

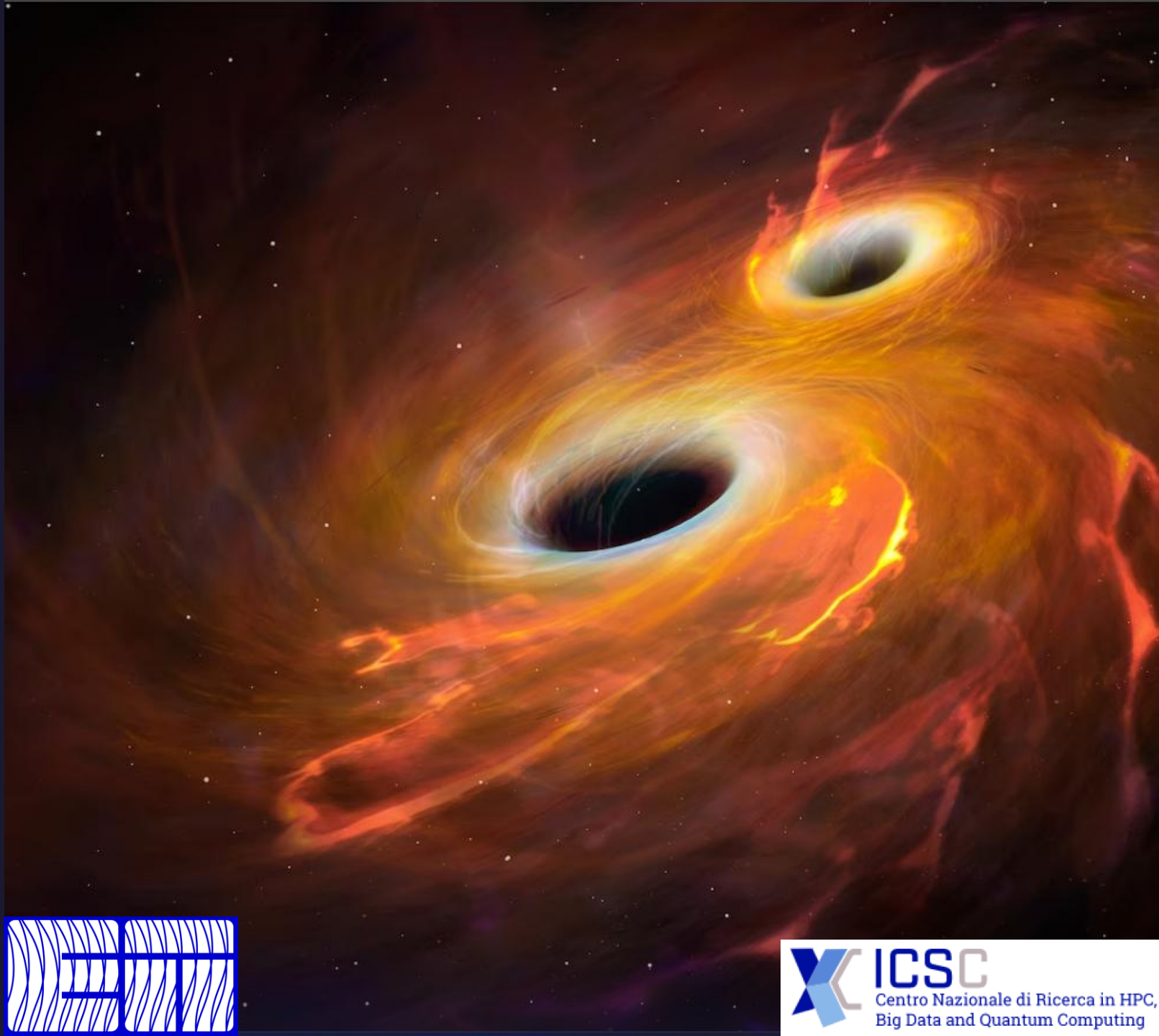
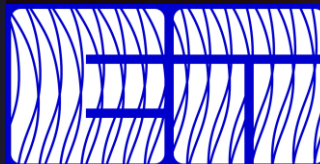
# Beyond GR Tests with the Einstein Telescope

Andrea Begnoni

University of Padua

11/11/25

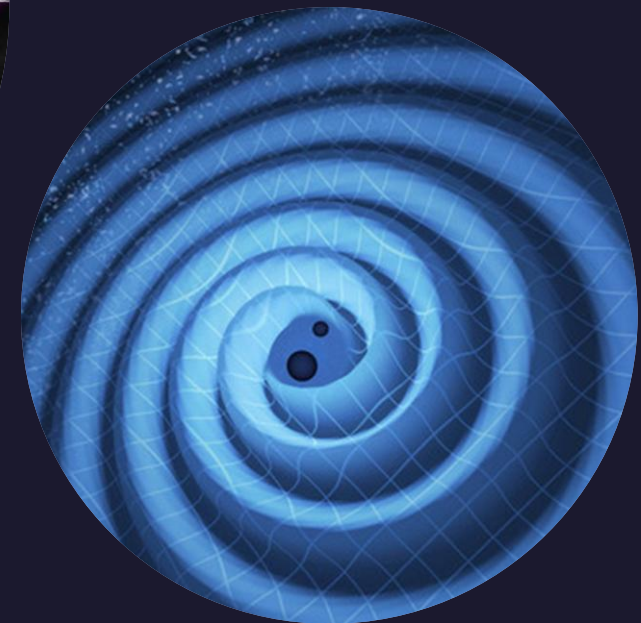
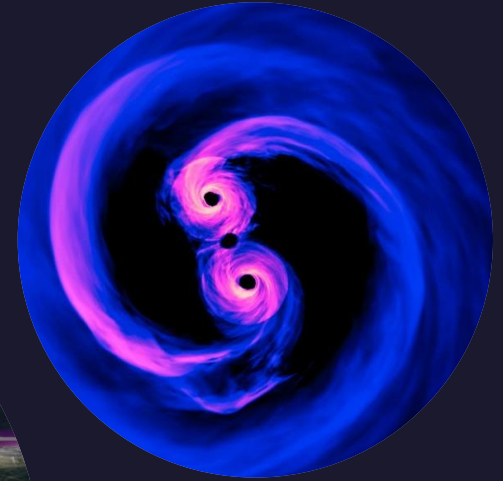
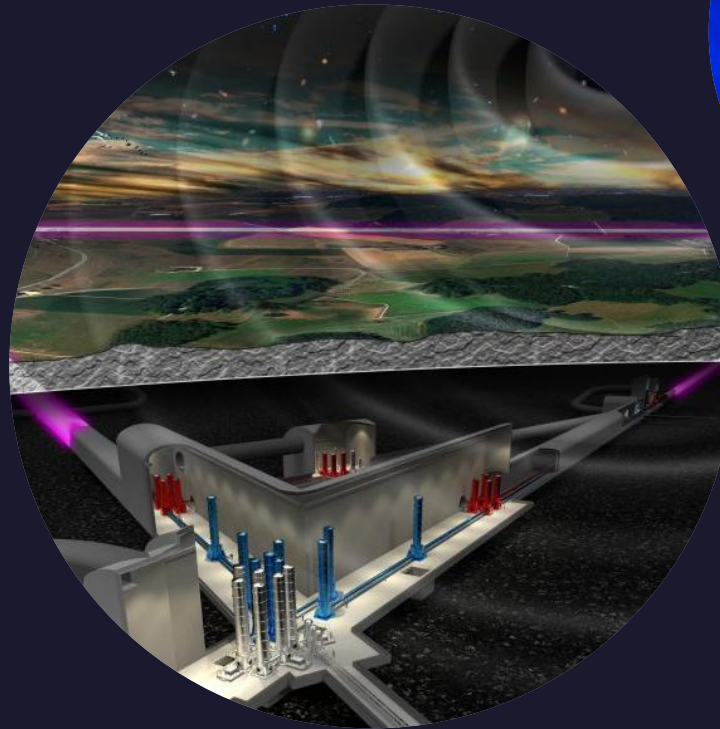
4<sup>th</sup> ET Meeting



# Collaborators

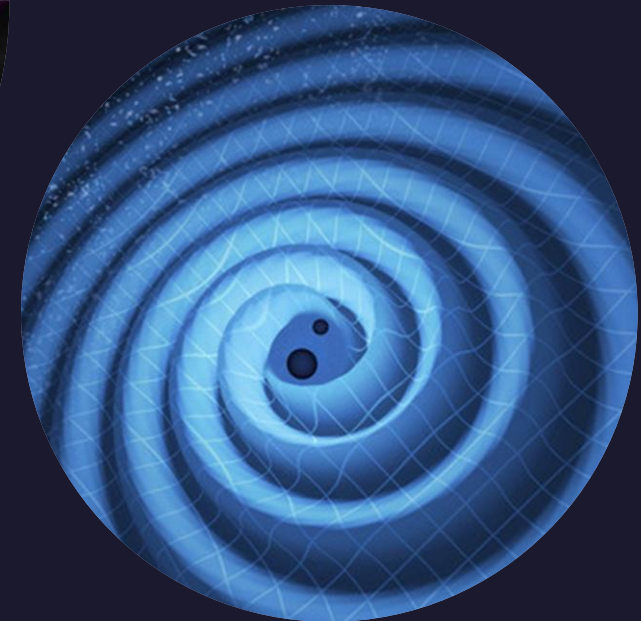
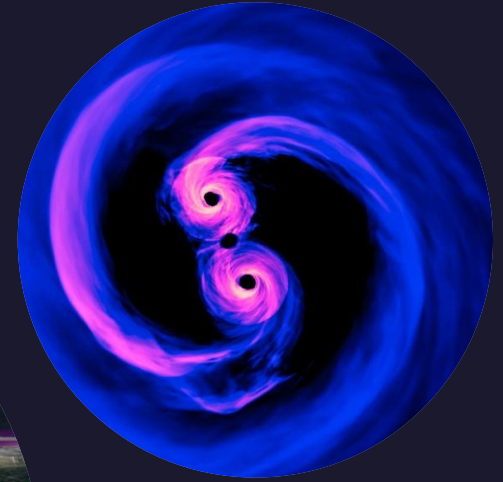
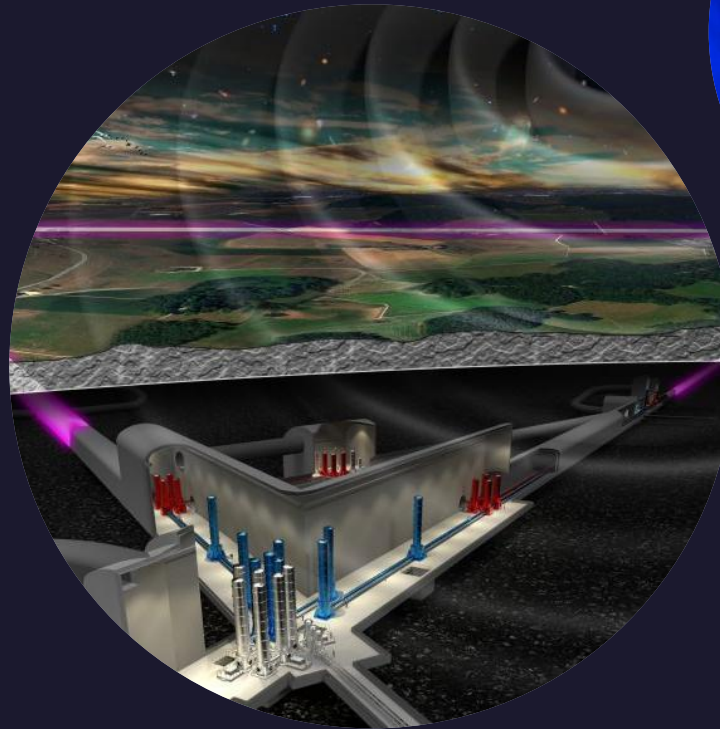
All the work presented was done in collaboration with:

- Stefano Anselmi, Unipd
- Walter Del Pozzo, Unipi
- Matteo Pegorin, Unipd
- Mauro Pieroni, IEM
- Joachim Pomper, Unipi
- Alessandro Renzi, Unipd
- Angelo Ricciardone, Unipi



# Einstein Telescope

- ET will be part of the third generation of ground-based interferometers
- Will be operative in the late 2030s
- The design is still under scrutiny (triangular vs 2L) [1-2]

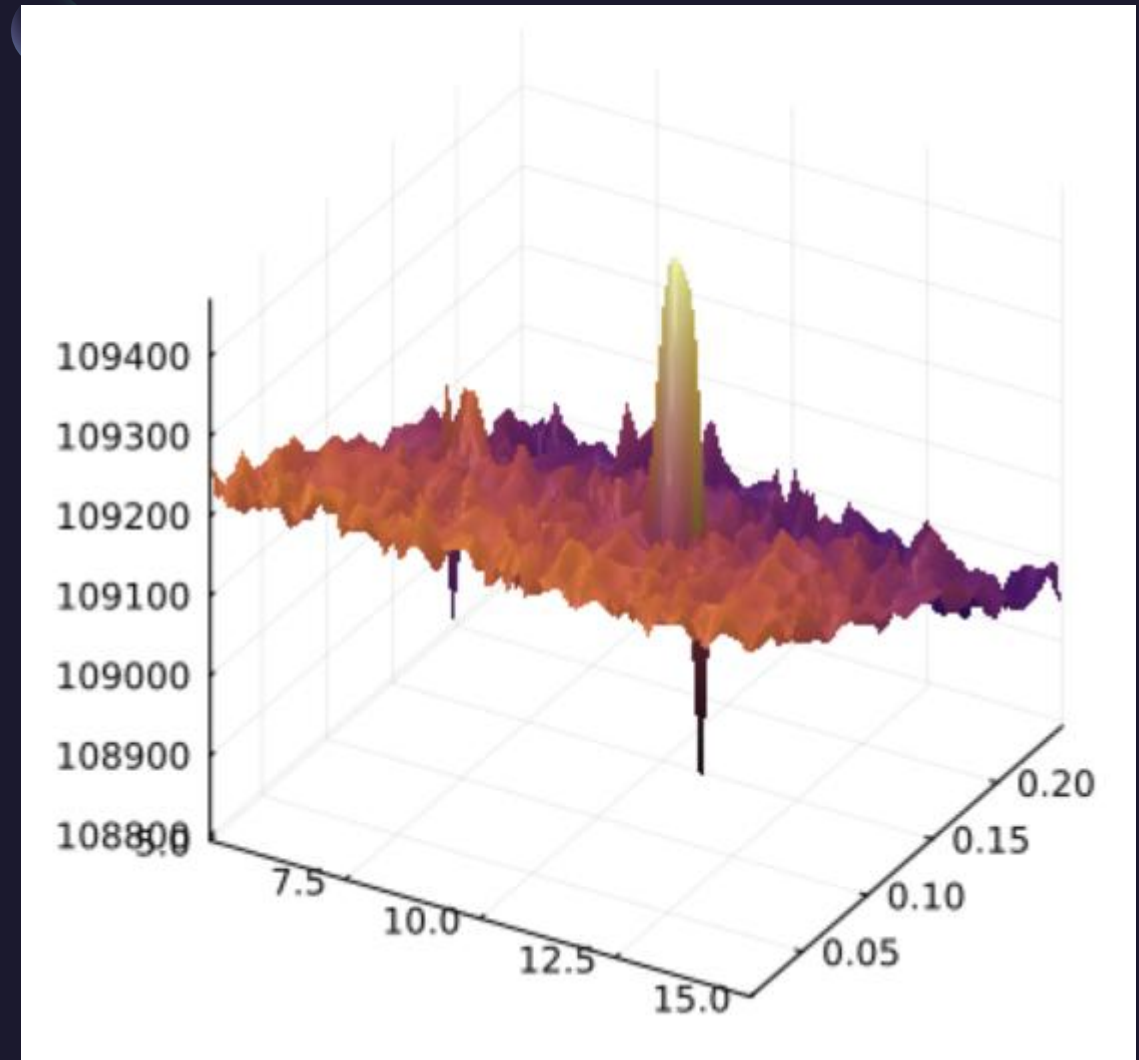


# Bayesian analysis

- The gaussian likelihood is expressed as

$$\mathcal{L}(d \mid \boldsymbol{\theta}) \propto \exp\{-(d - s(\boldsymbol{\theta}) \mid d - s(\boldsymbol{\theta}))/2\}$$

