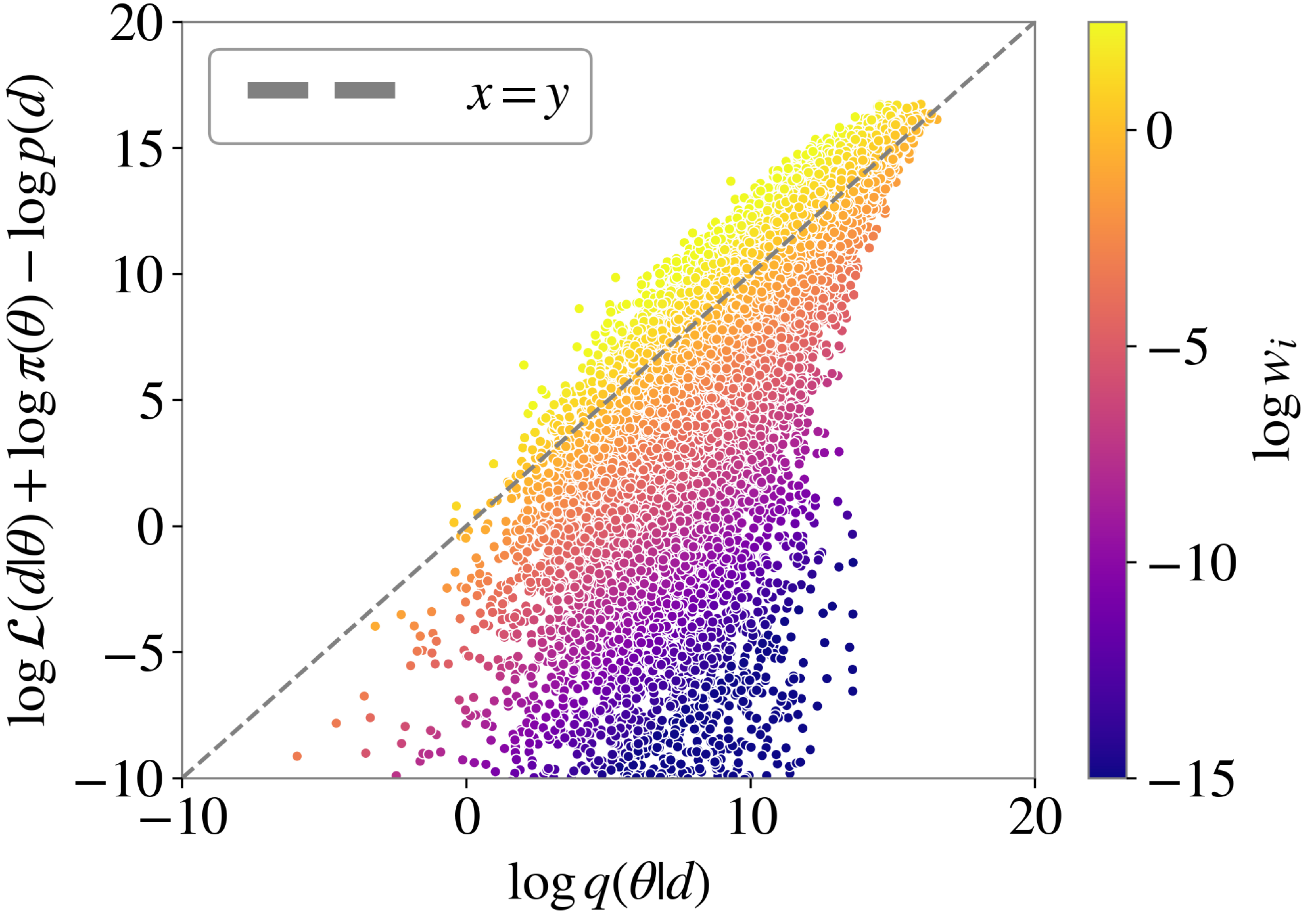
We can correct inaccuracies

$$w_i = \frac{\mathscr{L}(d\,|\,\theta)\pi(\theta)}{q(\theta\,|\,d)}$$

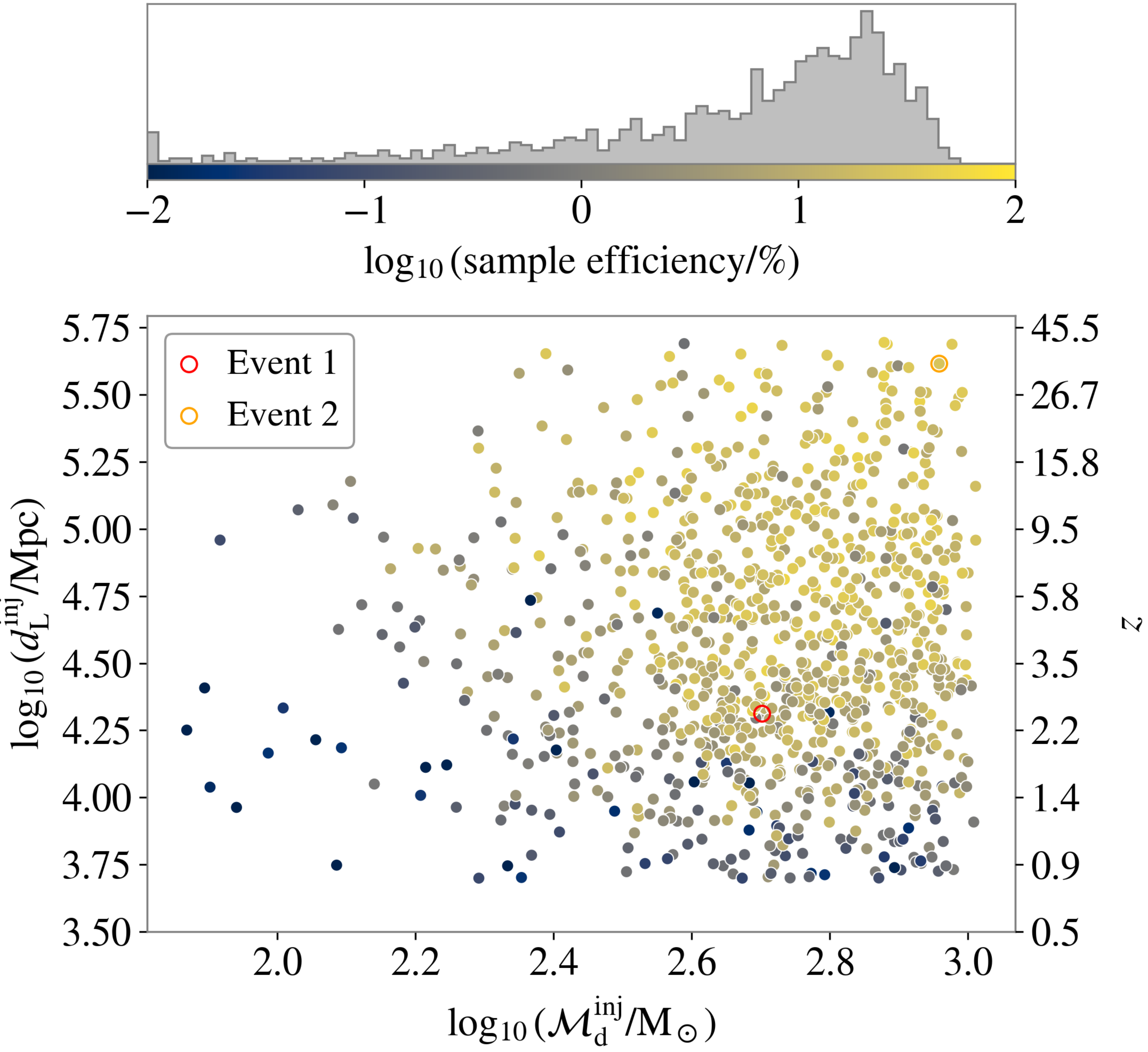
Target (likelihood x prior)

Dingo proposal

sample efficiency $\epsilon = \dfrac{\left(\sum_i w_i\right)^2}{N_s \sum_i w_i^2}$



Wide redshift range accuracy ➡️



85% of sources with sample efficiency > 1%

Sky modes in LISA

β_L, λ_L	injected sky location

Sky mode	*Full*	*Frozen*	*Low-f*	*Frozen low-f*
reflected: $-\beta_L,\ \lambda_L$	t-dep.	degen.	t-dep.	degen.
antipodal: $-\beta_L,\ \lambda_L+\pi$	f-dep.$+\Delta\Phi_R$	f-dep.	$\Delta\Phi_R$	degen.
$\beta_L,\ \lambda_L+\pi/2$	t-f-dep.	f-dep.	t-dep.	degen.
$\beta_L,\ \lambda_L+\pi$	t-f-dep.	f-dep.	t-dep.	degen.
$\beta_L,\ \lambda_L-\pi/2$	t-f-dep.	f-dep.	t-dep.	degen.
$-\beta_L,\ \lambda_L+\pi/2$	t-f-dep.	f-dep.	t-dep.	degen.
$-\beta_L,\ \lambda_L-\pi/2$	t-f-dep.	f-dep.	t-dep.	degen.

