

Probing the hadron-quark phase transition in twin stars with f-modes



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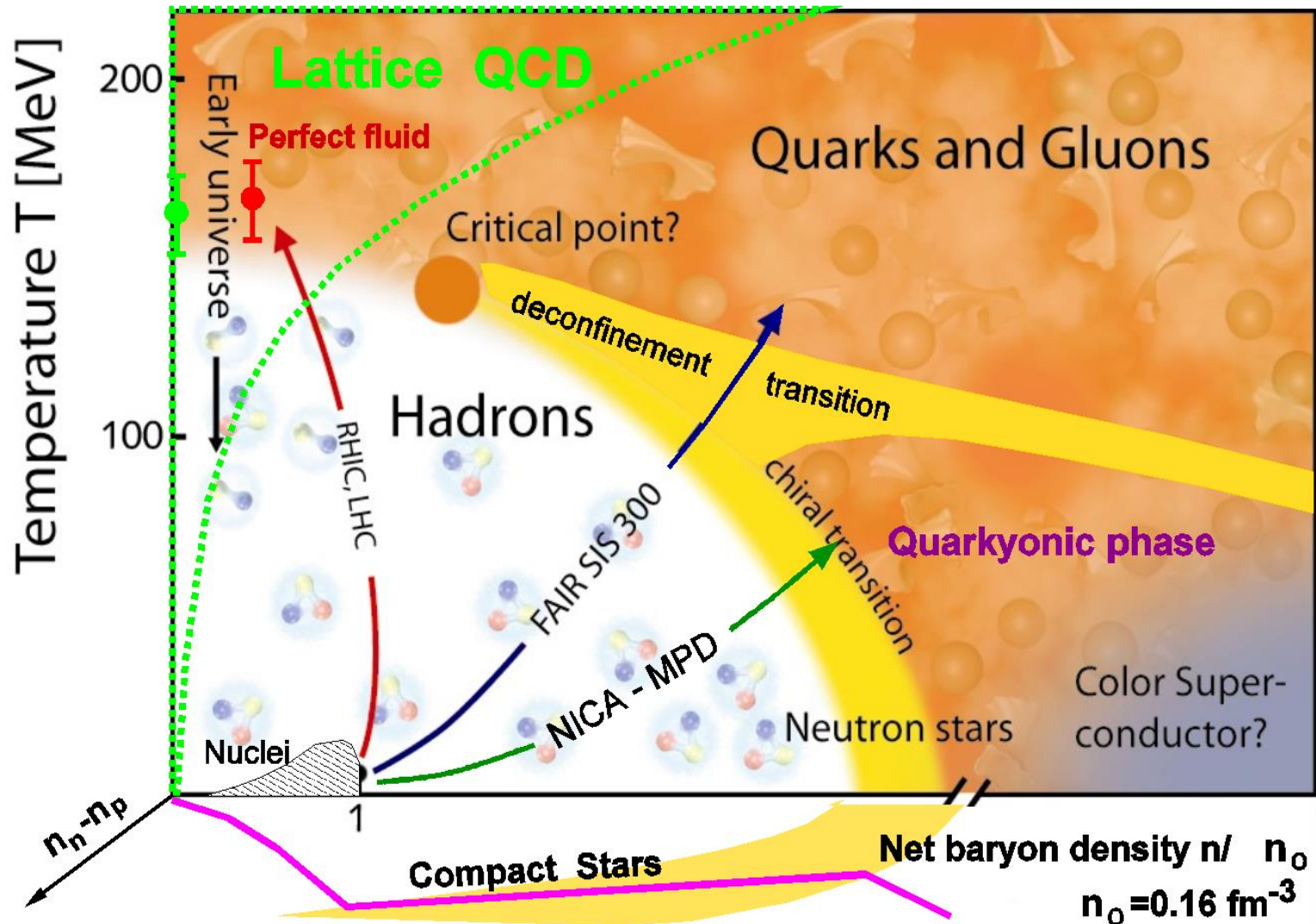
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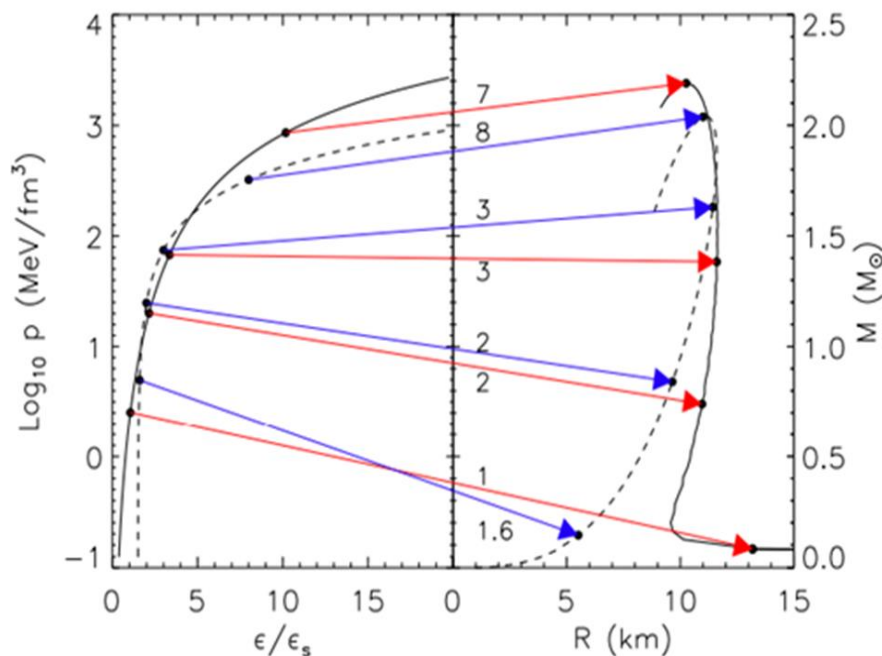
Outline

- A brief introduction to the physics of compact stars and their location in the QCD phase diagram.
- The compact star mass twins scenario.
- f-mode future detection with ET and implications for the EoS.
- Outlook and perspectives.

