

ET phone Home:
**Parity-Violation in the gravitational-wave background
with third-generation ground detectors**

Hannah Duval, Charles Badger, Mairi Sakellariadou



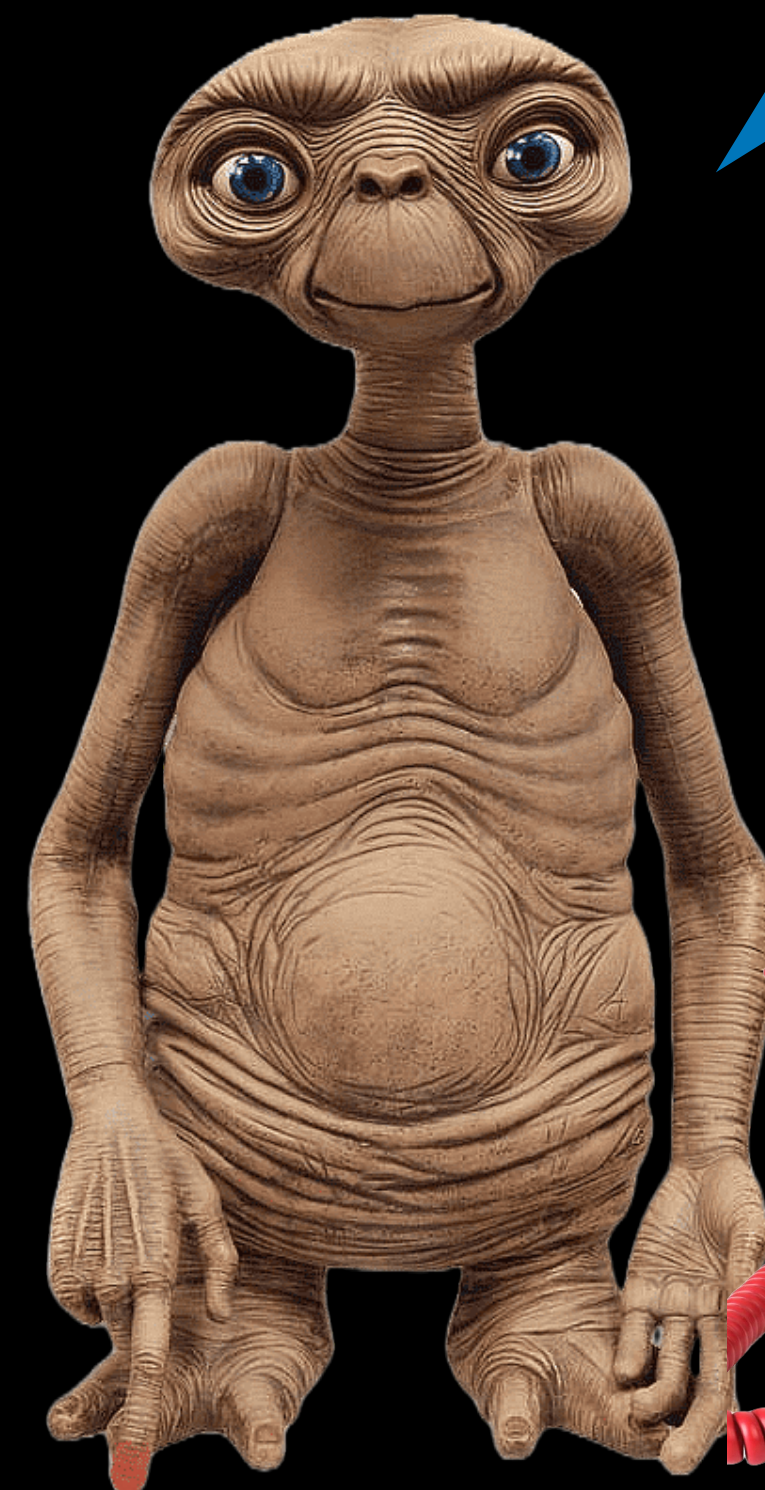
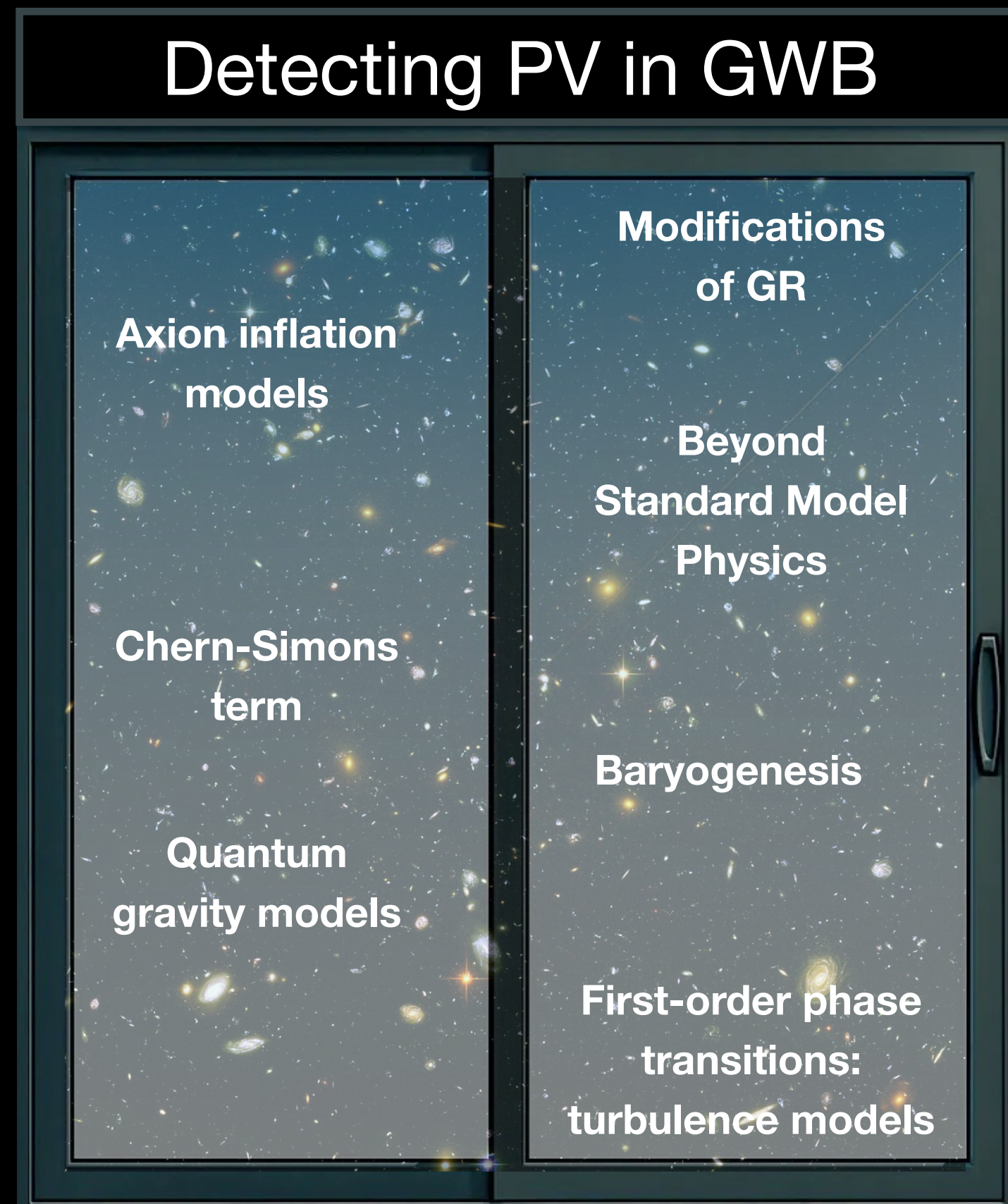
4th Einstein Telescope Annual Meeting

11 November 2025



The story of 2 kinds of aliens, helping us in our quest to detect parity violation (PV) in the gravitational-wave background (GWB).

Let's see when the phone connection is best.



I am ET!



I am ChewiE! But you can call me CE!

Outline

?



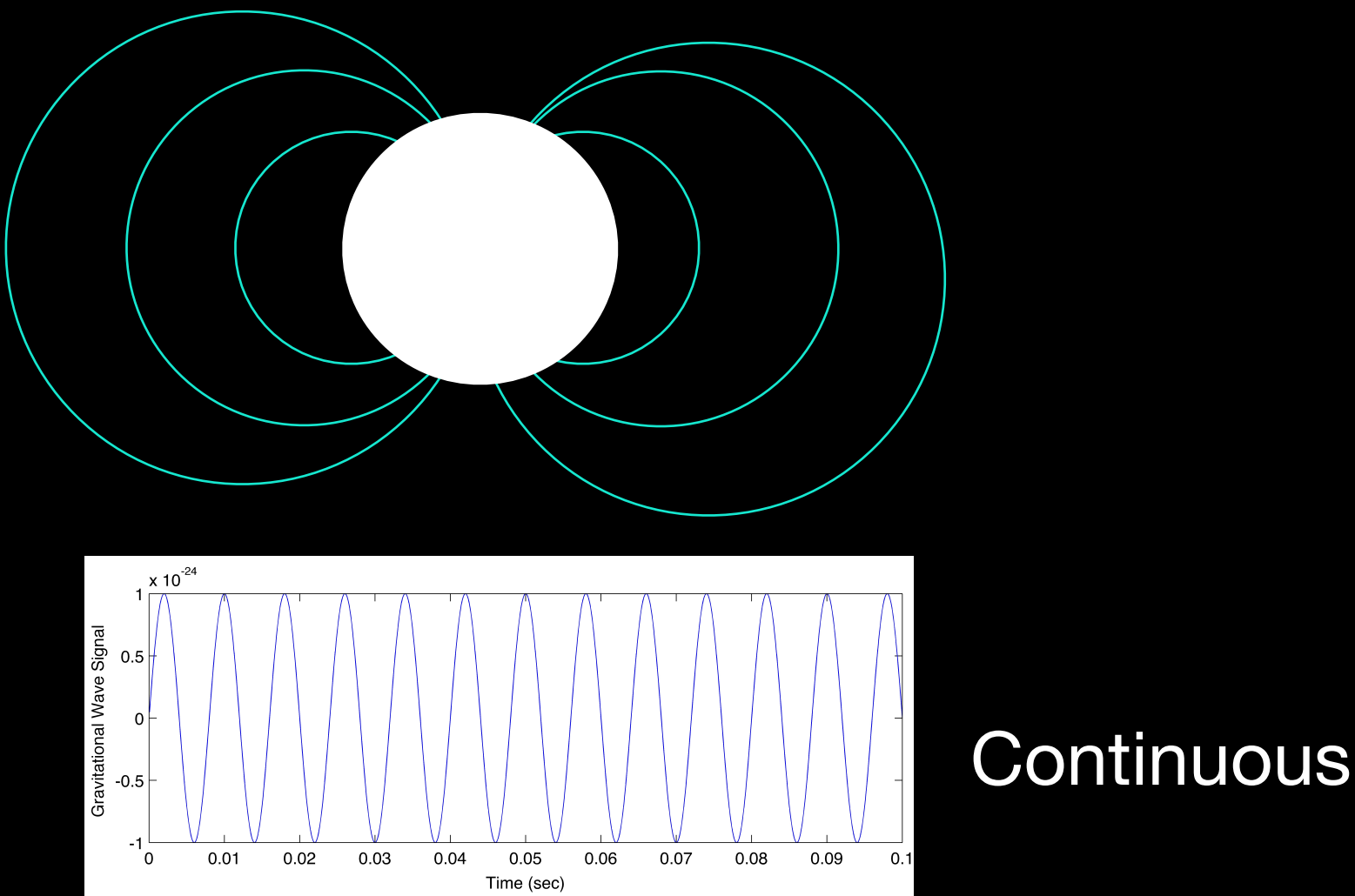
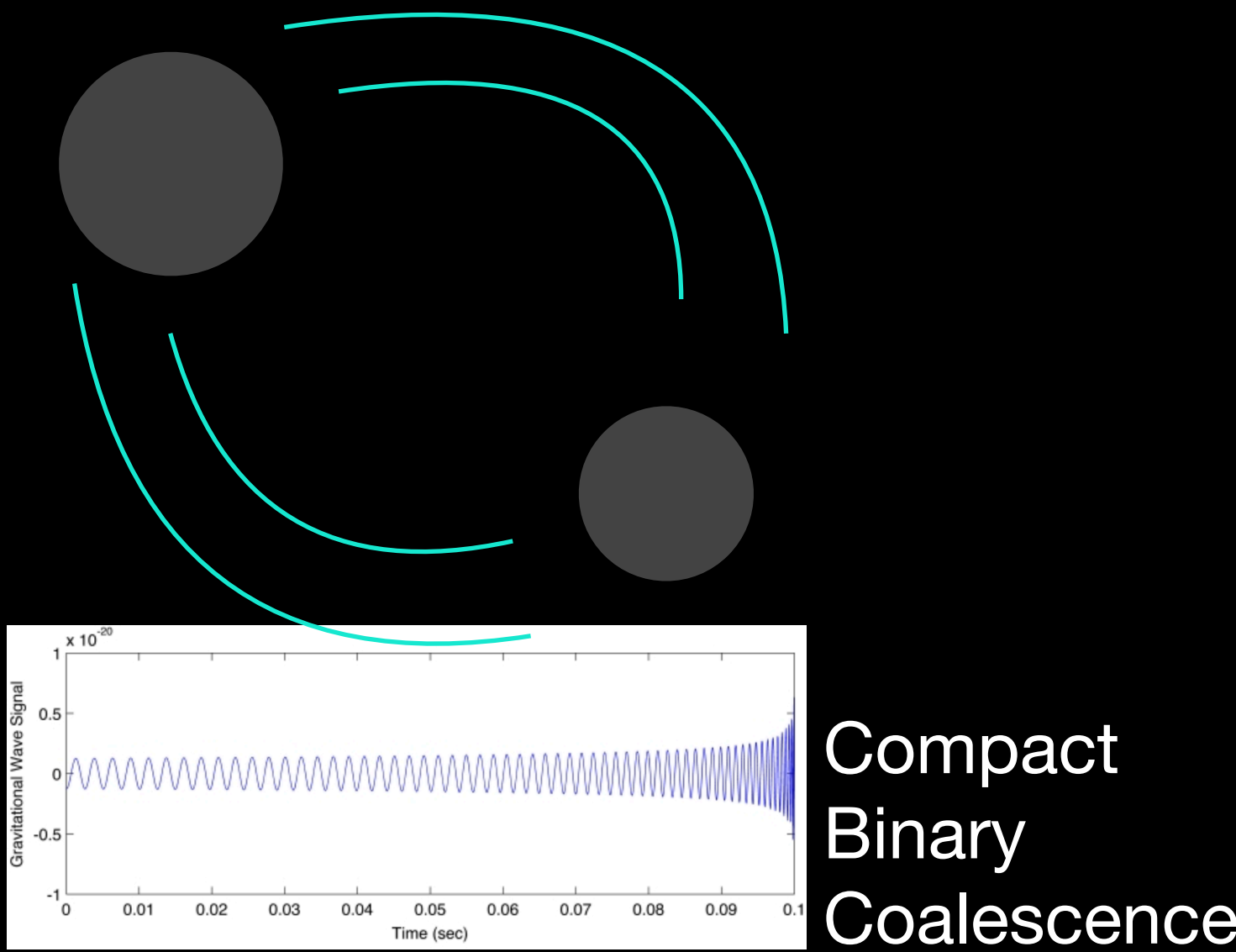
- Brief overview of gravitational-wave background
- Brief visit to Parity Violation
- 3G networks
- Detection Prospects for different configurations:
 - PI curves and SNR prospects
 - Bayesian analysis

Classification Gravitational Waves

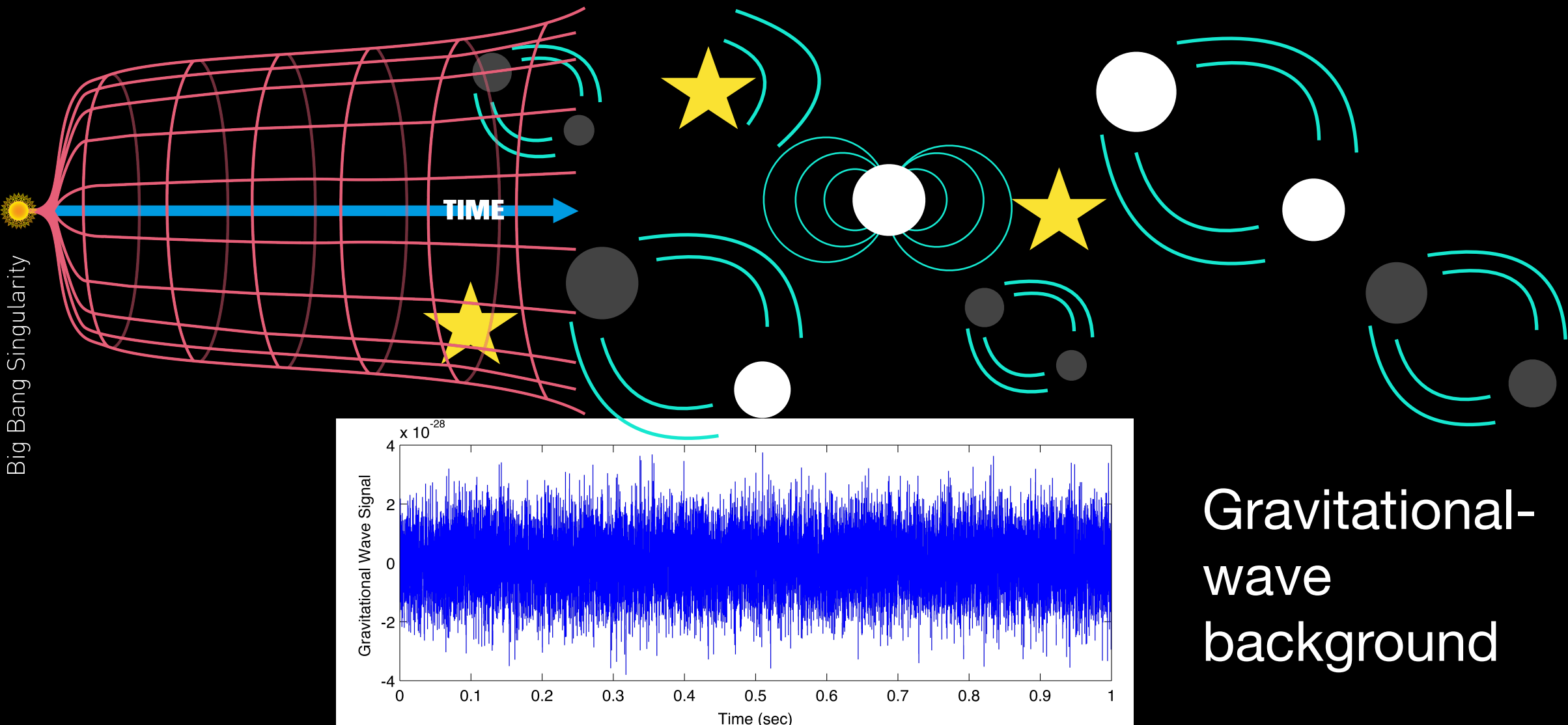
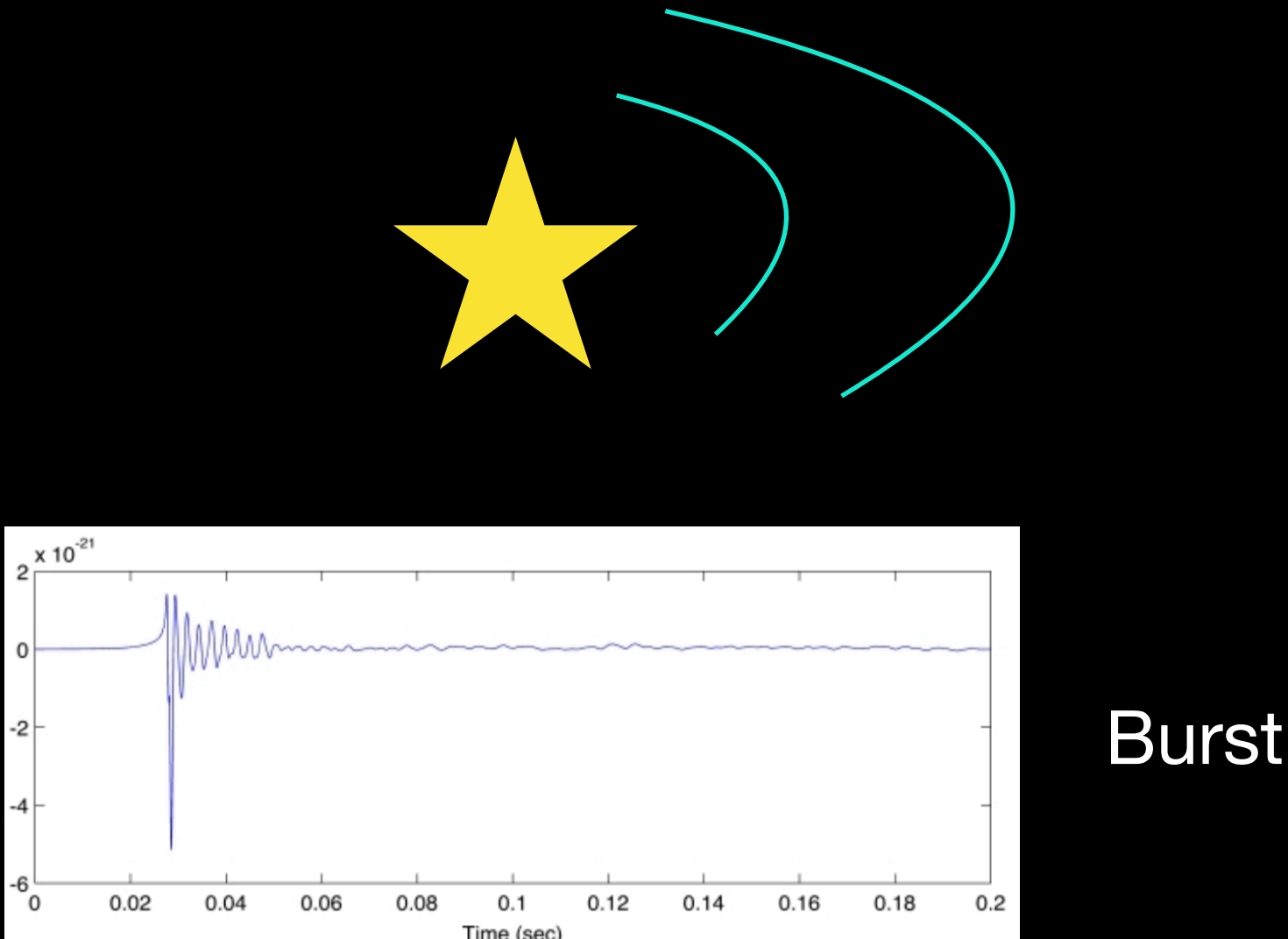
Short Duration

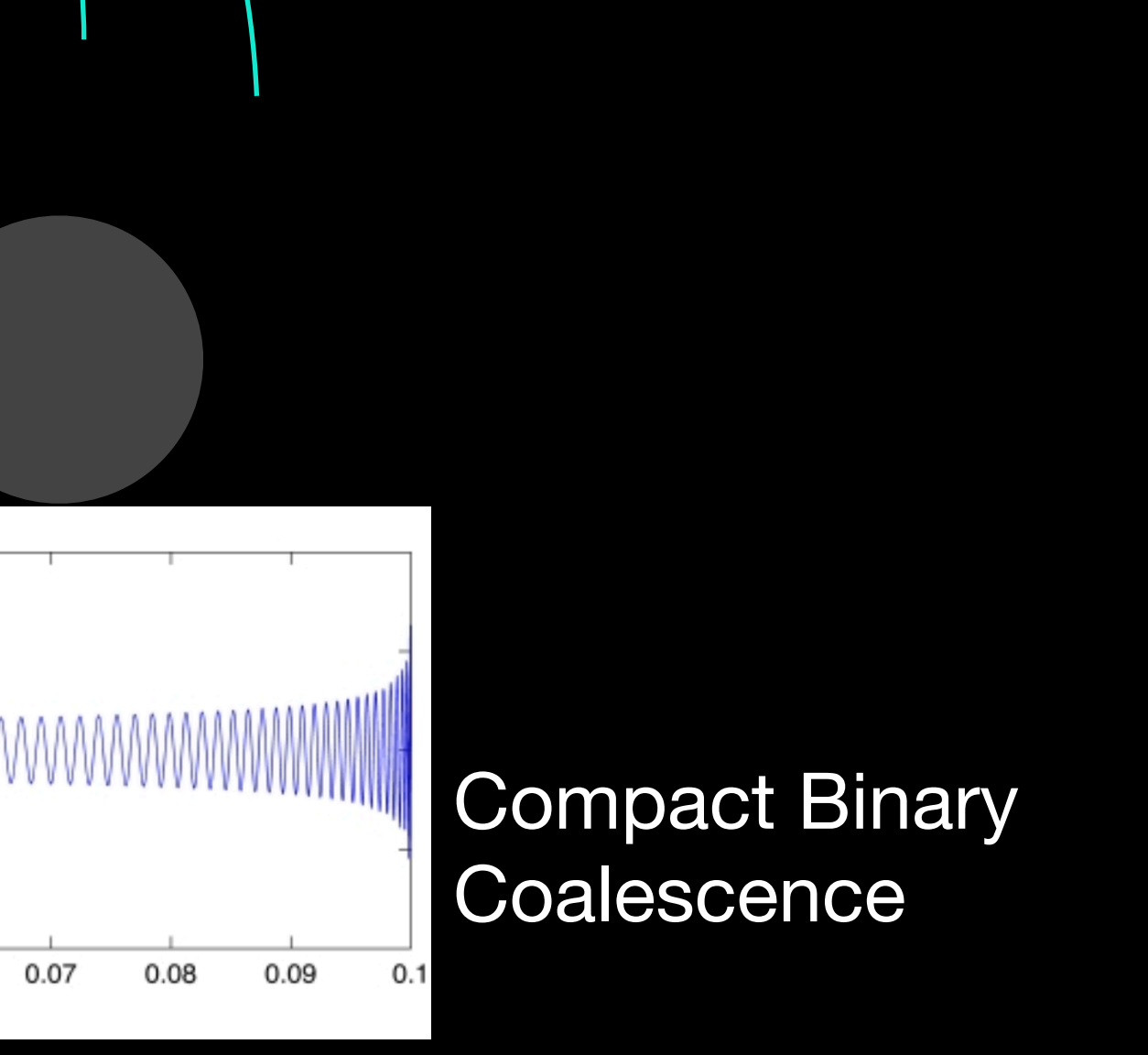
Long Duration

Modelled

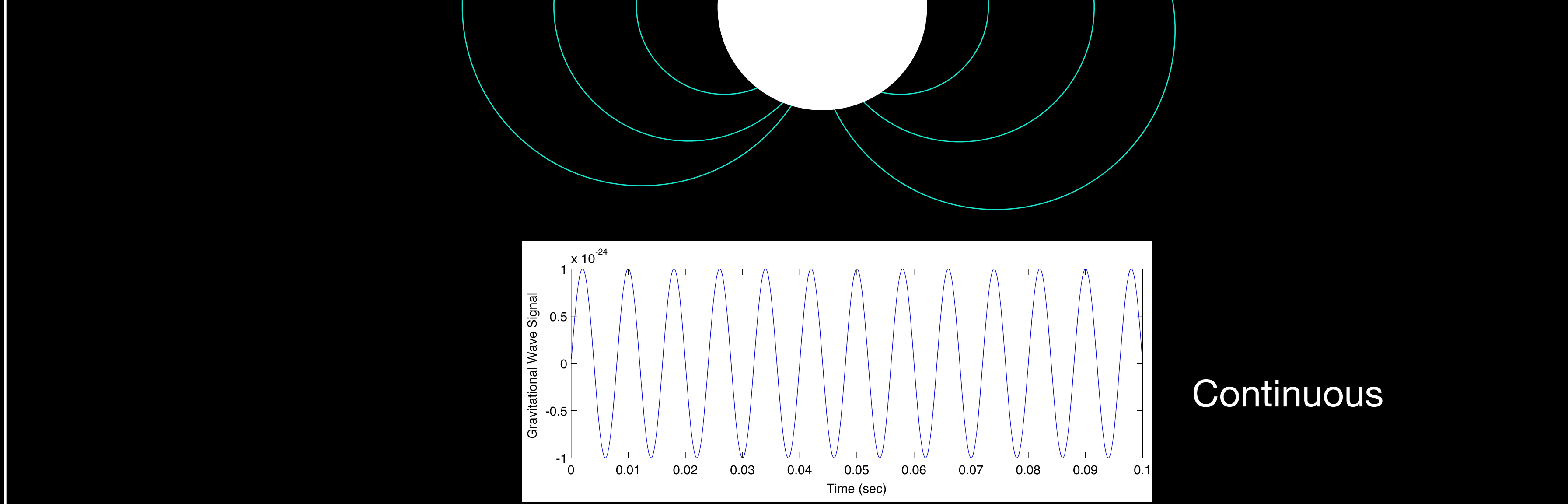


Unmodelled





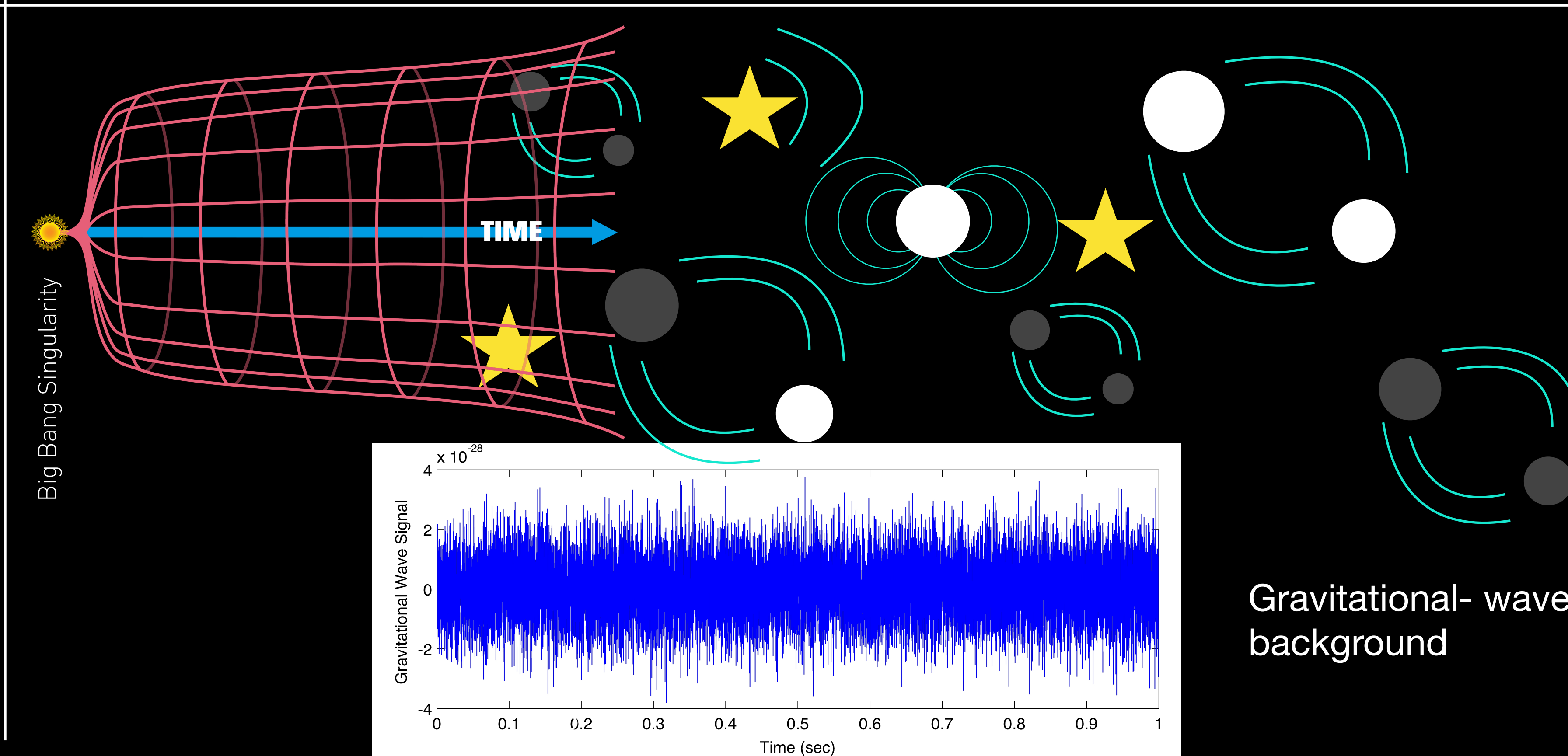
Compact Binary Coalescence



Continuous

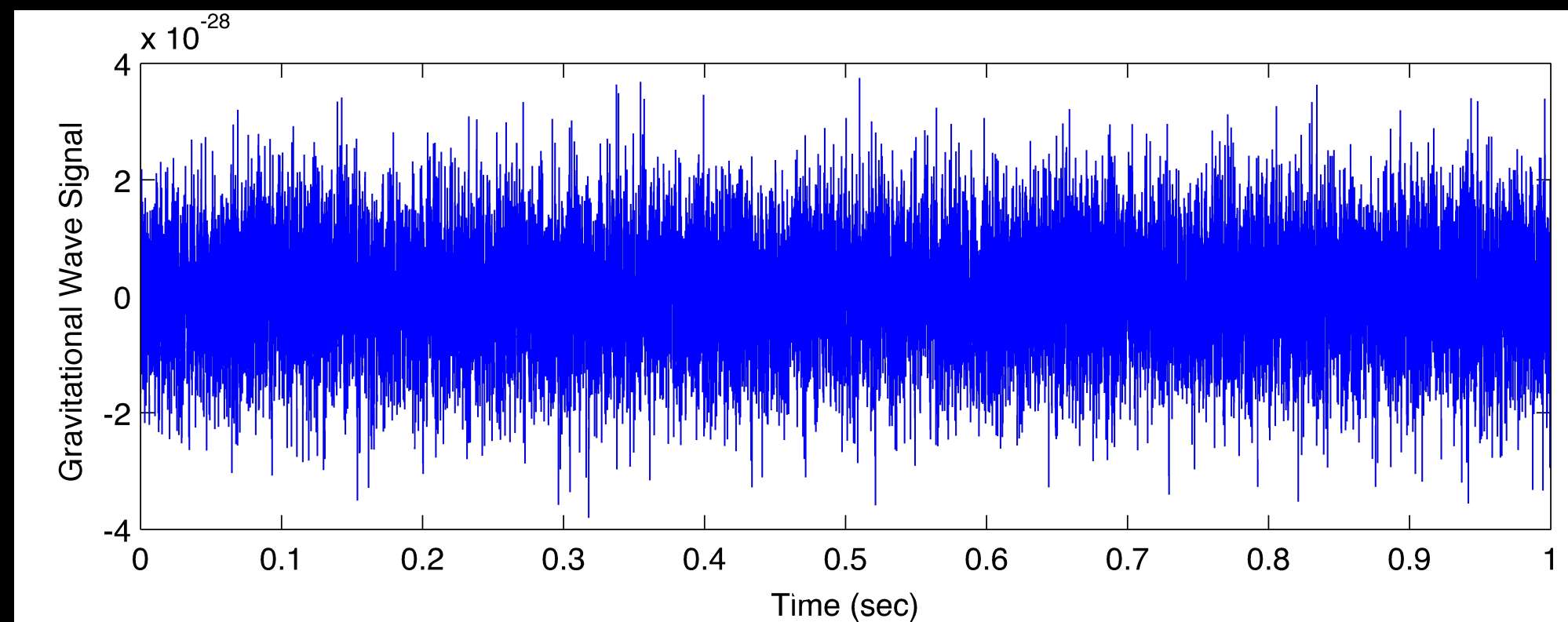
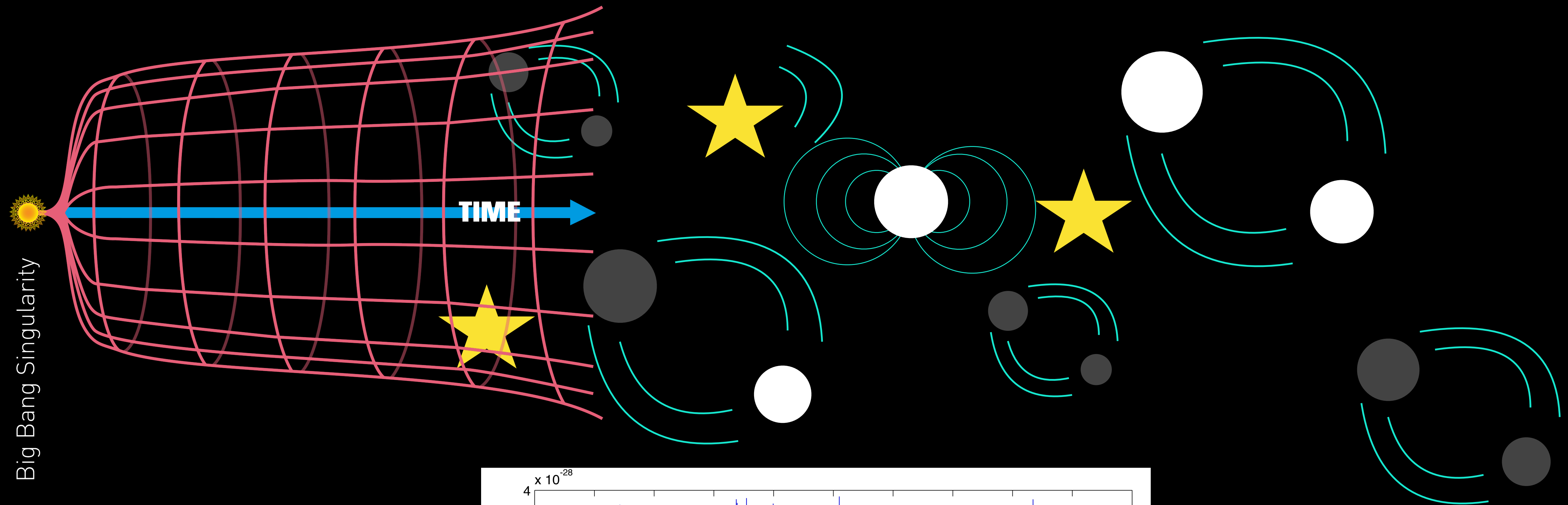


Burst



Gravitational- wave background

Gravitational-Wave background



Gravitational-Wave background

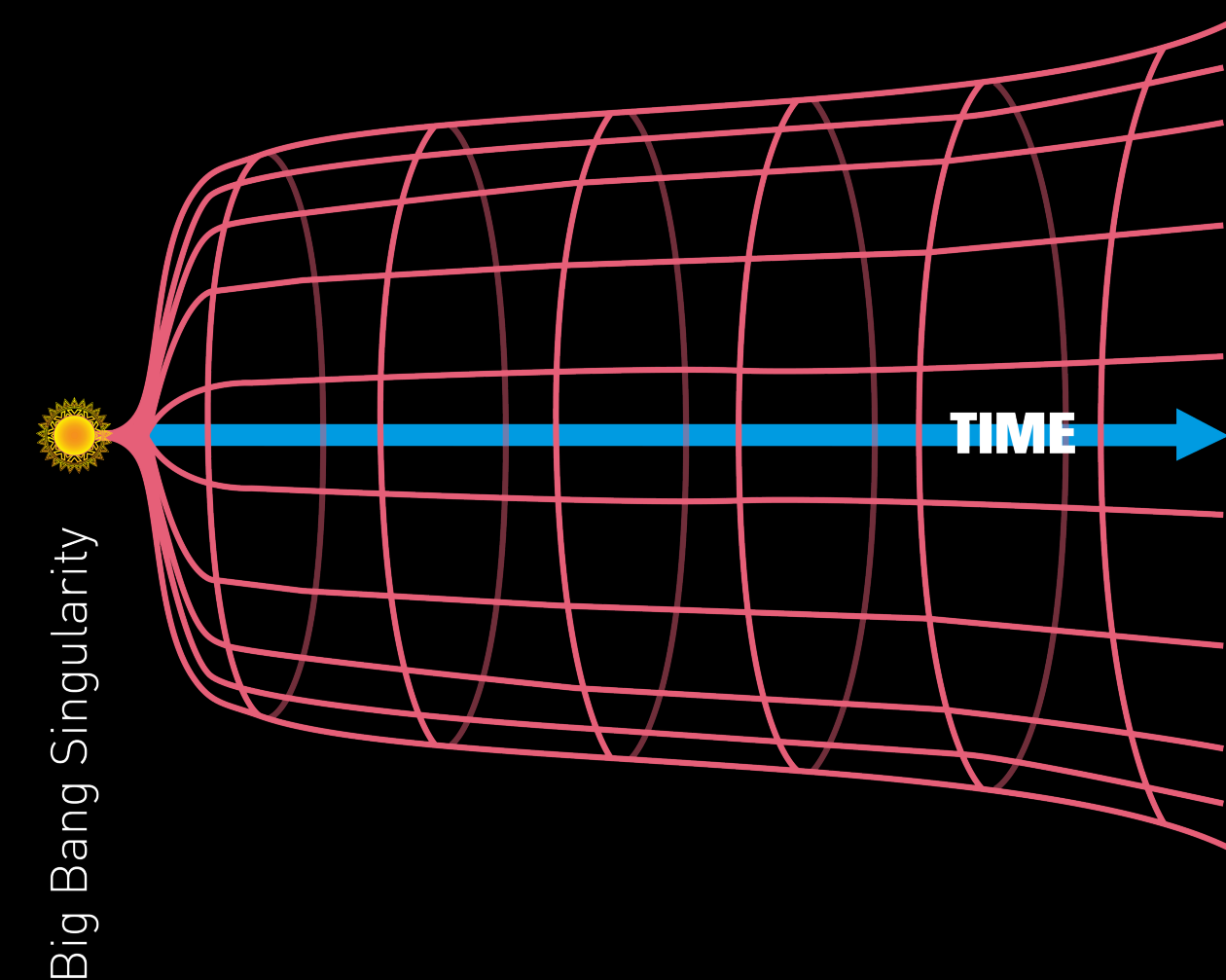
Gravitational Wave Background (GWB): superposition of large number of weak, independent and unresolved GW sources
→ best characterised statistically

Astrophysical



- Compact binaries: BBH, BNS, BHNS...
- Supermassive BH, rotating NS...

Cosmological



- Early Universe phenomena, such as inflation, phase transitions etc

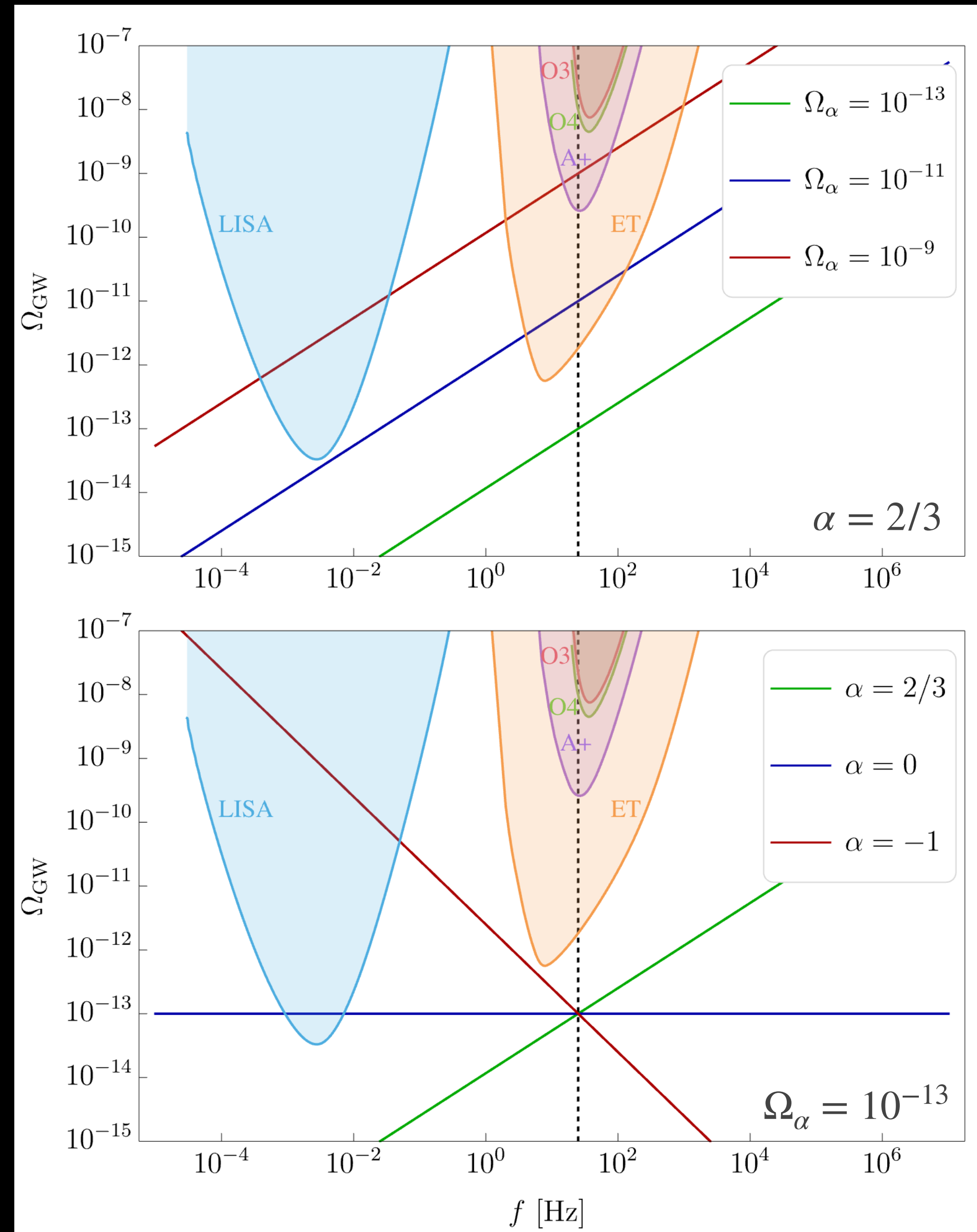
Cosmological GWB

- Many different cosmological sources for the GWB, (see e.g. [Cosmological implications from GWB searches for O1-O4a](#))
- General way to approximate cosmological GWB: power-law (PL) spectrum

$$\bullet \quad \Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln f} = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Critical energy density of the Universe today

GW energy density



Detecting a GWB: cross-correlation

- Given two detectors d_1 and d_2 the **output** of each detector is:
 - $s_1 = h + n_1$ & $s_2 = h + n_2$ with **GW signal component** & **Detector noise**
- Cross-correlation: $\langle C^{d_1 d_2}(f) \rangle \sim \langle s_1 s_2 \rangle$
- For an isotropic, stationary and unpolarised, and when noise between detectors is uncorrelated:

$$\langle C^{d_1 d_2}(f) \rangle = \frac{3H_0^2}{10\pi^2} \frac{\Omega_{\text{GW}}(f) \gamma_I^{d_1 d_2}(f)}{f^3}$$

Normalized overlap reduction function (ORF): how $d_1 - d_2$ pair responds to GWB (depends on detector geometry)

Outline



- Brief overview of gravitational-wave background
- Brief visit to Parity Violation
- 3G networks
- Detection Prospects for different configurations:
 - PI curves and SNR prospects
 - Bayesian analysis

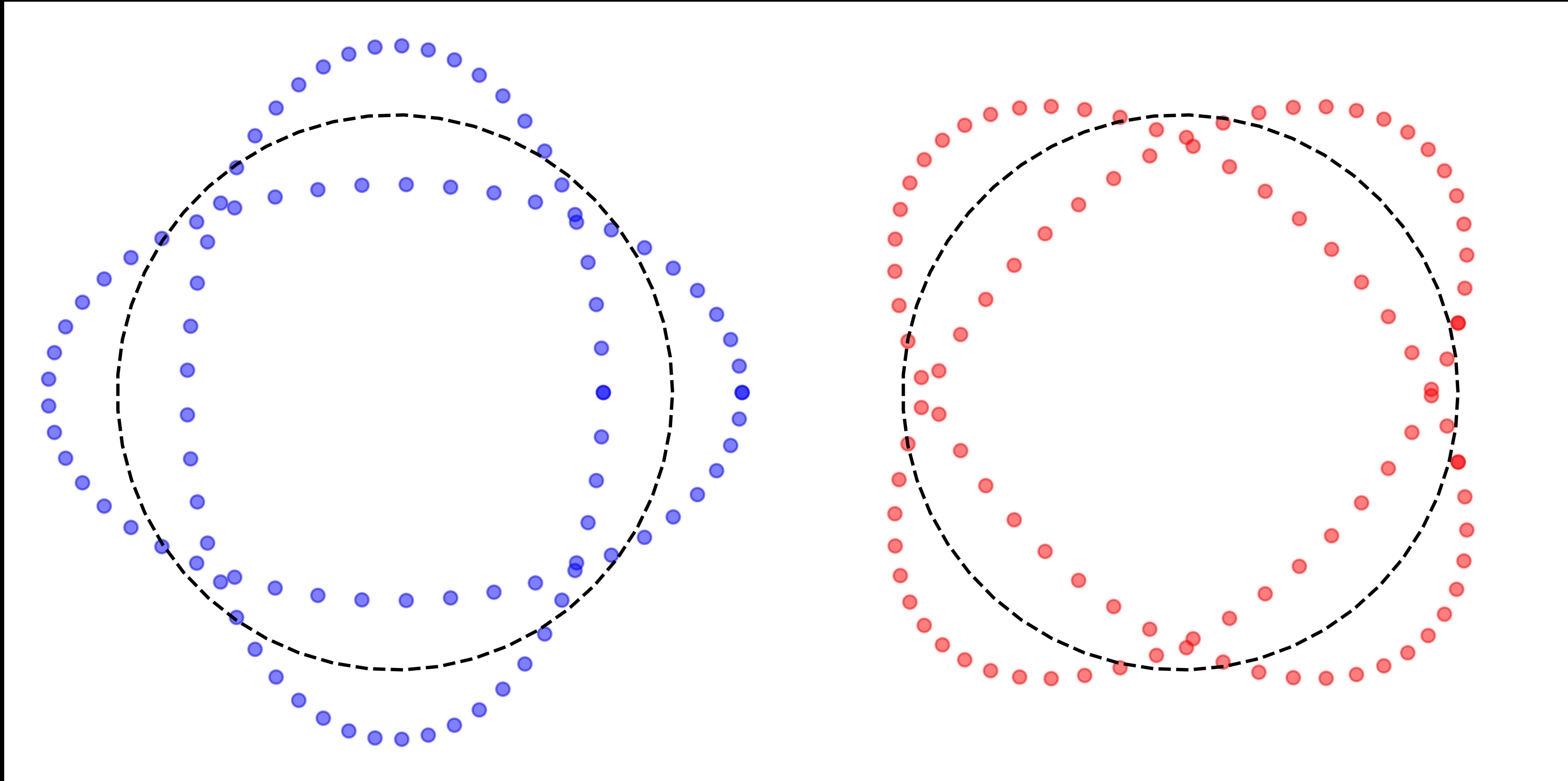
Outline

?



- Brief overview of gravitational-wave background
- Brief visit to Parity Violation
- 3G networks
- Detection Prospects for different configurations:
 - PI curves and SNR prospects
 - Bayesian analysis

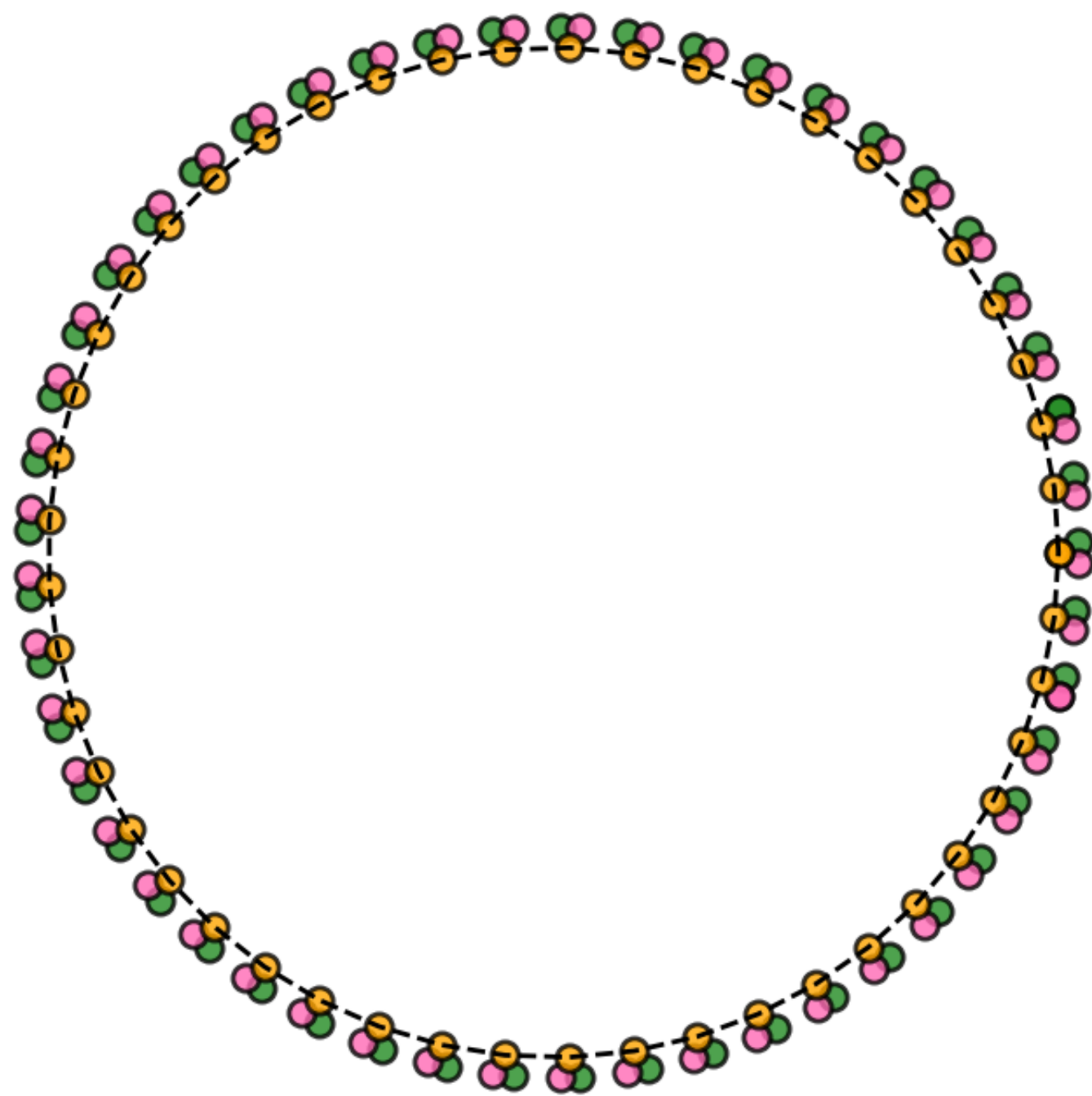
Back to the basics: polarization GW



h_+

h_x

Circular Polarization Basis

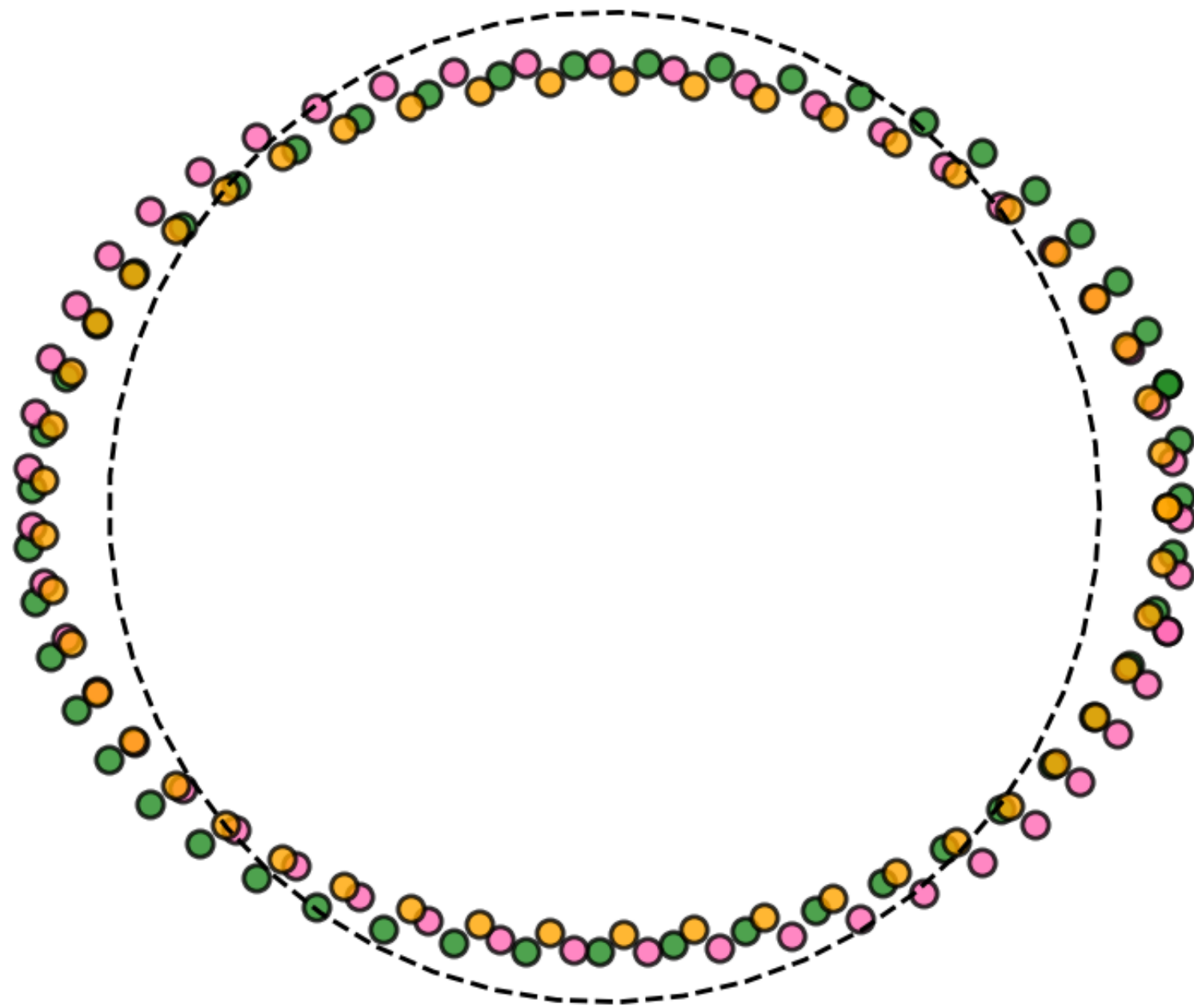


- $$h_R = \frac{h_+ - ih_x}{\sqrt{2}}$$

- $$h_L = \frac{h_+ + ih_x}{\sqrt{2}}$$

- $$h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$$

Circular Polarization Basis

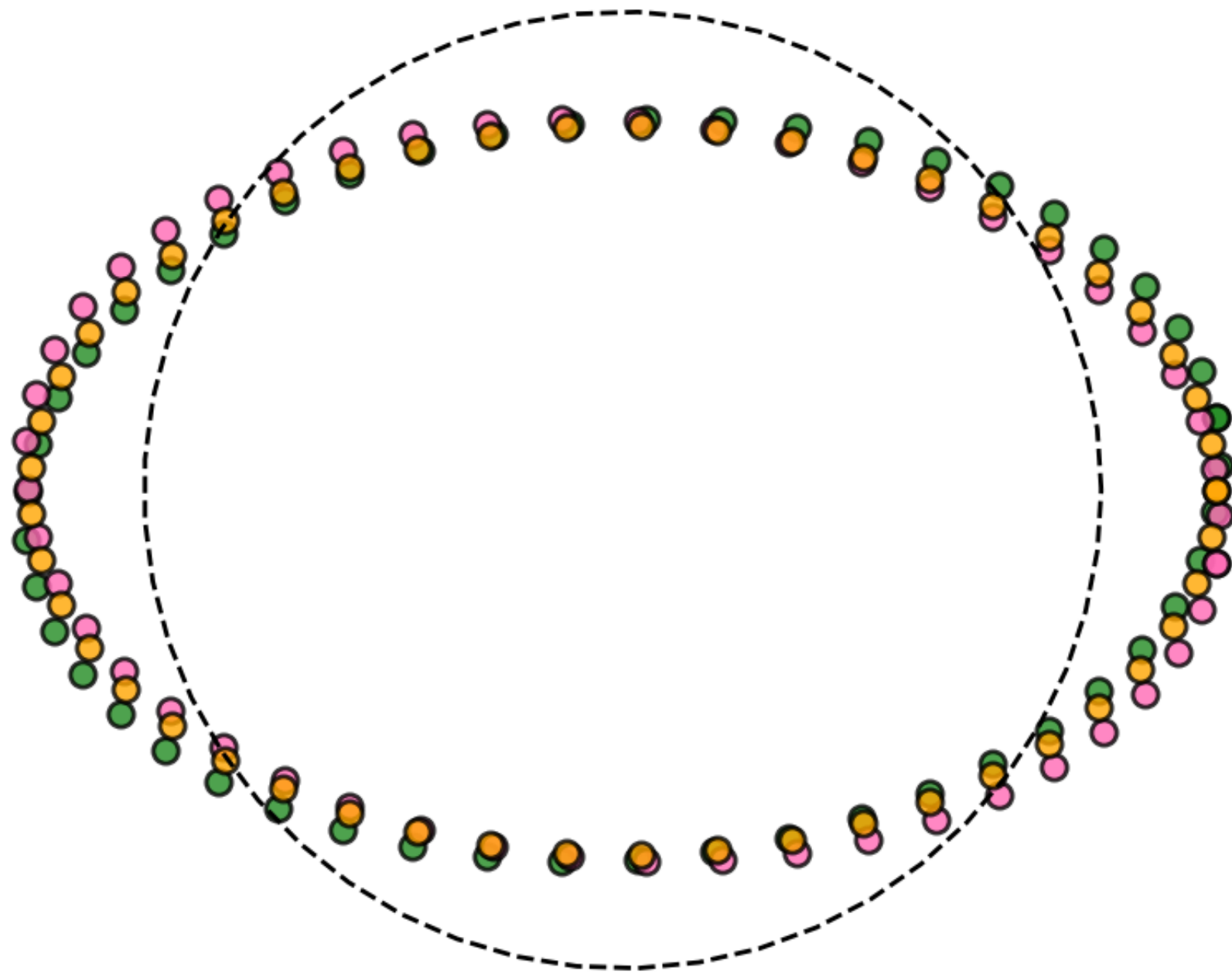


- $$h_R = \frac{h_+ - ih_-}{\sqrt{2}}$$

- $$h_L = \frac{h_+ + ih_-}{\sqrt{2}}$$

- $$h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$$

Circular Polarization Basis

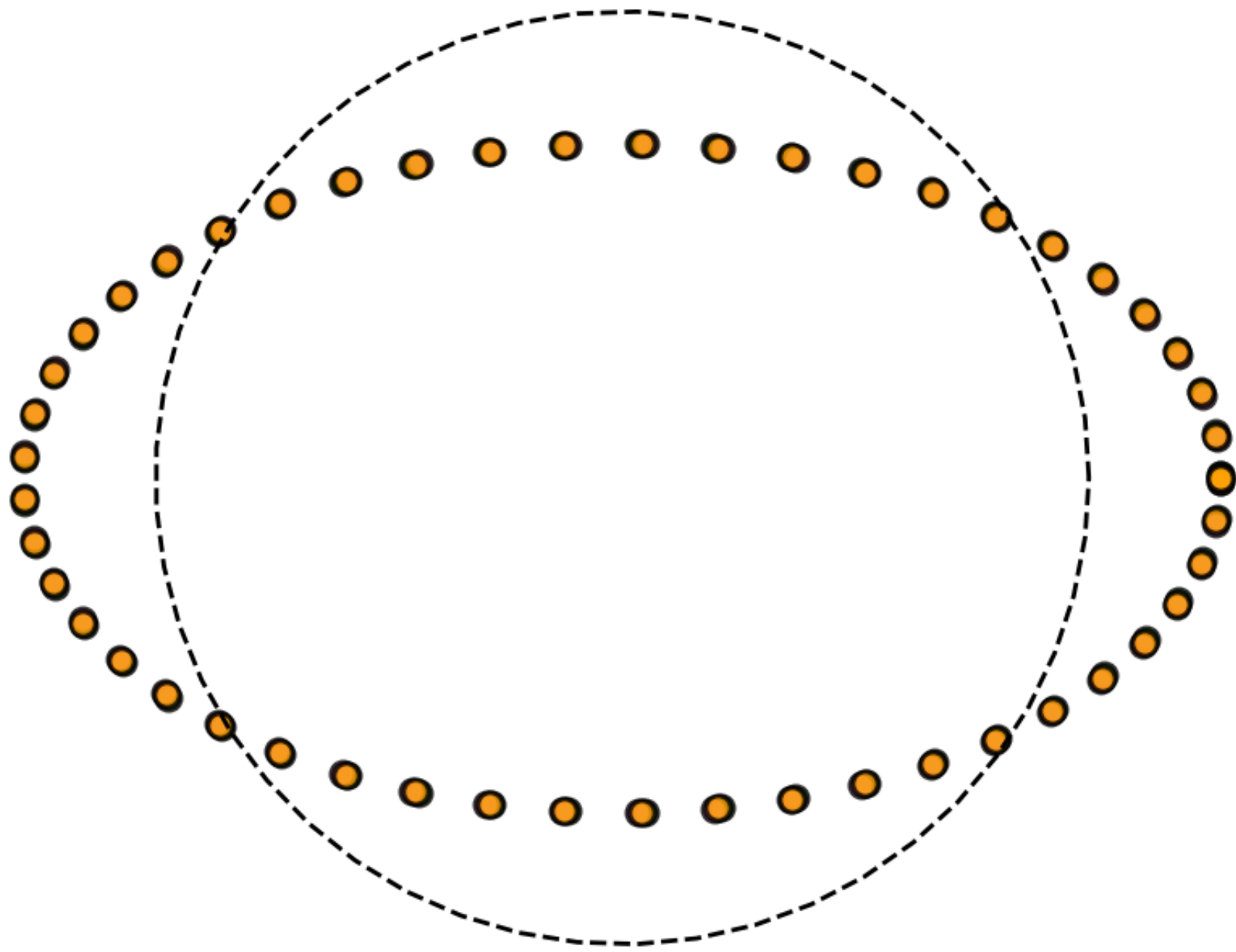


- $h_R = \frac{h_+ - ih_-}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_-}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

