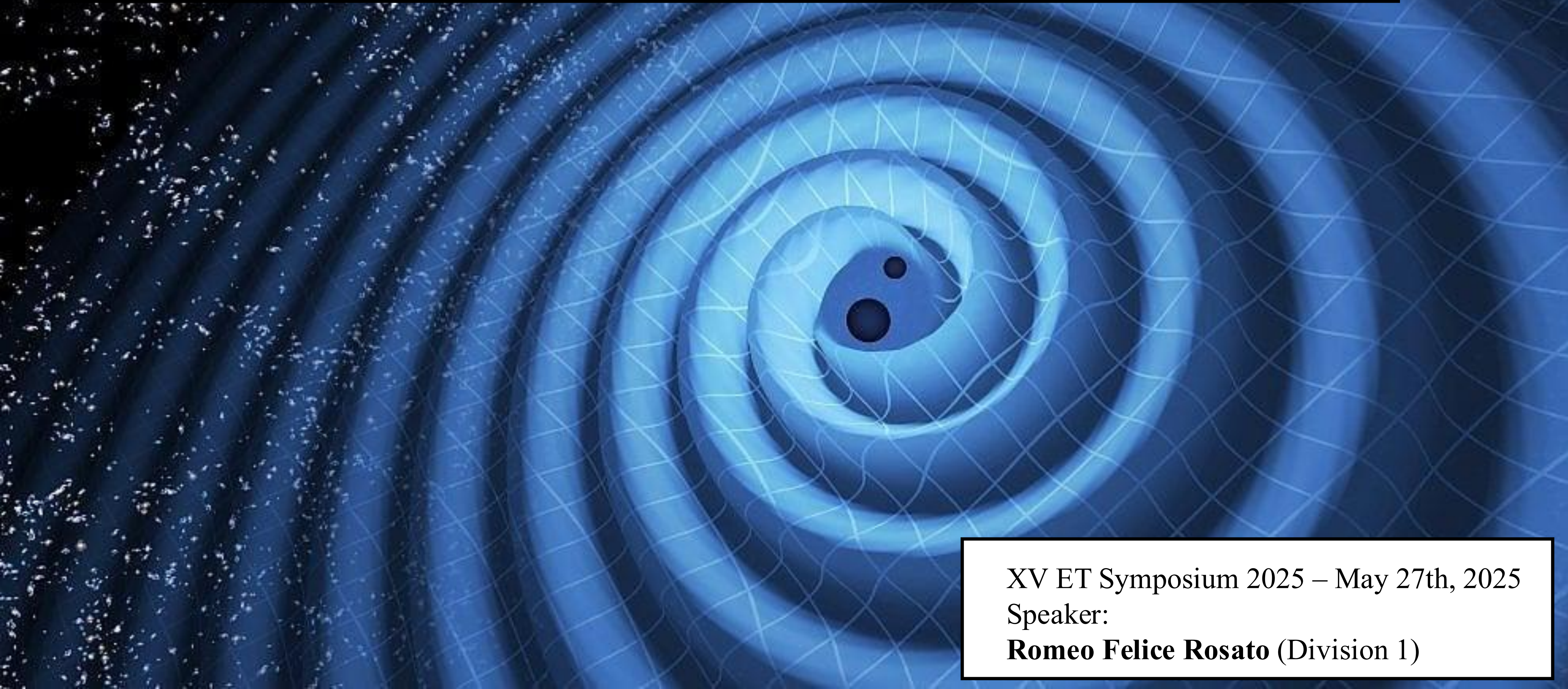
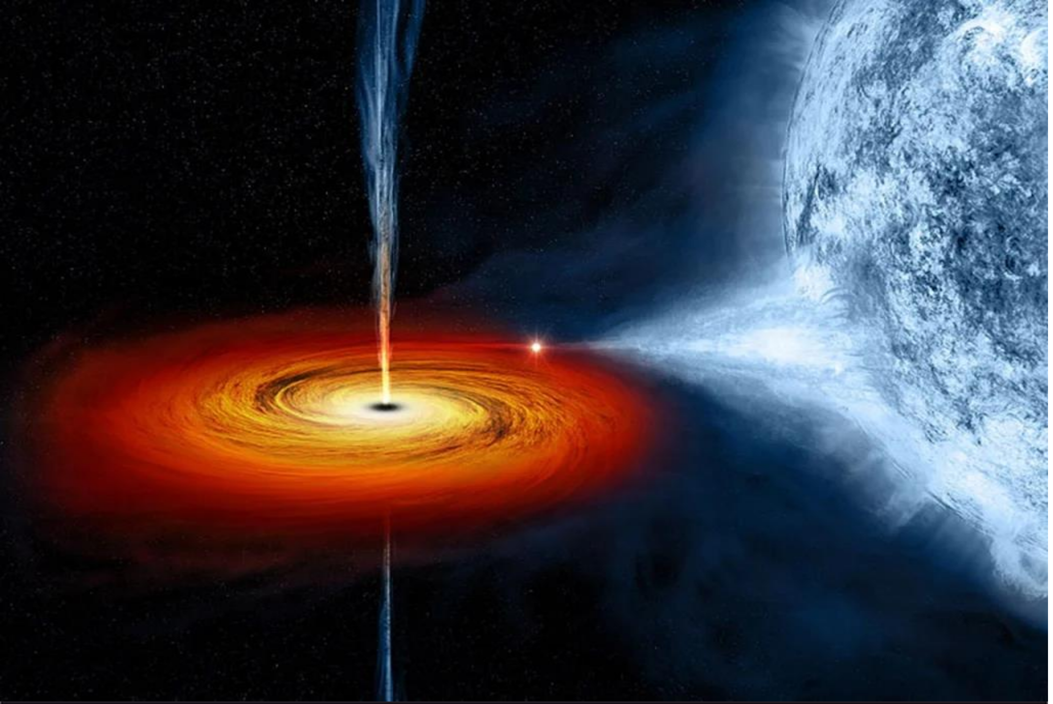
Greybody factors as robust gravitational observables

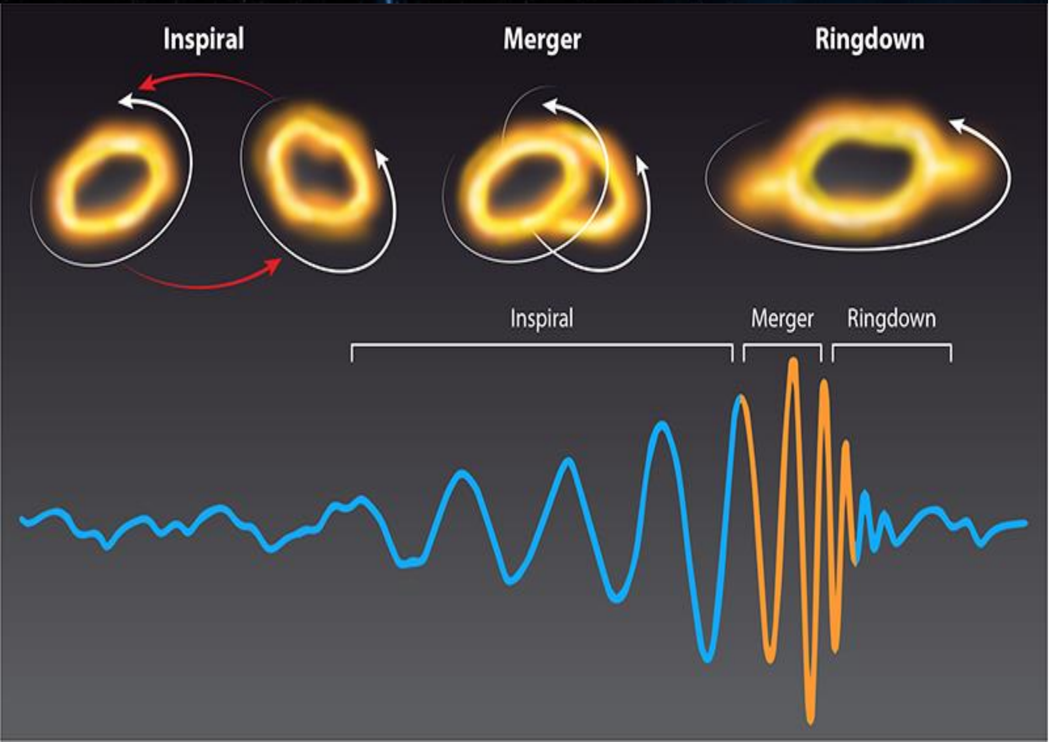
A complementary approach to quasinormal modes



XV ET Symposium 2025 – May 27th, 2025

Speaker:
Romeo Felice Rosato (Division 1)





OVERVIEW

1. **Ringdown signals**: greybody factors vs quasinormal modes for a Schwarzschild geometry.

2. **GR tests**: greybody factors as gravitational observables.

3. **Exotic compact objects (ECOs)**: geybody factors for ECOs.