Critical Endpoint in QCD



Compact Star Sequences (M-R $\Leftrightarrow$ EoS)



James Lattimer,
Annu. Rev. Nucl. Part. Sci. 62,
485 (2012), arXiv:1305.3510

- TOV Equations

- Equation of State (EoS)

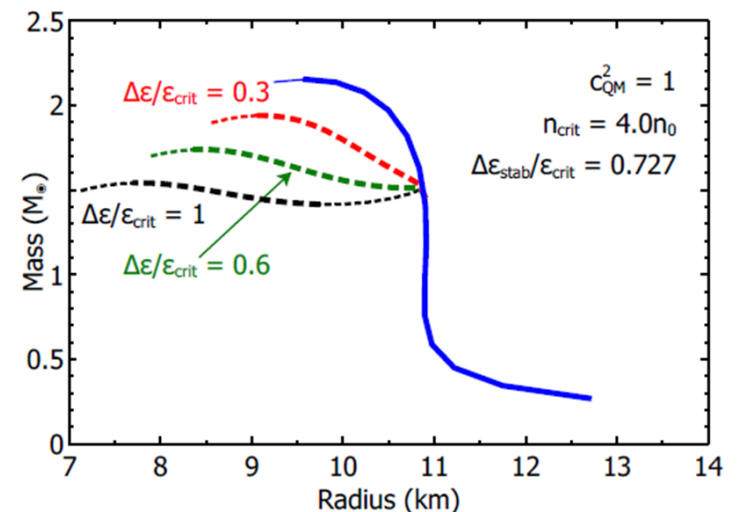
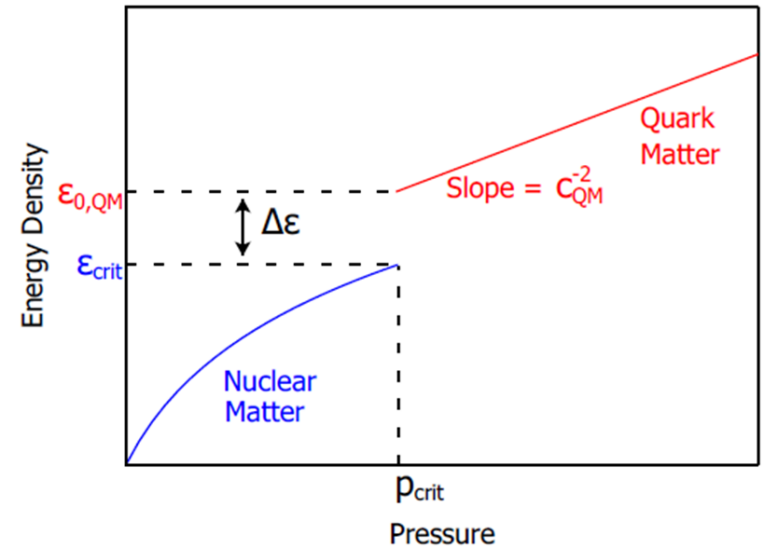
$$\frac{dp}{dr} = - \frac{(\varepsilon + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon \quad p(\varepsilon)$$



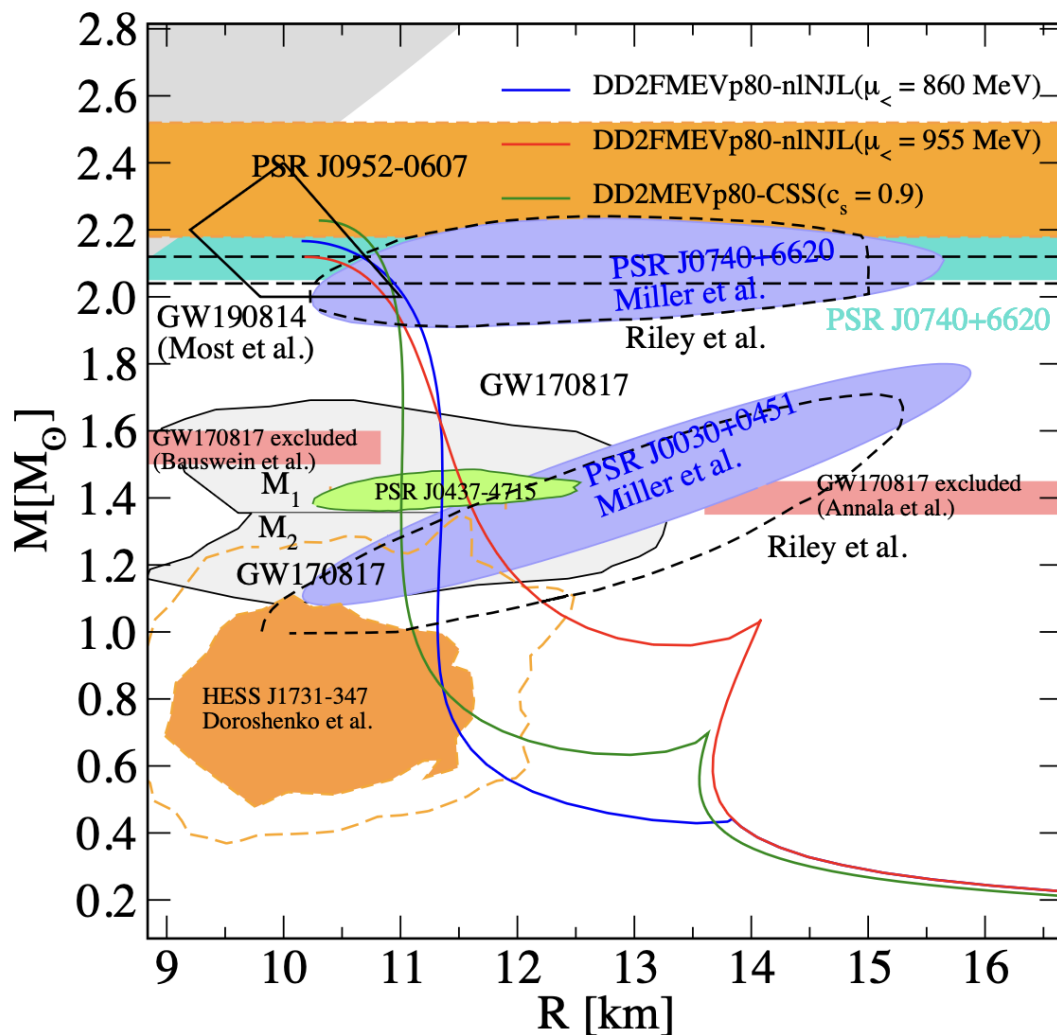
Compact Star Mass Twins and the AHP scheme

- First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → “third family of CS”.
- Measuring two **disconnected** populations of compact stars in the M-R diagram would represent **the detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP)** in the QCD phase diagram!