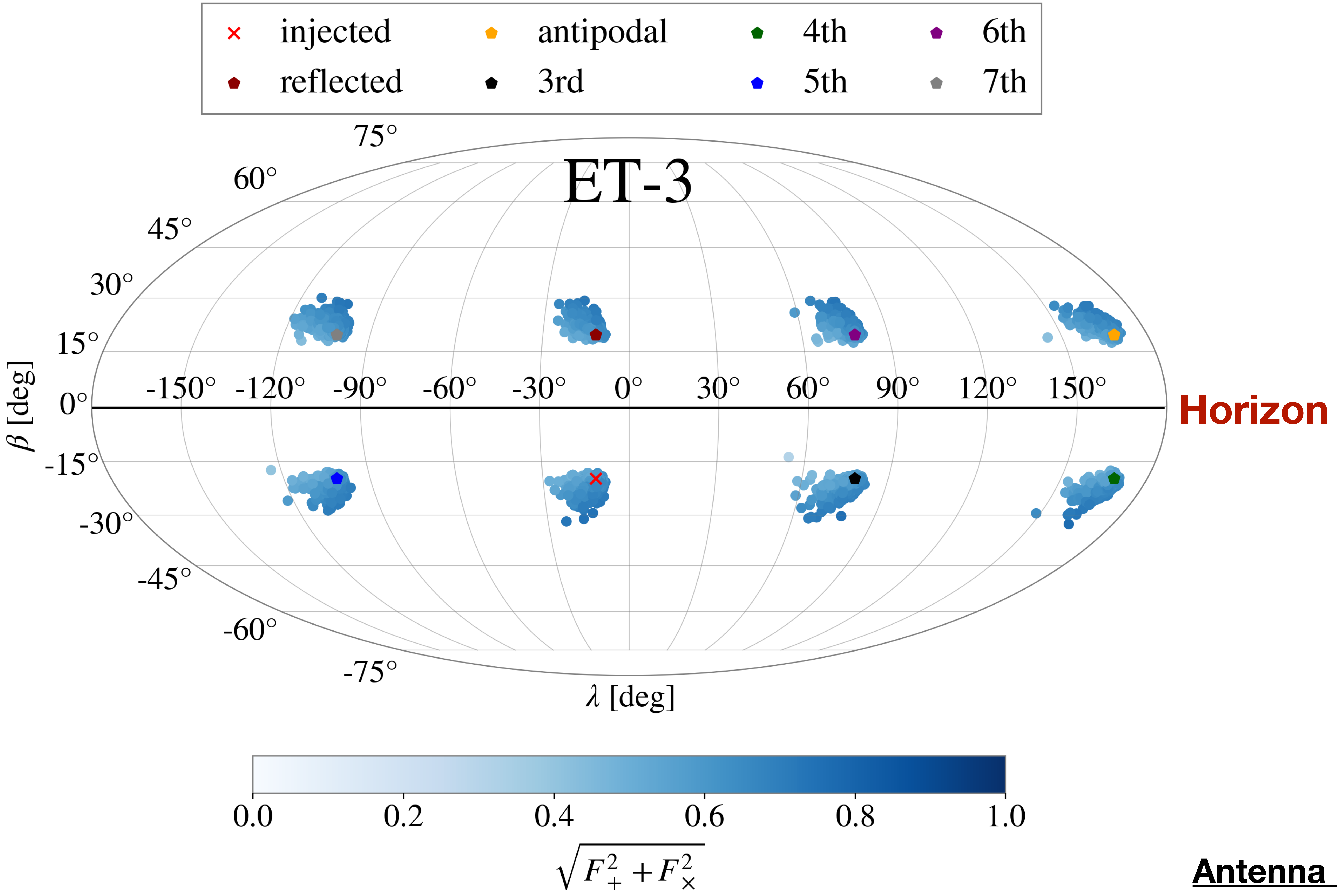
Ref. Vishal et al. 2020, Sylvain et al. 2021, Singh and Bulik 2021, Singh and Bulik 2022

Energy cost

- Hardware: **GPU** NVIDIA A100 80GB, **CPU** AMD EPYC 7513 32-core
- `Dingo-IS`:
  - Training: 970 kWh {1 GPU + 32 CPUs running for 6 days}
  - Inference: 45 kWh {8 CPUs, 1.7 minutes per source on average}
    - Total: **1015 kWh**
- `Bilby`:
  - Inference: **4000 kWh** {4 CPUs, ~5 hours per source on average}

Ref. Wouters et al. 2024, Hu et al. 2024

Sky modes in ET



Antenna power pattern function

Ref. [Santoliquido et al. 2025]