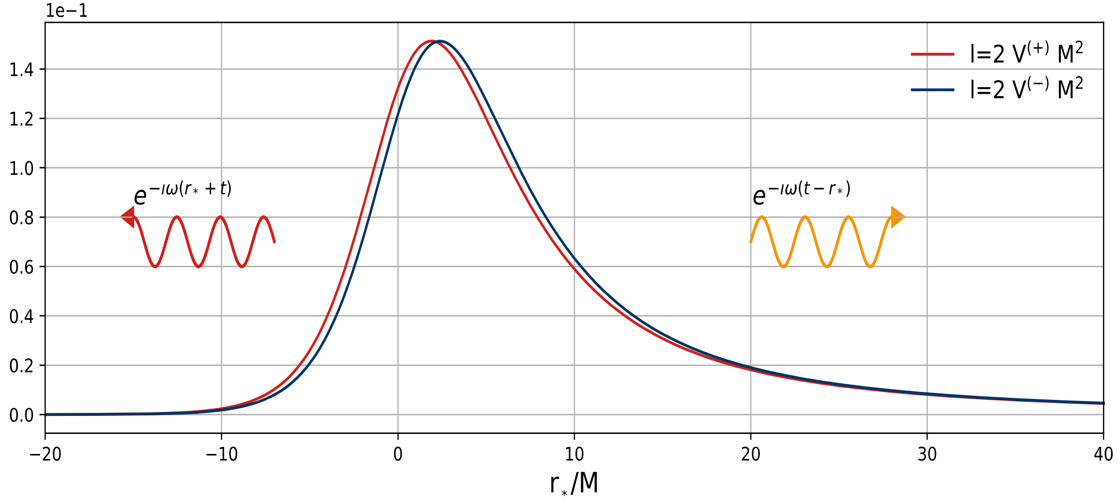
QUASINORMAL MODES vs GREYBODY FACTORS

Perturbations of a Schwarzschild black hole can be described through **Regge-Wheeler** and **Zerilli** equations.

$$\left(\frac{d^2}{dr_*^2} - V_l^{(\pm)}(r) + \omega^2\right) Z_l^{(\pm)}(r_*) = 0$$

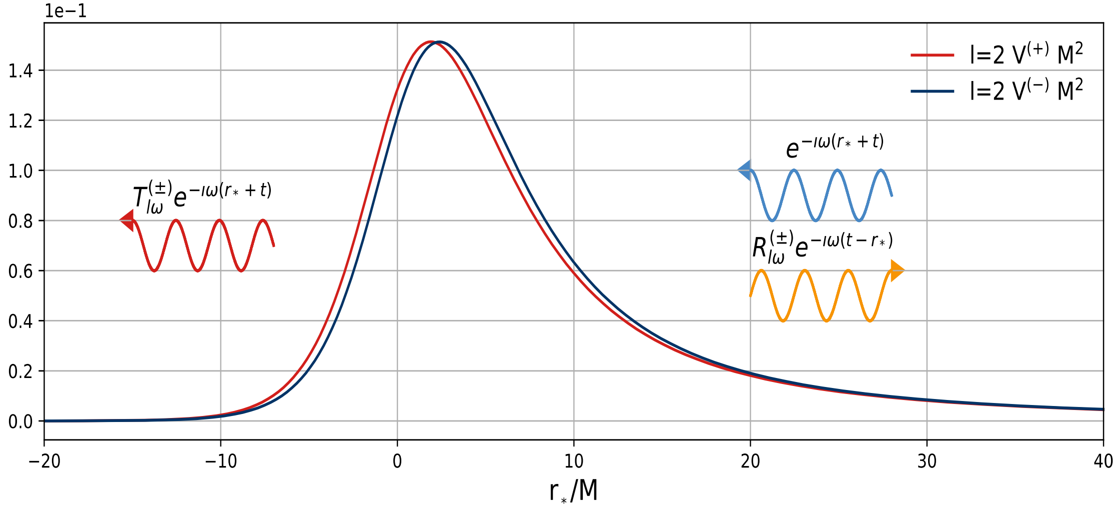
Quasinormal modes (QNMs) are the natural oscillation frequencies of the black hole.

$$Z_l^{(\pm)}(r_*) \begin{cases} e^{\imath\omega_{ql} r_*} & r_* \to \infty \\ e^{-\imath\omega_{ql} r_*} & r_* \to -\infty \end{cases}$$



Greybody factors (GFs) describe how the black hole reacts to a scattering process.

$$Z_l^{(\pm)}(r_*) \begin{cases} A_{l\omega}^{(\pm),out}\, e^{\imath\omega r_*} + A_{l\omega}^{(\pm),in}\, e^{-\imath\omega r_*} & r_* \to \infty \\ e^{-\imath\omega r_*} & r_* \to -\infty \end{cases}$$



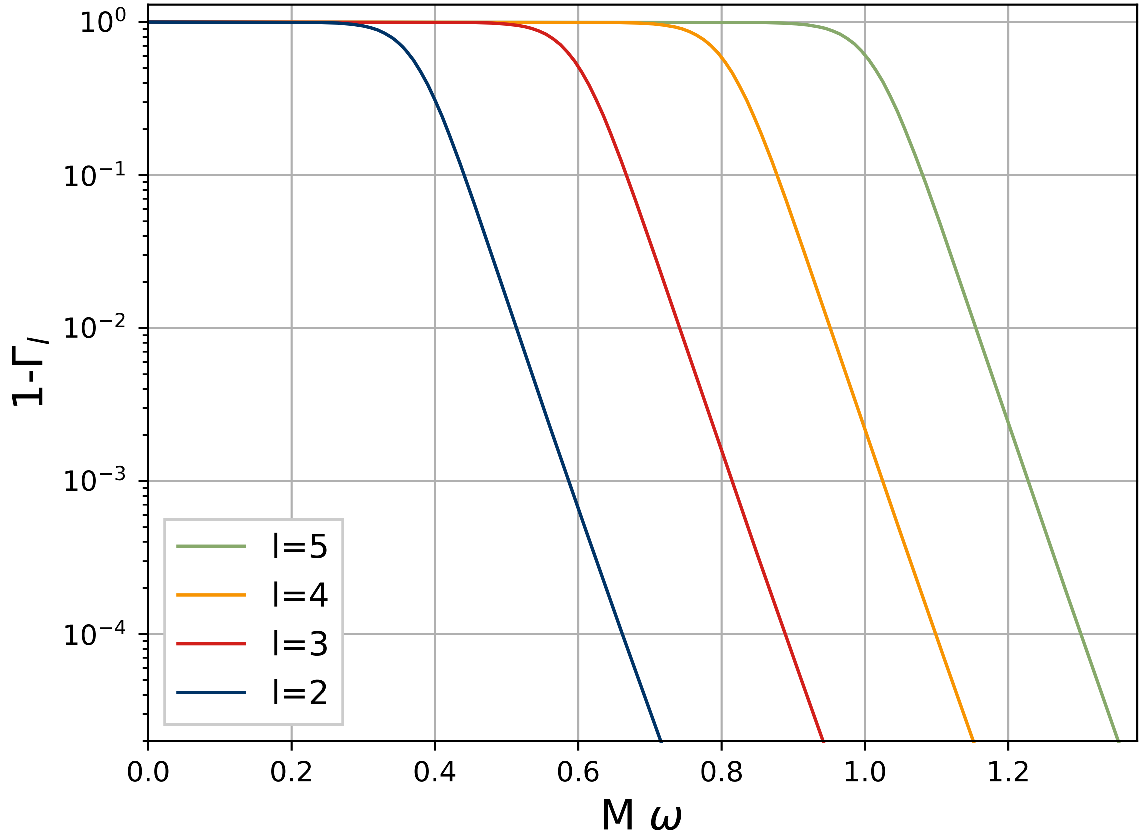
GREYBODY FACTORS

From this, one can define black hole reflectivity and black hole transmission amplitude (i.e. the **greybody factor** $\Gamma_l(\omega)$).

GFS AND REFLECTIVITY

$$\Gamma_l(\omega) = \left|\mathrm{T}_{\mathrm{l}\omega}^{(\pm)}\right|^2 = \left|\frac{1}{A_{l\omega}^{(\pm),in}}\right|^2$$

$$\mathcal{R}_l(\omega) = \left|\mathrm{R}_{\mathrm{l}\omega}^{(\pm)}\right|^2 = \left|\frac{A_{l\omega}^{(\pm),out}}{A_{l\omega}^{(\pm),in}}\right|^2 = 1 - \Gamma_l(\omega)$$



QUASINORMAL MODES vs GREYBODY FACTORS

Consider the infall of a mass μ (with initial angular momentum L and initial energy E) inside the black hole as a model for **ringdown**.