- It is very hard to sample (extremely multimodal and long evaluation)
- In LVK nested sampling is used



# Fisher Matrix

- To compute the Fisher Matrix which is defined as

$$\Gamma_{ab} = -\langle \partial_a \partial_b \log \mathcal{L}(d|\theta) \rangle \big|_{\theta=\theta_0} = (h_a | h_b)$$

where we need to derive the likelihood of the data realization

- Each derivative requires calling the expensive waveform function at least a few times (for numerical diff methods)





# Automatic Differentiations

```
1 x = Dual(1.0, 1.0, 0.)  
2 y = Dual(2.0, 0.0, 1.)  
3 z = x*y + 3y^2
```

✓ 1.6s

```
Dual{Nothing}(14.0,2.0,13.0)
```



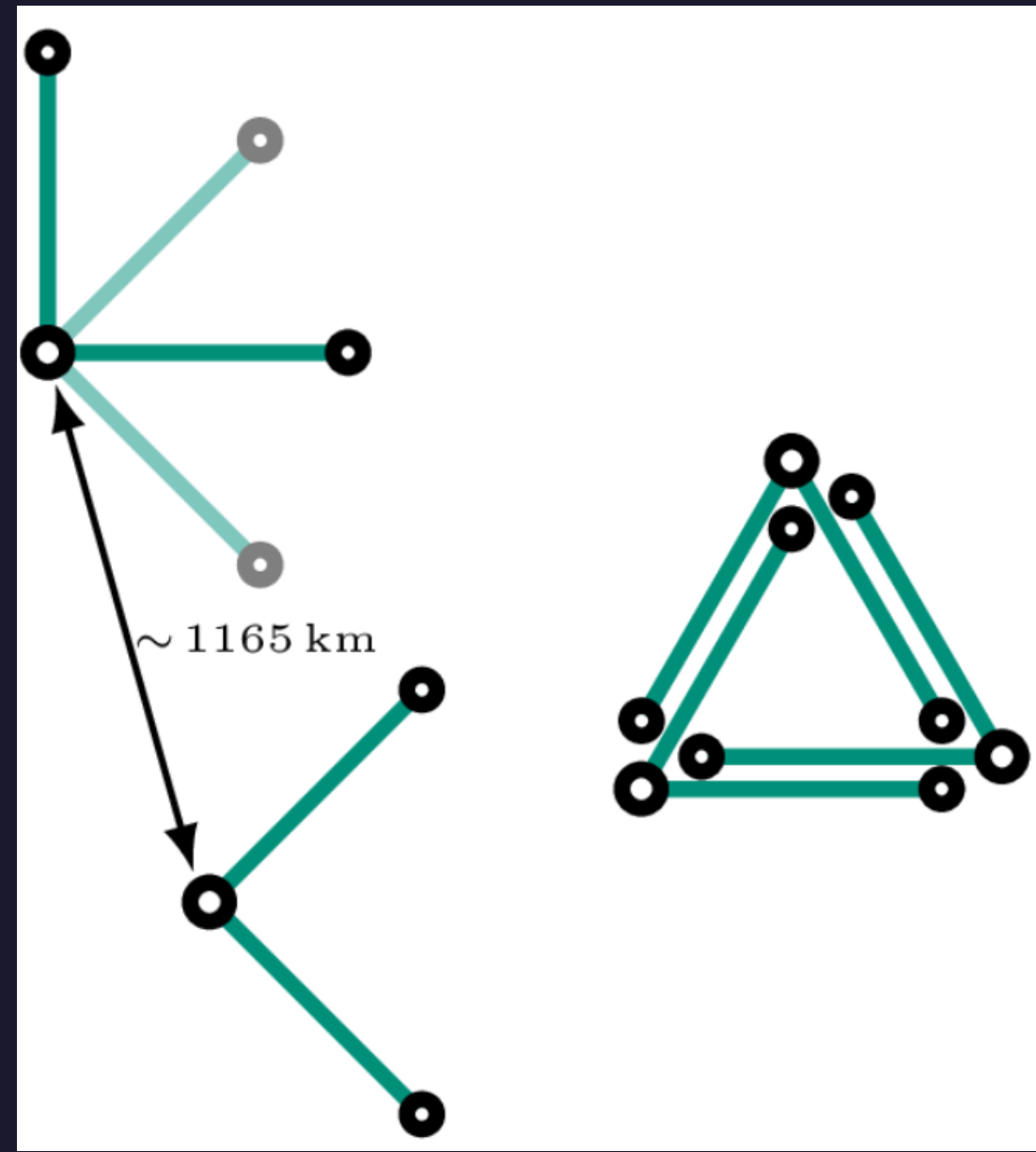
- Easy to learn
- Using automatic differentiation (superior to numerical differentiation)
- Features advanced waveforms (e.g. PhenomXHM, PhenomD), all written in Julia
- Very fast, e.g., using XHM 3 detectors  $\sim 0.5$  sec per core )
- Does not rely on external packages (all is written in Julia)
- Based on arXiv:2506.21530

```
using GWInference
```

```
parameters = GenerateCatalog(1_000, "BBH")  
FisherMatrix(PhenomD(), [CE1Id, CE2NM, ETS], parameters...)
```

# Case study – Detector design comparison

- **T**: triangular ET with 10 km arms featuring cryogenic technology.
- **2L\_0**: two aligned 15 km L-shape interferometers one in Sardinia, one in the Meuse–Rhine (MR) Euroregion, both with the cryogenic technology.
- **2L\_45**: same as 2L\_0 with the exception that the orientations lead to  $\beta = 45^\circ$ .

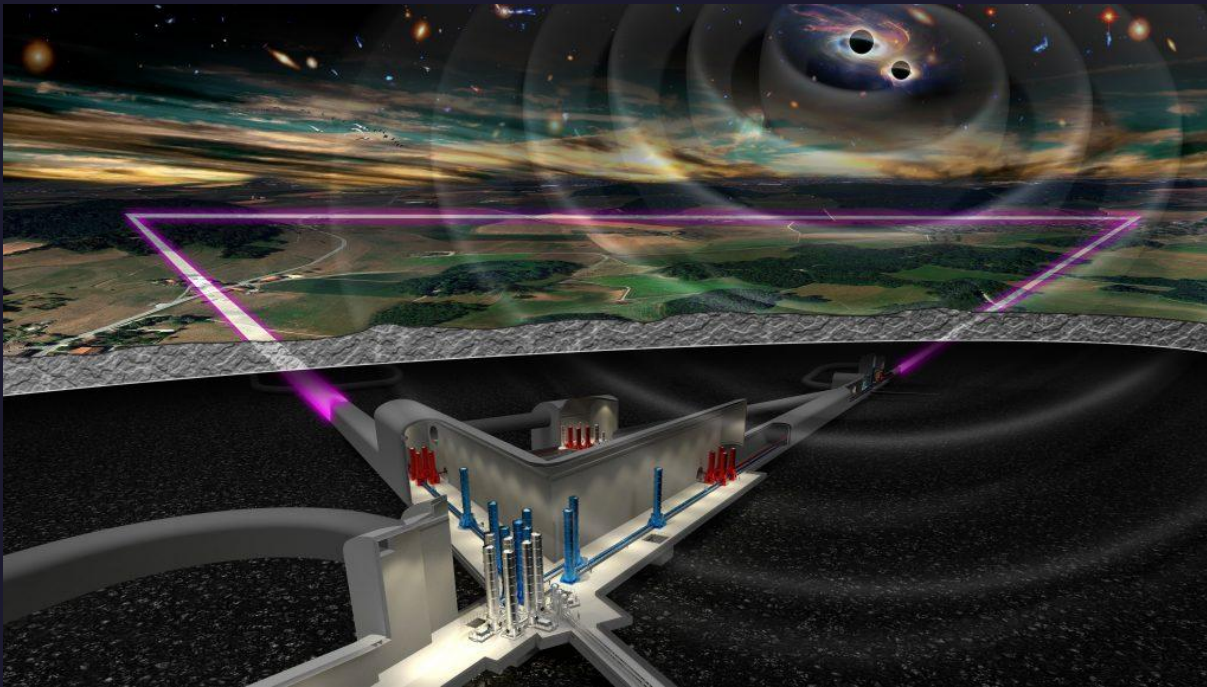


Credits: [1]

[1] Branchesi et al. 2023  
[2] Abac et al, 2025



# Detector design comparison - What to expect



What are we interested in?

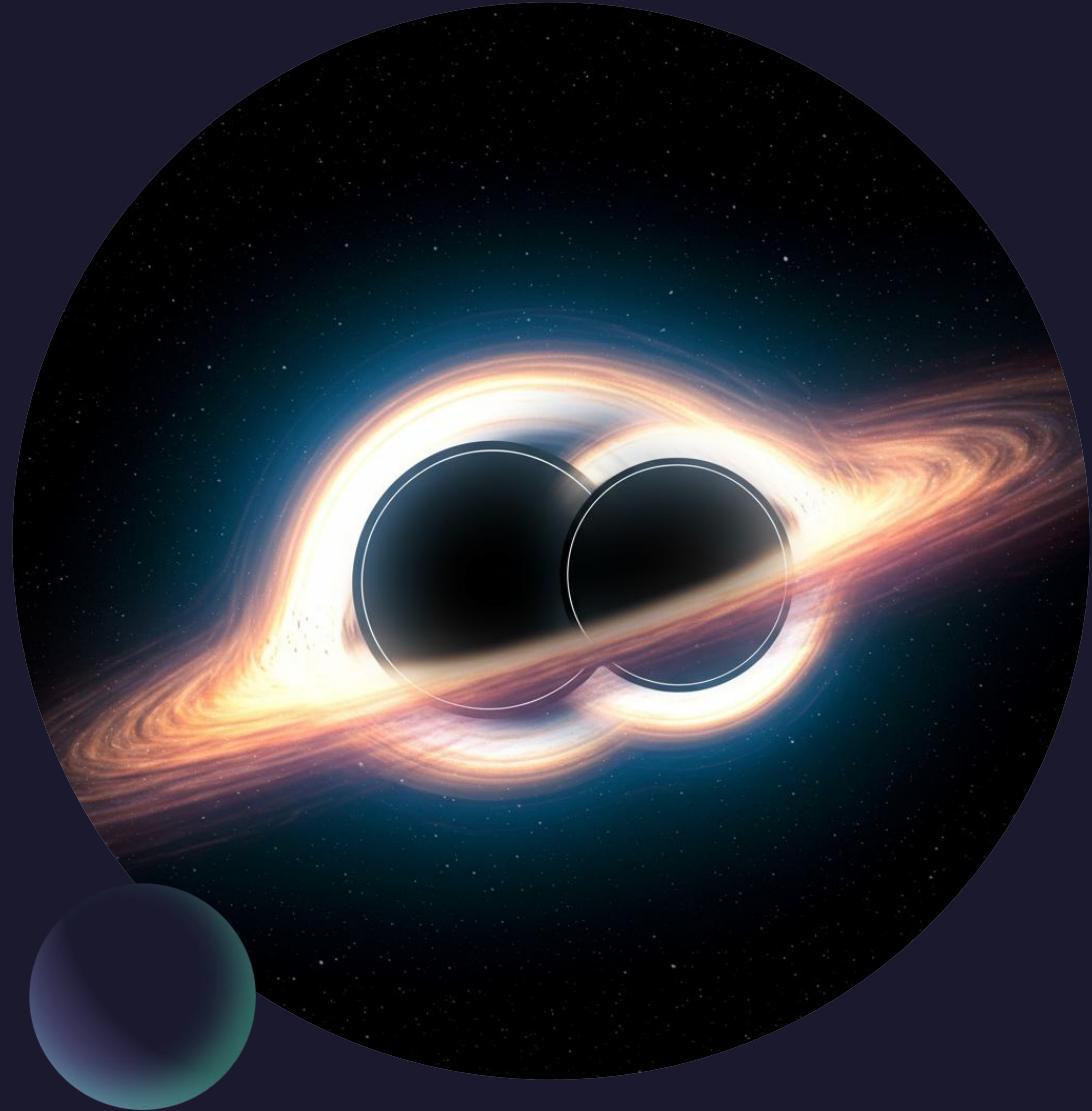
- Cosmology with CBC
- Multimessenger with CBC
- Source properties of CBC
- SGWB

Detector preferences: [1,2]

- CBC :  $2L_{45} > 2L_0 > T$
- SGWB :  $T \approx 2L_0 > 2L_{45}$

# Beyond GR

- GW from CBC can be important test of GR
- Many possible aspects of GR can be tested, e.g., tidal deviations, spin-induced quadrupole moment
- In this work, we focus on the Post-Newtonian terms