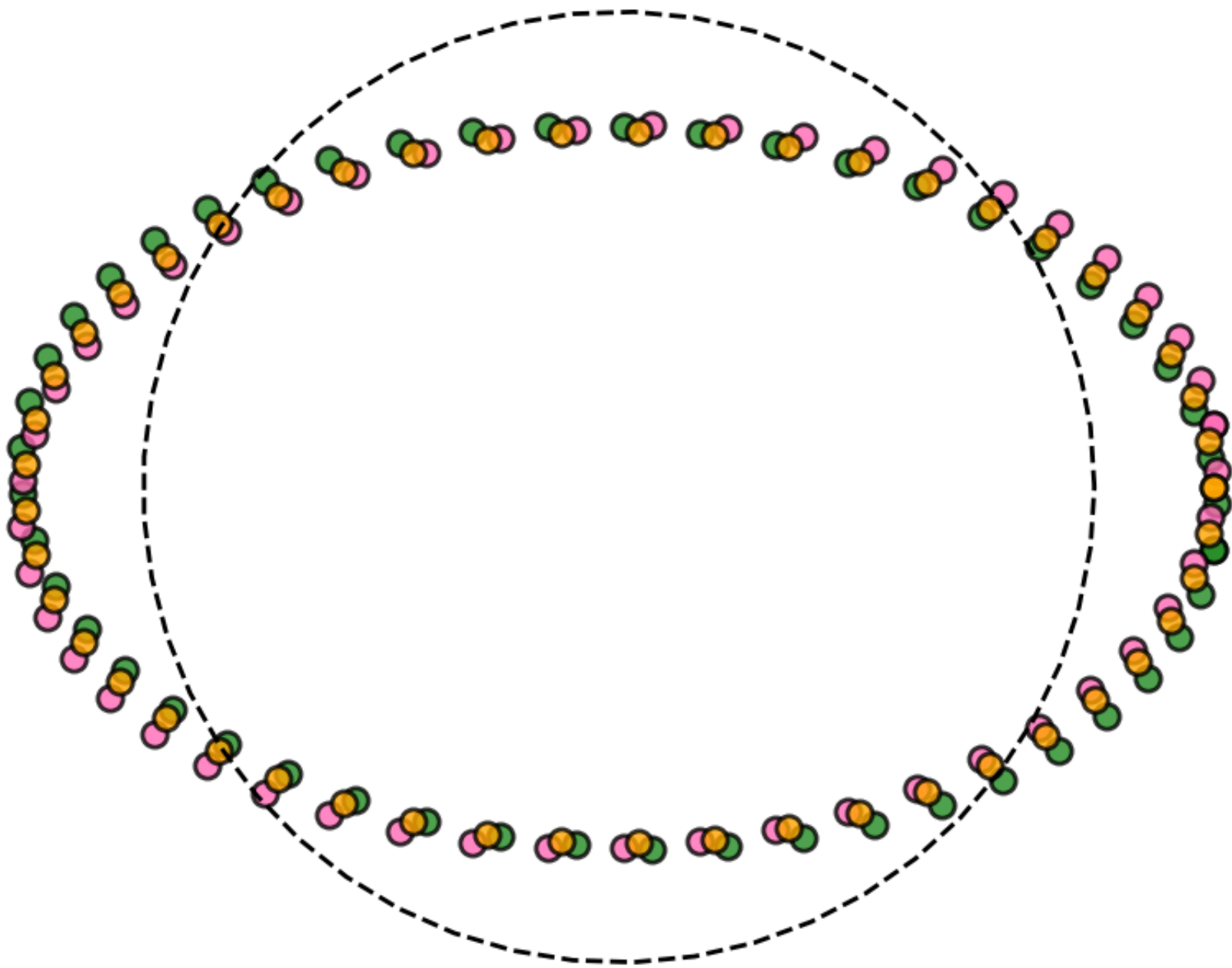


- $h_R = \frac{h_+ - ih_-}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_-}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

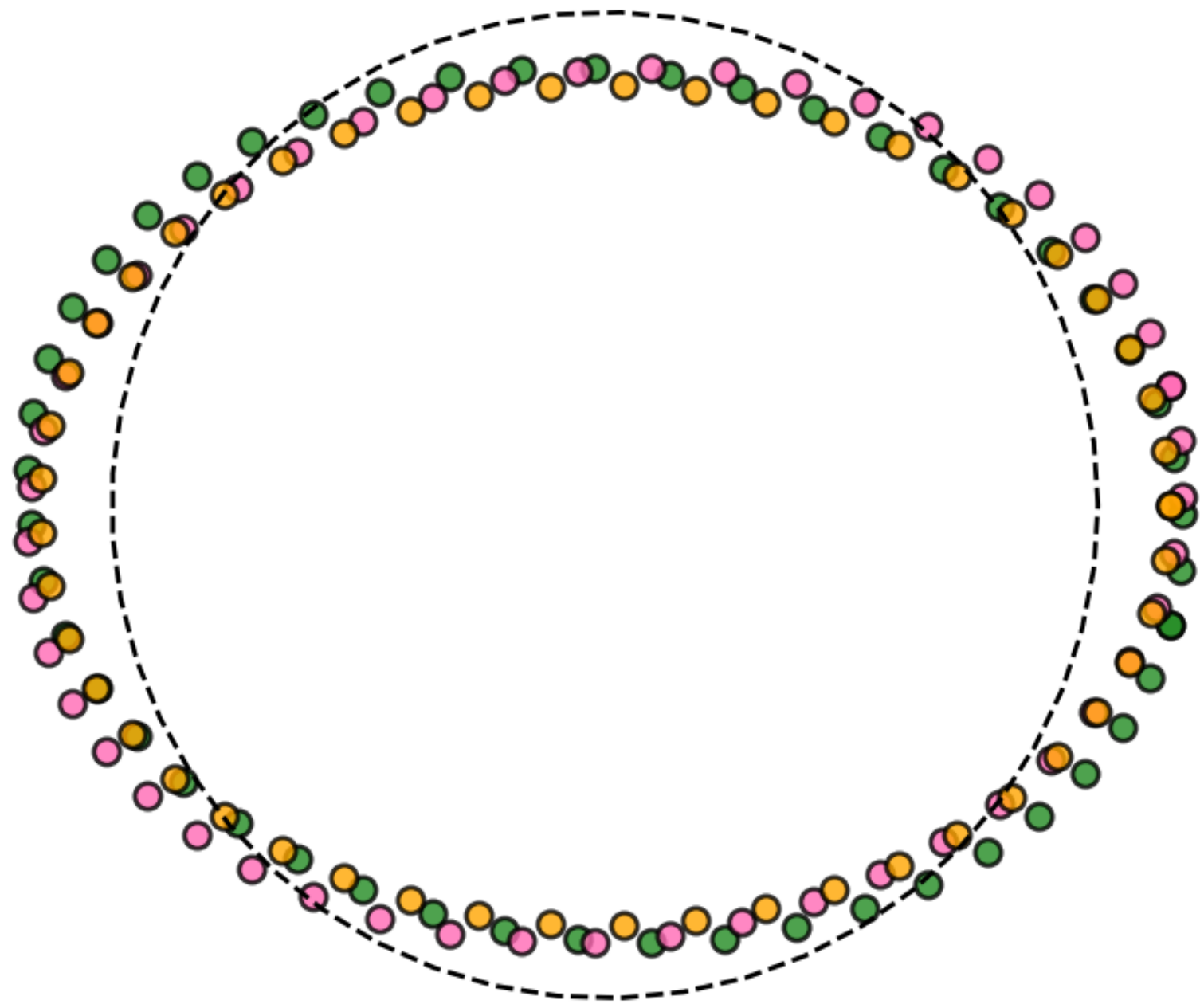


- $h_R = \frac{h_+ - ih_-}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_-}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

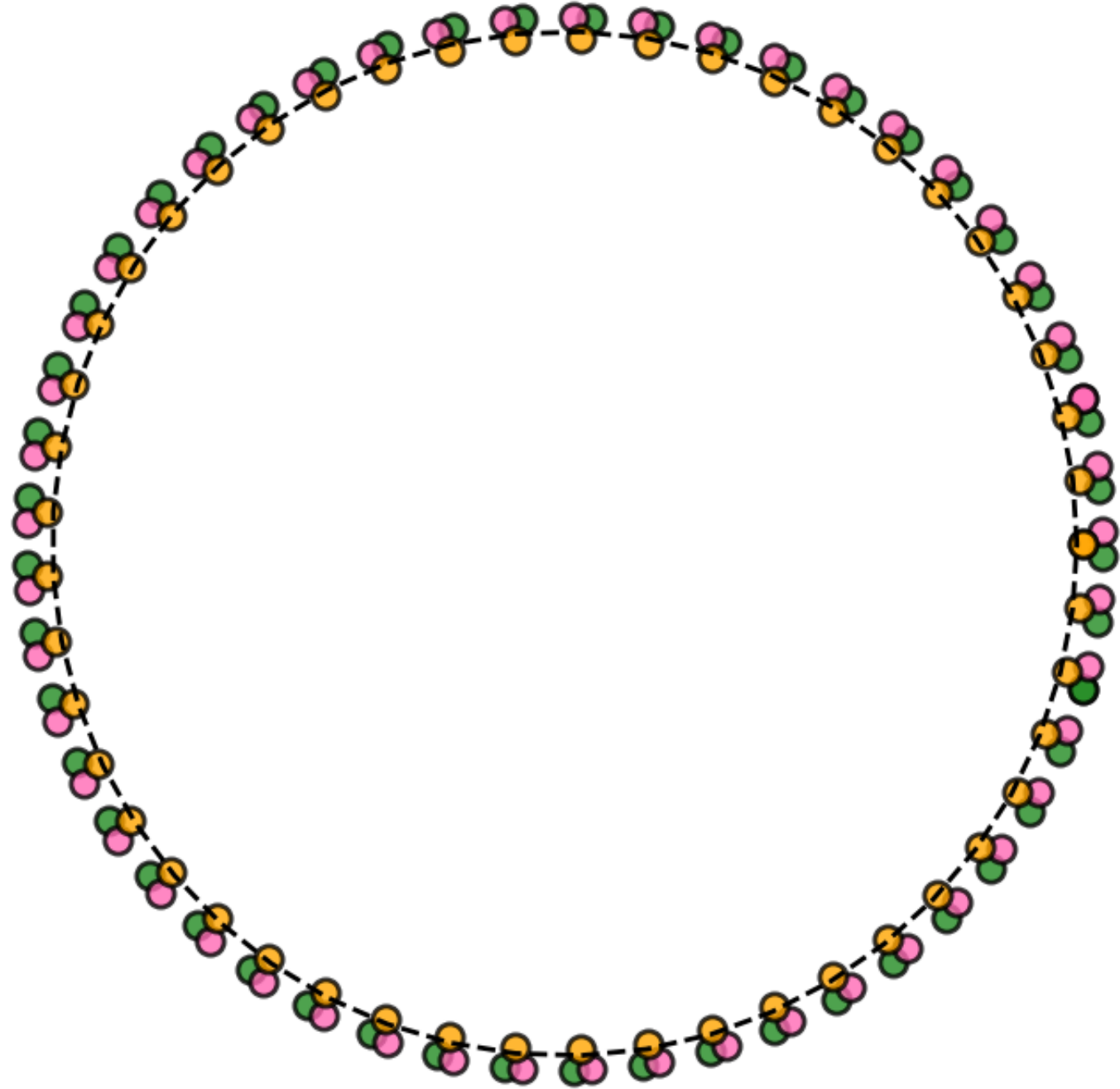


- $h_R = \frac{h_+ - ih_-}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_-}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

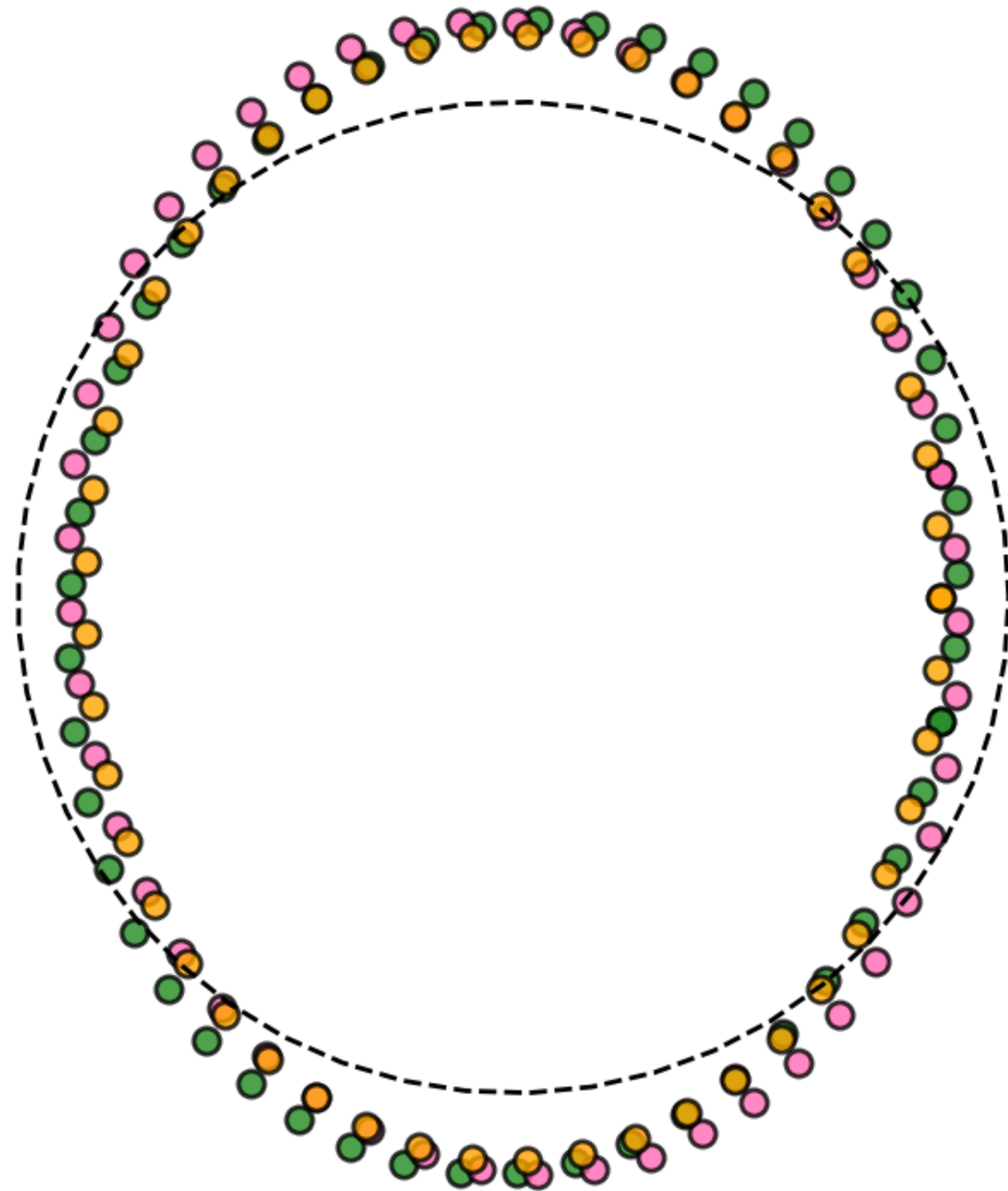


- $$h_R = \frac{h_+ - ih_x}{\sqrt{2}}$$

- $$h_L = \frac{h_+ + ih_x}{\sqrt{2}}$$

- $$h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$$

Circular Polarization Basis

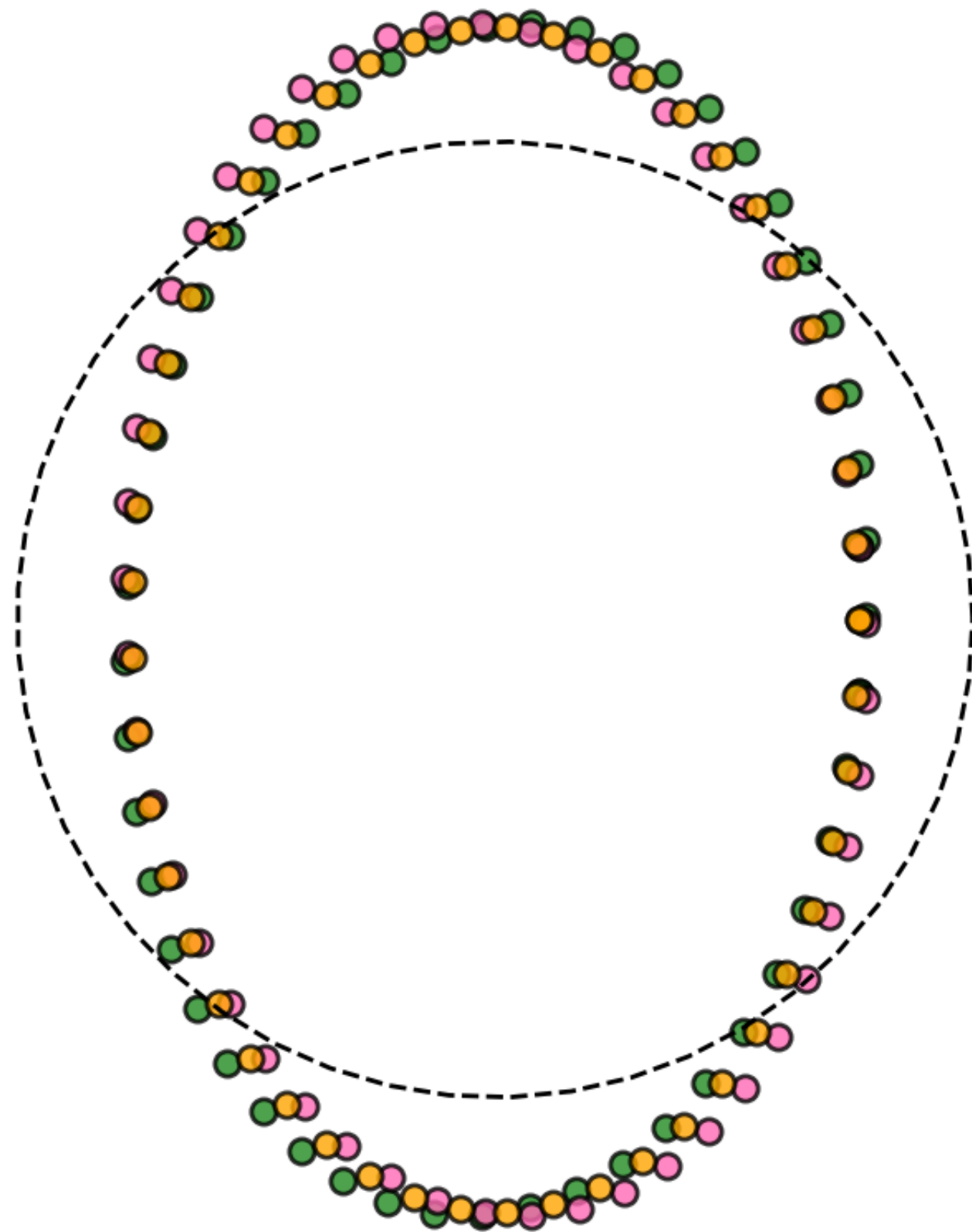


- $h_R = \frac{h_+ - ih_-}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_-}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

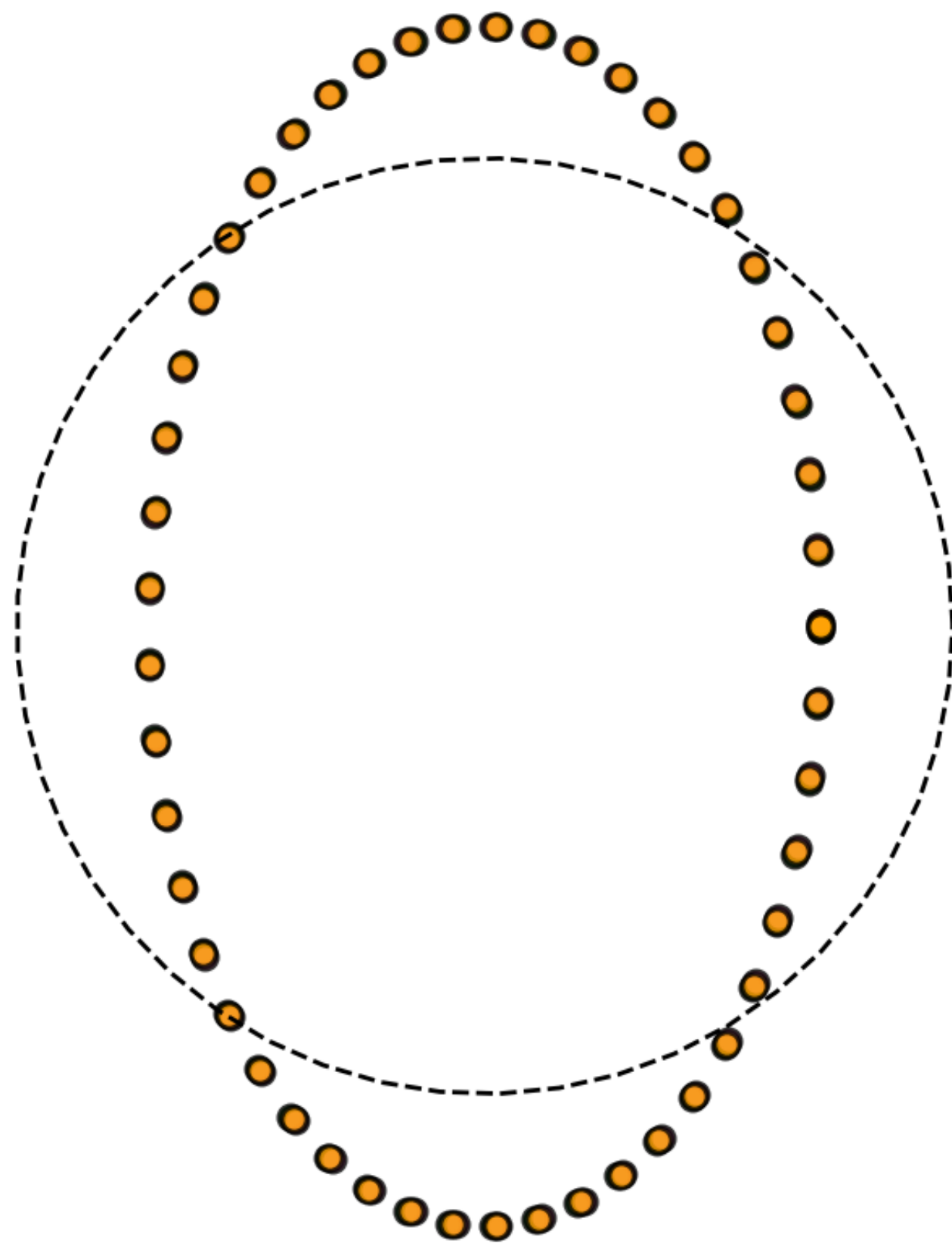


- $h_R = \frac{h_+ - ih_x}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_x}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

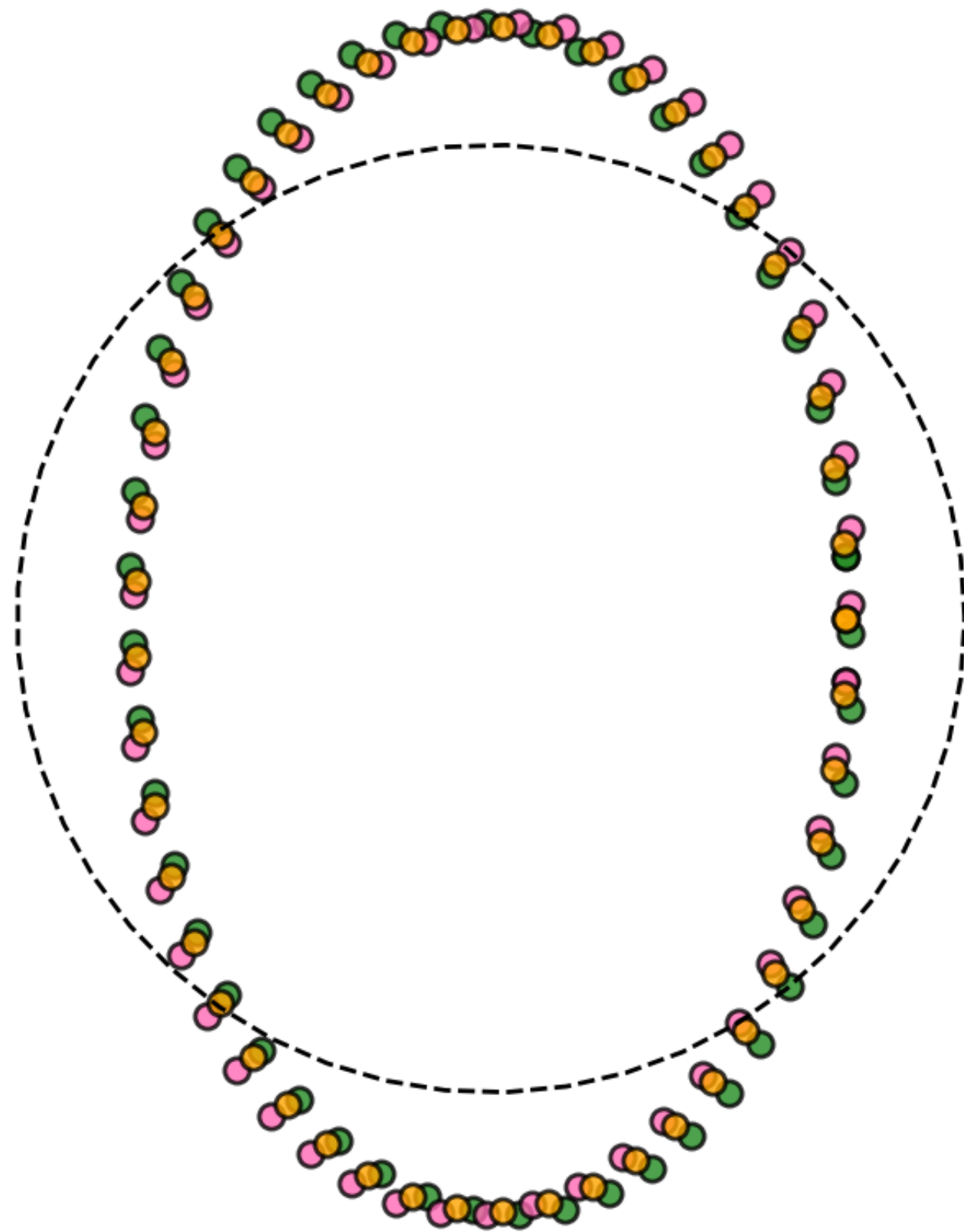


- $h_R = \frac{h_+ - ih_x}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_x}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

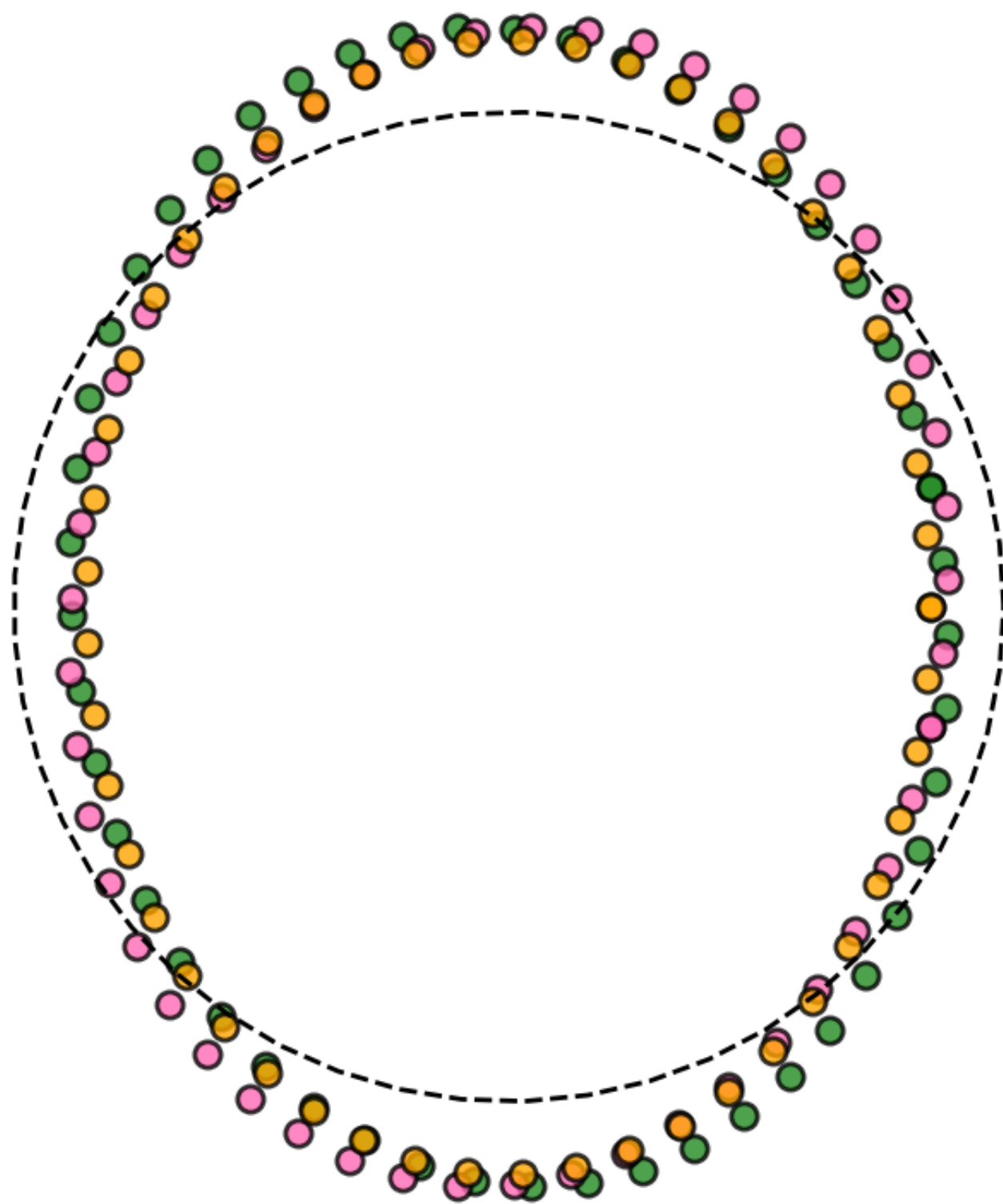


- $$h_R = \frac{h_+ - ih_x}{\sqrt{2}}$$

- $$h_L = \frac{h_+ + ih_x}{\sqrt{2}}$$

- $$h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$$

Circular Polarization Basis

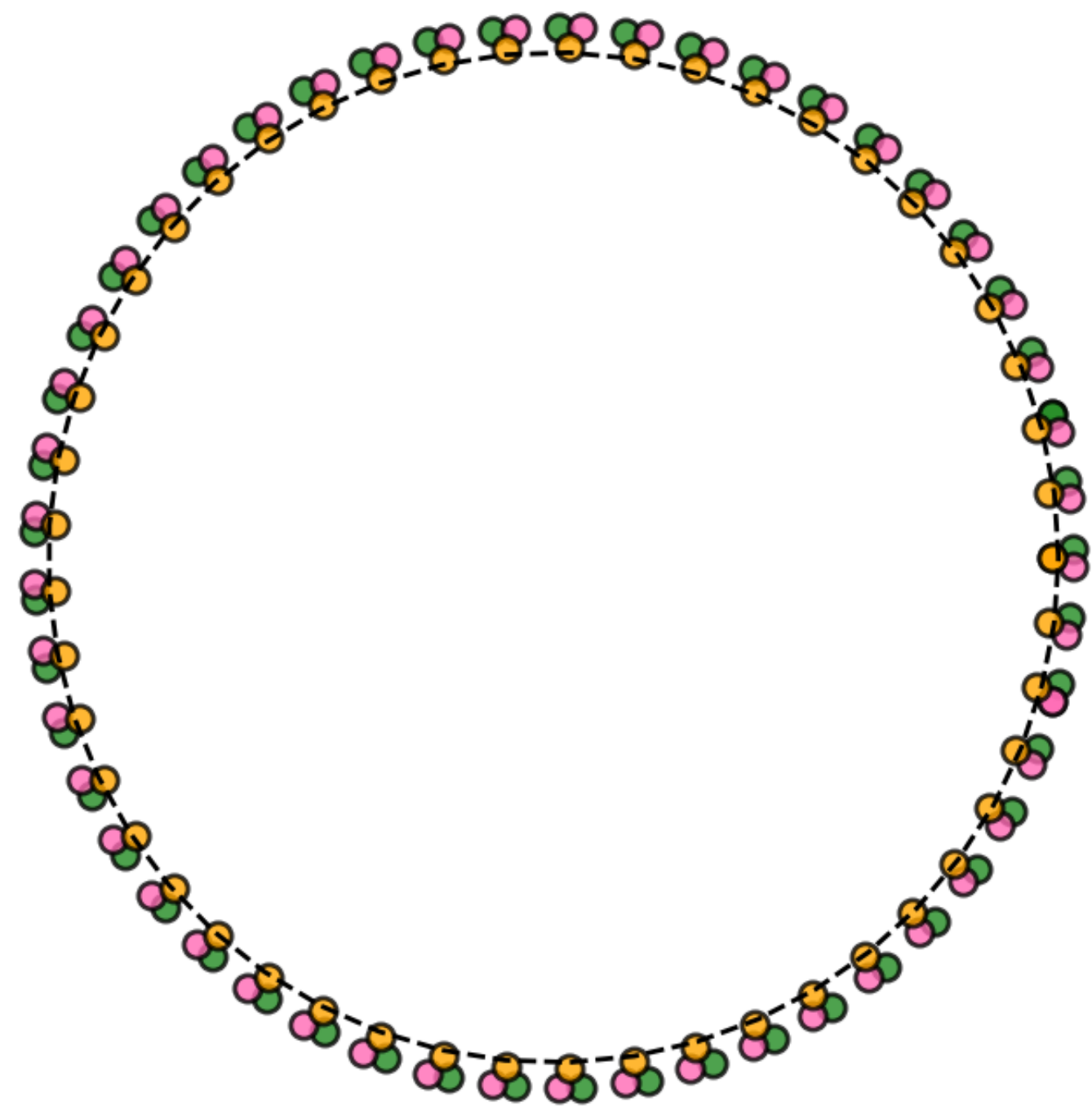


- $h_R = \frac{h_+ - ih_-}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_-}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

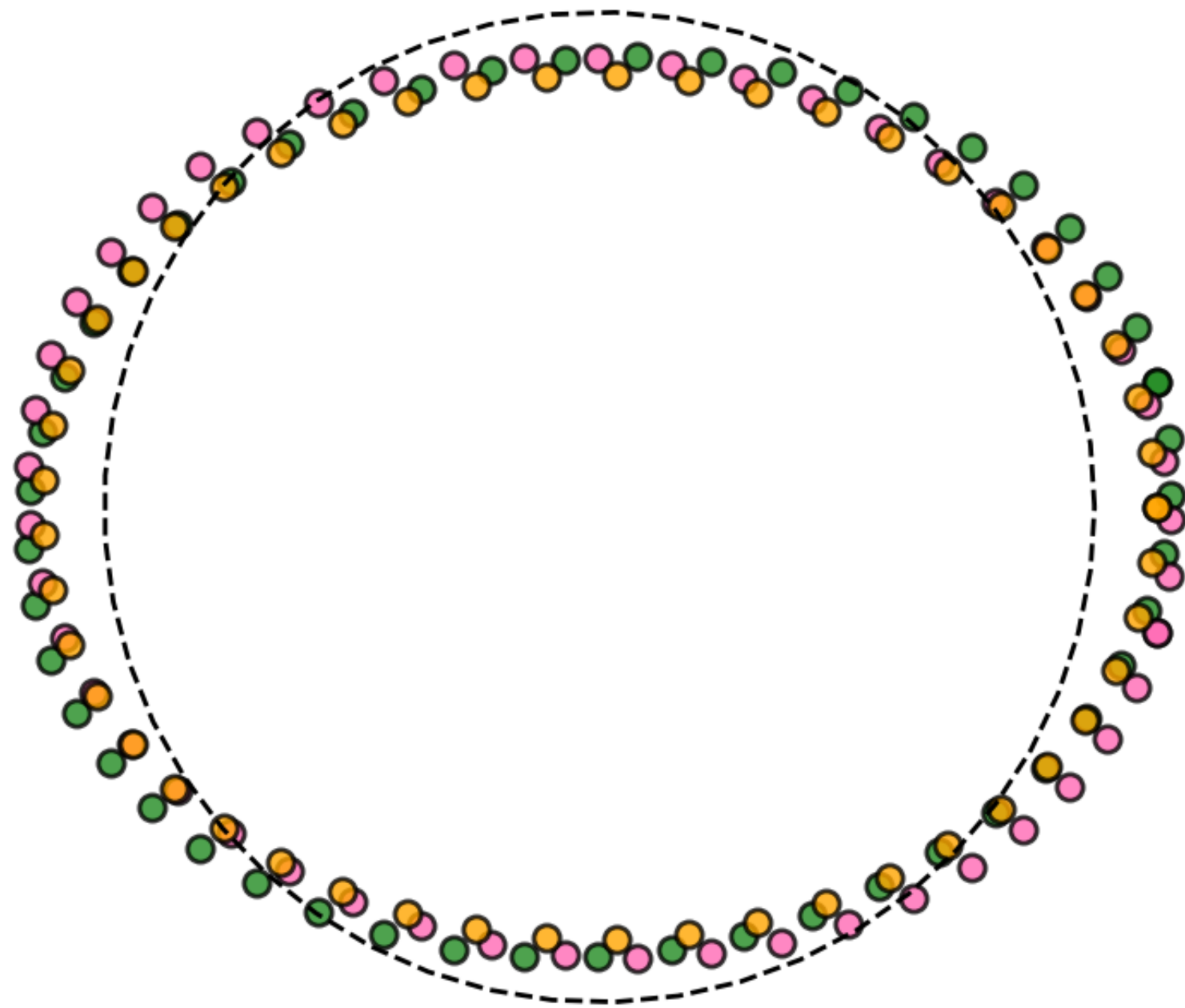


- $$h_R = \frac{h_+ - ih_x}{\sqrt{2}}$$

- $$h_L = \frac{h_+ + ih_x}{\sqrt{2}}$$

- $$h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$$

Circular Polarization Basis

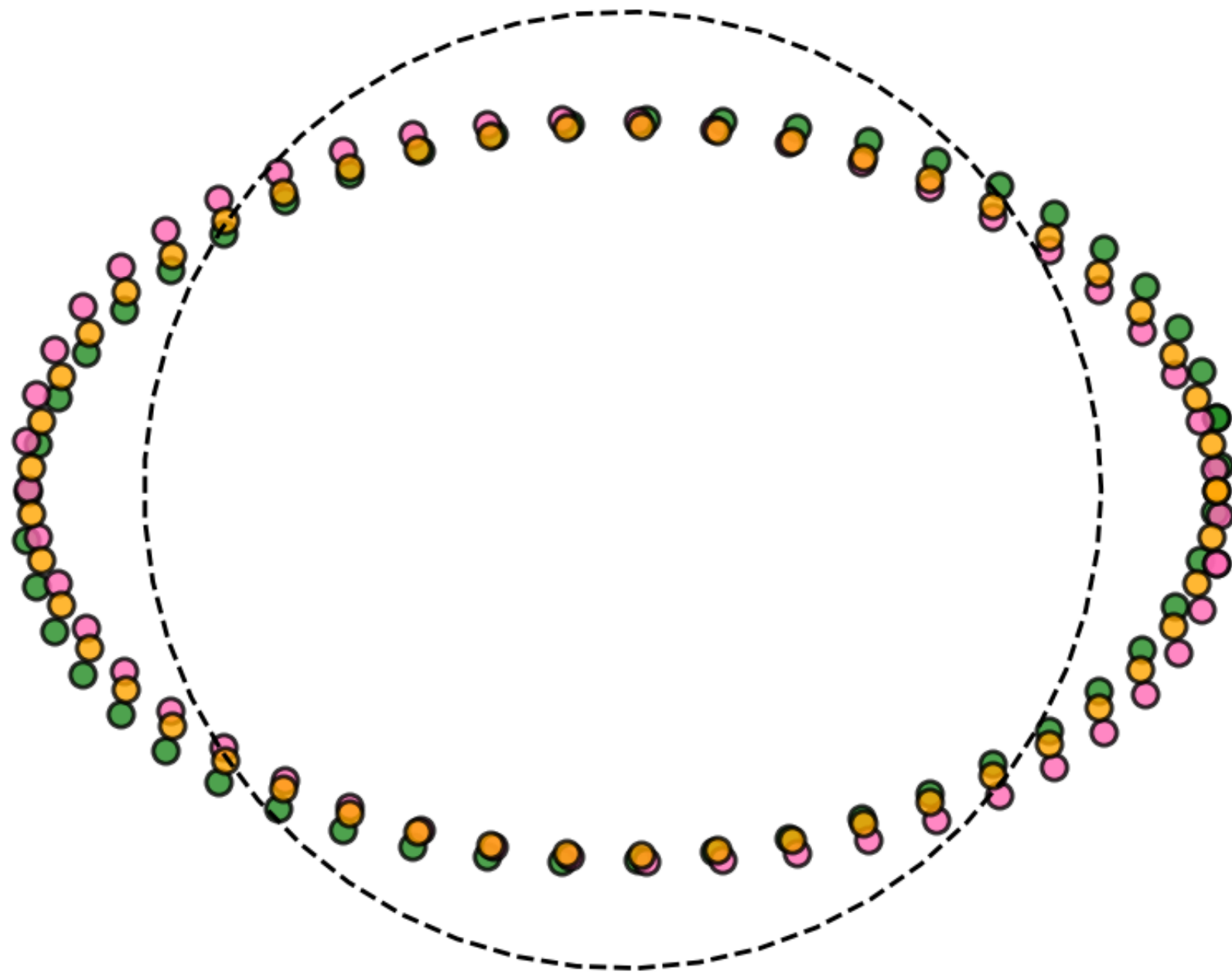


- $h_R = \frac{h_+ - ih_-}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_-}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

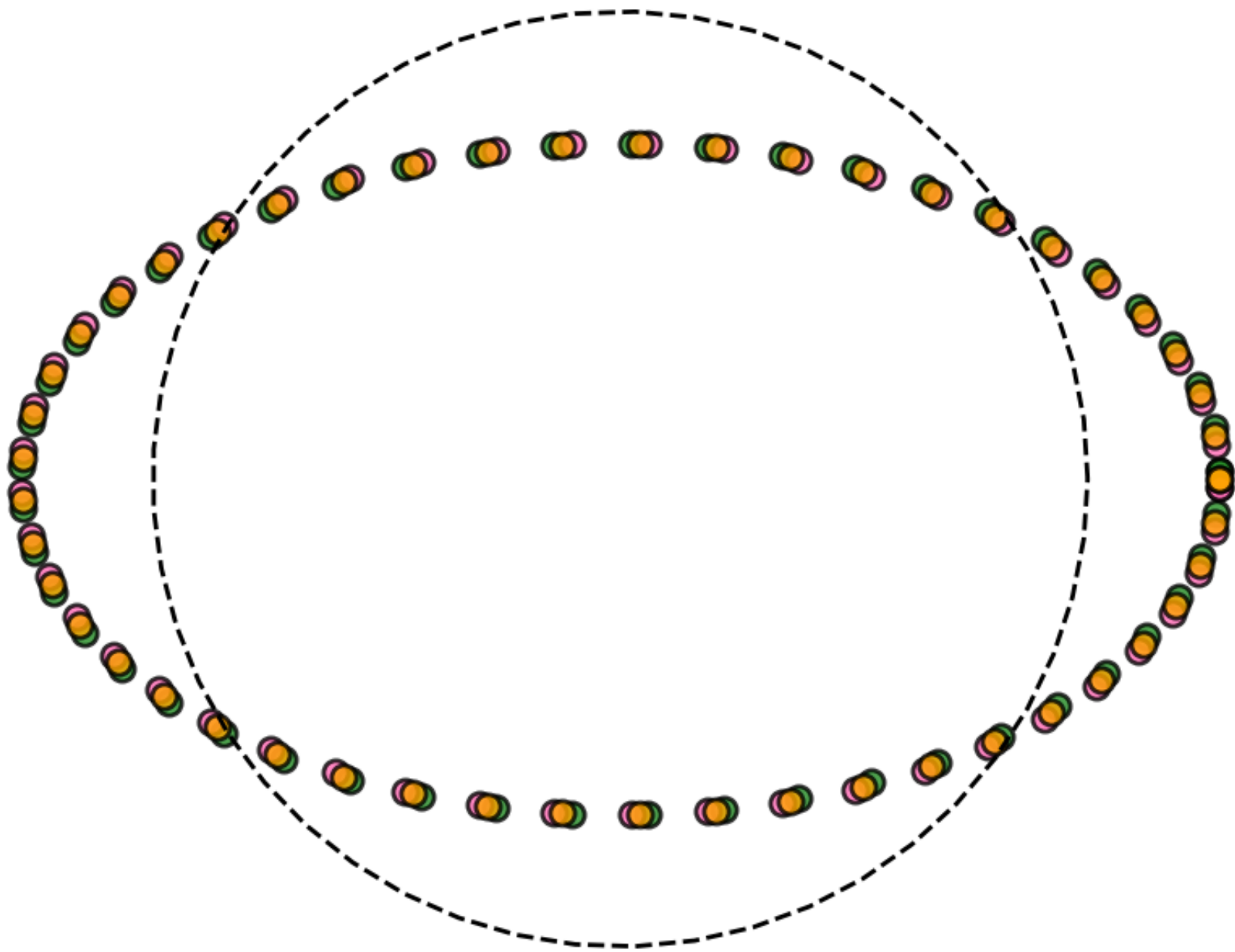


- $h_R = \frac{h_+ - ih_-}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_-}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

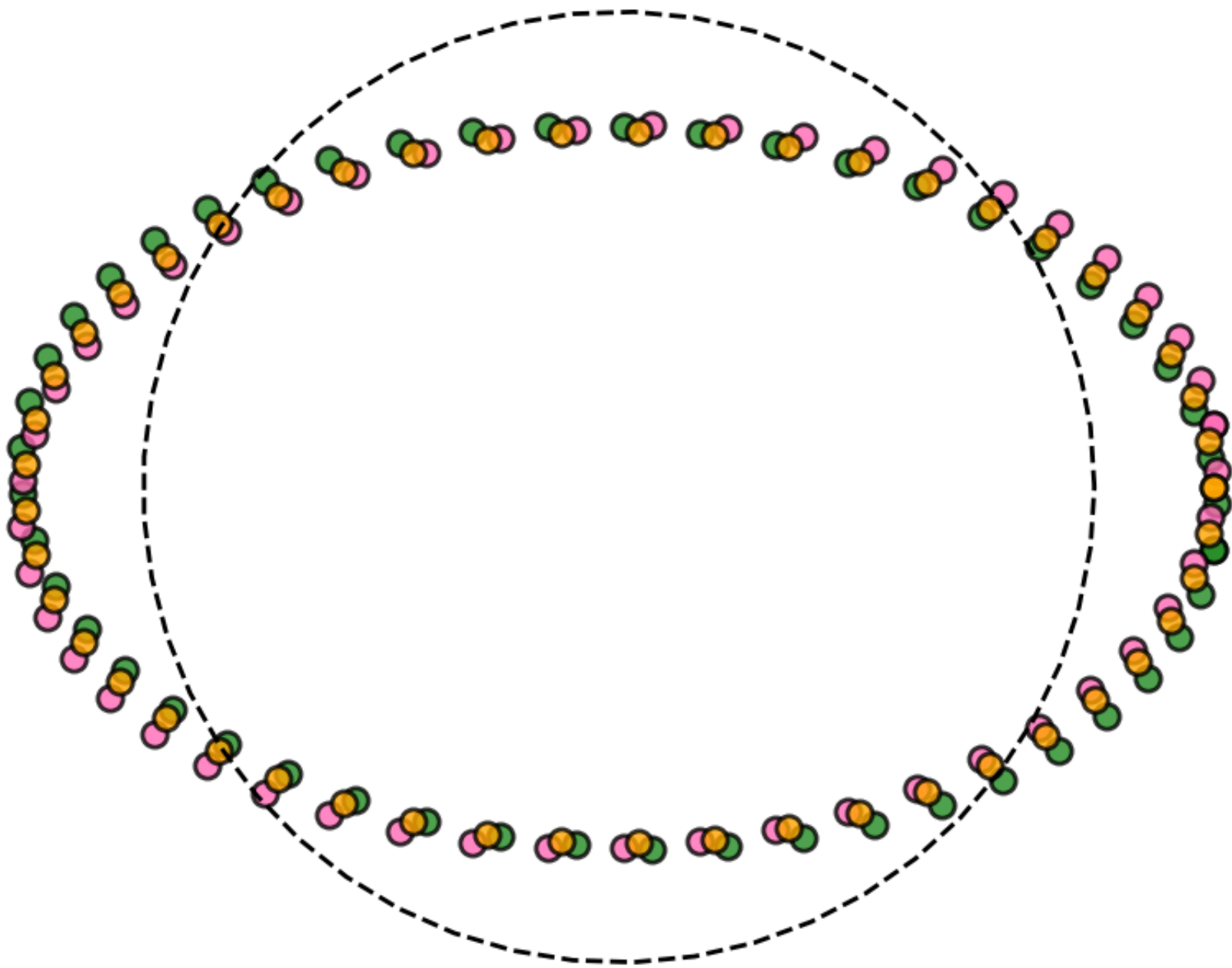


- $h_R = \frac{h_+ - ih_-}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_-}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

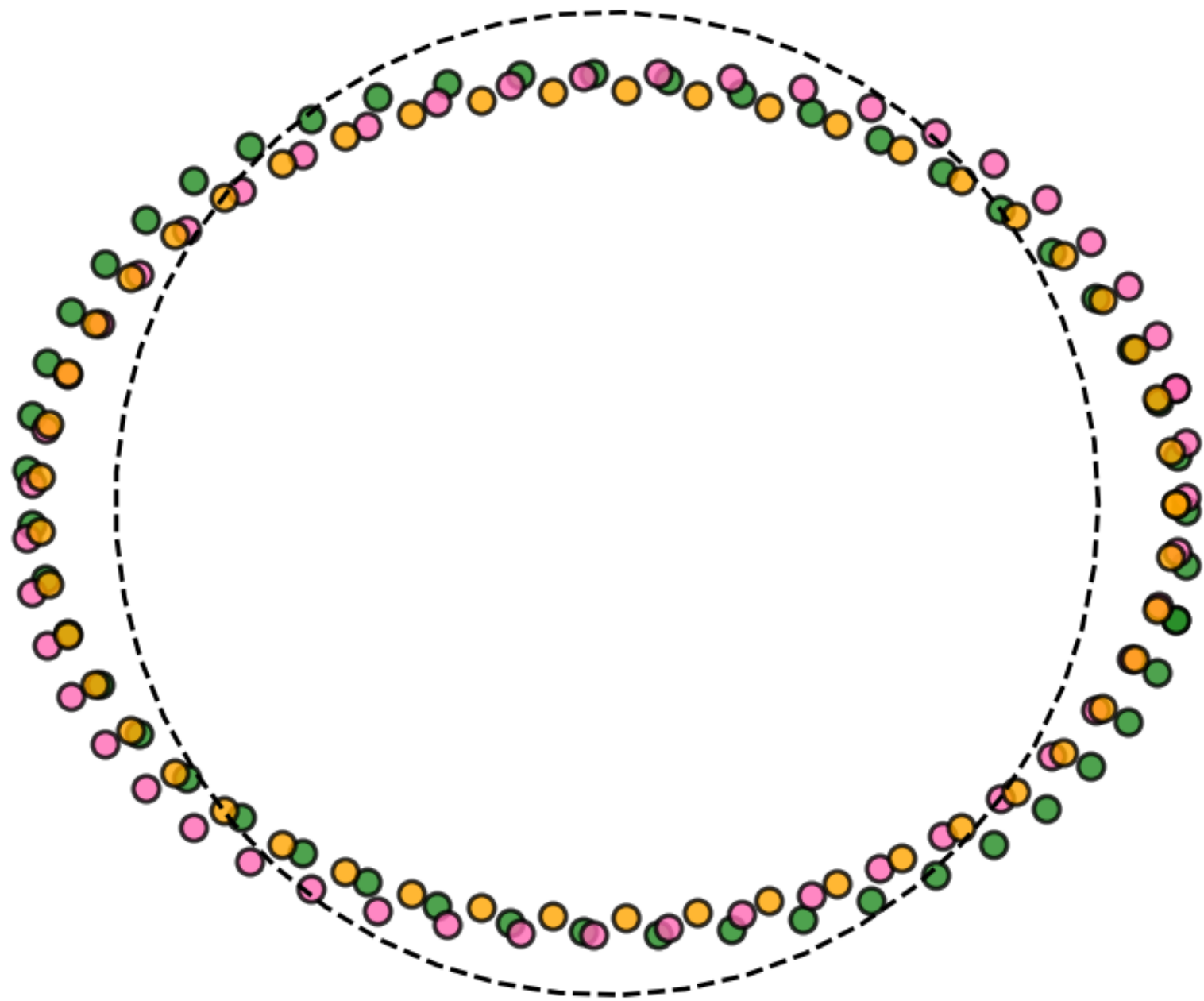


- $h_R = \frac{h_+ - ih_x}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_x}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

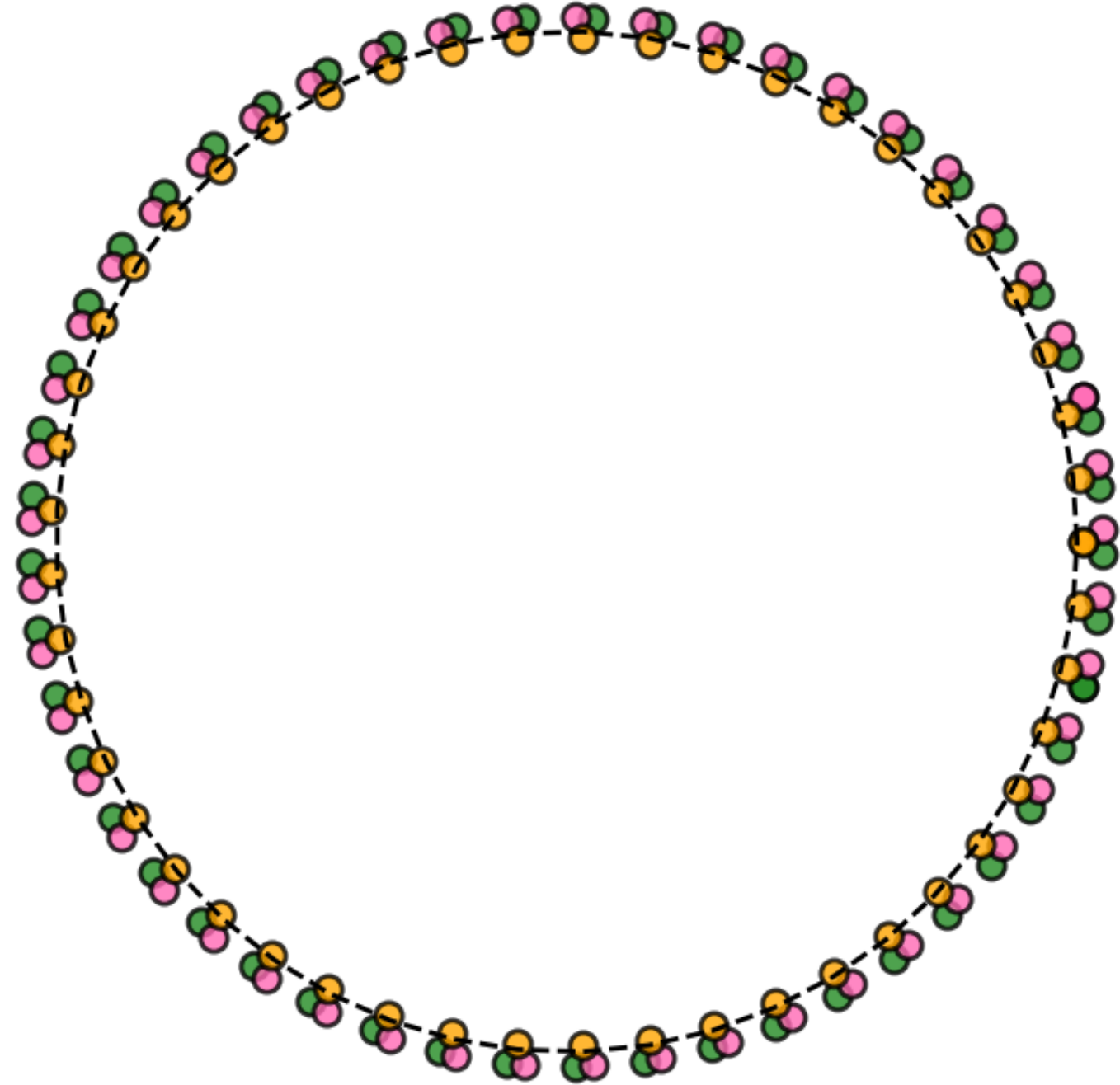


- $h_R = \frac{h_+ - ih_-}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_-}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

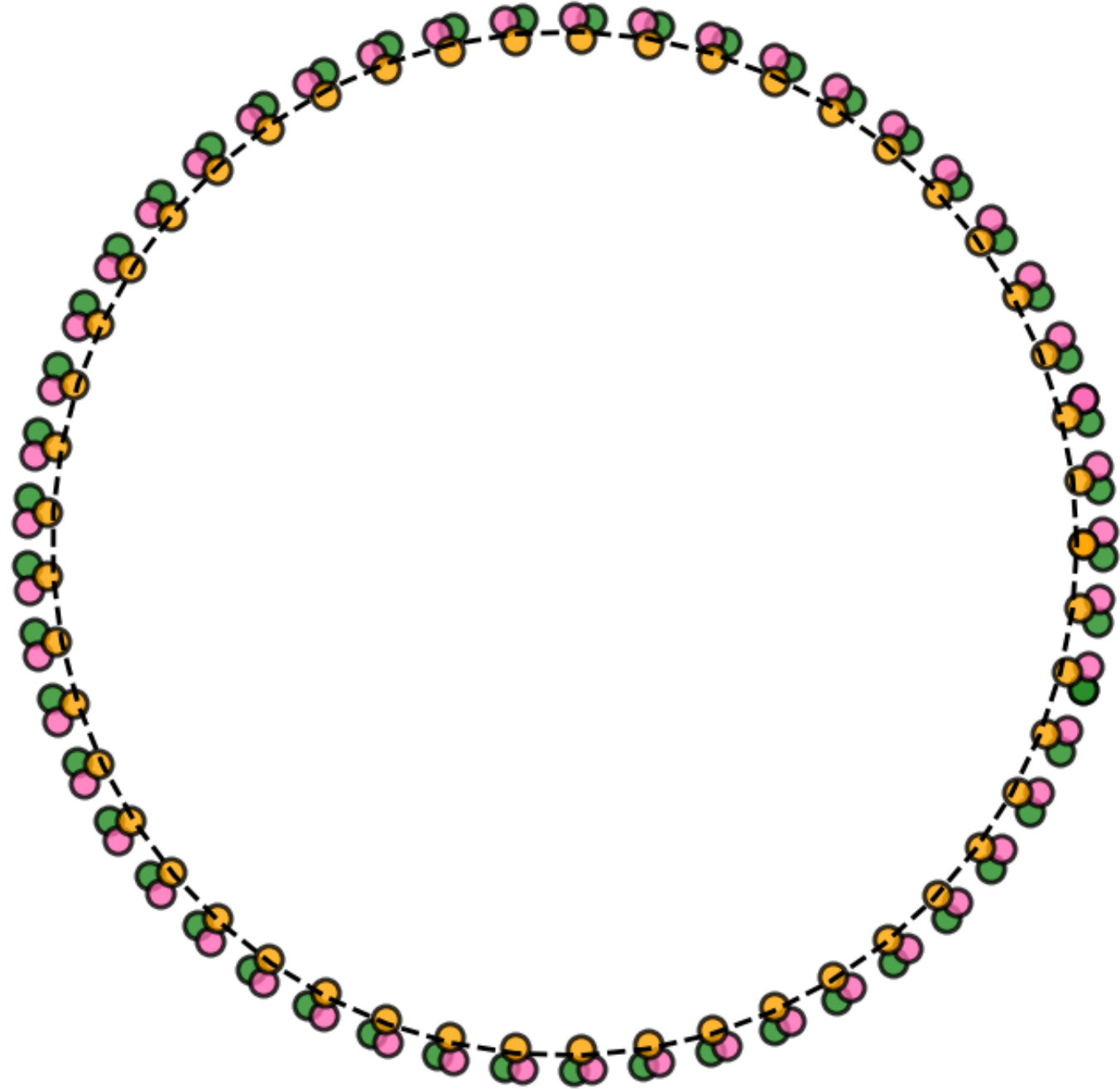


- $h_R = \frac{h_+ - ih_x}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_x}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