EMITTED GRAVITATIONAL WAVES

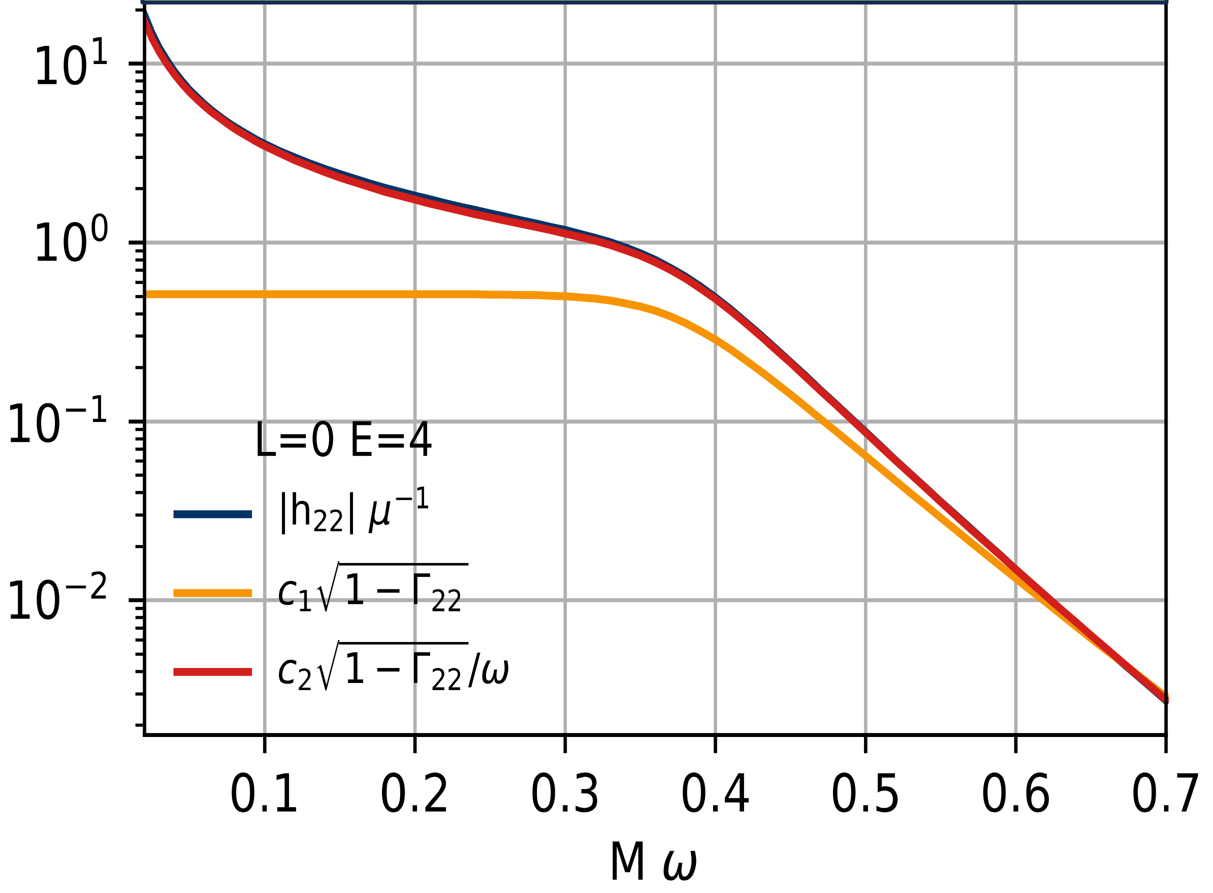
$$h_+ + \imath h_\times = \frac{1}{r}\sum_{lm}\int d\omega e^{\imath\omega(r_*-t)} H_{lm}(\omega)$$

$$H_{lm}(\omega) = \frac{{}_{-2}Y_{lm}(\theta,\phi)}{16\omega A^{in}_{lm}(\omega)}\int dr_*\, X^{hom}_{lm} S_{lm\omega}$$

TIME DOMAIN

$$h_+ + \imath h_\times = \sum_q \sum_{lm} A_{lm} e^{-\imath\omega_{q,R}t} e^{-|\omega_{q,I}|t}$$

FREQUENCY DOMAIN



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Signal decomposed as a superposition of quasinormal (damped) oscillations.

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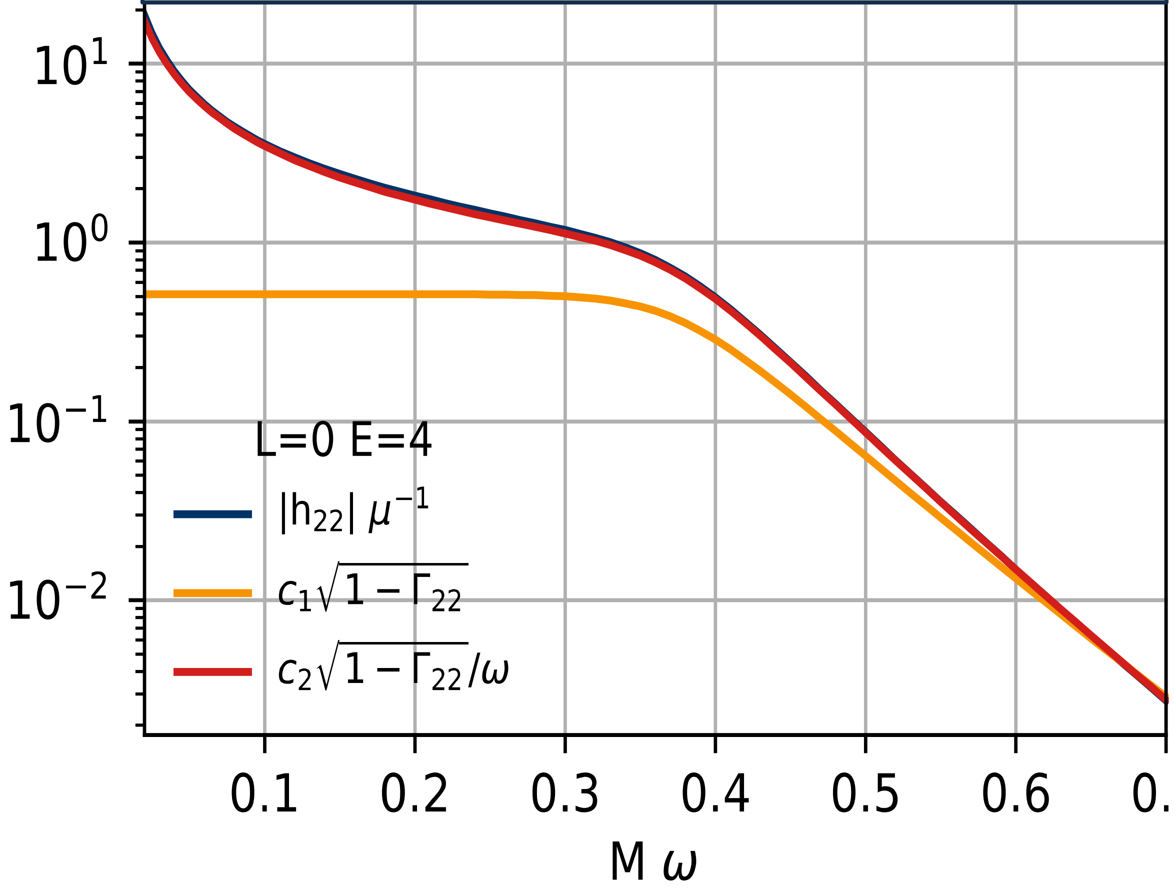

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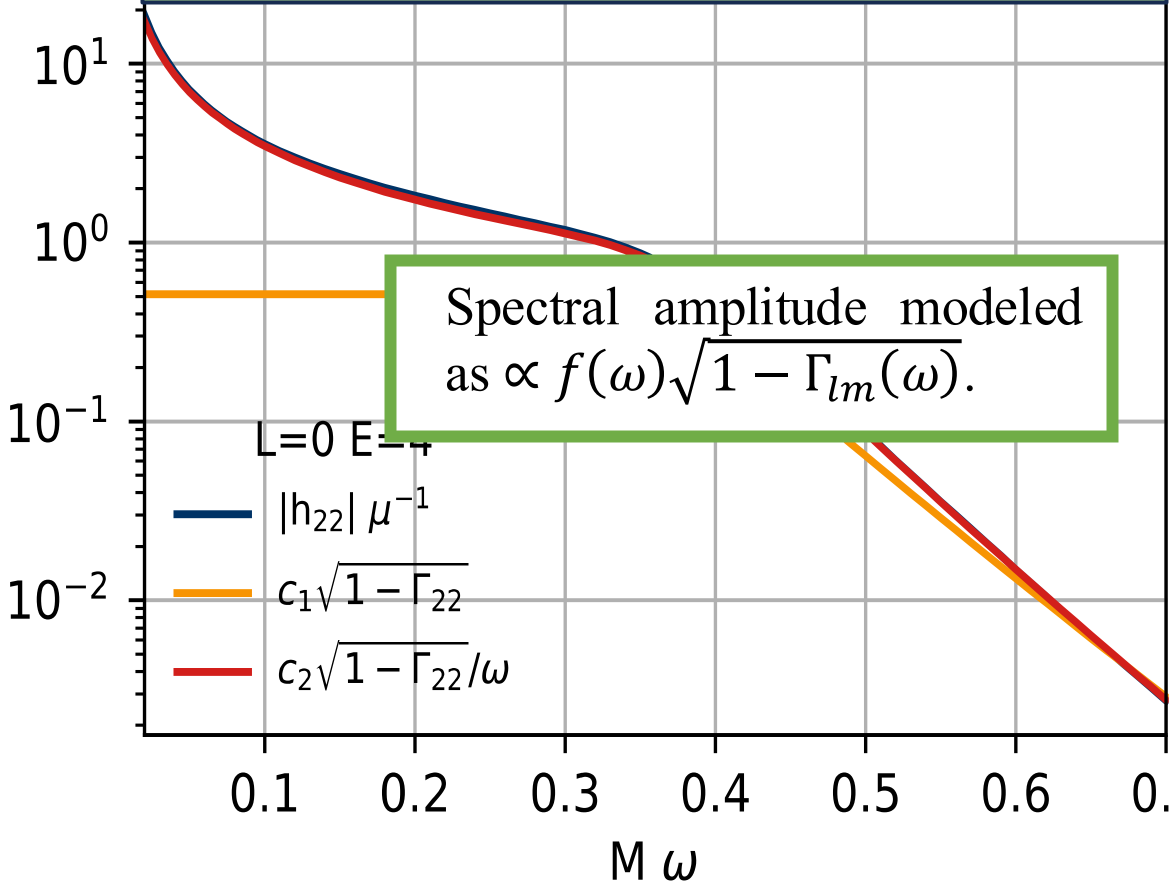

Spectral amplitude modeled as $\propto f(\omega)\sqrt{1-\Gamma_{lm}(\omega)}$.

GREYBODY FACTORS as observable

The imprint of greybody factors in the ringdown signal, visible in the frequency domain, motivates the idea of proposing an alternative test of General Relativity based on them.

GREYBODY FACTORS
GFs can provide an alternative approach to QNMs.
GFs are stable under small perturbations of the system, while QNMs are not. See [https://arxiv.org/pdf/2406.01692] (**R. F. Rosato**, K.Destounis and P. Pani).
GFs are no-hair quantities, (for a Kerr black hole they depend solely on M and χ) . This property makes them valuable tools to probe the properties of compact remnants.
Greybody factors may provide the foundation for alternative tests of General Relativity.