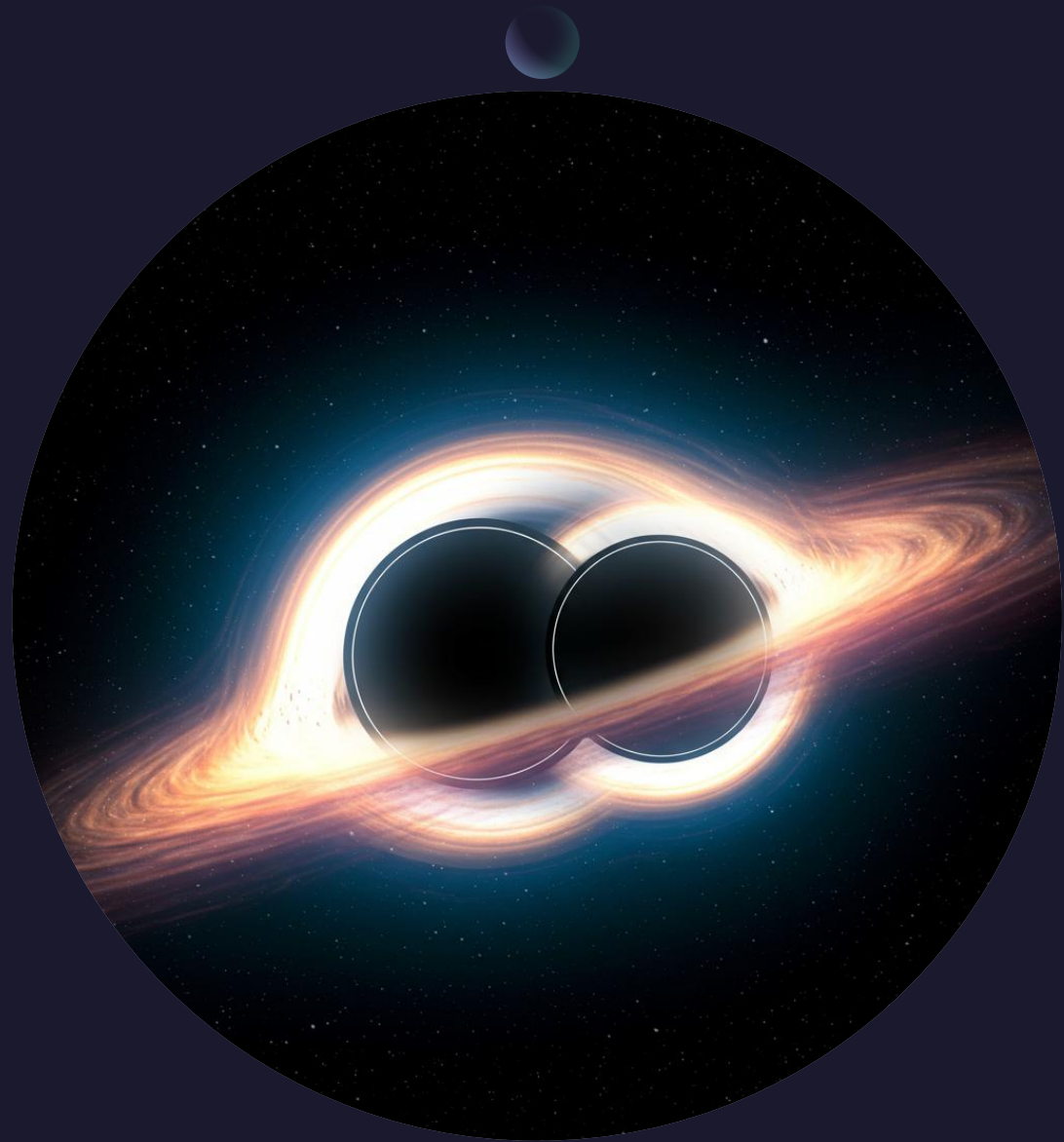


# Post-Newtonian terms

- In frequency domain the phase of a GW can be expressed as

$$\Phi(f) \propto \sum_{j=0}^7 \left[ \varphi_j + \varphi_j^{(l)} \ln f \right] f^{(j-5)/3}$$

- Approximation valid for weak gravitational fields and low velocities
- Used for the inspiral phase of comparable mass CBC

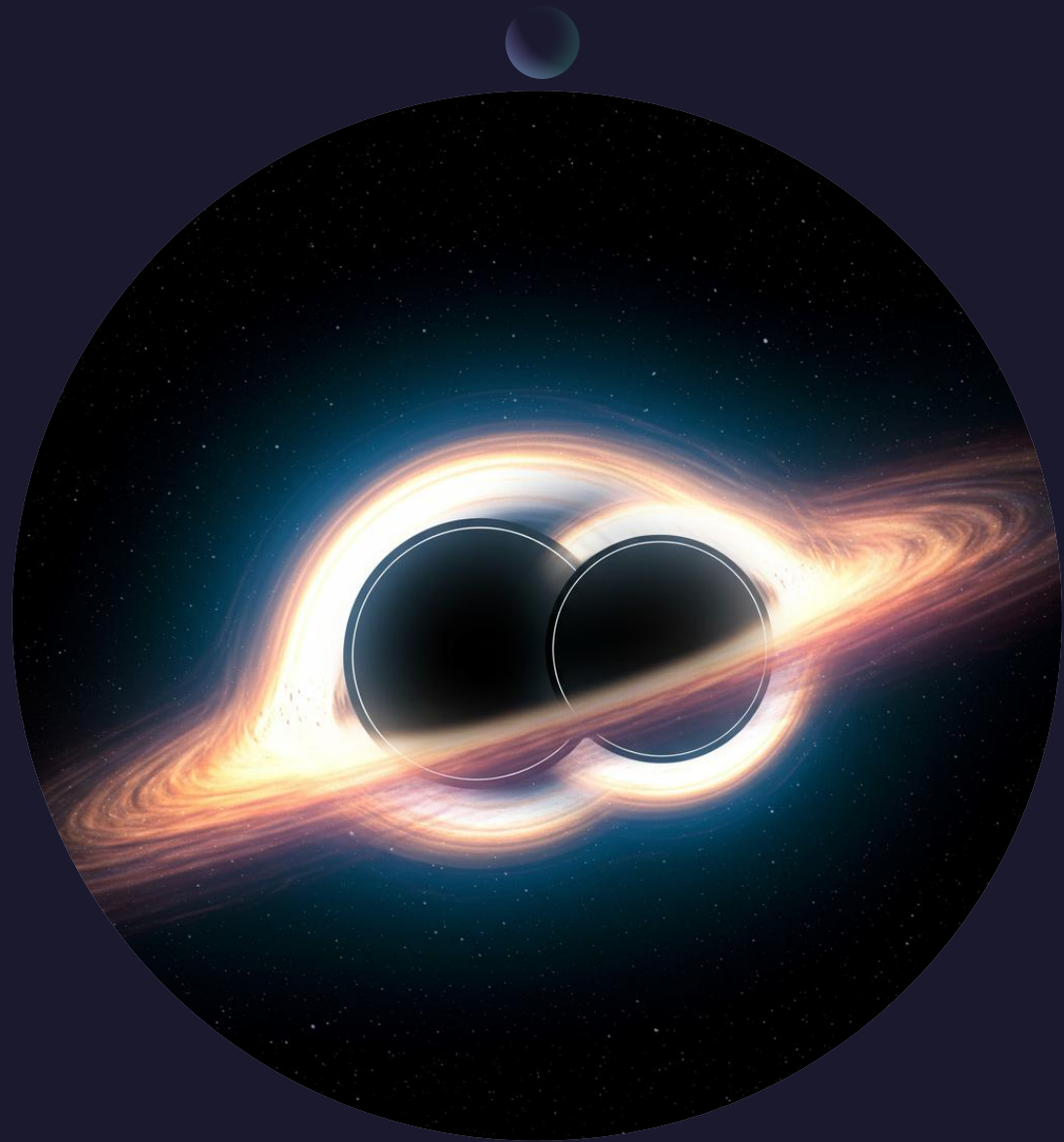


# Post-Newtonian expansion

- Expansion in terms of  $(v/c)$
- Analytical GR predictions for

$$\{\varphi_0, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_{5l}, \varphi_6, \varphi_{6l}, \varphi_7\}$$

where the index represents two times the PN order

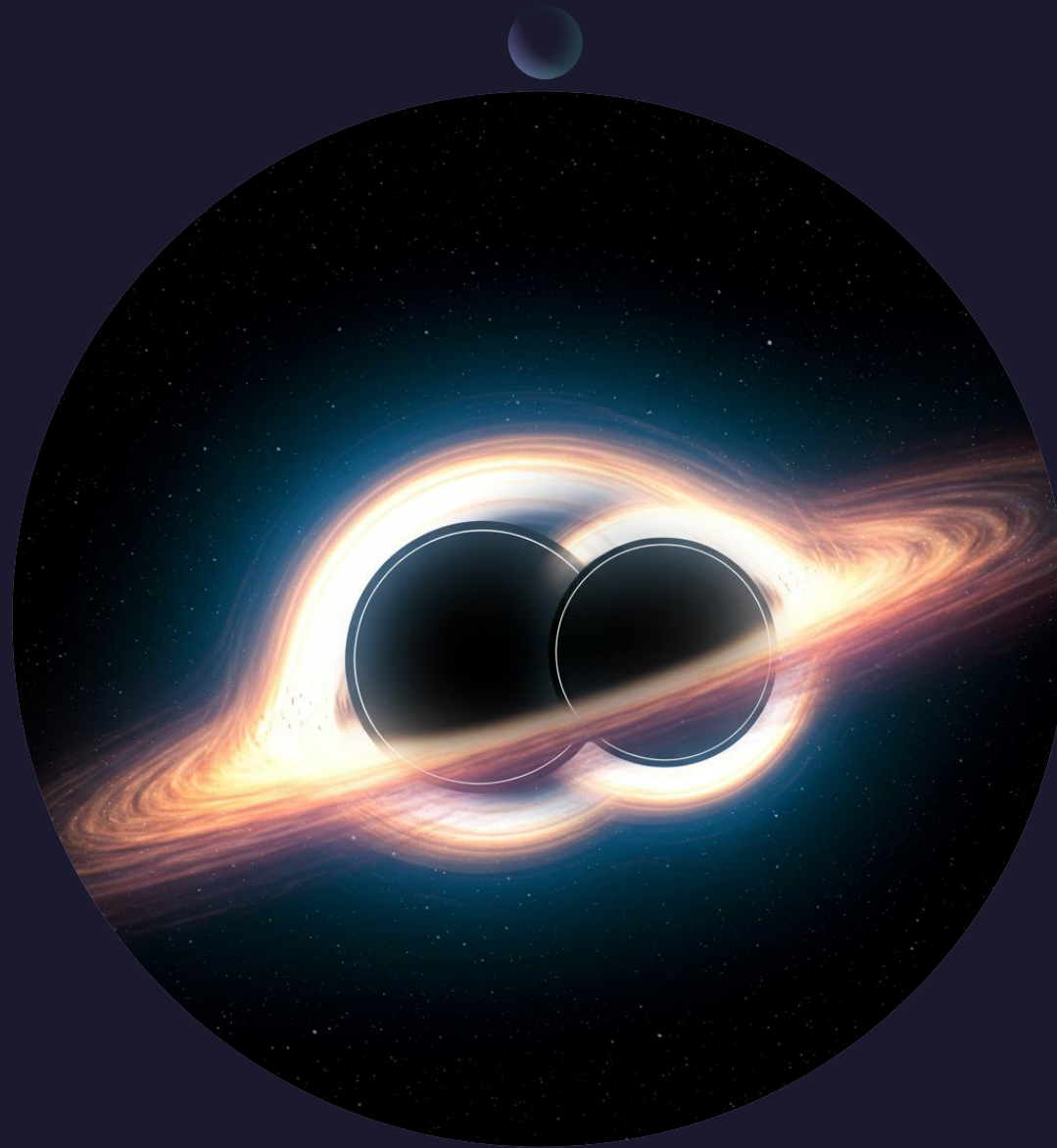


# TIGER framework

In the TIGER framework[9], *one PN term at a time* is modified as

$$\varphi_k^{\text{GR}} \rightarrow (1 + \delta\varphi_k) \varphi_k^{\text{GR}}$$

We modified accordingly the waveforms PhenomD and PhenomHM, ensuring that they remain  $C^1$ .