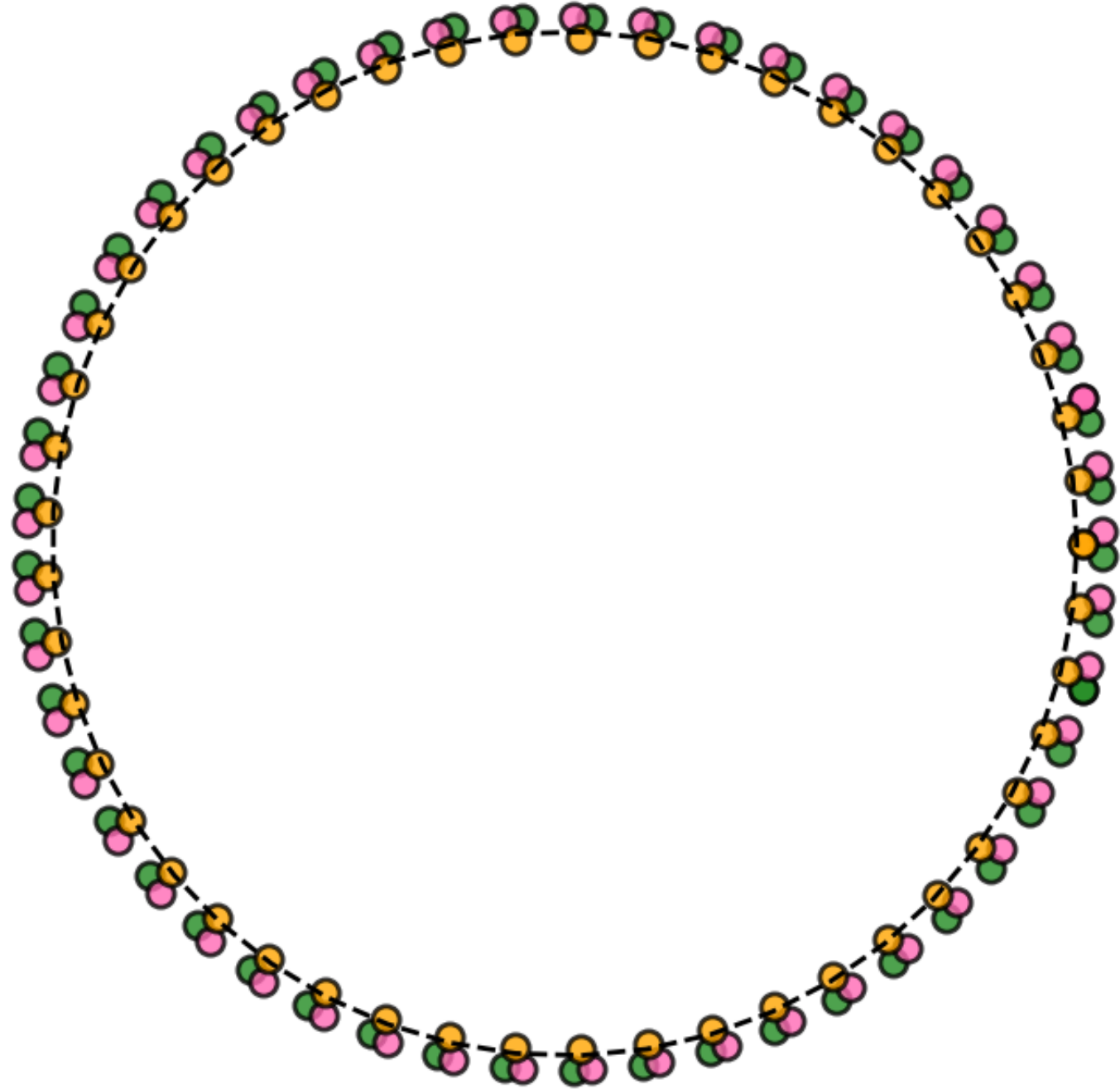


- $$h_R = \frac{h_+ - ih_x}{\sqrt{2}}$$

- $$h_L = \frac{h_+ + ih_x}{\sqrt{2}}$$

- $$h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$$

Circular Polarization Basis

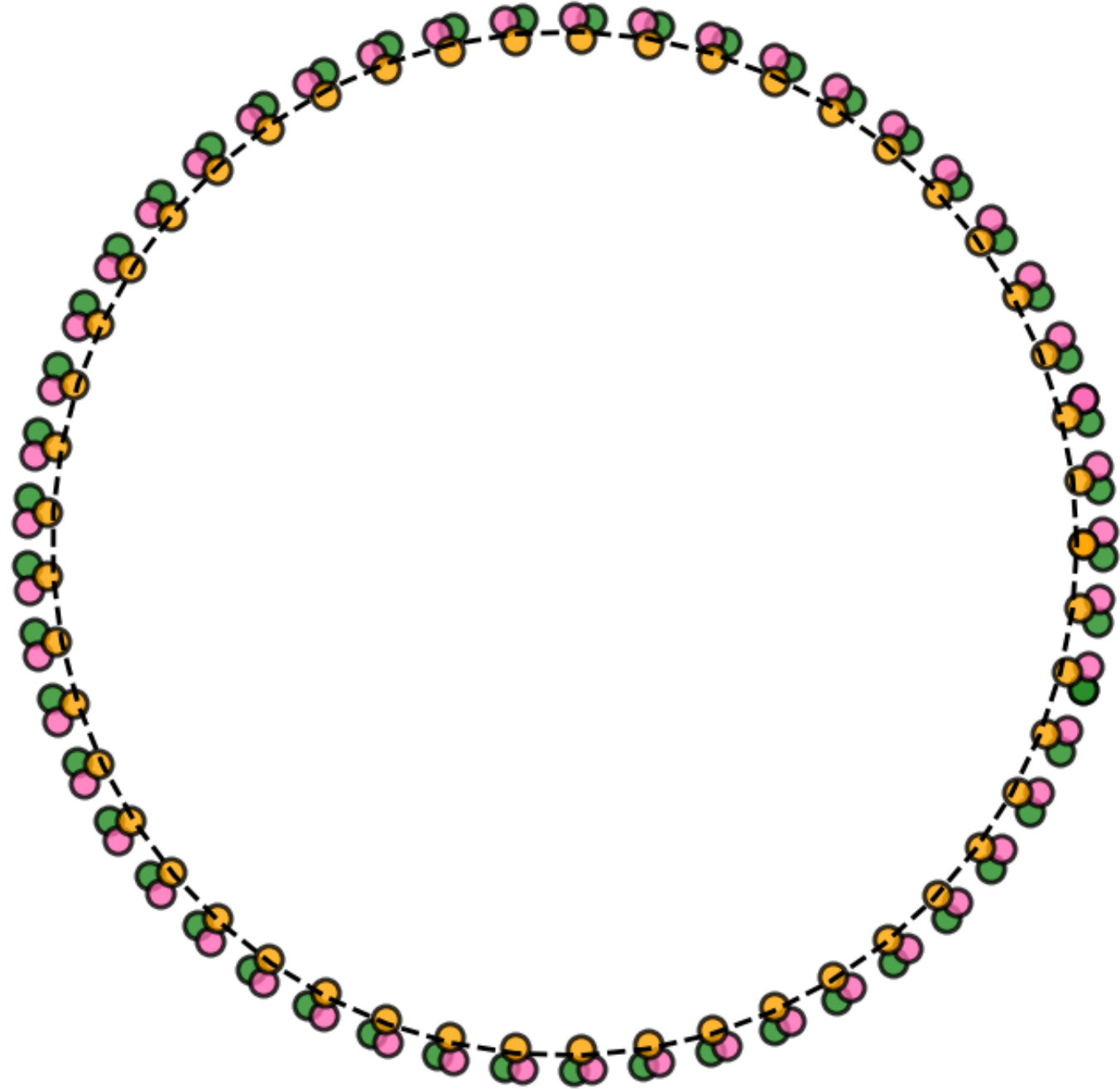


- $h_R = \frac{h_+ - ih_-}{\sqrt{2}}$

- $h_L = \frac{h_+ + ih_-}{\sqrt{2}}$

- $h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$

Circular Polarization Basis

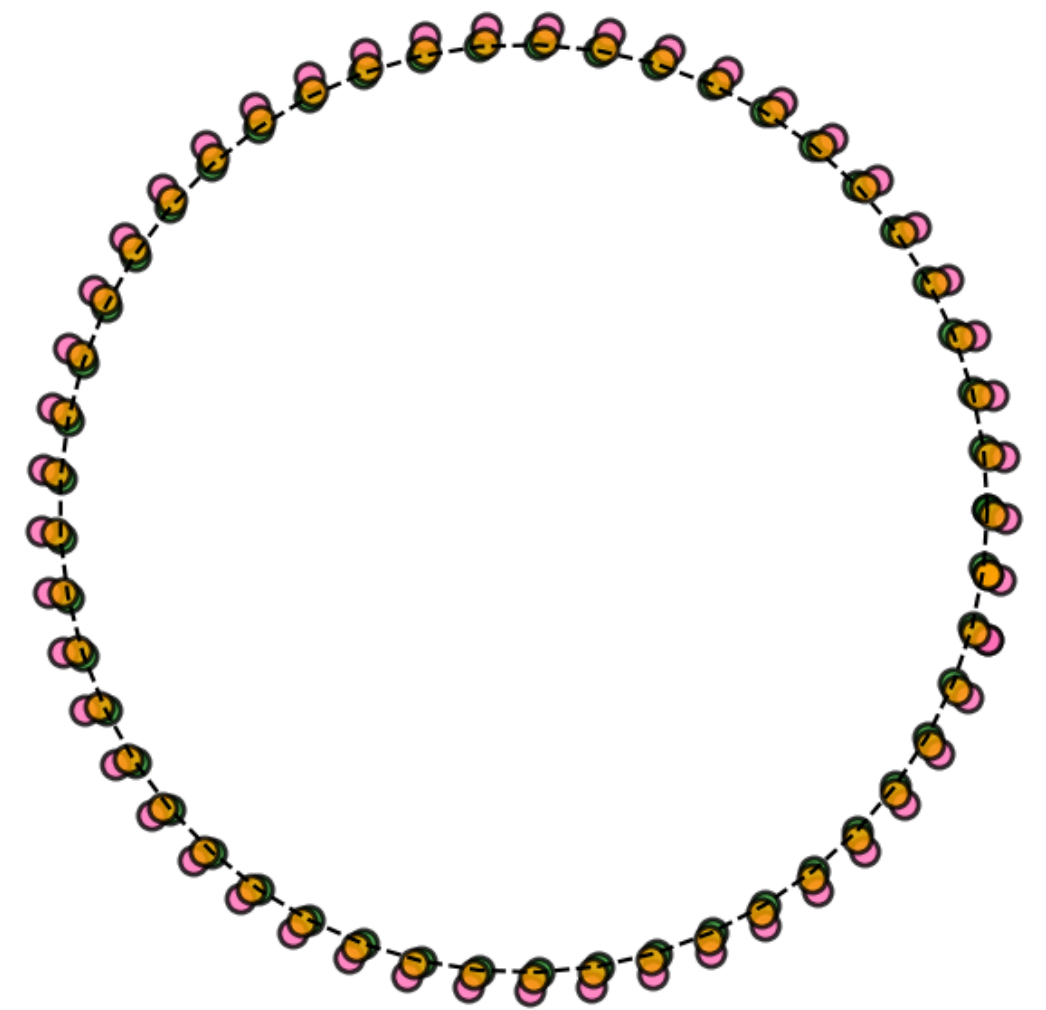
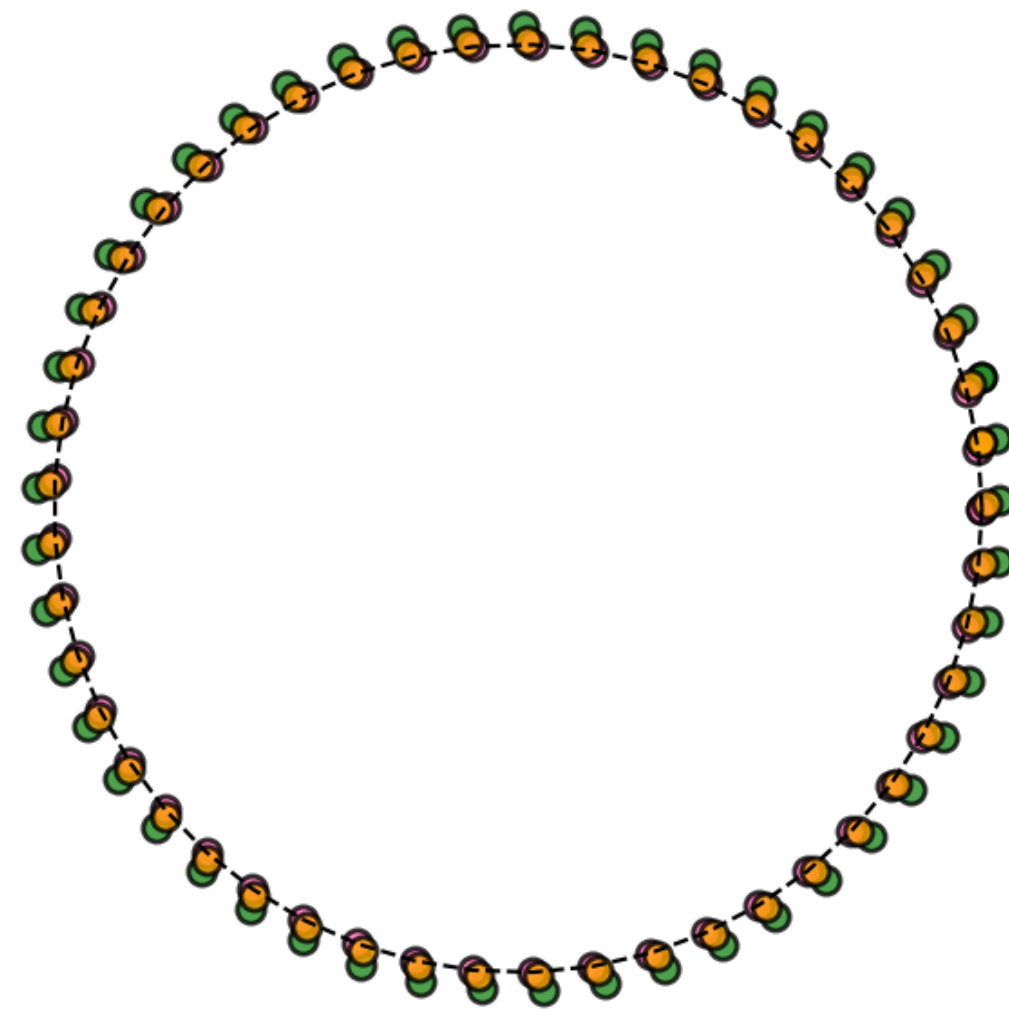
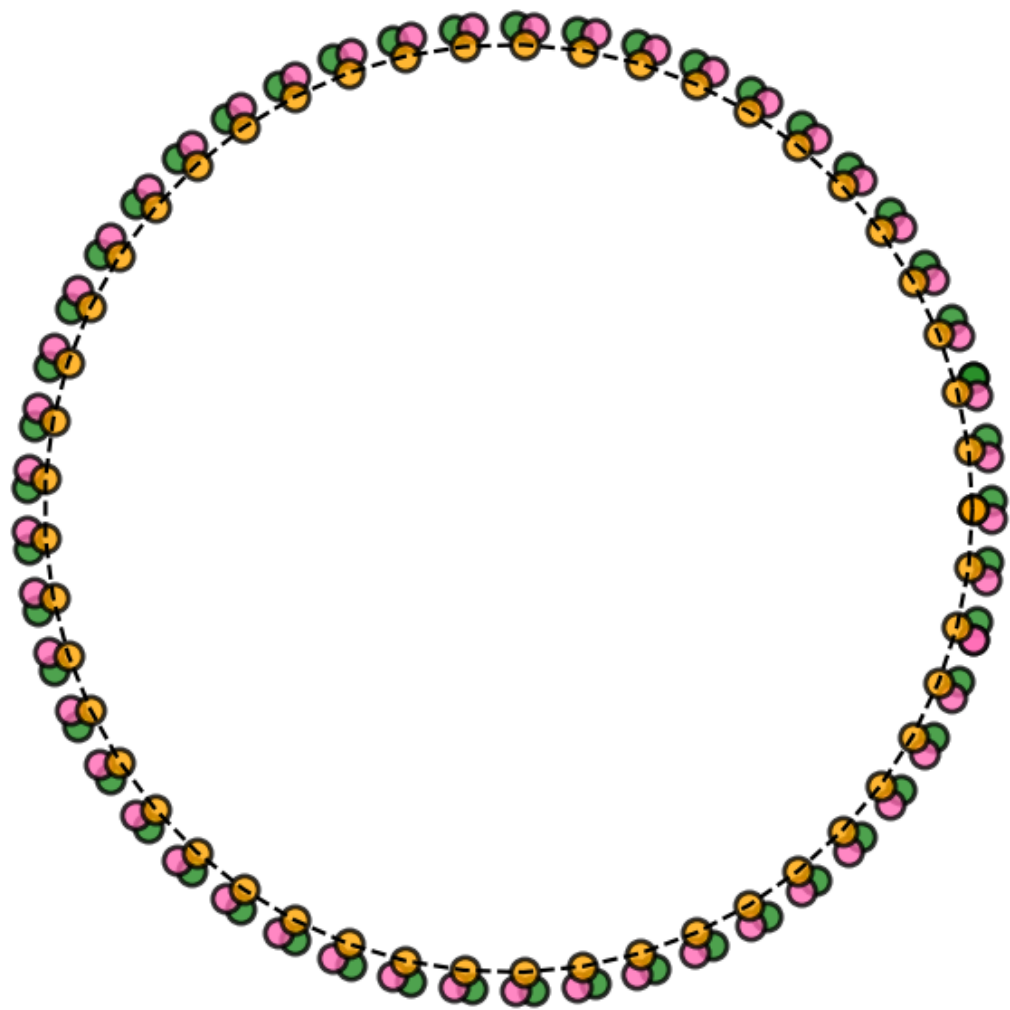


- $$h_R = \frac{h_+ - ih_x}{\sqrt{2}}$$

- $$h_L = \frac{h_+ + ih_x}{\sqrt{2}}$$

- $$h_{av} = \frac{h_R + h_L}{2} = \frac{h_+}{2}$$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

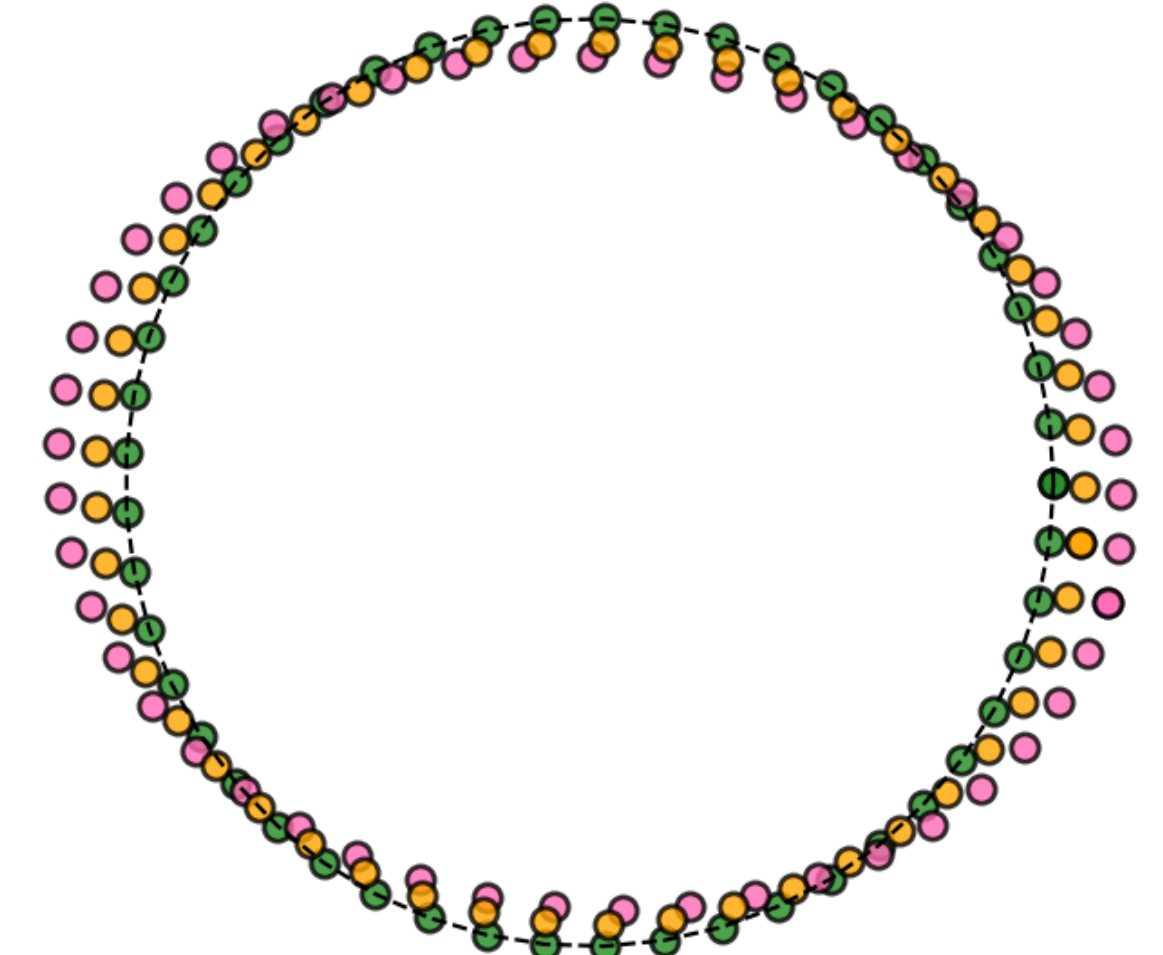
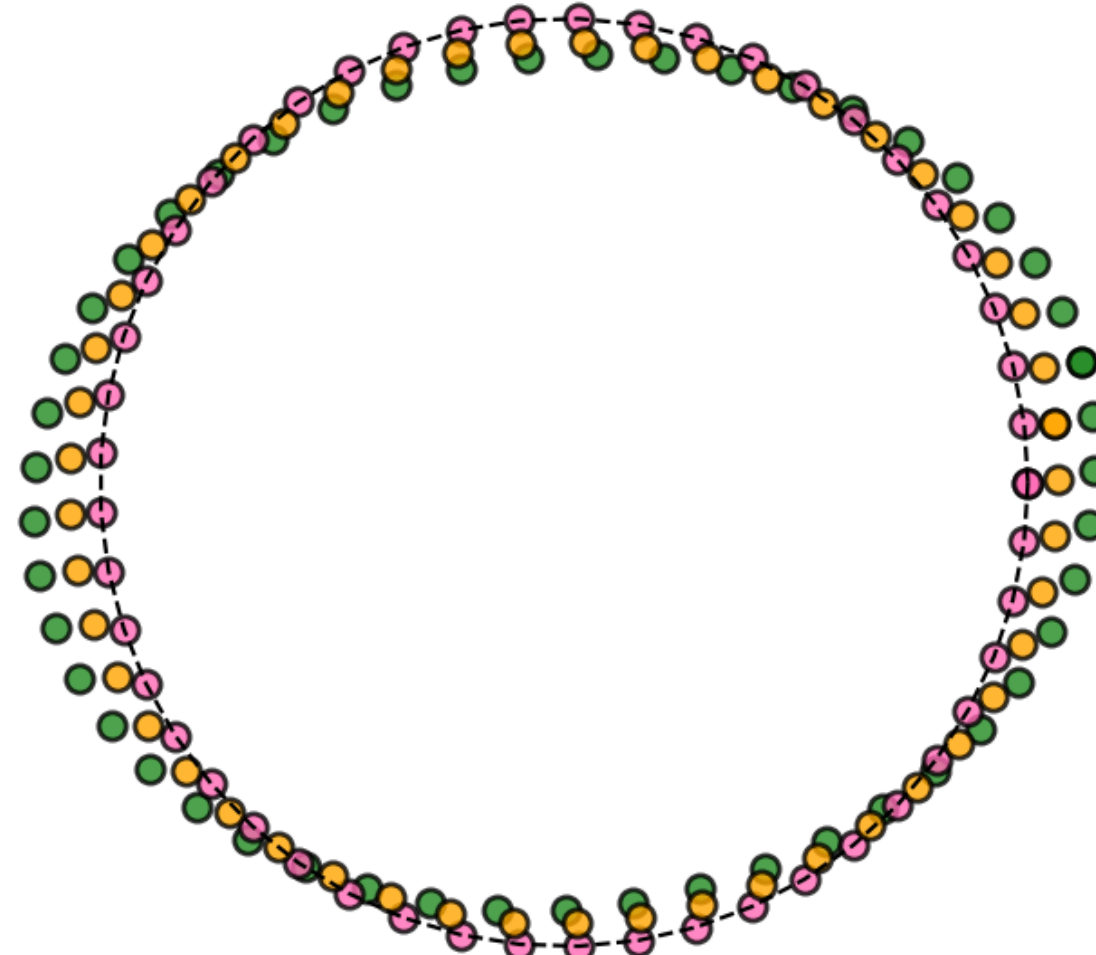
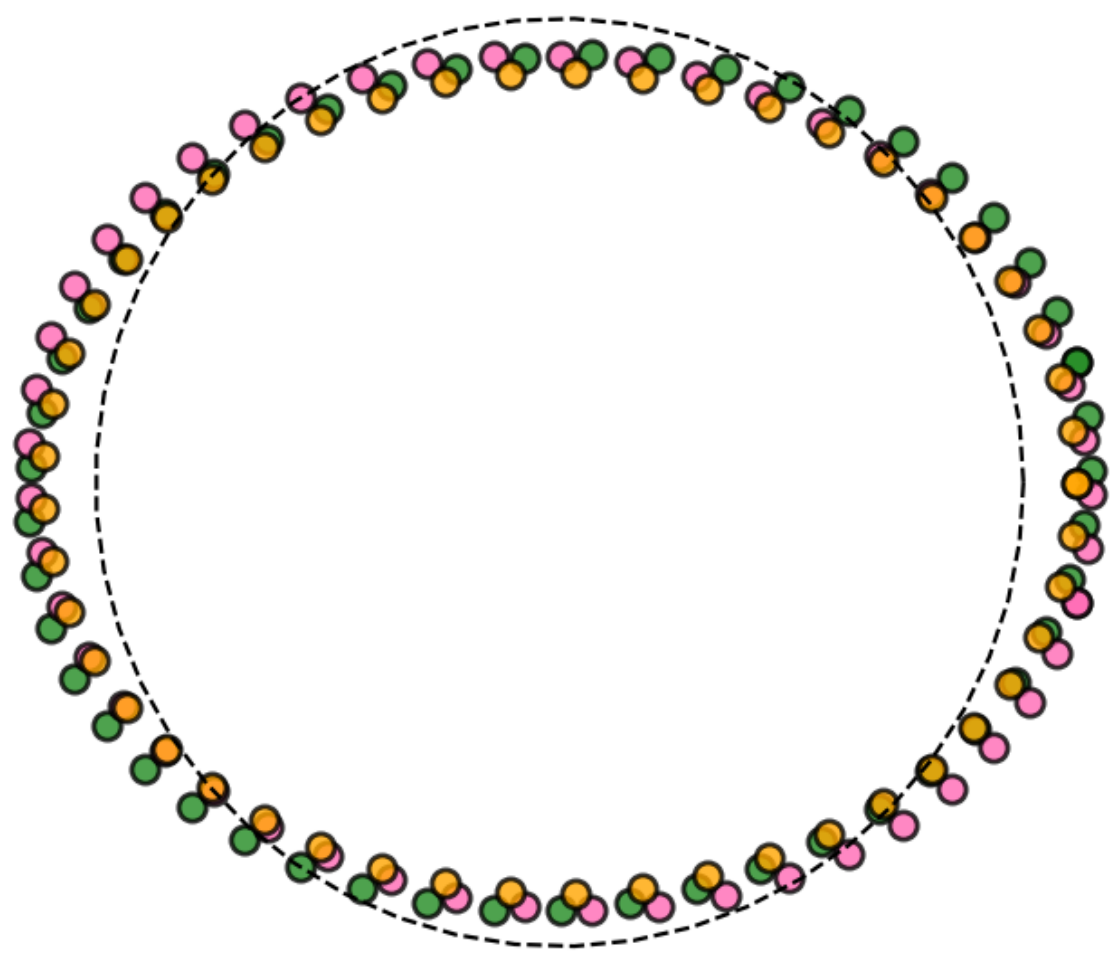
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

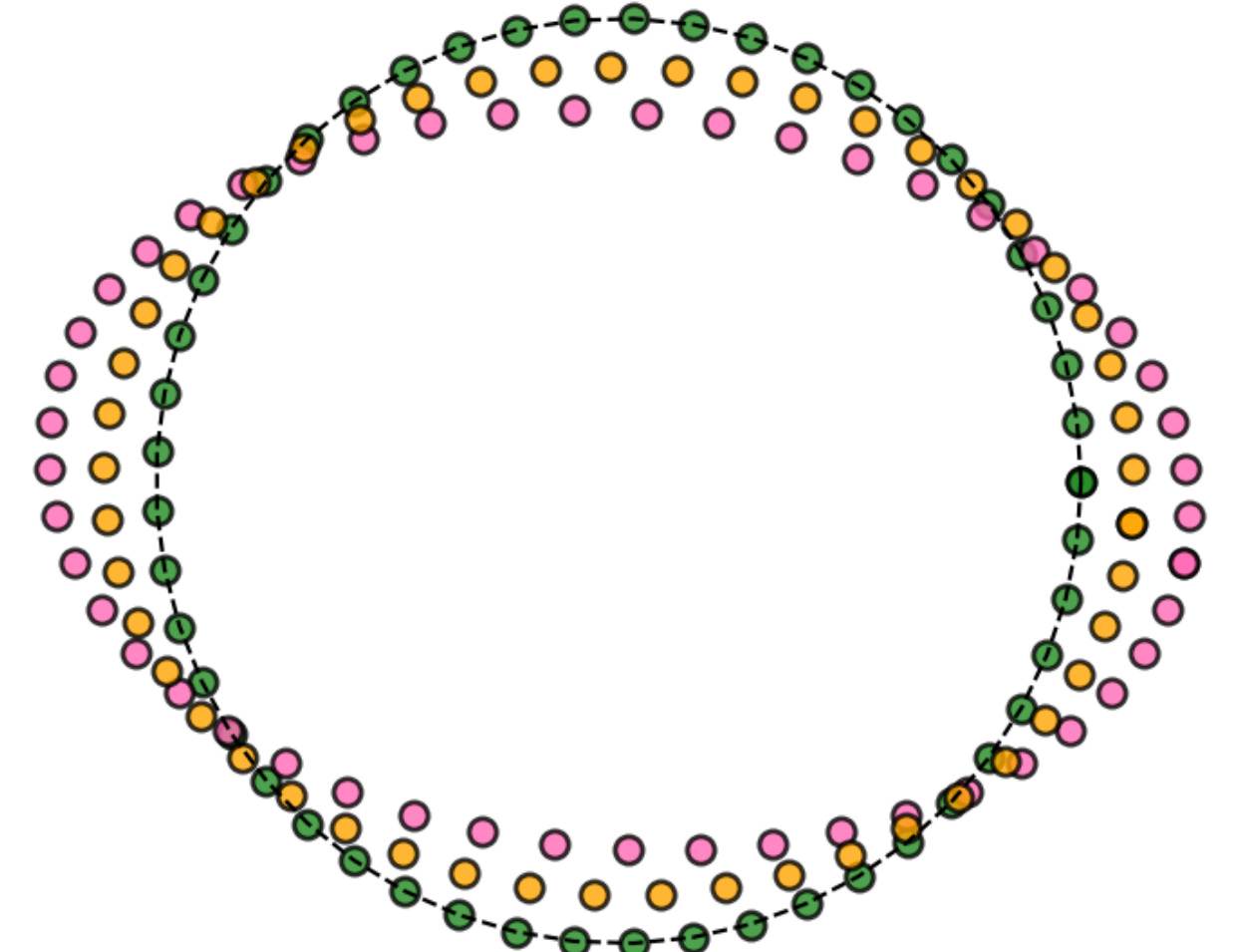
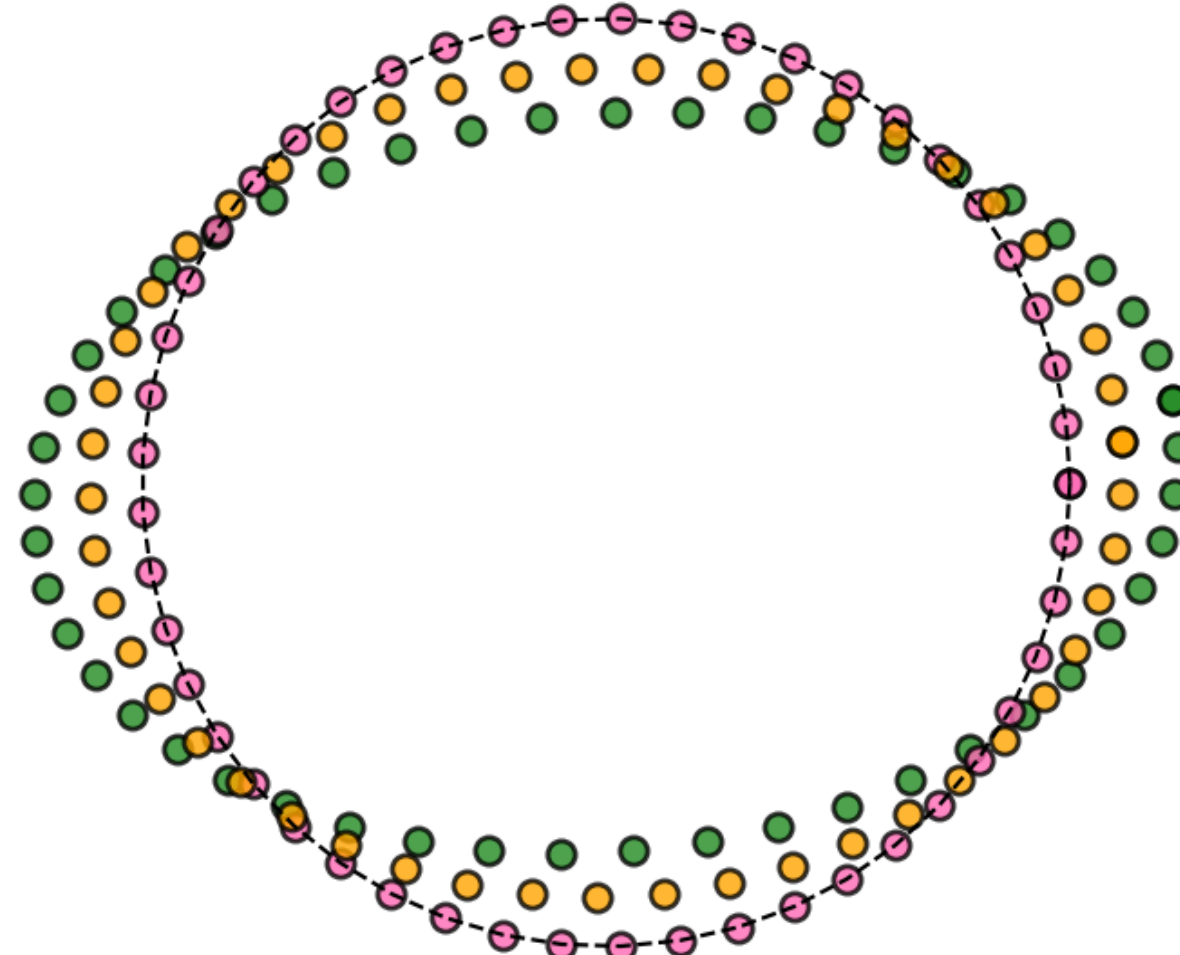
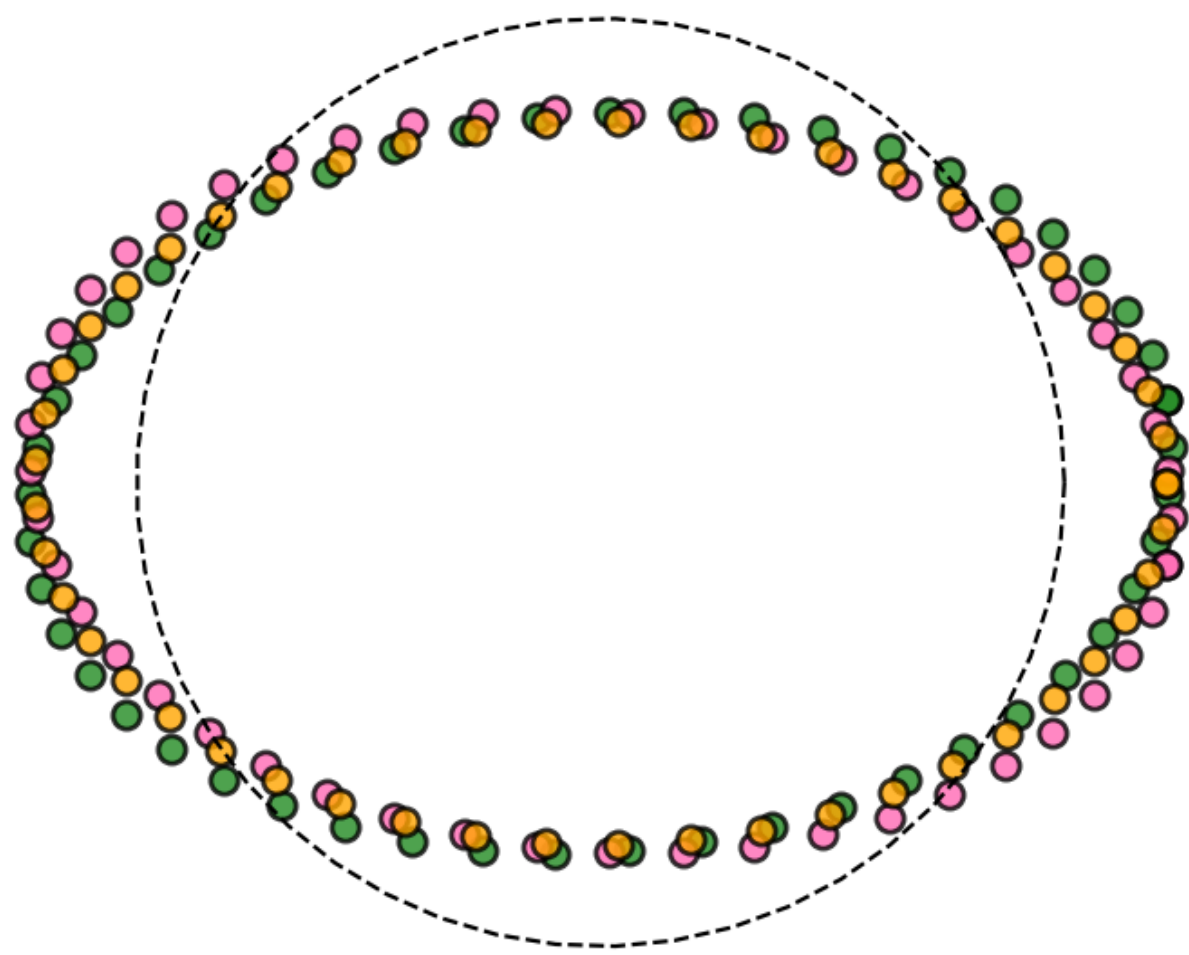
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

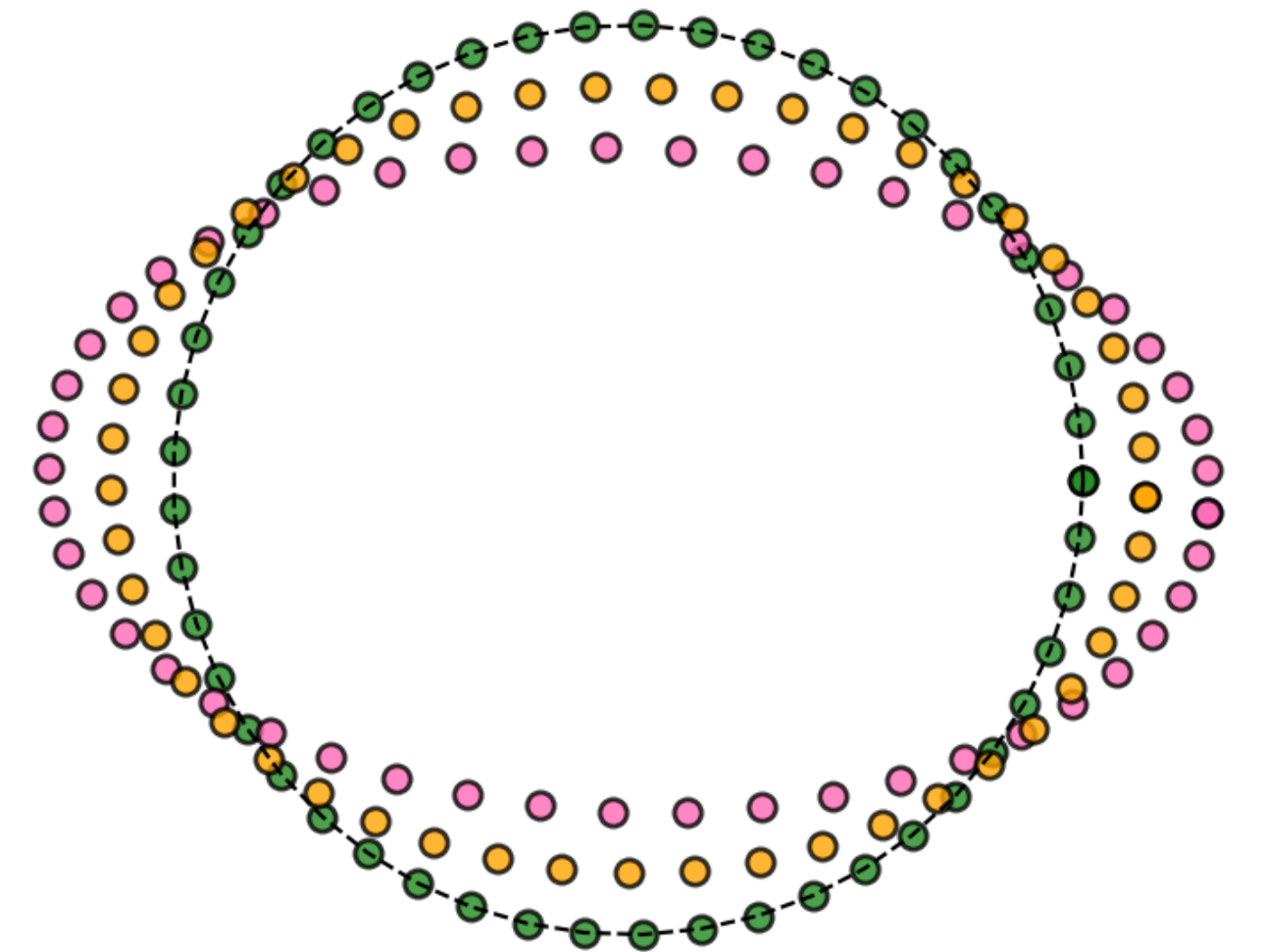
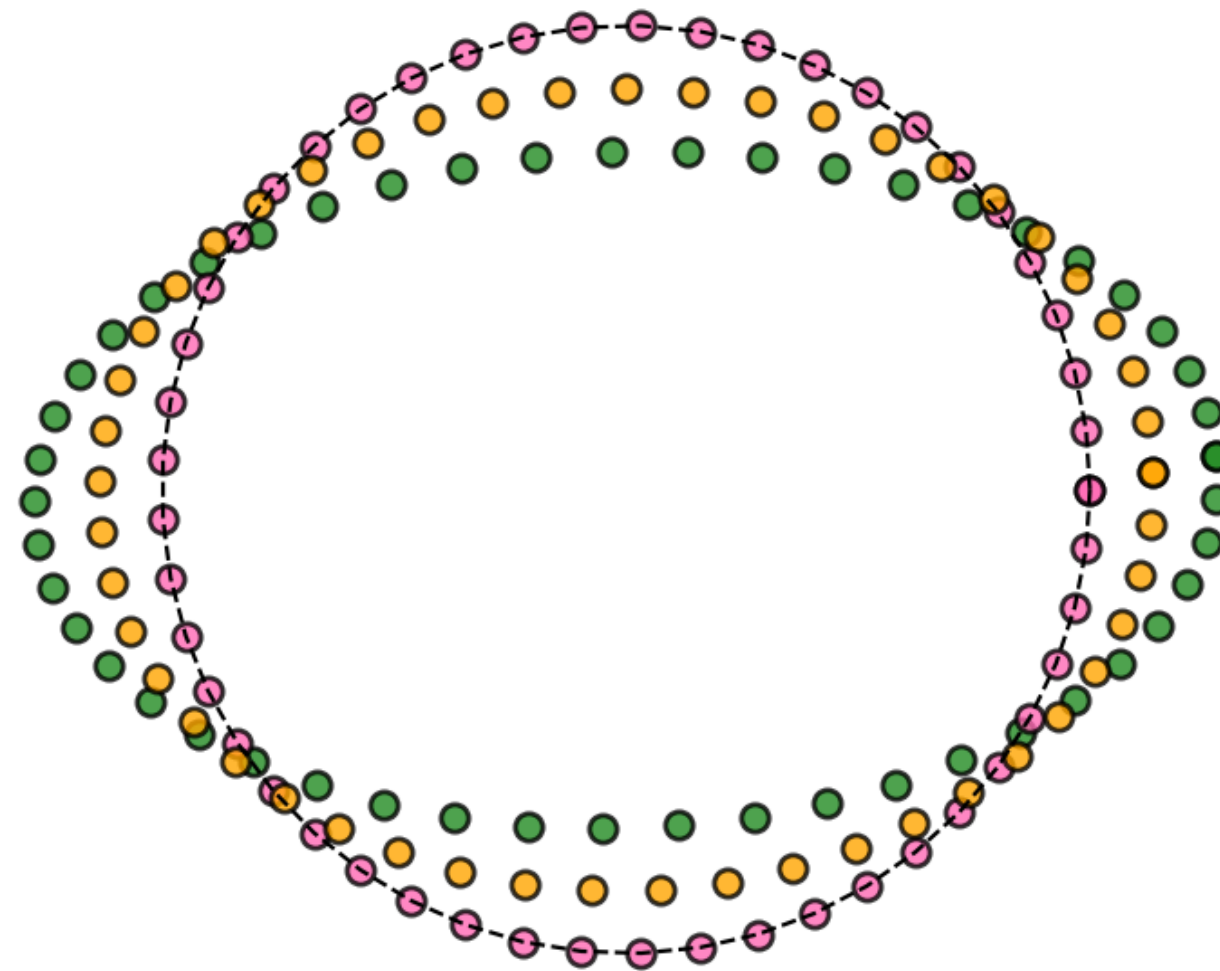
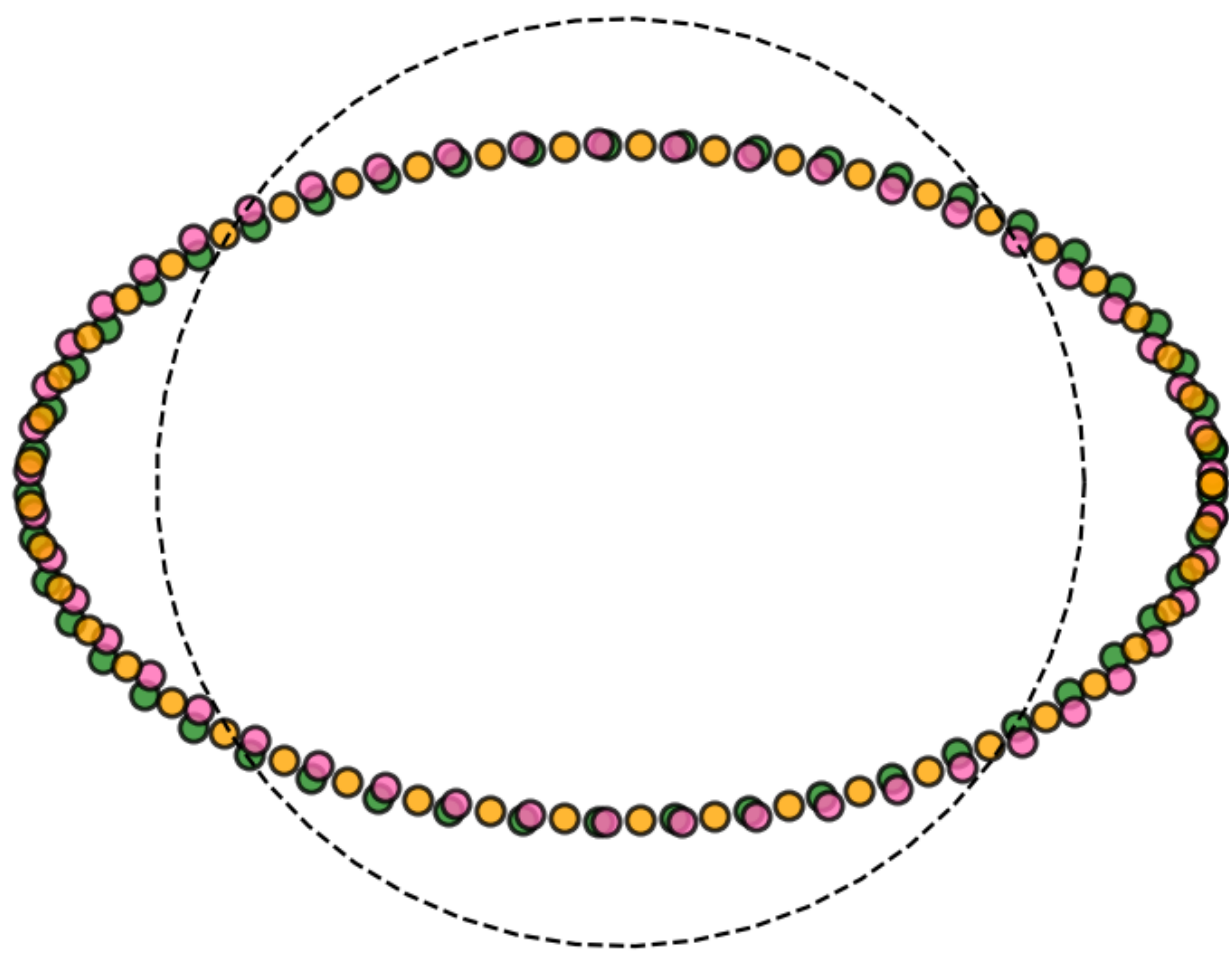
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

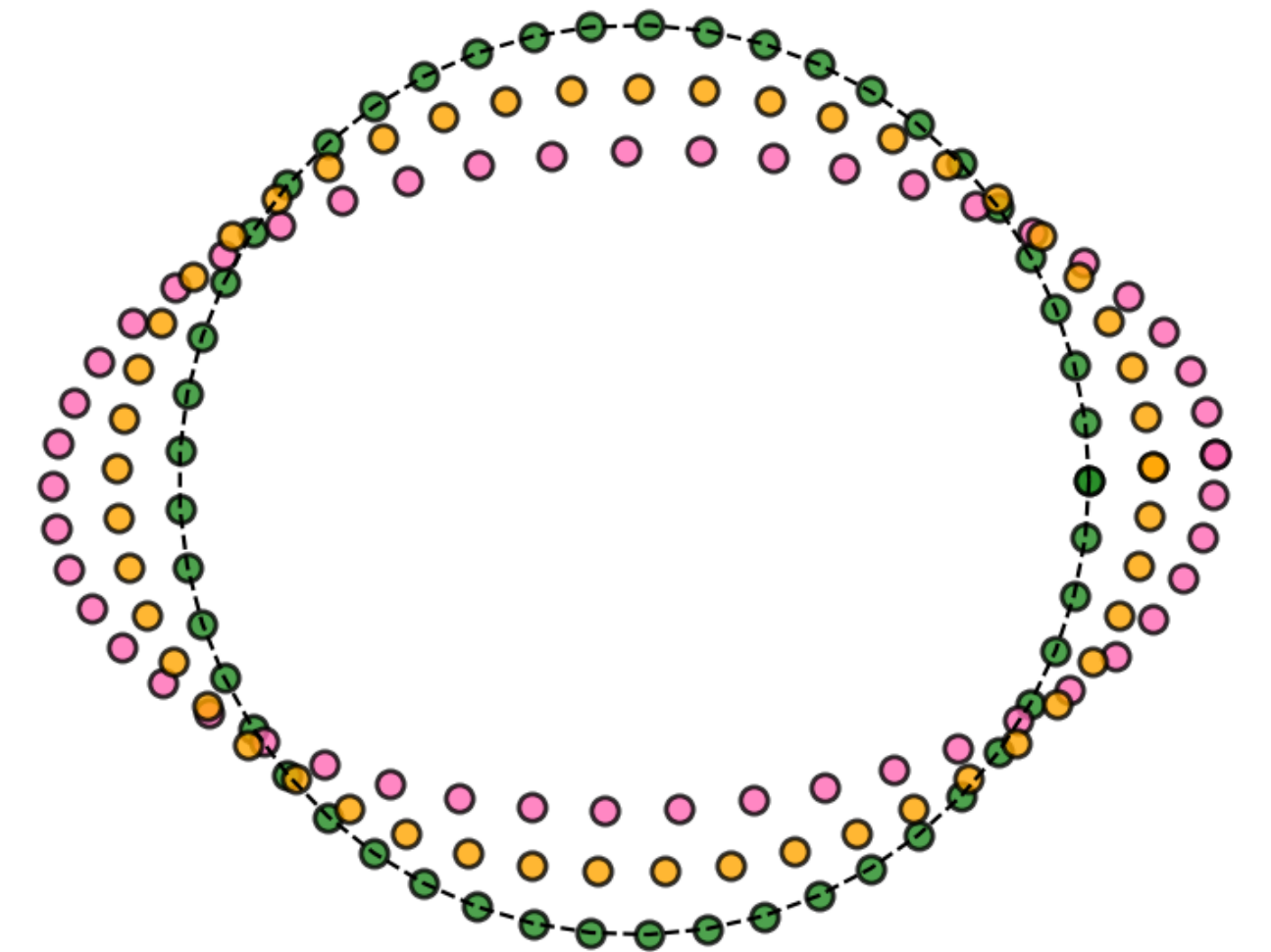
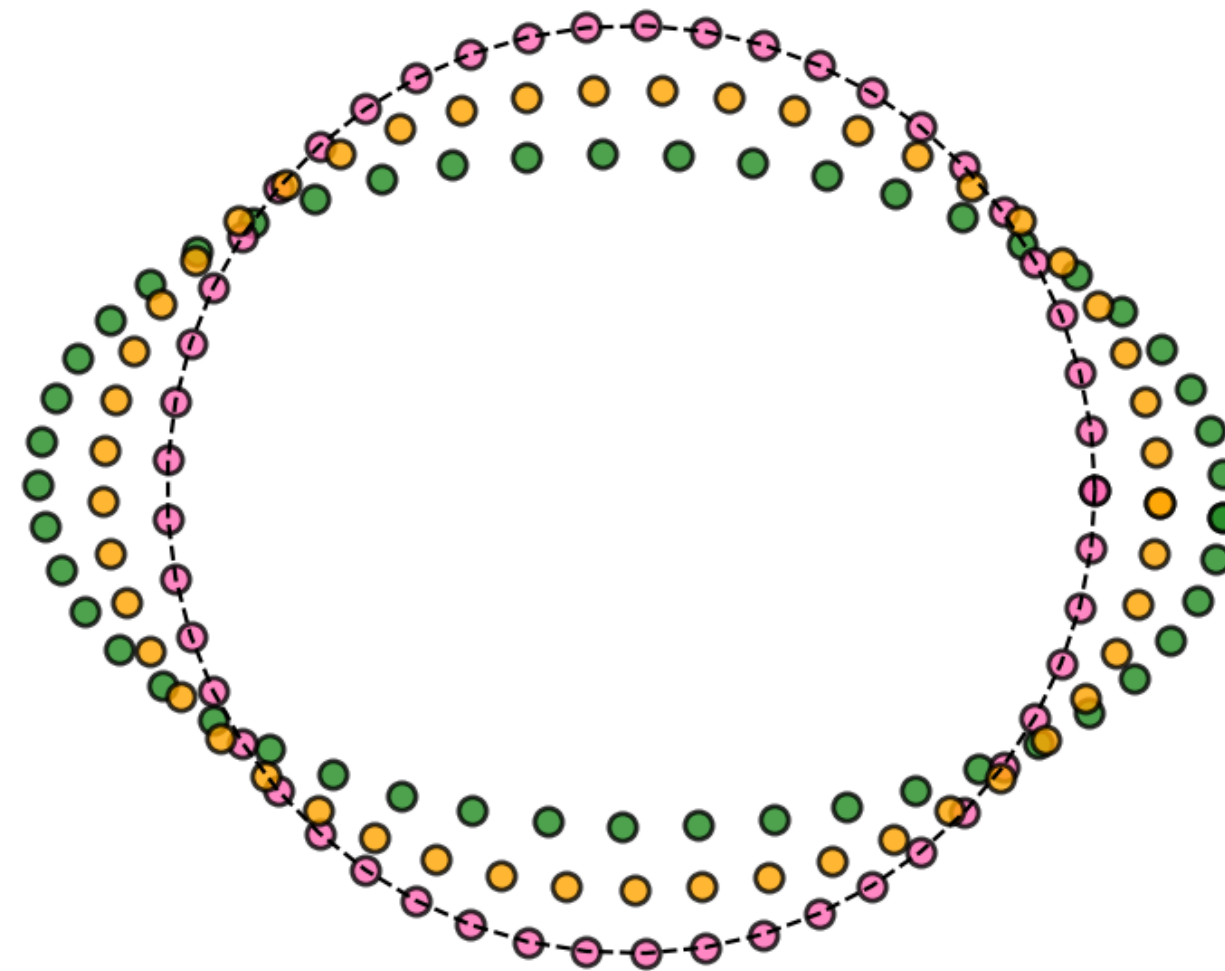
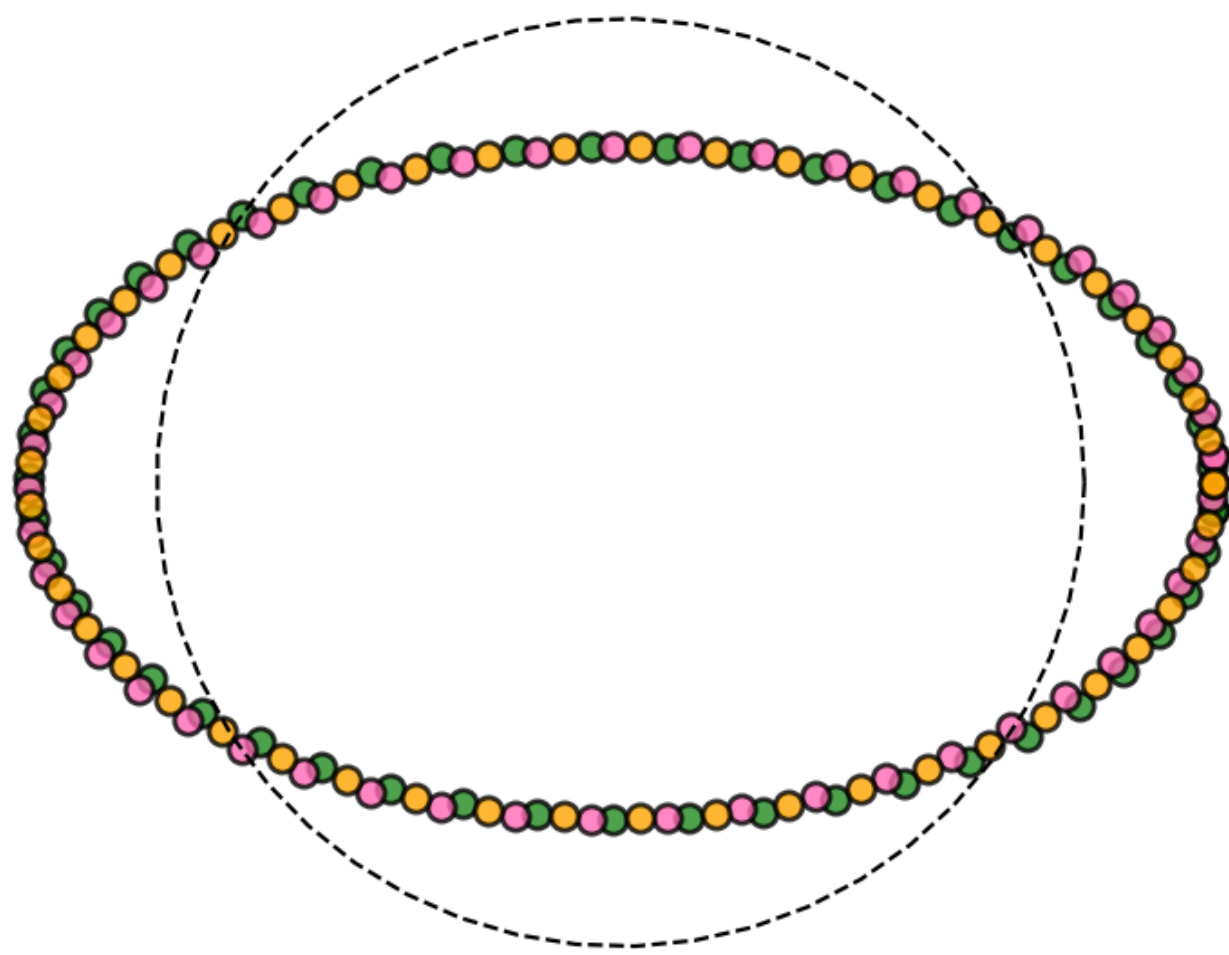
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

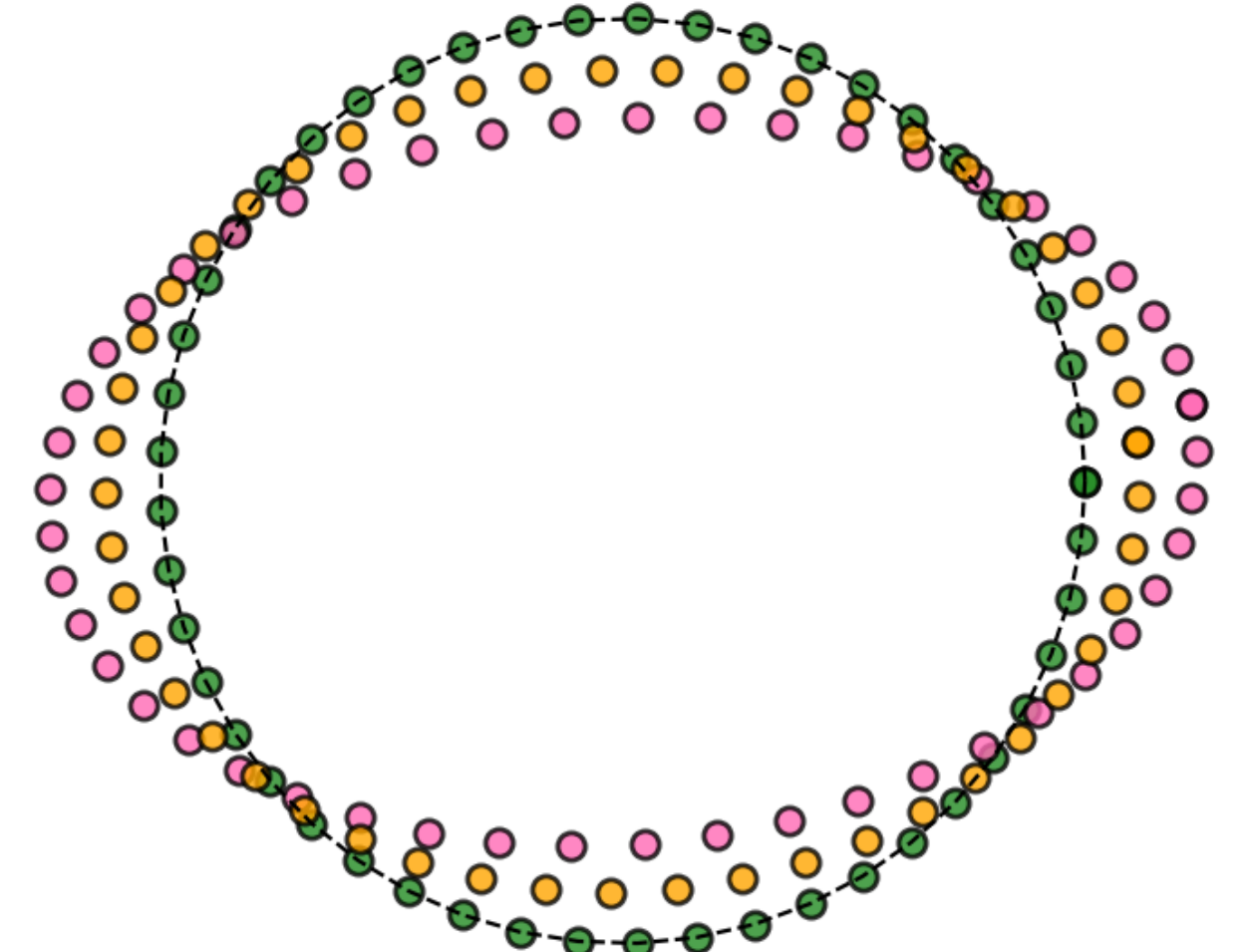
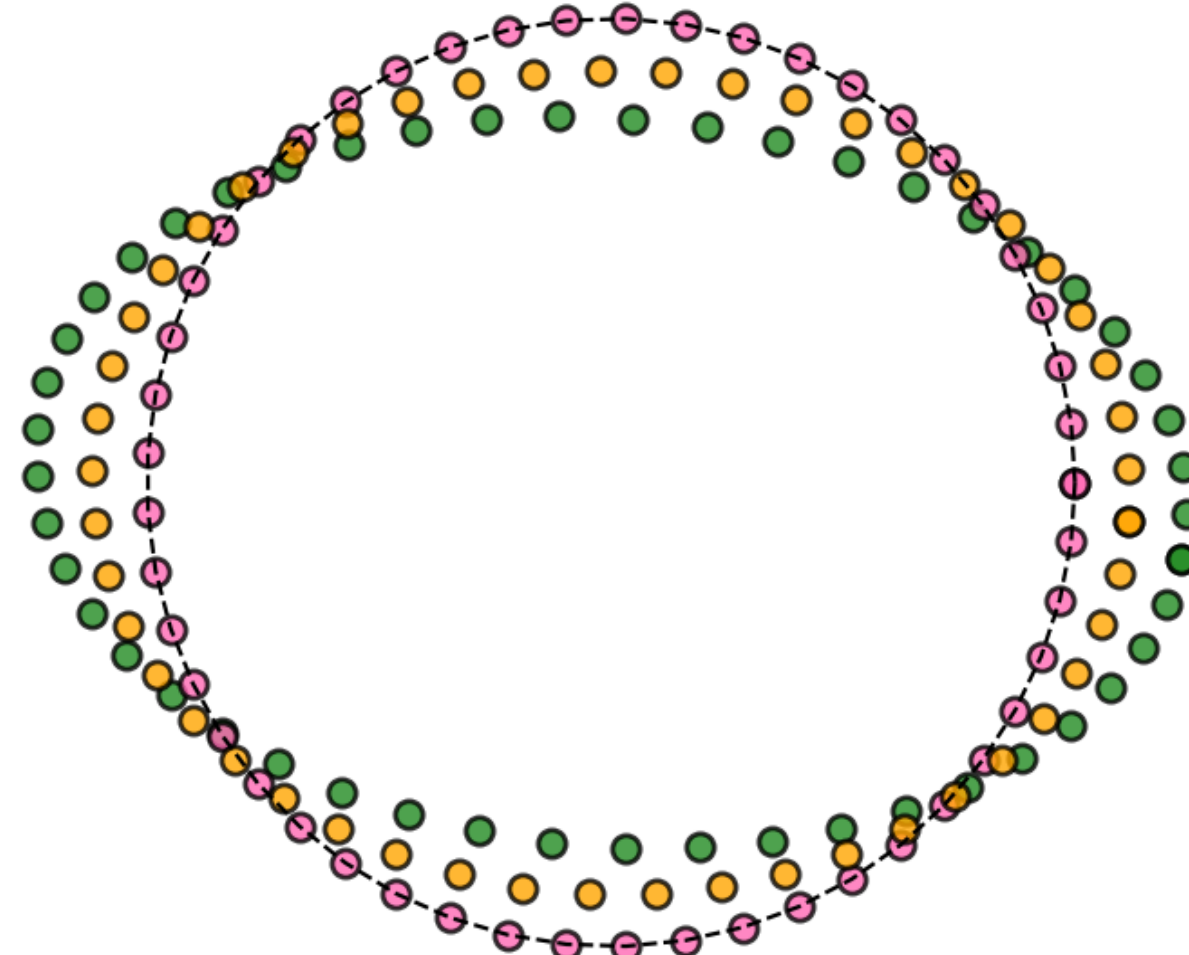
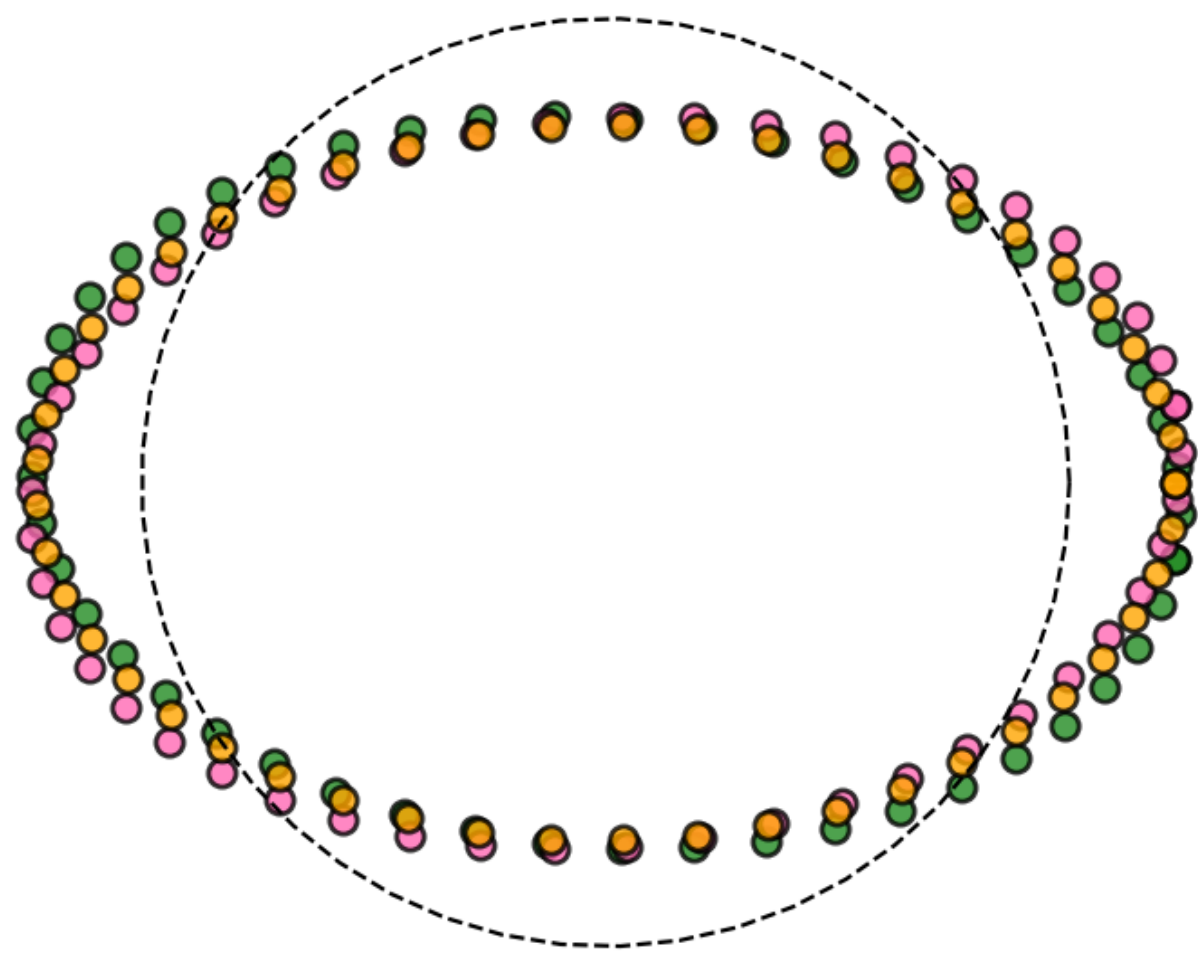
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