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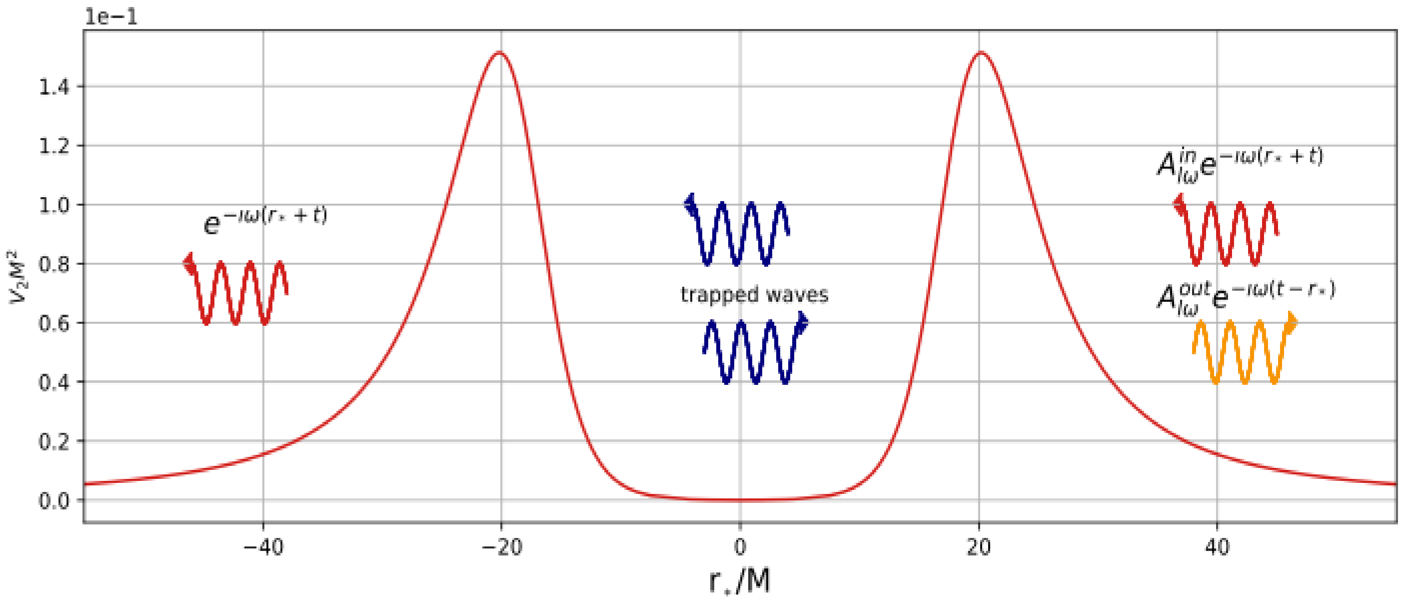
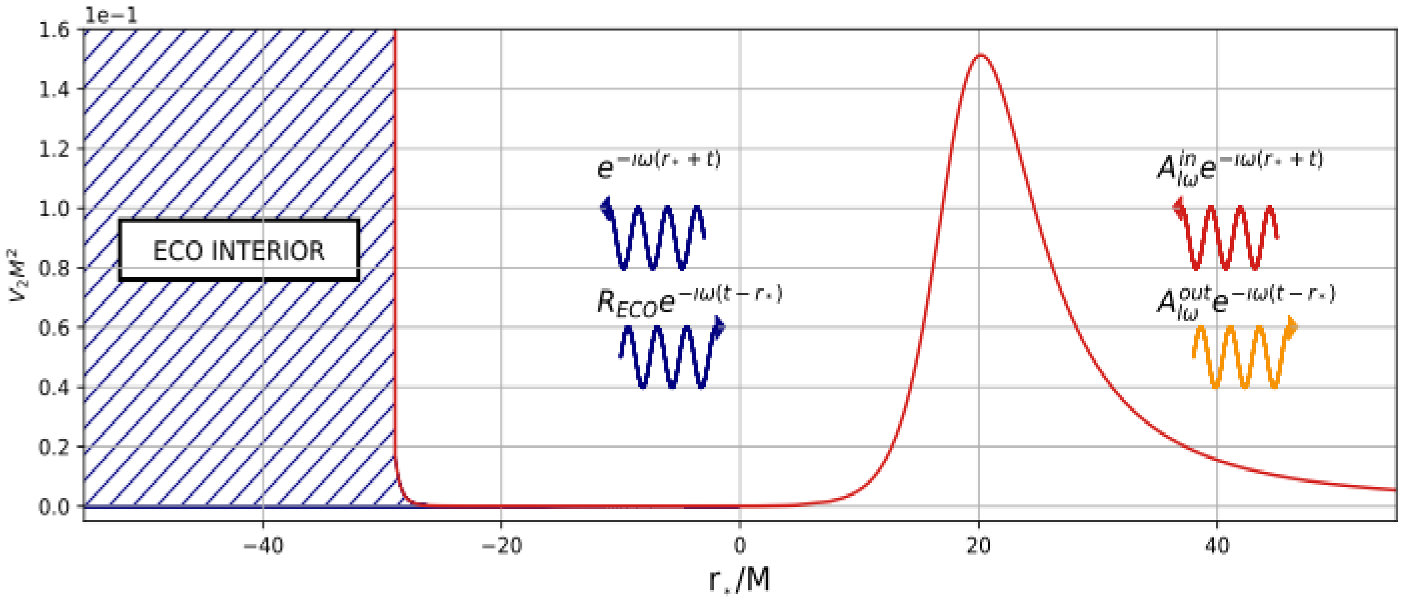
EXOTIC HORIZONLESS ULTRACOMPACT OBJECT

Exotic Horizonless Ultracompact Objects (ECOS) are hypothetical astrophysical objects that are as nealry compact as black holes -meaning they can have a radius very close to their Schwarzschild radius- but **do not possess an event horizon.**

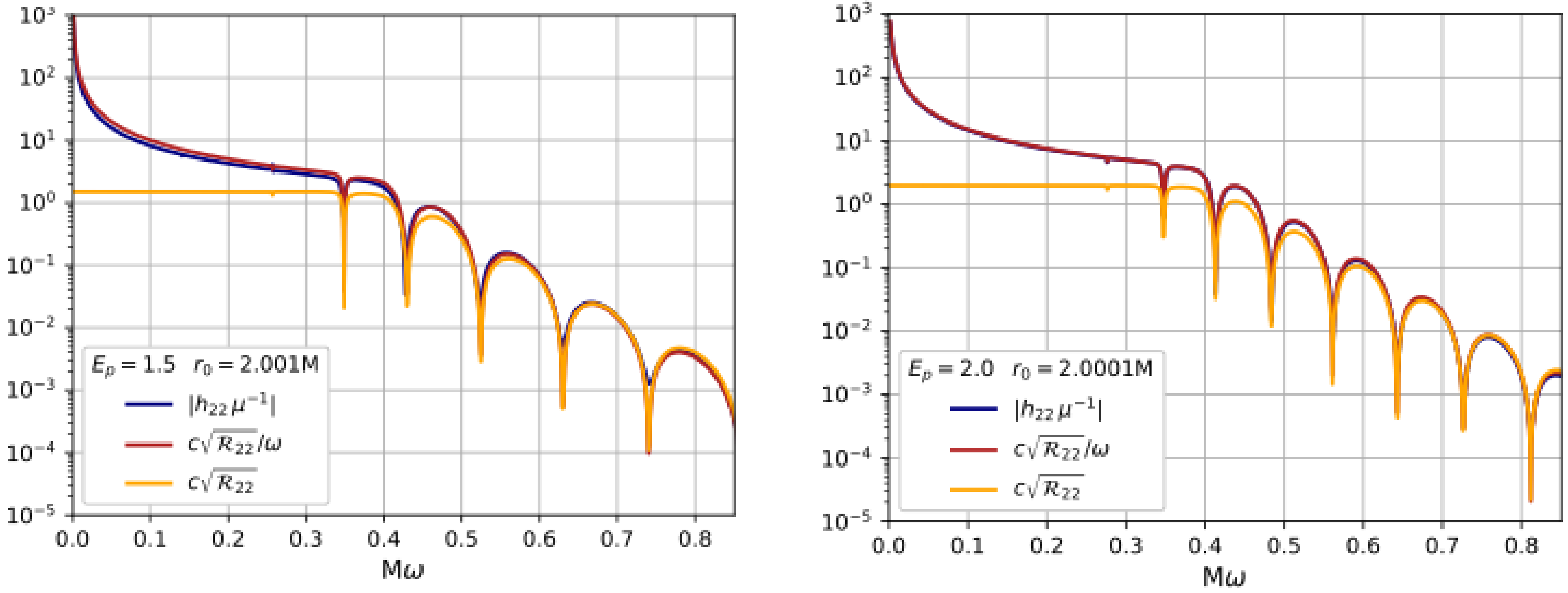


ECO Model	Surface reflection amplitude
Wormhole	$R_{\rm ECO}(\omega) = R'_{\rm BH}(\omega)e^{-2i\omega r^0_*}$
Constant $R_{\rm ECO}$	$R_{\rm ECO}(\omega) = {\rm const}$
Boltzmann $R_{\rm ECO}$	$R_{\rm ECO}(\omega) = e^{-\lvert\omega\rvert/T_H}$