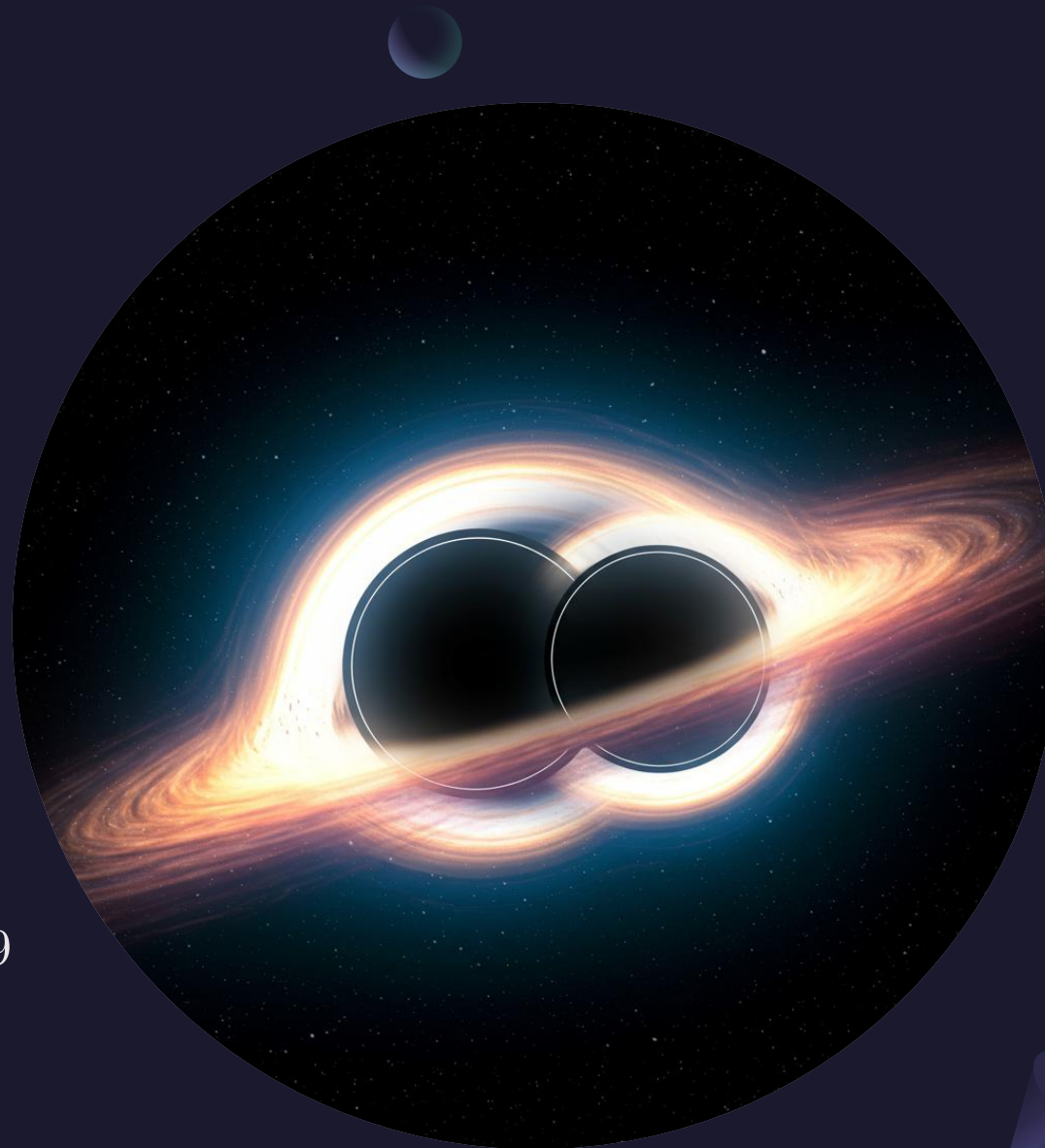


[9] Agathos et al. 2014

# Hierarchical framework

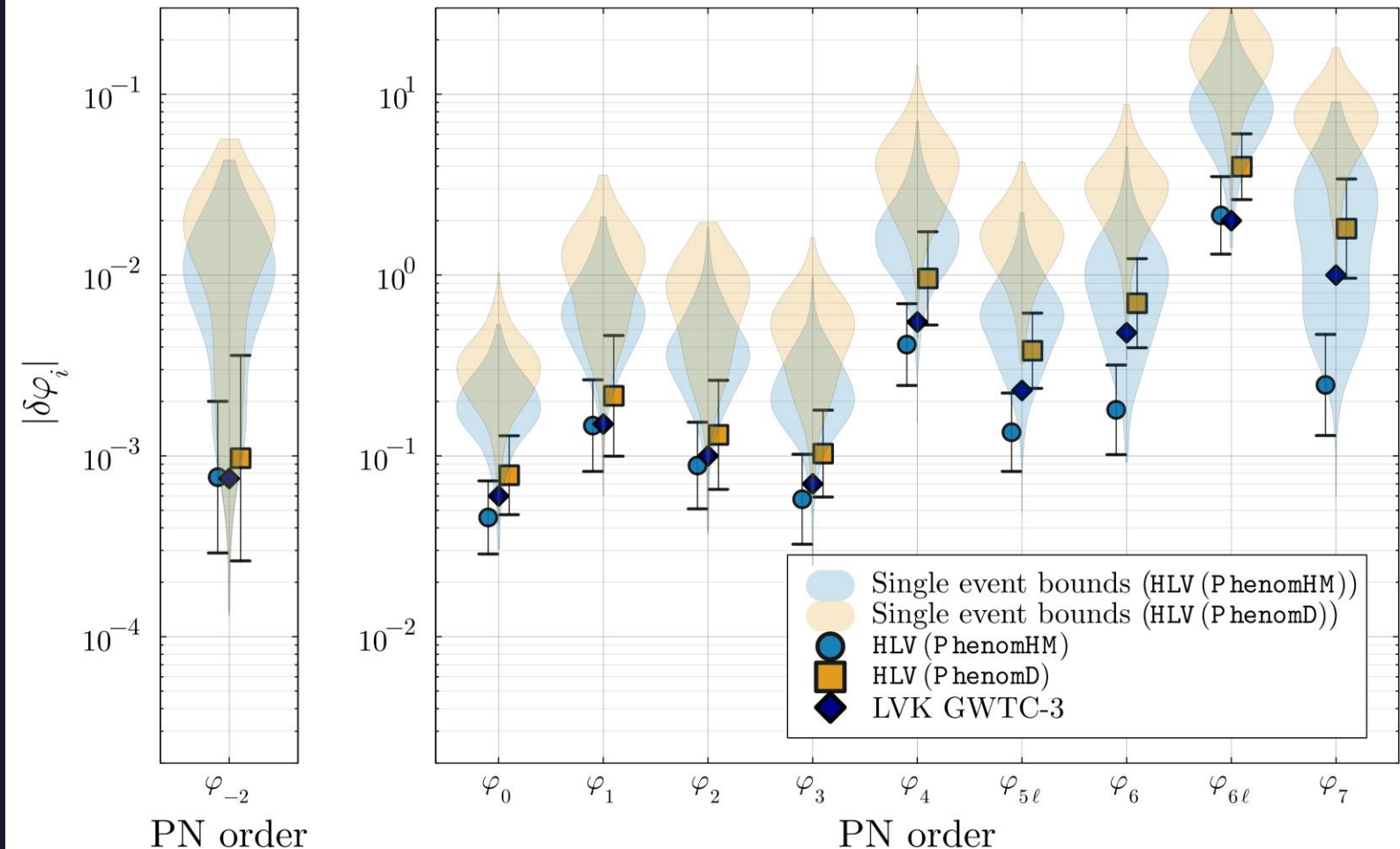
$$P(\delta\varphi_p \mid D^{N_{\text{obs}}}, I) = \int d\mu d\sigma \mathcal{N}(\delta\varphi_p \mid \mu, \sigma) P(\mu, \sigma \mid D^{N_{\text{obs}}}, I)$$

$$|\delta\varphi_p| \quad \text{such that} \quad \int_{-|\delta\varphi_p|}^{+|\delta\varphi_p|} P(\delta\varphi_p \mid D^{N_{\text{obs}}}, I) = 0.9$$



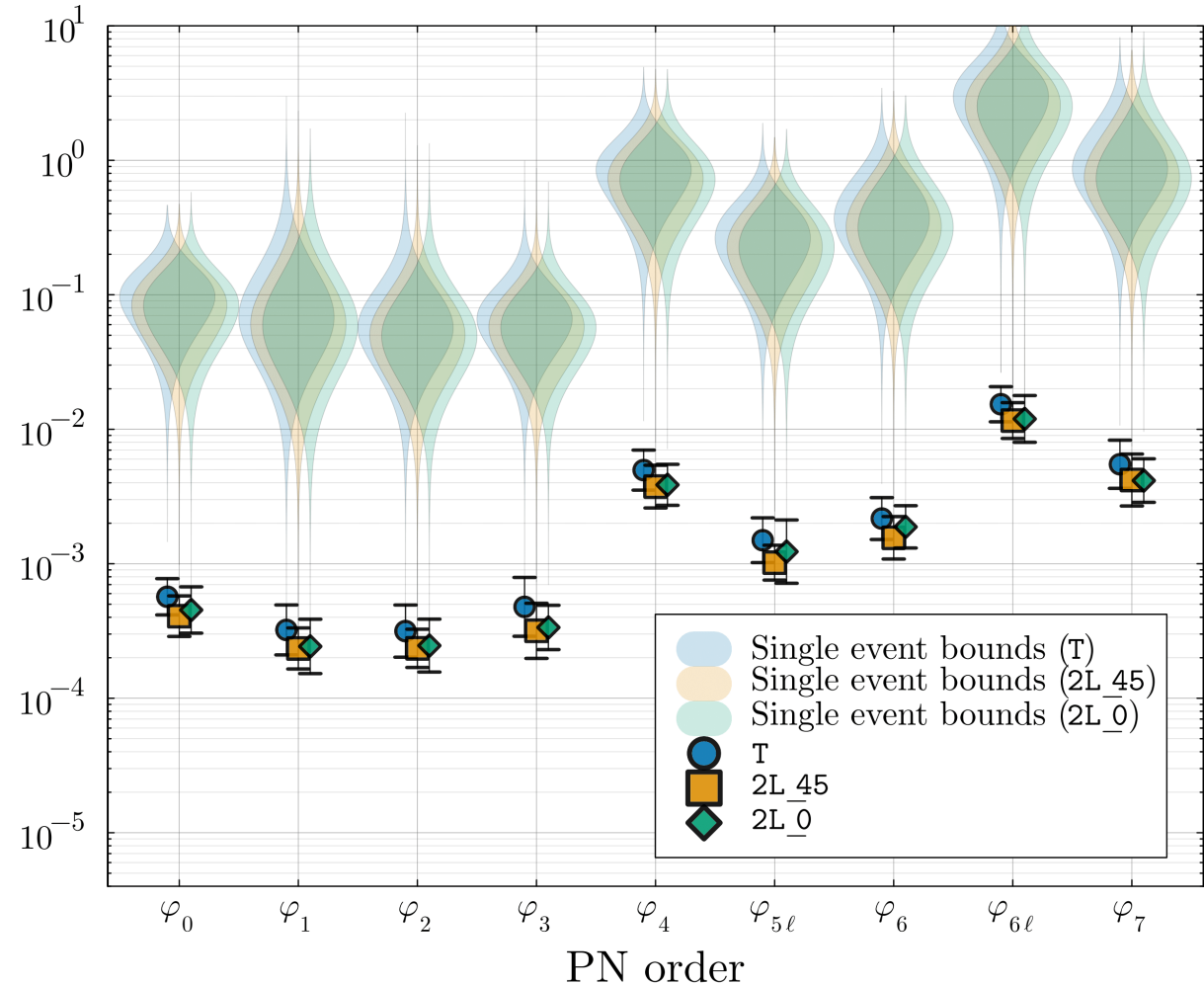
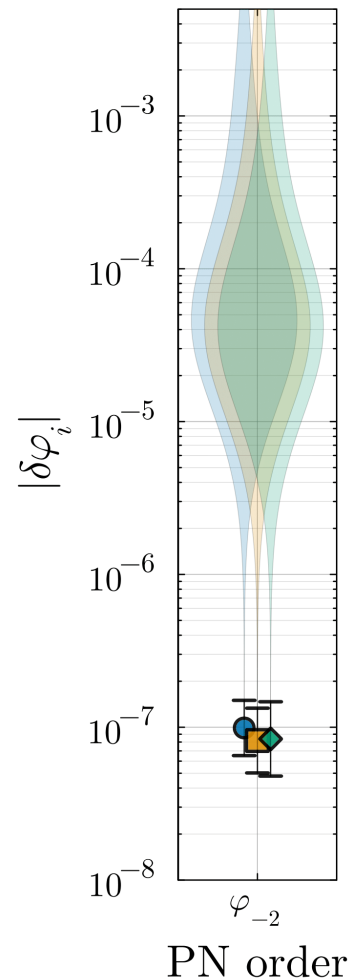
# Comparison with GWTC-3

- We test the 90% upper bound from the Fisher analysis with the actual LVK results. The events analyzed are 9 and the different catalog realizations give the error bars. The violin plots are obtained by the single events bounds.
- HLV indicates that the detectors used are Livingston, Hanford and Virgo.
- There is great agreement between the Fisher and the GWTC-3 collaboration results



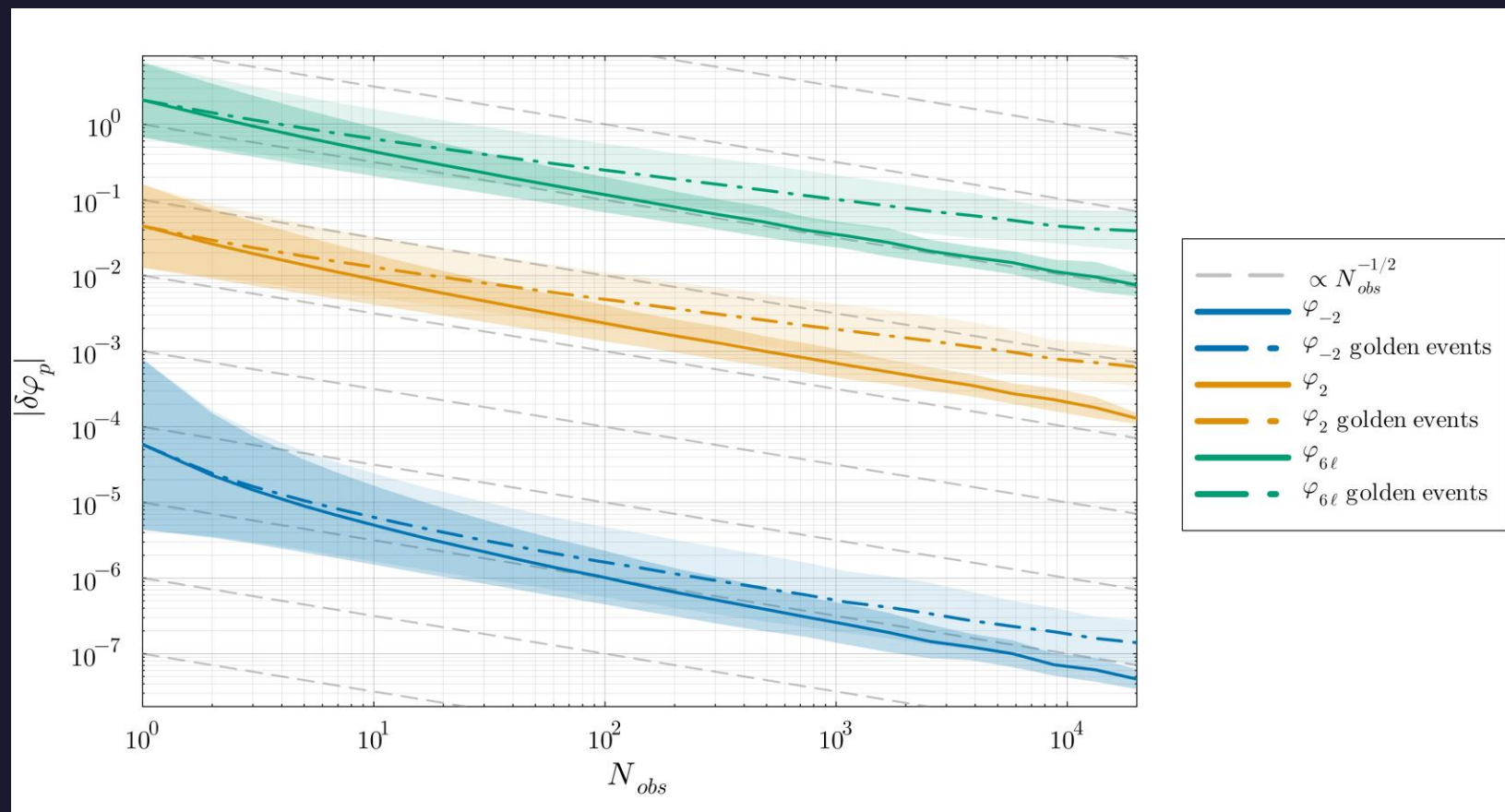
# Forecast for ET

- We test 3 detector designs for ET
- All detectors improve significantly the LVK constraint (2 orders of magnitude) while -1 PN improves by 4 orders due to the lower frequency minimum of ET
- These results correspond to approx. 4 months of observations