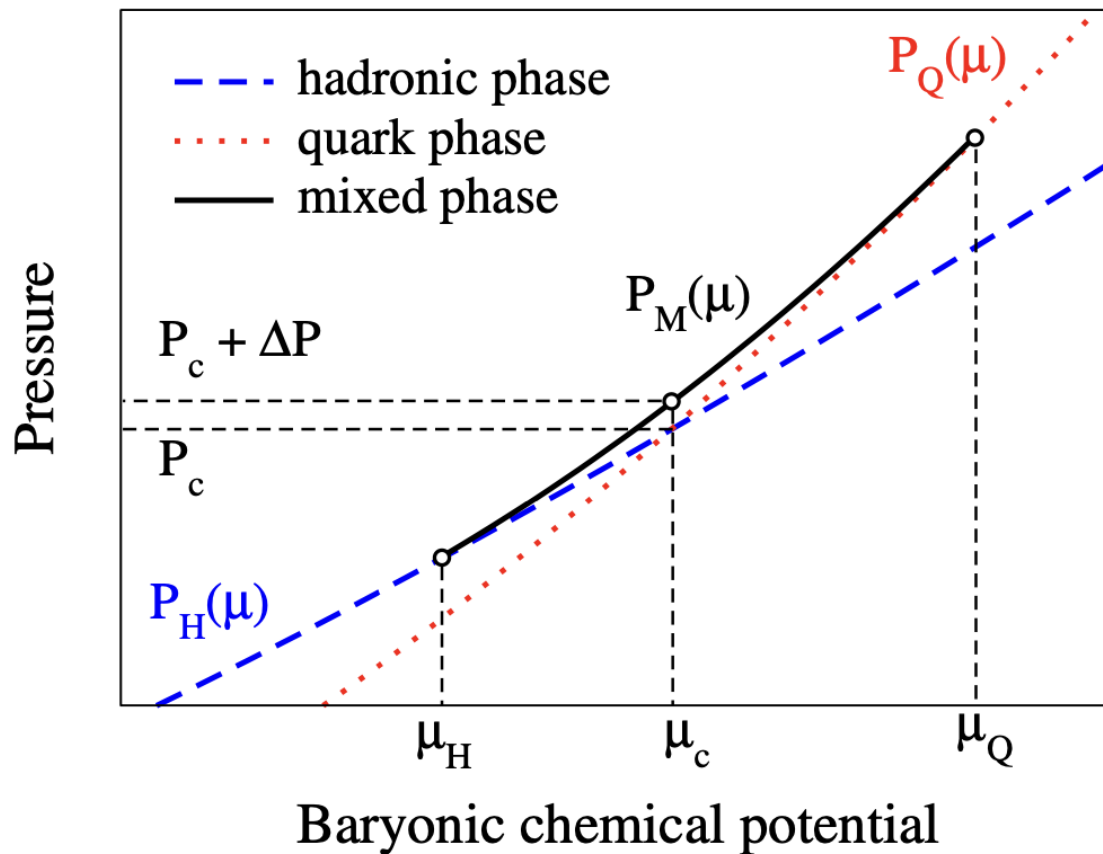


Alford, Han, Prakash,
Phys. Rev. D 88, 083013 (2013)
arxiv:1302.4732

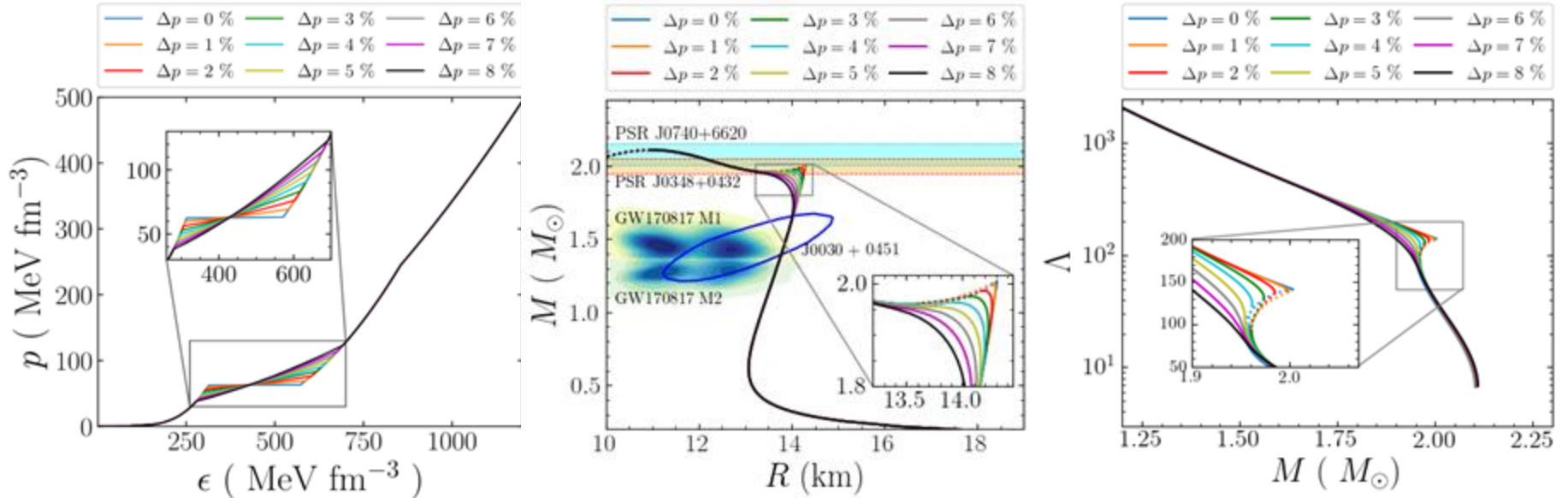
Compact Star Twins



Compact Star Twins (Pasta Phases)



Stellar Properties

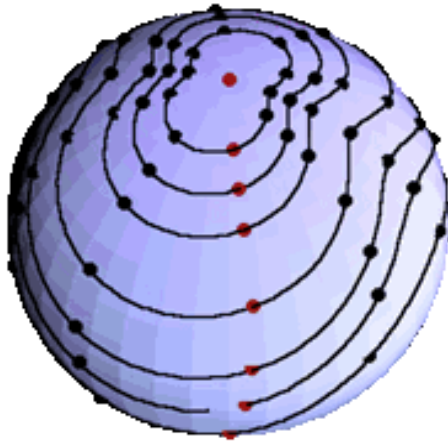


- ❖ The second and third family merge to form a single branch for $\Delta p > 4\%$.
- ❖ Precise measurement of M - R required for detection of twin star.
- ❖ The jump Δp (if any) can be measured $\sim 15\%$ ($< 90\%$ CI) with next-generation GW detectors ([P. Landry & K. Chakravarti, arXiv:2212.09733, 2022](#)).