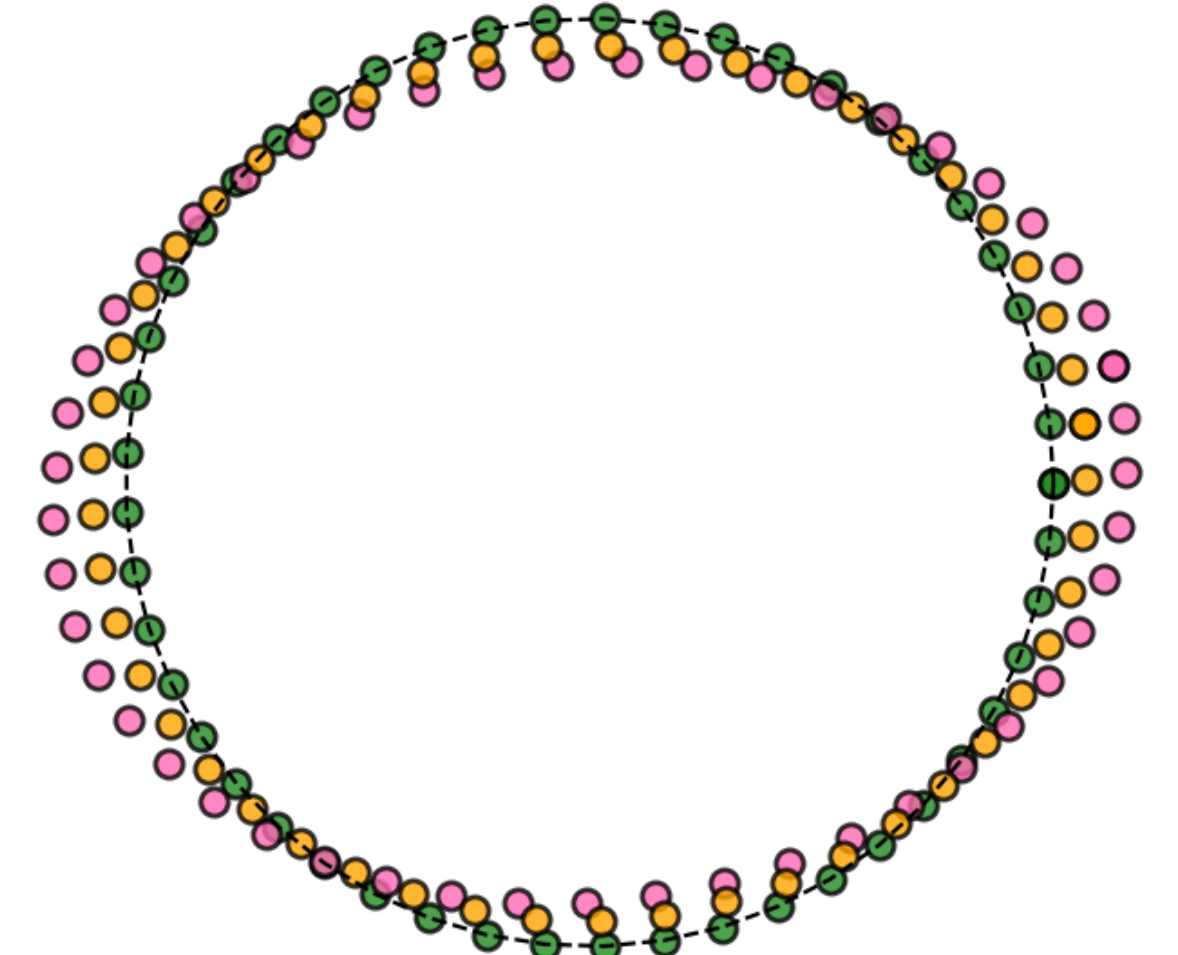
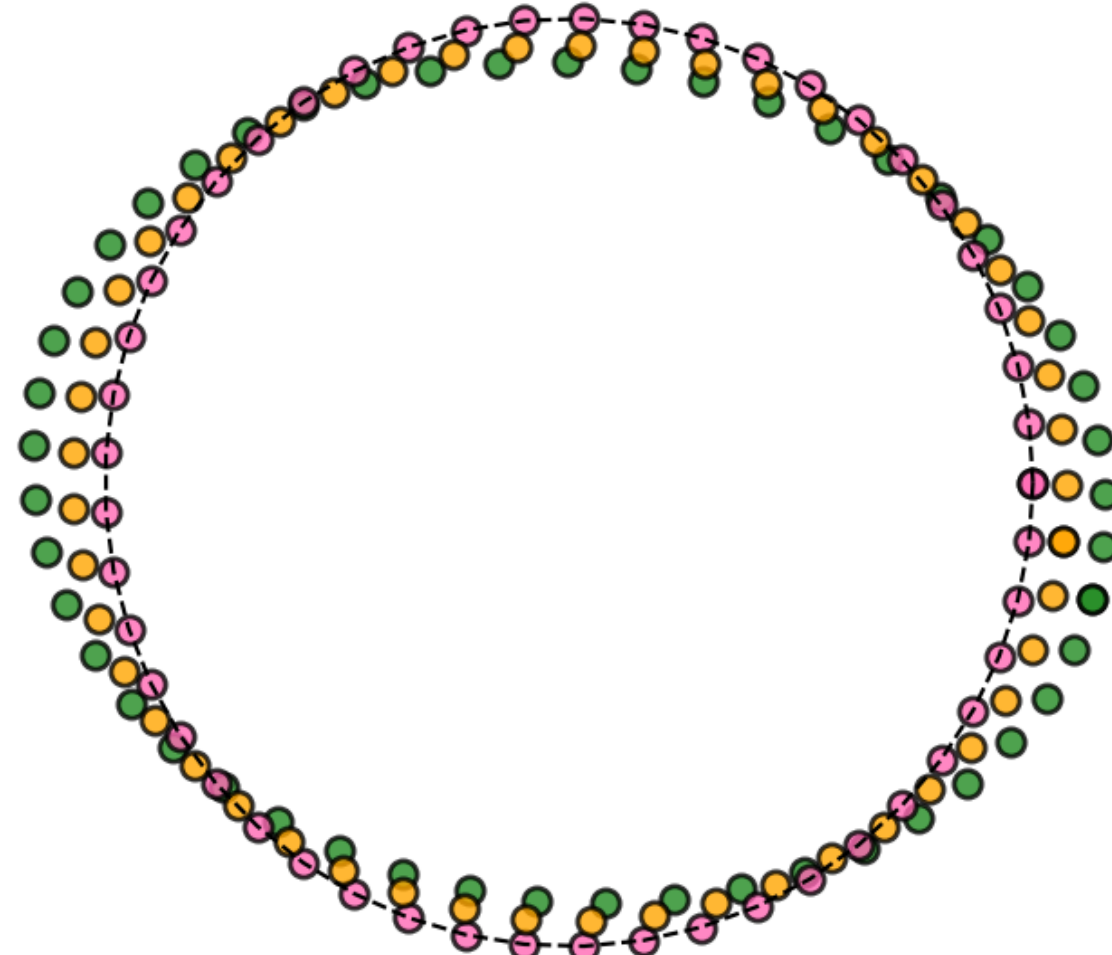
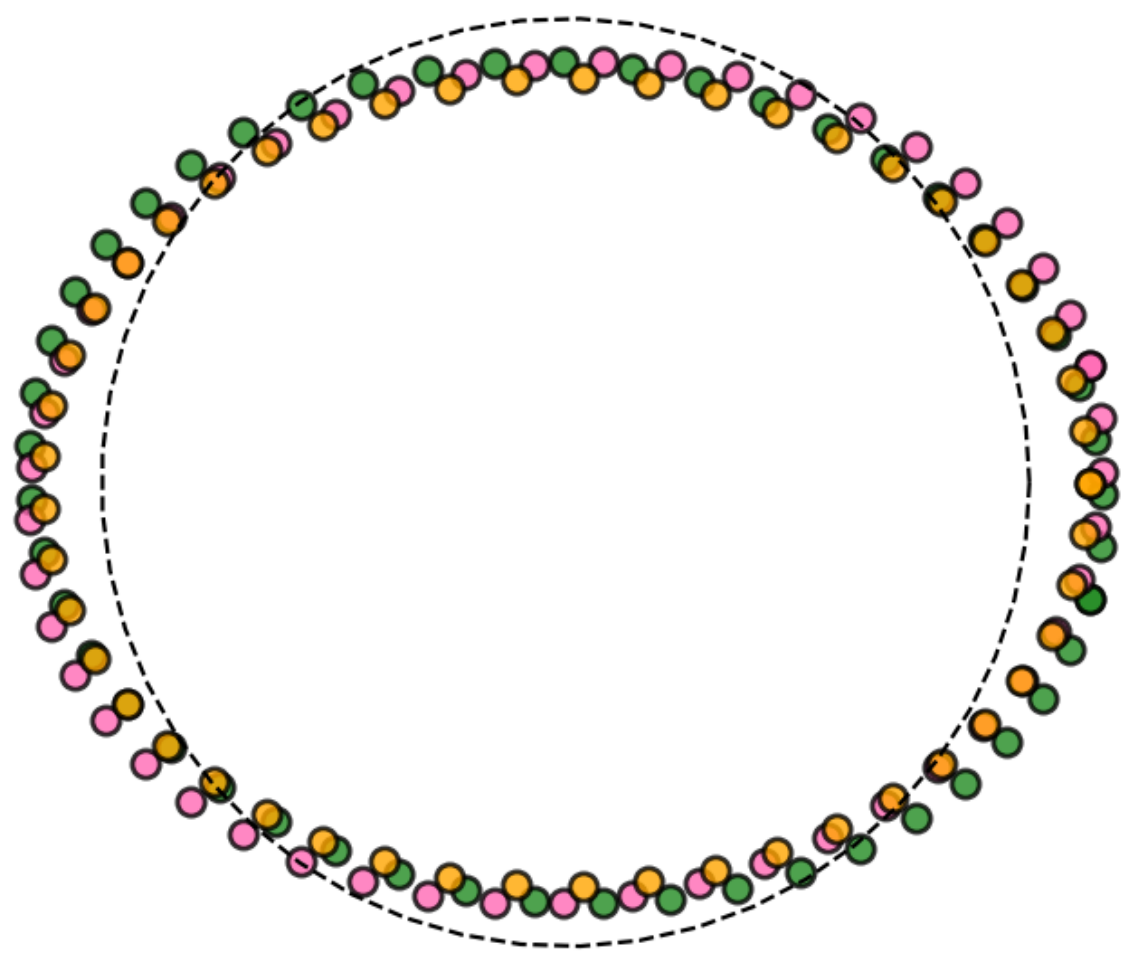
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

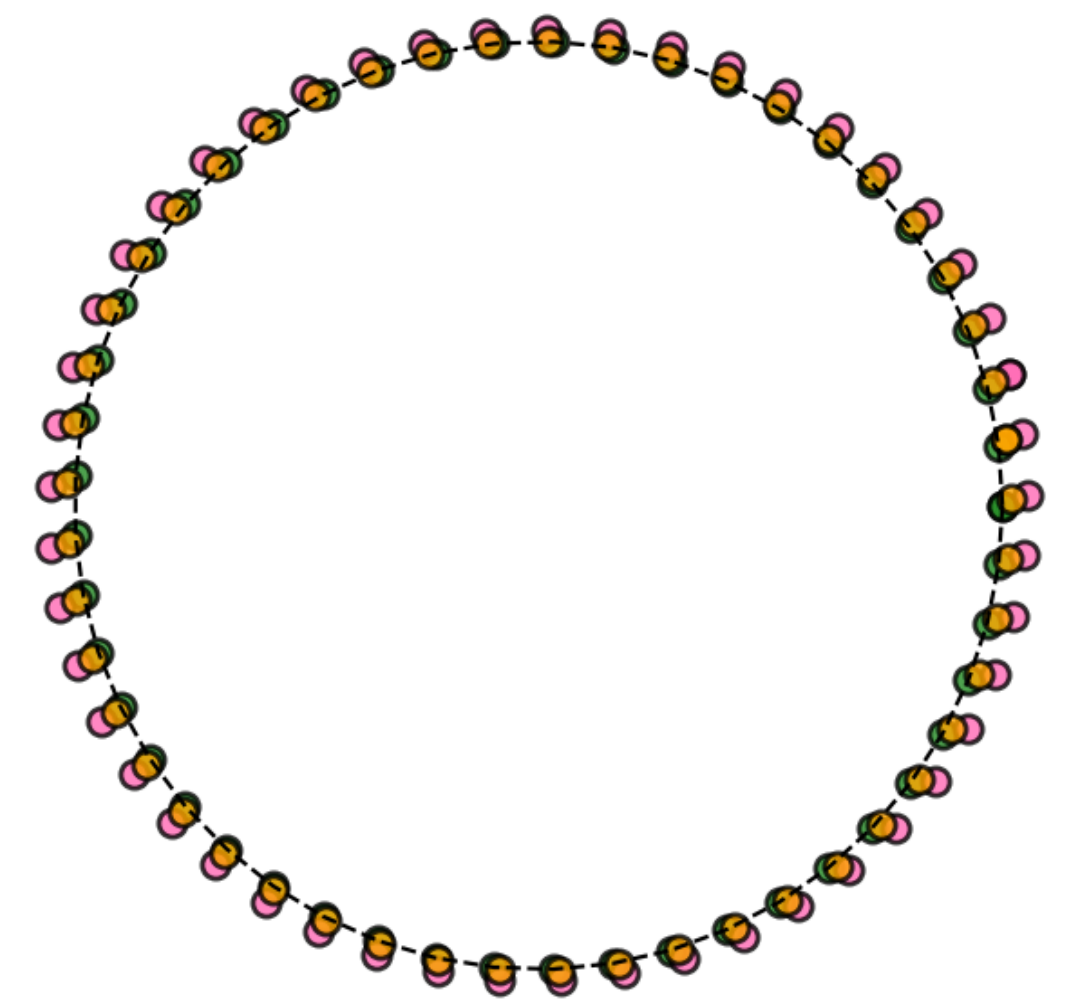
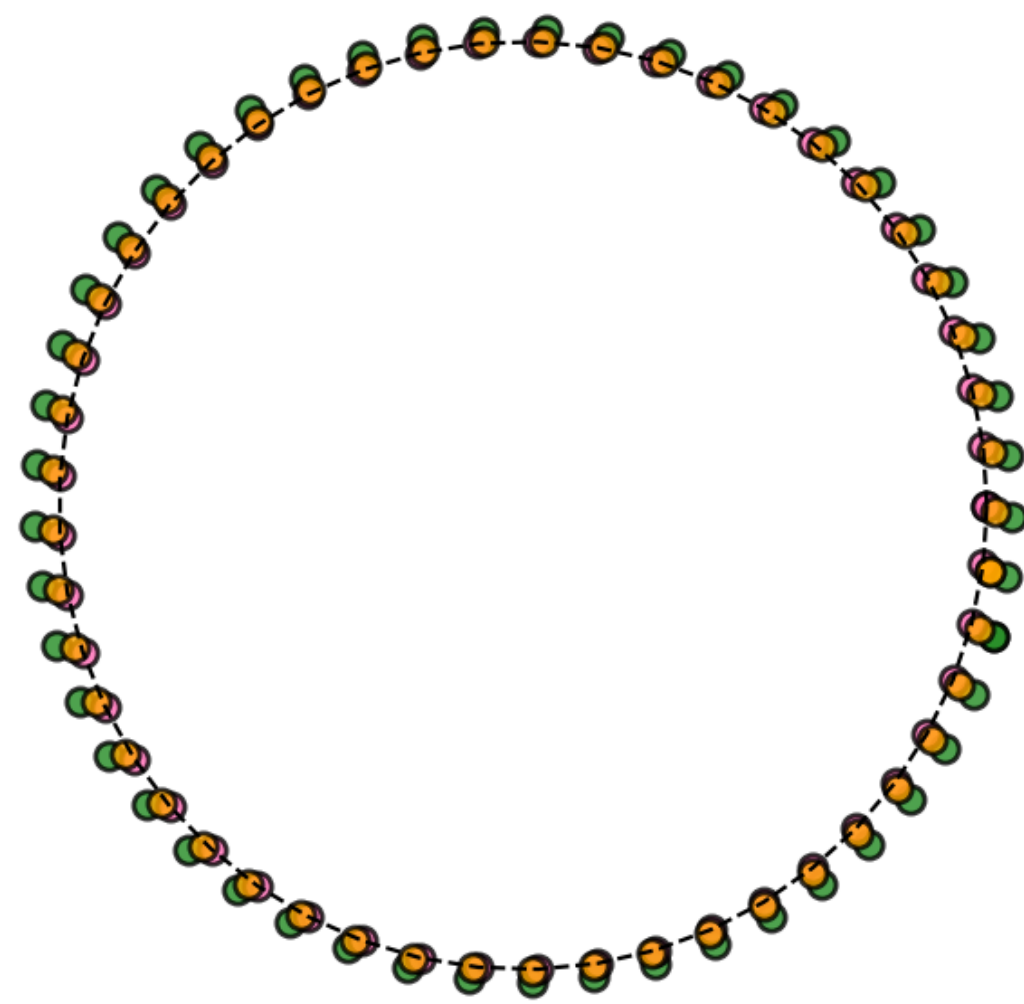
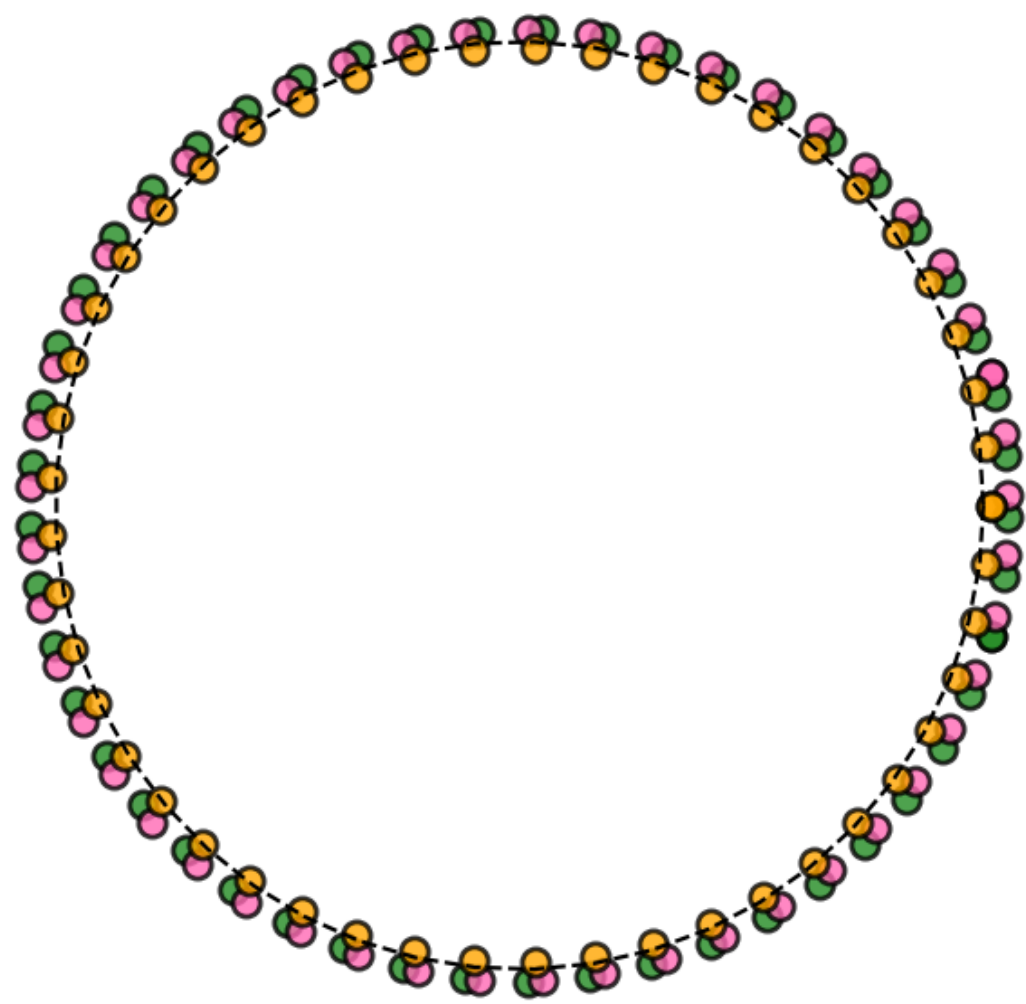
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

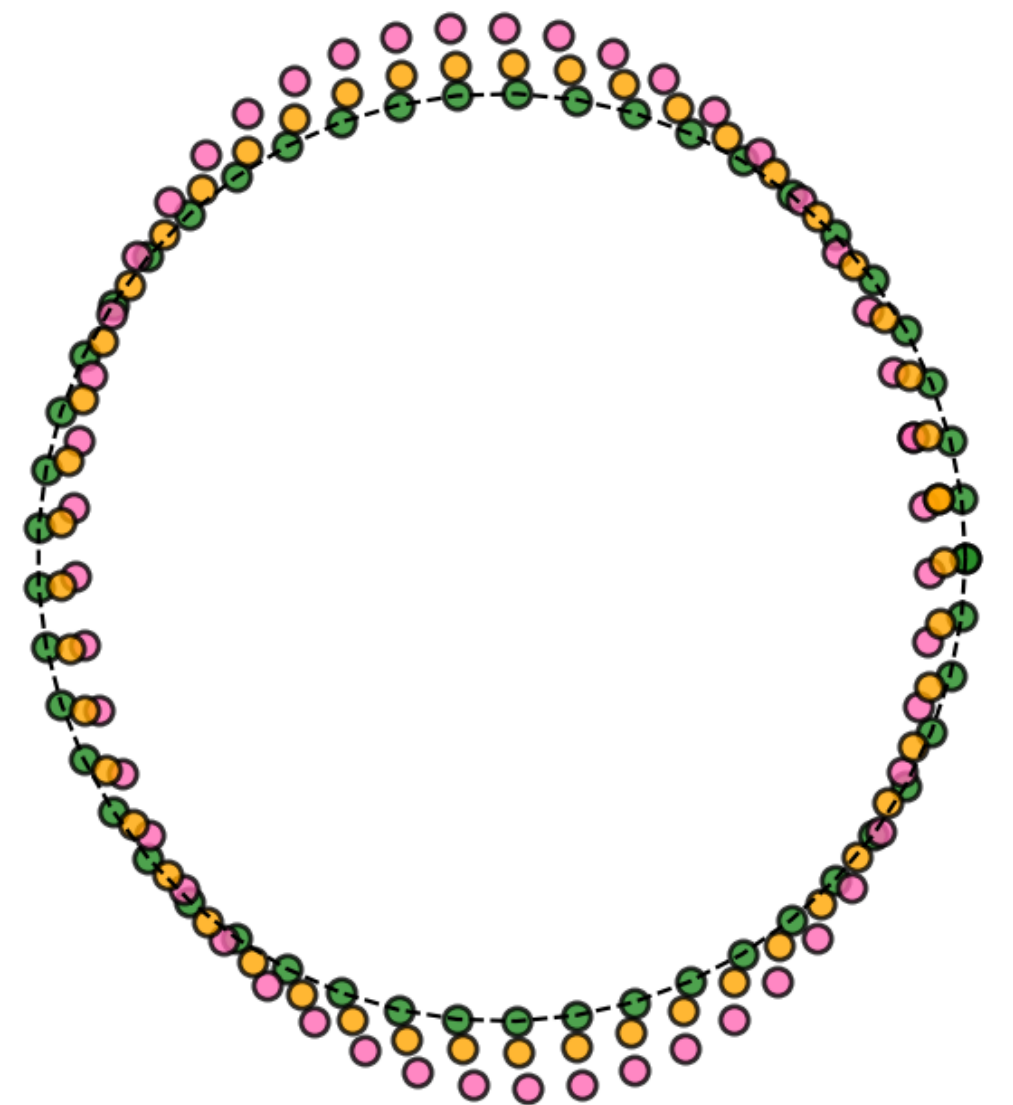
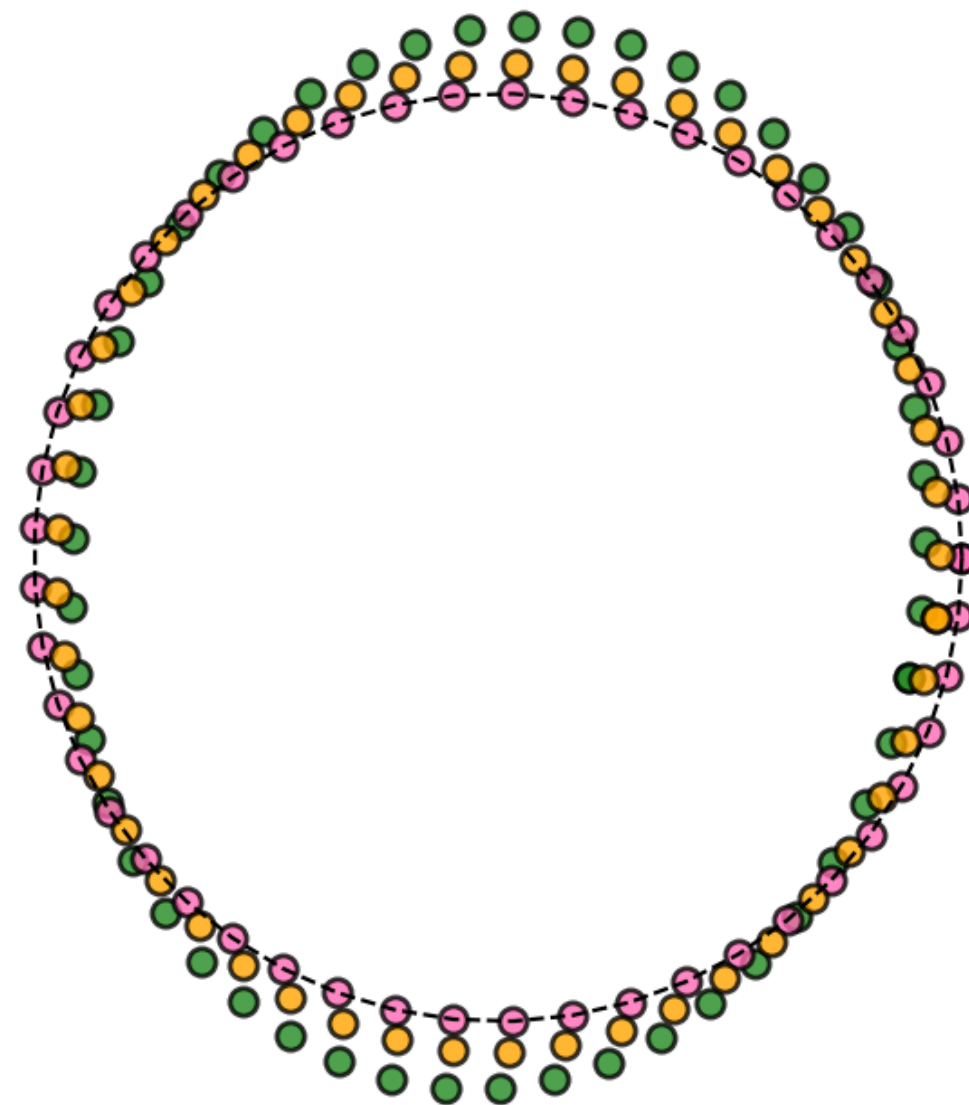
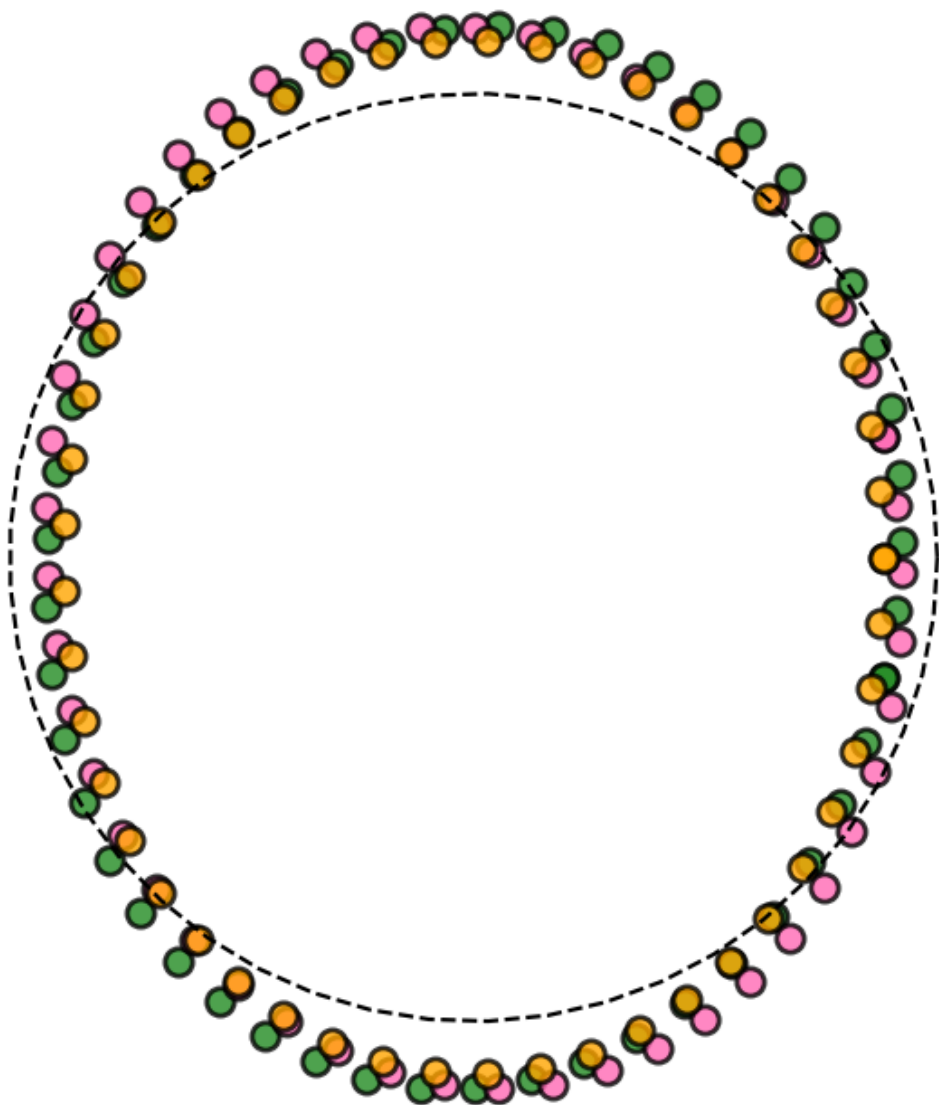
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

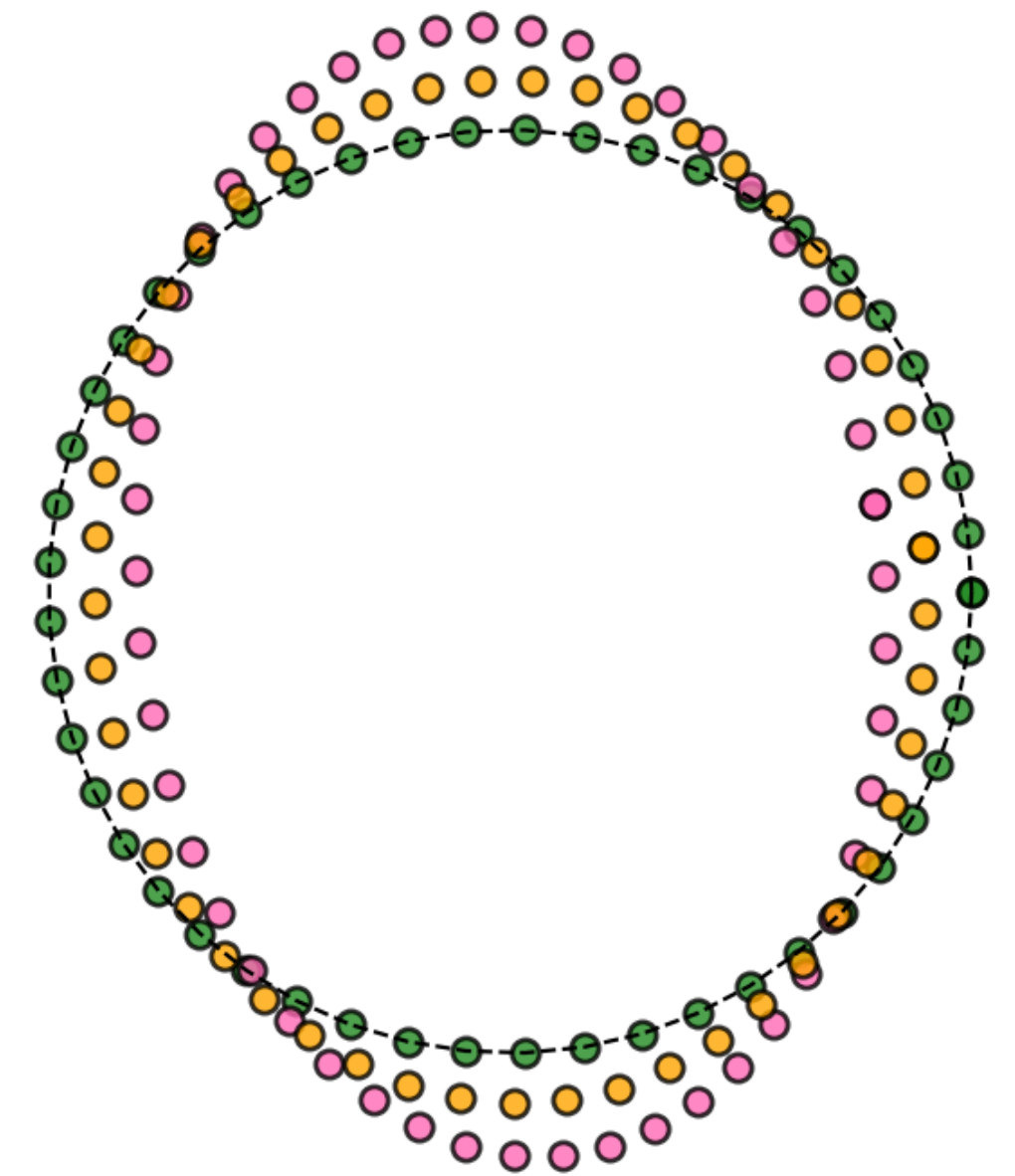
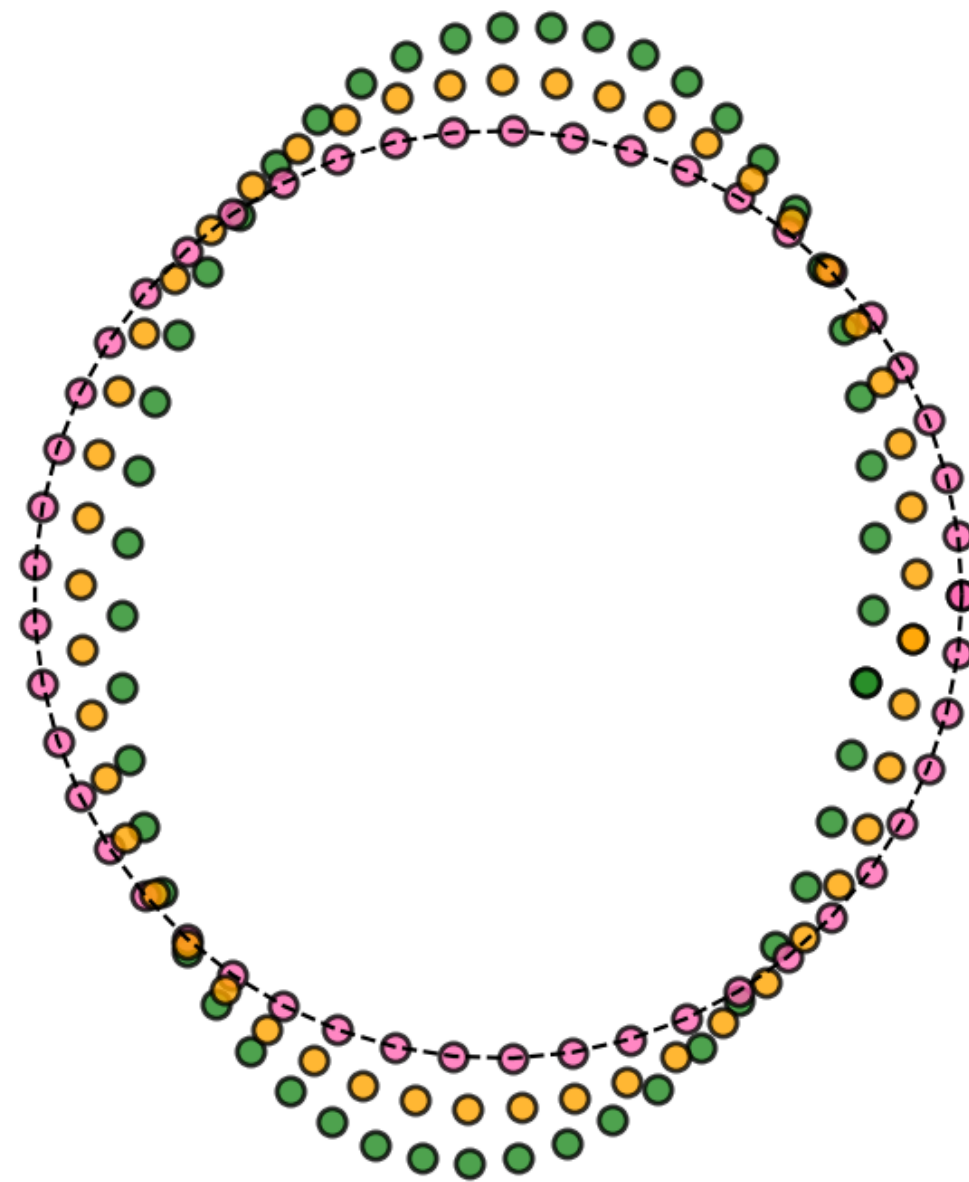
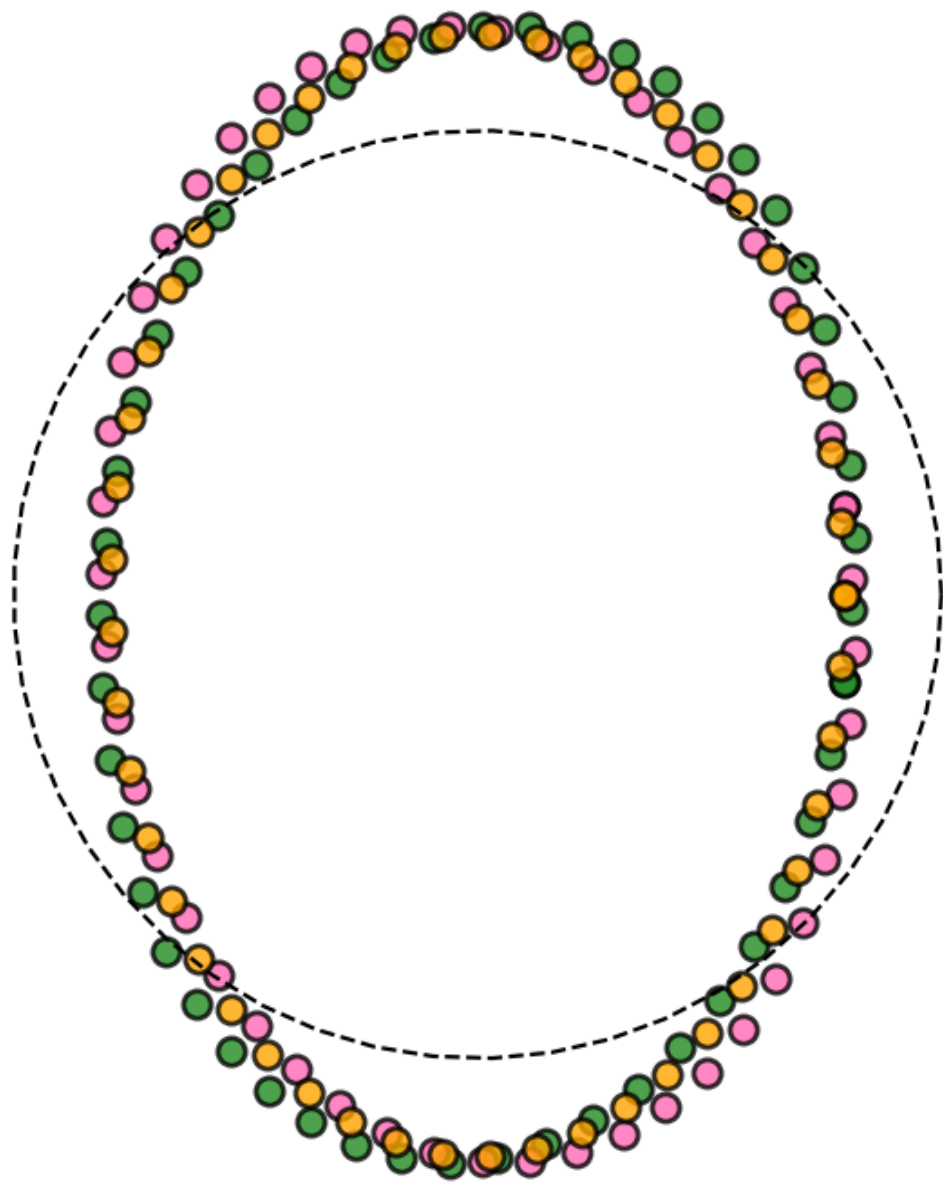
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+ / 2$

LH: $h_{av} = (h_+ - ih_x) / \sqrt{2}$

RH: $h_{av} = (h_+ + ih_x) / \sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

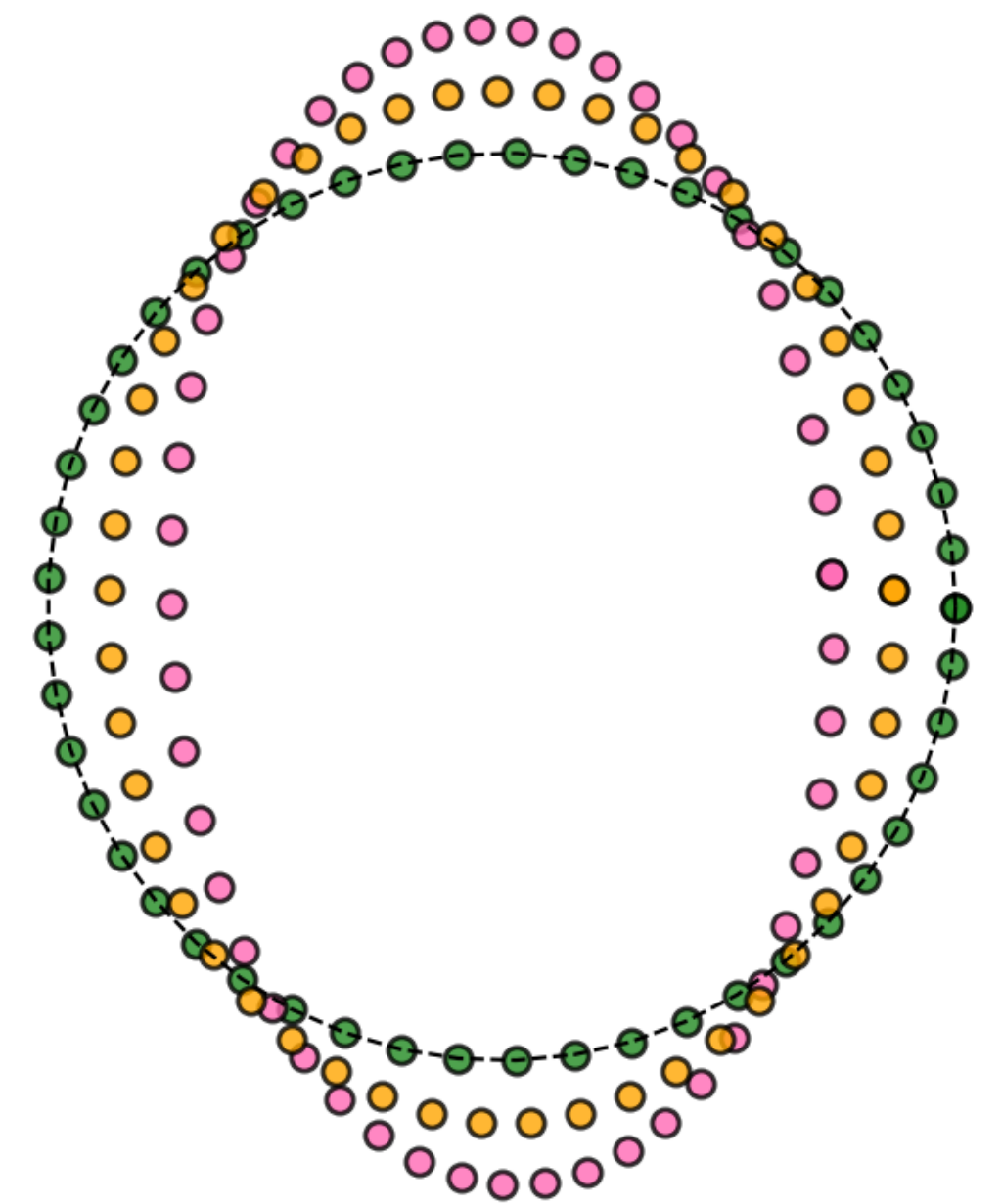
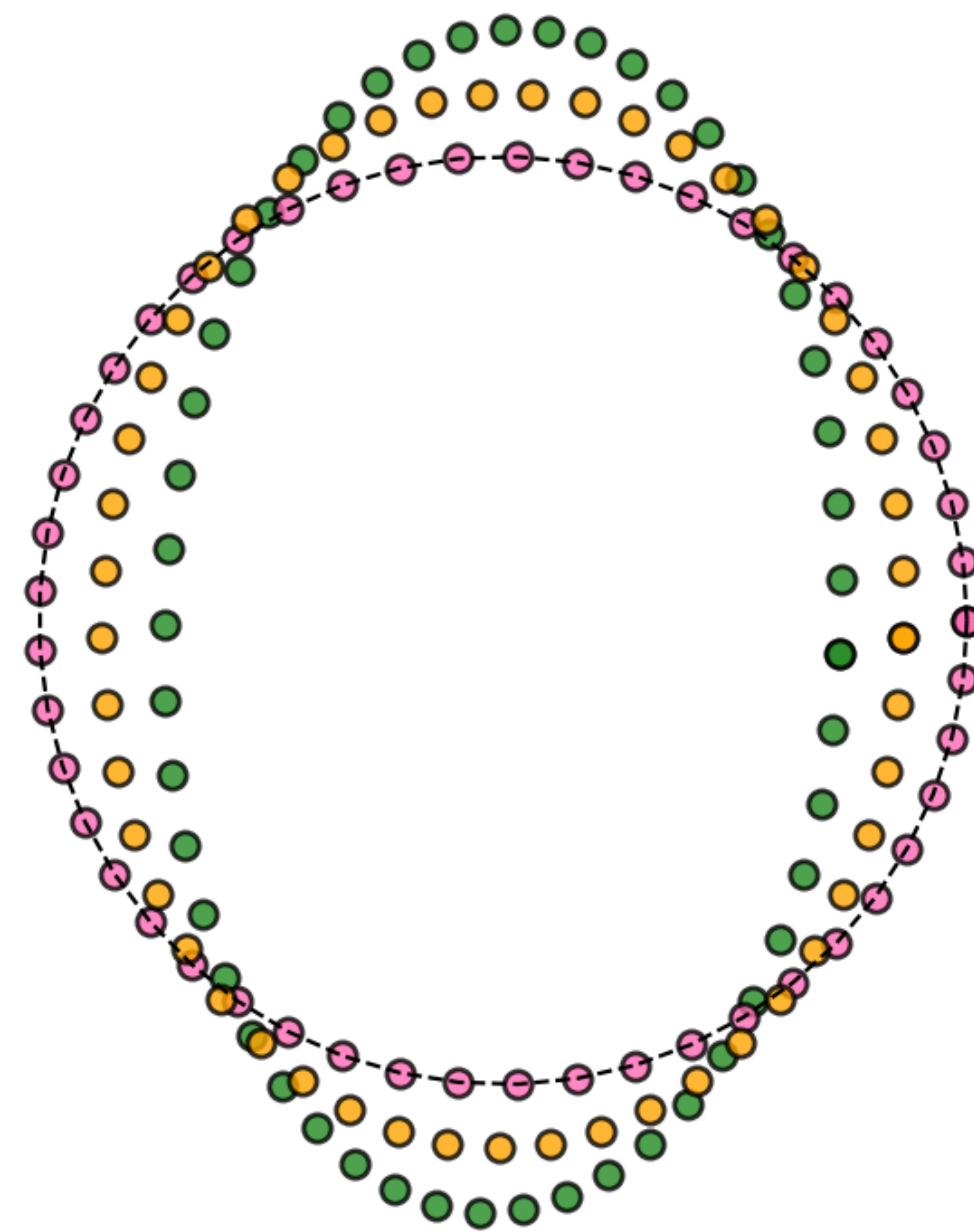
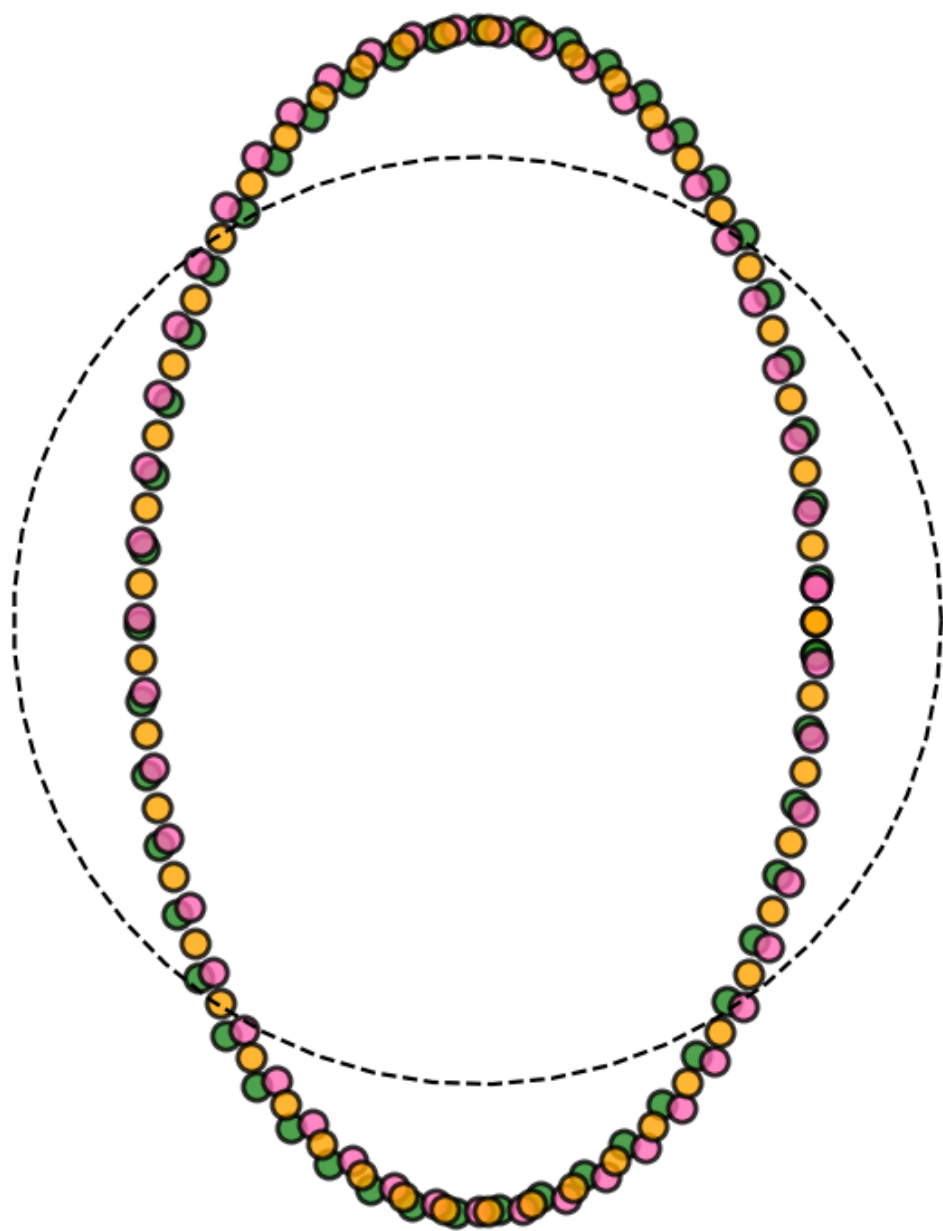
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

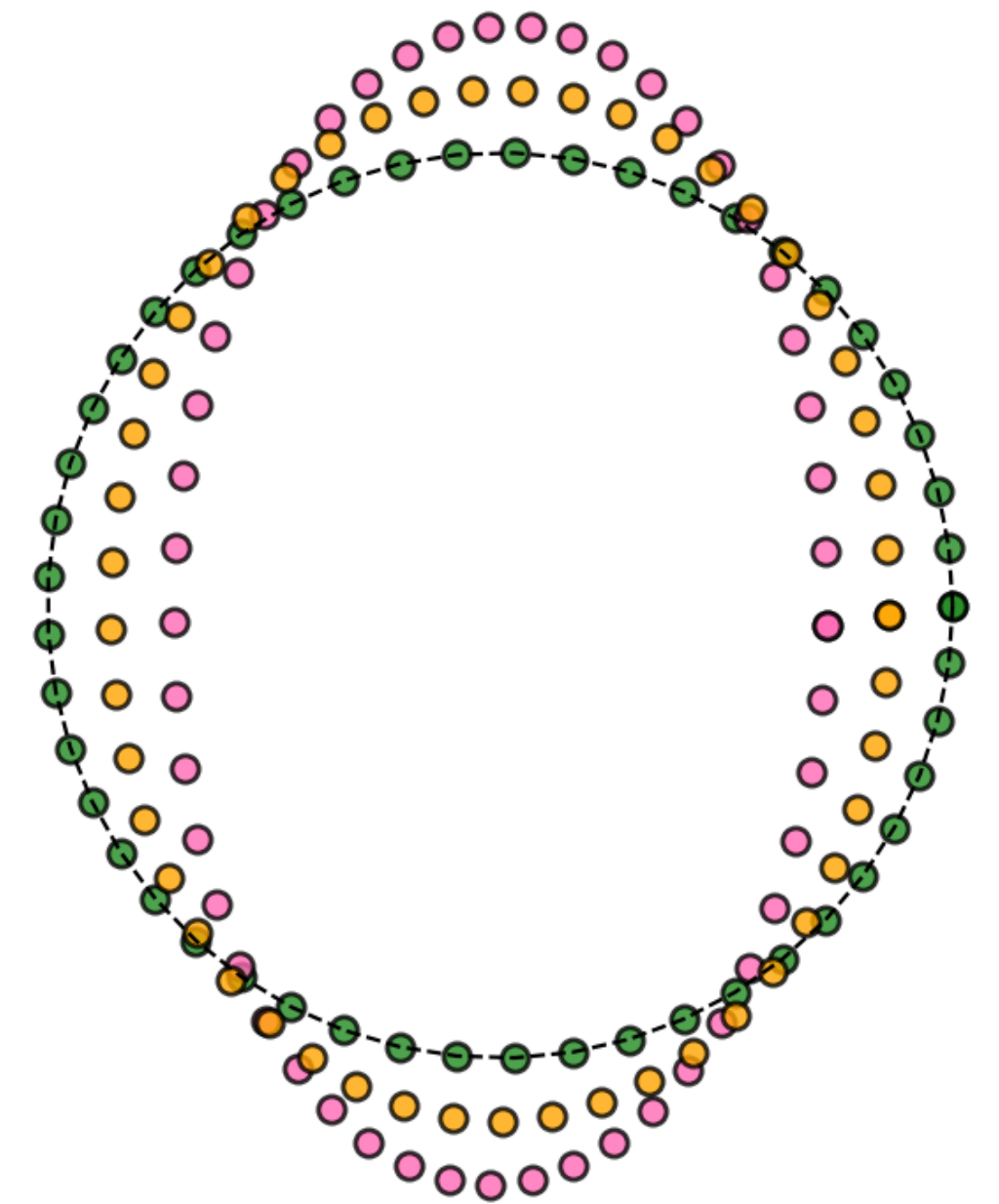
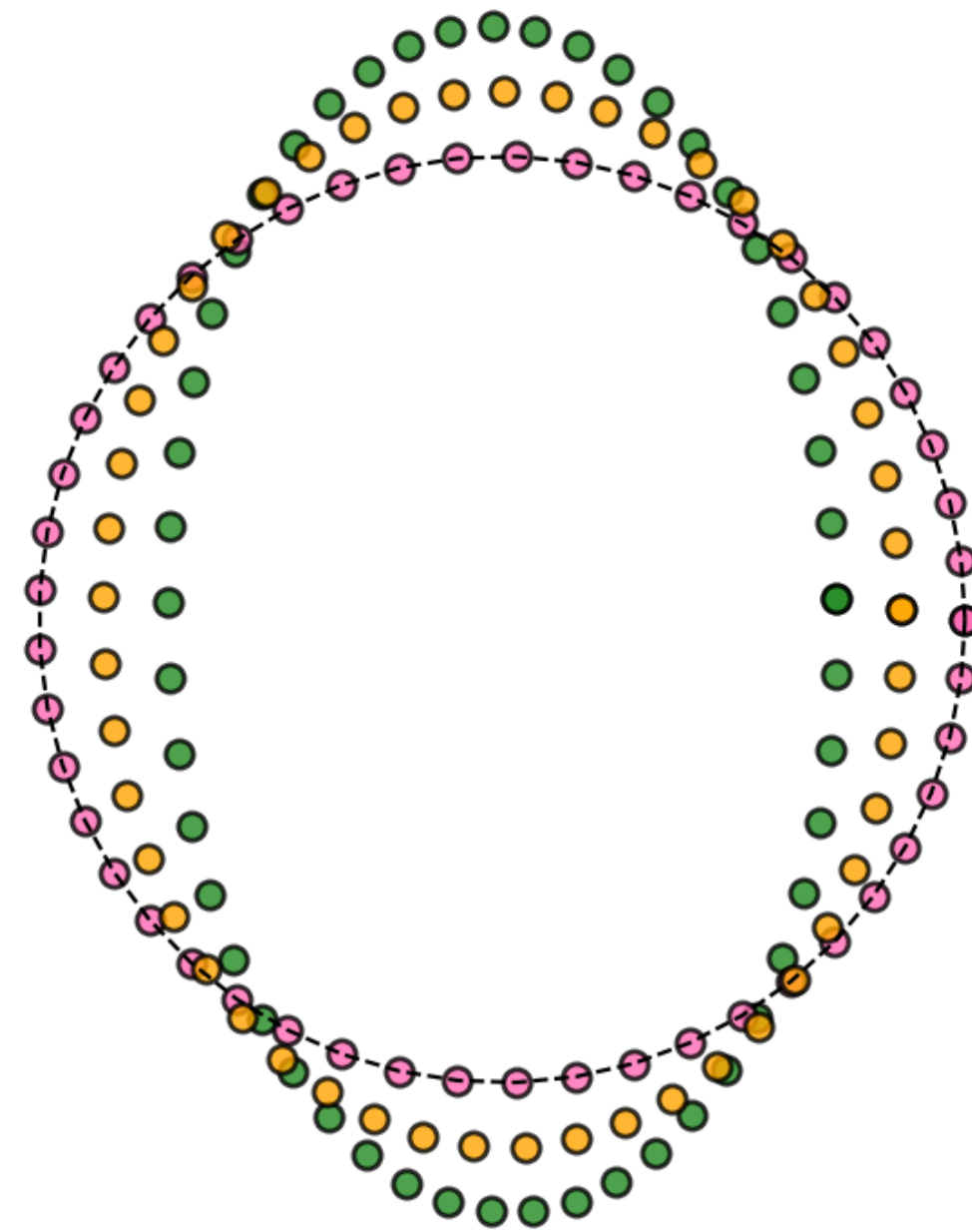
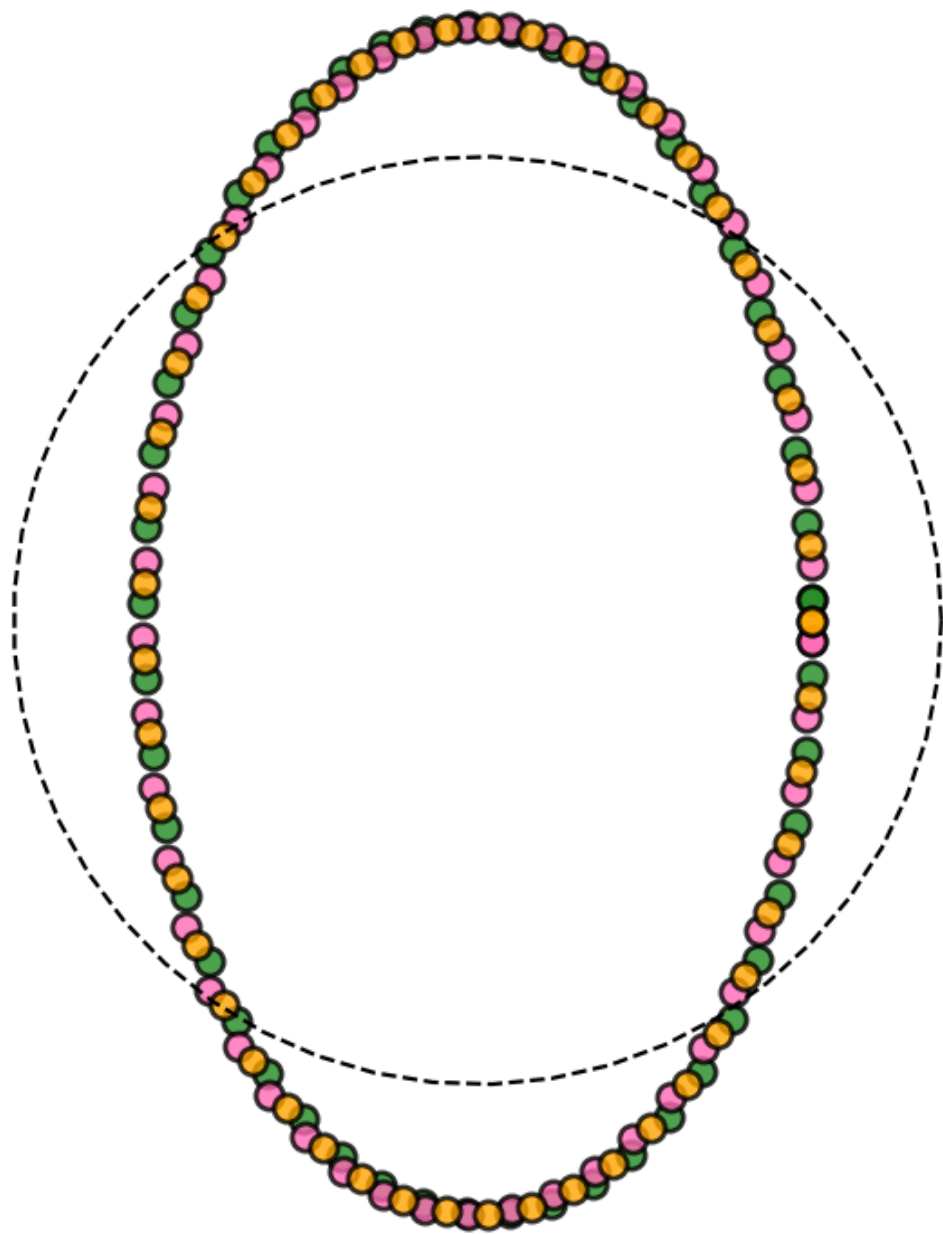
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