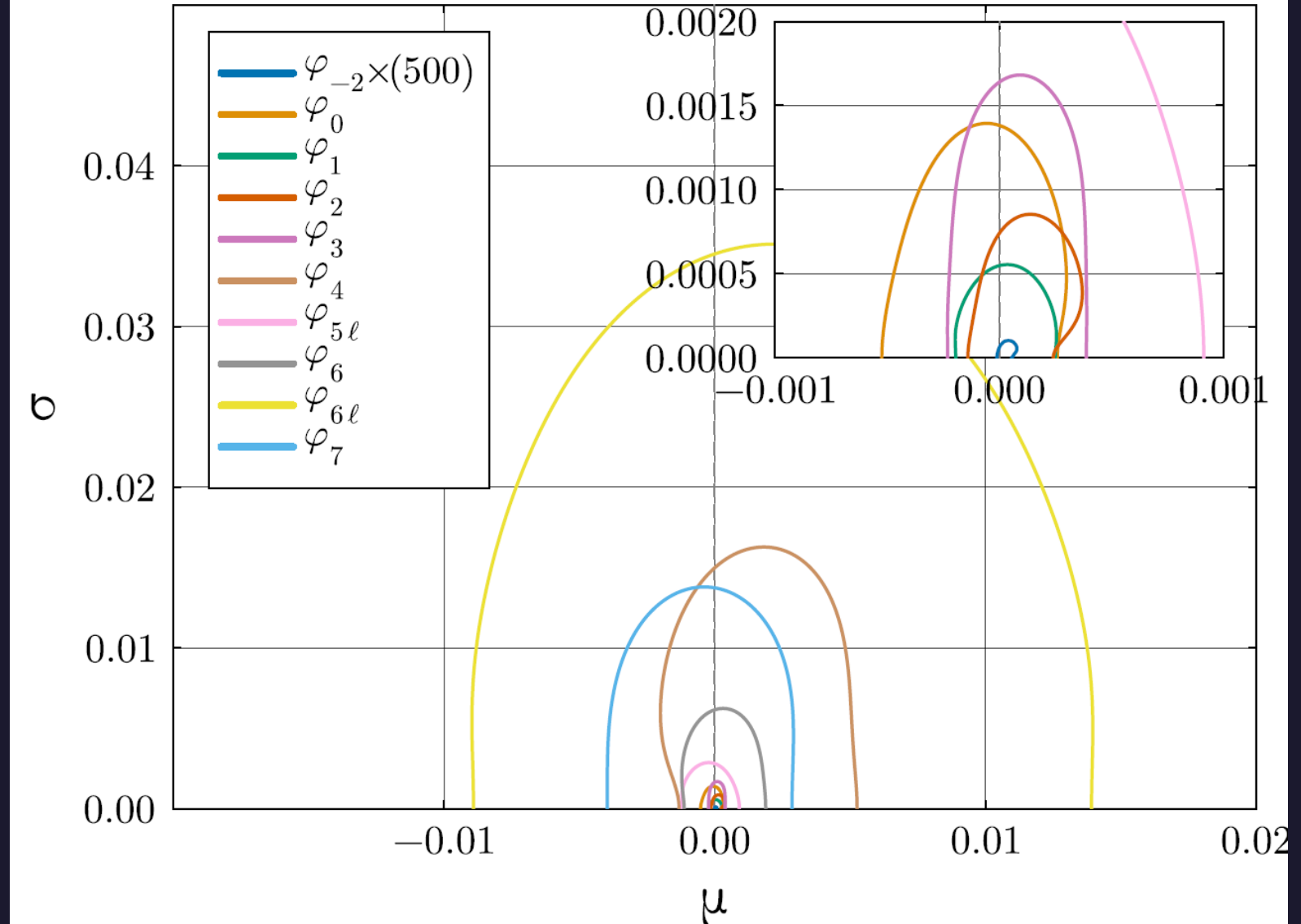
# Forecast for ET

- “How do the bounds scale with the number of sources?”
- “What is the role of the golden events compared to the larger statistics?”



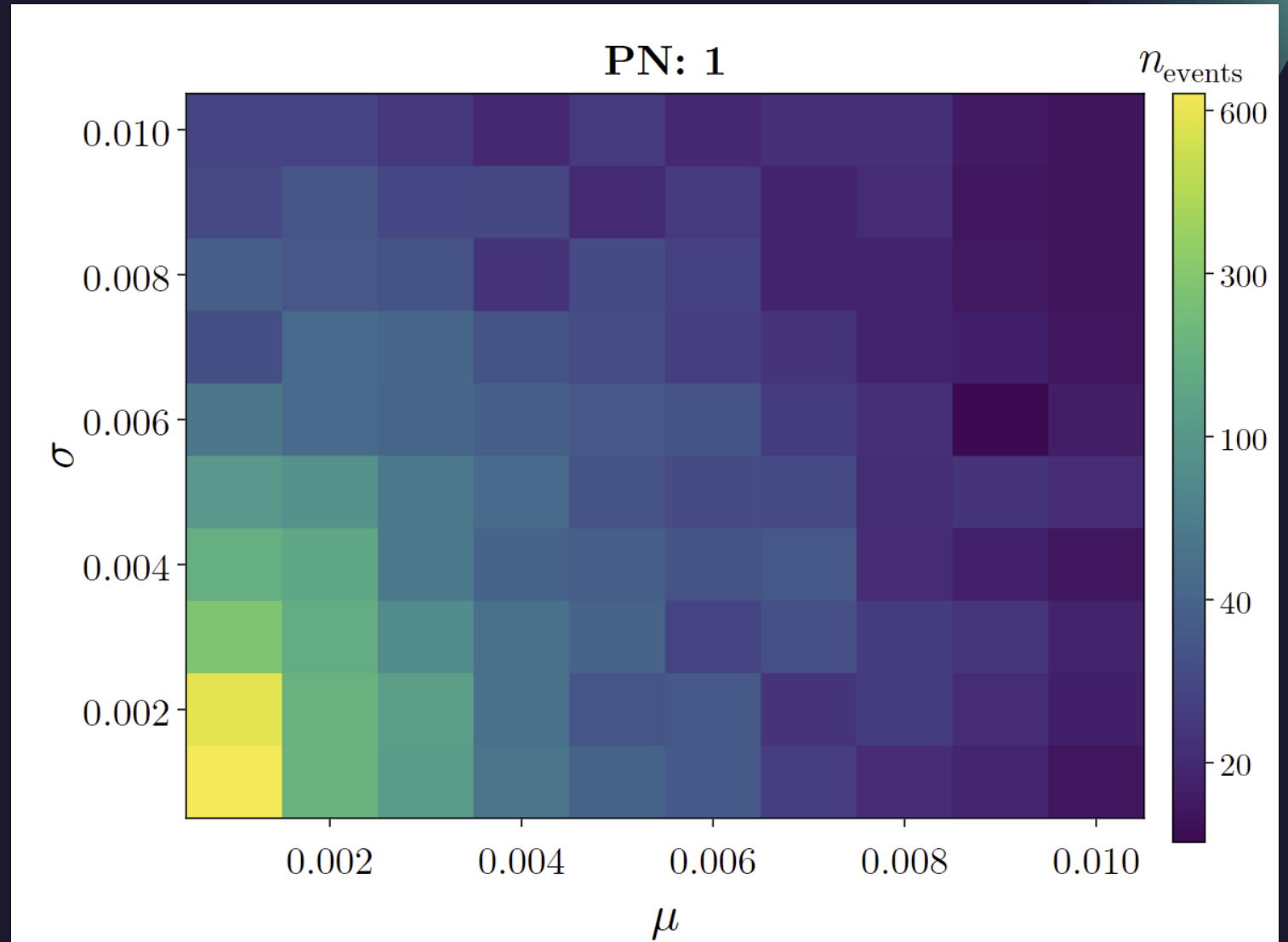
# Forecast for ET

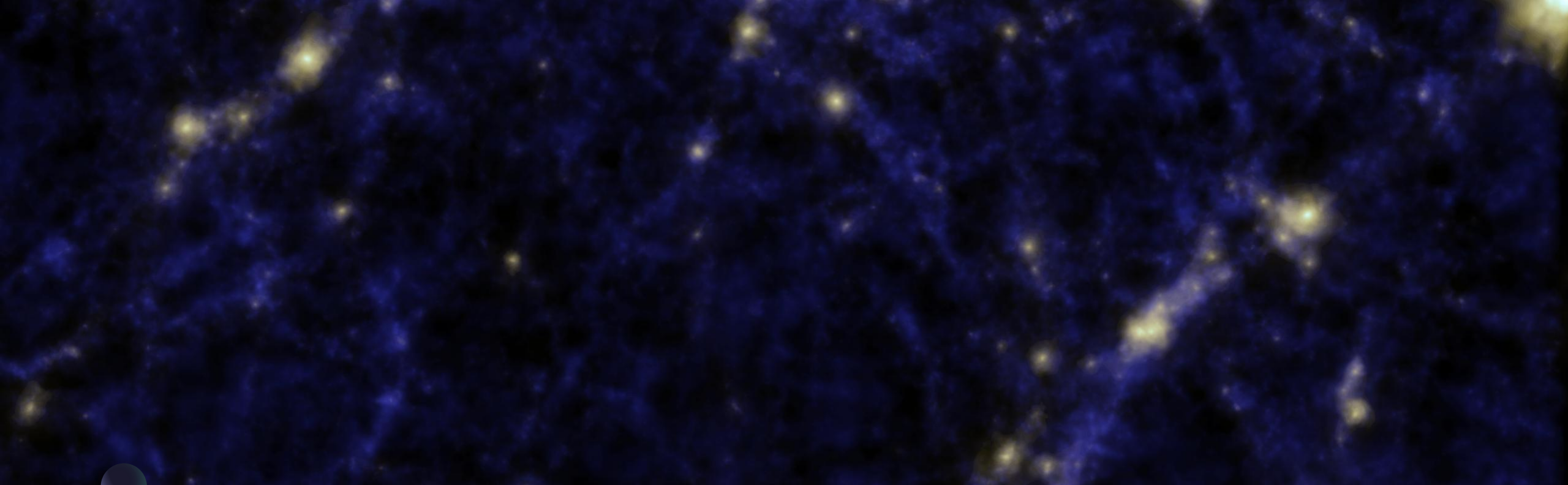
- We model the deviation as a gaussian with mean  $\mu$  and std  $\sigma$ .
- We inject events with no GR deviations, and we recover the hyperparameters ( $\mu$ ,  $\sigma$ ).
- We combine the observations in a hyperparameter framework
- These results corresponds to 4/5 months of observations



# Forecast for ET

- How many detections are needed to detect a GR deviation at 90% confidence level?
- We inject events with GR deviations drawn from a gaussian with given  $\mu$  and  $\sigma$
- These constrains can be achieved in weeks or days of observations
- We used 2L\_45 and the PN I term