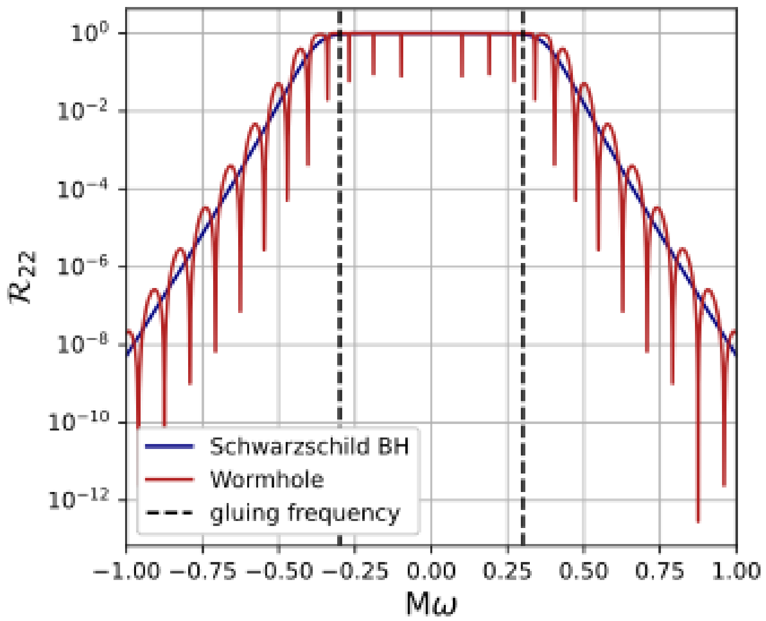
GREYBODY FACTORS AND GRAVITATIONAL ECO SIGNAL

Greybody factors model the signal spectral amplitude also for ECOs. In figure, the wormhole case is showed.

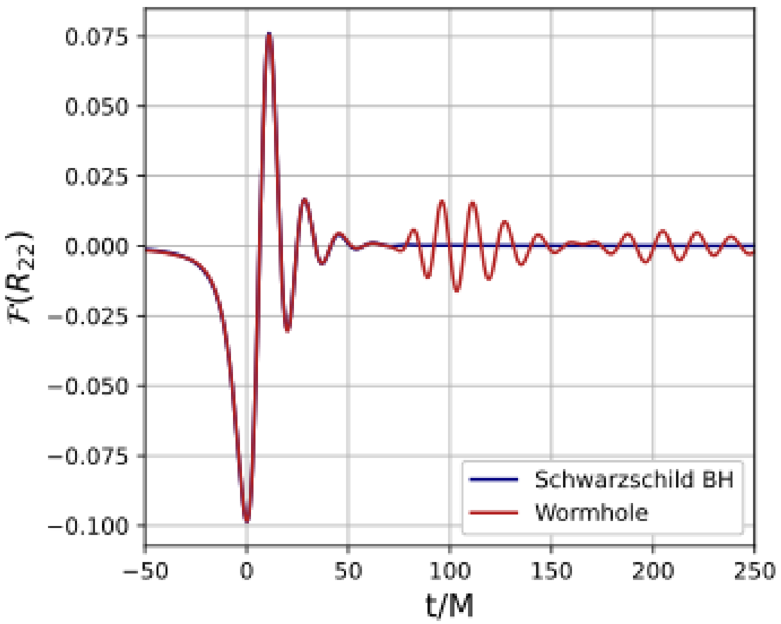


ECHOES IN THE GRAVITATIONAL WAVE SIGNAL OF WORMHOLES

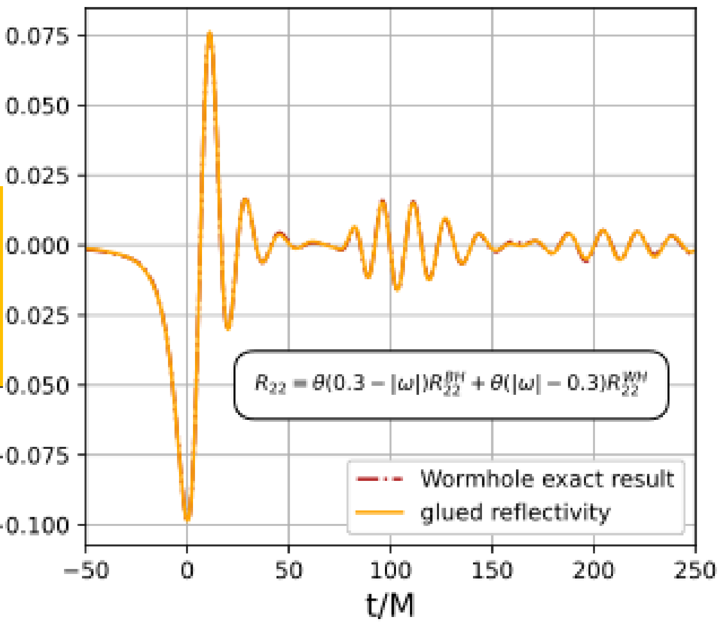
REFLECTIVITY



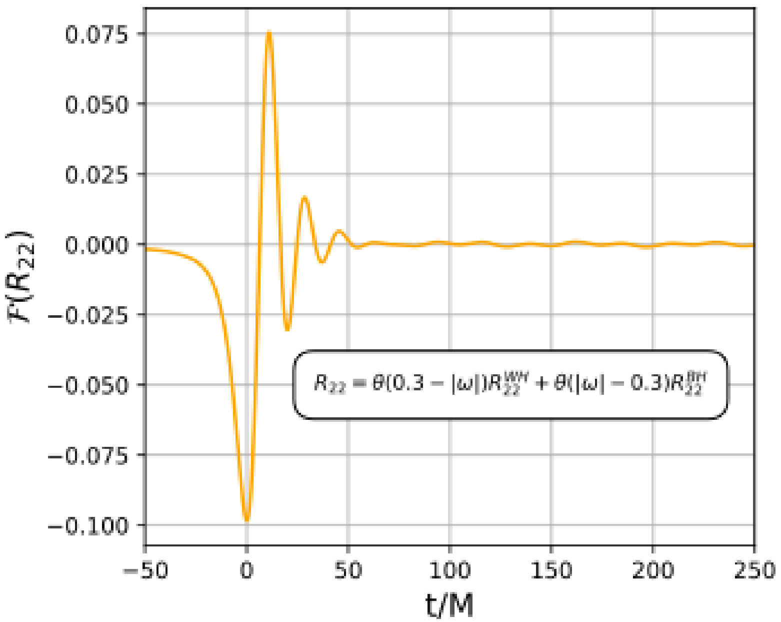
FULL SIGNAL



SINGAL CONSIDERING ONLY HIGH FREQUENCY OSCILLATIONS.



SINGAL CONSIDERING ONLY LOW FREQUENCY RESONANCES (QNMS).



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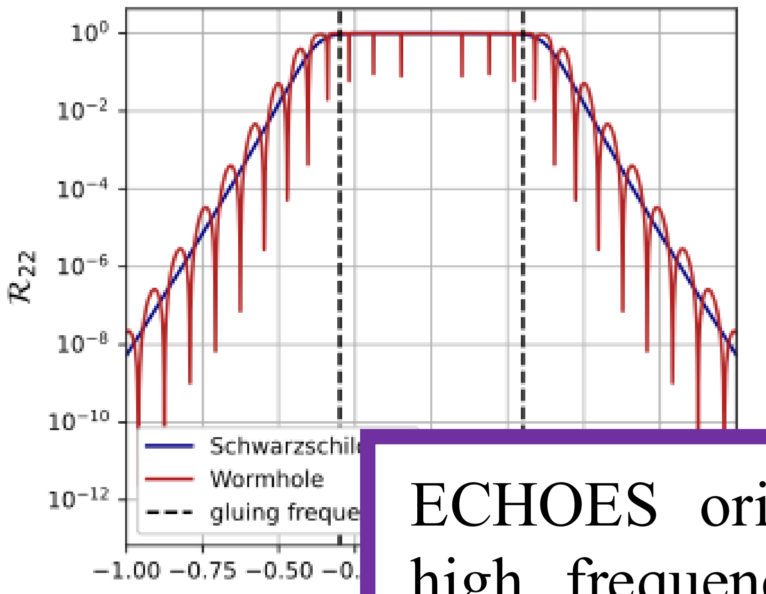

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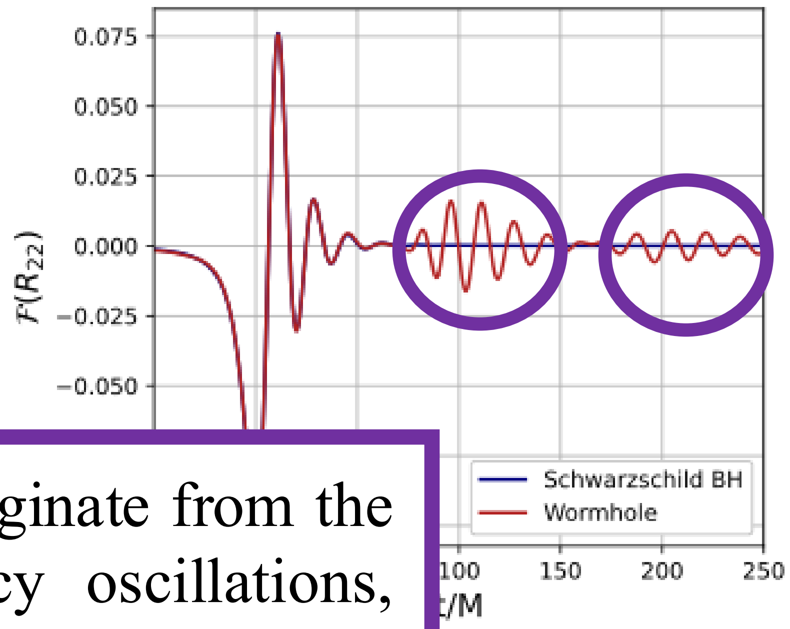

ECHOES originate from the high frequency oscillations, not from QNMs!