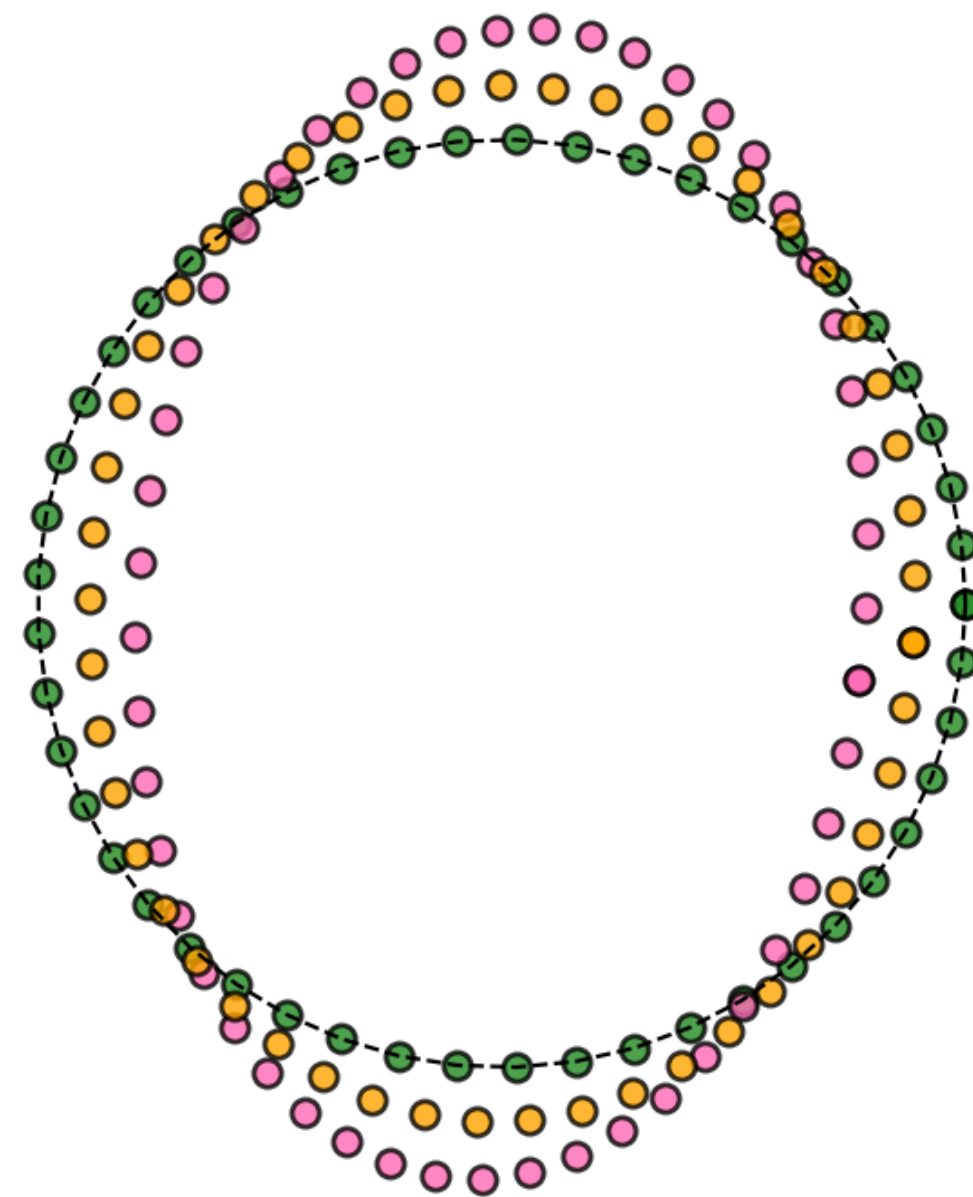
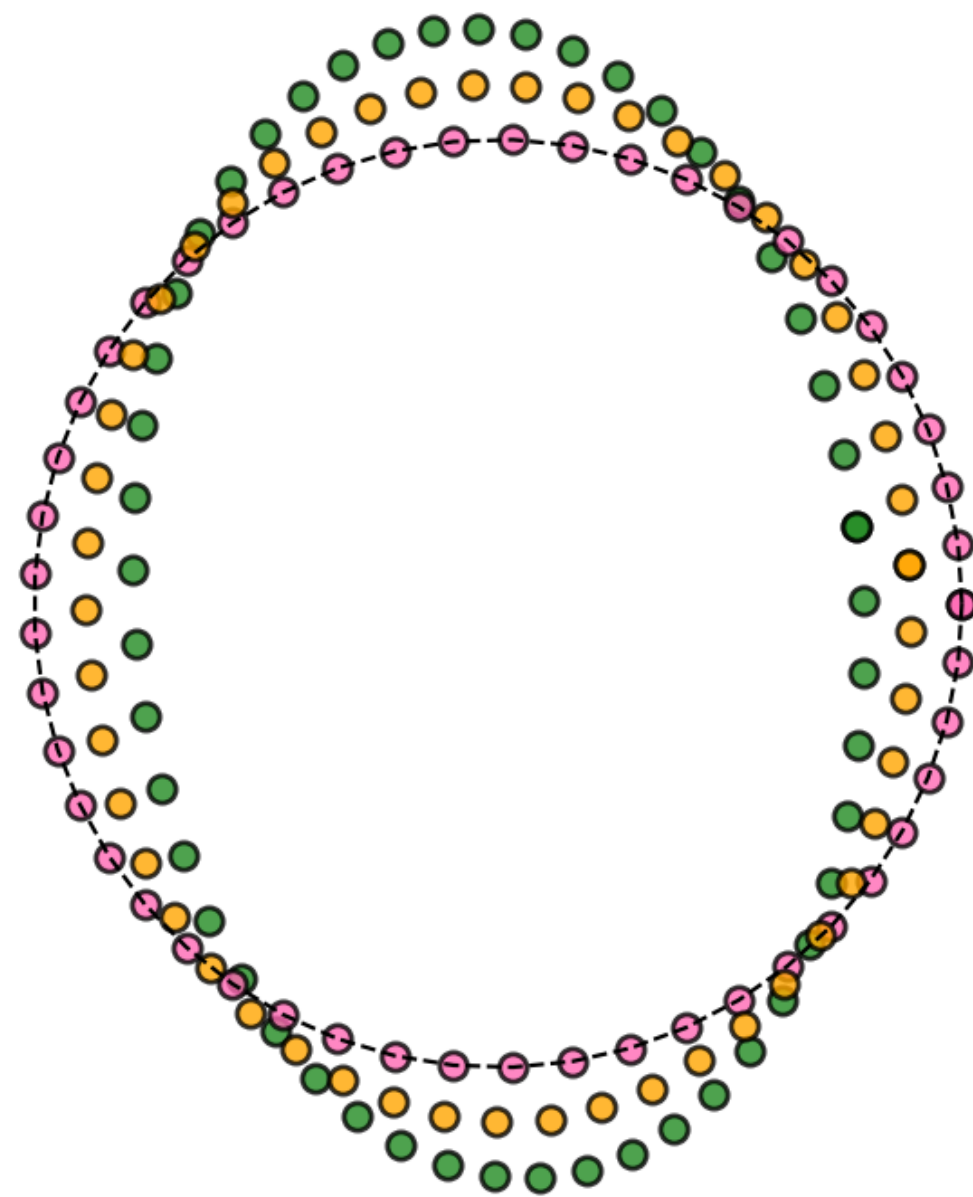
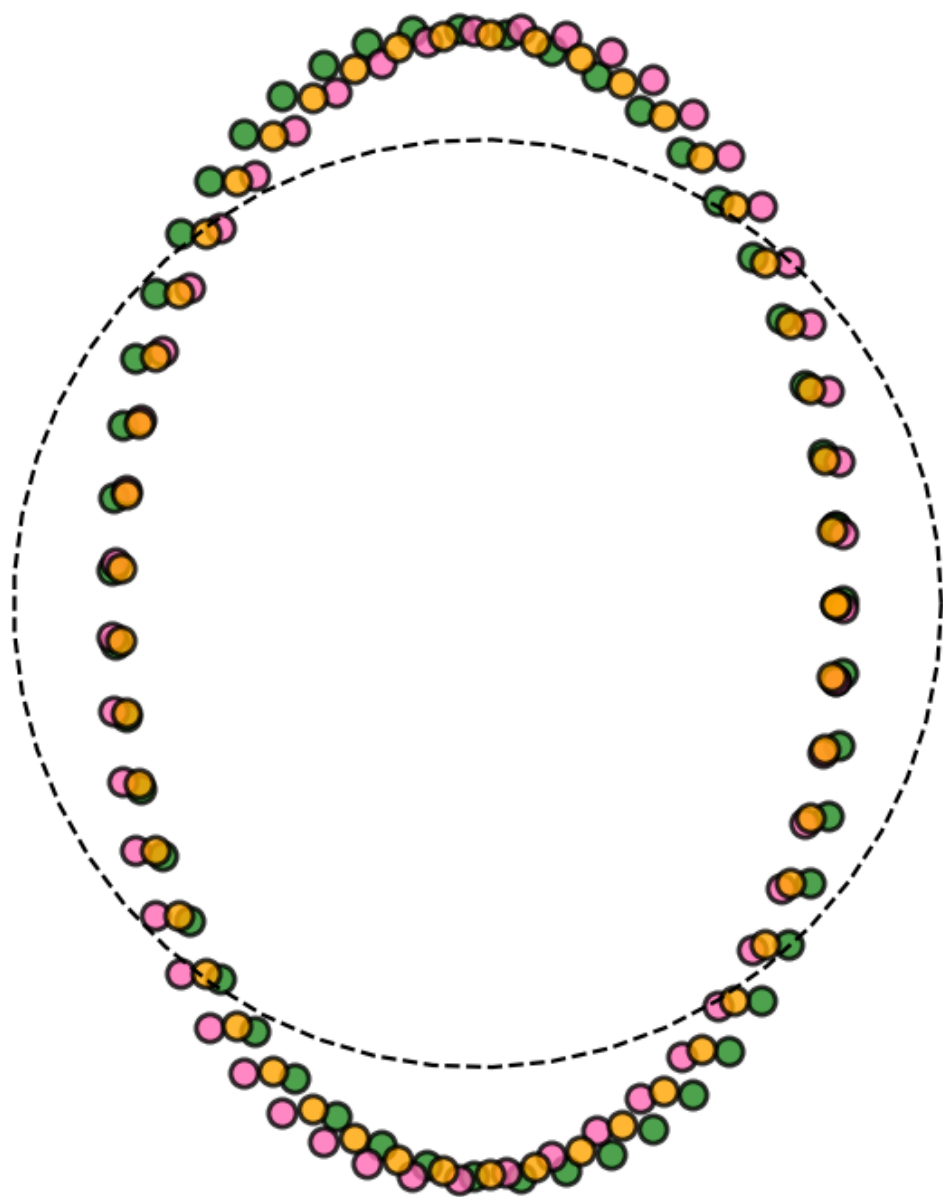
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

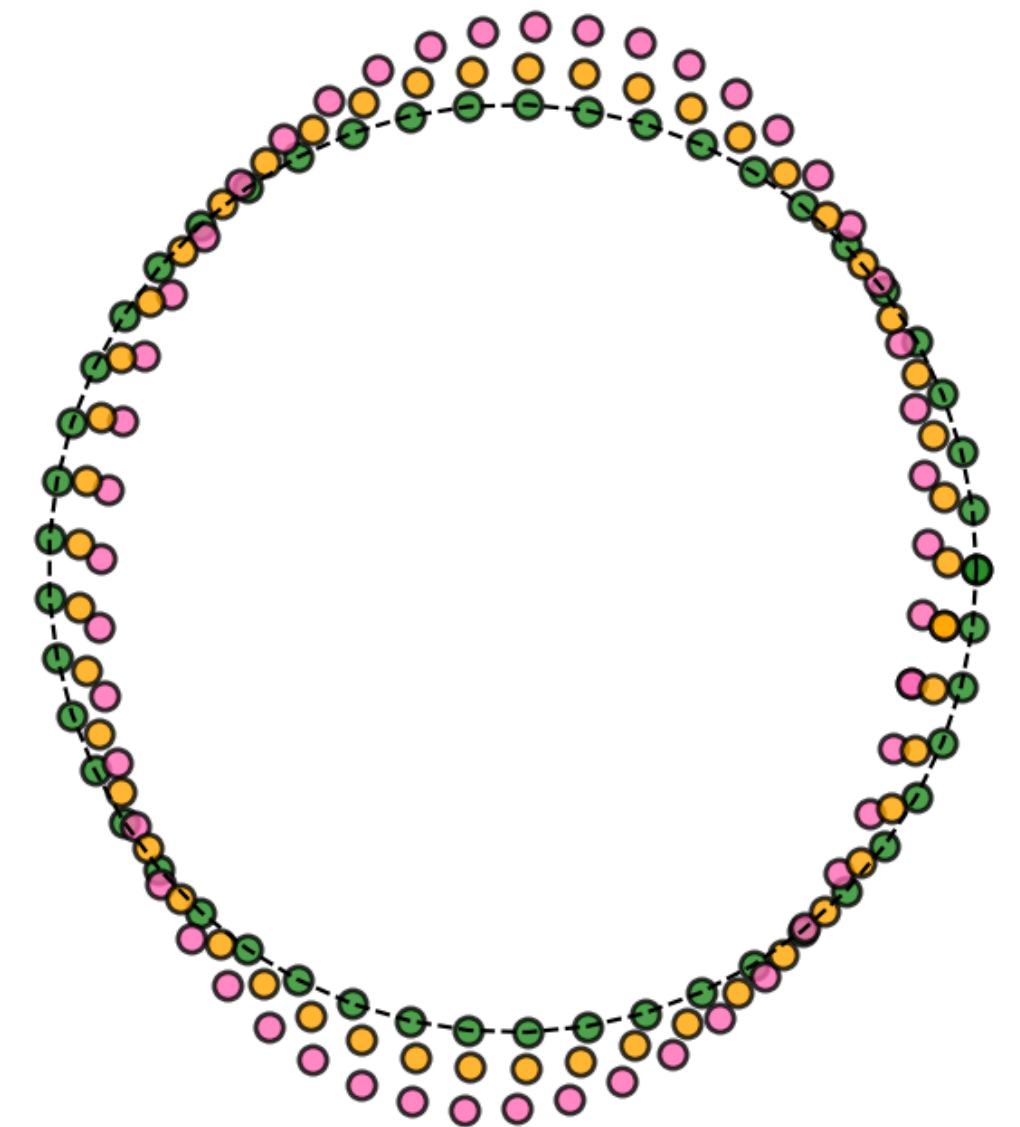
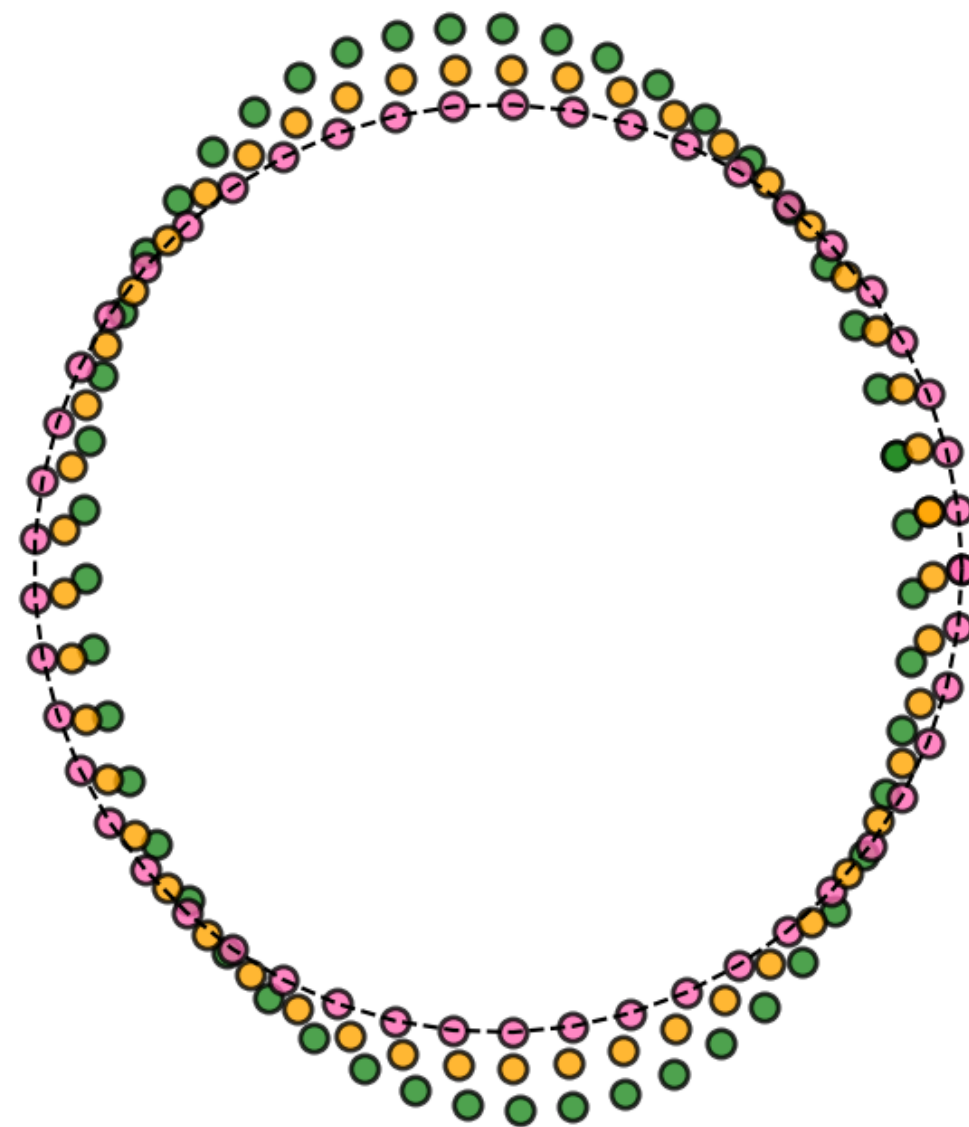
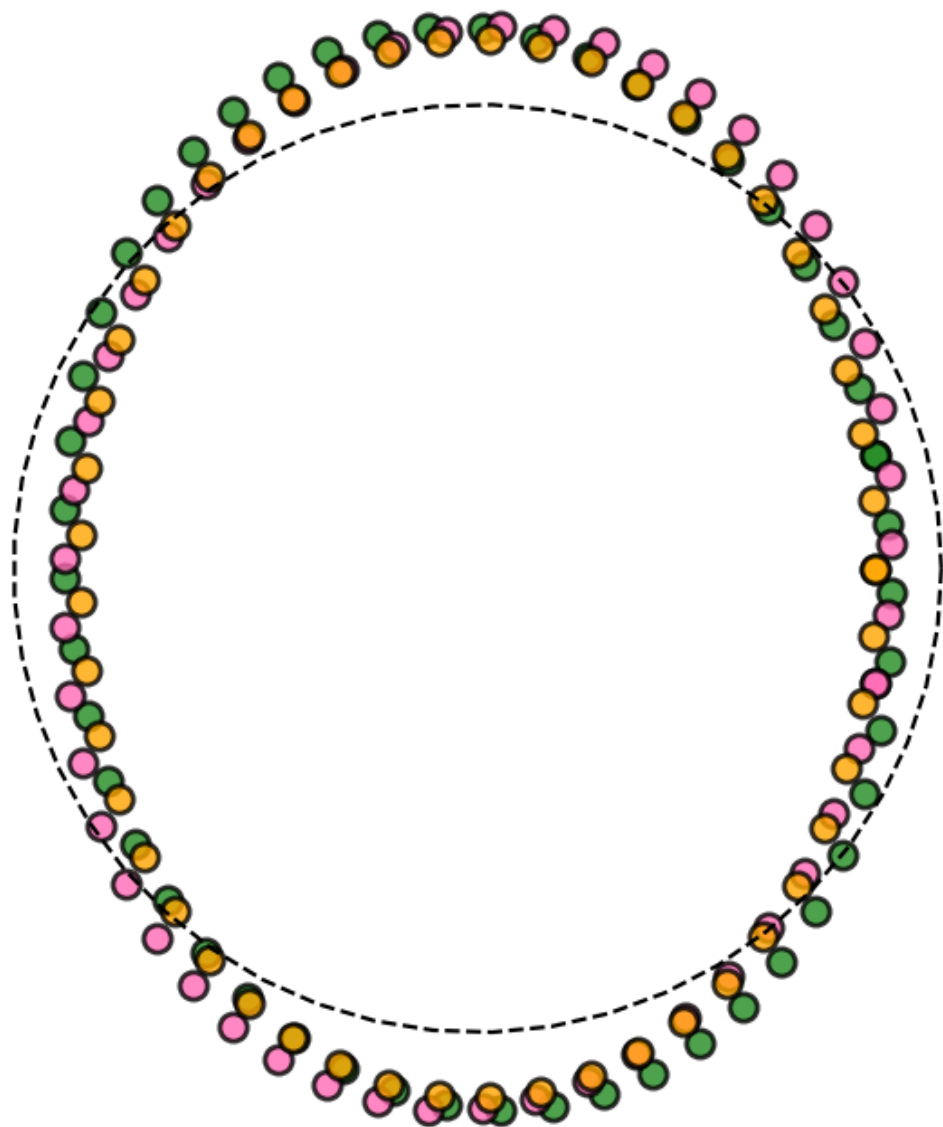
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

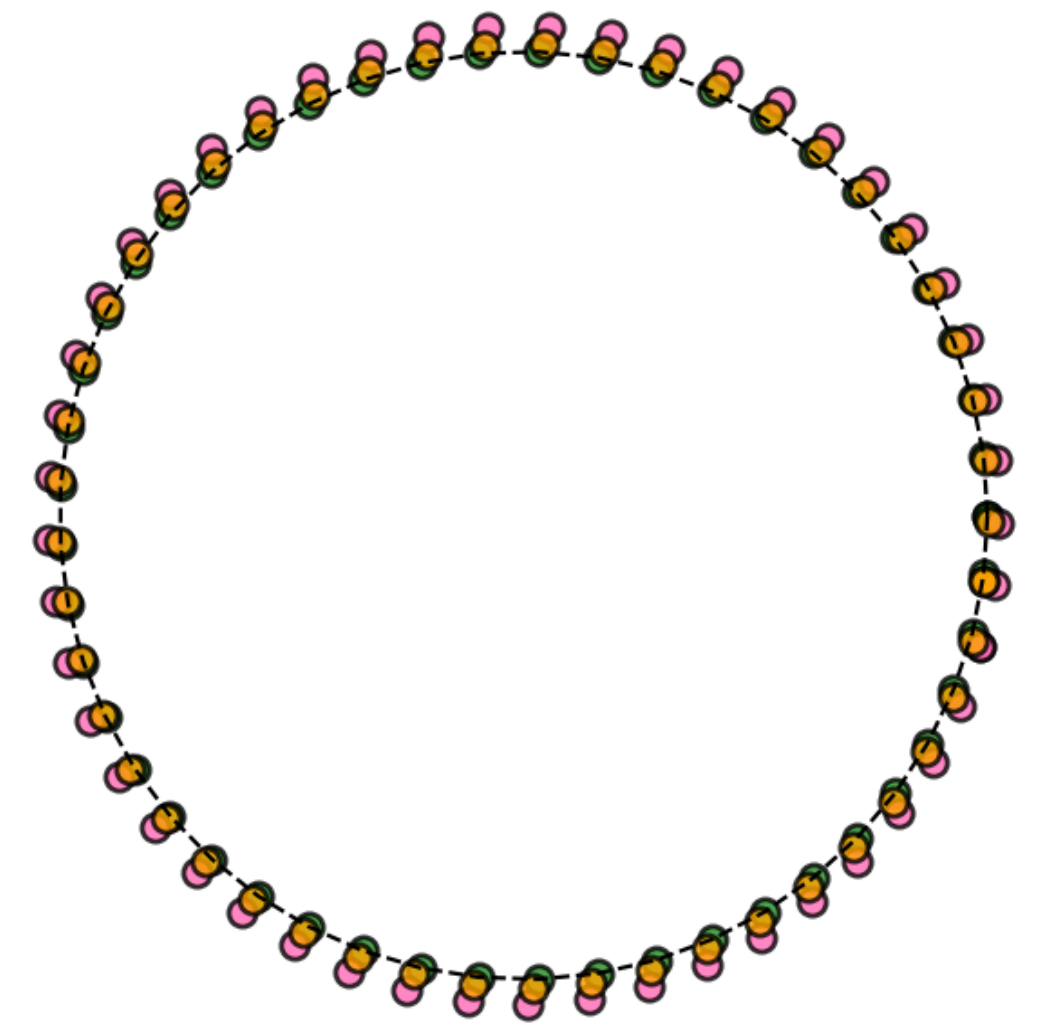
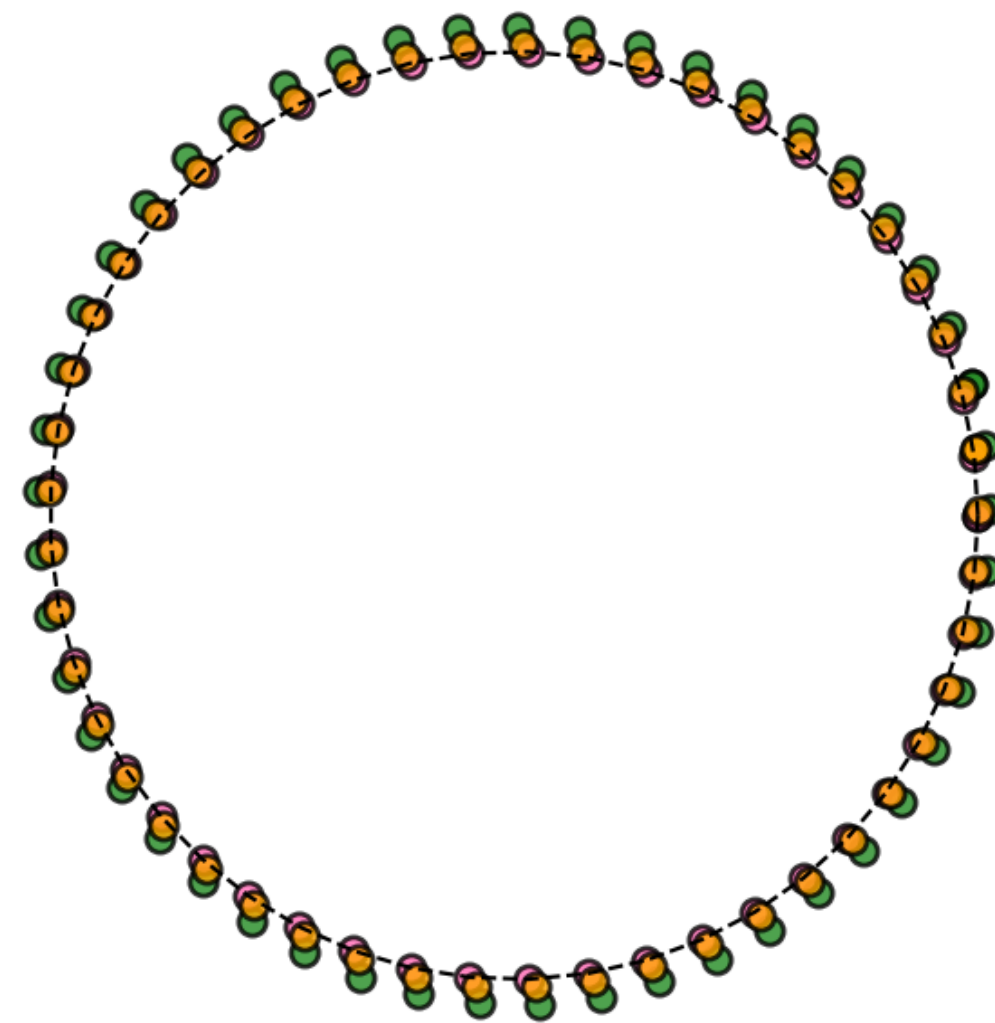
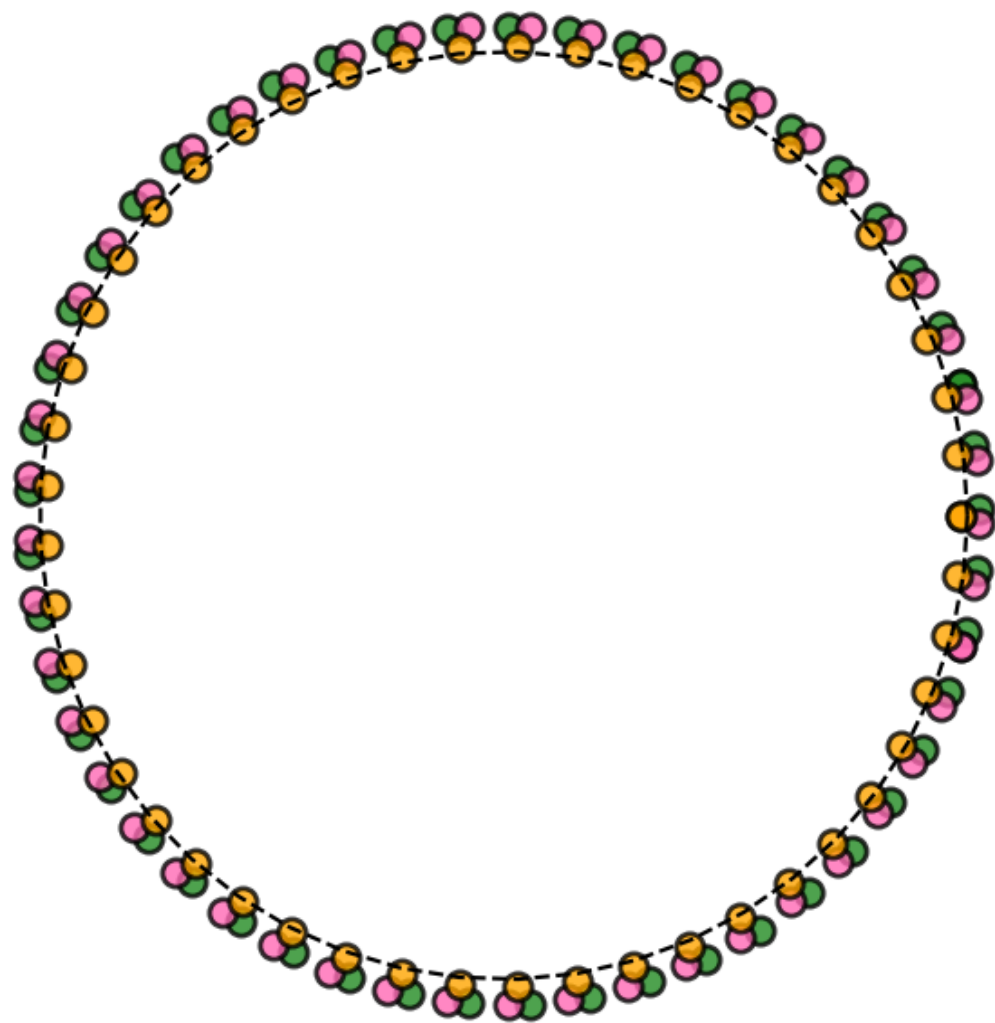
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

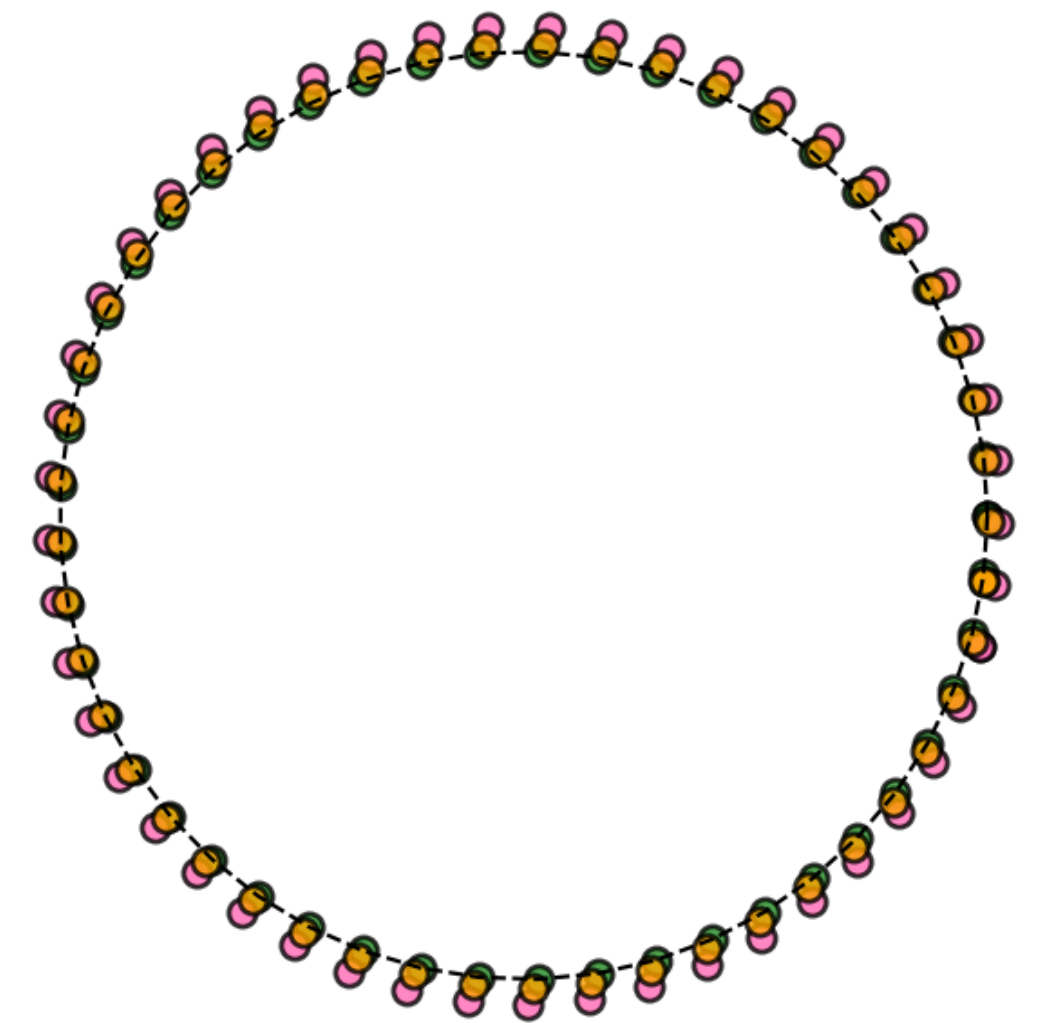
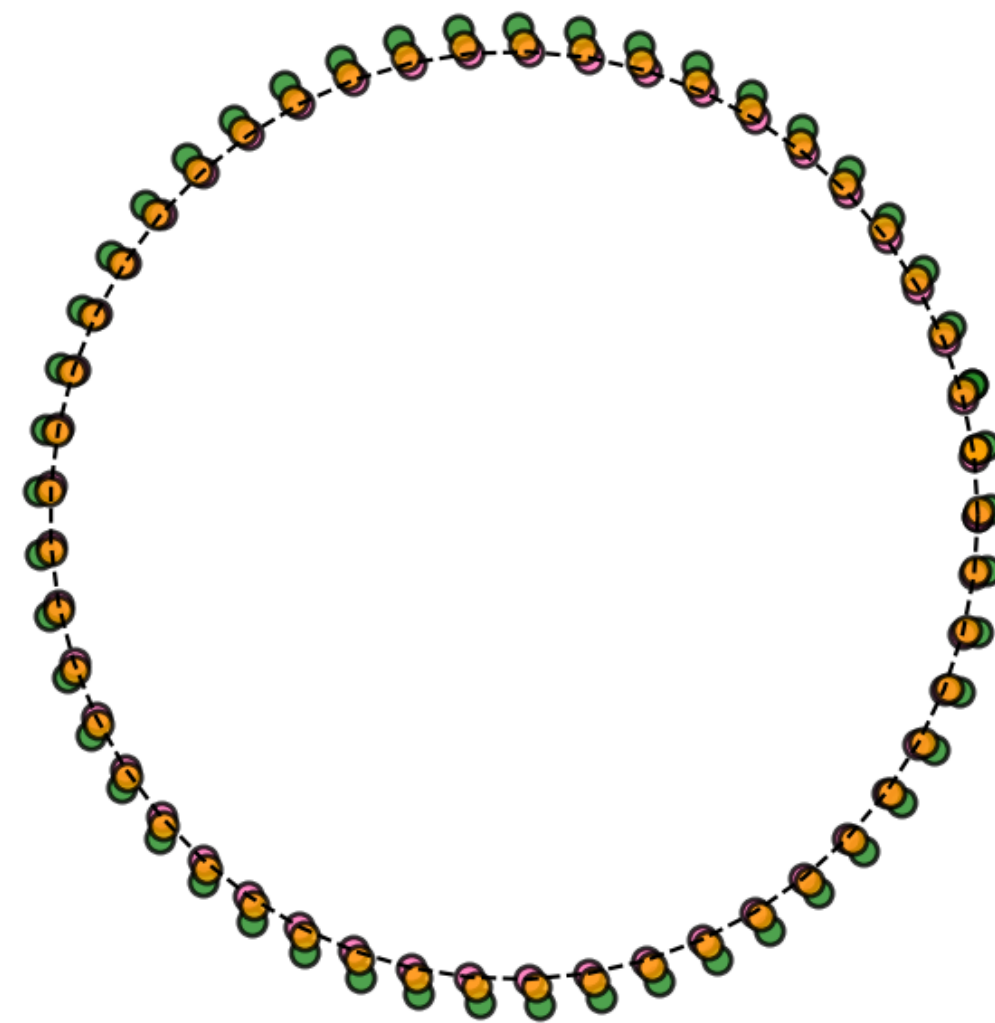
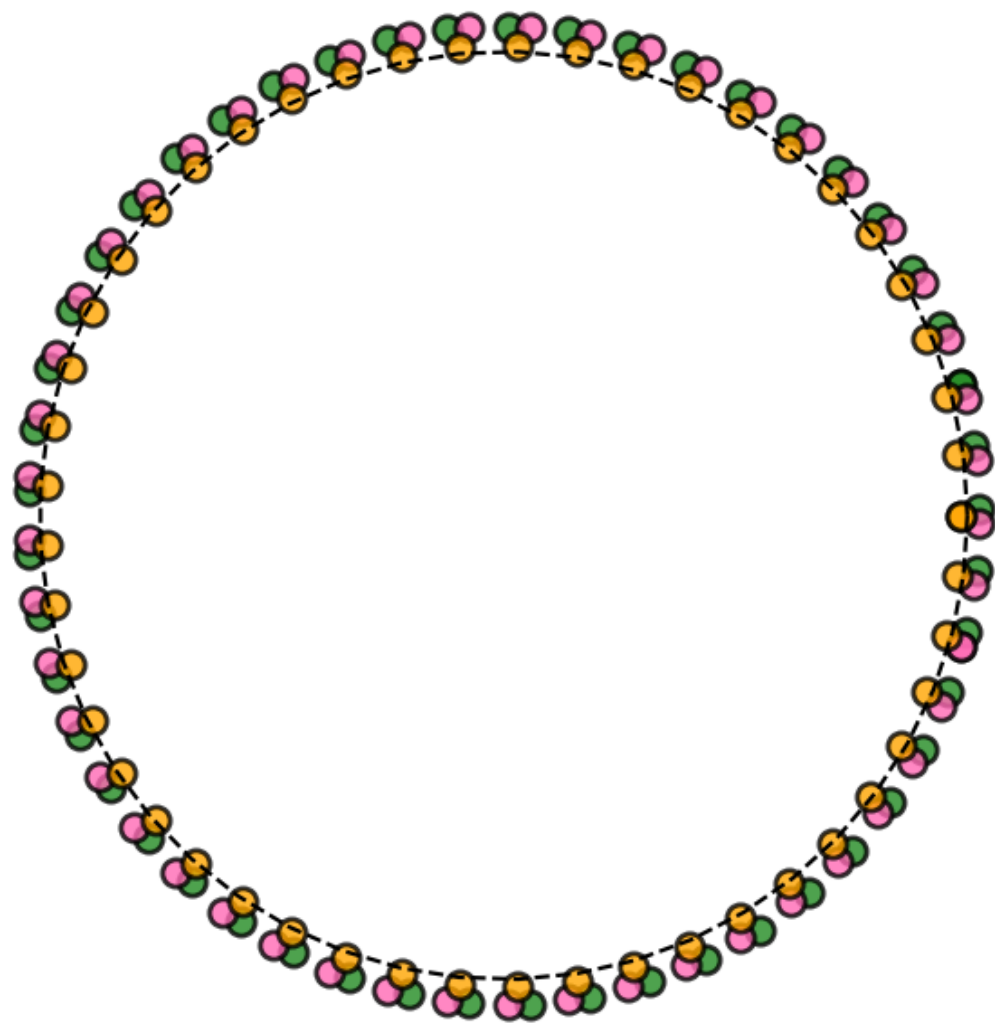
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

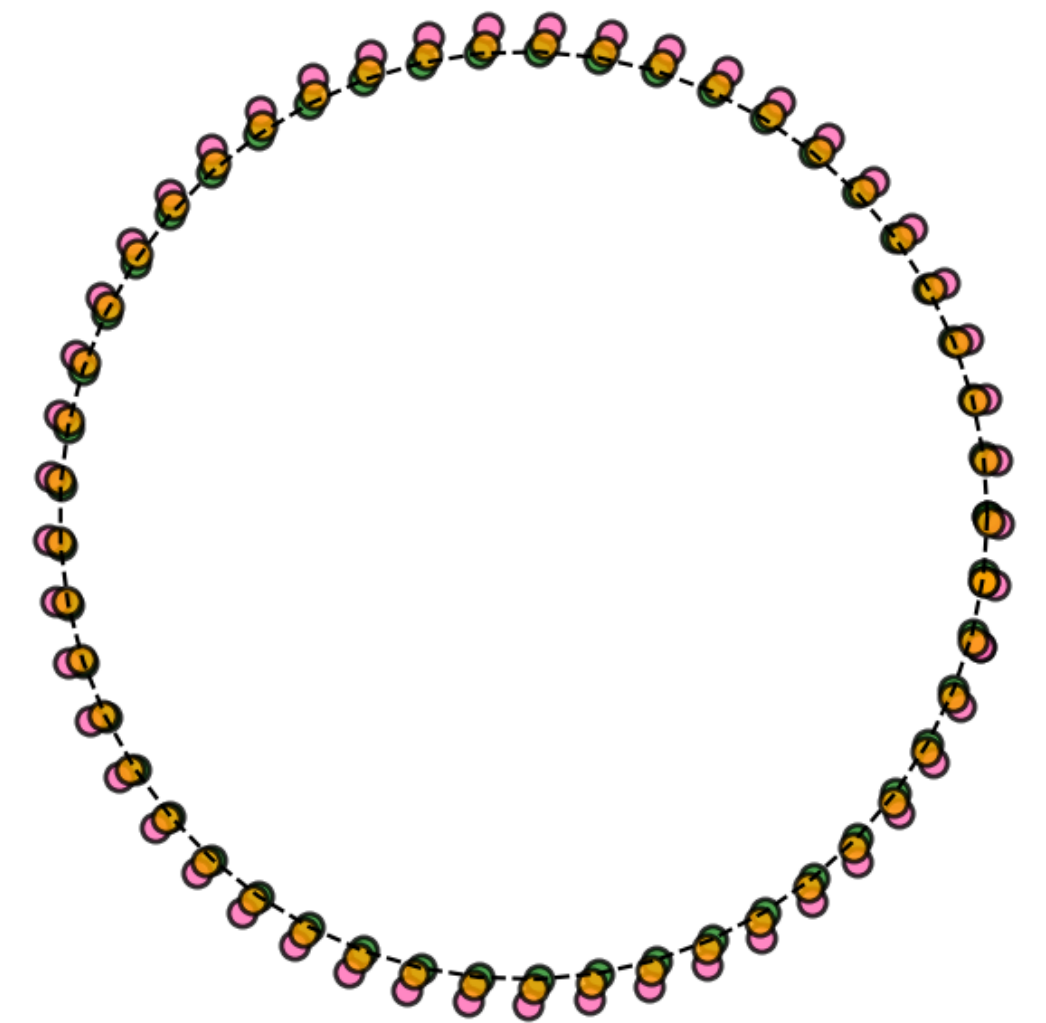
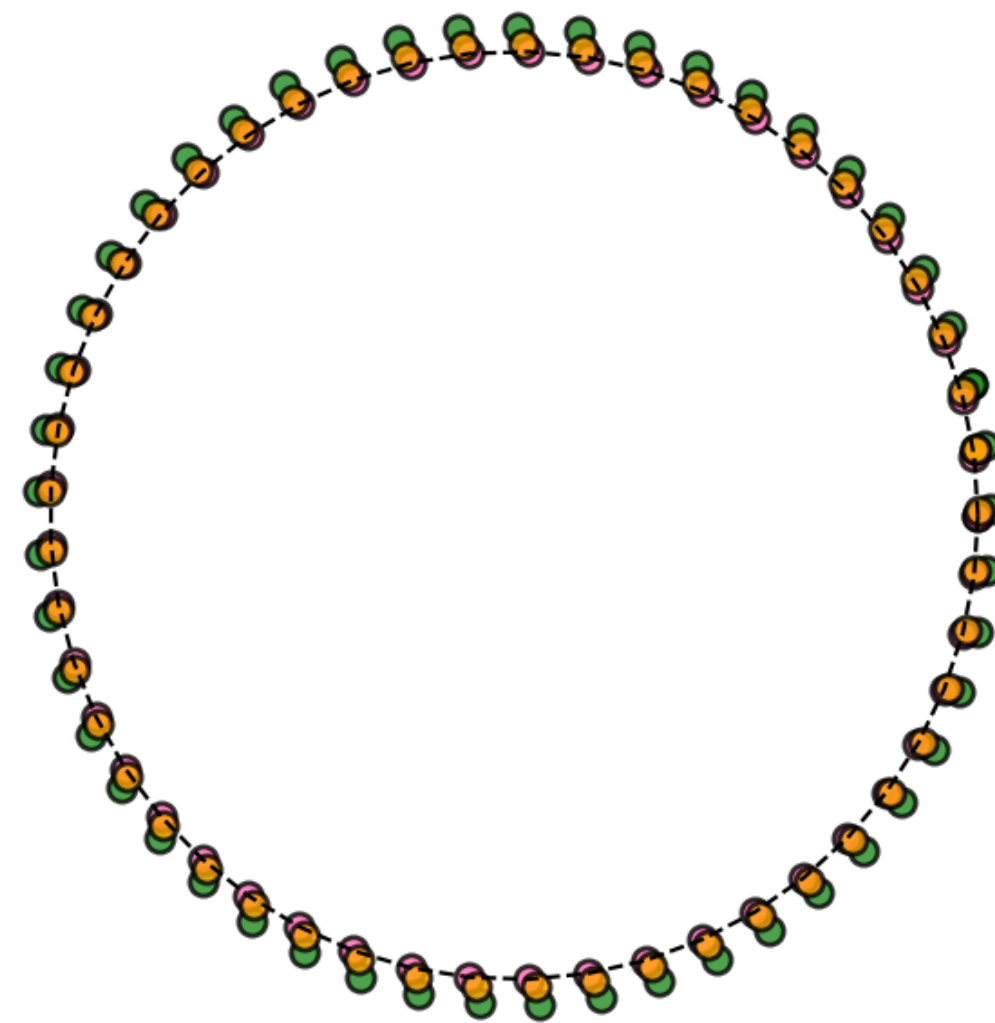
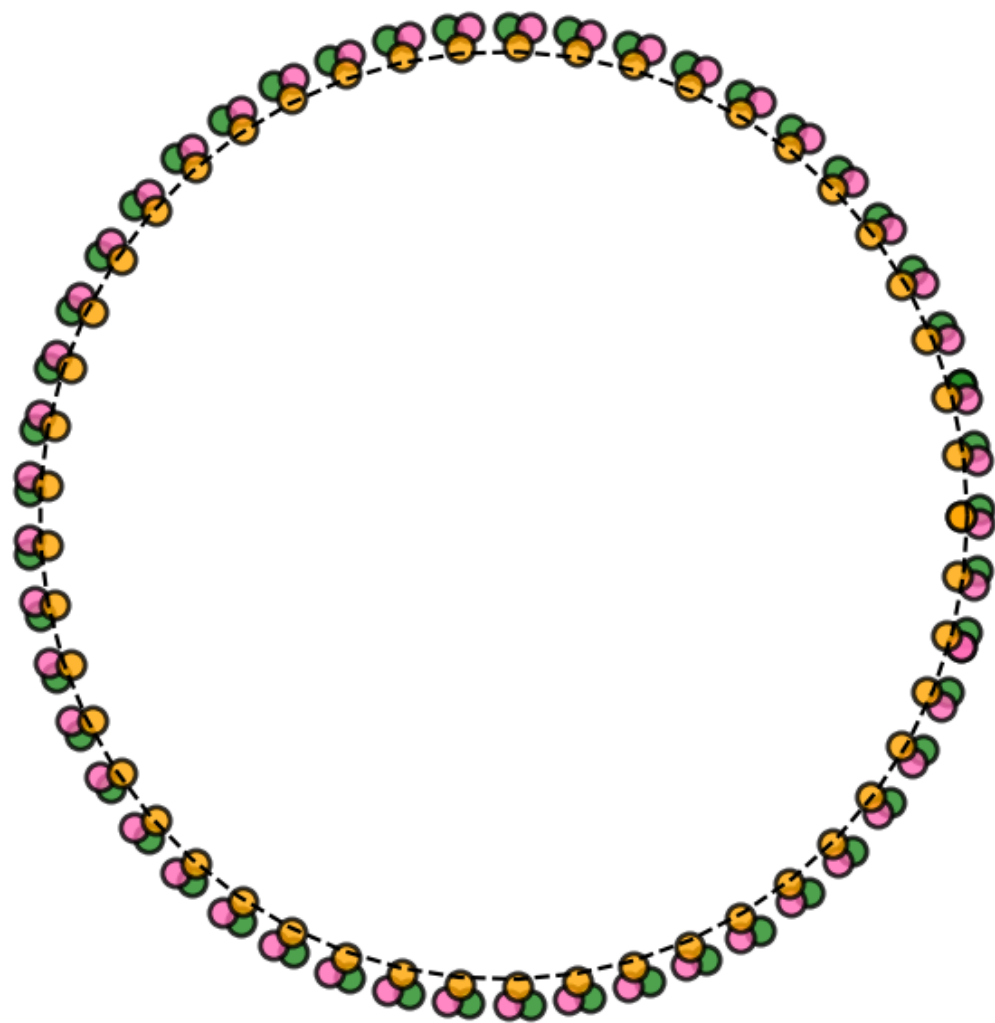
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

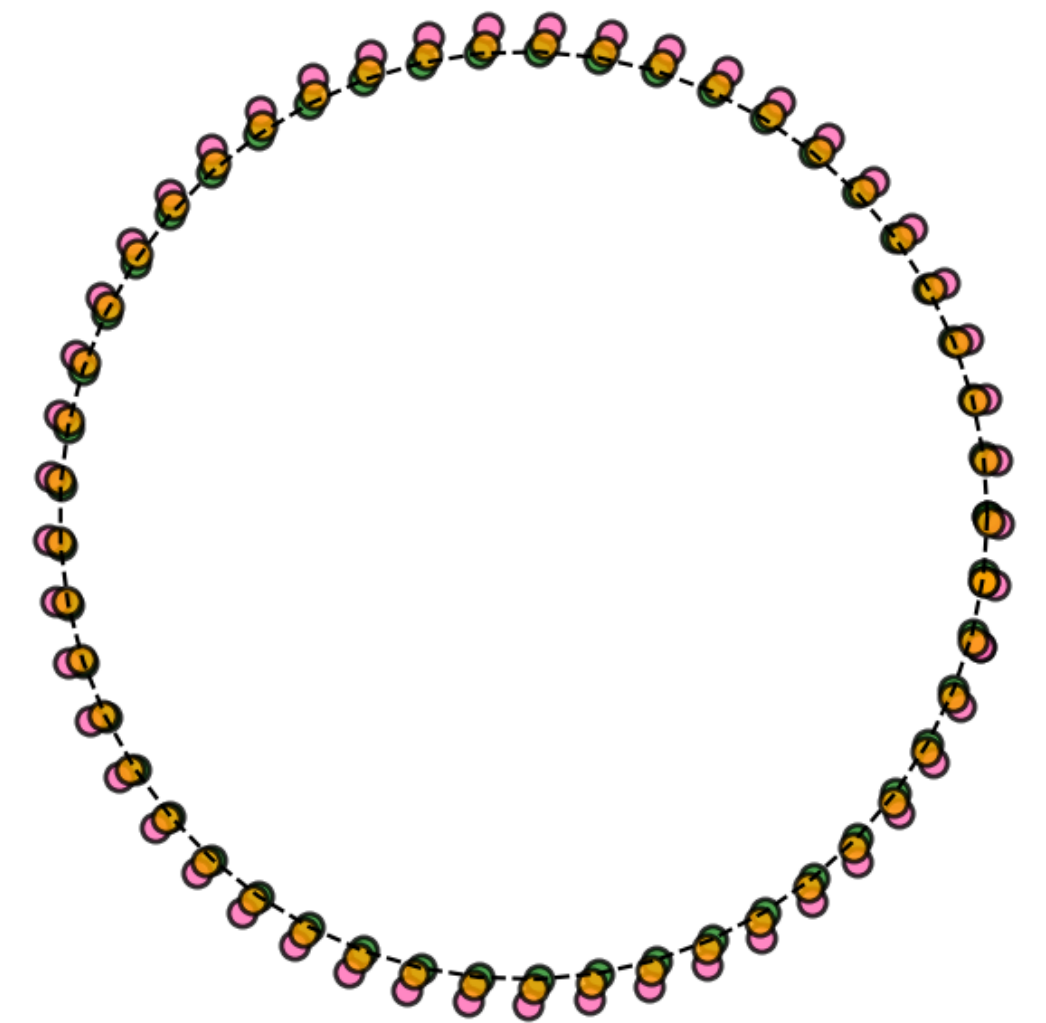
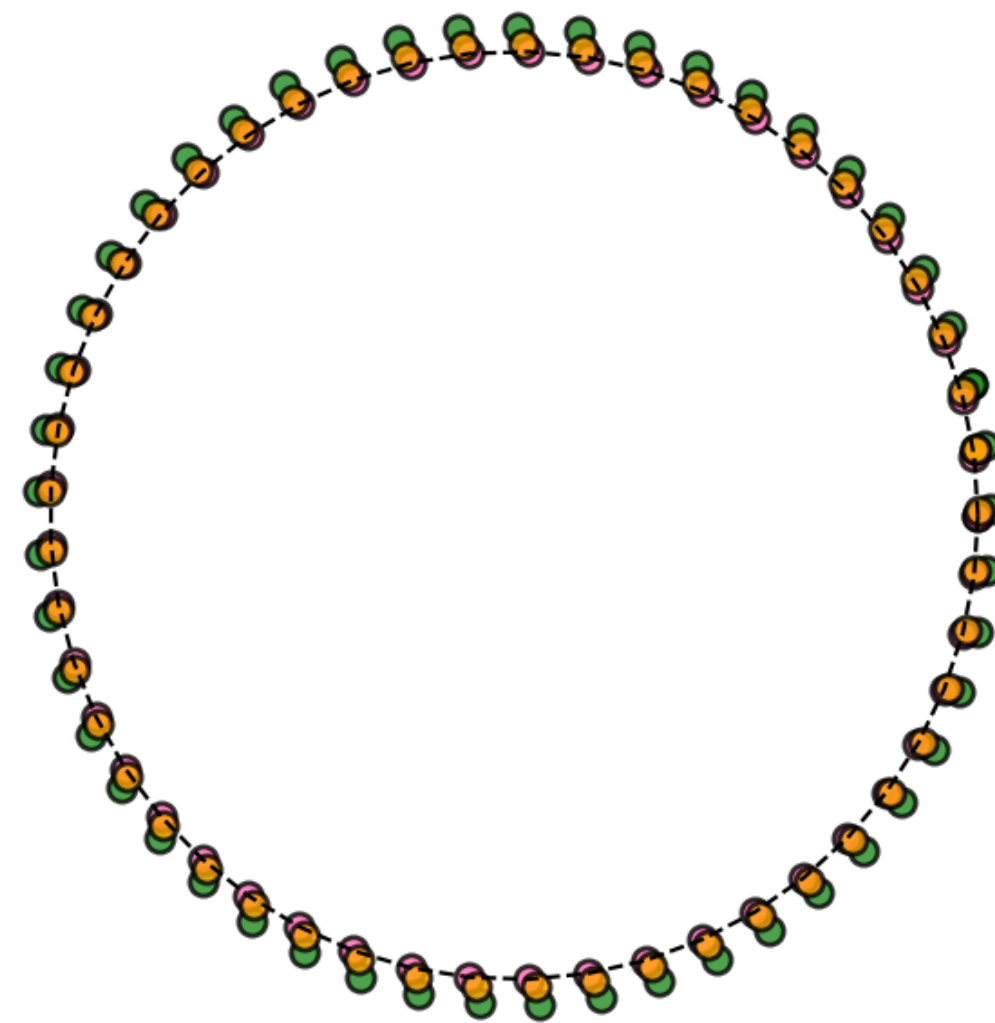
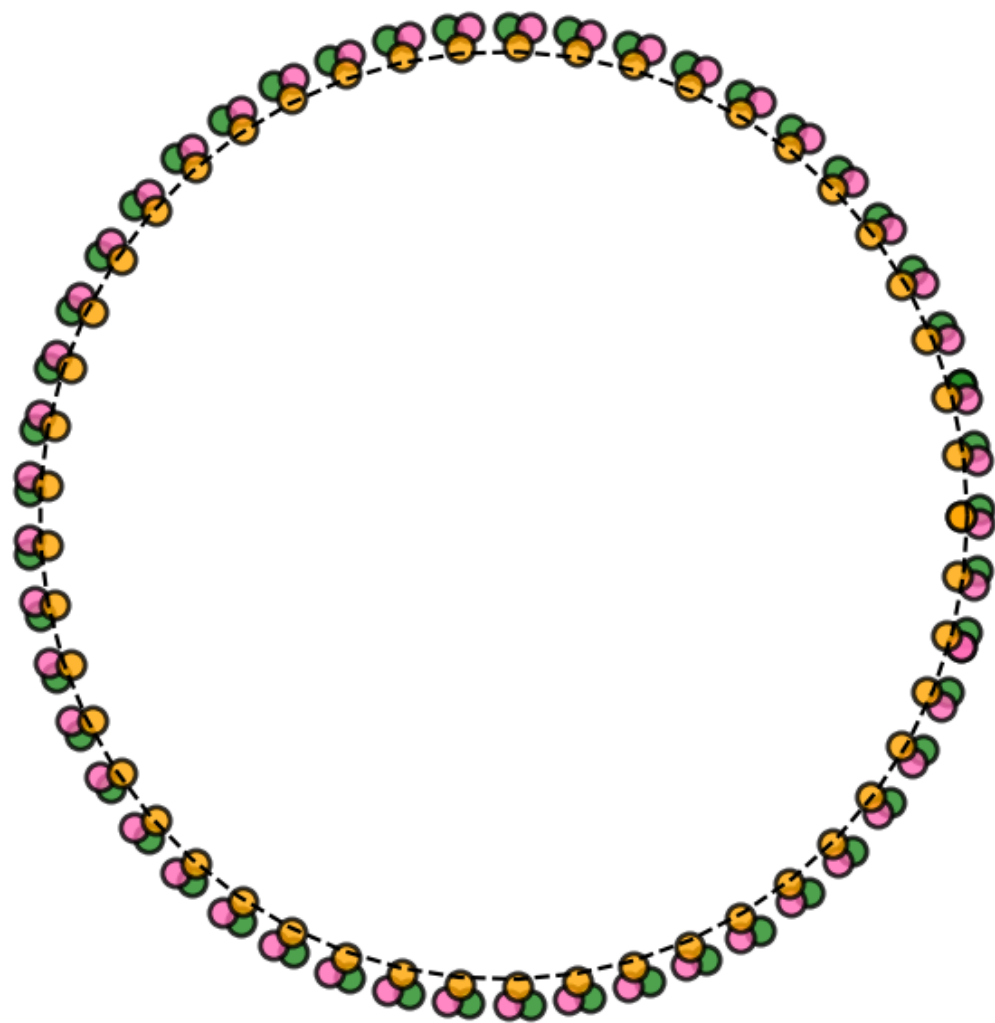
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