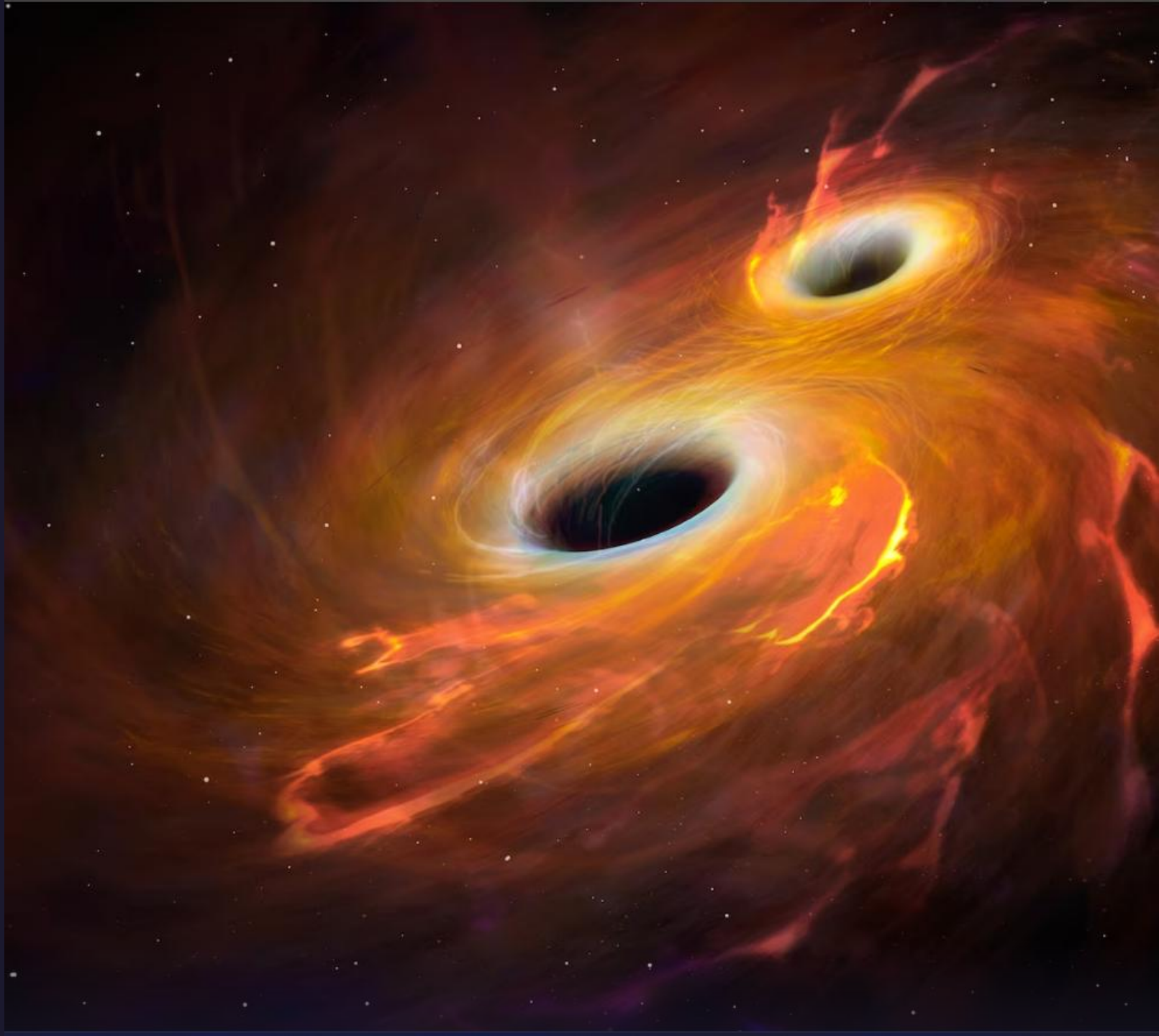




# Take home messages

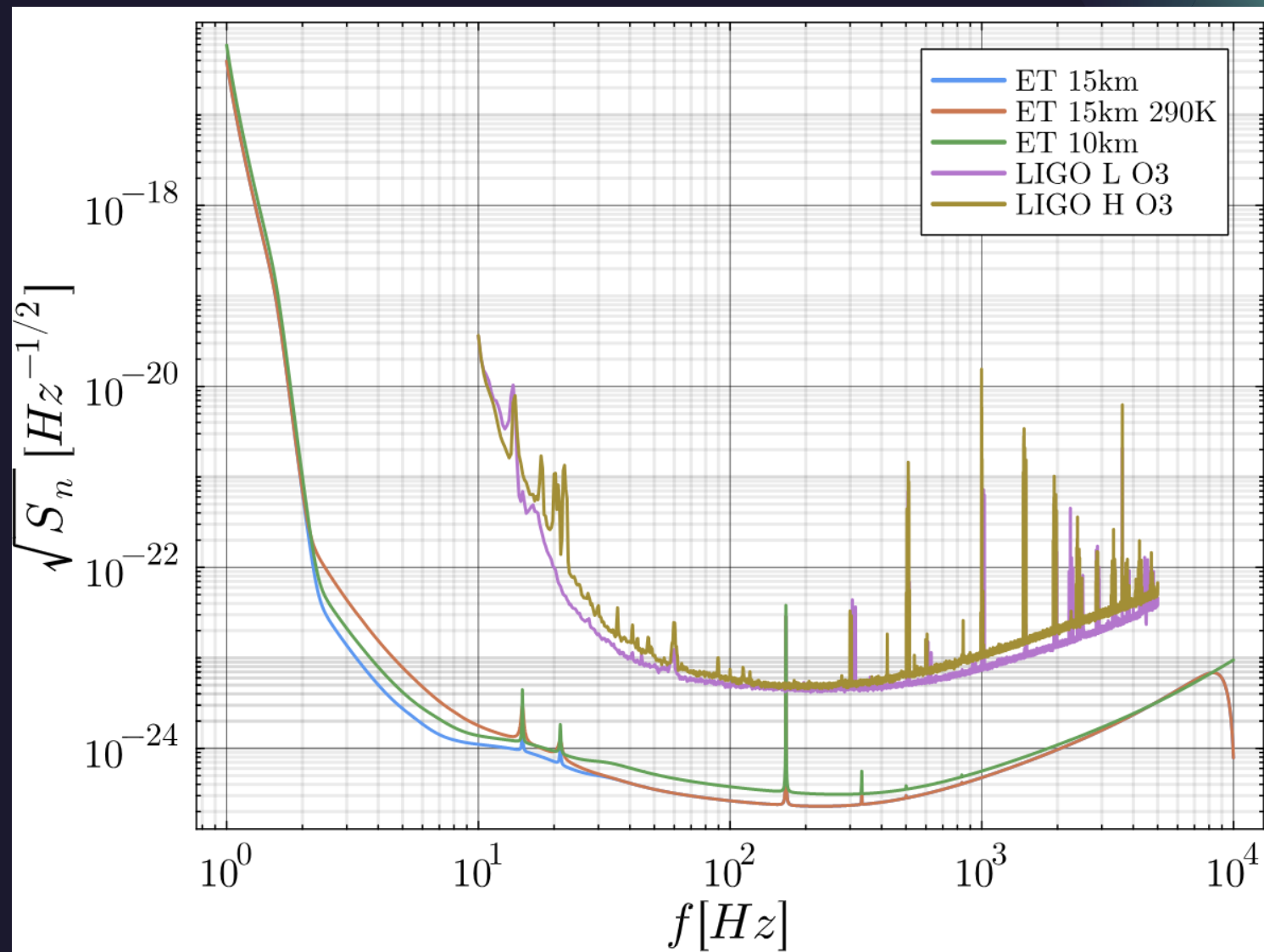
- GWJulia is a fast open-source tool to evaluate Fisher matrices of CBC
- BGR deviations are SNR driven thus the different networks produce comparable results
- With ET, BGR test of the PN terms will reach an unprecedented level of precision with just a handful of sources

Thank you!



# What ET brings to the table

- Larger frequency space
- Detections up to 50k BBH per year
- Detections up to  $z \sim 10$
- SGWB:  $\Omega_{\text{GW}} \sim 10^{-12}$



# Case study – SNR

| Network    | SNR > 8 | SNR > 12 | SNR > 20 | SNR > 50 | SNR > 100 |
|------------|---------|----------|----------|----------|-----------|
| T          | 87.6 %  | 71.1 %   | 43.3 %   | 9.1 %    | 1.7 %     |
| 2L_0       | 89.3 %  | 78.6 %   | 56.5 %   | 15.7 %   | 3.6 %     |
| 2L_45      | 94.1 %  | 82.9 %   | 58.1 %   | 15.6 %   | 3.5 %     |
| 2L_290K_0  | 87.4 %  | 75.5 %   | 52.3 %   | 13.4 %   | 2.9 %     |
| 2L_290K_45 | 92.3 %  | 79.6 %   | 53.5 %   | 13.3 %   | 2.8 %     |

The table represents the percentage of events above the threshold written on top of the column



# Case study – Angular Precision

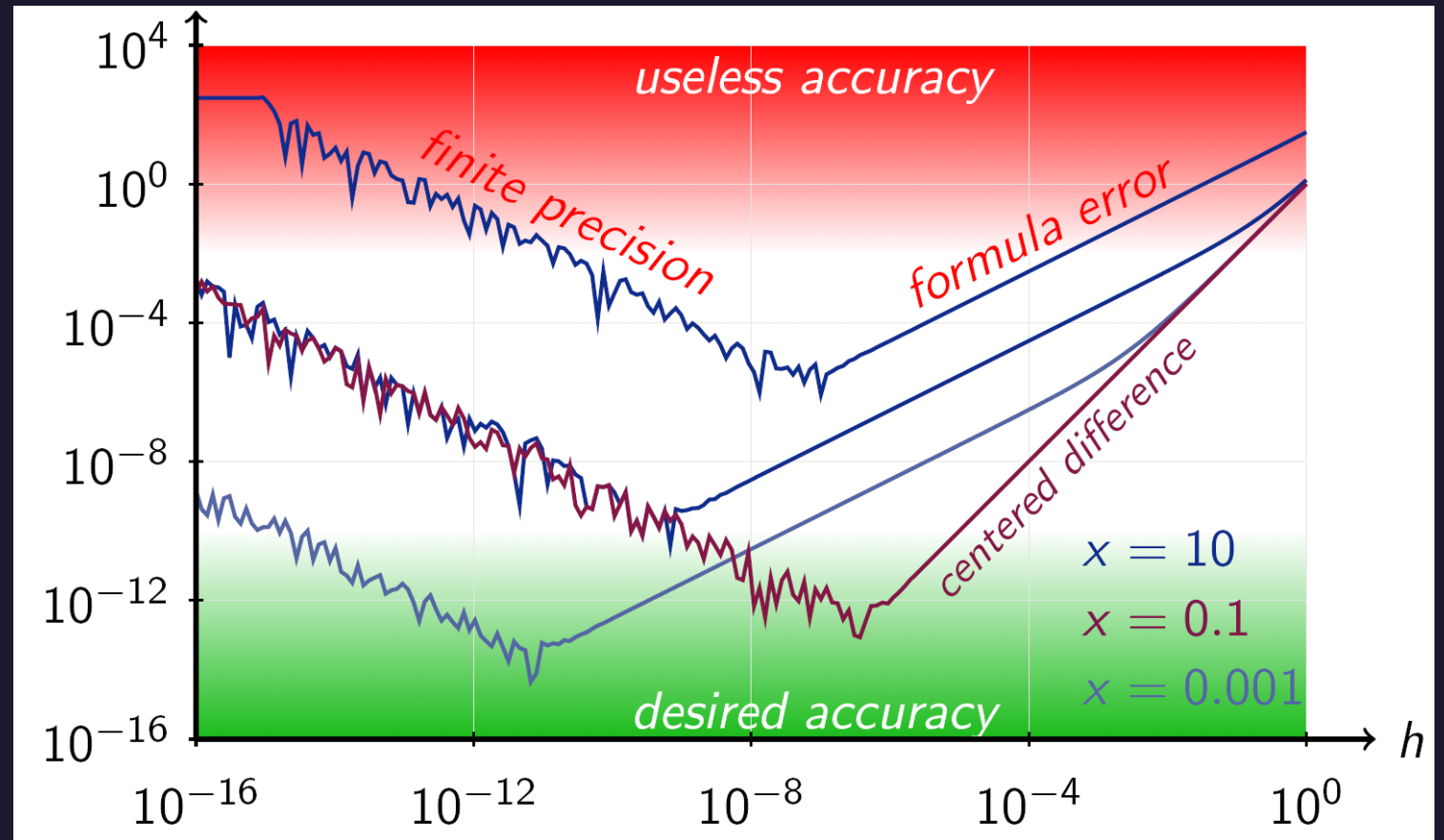
| Network    | $\Omega < 10 \text{ deg}^2$ | $\Omega < 100 \text{ deg}^2$ | $\Omega < 1000 \text{ deg}^2$ | $\Omega < \text{Whole sky}$ |
|------------|-----------------------------|------------------------------|-------------------------------|-----------------------------|
| T          | 0.31 %                      | 3.86 %                       | 12.65 %                       | 34.24 %                     |
| 2L_0       | 0.23 %                      | 4.68 %                       | 32.91 %                       | 75.43 %                     |
| 2L_45      | 1.01 %                      | 9.63 %                       | 34.04 %                       | 72.22 %                     |
| 2L_290K_0  | 0.18 %                      | 3.79 %                       | 30.14 %                       | 72.49 %                     |
| 2L_290K_45 | 0.81 %                      | 8.67 %                       | 32.18 %                       | 68.72 %                     |

The table represents the percentage of events of which the 90% sky areas are less than the threshold indicated in each column. The 45° networks outperform the other networks and the T configuration. In particular, the T is comparable when considering the few high-precision sources, however, the performance degrades significantly for sources with precision worse than  $1000 \text{ deg}^2$ .



# Numerical derivatives

- There is an optimal step size to maximize the accuracy
- The step size is usually dependent on both the function and the point where the derivative is calculated
- There are better alternatives to finite difference methods



Credits: Wikipedia



# Automatic Differentiations

- Accurate (at machine level)
- If a derivative exists, it will find it
- Very fast (2x the evaluation of the target function in our case)
- Adopted from ML



# Automatic Differentiations

Variable = [ value,  $\partial_x$ variable,  $\partial_y$ variable ]

$$\begin{aligned}\langle u, u' \rangle + \langle v, v' \rangle &= \langle u + v, u' + v' \rangle \\ \langle u, u' \rangle - \langle v, v' \rangle &= \langle u - v, u' - v' \rangle \\ \langle u, u' \rangle * \langle v, v' \rangle &= \langle uv, u'v + uv' \rangle \\ \langle u, u' \rangle / \langle v, v' \rangle &= \left\langle \frac{u}{v}, \frac{u'v - uv'}{v^2} \right\rangle \quad (v \neq 0)\end{aligned}$$

# Waveforms

- The measured data consists of the GW signal plus a noise

$$d(t) = s(t) + n(t)$$

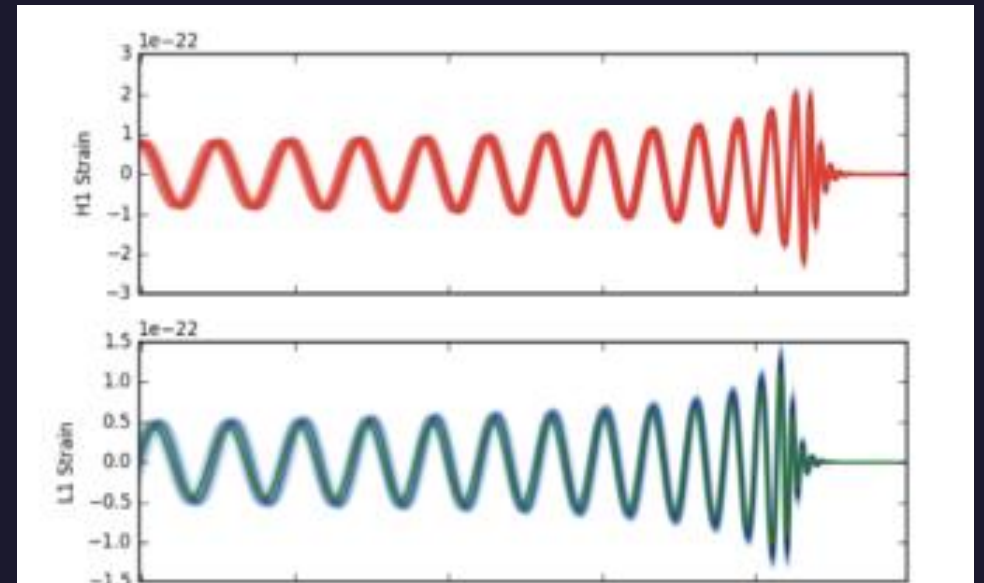
- Where the signal is the projection of the strain onto the detector

$$s = h_+ e_{+ij} D^{ij} + h_{\times} e_{\times ij} D^{ij} \equiv h_+ F_+ + h_{\times} F_{\times}$$

- A waveform is a function that associates to the source parameters the GW, in time or frequency domain
- Here we use the IMRPhenom waveforms [3-8] (up to XHM) that work in the frequency domain.

$$h = \sum_k A_k(f) e^{i\Phi(f)}$$

- [3] Khan et al. 2015
- [4] London et al. 2018
- [5] García-Quirós et al. 2020
- [6] Pratten et al. 2020a
- [7] Pratten et al. 2020b
- [8] Dietrich et al. 2019



# Julia

- Compiled (Python is interpreted)
- Designed for scientific computing
- As easy to write as Python

```
def factorial(n):  
    if n == 0 or n == 1:  
        return 1  
    else:  
        return n * factorial(n - 1)  
  
# Example usage  
num = 5  
print(f"The factorial of {num} is {factorial(num)}")
```

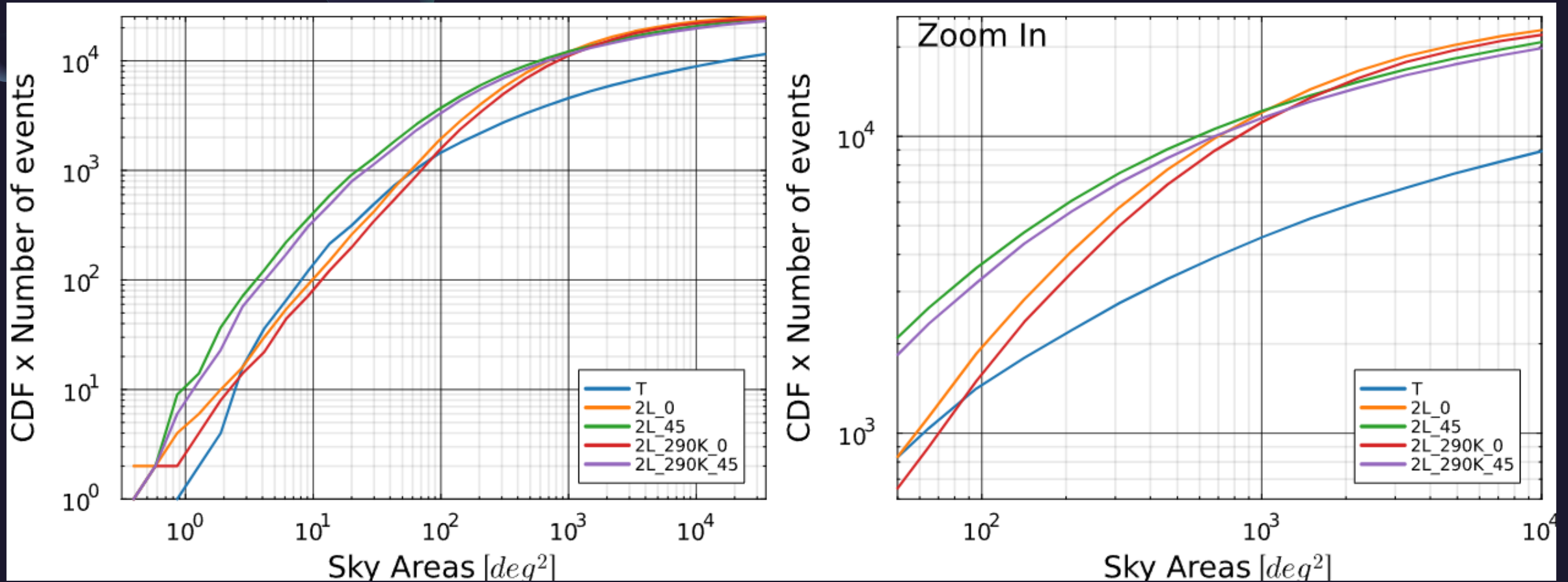
```
function factorial(n)  
    if n == 0 || n == 1  
        return 1  
    else  
        return n * factorial(n - 1)  
    end  
end  
  
# Example usage  
num = 5  
println("The factorial of $num is $(factorial(num))")
```