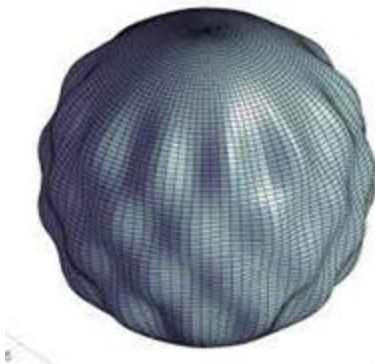
David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee – [arXiv:2309.08775](#)

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Gravitational Waves from NS



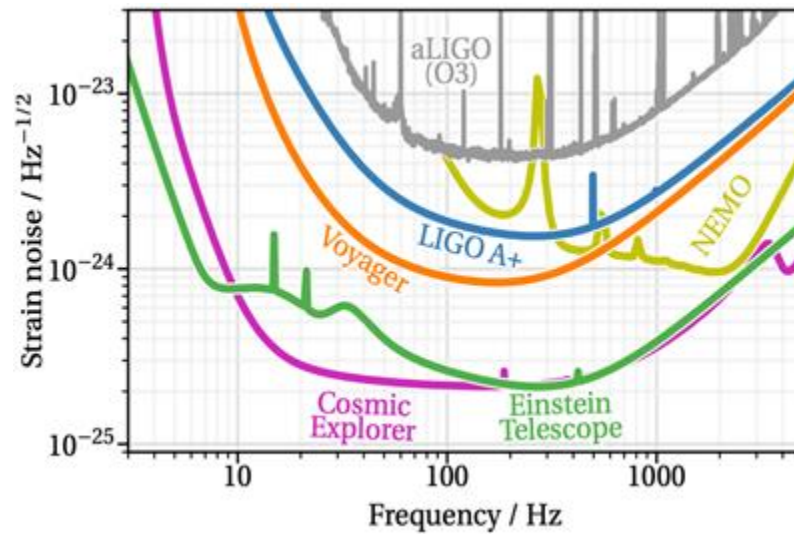
Credit: C. Hanna and B. Owen



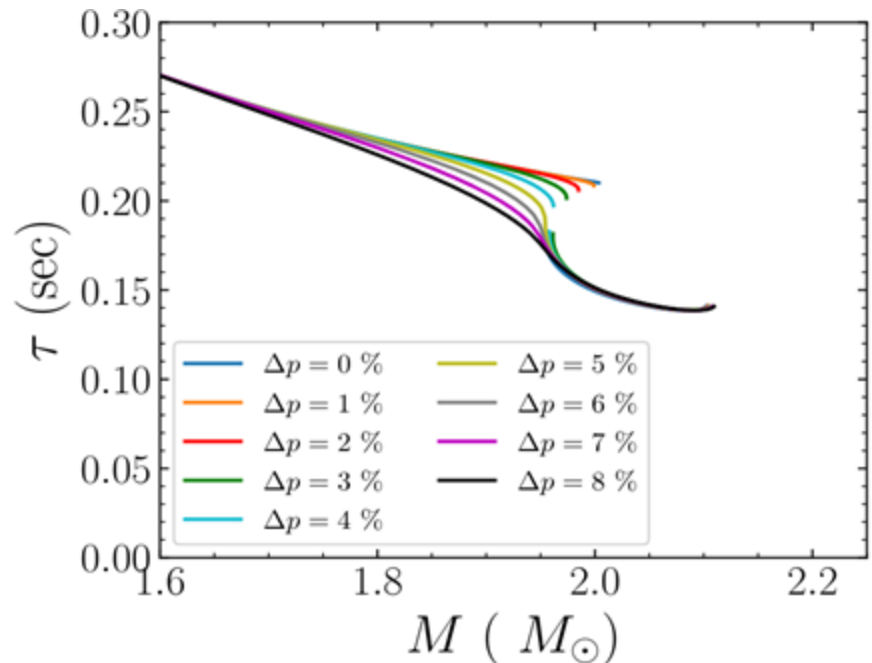
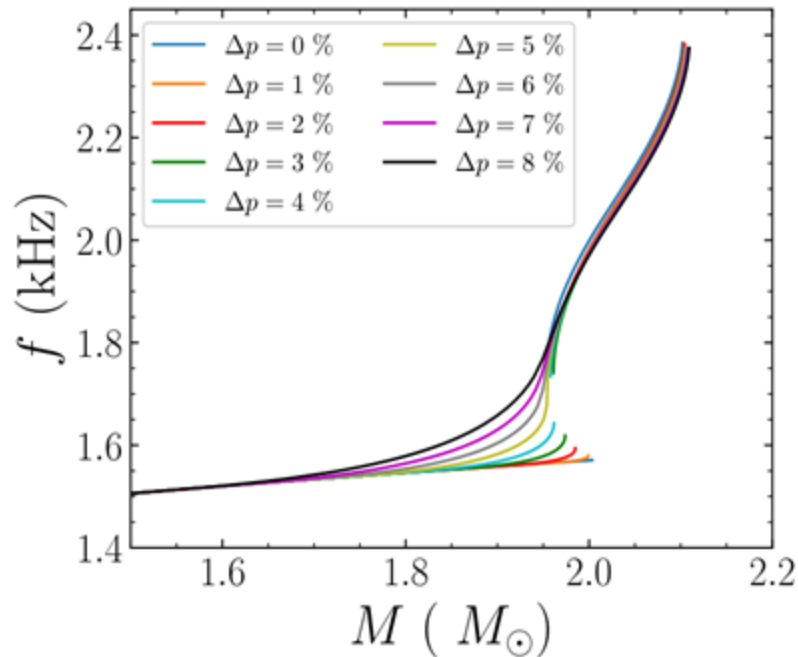
Credit: CERN/Indico

PC: cosmicexplorer.org/sensitivity

- Non-radial QNMs raised from time varying quadrupole deformations are source of GWs.
 - fundamental (f) mode,
 - no node, probe for mean density, ($1 \text{ kHz} < f < 3 \text{ kHz}$)
 - pressure (p) mode,
 - Sound speed, ($5 \text{ kHz} < f < 10 \text{ kHz}$)
 - gravity (g) mode,
 - ($50 \text{ Hz} < f < 500 \text{ Hz}$)
- R-mode, for rotating stars only.
 - Viscosity, ($0.5 \text{ kHz} < f < 2 \text{ kHz}$)
- Space-time (w) mode.
 - $5 \text{ kHz} < f$



f-mode characteristics



- f-mode characteristics are obtained within **General relativistic** formalism.
- Sudden increase (decrease) in the frequency (damping time) observed with appearance of twin star.
- Detections of f-mode GWs from compact stars with known mass may reveal the presence of twin stars.
- Simultaneous measurement of M - f (from binary system) can be used to comment on twin stars.