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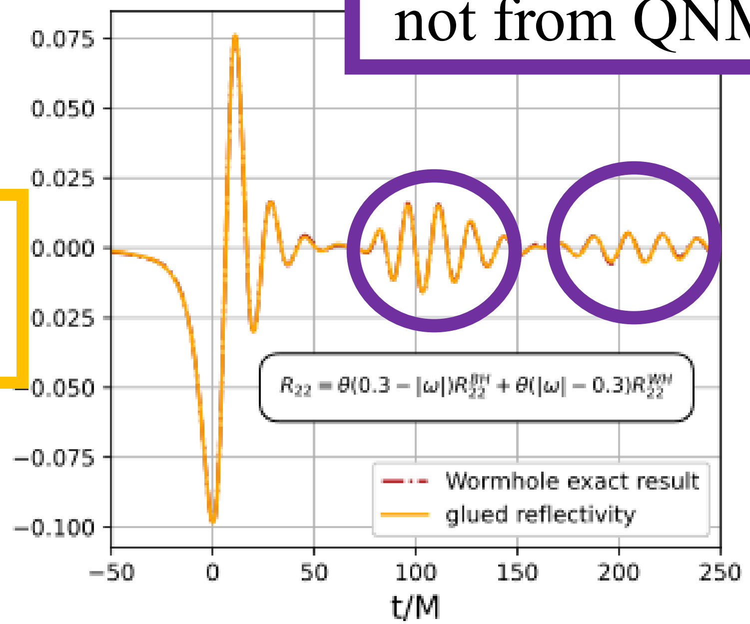

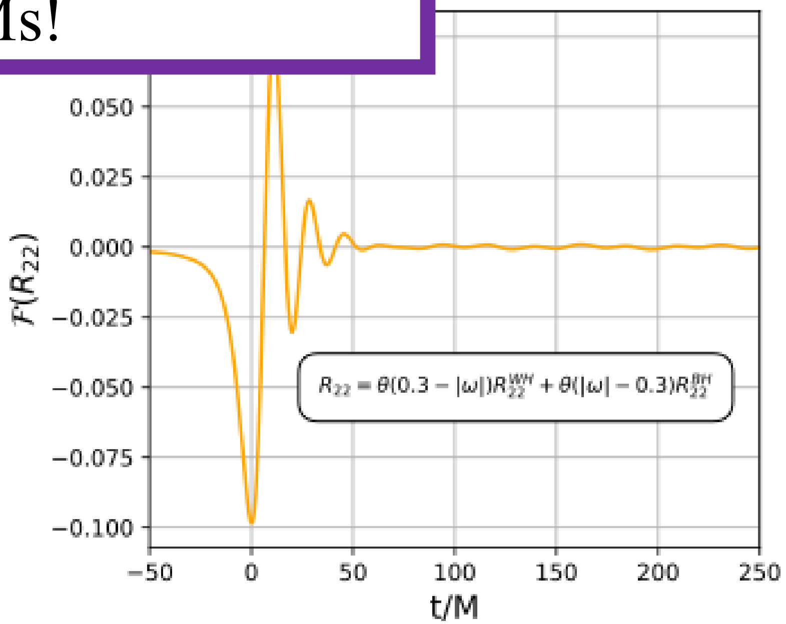

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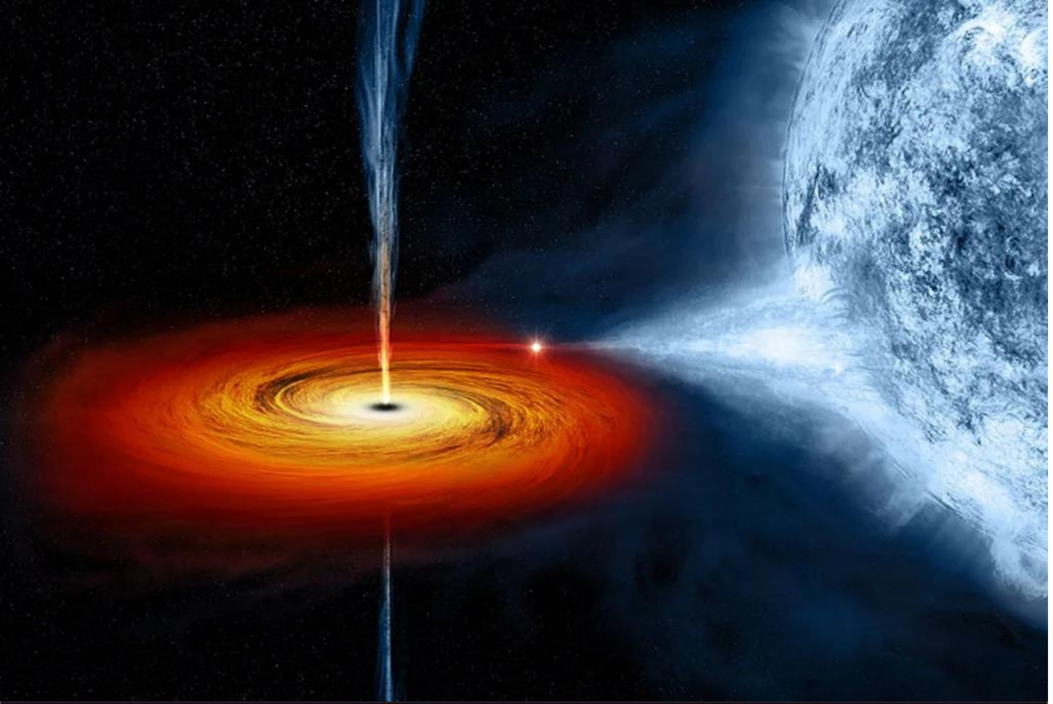
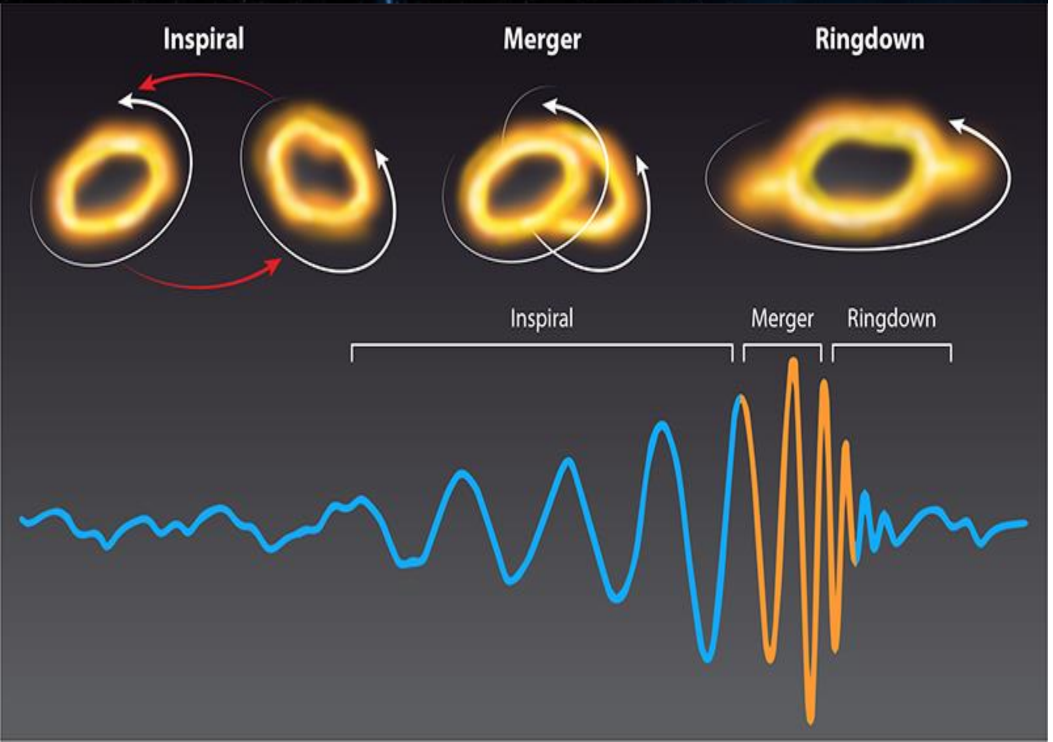
MAIN TAKE-AWAYS





1. **greybody factors** have been characterized as gravitational wave observables, furnishing a complementary approach to quasinormal modes.

2. **greybody factors** play an important role in modeling the physics of hypothetical exotic ultracompact objects.