# Future plans – Fisher informed HMC

- Bayesian data analysis of CBC is very computationally demanding (e.g., using nested sampling)
- If we are interested in future observations, we can simplify the inference problem, i.e., we know an approximate location of the maximum posterior
- With automatic differentiation we can perform fast and accurate gradients
- We can perform Hamiltonian Monte Carlo sampling, injecting information from the Fisher Matrix

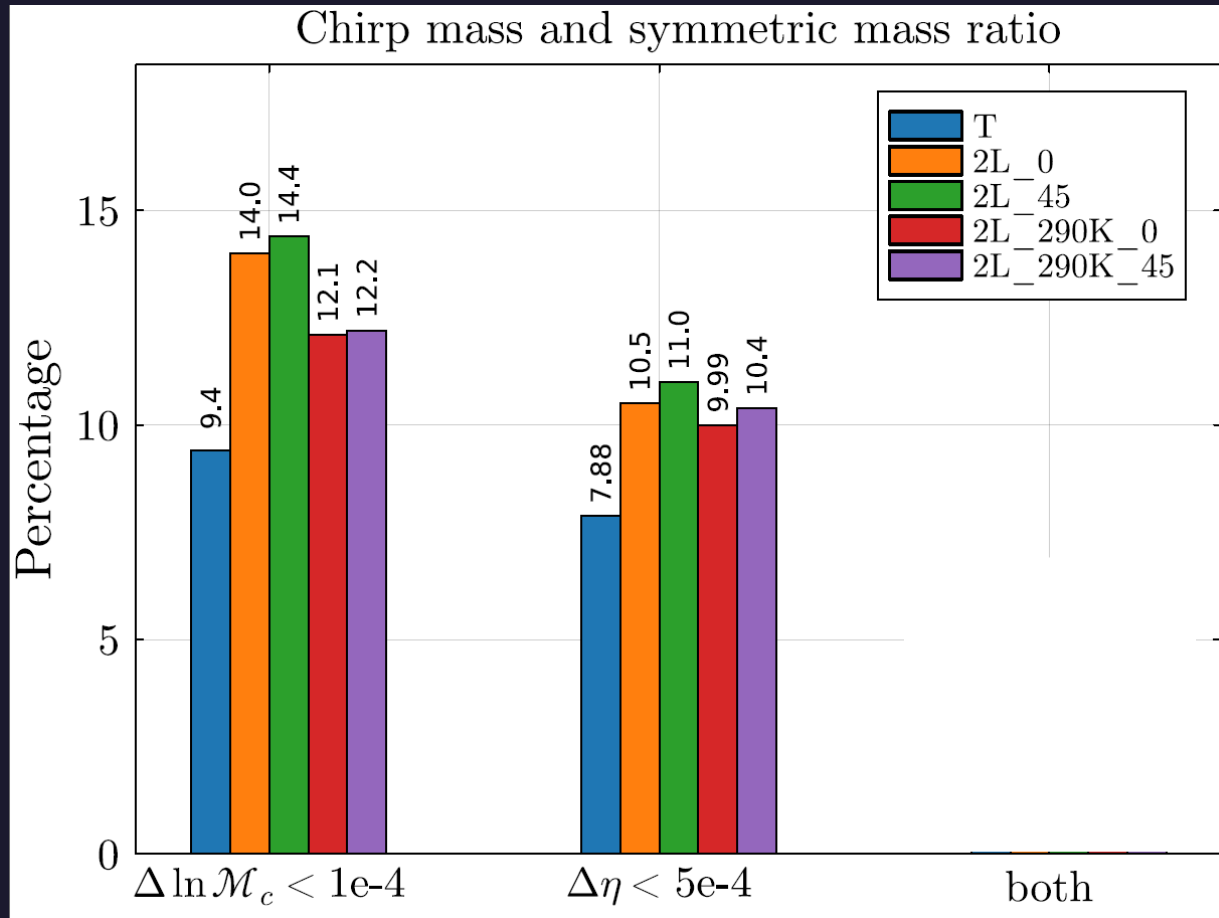


# Case study – Angular Precision



Cumulative density function times the number of events as a function of the 90% sky area. *Right:* zoom in of the left plot

# Source properties



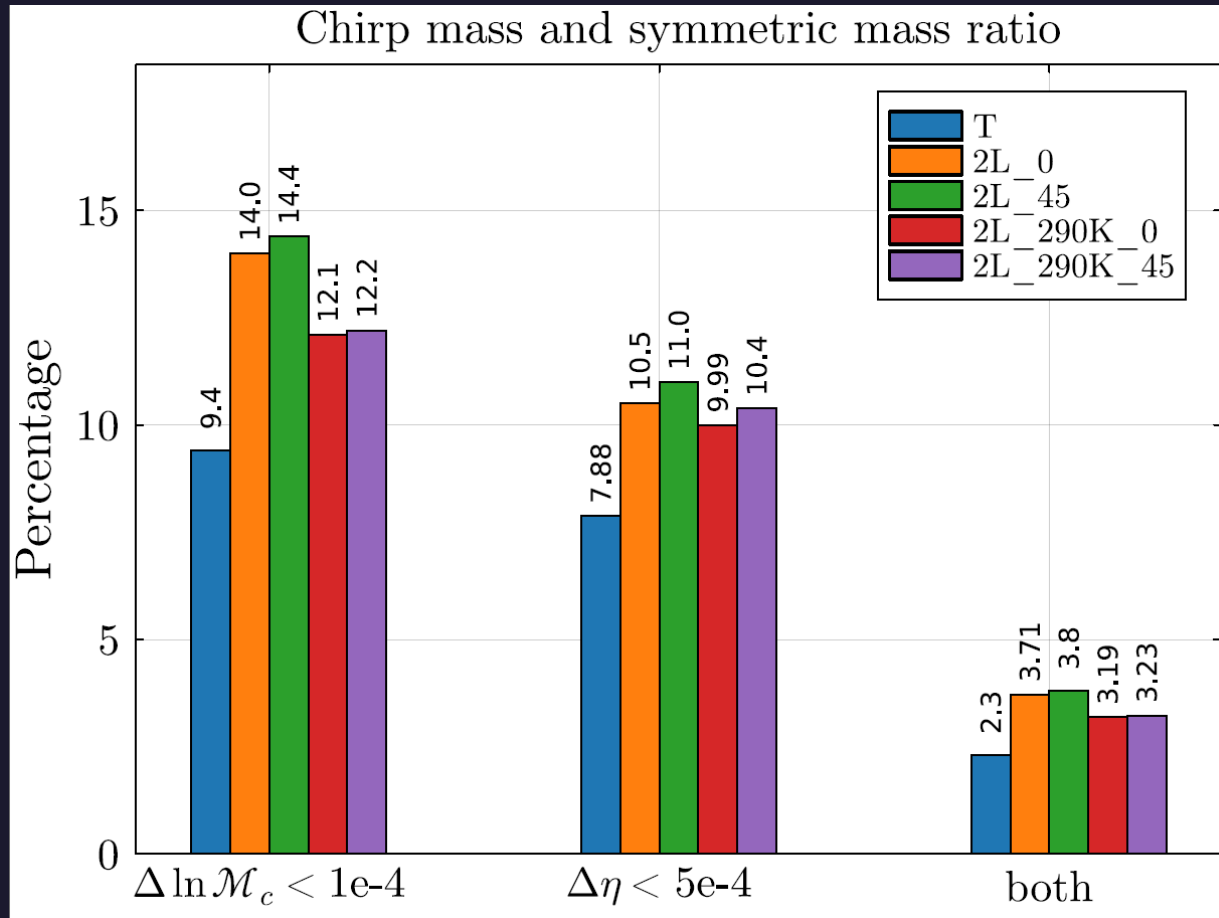
The plot represents the percentage of events of which the 1-sigma error is below  $1e-4$  for the detector chirp mass and  $5e-4$  for the symmetric mass ratio. Only  $\sim 1/3$  of sources that satisfy the mass ratio requirement satisfy both requirements.

$$\mathcal{M}_{c,source} = \frac{m_1^{3/5} m_2^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$\eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$



# Source properties



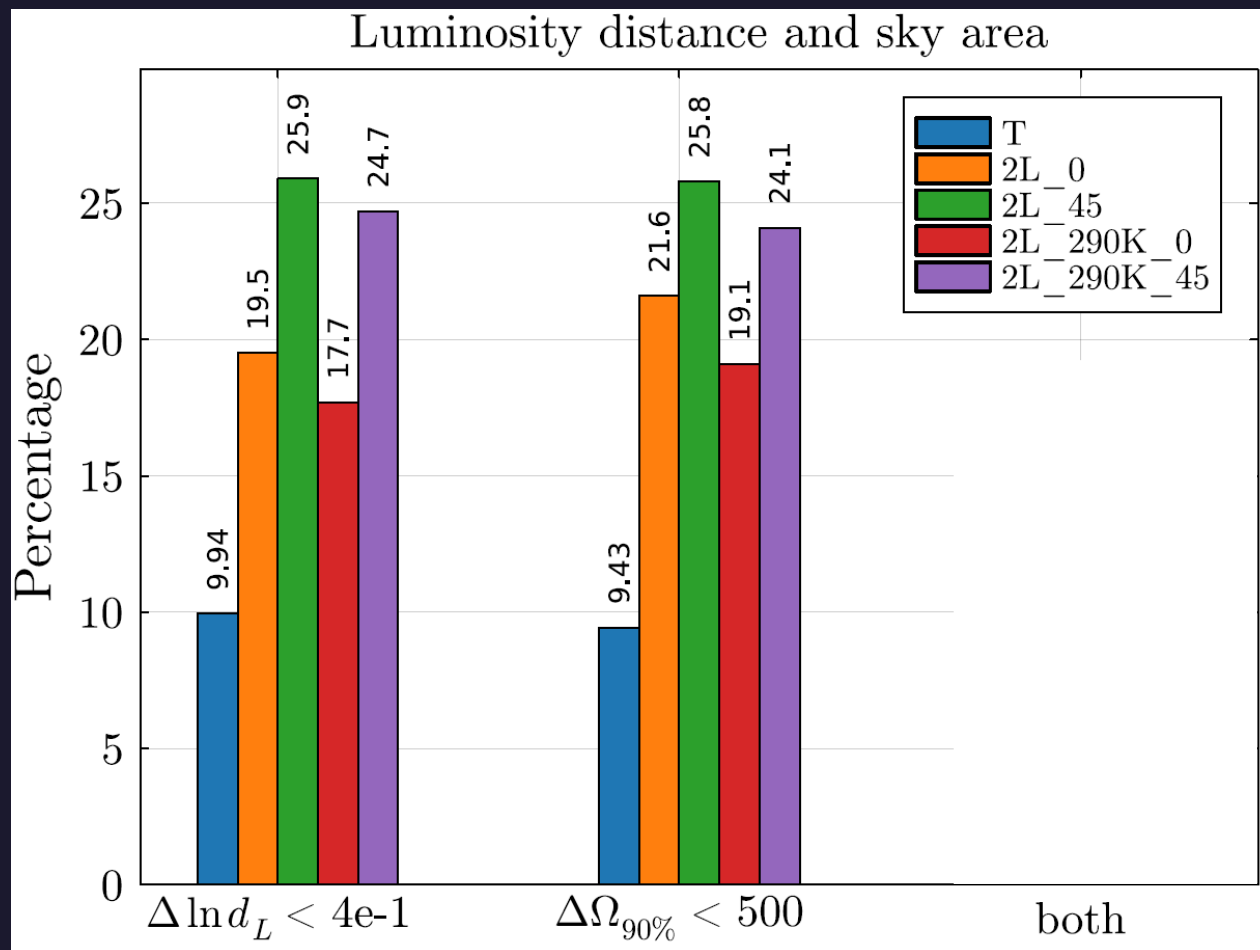
The plot represents the percentage of events of which the 1-sigma error is below  $1e-4$  for the detector chirp mass and  $5e-4$  for the symmetric mass ratio. Only  $\sim 1/3$  of sources that satisfy the mass ratio requirement satisfy both requirements.

$$\mathcal{M}_{c,source} = \frac{m_1^{3/5} m_2^{3/5}}{(m_1 + m_2)^{1/5}}$$

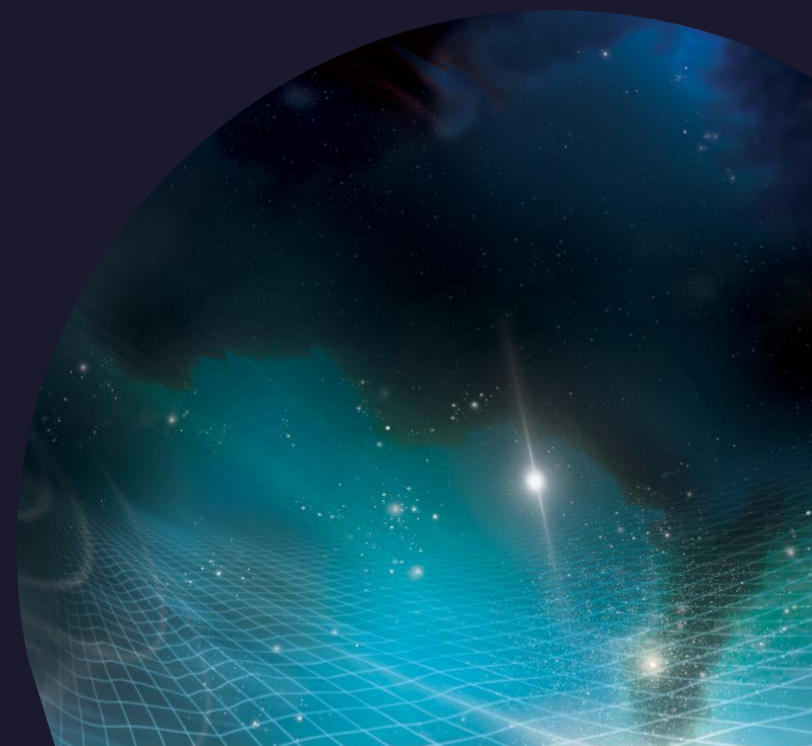
$$\eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$