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee – [arXiv:2309.08775](https://arxiv.org/abs/2309.08775)

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Asteroseismology and Universal Relations

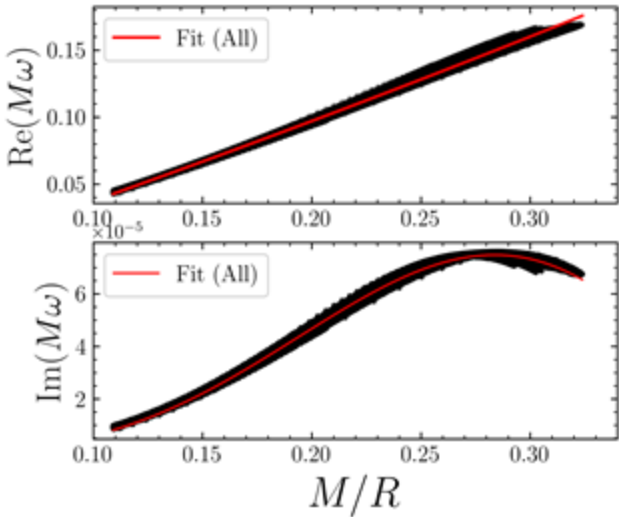
- URs among f-mode characteristics (f , τ_f or $\omega=2\pi f+1/\tau_f$) and NS observables.

$$f(\text{kHz}) = a_r + b_r \sqrt{\frac{M}{R^3}}$$

Empirical relations (EOS dependent)

$$\text{Re}(M\omega) = a_0 + a_1 \left(\frac{M}{R}\right) + a_2 \left(\frac{M}{R}\right)^2$$

$$\text{Im}(M\omega) = b_0 \left(\frac{M}{R}\right)^4 + b_1 \left(\frac{M}{R}\right)^5 + b_2 \left(\frac{M}{R}\right)^6$$



	Re($M\omega$)		Im($M\omega$)
a_0	$-0.027 \pm 9 \times 10^{-5}$	b_0	$(9.81 \pm 0.004) \times 10^{-2}$
a_1	0.610 ± 0.0015	b_1	$(-4.444 \pm 0.003) \times 10^{-1}$
a_2	0.049 ± 0.002	b_2	$(4.91 \pm 0.0045) \times 10^{-1}$

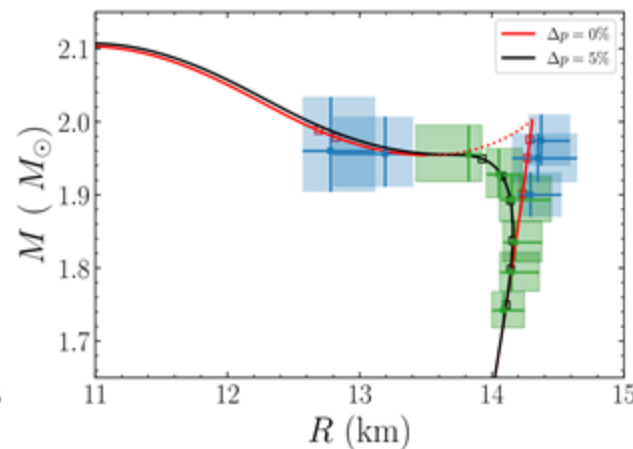
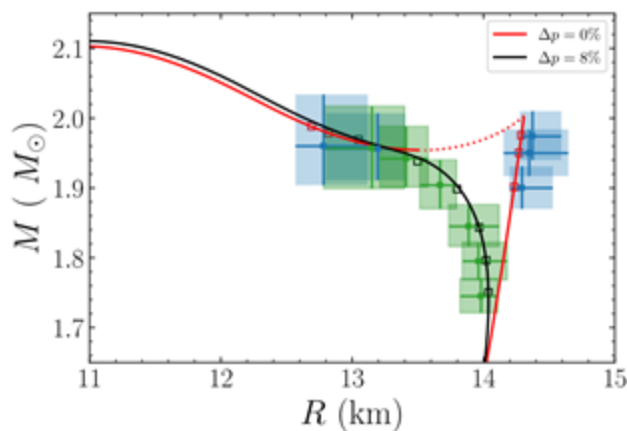
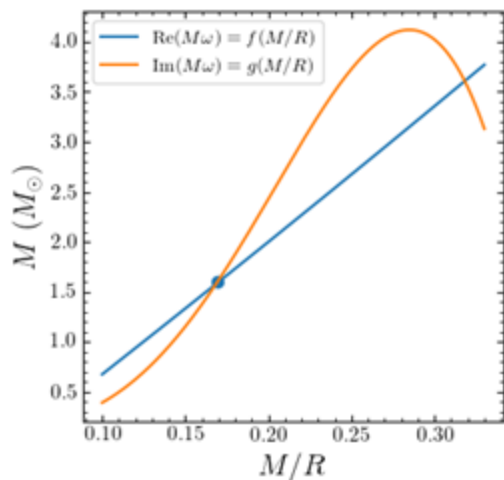
- Scaled Universal relations are more useful.
- The URs can be used for EoS inference.
- URs involving tidal deformability have also been examined.

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee – [arXiv:2309.08775](https://arxiv.org/abs/2309.08775)

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Compact star observables from f-mode observations: the role of UR Uncertainty

- ❖ Determination under the assumption that f , τ are measured precisely.
- ❖ Errors on UR results uncertainties on M - R .



- ★ The presence of the twins maybe confirmed with exact measurement of f , and τ .
- ★ The unstable branch of $\Delta p = 0\%$ can be distinguished from the connecting stable branch of $\Delta p = 8\%$. Differentiating among $\Delta p = 0\%$ and $\Delta p = 5\%$ is more challenging.
- ★ We perform a parameter estimation using Bilby to get the posteriors of (f, τ) . From the recovered (f, τ) , we reconstruct the (M, R) using Universal relations.