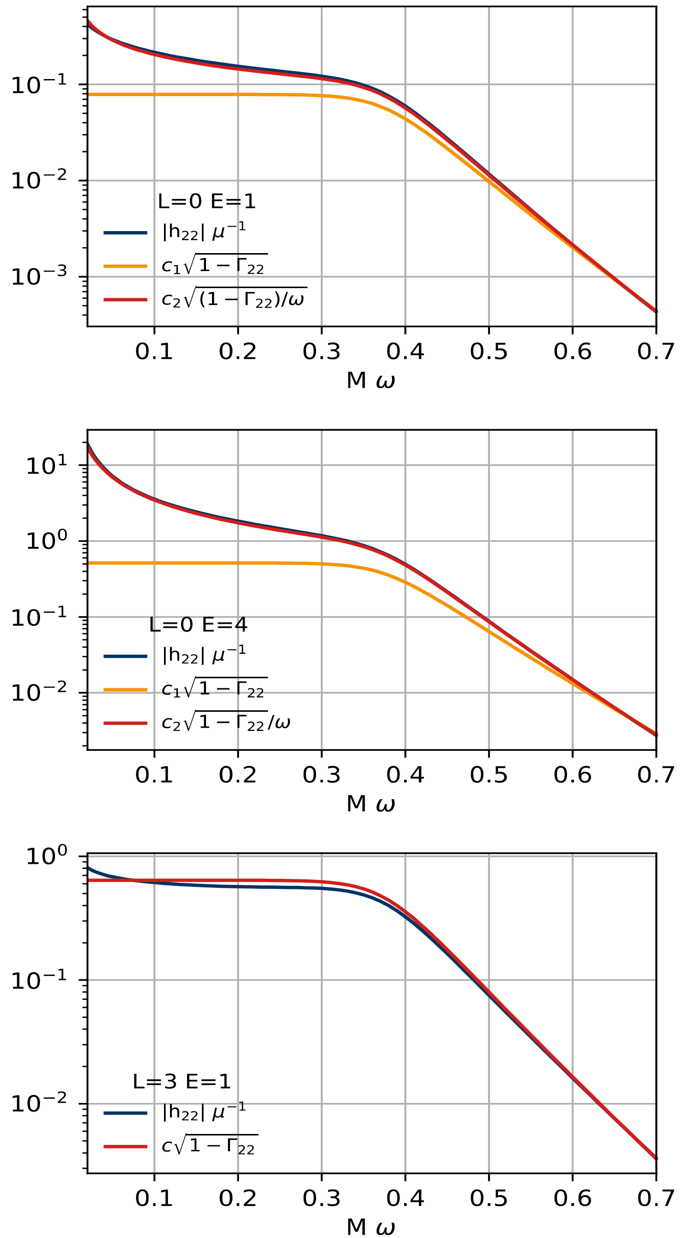
Thank you for listening ☺

You can find everything here:

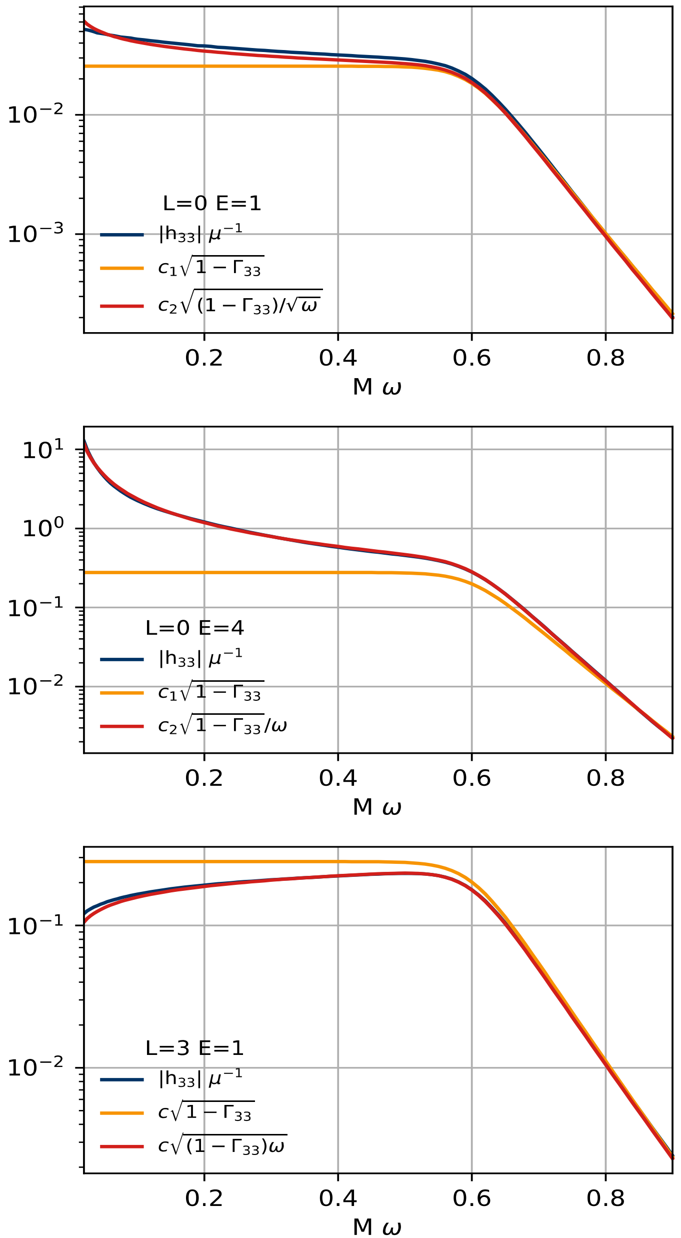
- Ringdown stability: greybody factors as stable gravitational-wave observables. **R.F. Rosato,** K. Destounis, P. Pani. https://arxiv.org/pdf/2406.01692 .
- Greybody factors, reflectionless scattering modes, and echoes of ultracompact horizonless objects. **R. F. Rosato**, Shauvik Biswas, Sumanta Chakraborty, and Paolo Pani. https://arxiv.org/pdf/2501.16433 .

BACKUP: plunging particle model

$|h_{22}(\omega)|$



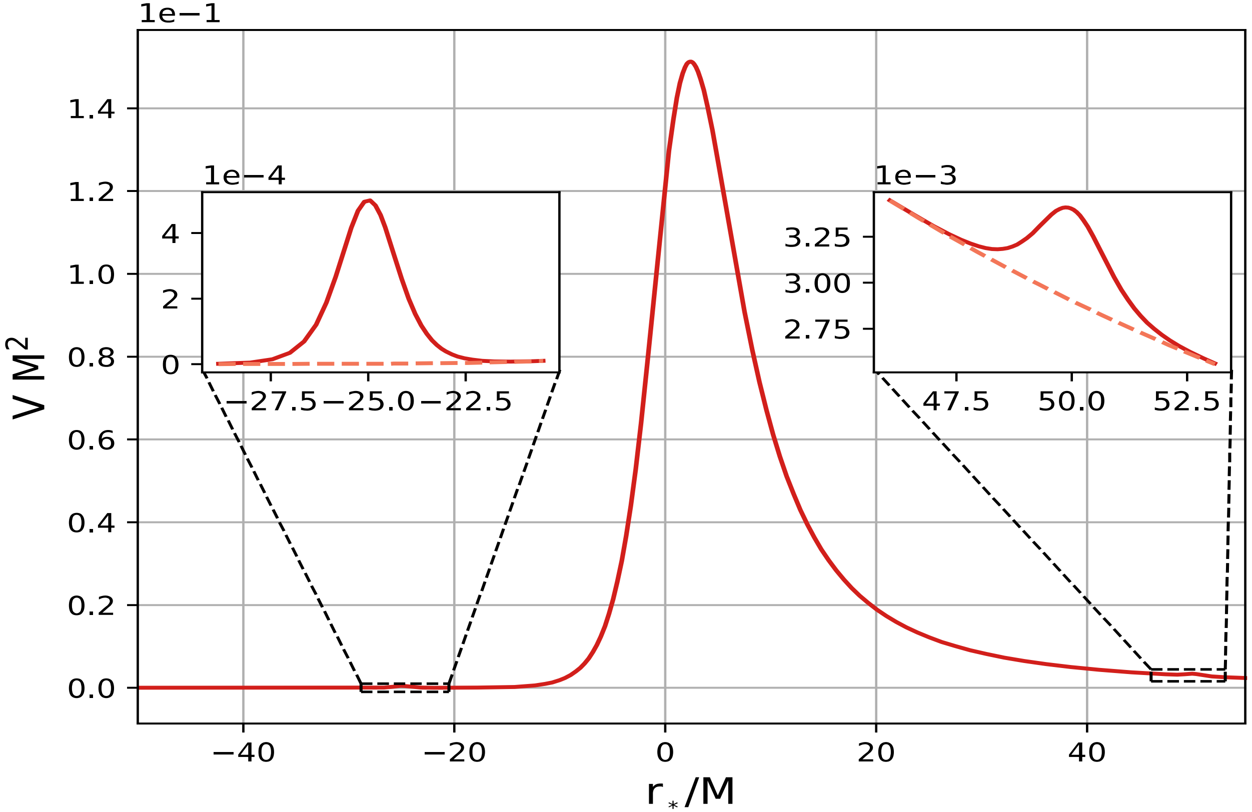
$|h_{33}(\omega)|$



BACKUP: environmental effects

TOY MODEL
$$V_l^{new}(r) = V_l(r) + \frac{\epsilon}{M^2}\,\mathrm{sech}^2\left(\frac{r_* - c}{M}\right)$$