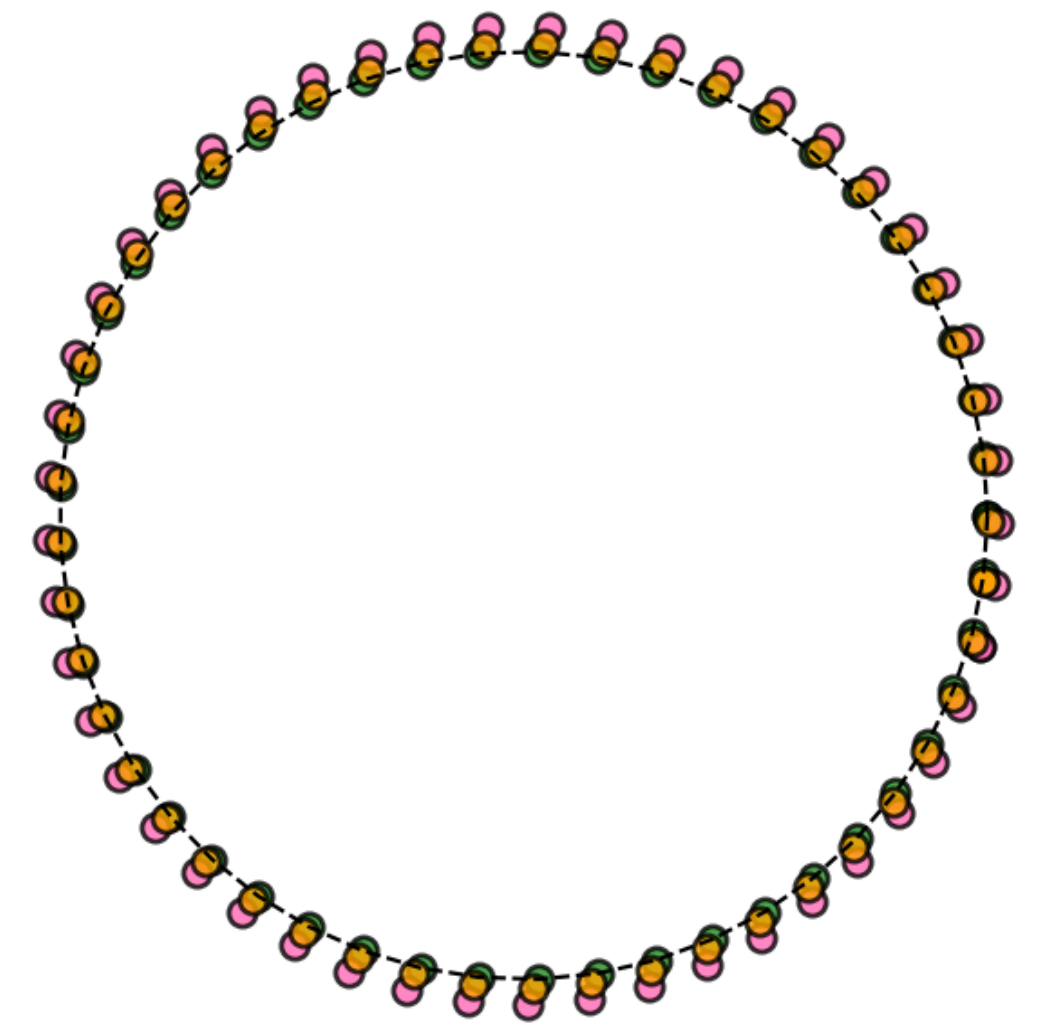
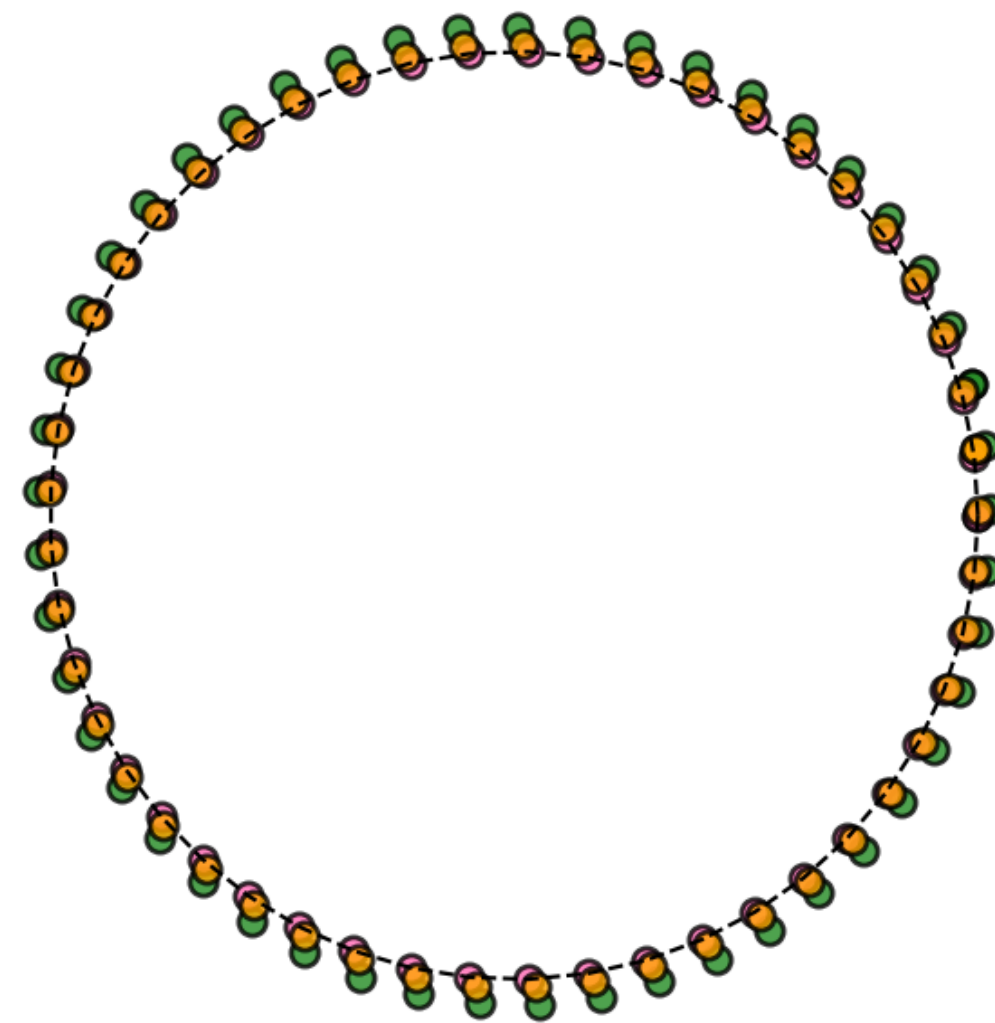
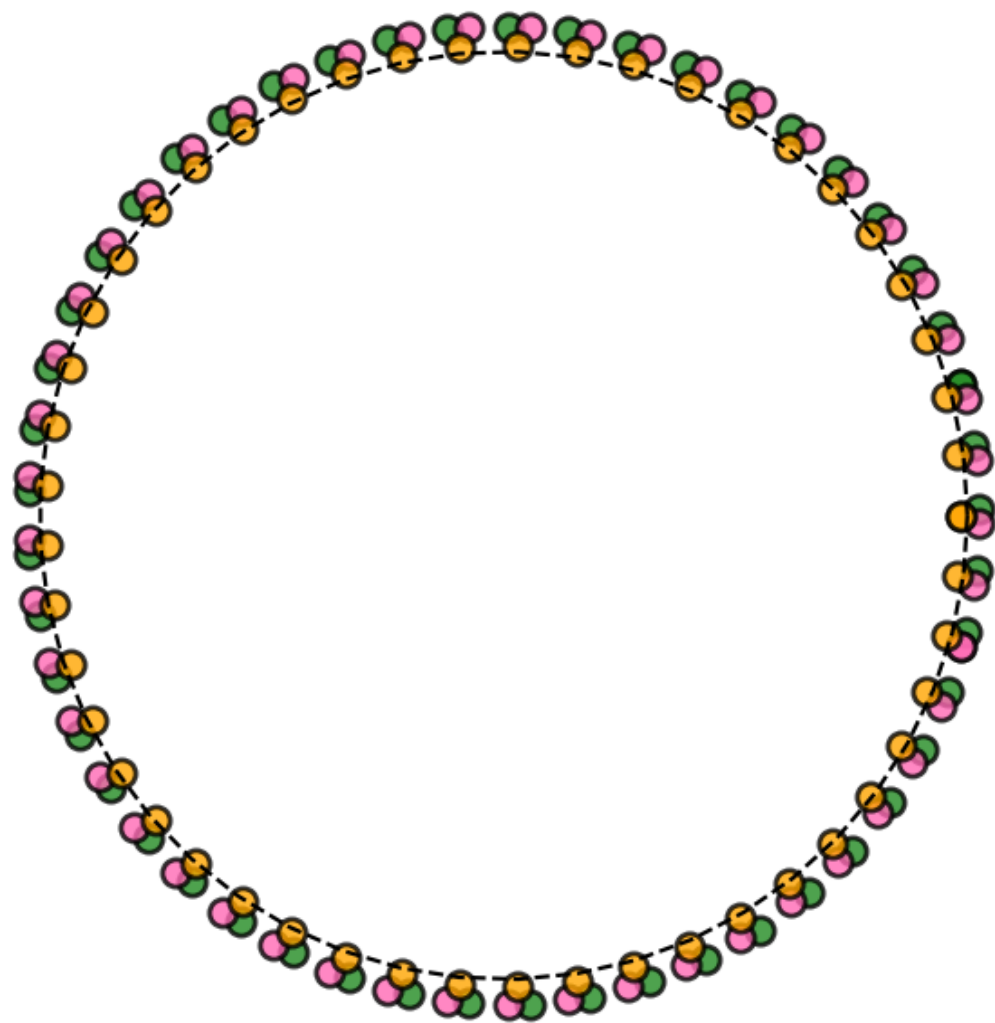
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+ / 2$

LH: $h_{av} = (h_+ - ih_x) / \sqrt{2}$

RH: $h_{av} = (h_+ + ih_x) / \sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

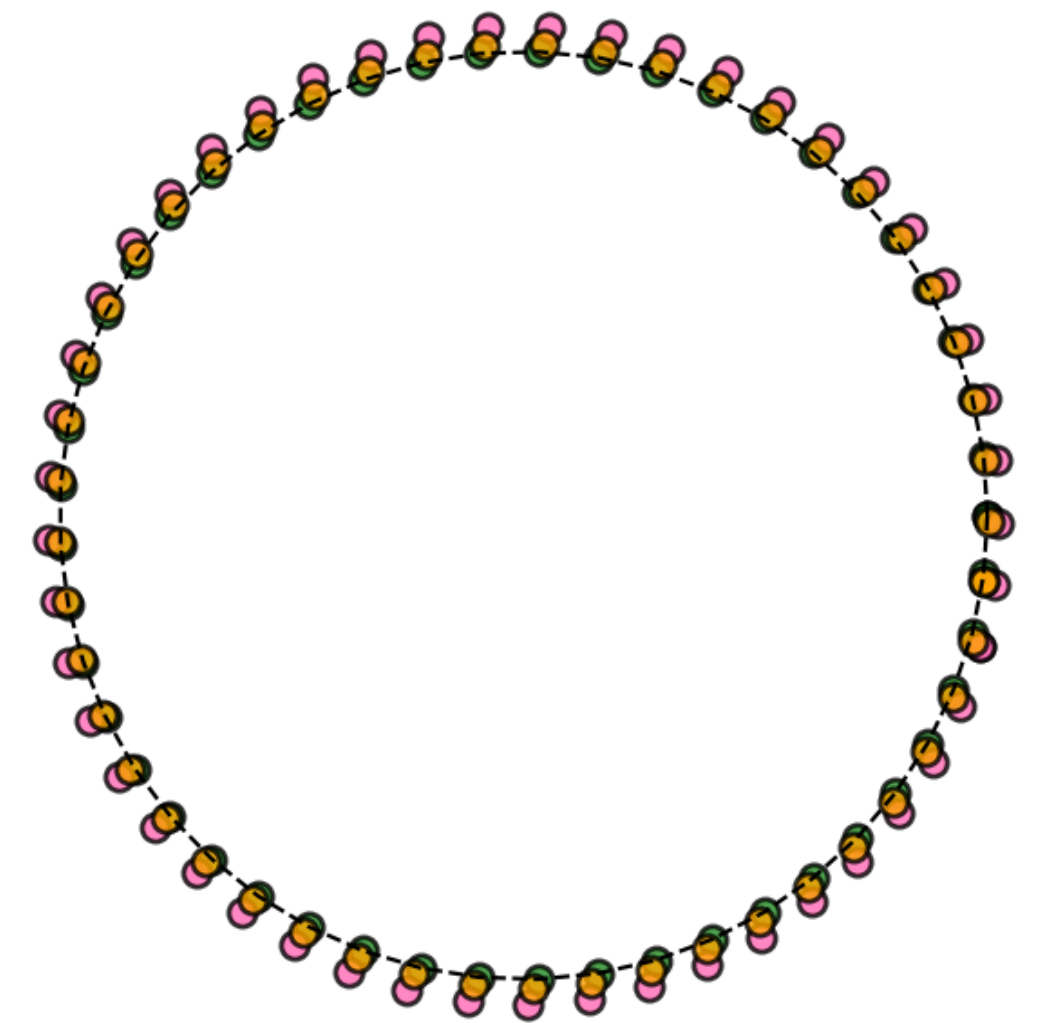
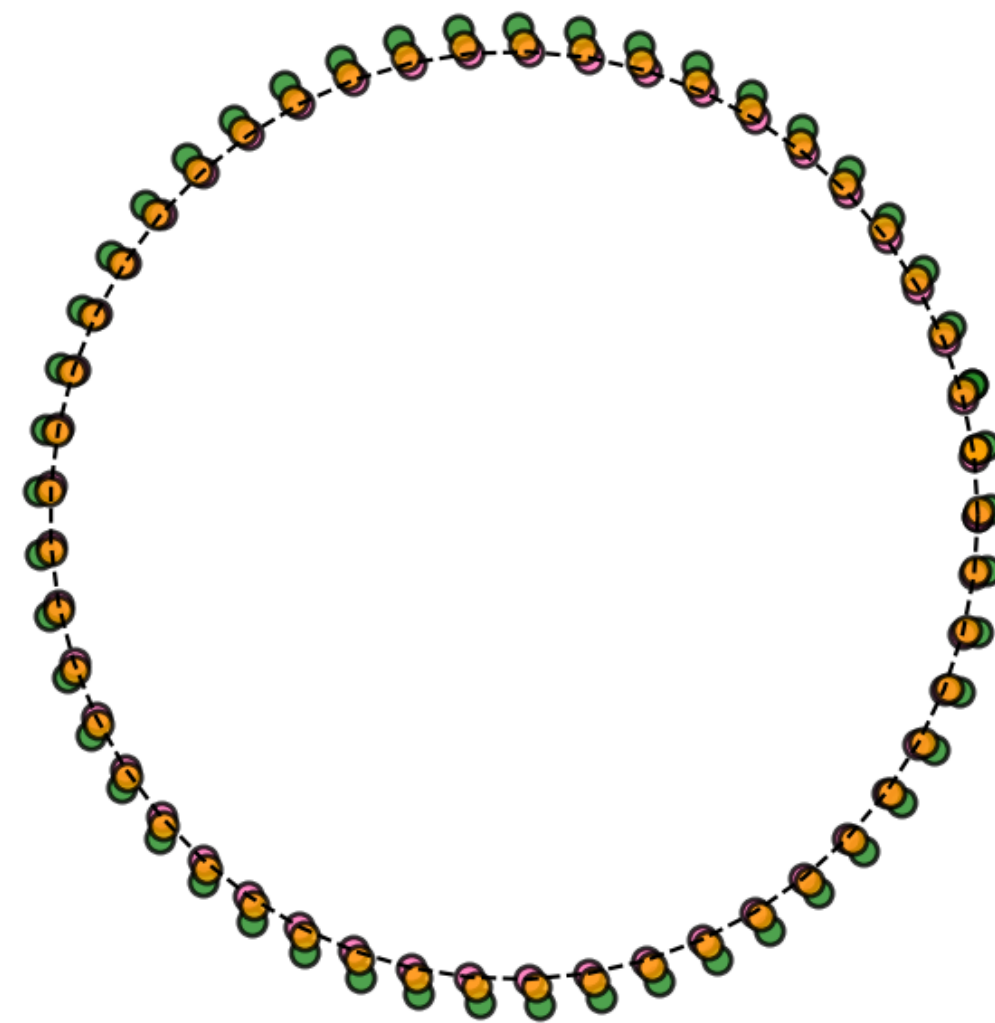
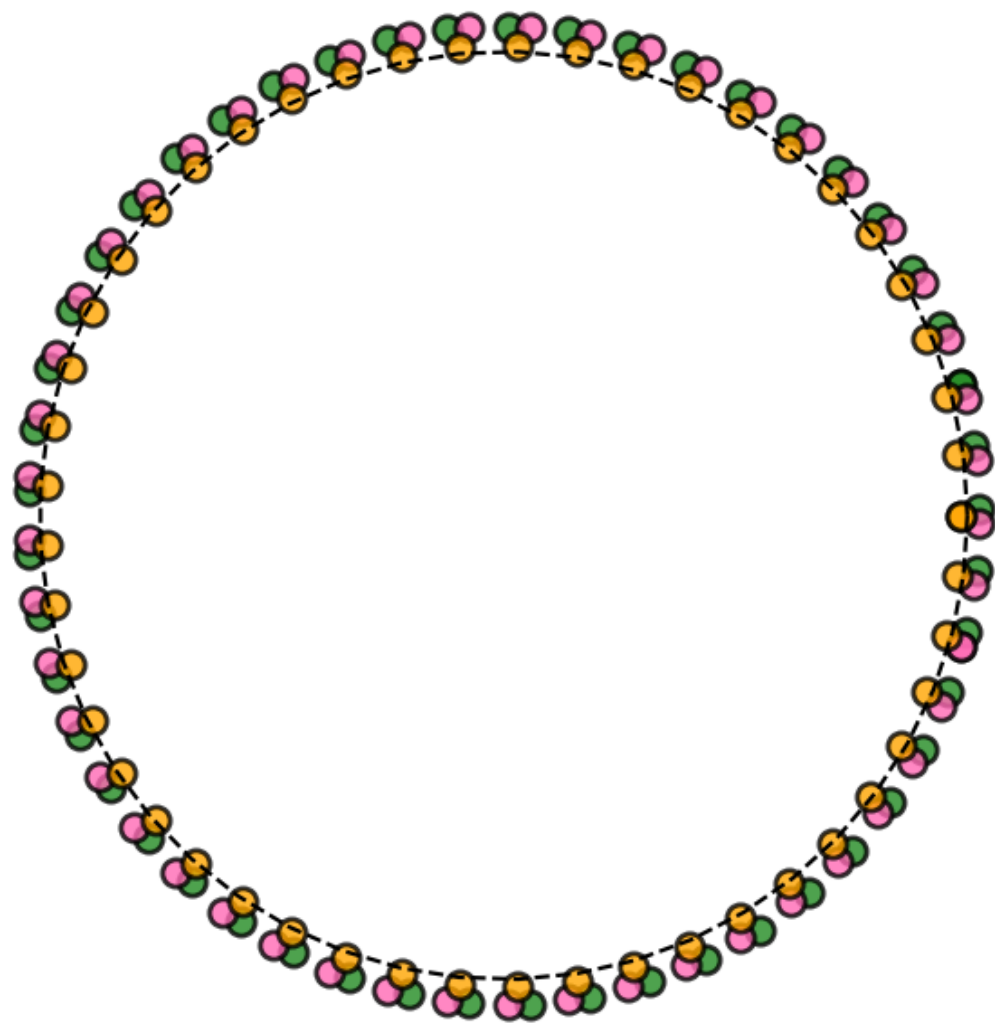
Fully RH: $h_L = 0$

No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Circular Polarization Basis



No PV: $h_L = h_R$

Fully LH: $h_R = 0$

Fully RH: $h_L = 0$

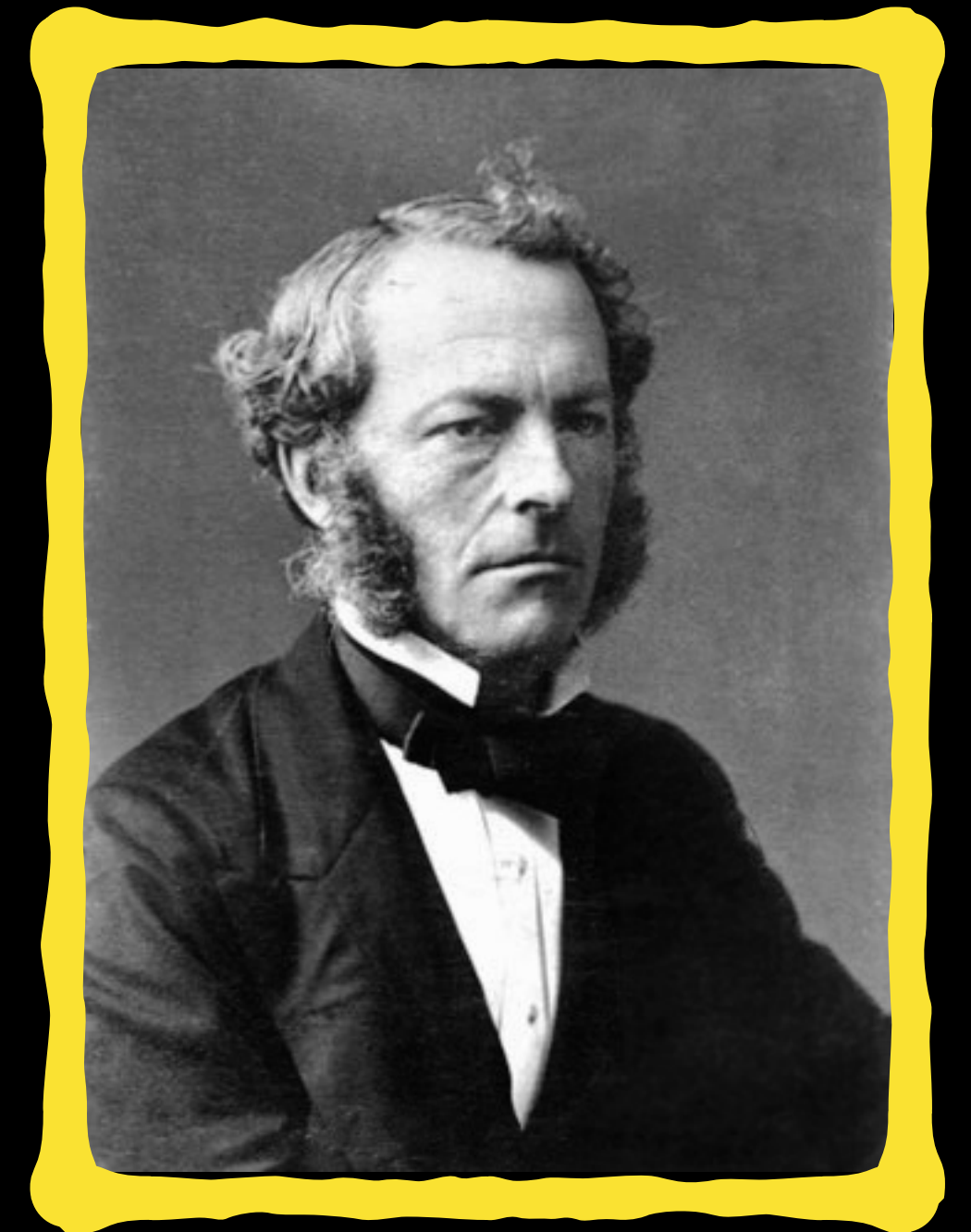
No PV: $h_{av} = h_+/2$

LH: $h_{av} = (h_+ - ih_x)/\sqrt{2}$

RH: $h_{av} = (h_+ + ih_x)/\sqrt{2}$

Polarization in the GWB

- Stokes parameters
 - $I = \langle |h_R|^2 \rangle + \langle |h_L|^2 \rangle$
 - $V = \langle |h_R|^2 \rangle - \langle |h_L|^2 \rangle$
- PV parameter $\Pi = \frac{V}{I}$, where
 - Fully RH: $\Pi = 1$,
 - Fully LH: $\Pi = -1$,
 - No net circular polarization: $\Pi = 0$



Detecting Parity Violation in the GWB: cross-correlation

- For a **standard GWB** between two detectors d_1 and d_2 :

$$\bullet \quad \langle C^{d_1 d_2}(f) \rangle = \frac{3H_0^2}{10\pi^2} \frac{\gamma_I^{d_1 d_2}(f) \Omega_{\text{GW}}(f)}{f^3}$$

$$\text{with } \Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln f} = \Omega_\alpha \left(\frac{f}{f_{\text{ref}}} \right)^\alpha$$

- For a **parity-violated GWB** between two detectors d_1 and d_2 :

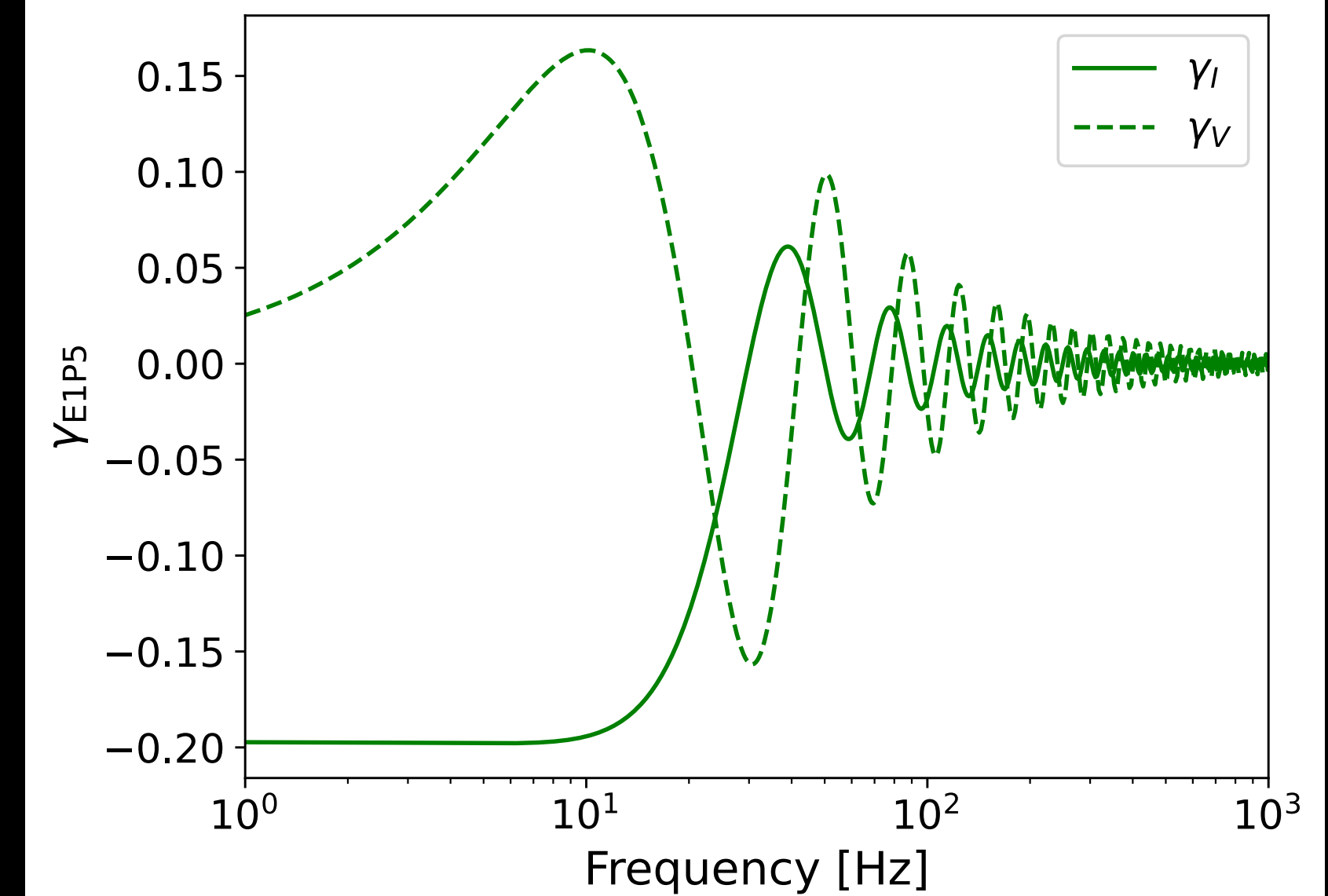
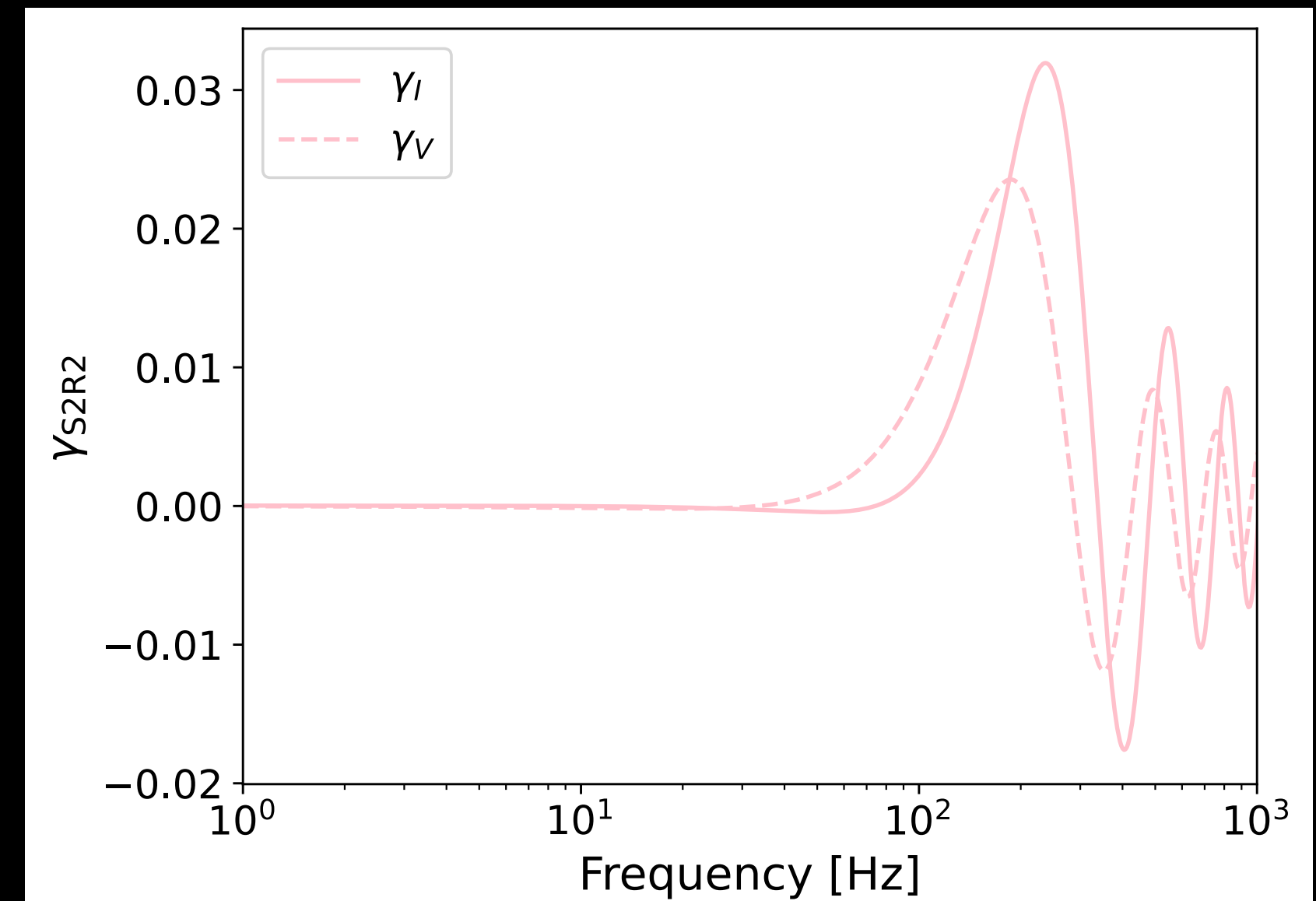
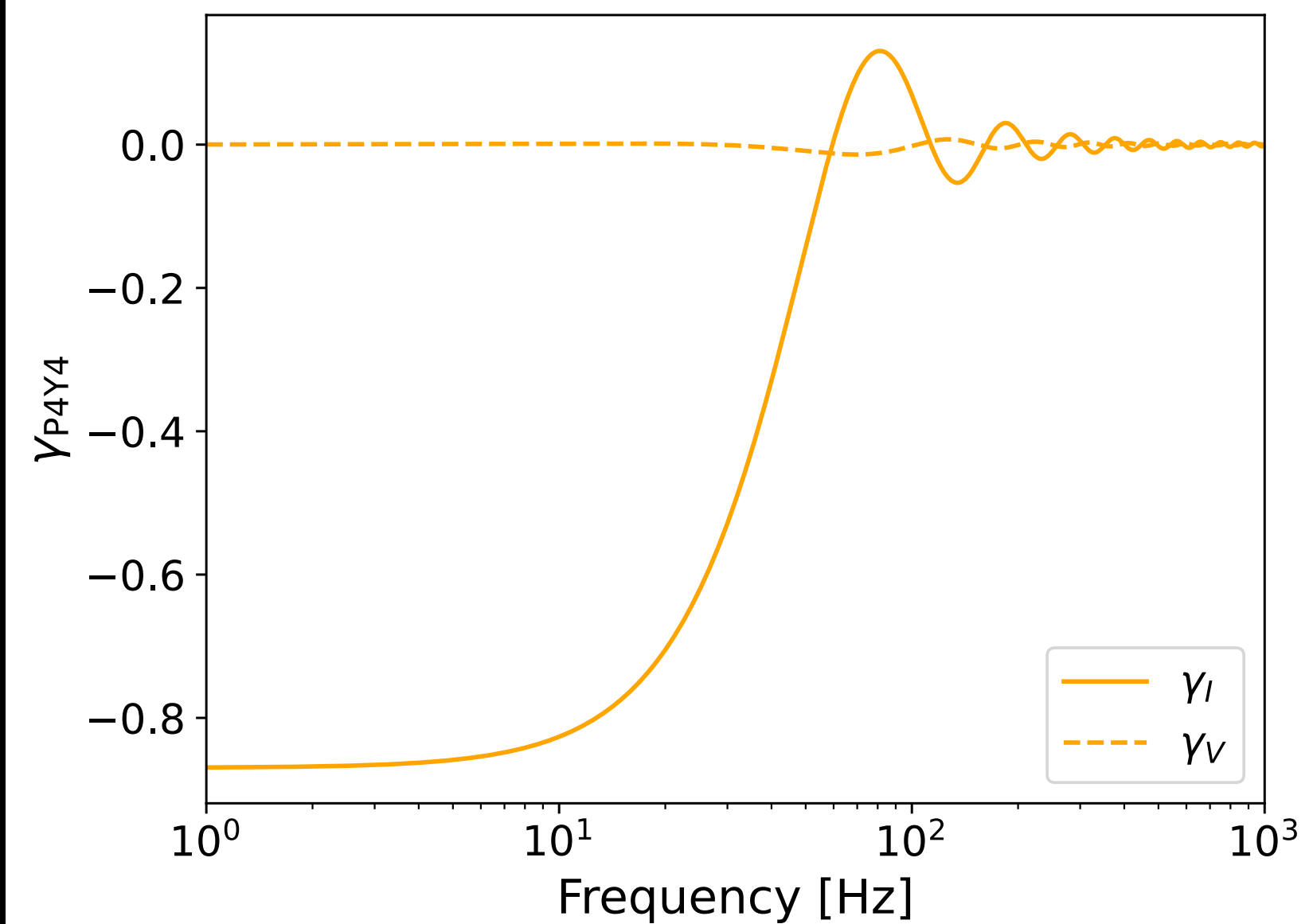
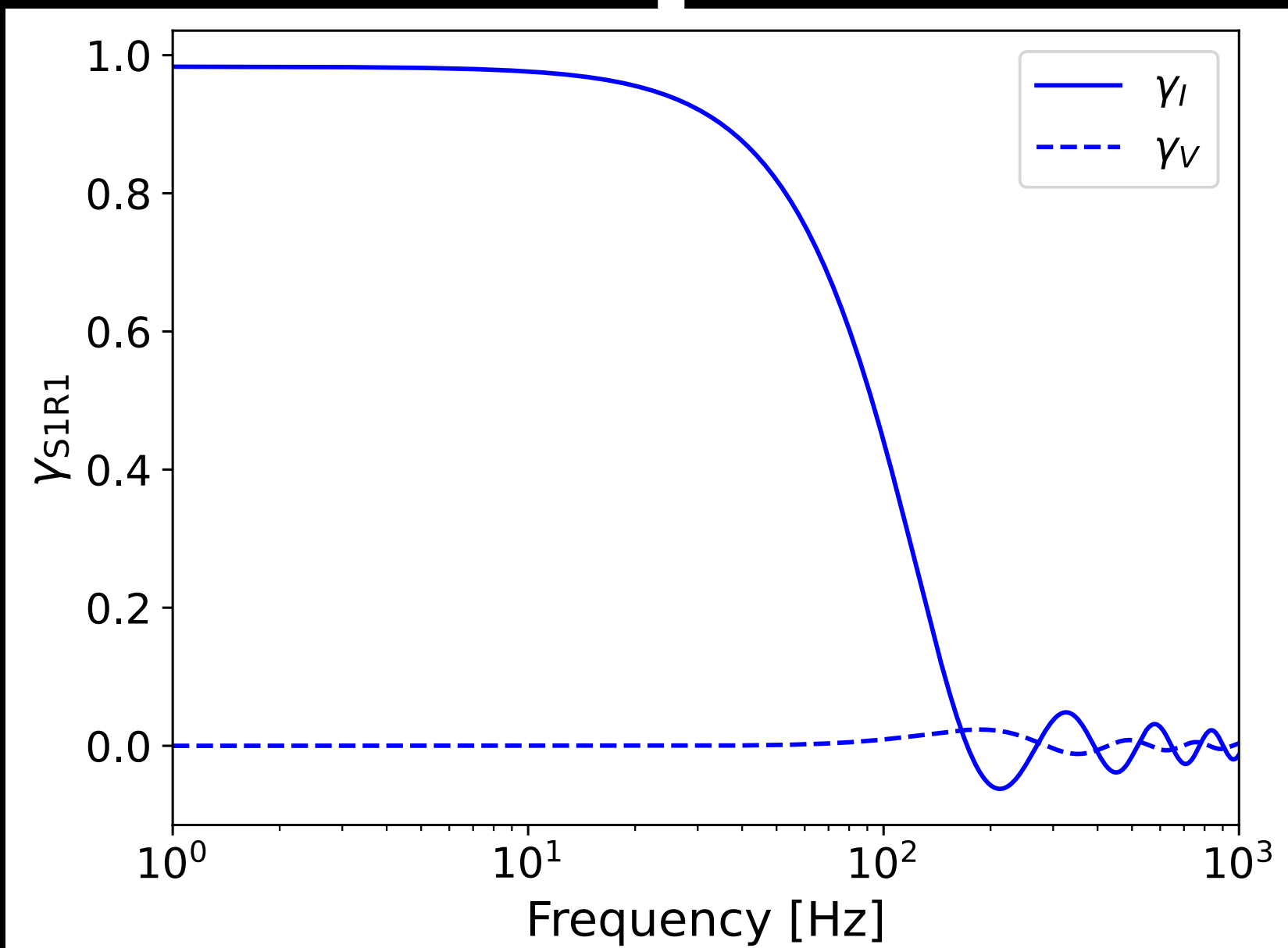
$$\bullet \quad \langle C^{d_1 d_2}(f) \rangle = \frac{3H_0^2}{10\pi^2} \frac{\gamma_I^{d_1 d_2}(f) \Omega'_{\text{GW}}(f)}{f^3}$$

$$\text{with } \Omega'_{\text{GW}} = \Omega_{\text{GW}} \left(1 + \Pi \frac{\gamma_V^{d_1 d_2}}{\gamma_I^{d_1 d_2}} \right)$$

$\gamma_I^{d_1 d_2}$ = Normalized ORF for I-modes

$\gamma_V^{d_1 d_2}$ = normalized ORF for V-modes

Overlap reduction functions



Quantifying Sensitivity to Parity Violation

- Signal-to-noise ratio (SNR):

$$\text{SNR}^2 = \rho^2 = 2T \left(\frac{3H_0^2}{10\pi^2} \right)^2 \int_0^\infty df \frac{\Omega_{\text{GW}}^2(f)}{f^6} \sum_{(a,b)} \theta_{ab}^2 \frac{(\gamma_{I,ab} + \Pi \gamma_{V,ab})^2}{N_a N_b}$$

- Separation for I and V modes ([Seto et al.](#))

$$\bullet \rho_I^2 = 2T \left(\frac{3H_0^2}{10\pi^2} \right)^2 \int_0^\infty df \frac{\Omega_{\text{GW}}^2(f)}{f^6} \frac{\Gamma_{\text{eff}}(f)}{\Gamma_{VV}(f)}$$

$$\bullet \rho_V^2 = 2T \left(\frac{3H_0^2}{10\pi^2} \right)^2 \int_0^\infty df \frac{\Omega_{\text{GW}}^2(f)}{f^6} \Pi^2(f) \frac{\Gamma_{\text{eff}}(f)}{\Gamma_{II}(f)}$$

With

N_a detector noise PSD

$$\Gamma_{\text{eff}}(f) \equiv \Gamma_{II}(f)\Gamma_{VV}(f) - \Gamma_{IV}^2(f)$$

$$\Gamma_{AB}(f) \equiv \sum_{(a,b)} \theta_{ab}^2 \frac{\gamma_{A,ab}(f)\gamma_{B,ab}(f)}{N_a(f)N_b(f)}$$

with $A, B = I, V$

$$\theta_{ab} \equiv \frac{\sin \theta_a \sin \theta_b}{5}$$

with θ_a opening angle of detector a

Outline



- Brief overview of gravitational-wave background
- Brief visit to Parity Violation
- 3G networks
- Detection Prospects for different configurations:
 - PI curves and SNR prospects
 - Bayesian analysis

Outline

- Brief overview of gravitational-wave background

- Brief visit to Parity Violation

- 3G networks

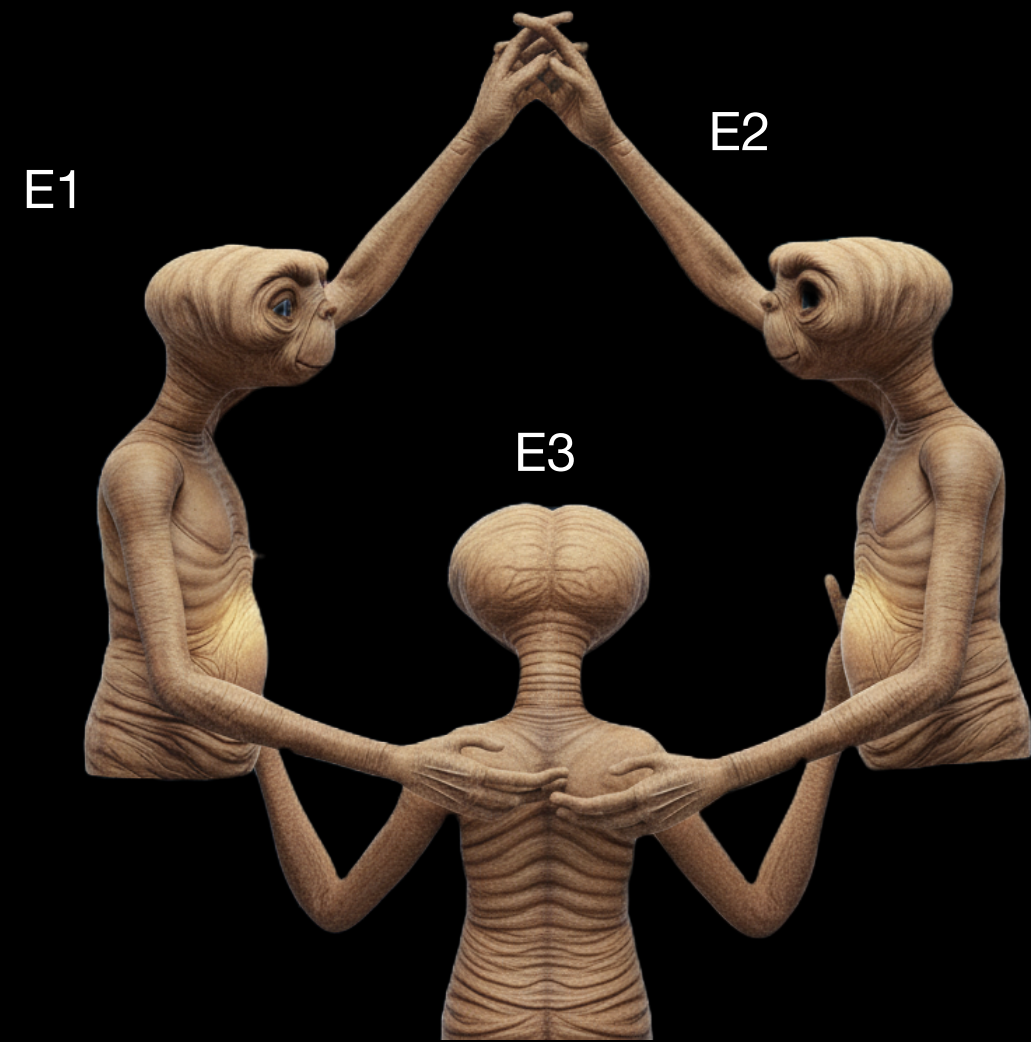
- Detection Prospects for different configurations:

- PI curves and SNR prospects

- Bayesian analysis



More on ET



- Δ
- 10 km arms
(we will also study 15 km arms)
- Virgo site, Pisa, Italy



- 2L: S and R
- 15 km arms
- S = L-shaped ET in Sardinia
- R = L-shaped ET in three-border region:
Belgium, the Netherlands, Germany

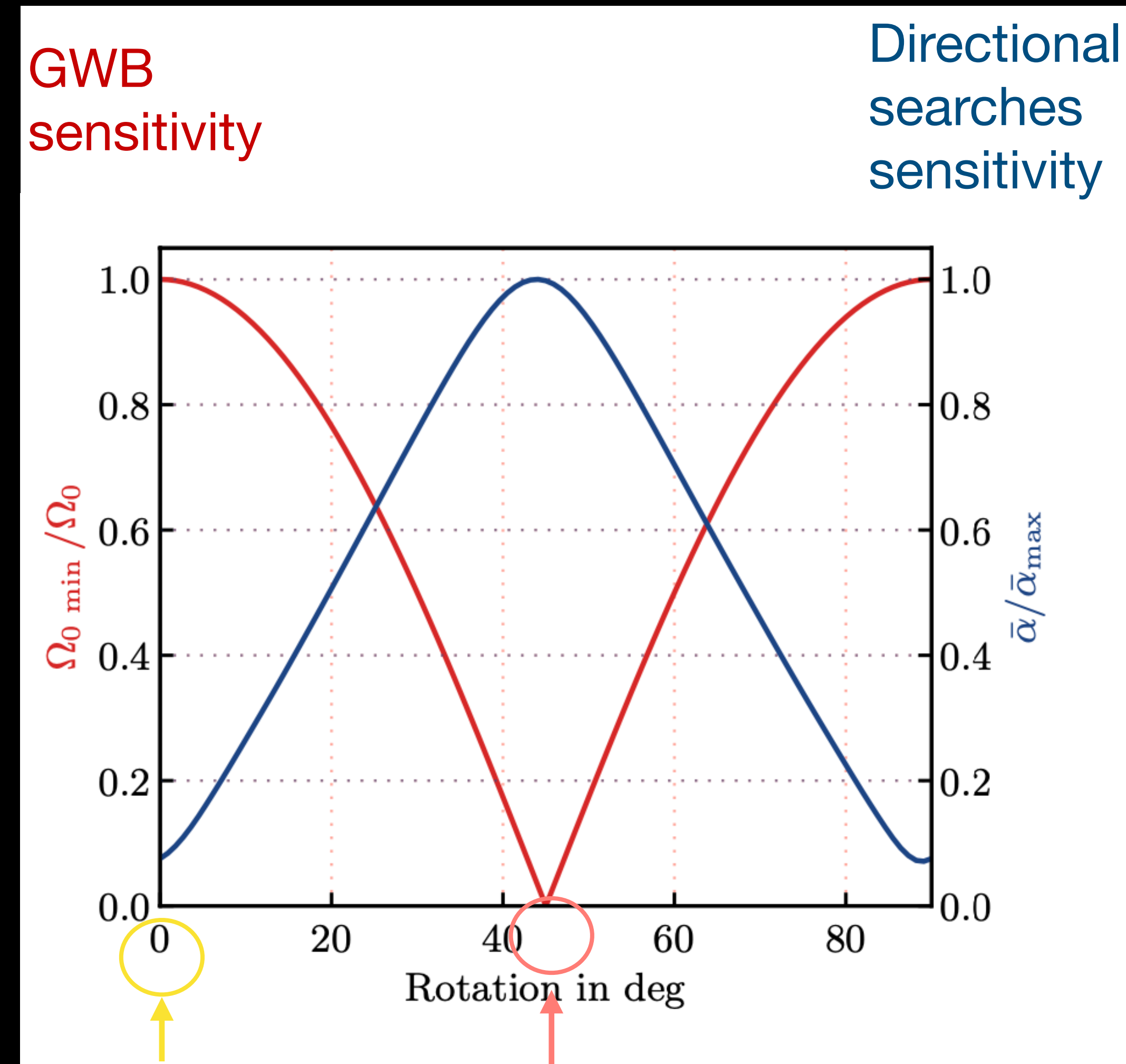
More on CE

- United States
- 2 L-shaped detectors
- P = CE in Pacific Ocean, 20 km arms
- Y = CE in Atlantic Ocean, 40 km arms



Configuration orientation

- The orientation angle between the 2L detectors matters... A LOT
- For one 2L set-up:
 - Orientation angle of 0° (completely alignment)
→ optimal scenario for detecting GWB (GWB optimized)
 - Orientation angle of 45° (completely misaligned)
→ worst-case scenario for detecting GWB (localization optimized)
- For a global network of 2L ET + 2L CE:
 - Orientation angles are no longer just 0° and 45°
 - Configuration angles for global network computed in [Ebersold et al.](#)
- Idea remains: relative orientation angle drives sensitivity to GWB!



Different third-generation configurations

$\Delta\text{ET} + 2\text{L CE}$	$2\text{L ET} + 2\text{L CE}$
• $\Delta_{10}(\text{PY})_4$: 10 km Δ + 2CE, GWB-optimized • $\Delta_{10}(\text{PY})_5$: 10 km Δ + 2CE, Localization-optimized	

Different third-generation configurations

$\Delta\text{ET} + 2\text{L CE}$

- $\Delta_{10}(\text{PY})_4$: 10 km $\Delta + 2\text{CE}$,
GWB-optimized
- $\Delta_{10}(\text{PY})_5$: 10 km $\Delta + 2\text{CE}$,
Localization-optimized
- $\Delta_{15}(\text{PY})_4$: 15 km $\Delta + 2\text{CE}$,
GWB-optimized
- $\Delta_{15}(\text{PY})_5$: 15 km $\Delta + 2\text{CE}$,
Localization-optimized

$2\text{L ET} + 2\text{L CE}$

Different third-generation configurations

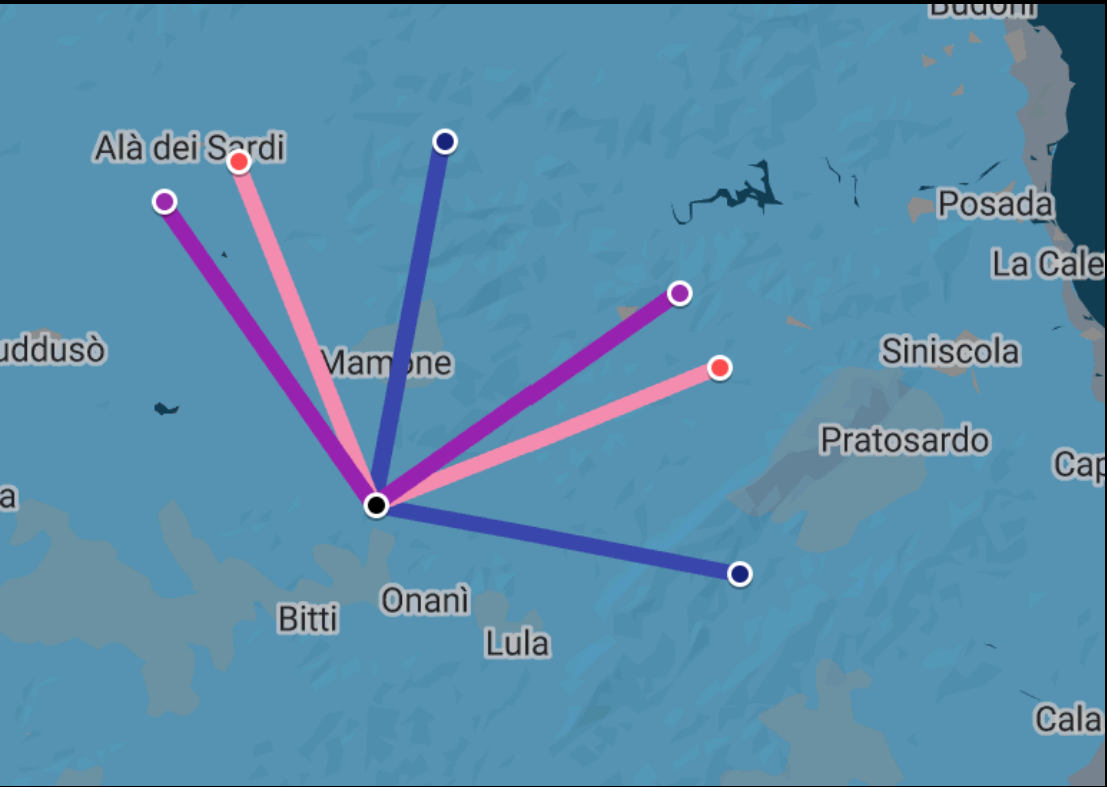
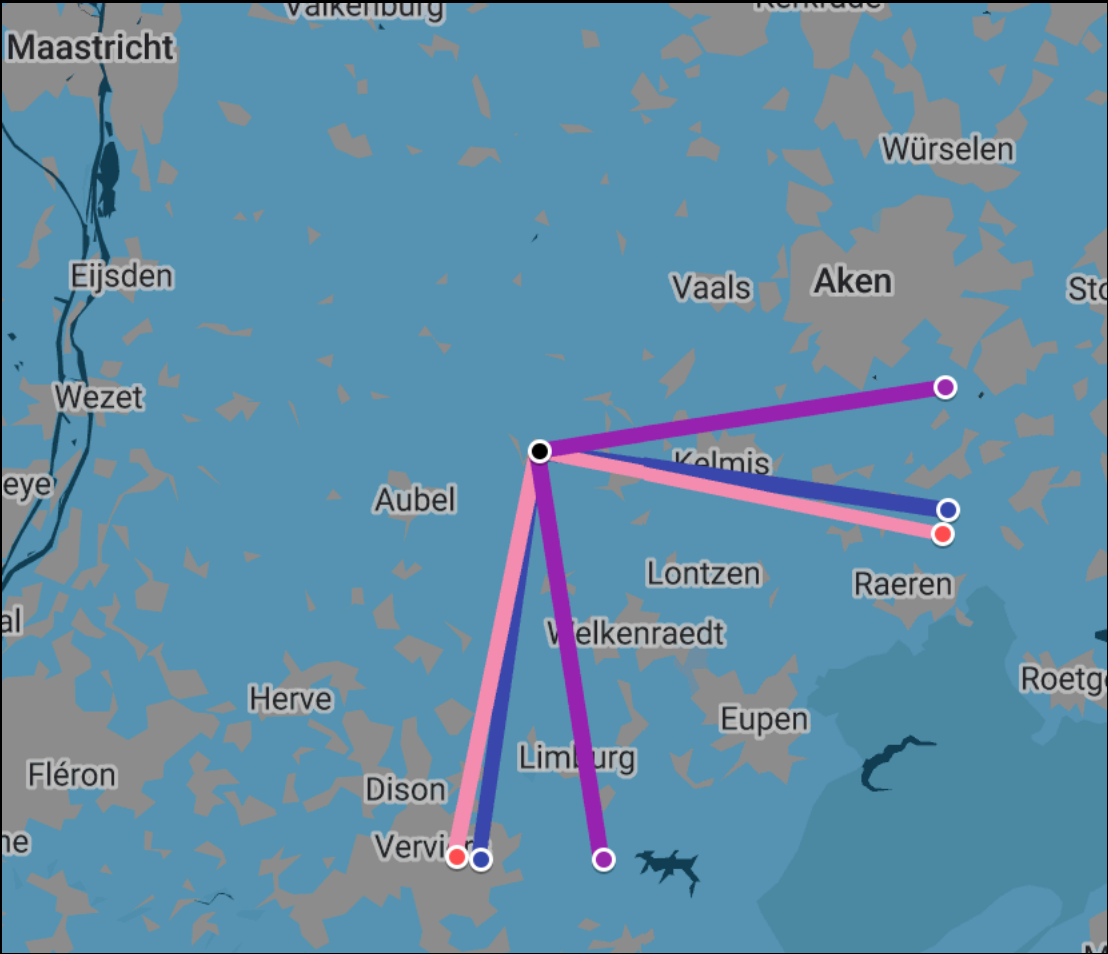
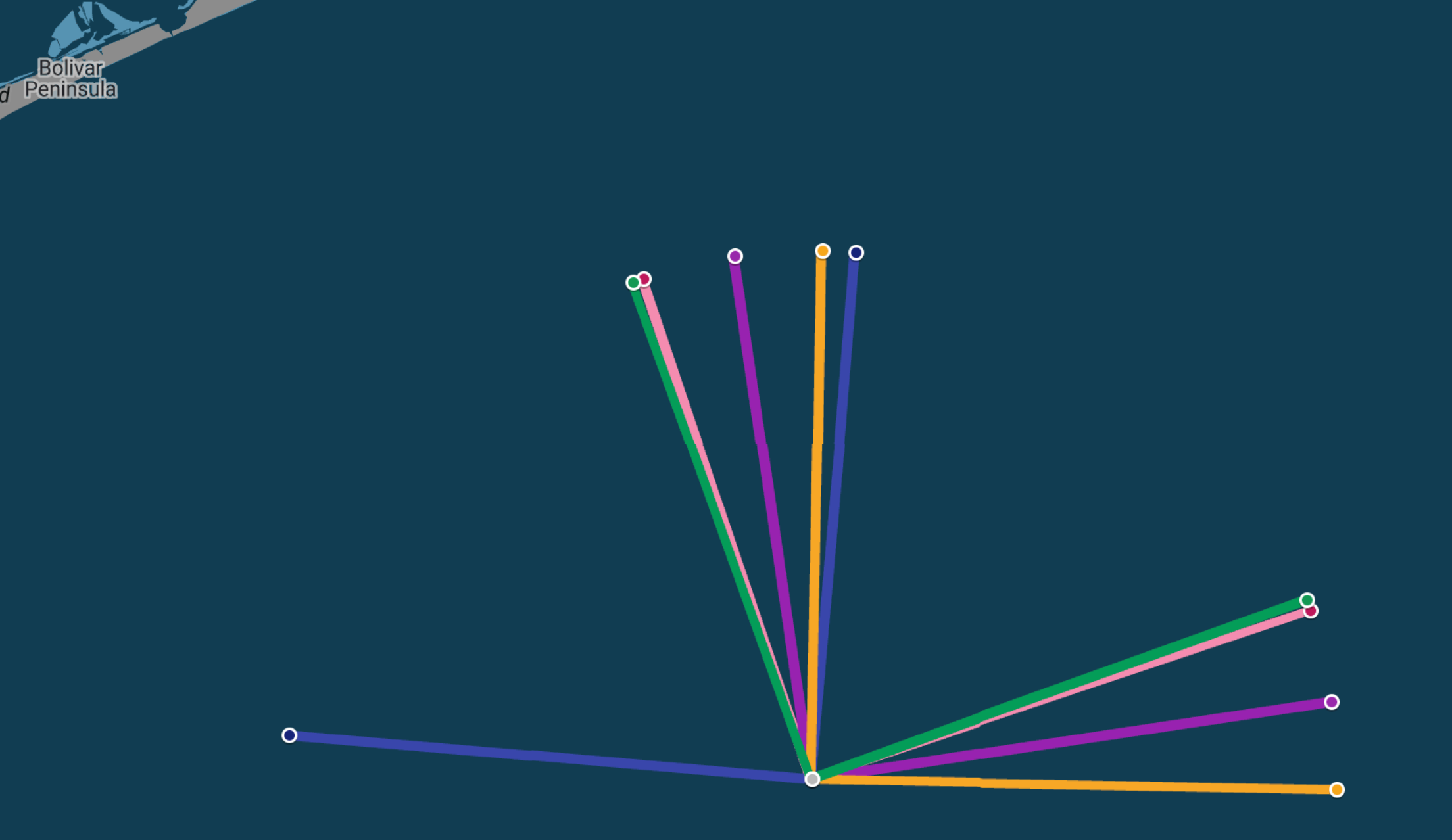
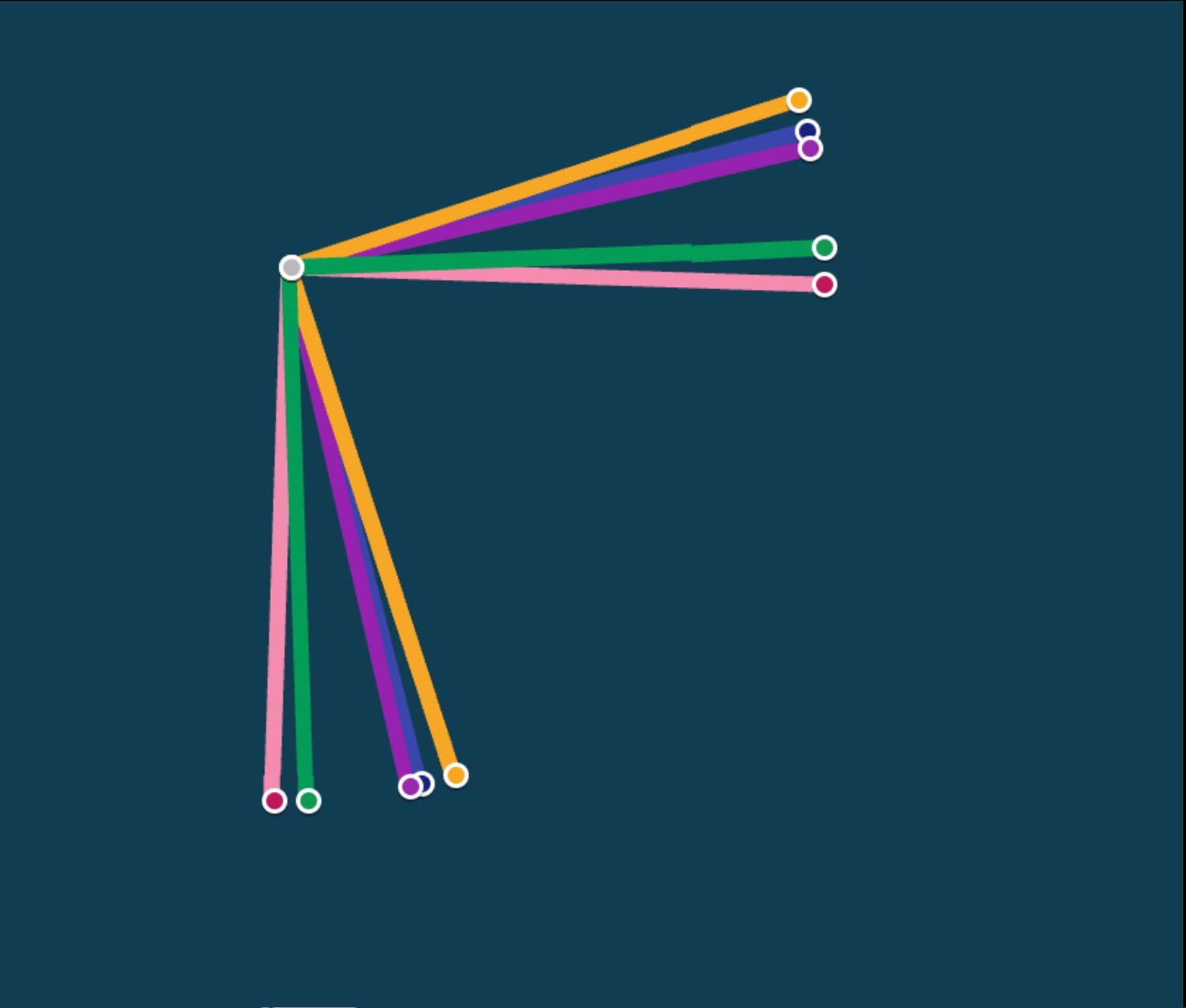
Δ ET + 2L CE

- $\Delta_{10}(\text{PY})_4$: 10 km Δ + 2CE, GWB-optimized
- $\Delta_{10}(\text{PY})_5$: 10 km Δ + 2CE, Localization-optimized
- $\Delta_{15}(\text{PY})_4$: 15 km Δ + 2CE, GWB-optimized
- $\Delta_{15}(\text{PY})_5$: 15 km Δ + 2CE, Localization-optimized

2L ET + 2L CE

- $(\text{SR})_1(\text{PY})_1$: 15 km 2L + 2CE, GWB-optimized
- $(\text{SR})_2(\text{PY})_2$: 15 km 2L + 2CE, Localization-optimized
- $(\text{SR})_3(\text{PY})_3$: 15 km 2L + 2CE, Hybrid

World map



Outline

- Brief overview of gravitational-wave background
- Brief visit to Parity Violation
- 3G networks
- Detection Prospects for different configurations:
 - PI curves and SNR prospects
 - Bayesian analysis



Outline

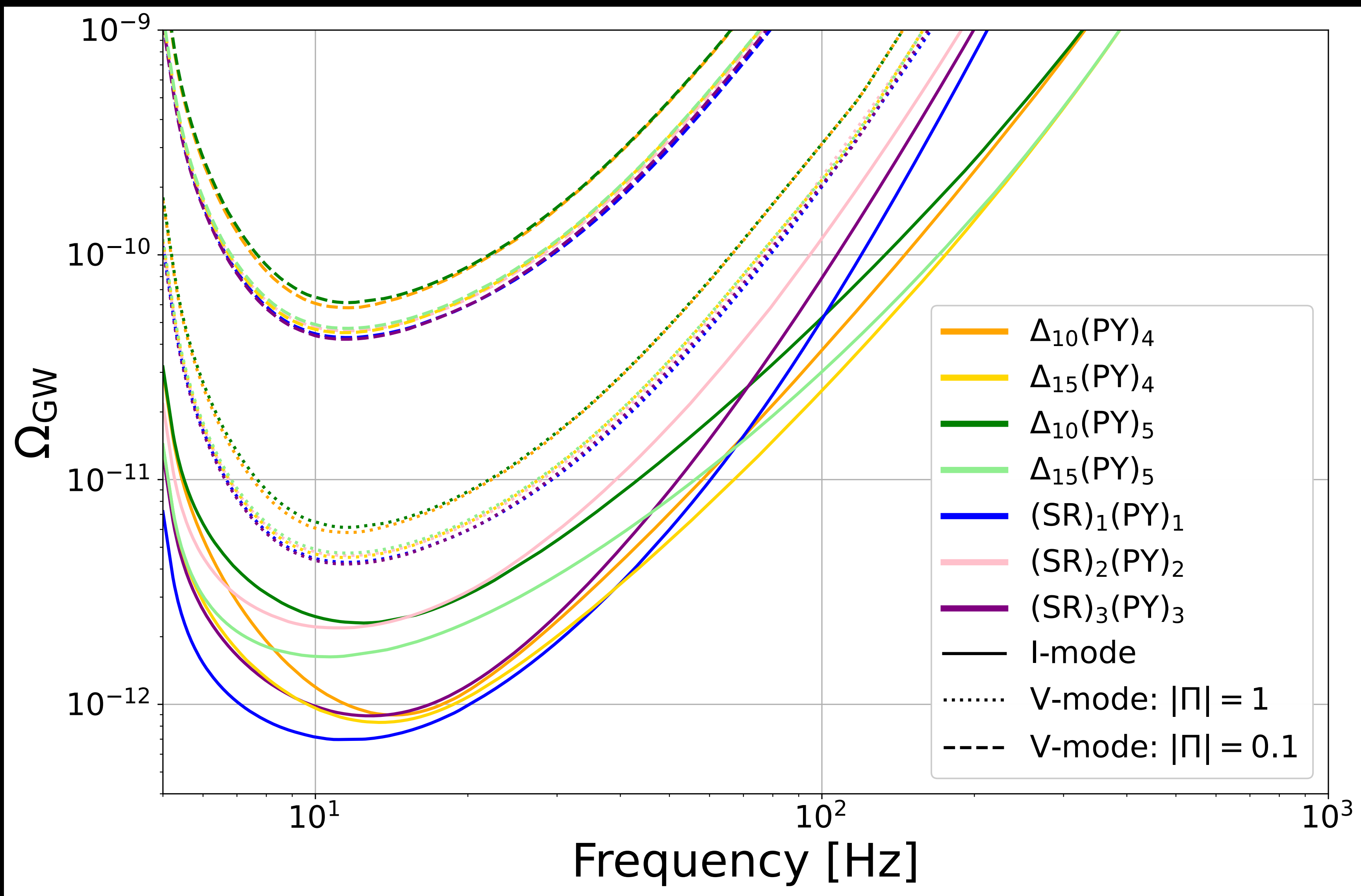
- Brief overview of gravitational-wave background
- Brief visit to Parity Violation
- 3G networks
- Detection Prospects for different configurations:
 - PI curves and SNR prospects
 - Bayesian analysis

?