Inclusion of Observational Uncertainties

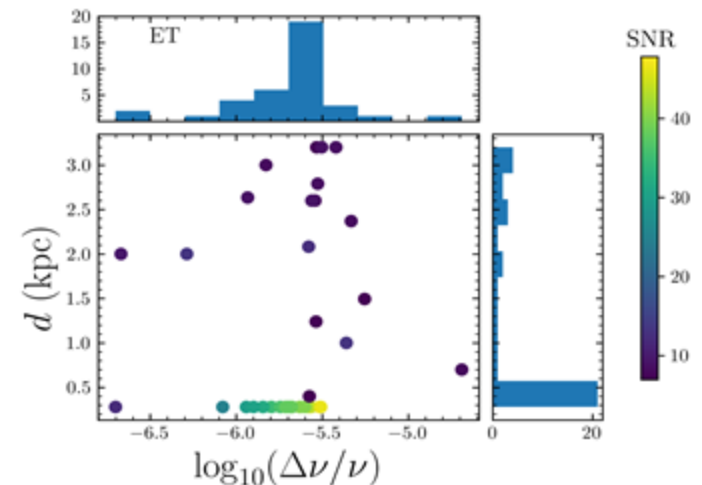
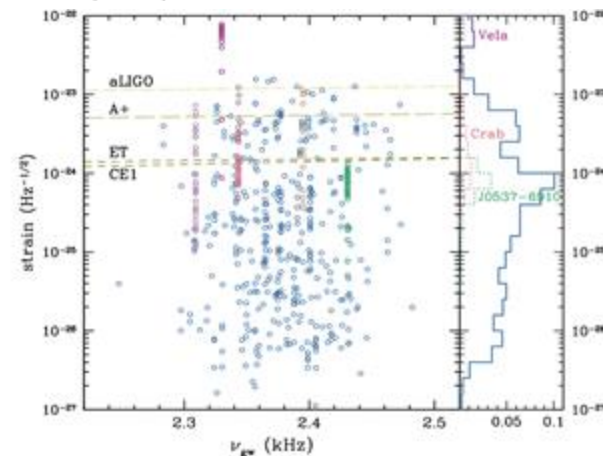
- F-mode being excited during pulsar glitches. All the energy radiated through GW.
- The burst waveform is modelled as an exponentially damped oscillation.

$$h(t) = h_0 \exp(-t/\tau_f) \sin(2\pi\nu_f t), \quad t > 0 \quad (\text{B.J. Owen, 2010, Ho et al. 2020})$$

$$h_0 = 4.85 \times 10^{-17} \sqrt{\frac{E_{\text{gw}}}{M_{\odot} c^2}} \sqrt{\frac{0.1 \text{ sec kpc}}{\tau_f d}} \left(\frac{1 \text{ kHz}}{\nu_f} \right)$$

f-modes GW

$$E_{\text{gw}} = E_{\text{glitch}} = 4\pi^2 I \nu^2 \left(\frac{\Delta\nu}{\nu} \right)$$



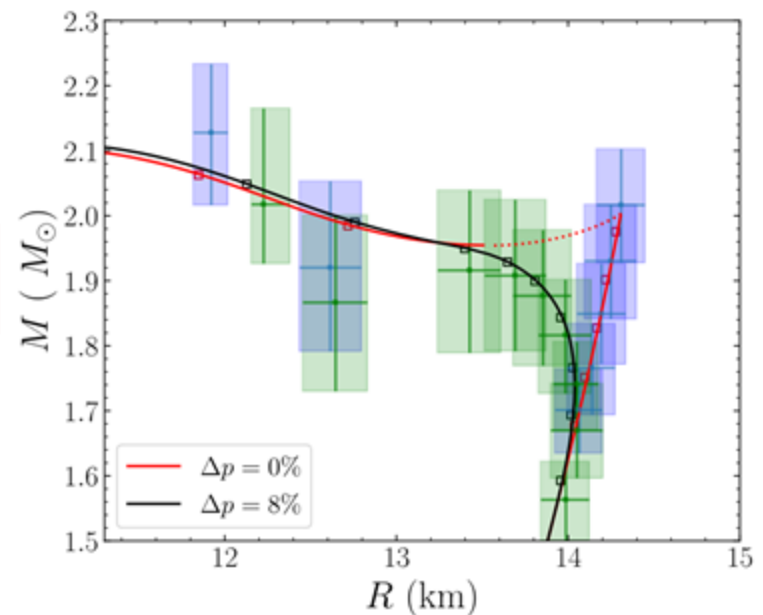
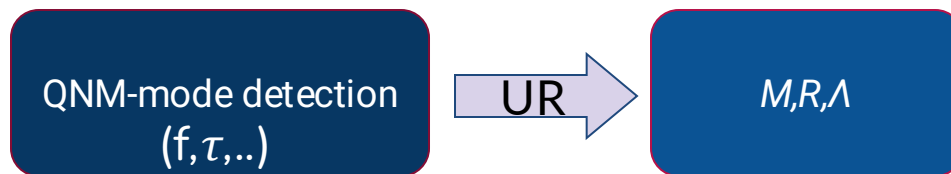
- B. Abbott et al., LVC, [ApJ 874 163, 2019.](#)
- R. Abbott et al., LVC, [PhRvD, 104, 122004, 2021.](#)
- R. Abbott, et al., LVC, [arXiv:2210.10931, 2022.](#)
- R. Abbott, et al., LVC, [arXiv:2203.12038, 2022.](#)
- D. Lopez et al., [PhRvD, 106, 103037, 2022](#)
- Ho et al., [PRD 101, 103009 \(2020\)](#)
- B. K. Pradhan, D. Pathak, and D. Chatterjee, [ApJ 956 38, \(2023\)](#)

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee – [arXiv:2309.08775](#)

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Inclusion of Observational Uncertainties

- Glitching pulsars data taken from the [Jodrell Bank Glitch catalogue](#).
- Spin frequency, distance (d) and sky position to each pulsar are assigned from [ATNF Pulsar Catalogue](#).
- Consider few random mass configurations with an assumed EOS model.
- Then f-mode frequency, damping time, moment of inertia to pulsars from the assumed EoS model.