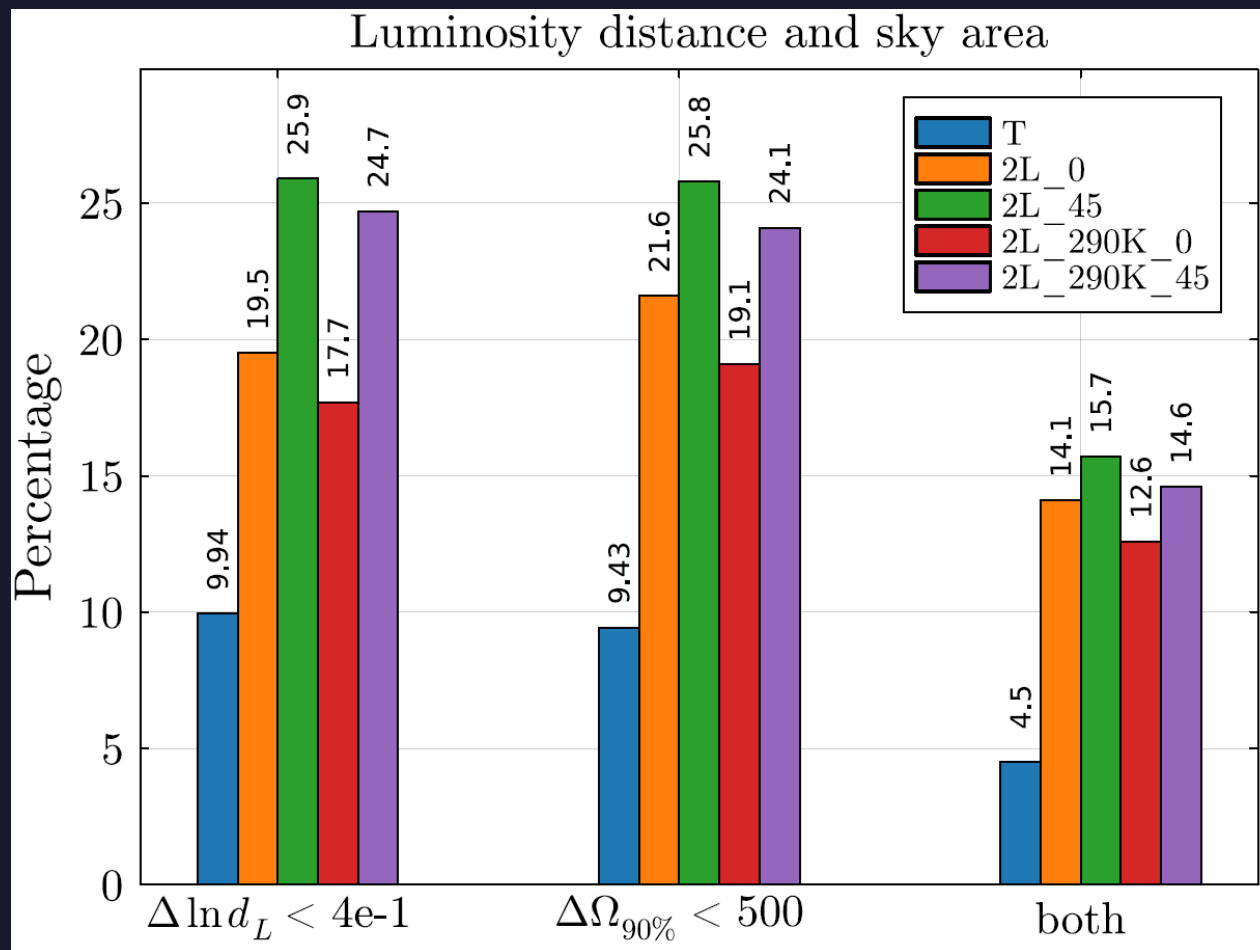
# Cosmology



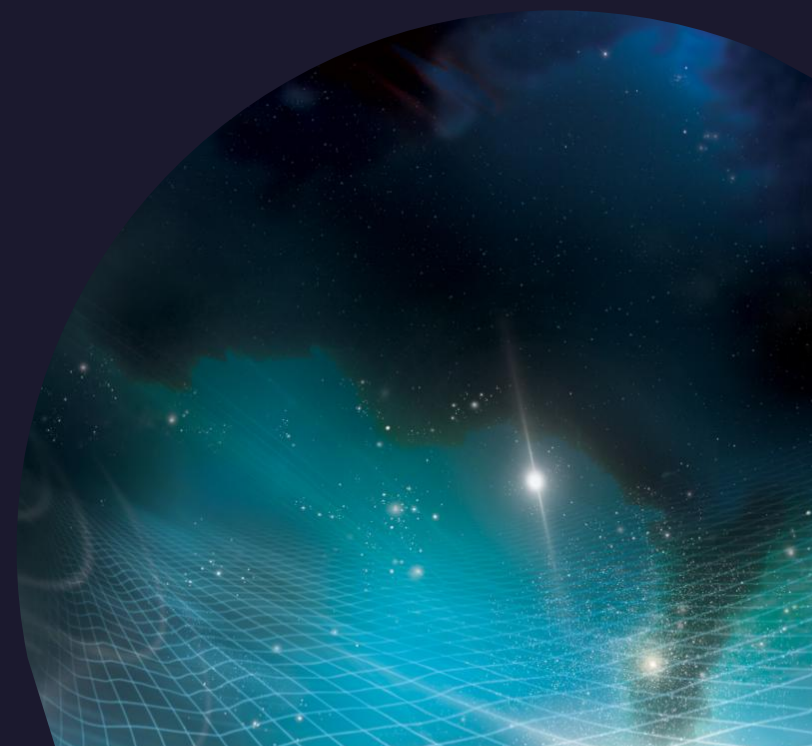
The advantage of the 2L\_45 over 2L\_0 in the luminosity distance and in the sky area is erased when we require both requirements to be satisfied at the same time.



# Cosmology



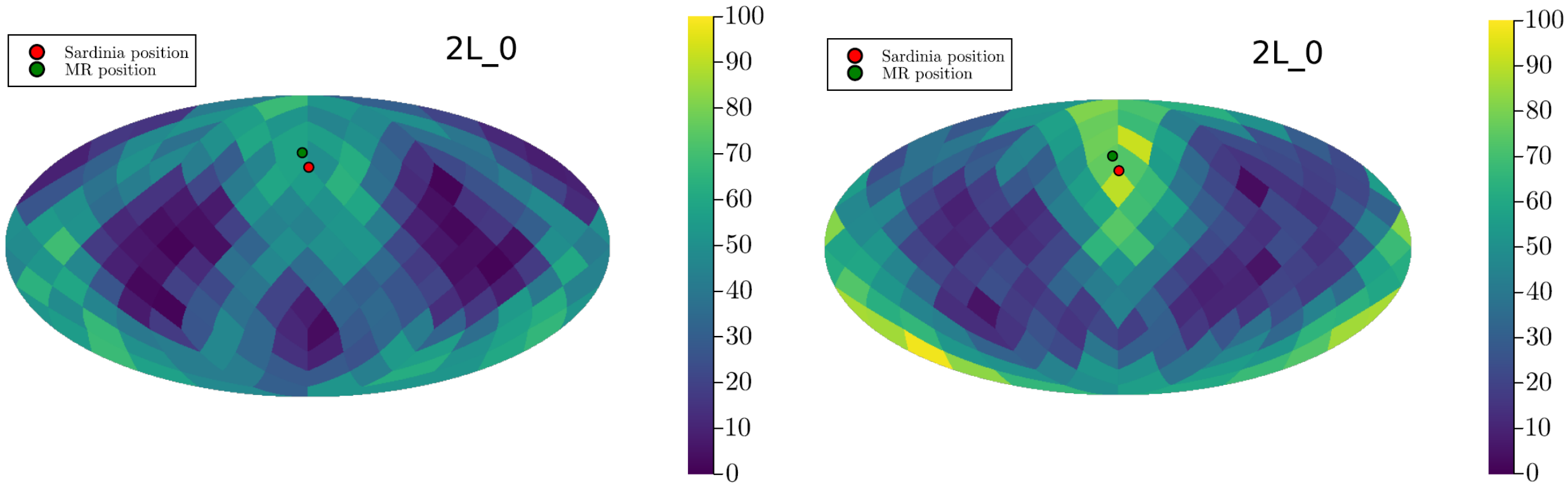
The advantage of the 2L\_45 over 2L\_0 in the luminosity distance and in the sky area is erased when we require both requirements to be satisfied at the same time.



# Sky area and luminosity distance – 2L\_0

Number of events with  $\Delta \ln d_L < 0.4$

Number of events with sky area  $\Omega_{90\%} < 500 \text{ deg}^2$



Sky maps in the detector frame for 2L\_0.

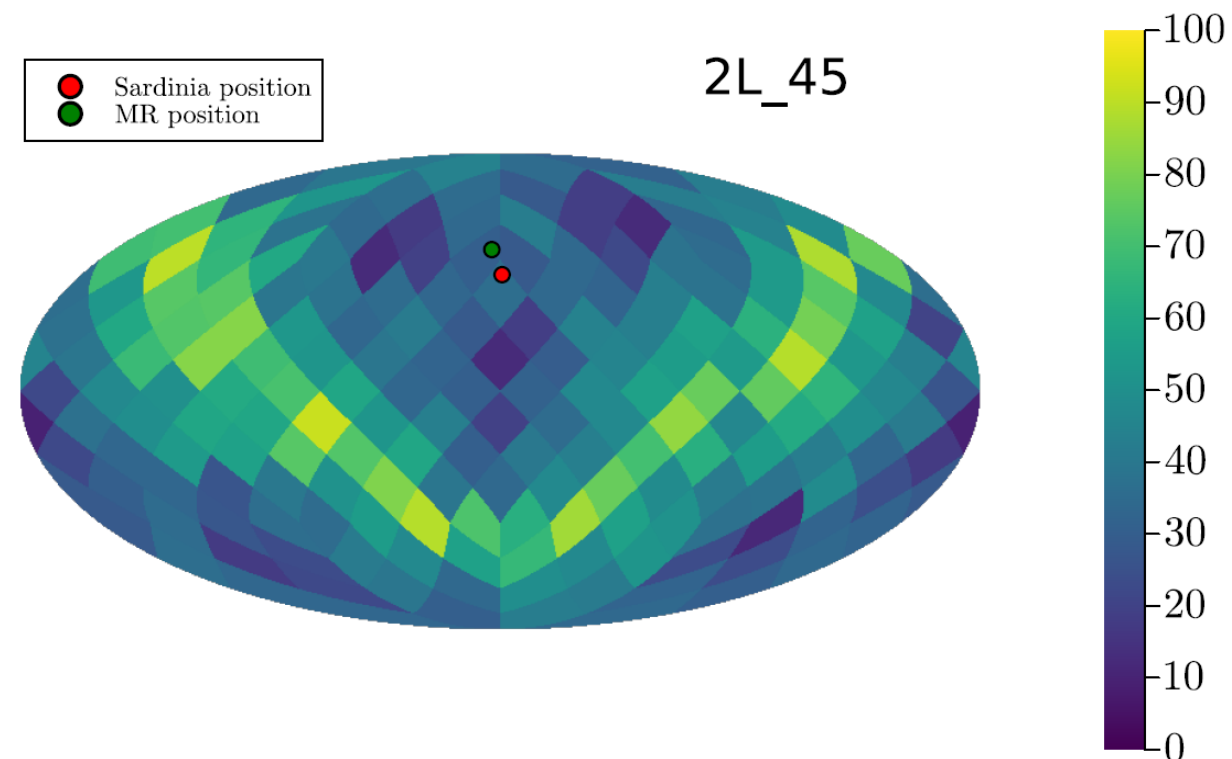
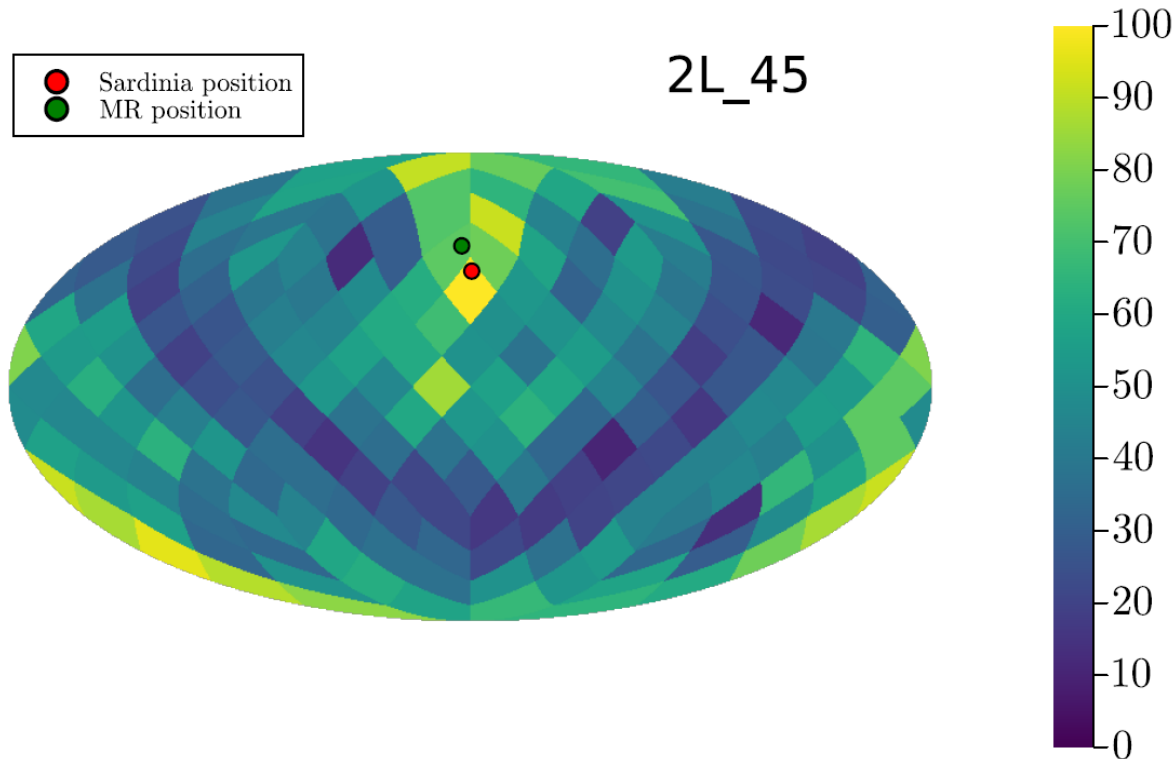
*Left:* number of events for which the relative luminosity distance 1-sigma error is under 40%

*Right:* number of events for which the 90% sky area is under 500 sqdeg.

# Sky area and luminosity distance – 2L\_45

Number of events with  $\Delta \ln d_L < 0.4$

Number of events with sky area  $\Omega_{90\%} < 500 \text{ deg}^2$



Sky maps in the detector frame for 2L\_45.

*Left:* number of events for which the relative luminosity distance 1-sigma error is under 40%

*Right:* number of events for which the 90% sky area is under 500 sqdeg.