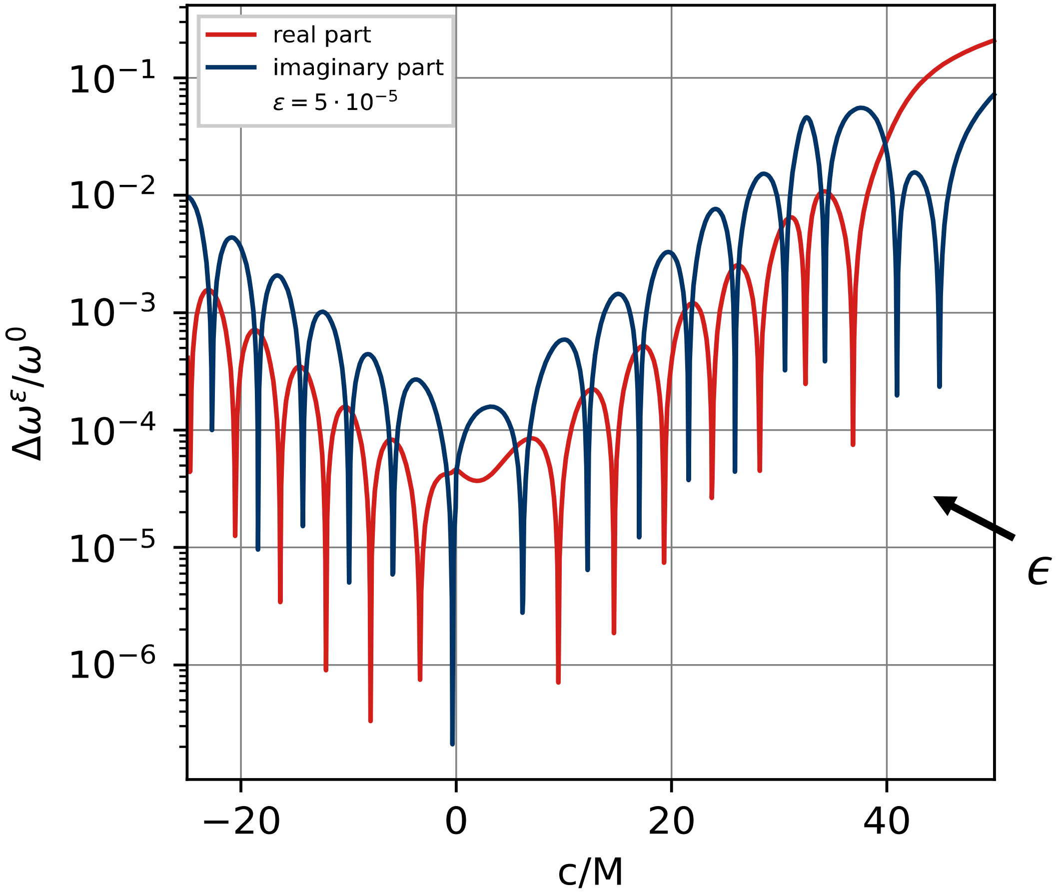
Black holes are immersed in an **astrophysical environment**. Such environment can affect the study of perturbations.

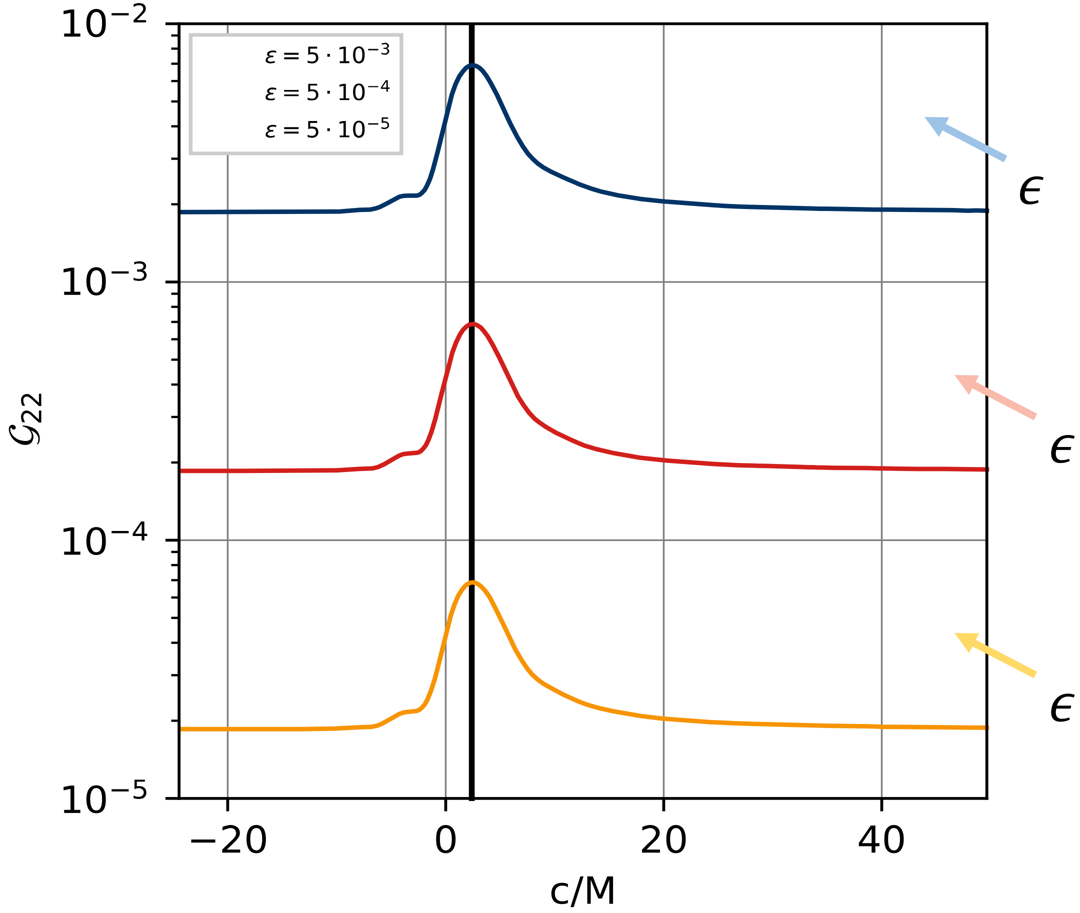


BACKUP: stability

GFS VARIATION

$$\mathcal{G}_{lm} = \frac{\int d\omega \, |\Gamma^{\epsilon}_{lm}(\omega, c) - \Gamma_{lm}(\omega)|}{\int d\omega \, |\Gamma_{lm}(\omega)|}$$





BACKUP: time domain decomposition

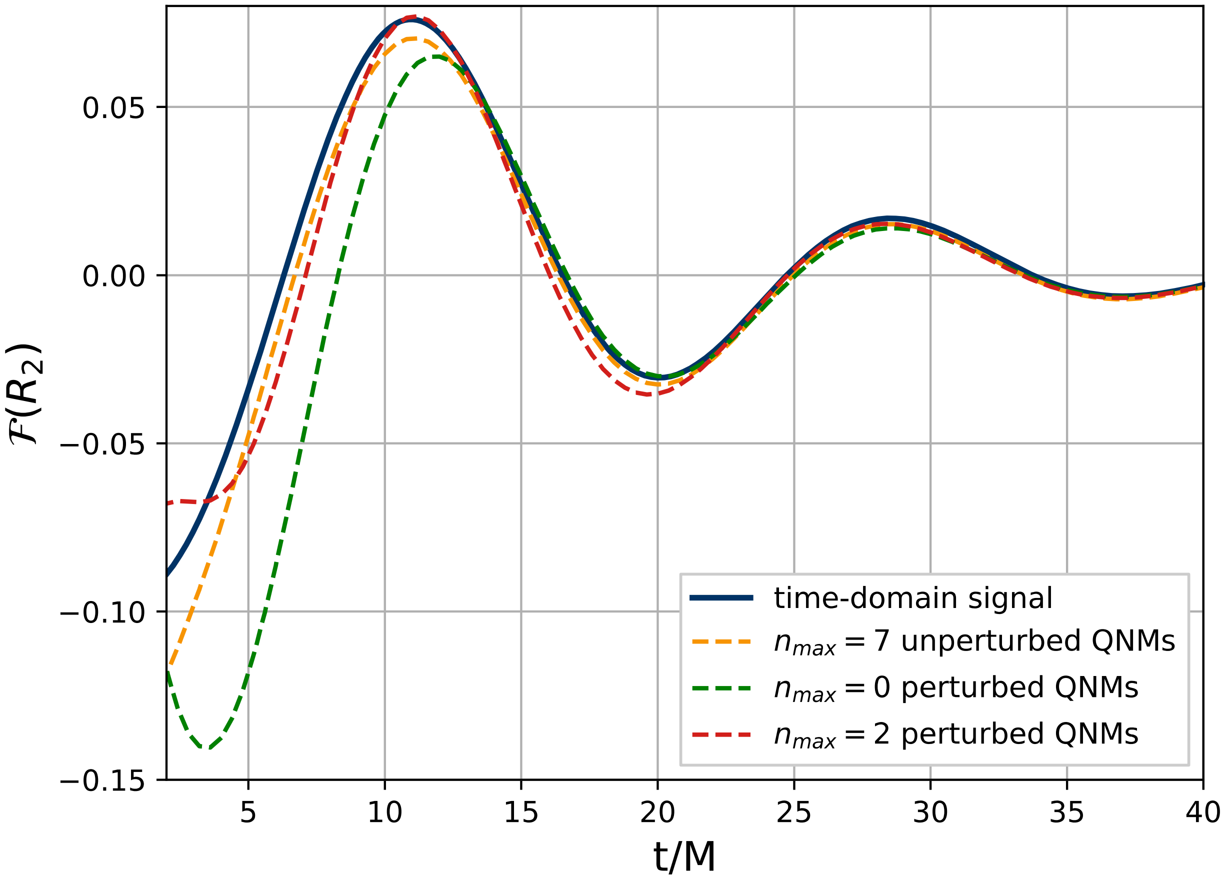
$$\mathcal{F}(R_{lm}) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{A^{out}_{lm\omega}}{A^{in}_{lm\omega}} e^{-\iota\omega t}$$

Poles for quasinormal frequencies ω_{nlm}, where:

$$A^{in}_{lm\omega_n} \equiv \gamma_{nlm}(\omega - \omega_{nlm}).$$

FOURIER-TRANSFORM DECOMPOSITION

$$\mathcal{F}(R_{22}) \simeq -2\, Re\left[\iota \sum_n \frac{A^{out}_{lm\omega_n}}{\gamma_{nlm}} e^{-\iota\omega_{nlm}t}\right]$$



$c = 15M$

$\epsilon = 10^{-3}$