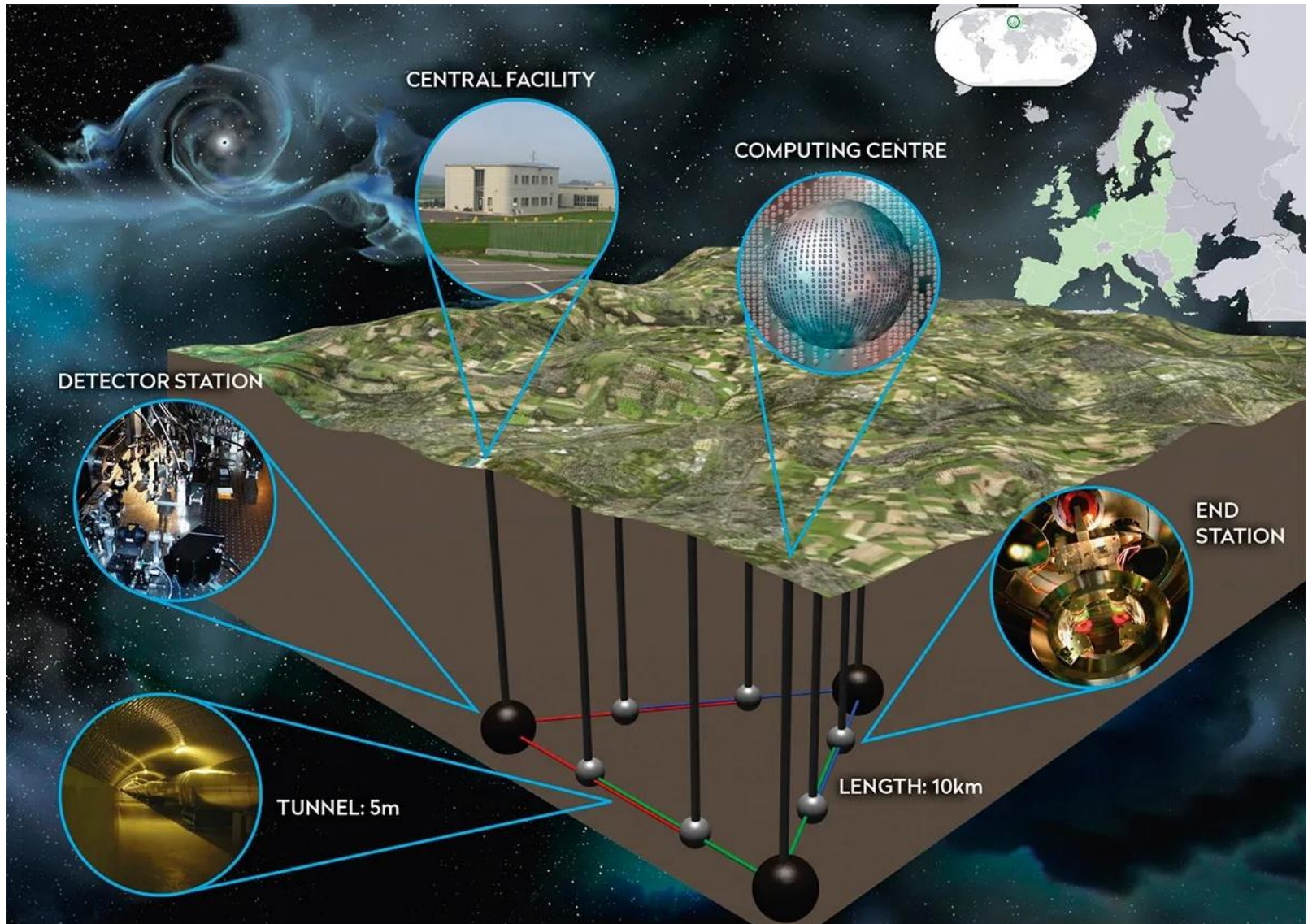
- The measurement of R from f-mode observation may confirm the presence of twins.
- More challenging for low mass twins. However, we have more observations at low masses.
- Differentiating the nature of Δp is more challenging.

Outlook

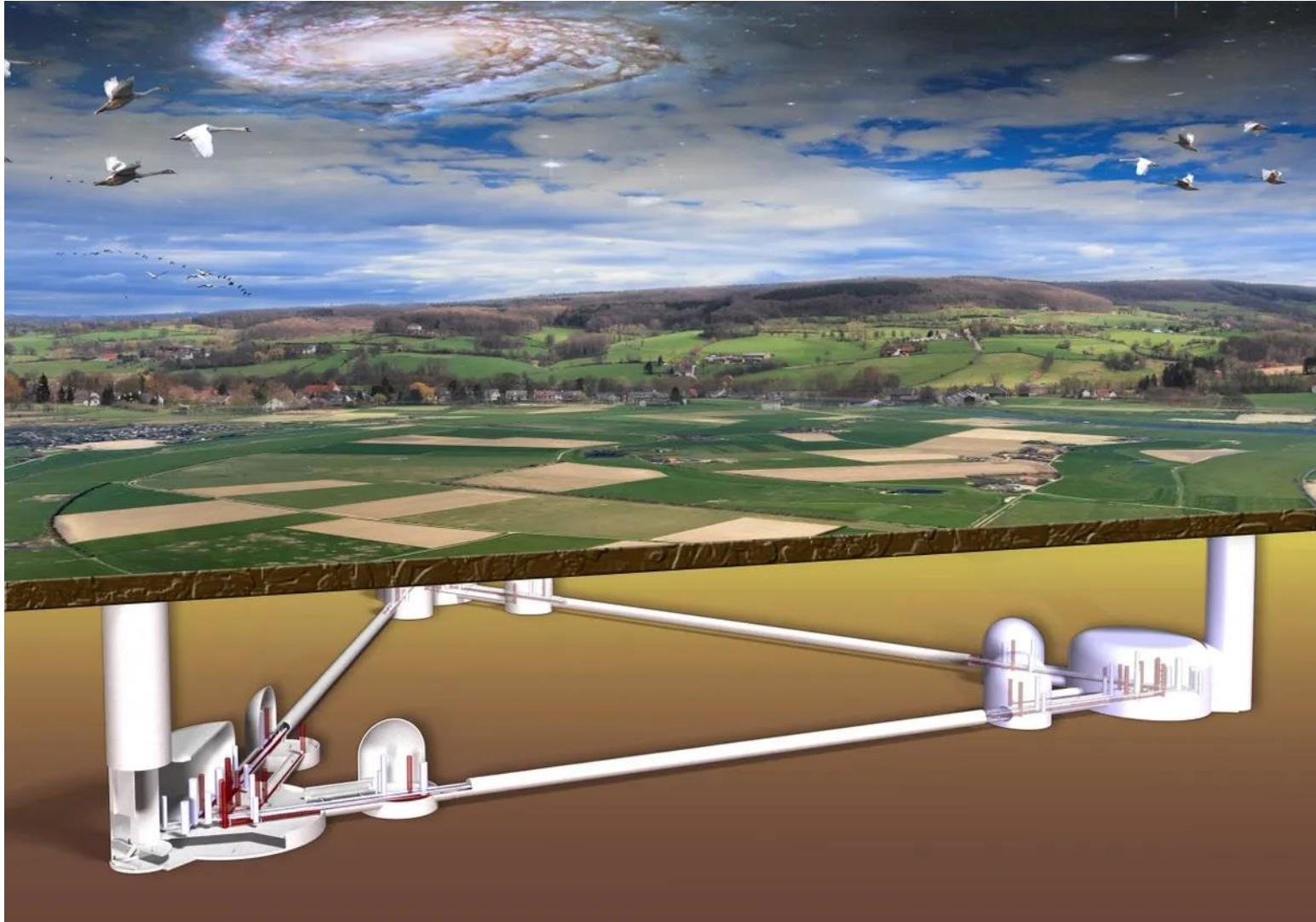
- Multi-messenger astronomy and collider experiments will continue probing the properties of dense matter.
- f-mode oscillation of hybrid stars and twin stars involving the “pasta phase” has been investigated leading to predictions for future potential gravitational wave detections. Nevertheless, complementary compact star measurements are necessary to precisely confirm the nature of the quark-hadron phase transition. Potential for radio counterpart glitch detections.
- Bayesian Analysis and Machine Learning methods including compact star constraints and from laboratory experiment are useful for estimation of unknown physical parameters.
- Other physical effects can be studied with f-mode detections, like possible dark matter content in compact stars.

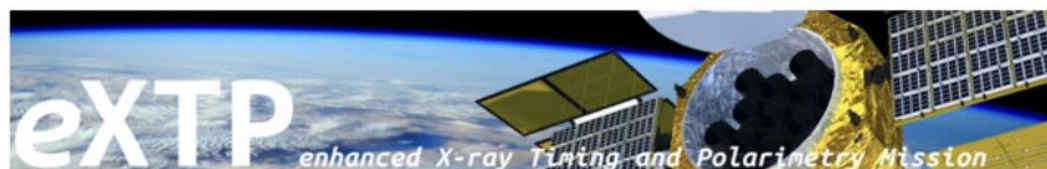
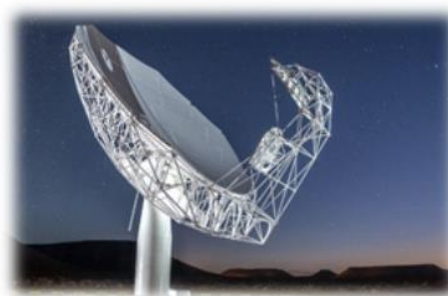
Gracias

Einstein Telescope



Einstein Telescope





Neutron Star Equation of State

The energy per nucleon in neutron star core matter is given by:

$$\begin{aligned} E_{\text{tot}}(n, \{x_i\}) &= E_{\text{b}}(n, x_p) + E_{\text{lep}}(n, x_e, x_\mu) , \\ E_{\text{b}}(n, x_p) &= E_0(n) + S(n, x_p) \\ E_{\text{lep}}(n, x_e, x_\mu) &= E_e(n, x_e) + E_\mu(n, x_\mu) , \end{aligned}$$

where $n = n_p + n_n$ is the total baryon density and $x_i = n_i/n$, $i = p, e, \mu$ are the fractions of protons, electrons and muons, respectively. The baryonic part is very well described by the parabolic approximation w.r.t. the asymmetry

$$\alpha = \frac{n_n - n_p}{n_n + n_p} = 1 - 2x_p,$$

resulting in $S(n, x_p) = (1 - 2x_p)^2 E_s(n)$. The leptonic contribution is a sum of the Fermi gas expressions for the contributing leptons $l = e, \mu$

$$E_l(n, x_l) = \frac{1}{n} \frac{p_{F,l}^4}{4\pi^2} \left[\sqrt{1 + z_l^2} \left(1 + \frac{z_l^2}{2} \right) - \frac{z_l^4}{2} \text{Arsinh} \left(\frac{1}{z_l} \right) \right] ,$$

where $z_l = m_l/p_{F,l}$. For massless leptons ($z_l \rightarrow 0$), this expression goes over to

$$E_l(n, x_l) \Big|_{m_l=0} = \frac{1}{n} \frac{p_{F,l}^4}{4\pi^2} = \frac{3}{4} (3\pi^2 n)^{1/3} x_l^{4/3} .$$

Charge neutrality and β -equilibrium

Under neutron star conditions charge neutrality holds,

$$x_p = x_e + x_\mu .$$

The β - equilibrium with respect to the weak interaction processes $n \rightarrow p + e^- + \bar{\nu}_e$ and $p + e^- \rightarrow n + \nu_e$ (and similar for muons), for cold neutron stars (temperature T below the neutrino opacity criterion $T < T_\nu \sim 1$ MeV) implies

$$\mu_n - \mu_p = \mu_e = \mu_\mu .$$

The chemical potentials are defined as

$$\mu_i = \frac{\partial \varepsilon_i}{\partial n_i} = \frac{\partial}{\partial x_i} E_i(n, \{x_j\}) , \quad i, j = n, p, e, \mu ,$$

where $\varepsilon_i = n E_i(n, \{x_j\})$ is the partial energy density of species i in the system. From the above equations:

$$\mu_e = 4(1 - 2x)E_s(n) .$$

Since electrons in neutron star interiors are ultrarelativistic,

$$\mu_e = \sqrt{p_{F,e}^2 + m_e^2} \approx p_{F,e}, \text{ and } p_{F,e} = (3\pi^2 n_e)^{1/3} = (3\pi^2 n)^{1/3} (x - x_\mu)^{1/3} ,$$

$$\frac{x - x_\mu}{(1 - 2x)^3} = \frac{64 E_s^3(n)}{3\pi^2 n} , \quad (x - x_\mu)^{2/3} - x_\mu^{2/3} = \frac{m_\mu^2}{(3\pi^2 n)^{2/3}} .$$

The total pressure is then given as $P(n) = n^2 \left(\frac{\partial E_{\text{tot}}}{\partial n} \right) .$

NJL model with multiquark interactions

$$\mathcal{L} = \bar{q}(i\partial - m)q + \mu_q \bar{q}\gamma^0 q + \mathcal{L}_4 + \mathcal{L}_8, \quad \mathcal{L}_4 = \frac{g_{20}}{\Lambda^2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] - \frac{g_{02}}{\Lambda^2}(\bar{q}\gamma_\mu q)^2,$$

$$\mathcal{L}_8 = \frac{g_{40}}{\Lambda^8}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]^2 - \frac{g_{04}}{\Lambda^8}(\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^8}(\bar{q}\gamma_\mu q)^2[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

Meanfield approximation: $\mathcal{L}_{\text{MF}} = \bar{q}(i\partial - M)q + \tilde{\mu}_q \bar{q}\gamma^0 q - U,$

$$M = m + 2\frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle + 4\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle\langle q^\dagger q\rangle^2,$$

$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle - 4\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle,$$

$$U = \frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle^2 + 3\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^4 - 3\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle^2 - \frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle^2 - 3\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^4.$$

Thermodynamic Potential:

$$\Omega = U - 2N_f N_c \int \frac{d^3 p}{(2\pi)^3} \left\{ E + T \log[1 + e^{-\beta(E - \tilde{\mu}_q)}] + T \log[1 + e^{-\beta(E + \tilde{\mu}_q)}] \right\} + \Omega_0$$