How to quantify PV detection prospects?

Δ GWB-optimized
 Δ GWB-least optimal
 Δ GWB-optimized
 Δ Localization-least optimal
2L GWB-optimized
2L GWB-least optimal
2L Hybrid



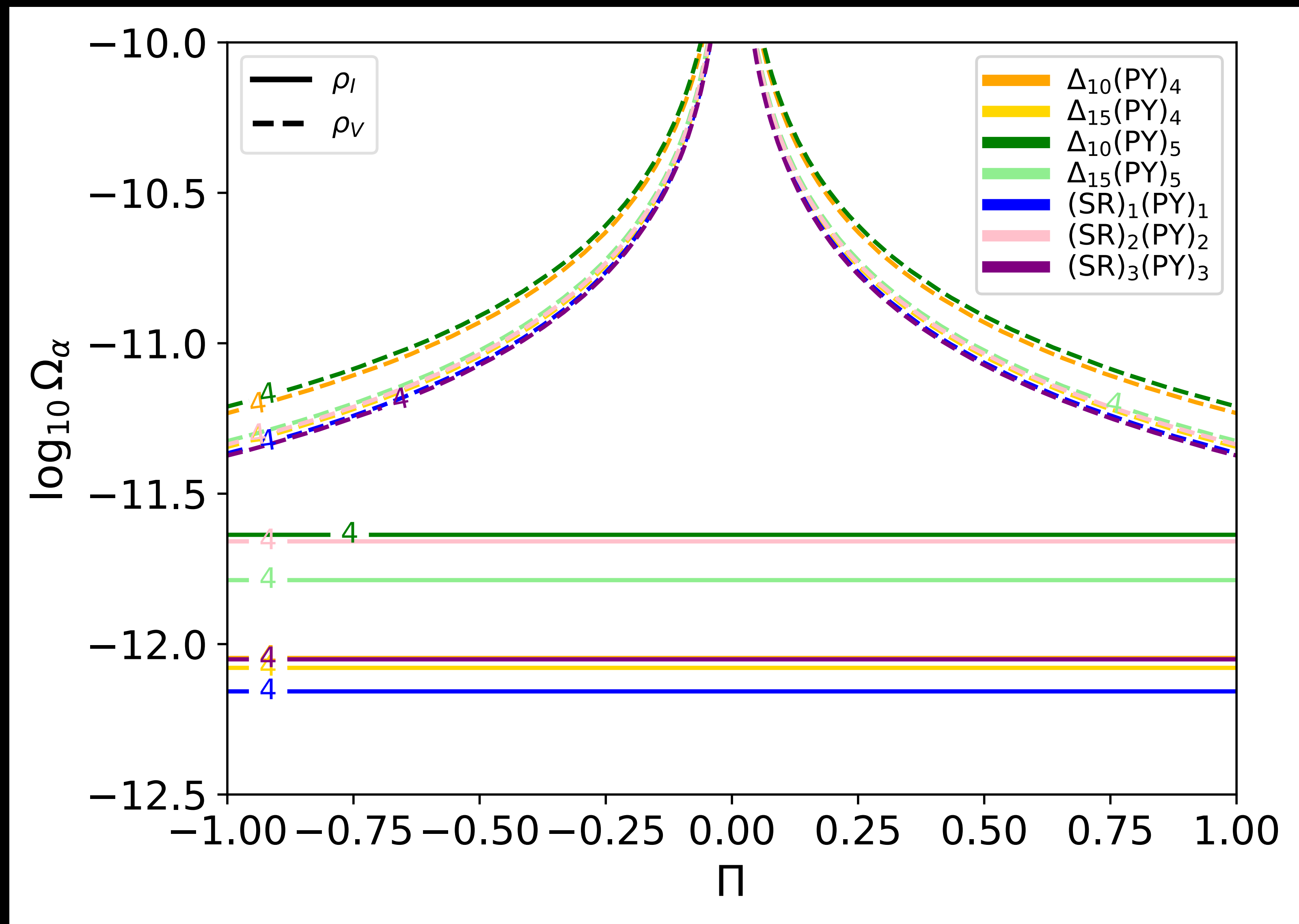
PI curve prospects

For $\rho = 4$

$T_{\text{obs}} = 1 \text{ year}$

How to quantify PV detection prospects?

Δ GWB-optimized
Δ GWB-least optimal
Δ GWB-optimized
Δ Localization-least optimal
2L GWB-optimized
2L GWB-least optimal
2L Hybrid



SNR prospects

$$\Omega_{\text{GW}}(f) = \Omega_\alpha \left(\frac{f}{f_{\text{ref}}} \right)^\alpha$$

For $\alpha = 0$

Battle of designs



Last

1th and 2nd

Bayesian Analysis

- Compare SNR findings to data-based analysis!
- Bayes Theorem: $P(\theta | \text{data}) \sim P(\text{data} | \theta)P(\theta)$



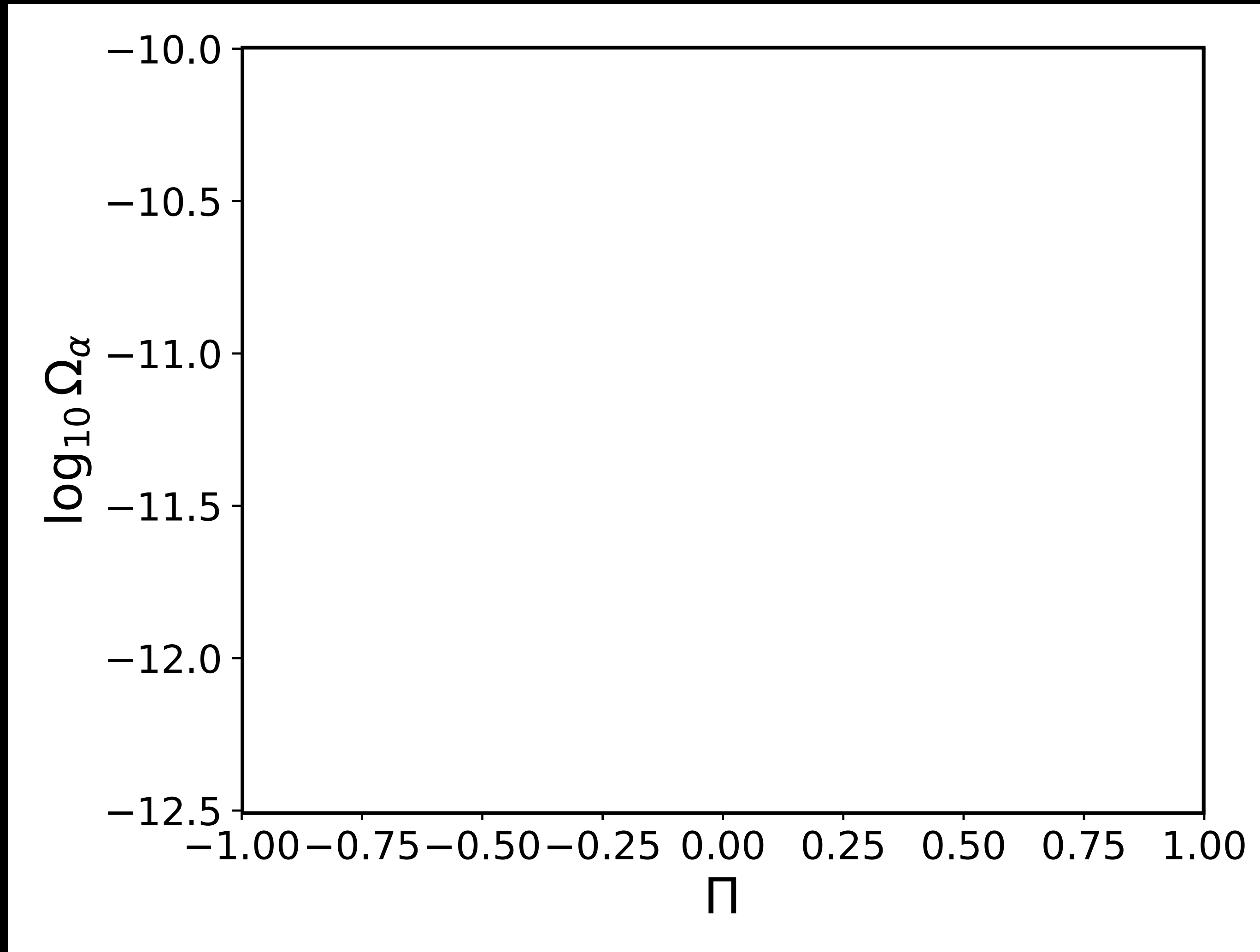
$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)



Bayesian Analysis

Make
injections for:





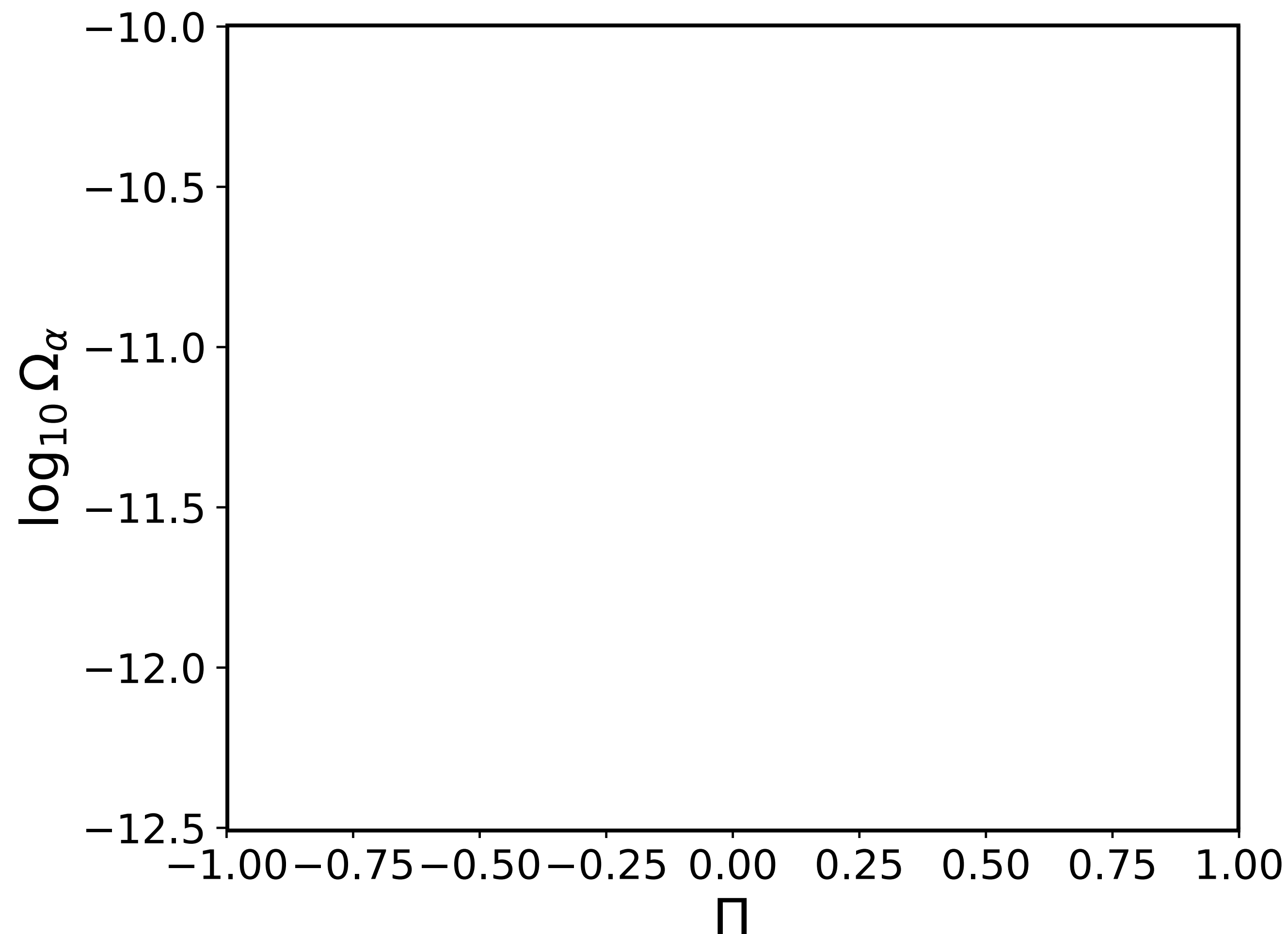
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Make
injections for:

1. Fix α value per network & create random detector noise



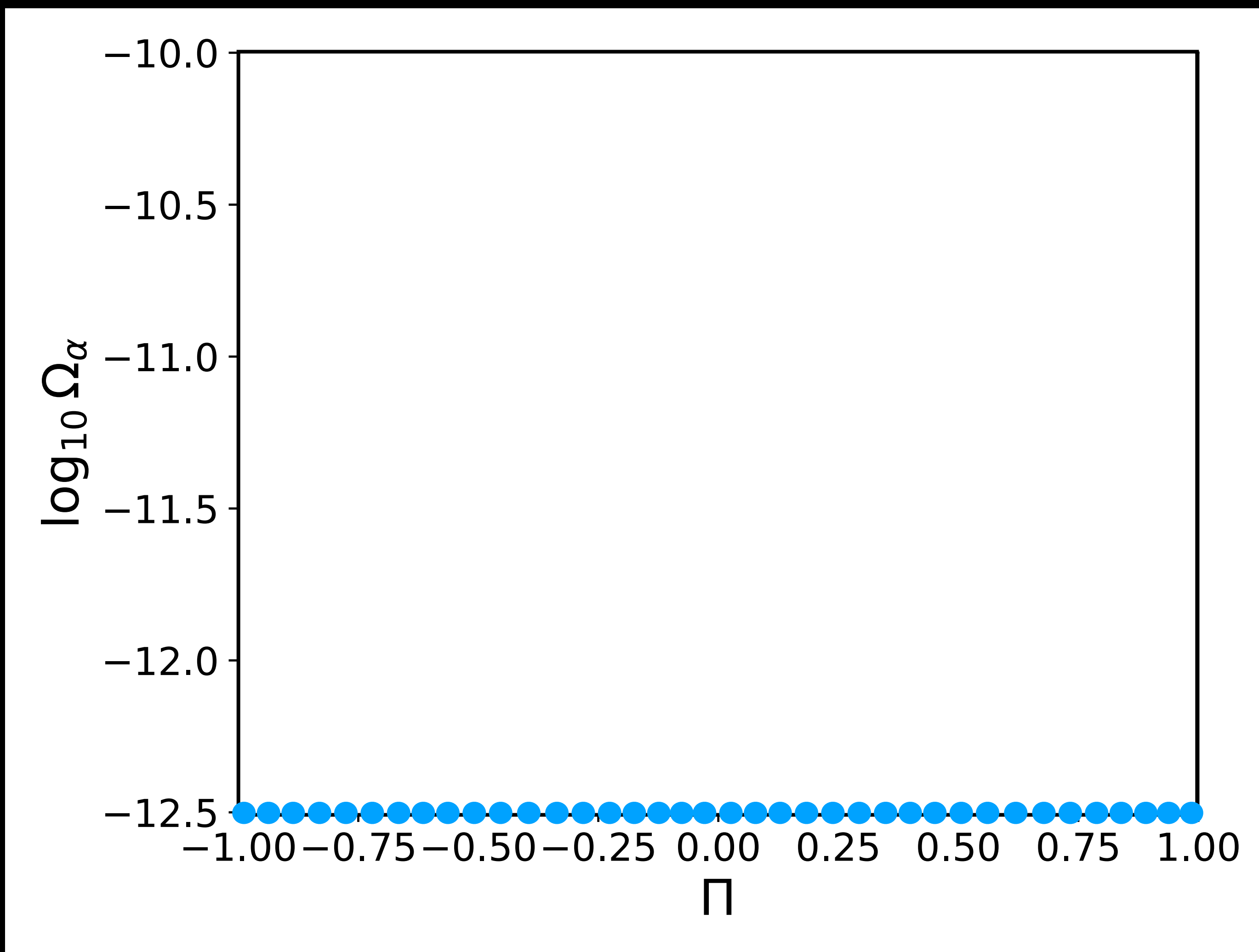
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. Π :
 $-1 \leq \Pi \leq 1$
(40 values)



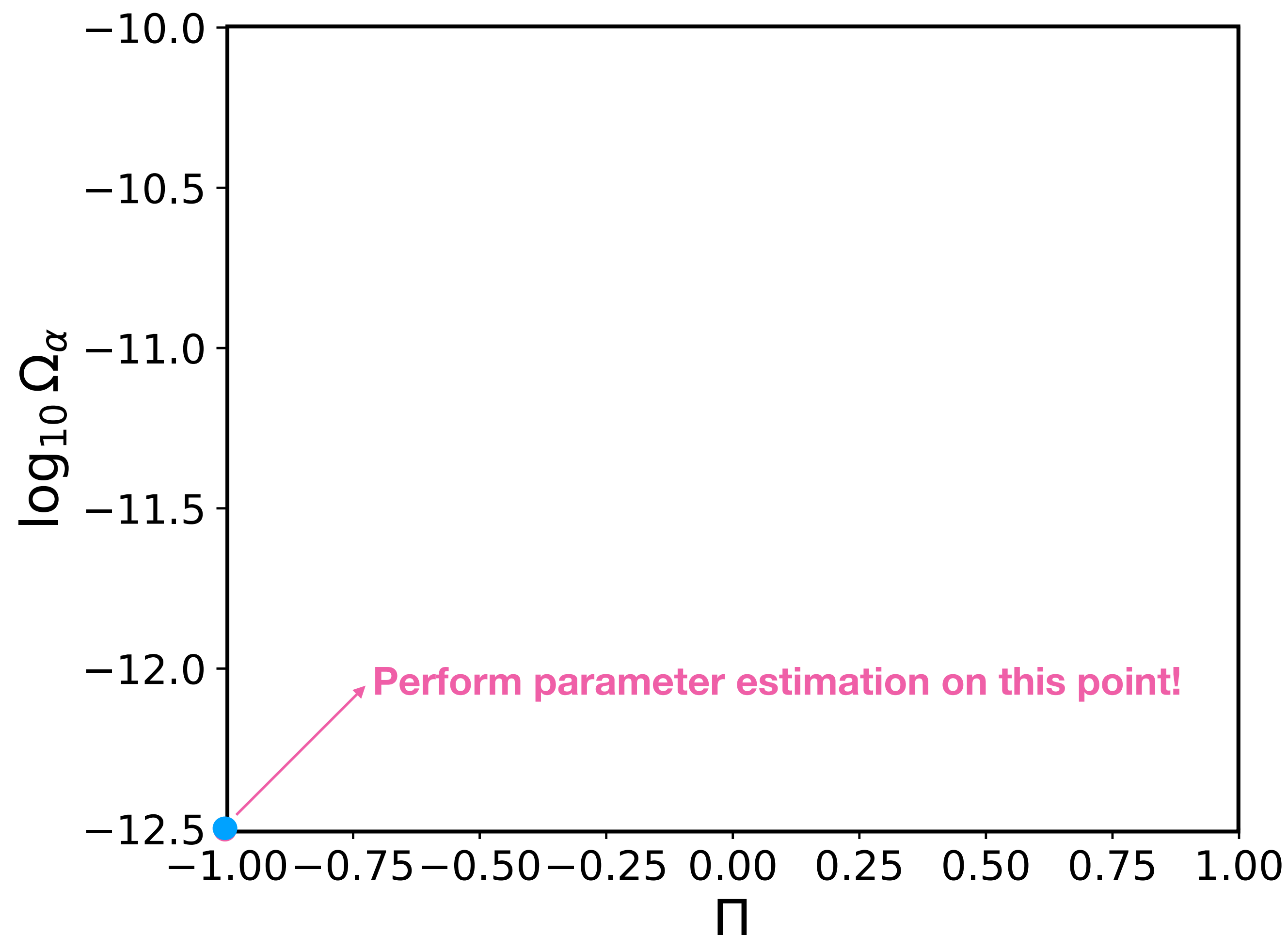
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

3. $\log \Omega_{\alpha}$:
 $-12.5 \leq \log \Omega_{\alpha} \leq -10$
(40 values)



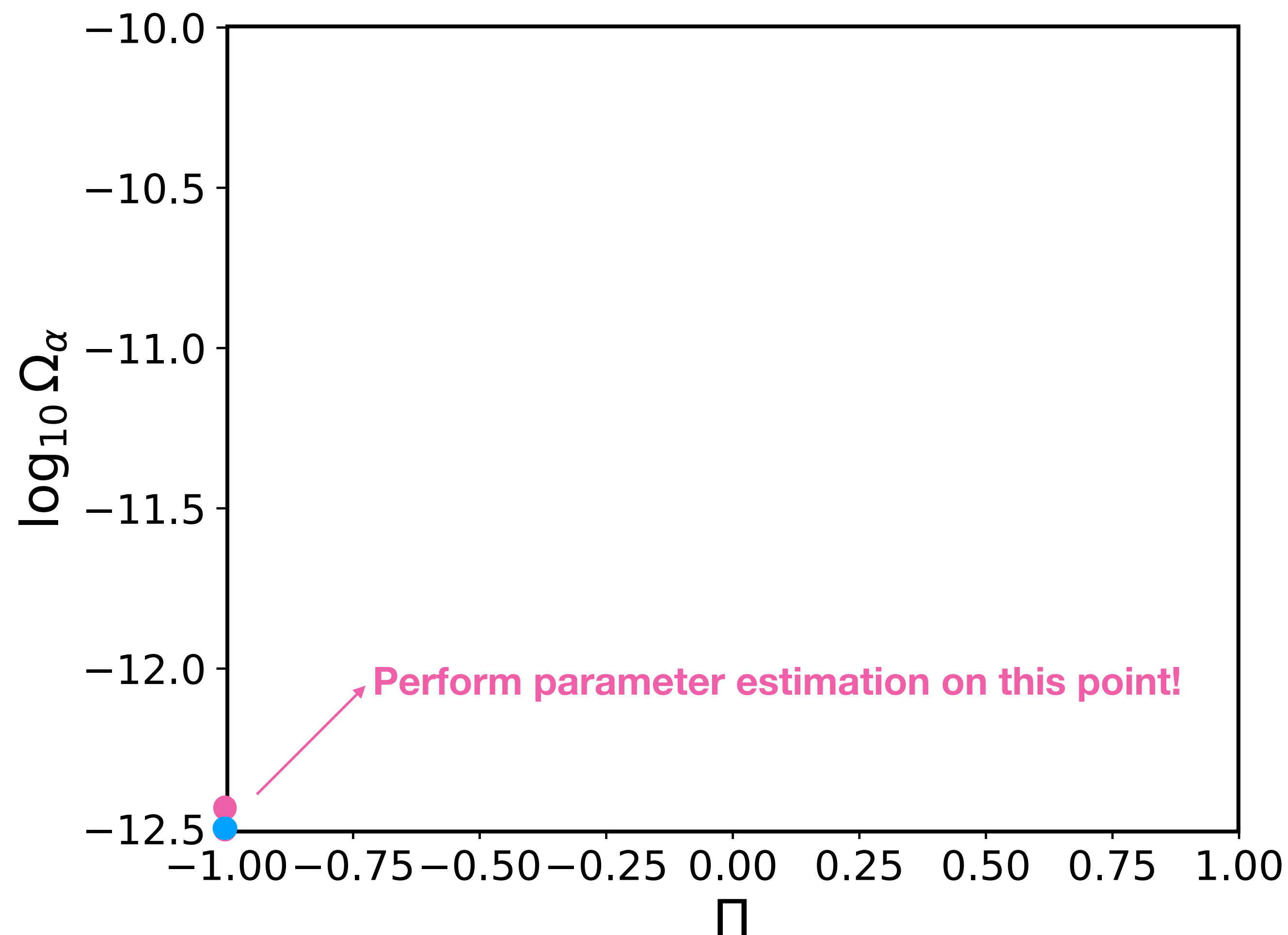
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

3. $\log \Omega_{\alpha}$:
 $-12.5 \leq \log \Omega_{\alpha} \leq -10$
(40 values)



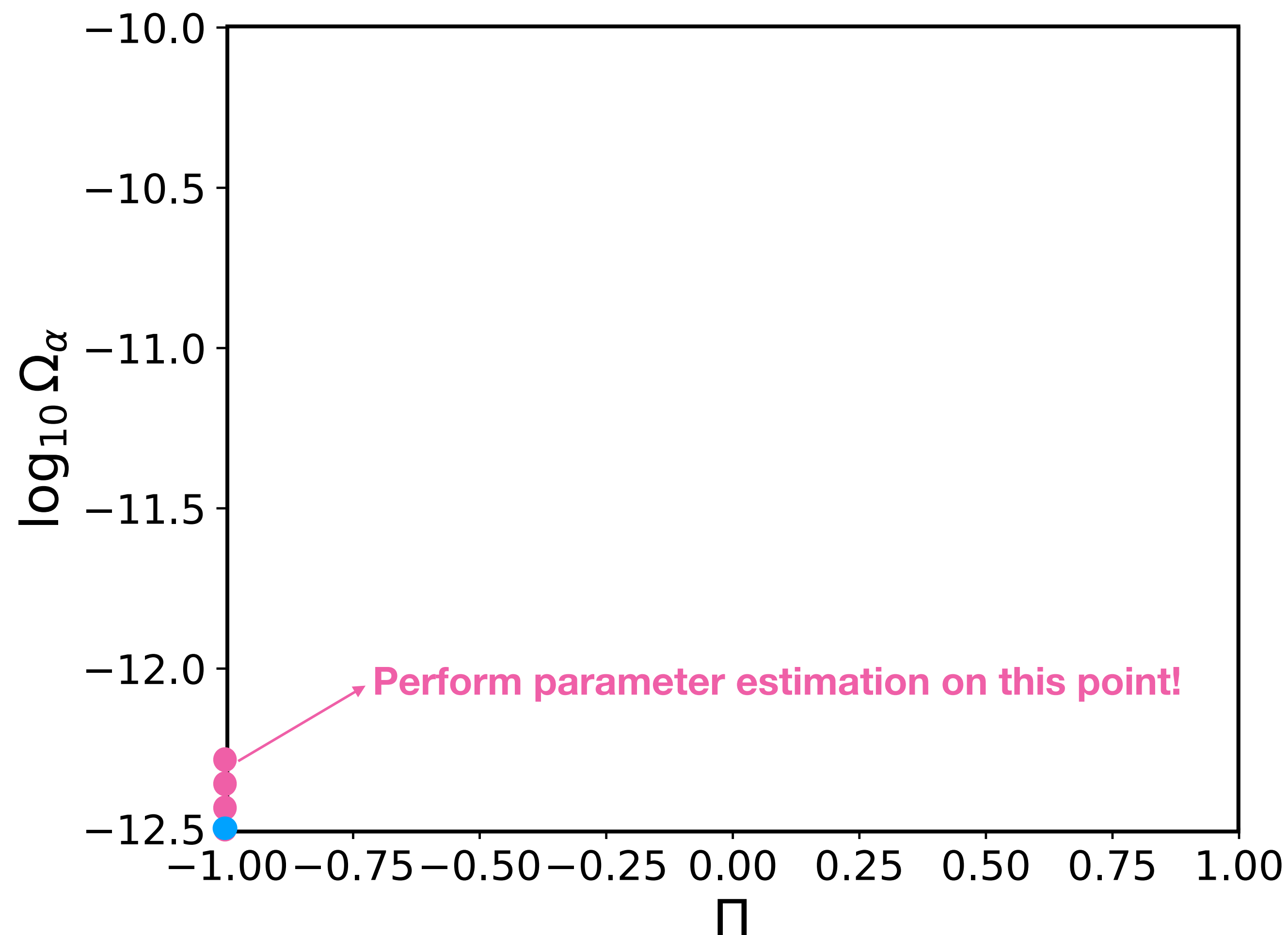
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

3. $\log \Omega_{\alpha}$:
 $-12.5 \leq \log \Omega_{\alpha} \leq -10$
(40 values)



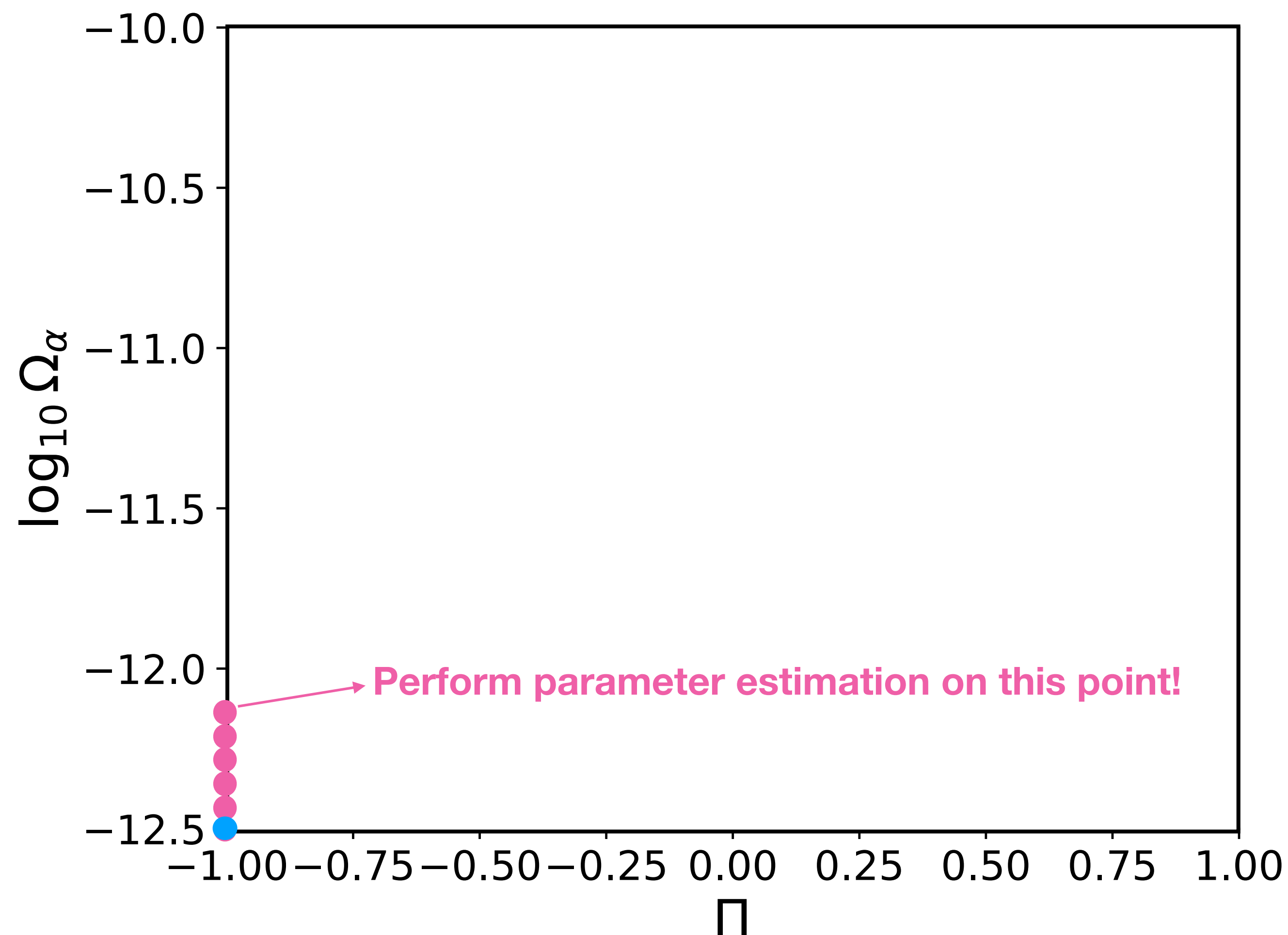
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. For each Π :

$$-1 \leq \Pi \leq 1$$

(40 values)

3. $\log \Omega_{\alpha}$:

$$-12.5 \leq \log \Omega_{\alpha} \leq -10$$

(40 values)



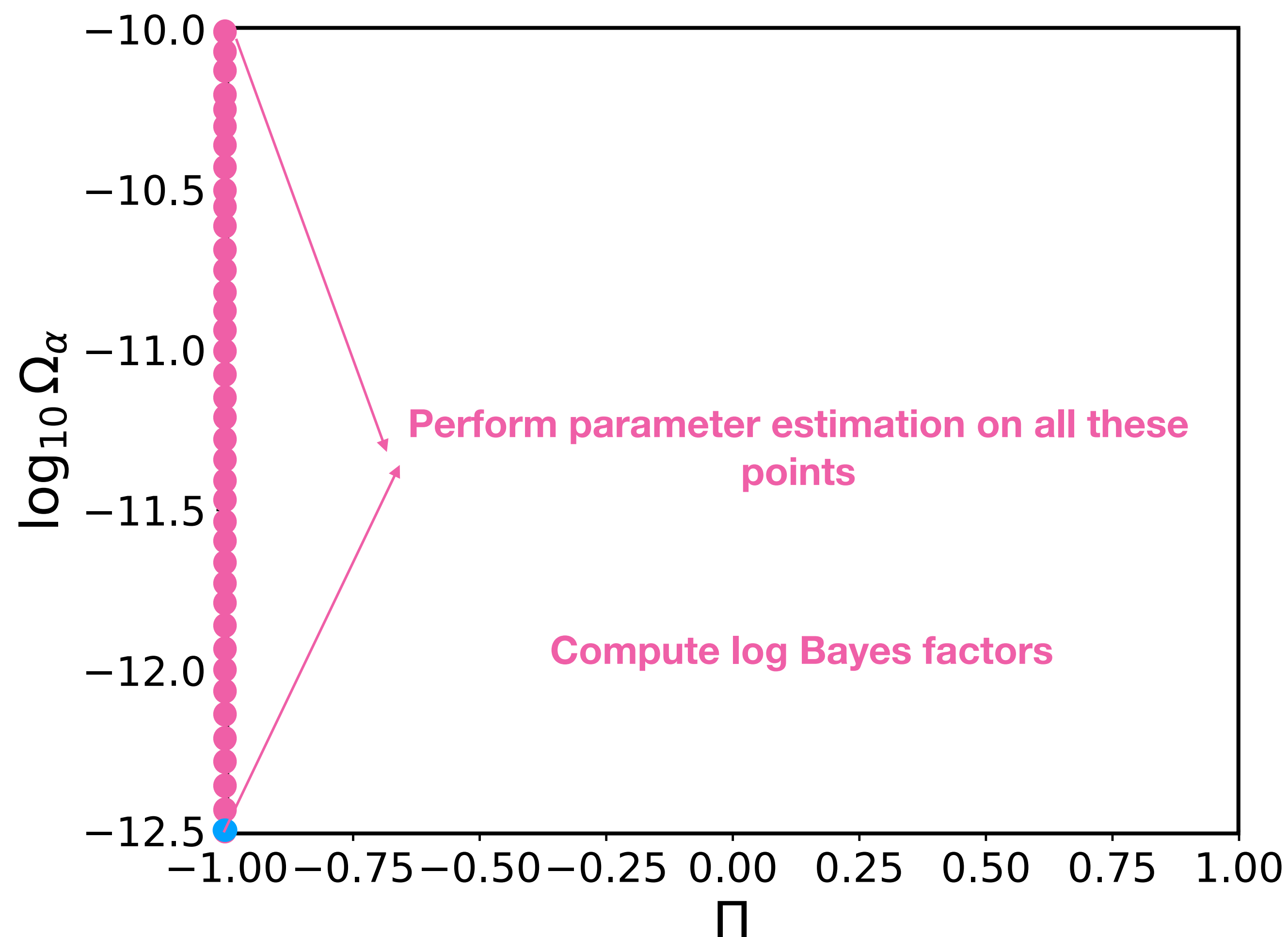
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

3. $\log \Omega_{\alpha}$:
 $-12.5 \leq \log \Omega_{\alpha} \leq -10$
(40 values)



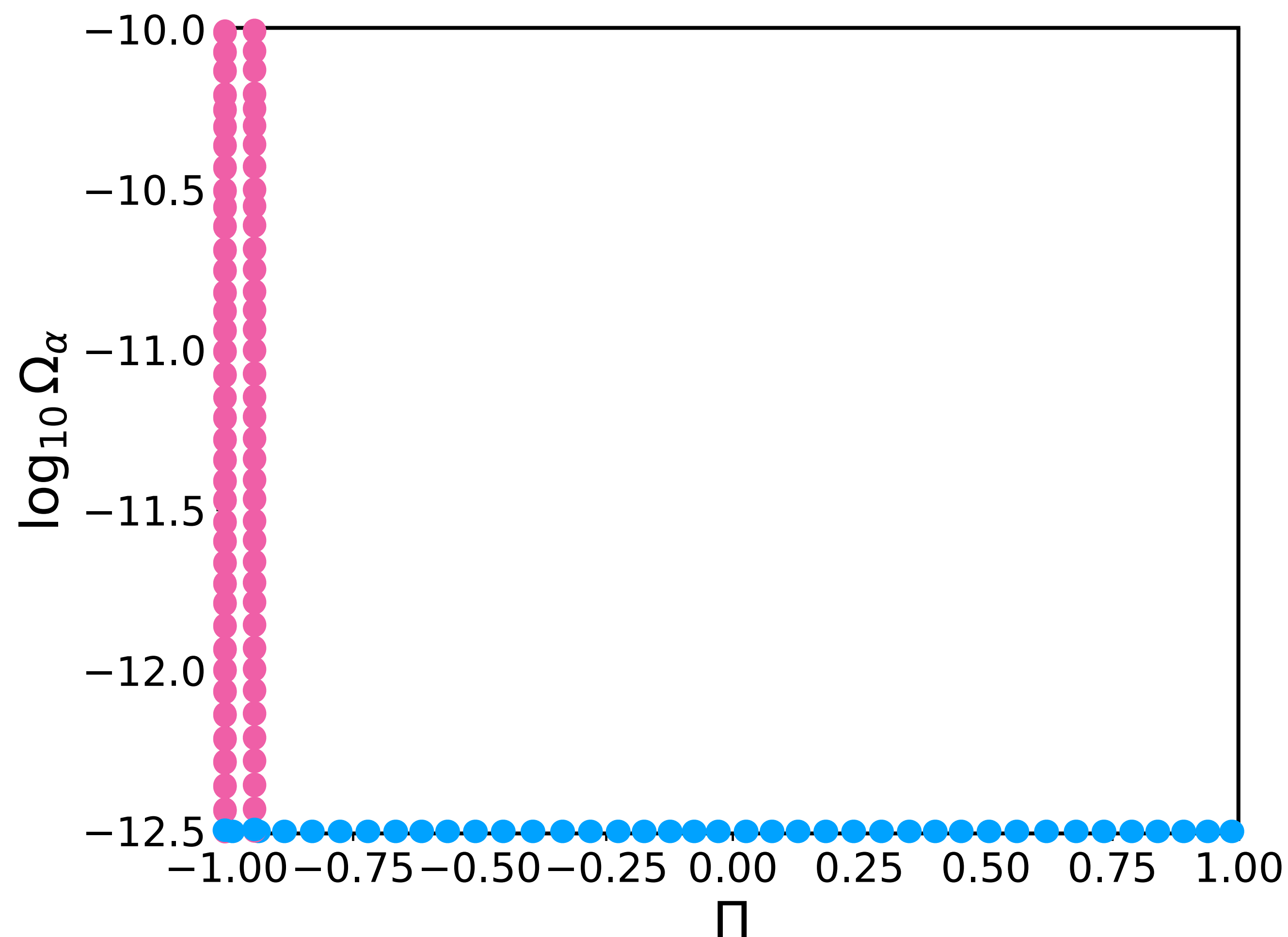
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

3. $\log \Omega_{\alpha}$:
 $-12.5 \leq \log \Omega_{\alpha} \leq -10$
(40 values)

4. Repeat



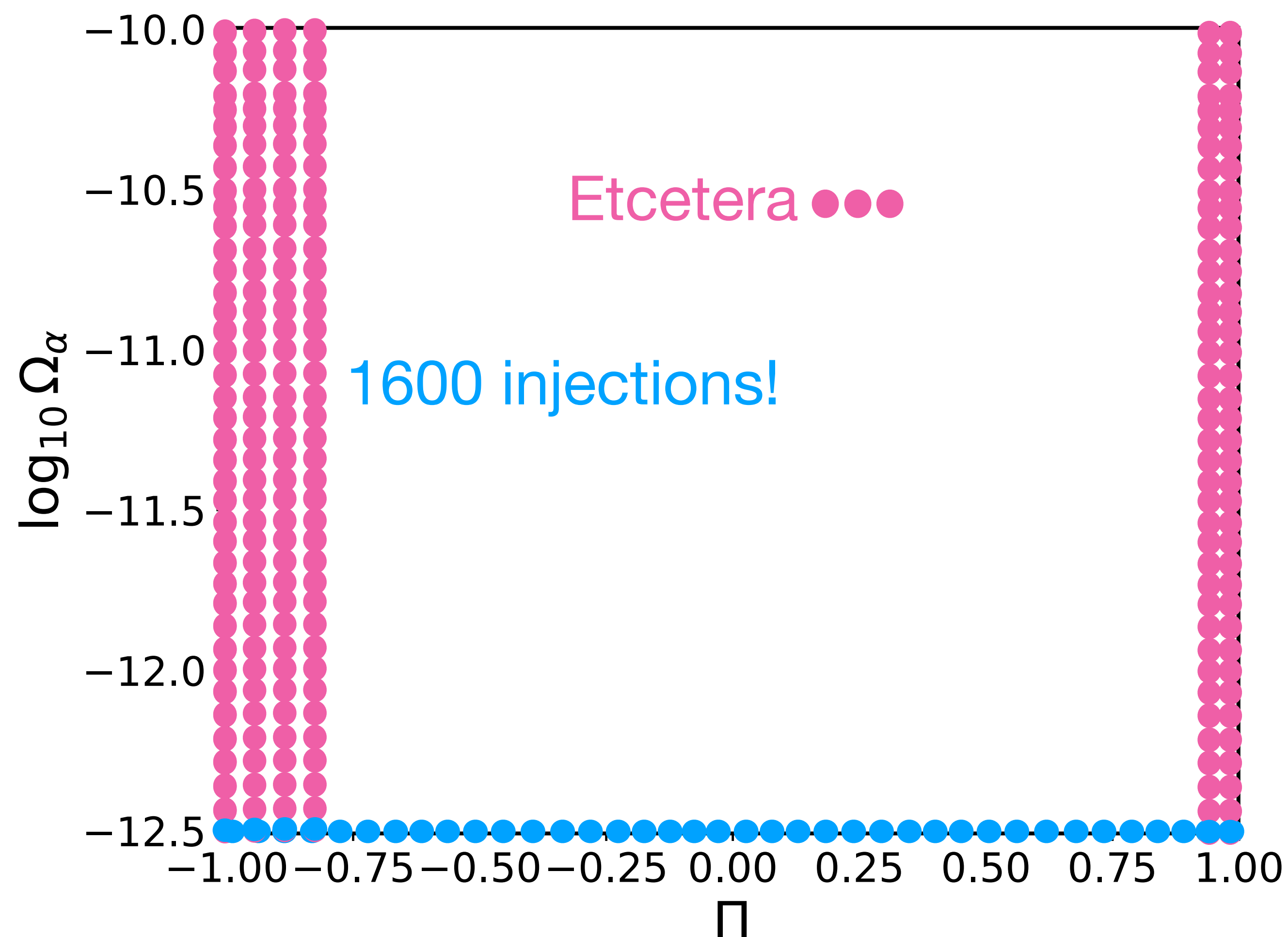
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

3. $\log \Omega_{\alpha}$:
 $-12.5 \leq \log \Omega_{\alpha} \leq -10$
(40 values)

4. Repeat



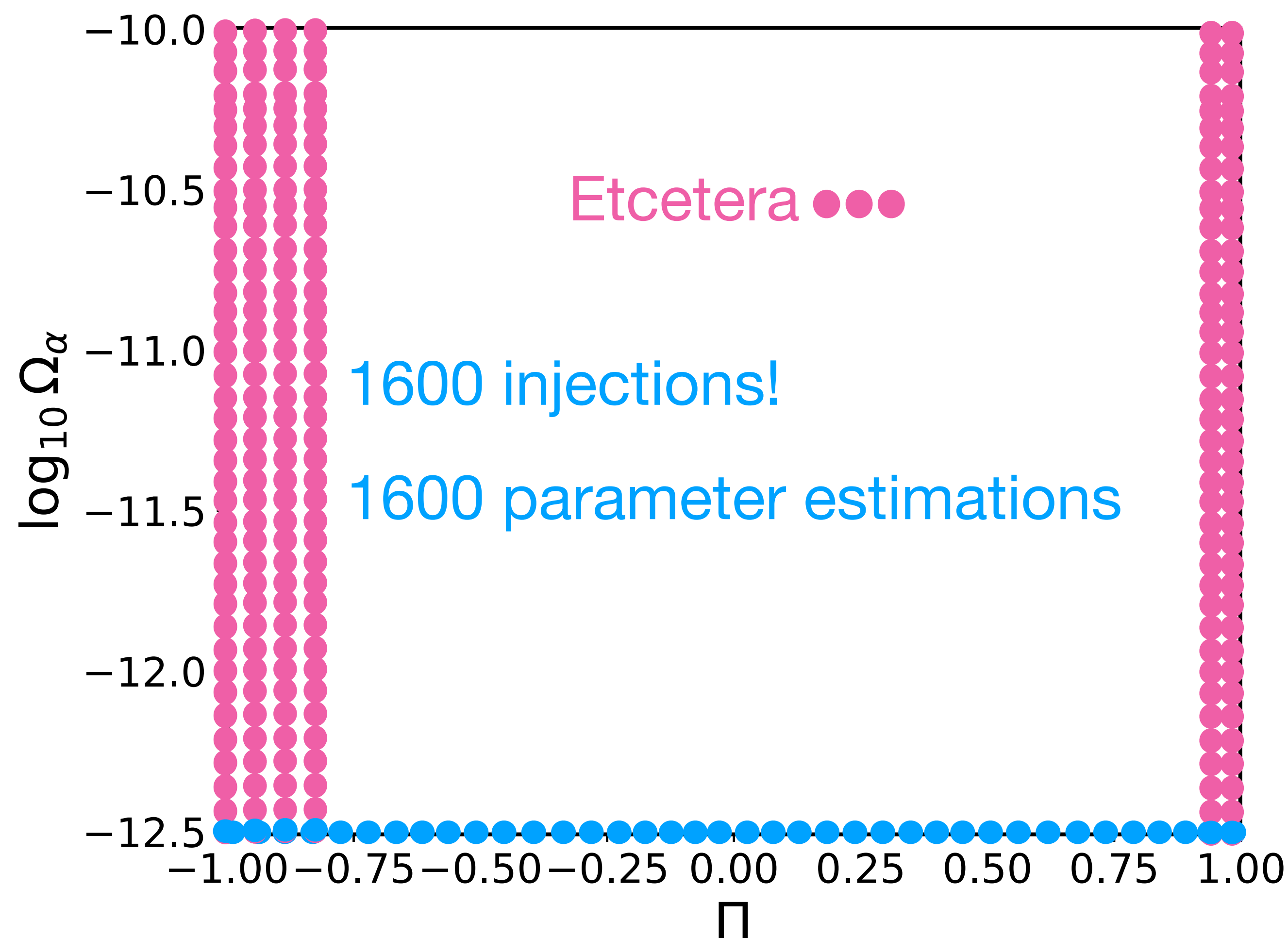
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

3. $\log \Omega_{\alpha}$:
 $-12.5 \leq \log \Omega_{\alpha} \leq -10$
(40 values)

4. Repeat



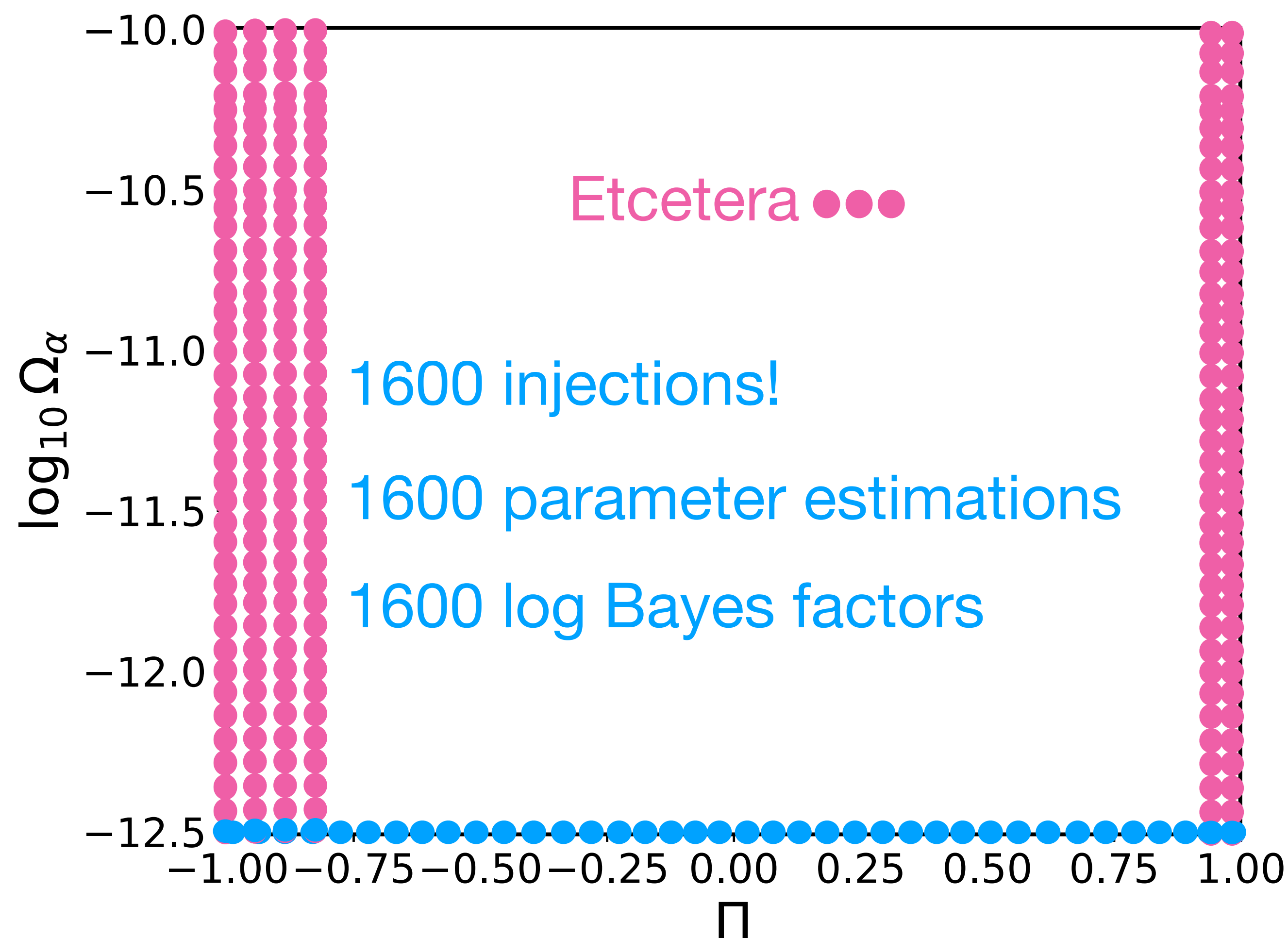
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

3. $\log \Omega_{\alpha}$:
 $-12.5 \leq \log \Omega_{\alpha} \leq -10$
(40 values)

4. Repeat



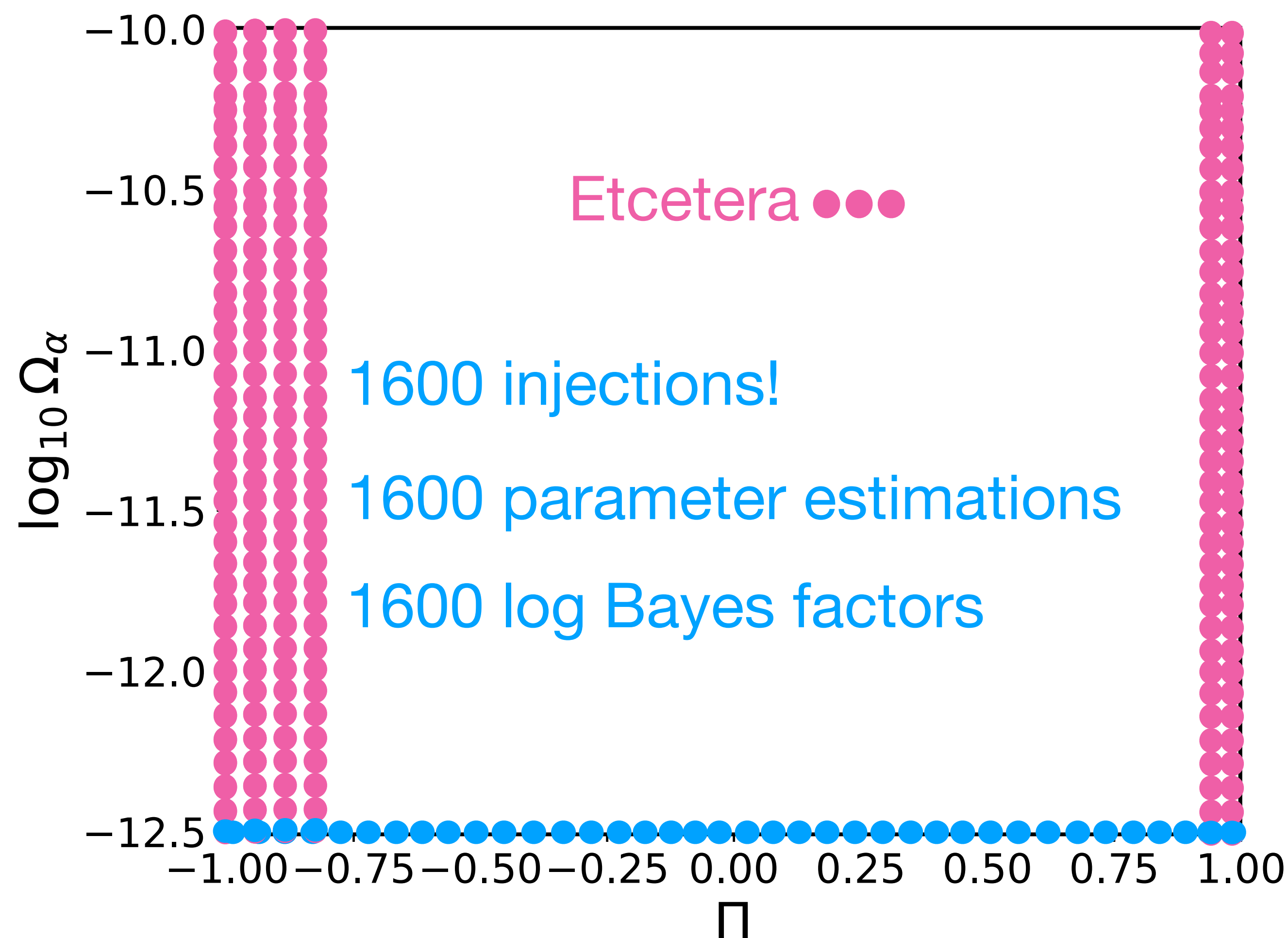
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Interested in points for which $\log \text{Bayes} \geq 8$

Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

3. $\log \Omega_{\alpha}$:
 $-12.5 \leq \log \Omega_{\alpha} \leq -10$
(40 values)

4. Repeat



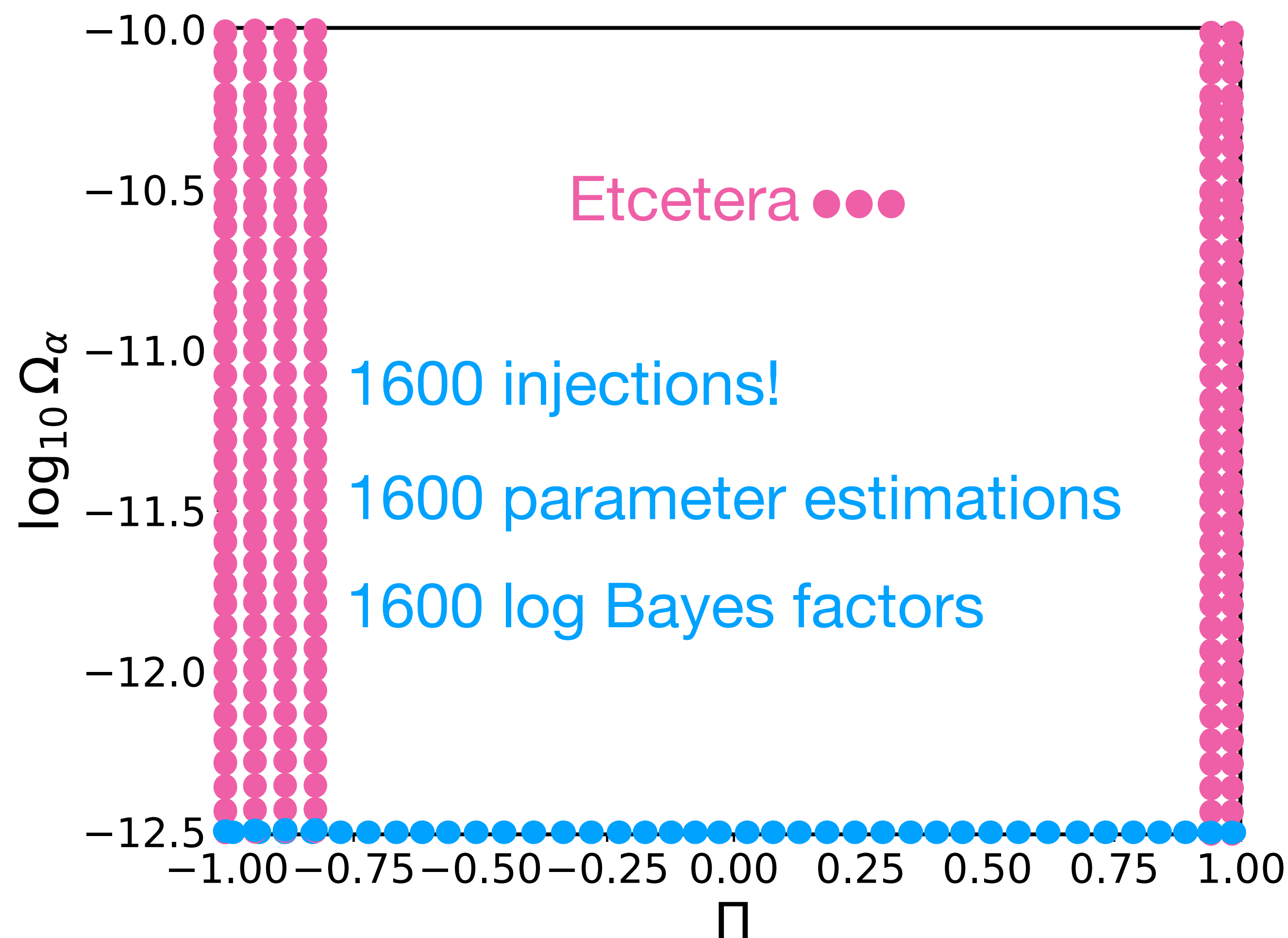
$$f_{\text{ref}} = 25 \text{ Hz}$$

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

Perform
parameter
estimation on
each injection

Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Bayesian Analysis



Interested in points for which log Bayes ≥ 8

Make
injections for:

1. Fix α value per
network & create
random detector
noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

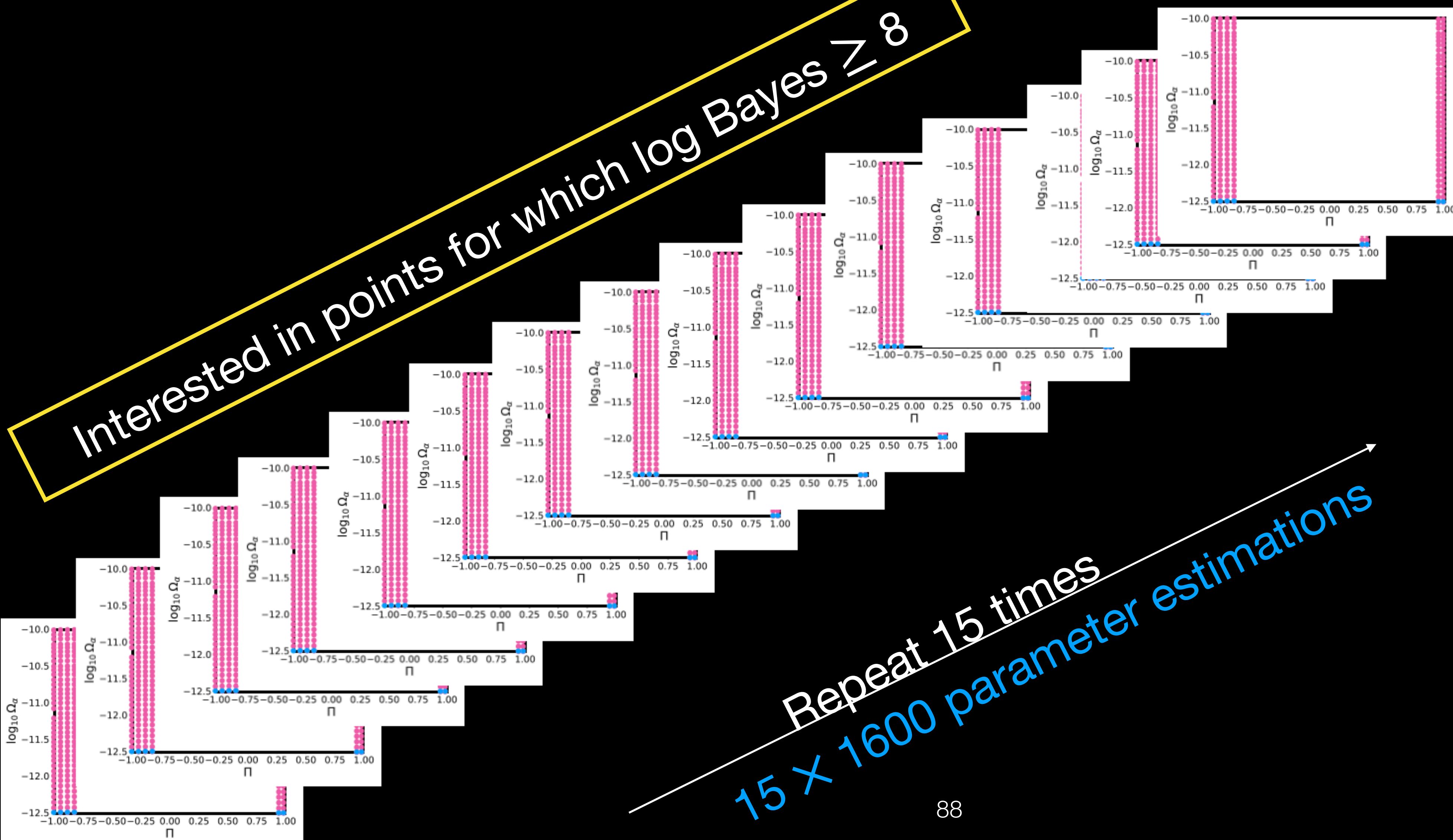
3. $\log \Omega_{\alpha}$:
 $-12.5 \leq \log \Omega_{\alpha} \leq -10$
(40 values)

4. Repeat



Bayesian Analysis

Interested in points for which $\log \text{Bayes} \geq 8$



Repeat 15 times
 15×1600 parameter estimations

Make injections for:

1. Fix α value per network & create random detector noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

3. $\log \Omega_\alpha$:
 $-12.5 \leq \log \Omega_\alpha \leq -10$
(40 values)

4. Repeat



Bayesian Analysis

Make injections for:

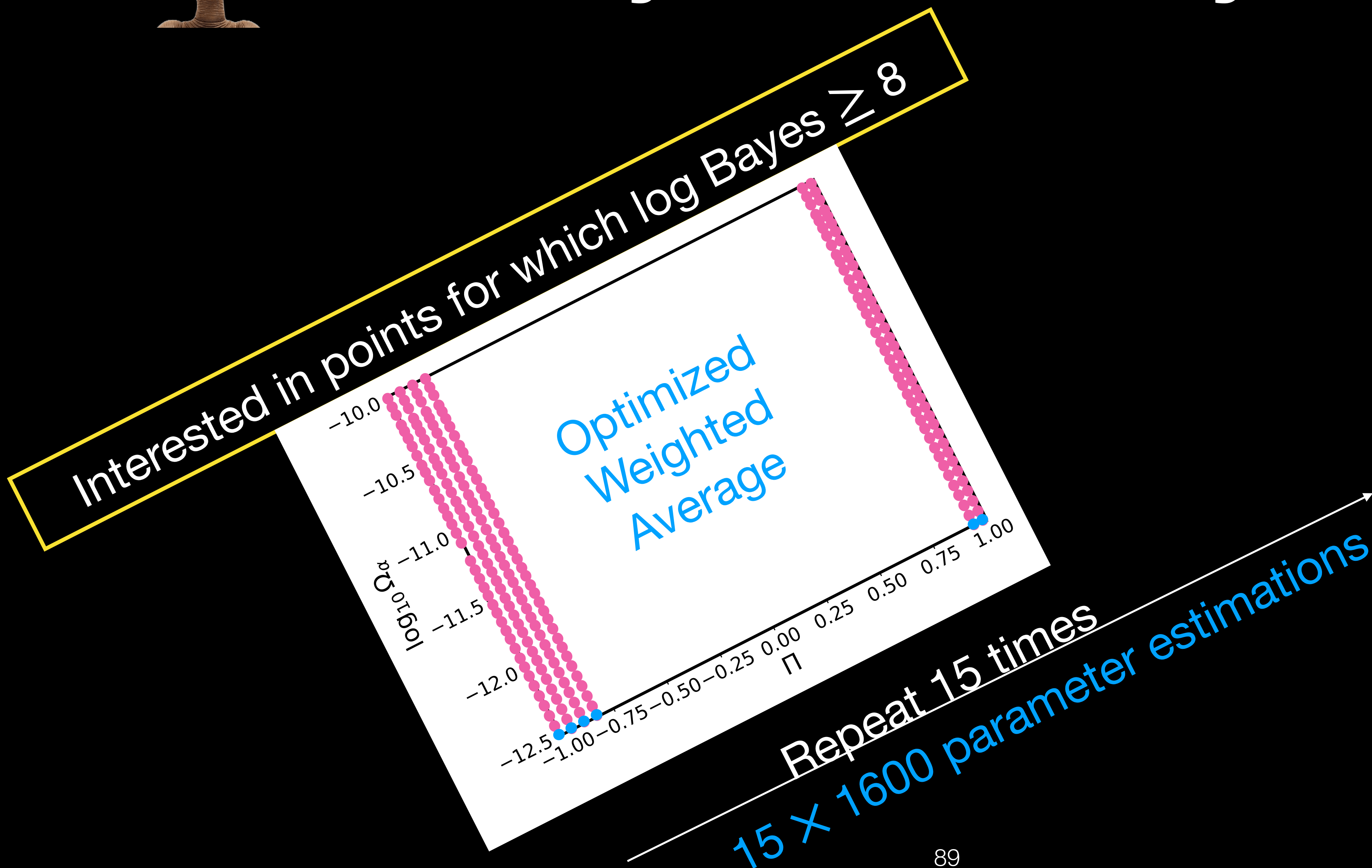
1. Fix α value per network & create random detector noise

2. For each Π :
 $-1 \leq \Pi \leq 1$
(40 values)

3. $\log \Omega_\alpha$:
 $-12.5 \leq \log \Omega_\alpha \leq -10$
(40 values)

4. Repeat

5. Optimized weighted average



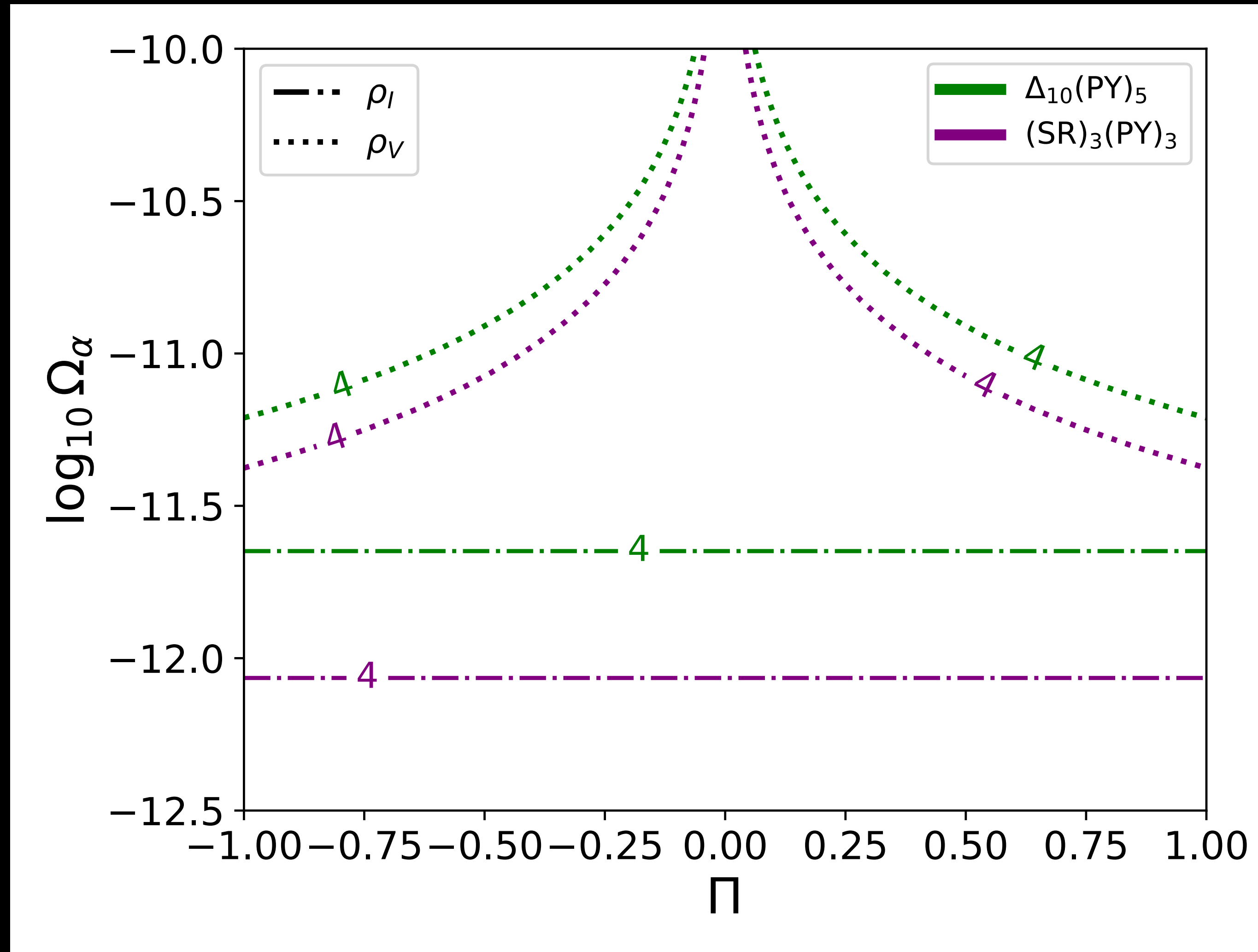
Bayesian Analysis

- Compare SNR findings to data-based analysis!
- Bayes Theorem: $P(\theta | \text{data}) \sim P(\text{data} | \theta)P(\theta)$



Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

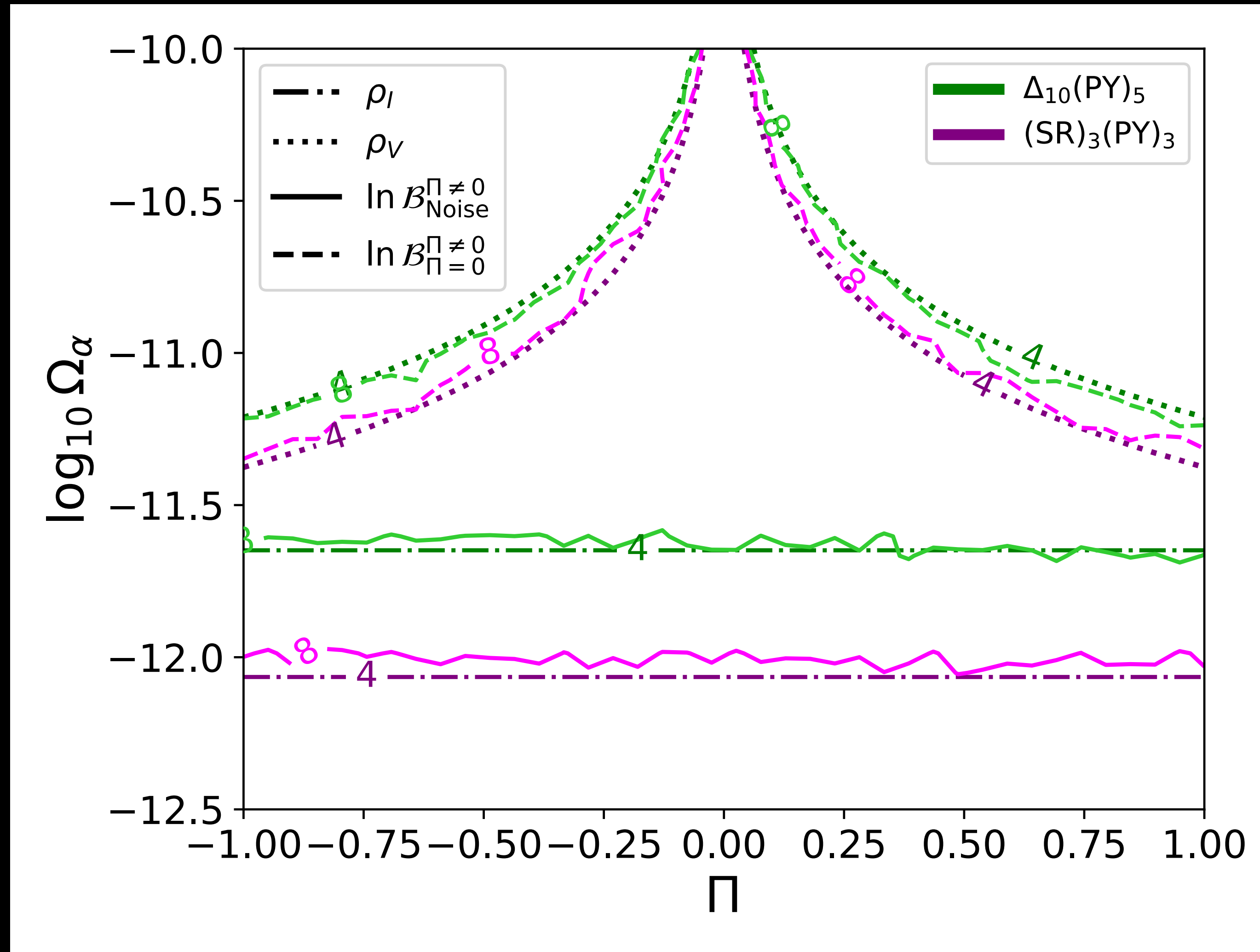
Comparison to Theory



For $\alpha = 0$

For best and worst network configurations

Comparison to Theory



Add log Bayes from data for both networks

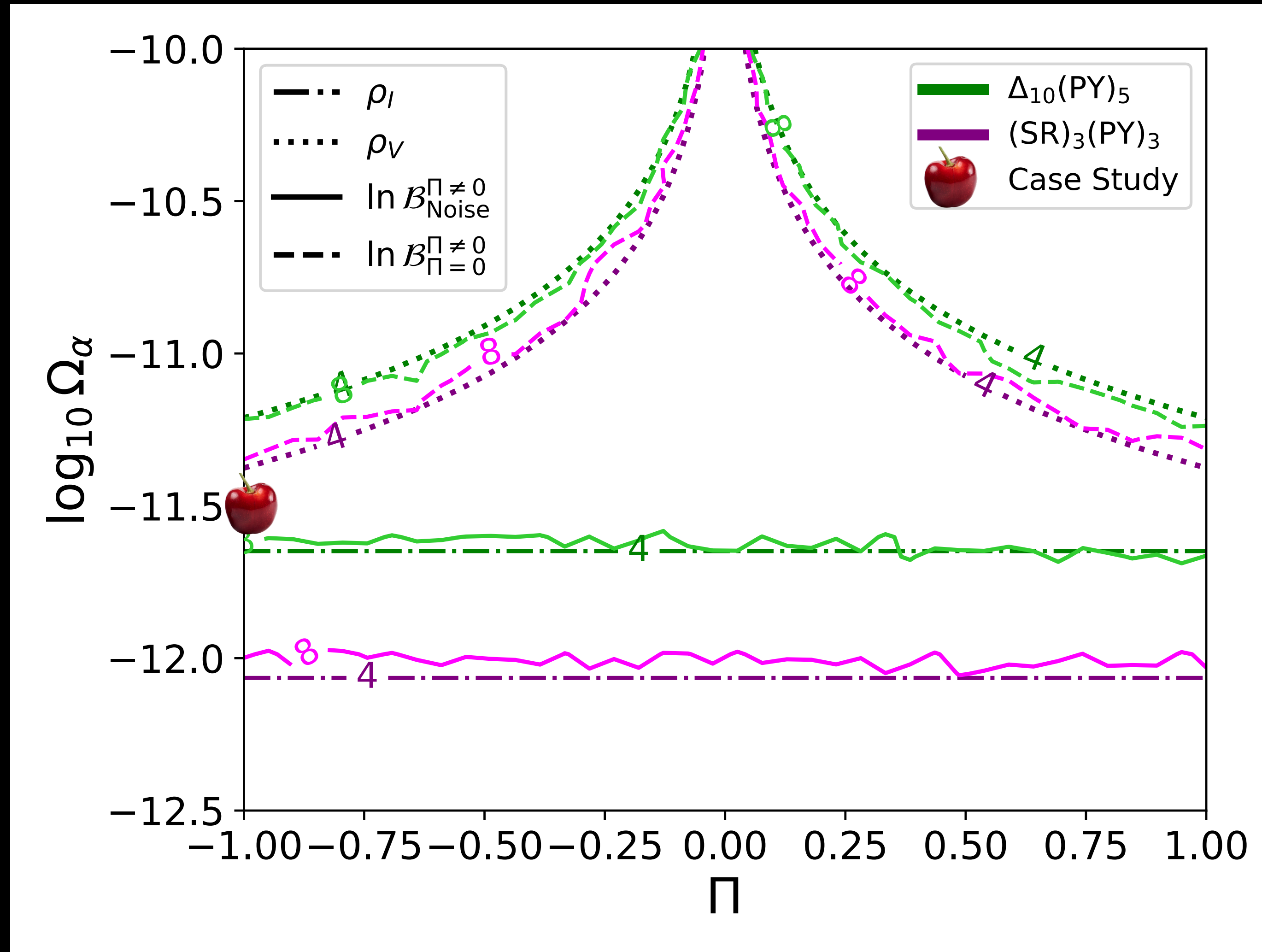
Bayesian Analysis

- Compare SNR findings to data-based analysis!
- Bayes Theorem: $P(\theta | \text{data}) \sim P(\text{data} | \theta)P(\theta)$



Parameters θ	Priors
$\log_{10} \Omega_{\alpha}$	Uniform($-18, -7$)
α	Gaussian($0, 3.5$)
Π	Uniform($-1, 1$)

Injection case



Combined for 15 runs!

Case study parameters:

$$\alpha = 0$$

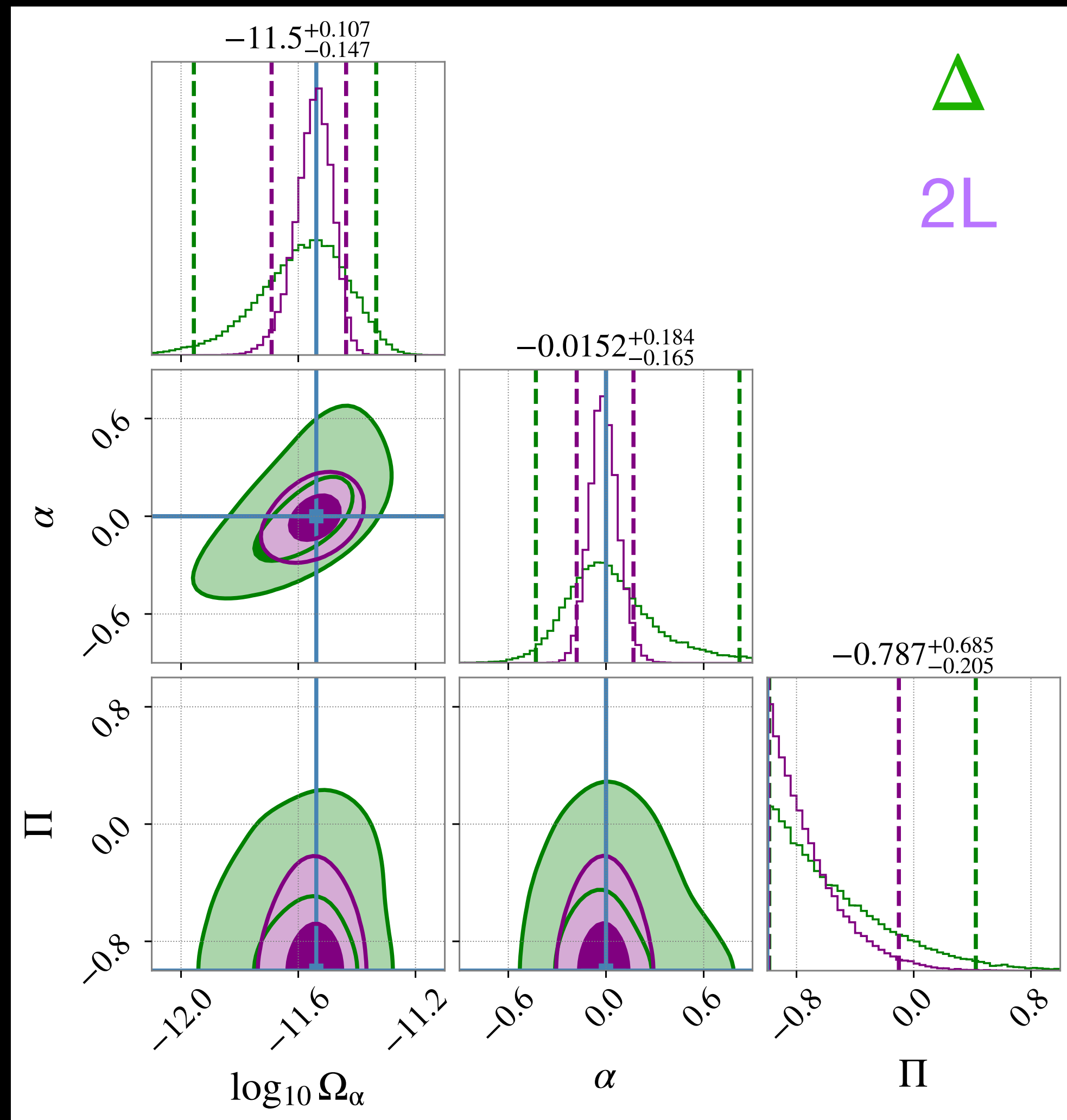
$$\log \Omega_\alpha = -11.54$$

$$\Pi = -1$$

$$\Omega_{\text{GW}}(f) = \Omega_\alpha \left(\frac{f}{f_{\text{ref}}} \right)^\alpha$$

Study specific injection for parameter estimation -> corner plot

Parameter Estimation



We are able to recover parameters within 1σ !

$$95\% \text{ UL } \Pi = 0.233$$

$$\ln \mathcal{B}_{\Pi=0}^{\Pi \neq 0} = 0.946 \pm 0.037$$

$$95\% \text{ UL } \Pi = -0.238$$

$$\ln \mathcal{B}_{\Pi=0}^{\Pi \neq 0} = 2.914 \pm 0.043$$

Case study parameters:

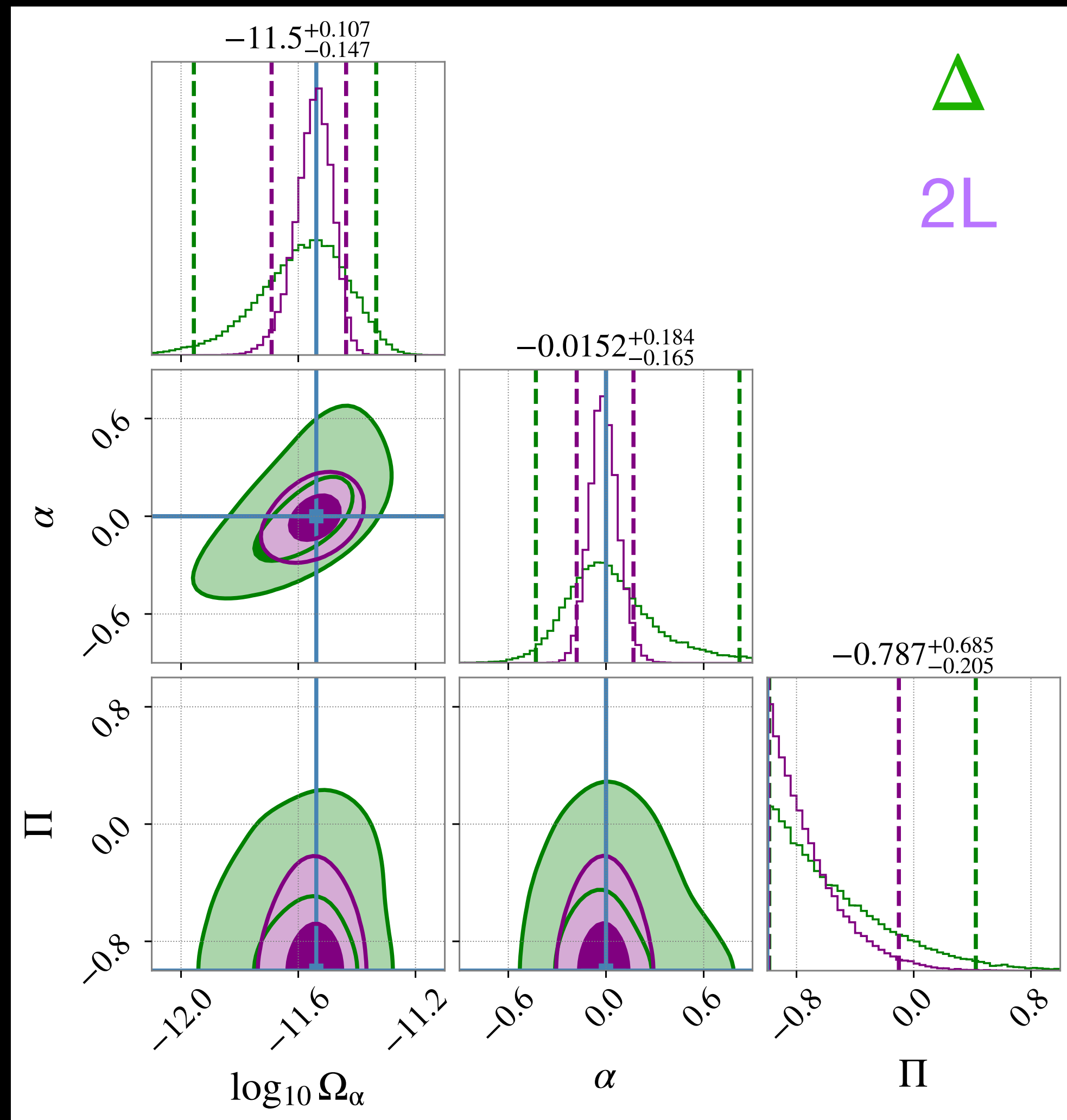
$$\alpha = 0$$

$$\log \Omega_\alpha = -11.54$$

$$\Pi = -1$$

$$\Omega_{\text{GW}}(f) = \Omega_\alpha \left(\frac{f}{f_{\text{ref}}} \right)^\alpha$$

Parameter Estimation



95 % UL $\Pi = -0.238$

95 % UL $\Pi = 0.233$

$\Pi = 0$ ruled out at 95 %

$\Pi = 0$ NOT ruled out at 95 %

Case study
parameters:

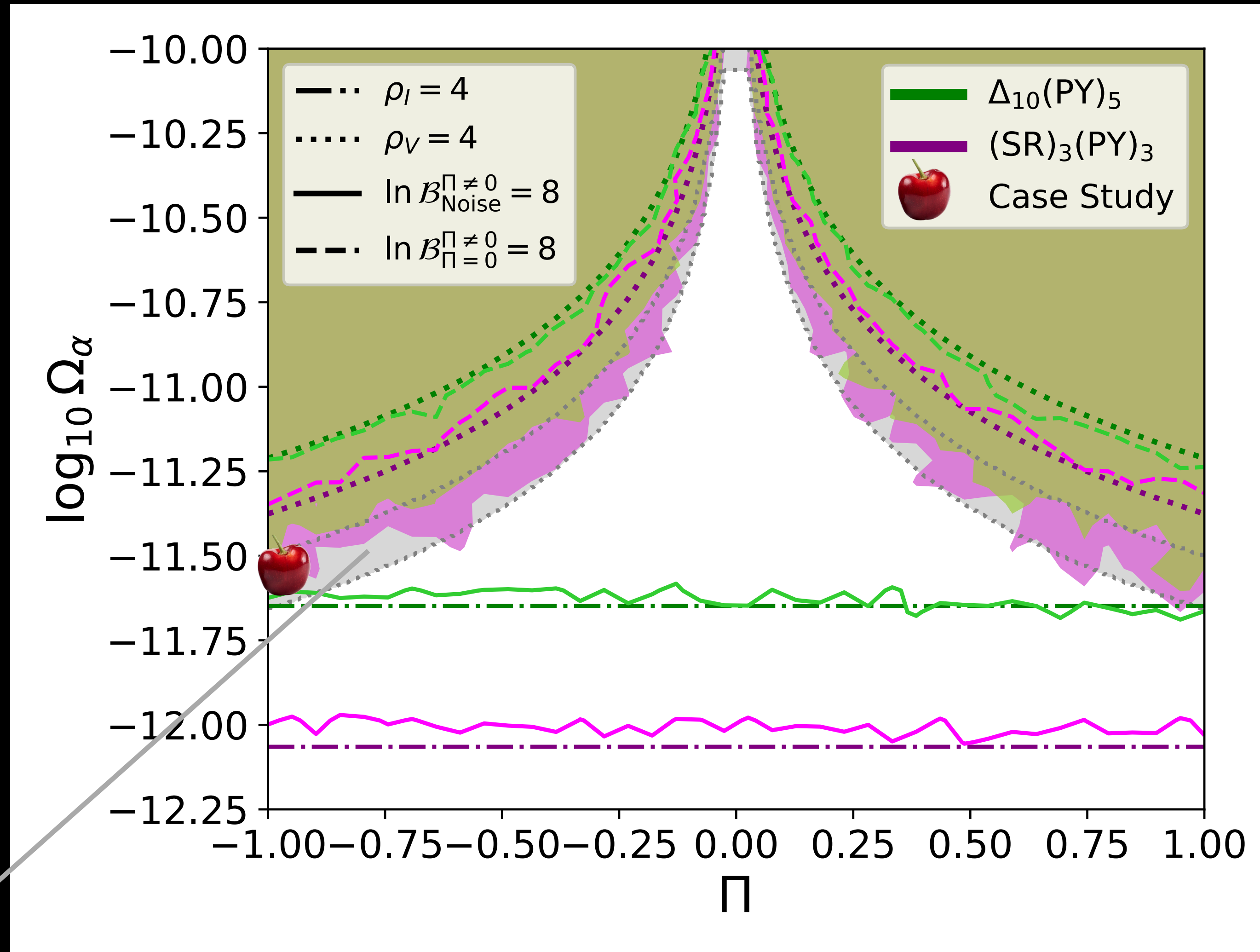
$$\alpha = 0$$

$$\log \Omega_\alpha = -11.54$$

$$\Pi = -1$$

95% exclusion regions for $\Pi = 0$

Combined for 15 runs!



Threshold region:

$$\rho_V^{\Delta_{10}(\text{PY})_5} \leq \rho_V^{95\%, \text{ crit}} \leq \rho_V^{(\text{SR})_3(\text{PY})_3}$$

$$\rho_V^{95\%, \text{ crit}} = 2.16$$

Conclusion

- We use formalism to quantify PV detection prospects.
- Both theoretically developed + tested with data-based formalism.
- Networks with a 2L ET configuration are better to constrain PV than Δ ET.

Detecting PV in GWB

Axion inflation
models

Chern-Simons
term

Quantum
gravity models

Modifications
of GR

Beyond
Standard Model
Physics

Baryogenesis

First-order phase
transitions:
turbulence models



Do YOU have Questions



Back-up slides

Polarization bases

- Linear polarization basis: e_{ij}^+ , $e_{ij}^\times$

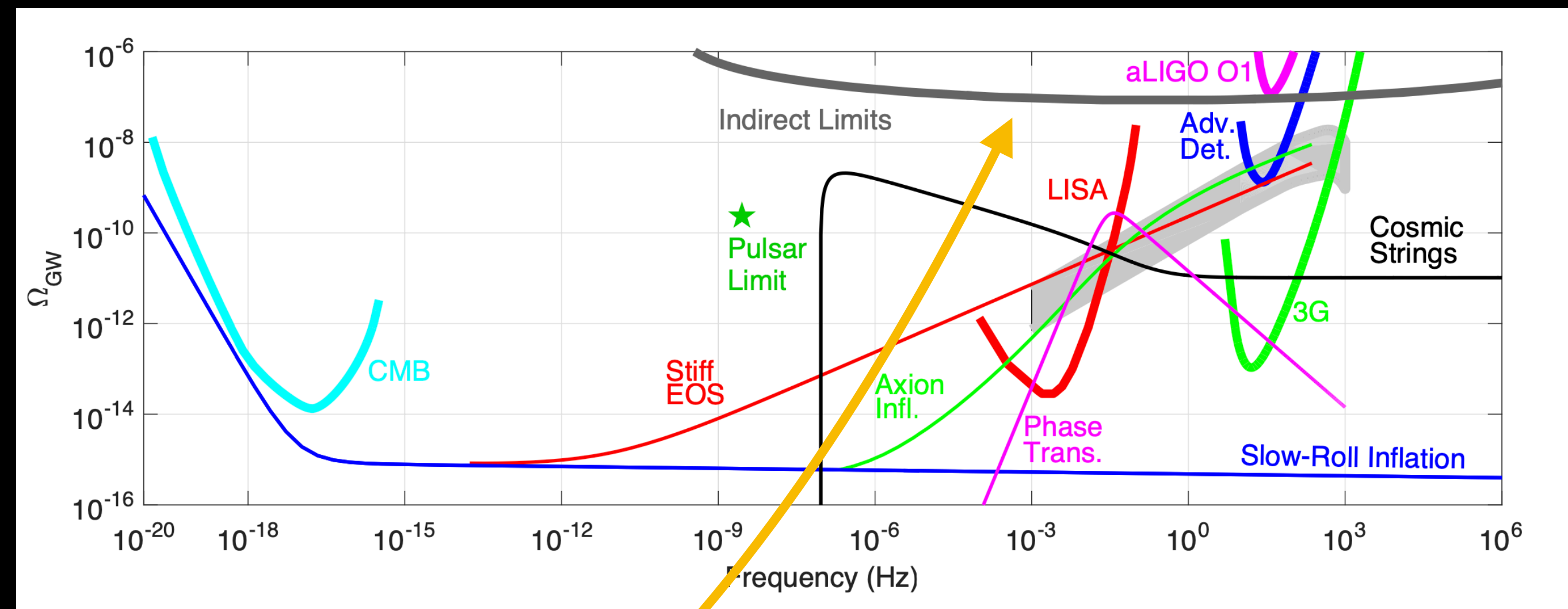
$$h_{ij}(t, \vec{x}) = h^+(t, \vec{x})e_{ij}^+ + h^\times(t, \vec{x})e_{ij}^\times$$

- Circular polarization basis: $e_{ij}^R = \frac{e_{ij}^+ + ie_{ij}^\times}{\sqrt{2}}$, $e_{ij}^L = \frac{e_{ij}^+ - ie_{ij}^\times}{\sqrt{2}}$

$$h_{ij}(t, \vec{x}) = h^R(t, \vec{x})e_{ij}^R + h^L(t, \vec{x})e_{ij}^L$$

Cosmological GWB

- Many different cosmological sources for the GWB, e.g.
 - First Order Phase Transitions - Phys. Rev. Lett. 126, 151301 – Published 16 April 2021
 - Cosmic strings - Phys. Rev. Lett. 126, 241102 – Published 16 June 2021
 - Formation of Primordial Black Holes - Phys. Rev. Lett. 128, 051301 – Published 1 February 2022
 - Stiff Equation of State (EOS) - JCAP 08 (2019) 011
 - Exotic cosmological histories - <https://arxiv.org/abs/2405.10201>



© <https://arxiv.org/abs/1903.09260>

-  Indirect limits from CMB & BBN measurements

Coordinates 3G network

Notation	Name	Origin (lat,lon)	X-arm end (lat,lon)	Y-arm end (lat,lon)	Azimuth [°] X-/Y-arm	ASD used
Δ	E1	43.6300, 10.5000	43.7149, 10.5413	43.6983, 10.4193	19.43 / 319.43	ET10kmcolumns.txt [51]
	E2	43.7200, 10.5500	43.7034, 10.4280	43.6351, 10.5088	259.43 / 199.43	
	E3	43.7000, 10.4200	43.6316, 10.5006	43.7164, 10.5420	139.43 / 79.43	
S	S1	40.5200, 9.4170	40.4950, 9.5909	40.6528, 9.4496	100.60 / 10.60	ET15kmcolumns.txt [51]
	S2		40.5699, 9.5815	40.6454, 9.3513	68.25 / 338.25	
	S3		40.5976, 9.5620	40.6305, 9.3151	54.90 / 324.90	
R	R1	50.7200, 5.9210	50.5865, 5.8910	50.7007, 6.1312	188.14 / 98.14	ET15kmcolumns.txt [51]
	R2		50.5878, 5.8794	50.6933, 6.1291	191.33 / 101.33	
	R3		50.5868, 5.9542	50.7409, 6.1309	170.99 / 80.99	
P	P1	46.0000, -125.0000	45.8257, -124.9361	46.0457, -124.7502	165.62 / 75.19	cosmic_explorer_20km_strain.txt [52]
	P2		45.8202, -125.0085	45.9937, -124.7420	181.90 / 91.90	
	P3		45.8246, -124.9426	46.0399, -124.7481	167.11 / 77.11	
	P4		45.8290, -124.9197	46.0559, -124.7545	161.82 / 71.82	
	P5		45.8202, -124.9909	46.0061, -124.7419	177.98 / 87.98	
Y	Y1	29.0000, -94.0000	29.3596, -93.9654	29.0297, -94.4092	4.82 / 274.82	cosmic_explorer_strain.txt [52]
	Y2		29.1152, -93.6108	29.3417, -94.1321	71.29 / 341.29	
	Y3		29.0526, -93.5938	29.3569, -94.0607	81.52 / 351.52	
	Y4		28.9921, -93.5896	29.3608, -93.9917	91.15 / 1.15	
	Y5		29.1225, -93.6136	29.3392, -94.1405	70.06 / 340.06	

ORFs

- Normalized ORFs

$$\Omega'_{\text{GW}}(f)\gamma_I(f) = \Omega_{\text{GW}}(f) [\gamma_I(f) + \Pi(f)\gamma_V(f)] \quad (5)$$

$$\gamma_I(f) = \frac{5}{8\pi} \int d\hat{\Omega} (F_1^+ F_2^{+*} + F_1^\times F_2^{\times*}) e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x}}$$

$$\gamma_V(f) = -\frac{5}{8\pi} \int d\hat{\Omega} i (F_1^+ F_2^{\times*} - F_1^\times F_2^{+*}) e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x}}.$$

Not-normalized ORFs: $\Gamma_A = \frac{\sin \theta_a \sin \theta_b}{5} \gamma_A$ with $A = I, V$

SNR

- Non-PV GWBs:

$$\text{SNR}^2 = \rho^2 = 2T \left(\frac{3H_0^2}{10\pi^2} \right)^2 \int_0^\infty df \frac{\Omega_{\text{GW}}^2(f)}{f^6} \sum_{(a,b)} \theta_{ab}^2 \frac{(\gamma_{I,ab})^2}{N_a N_b}$$

- PV GWBs:

$$\text{SNR}^2 = \rho^2 = 2T \left(\frac{3H_0^2}{10\pi^2} \right)^2 \int_0^\infty df \frac{\Omega_{\text{GW}}^2(f)}{f^6} \sum_{(a,b)} \theta_{ab}^2 \frac{(\gamma_{I,ab} + \Pi \gamma_{V,ab})^2}{N_a N_b}$$

PI curves

- $$\rho^2 = 2TC^2 \int_0^\infty df \frac{\Omega_{\text{GW}}^2(f)}{f^6} \sum_{(a,b)} \theta_{ab}^2 \frac{(\gamma_{I,ab} + \Pi \gamma_{V,ab})^2}{N_a N_b}$$

- $$\Omega_{\text{GW}} = \Omega_\beta \left(\frac{f}{f_{\text{ref}}} \right)^\beta$$

- $$\Rightarrow \rho^2 = 2TC^2 \Omega_\beta^2 \int_0^\infty df \left(\frac{f}{f_{\text{ref}}} \right)^{2\beta} \frac{1}{f^6} \sum_{(a,b)} \theta_{ab}^2 \frac{(\gamma_{I,ab} + \Pi \gamma_{V,ab})^2}{N_a N_b}$$

- $$\Omega_\beta^{(\text{total})} = \frac{\rho_{\text{target}}}{\sqrt{2T} C} \left[\int_{f_{\text{min}}}^{f_{\text{max}}} df \frac{(f/f_{\text{ref}})^{2\beta}}{f^6} \theta_{ab}^2 (\gamma_{II}(f) + 2\Pi(f)\gamma_{IV}(f) + \Pi^2(f)\gamma_{VV}(f)) \right]^{-1/2}$$

- $$\Omega_{\text{PI}}(f) = \max_\beta \left[\Omega_\beta \left(\frac{f}{f_{\text{ref}}} \right)^\beta \right]$$

With

$$C \equiv \frac{3H_0^2}{10\pi^2}$$

N_a detector noise PSD

$$\Gamma_{\text{eff}}(f) \equiv \Gamma_{II}(f)\Gamma_{VV}(f) - \Gamma_{IV}^2(f)$$

$$\Gamma_{AB}(f) \equiv \sum_{(a,b)} \theta_{ab}^2 \frac{\gamma_{A,ab}(f)\gamma_{B,ab}(f)}{N_a(f)N_b(f)}$$

with $A, B = I, V$

$$\theta_{ab} \equiv \frac{\sin \theta_a \sin \theta_b}{5}$$

PI curves- I modes

- $$\rho_I^2 = 2TC^2 \int_{f_{\min}}^{f_{\max}} df \frac{\Omega_{\text{GW}}^2(f)}{f^6} \frac{\Gamma_{\text{eff}}(f)}{\Gamma_{VV}(f)}$$
- $$\Rightarrow \rho_I^2 = 2TC^2 \Omega_\beta^2 \int_{f_{\min}}^{f_{\max}} df \left(\frac{f}{f_{\text{ref}}}\right)^{2\beta} \frac{1}{f^6} \frac{\Gamma_{\text{eff}}(f)}{\Gamma_{VV}(f)}$$
- $$\Omega_\beta^I = \frac{\rho_{\text{target}}}{\sqrt{2T} C} \left[\int_{f_{\min}}^{f_{\max}} df \frac{(f/f_{\text{ref}})^{2\beta}}{f^6} \frac{\Gamma_{\text{eff}}(f)}{\Gamma_{VV}(f)} \right]^{-1/2}$$
- $$\Omega_{\text{PI}}(f) = \max_\beta \left[\Omega_\beta \left(\frac{f}{f_{\text{ref}}}\right)^\beta \right]$$

With

$$C \equiv \frac{3H_0^2}{10\pi^2}$$

N_a detector noise PSD

$$\Gamma_{\text{eff}}(f) \equiv \Gamma_{II}(f)\Gamma_{VV}(f) - \Gamma_{IV}^2(f)$$

$$\Gamma_{AB}(f) \equiv \sum_{(a,b)} \theta_{ab}^2 \frac{\gamma_{A,ab}(f)\gamma_{B,ab}(f)}{N_a(f)N_b(f)}$$

with $A, B = I, V$

$$\theta_{ab} \equiv \frac{\sin \theta_a \sin \theta_b}{5}$$

PI curves- V-modes

- $\rho_V^2 = 2TC^2 \int_0^\infty df \frac{\Omega_{\text{GW}}^2(f)}{f^6} \Pi^2(f) \frac{\Gamma_{\text{eff}}(f)}{\Gamma_{II}(f)}$

- $\Rightarrow \rho_V^2 = 2TC^2 \Omega_\beta^2 \int_{f_{\min}}^{f_{\max}} df \left(\frac{f}{f_{\text{ref}}}\right)^{2\beta} \frac{\Pi^2(f)}{f^6} \frac{\Gamma_{\text{eff}}(f)}{\Gamma_{II}(f)}$

- $\Omega_\beta^V = \frac{\rho_{\text{target}}}{\sqrt{2T} C} \left[\int_{f_{\min}}^{f_{\max}} df \frac{(f/f_{\text{ref}})^{2\beta}}{f^6} \Pi^2(f) \frac{\Gamma_{\text{eff}}(f)}{\Gamma_{II}(f)} \right]^{-1/2}$

- For $\Pi=\text{constant}$: $\Omega_\beta^V = \frac{\rho_{\text{target}}}{|\Pi| \sqrt{2T} C} \left[\int_{f_{\min}}^{f_{\max}} df \frac{(f/f_{\text{ref}})^{2\beta}}{f^6} \frac{\Gamma_{\text{eff}}(f)}{\Gamma_{II}(f)} \right]^{-1/2}$

- $\Omega_{\text{PI}}(f) = \max_\beta \left[\Omega_\beta \left(\frac{f}{f_{\text{ref}}}\right)^\beta \right]$

With

$$C \equiv \frac{3H_0^2}{10\pi^2}$$

N_a detector noise PSD

$$\Gamma_{\text{eff}}(f) \equiv \Gamma_{II}(f)\Gamma_{VV}(f) - \Gamma_{IV}^2(f)$$

$$\Gamma_{AB}(f) \equiv \sum_{(a,b)} \theta_{ab}^2 \frac{\gamma_{A,ab}(f)\gamma_{B,ab}(f)}{N_a(f)N_b(f)}$$

with $A, B = I, V$

$$\theta_{ab} \equiv \frac{\sin \theta_a \sin \theta_b}{5}$$

Full SNR results

α	$-3, -2$	$-1, 0$	$\frac{1}{2}, \frac{2}{3}, 1, 2$	3
1st	$(\text{SR})_1(\text{PY})_1$	$(\text{SR})_1(\text{PY})_1$	$(\text{SR})_1(\text{PY})_1$	$\Delta_{15}(\text{PY})_4$
2nd	$(\text{SR})_3(\text{PY})_3$	$\Delta_{15}(\text{PY})_4$	$\Delta_{15}(\text{PY})_4$	$\Delta_{15}(\text{PY})_5$
3rd	$\Delta_{15}(\text{PY})_4$	$(\text{SR})_3(\text{PY})_3$	$\Delta_{10}(\text{PY})_4$	$\Delta_{10}(\text{PY})_4$
4th	$\Delta_{15}(\text{PY})_5$	$\Delta_{10}(\text{PY})_4$	$(\text{SR})_3(\text{PY})_3$	$\Delta_{10}(\text{PY})_5$
5th	$\Delta_{10}(\text{PY})_4$	$\Delta_{15}(\text{PY})_5$	$\Delta_{15}(\text{PY})_5$	$(\text{SR})_1(\text{PY})_1$
6th	$(\text{SR})_2(\text{PY})_2$	$(\text{SR})_2(\text{PY})_2$	$\Delta_{10}(\text{PY})_5$	$(\text{SR})_3(\text{PY})_3$
7th	$\Delta_{10}(\text{PY})_5$	$\Delta_{10}(\text{PY})_5$	$(\text{SR})_2(\text{PY})_2$	$(\text{SR})_2(\text{PY})_2$

Ranking of I-mode SNR

α	$-3 \text{ to } 1$	$2, 3$
1st	$(\text{SR})_3(\text{PY})_3$	$(\text{SR})_1(\text{PY})_1$
2nd	$(\text{SR})_1(\text{PY})_1$	$(\text{SR})_3(\text{PY})_3$
3rd	$\Delta_{15}(\text{PY})_4$	$(\text{SR})_2(\text{PY})_2$
4th	$(\text{SR})_2(\text{PY})_2$	$\Delta_{15}(\text{PY})_4$
5th	$\Delta_{15}(\text{PY})_5$	$\Delta_{15}(\text{PY})_5$
6th	$\Delta_{10}(\text{PY})_4$	$\Delta_{10}(\text{PY})_4$
7th	$\Delta_{10}(\text{PY})_5$	$\Delta_{10}(\text{PY})_5$

Ranking of V-mode SNR

Different configurations: ET only

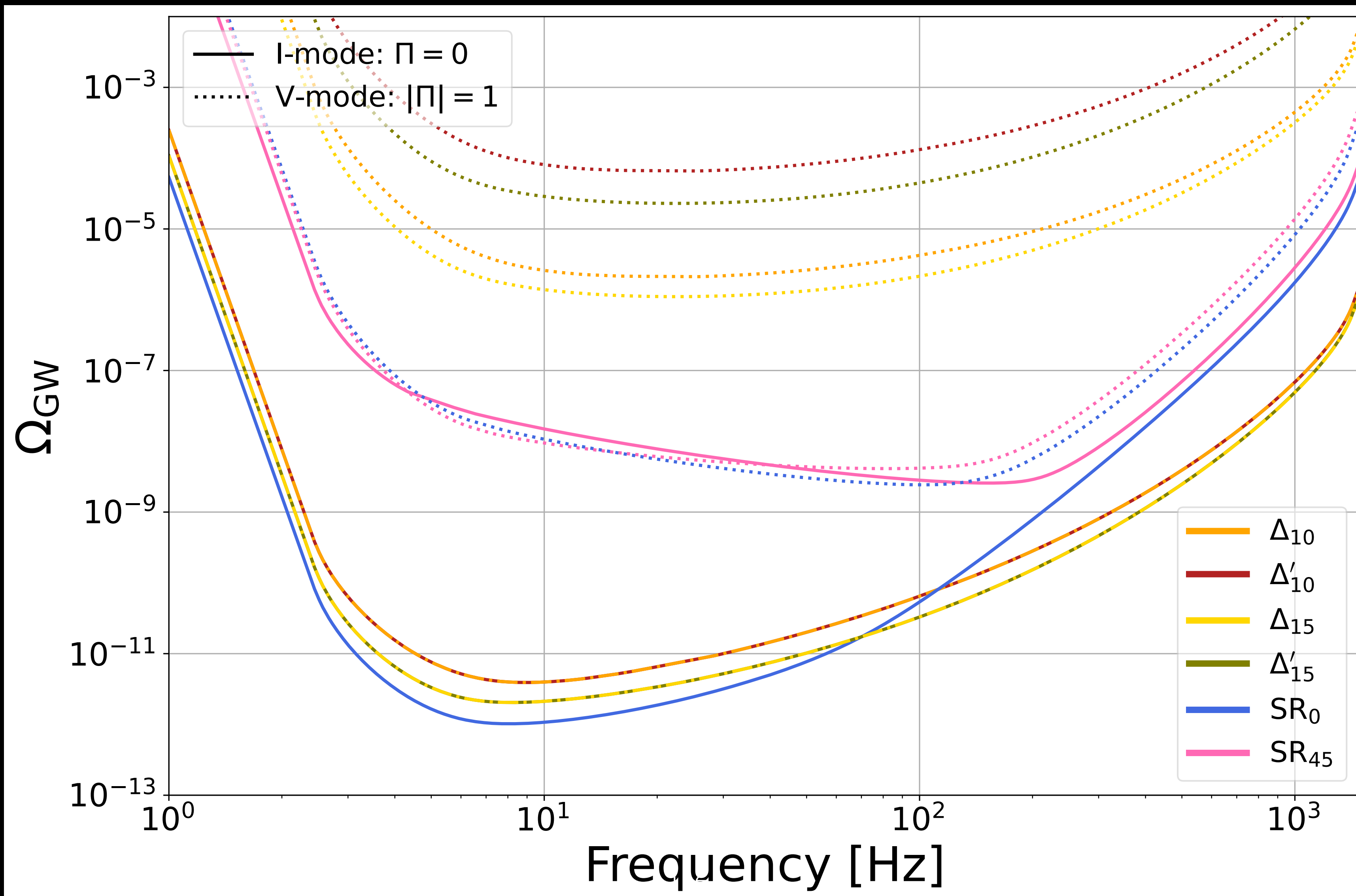
- Third-generation configurations:
 - Δ_{10} : 10 km Δ , same as before
 - Δ_{15} : 15 km Δ , same as before
 - Δ'_{10} : 10 km Δ , fully symmetric and closed
 - Δ'_{15} : 15 km Δ , fully symmetric and closed
 - $(SR)_0$: 15 km 2L, GWB-optimized: orientation angle 0 degrees
 - $(SR)_{45}$: 15 km 2L, Localization-optimized: : orientation angle 45 degrees

Coordinates

Name	Origin (lat, lon)	X-arm end (lat, lon)	Y-arm end (lat, lon)	Azimuth [°] X-/Y-arm
E1' ₁₀	43.6300, 10.5000	43.7149, 10.5413	43.6983, 10.4193	19.43 /319.43
E2' ₁₀	43.7149, 10.5413	43.6983, 10.4193	43.6300, 10.5000	259.46 /199.46
E3' ₁₀	43.6983, 10.4193	43.6300, 10.5000	43.7149, 10.5413	139.37 /79.37
E1' ₁₅	43.6300, 10.5000	43.7573, 10.5620	43.7325, 10.3789	19.43 /319.43
E2' ₁₅	43.7573, 10.5620	43.7325, 10.3789	43.6300, 10.5000	259.47 /199.47
E3' ₁₅	43.7325, 10.3789	43.6300, 10.5000	43.75730, 10.5620	139.35/79.35
S ₀	40.5200, 9.4170	40.3851, 9.4093	40.5258, 9.2402	182.51/272.51
S ₄₅	40.5200, 9.4170	40.4203, 9.5364	40.4287, 9.2867	137.51 /227.51
R ₀ =R ₄₅	50.7200, 5.9210	50.8548, 5.9210	50.7198, 6.1334	0.00/90.00

TABLE V: ET coordinates and azimuths (clockwise from North).

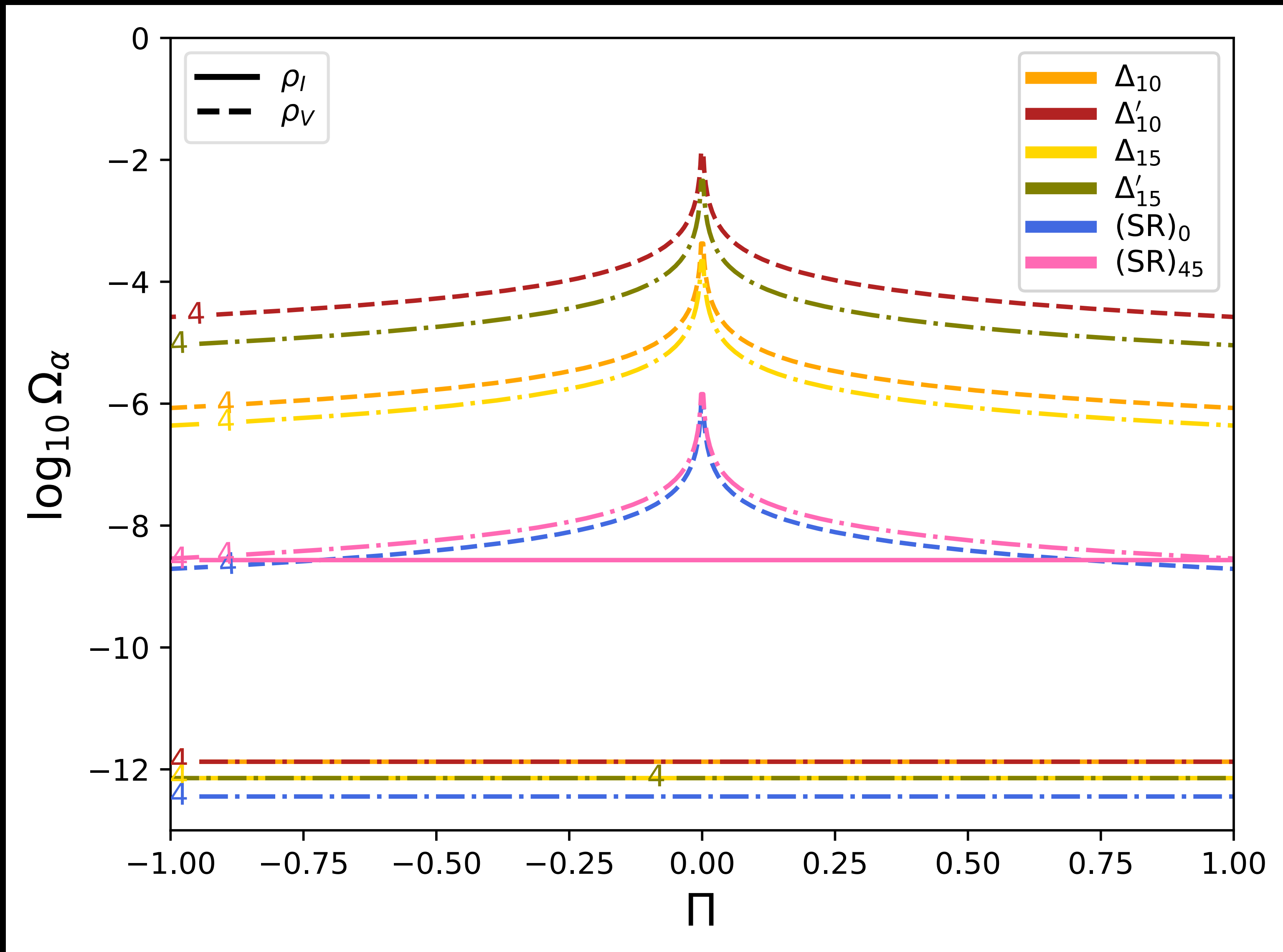
PI curves: ET only



For $\rho = 4$

$T_{\text{obs}} = 1 \text{ year}$

SNR prospects: ET only



- For $\alpha = 0$

Full SNR results

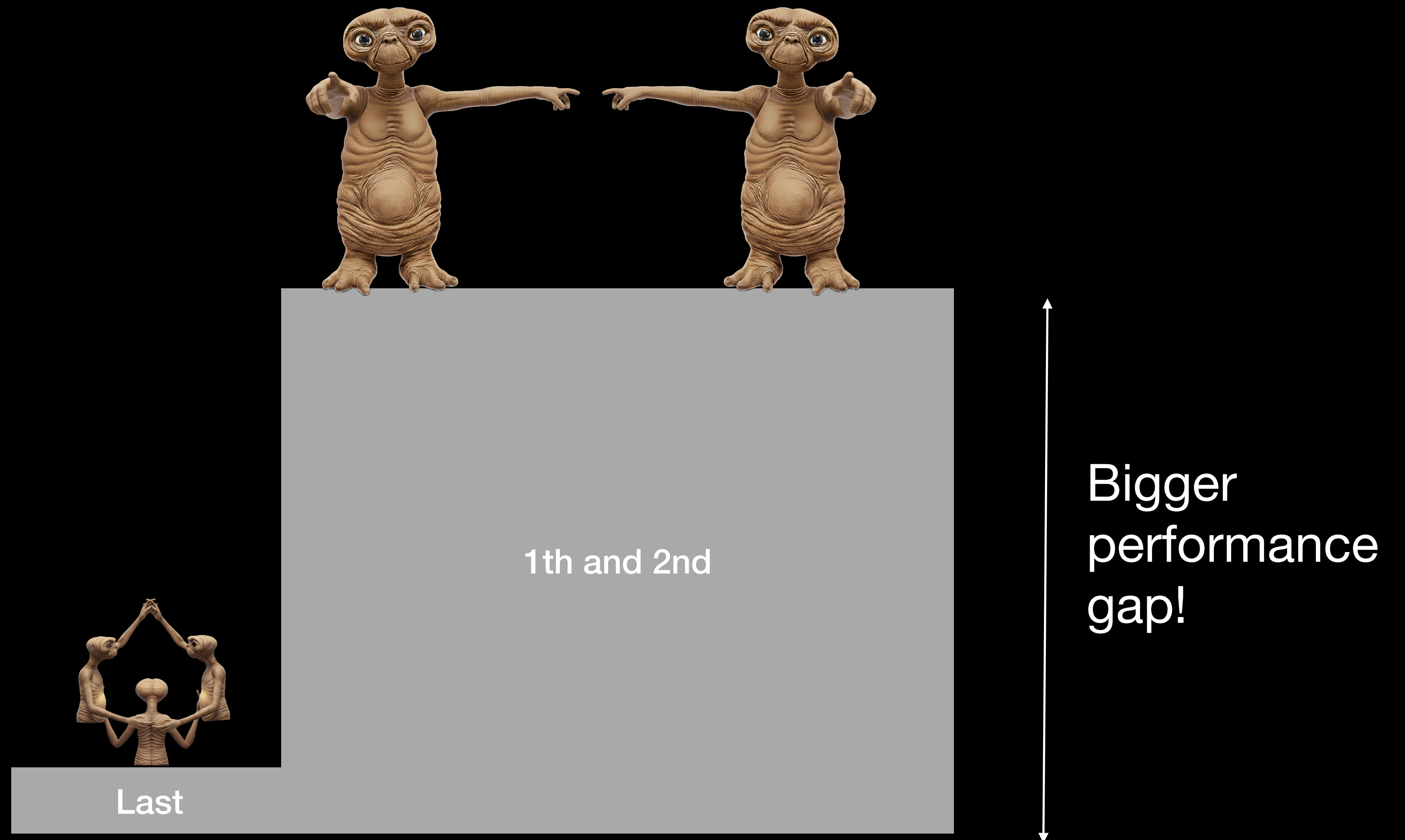
1st	$(\text{SR})_0$
2nd	$(\text{SR})_{45}$
3rd	$\Delta_{15} = \Delta'_{15}$
4th	$\Delta_{10} = \Delta'_{10}$

Ranking of I-mode SNR

α	$-3, -2, -1$	$0, \frac{1}{2}, 1, \frac{2}{3}, 2, 3$
1st	$(\text{SR})_{45}$	$(\text{SR})_0$
2nd	$(\text{SR})_0$	$(\text{SR})_{45}$
3rd	Δ_{15}	Δ_{15}
4th	Δ_{10}	Δ_{10}
5th	Δ'_{15}	Δ'_{15}
6th	Δ'_{10}	Δ'_{10}

Ranking of V-mode SNR

Repeat for ET only



How to quantify PV detection prospects?

- SNR: $\rho_A = \sqrt{2T} \left[\int_{f_{\min}}^{f_{\max}} df \frac{\Omega_{\text{GW}}^2(f)}{\Omega_{\text{eff},A}^2(f)} \right]^{1/2}$
- With $\Omega_{\text{eff},A}(f) = \frac{2\pi^2}{3H_0^2} f^3 \left[\frac{G_A(f)}{\Gamma_{\text{eff}}^{II}(f)\Gamma_{\text{eff}}^{VV}(f) - [\Gamma_{\text{eff}}^{IV}(f)]^2} \right]^{1/2}$
- $G_A(f) = \begin{cases} \Gamma_{\text{eff}}^{II}(f), & \text{if } A = I \\ \Gamma_{\text{eff}}^{VV}(f)/\Pi(f), & \text{if } A = V \end{cases}$
- $\Gamma_{\text{eff}}^{AB}(f) = \sum_{N=1} \sum_{M>N} \left(\frac{\sin \theta_N \sin \theta_M}{5} \right)^2 \cdot \frac{\gamma_A^{d_N d_M}(f) \gamma_B^{d_N d_M}(f)}{P_{nN}(f) P_{nM}(f)}$

How to quantify PV detection prospects?

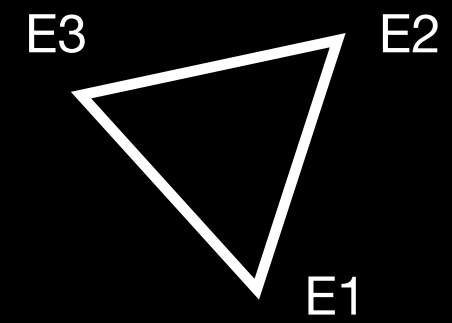
- $$\Omega_{\beta,A} = \frac{\rho_{\text{thr},A}}{\sqrt{2T}} \left[\int_{f_{\min}}^{f_{\max}} df \frac{(f/f_{\text{ref}})^{2\beta}}{\Omega_{\text{eff},A}^2(f)} \right]^{-1/2}.$$

- $$\Omega_{\text{PI},A}(f) = \max_{\beta} \left[\Omega_{\beta,A} \left(\frac{f}{f_{\text{ref}}} \right)^{\beta} \right],$$

More on ET

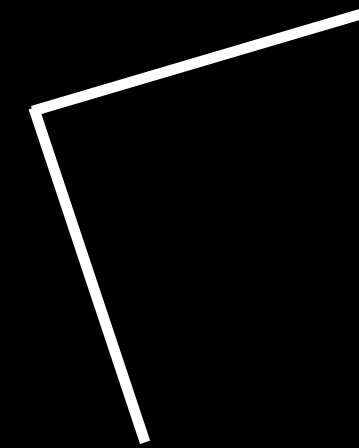
- Europe... but where?
- 10/15 km arms
- Configuration?

Schematic



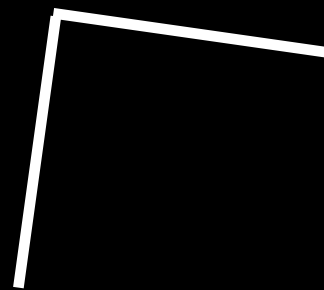
Δ = in Virgo's location
10 km arms

2 x CE



P = CE in Pacific Ocean
20 km arms

2 x ET



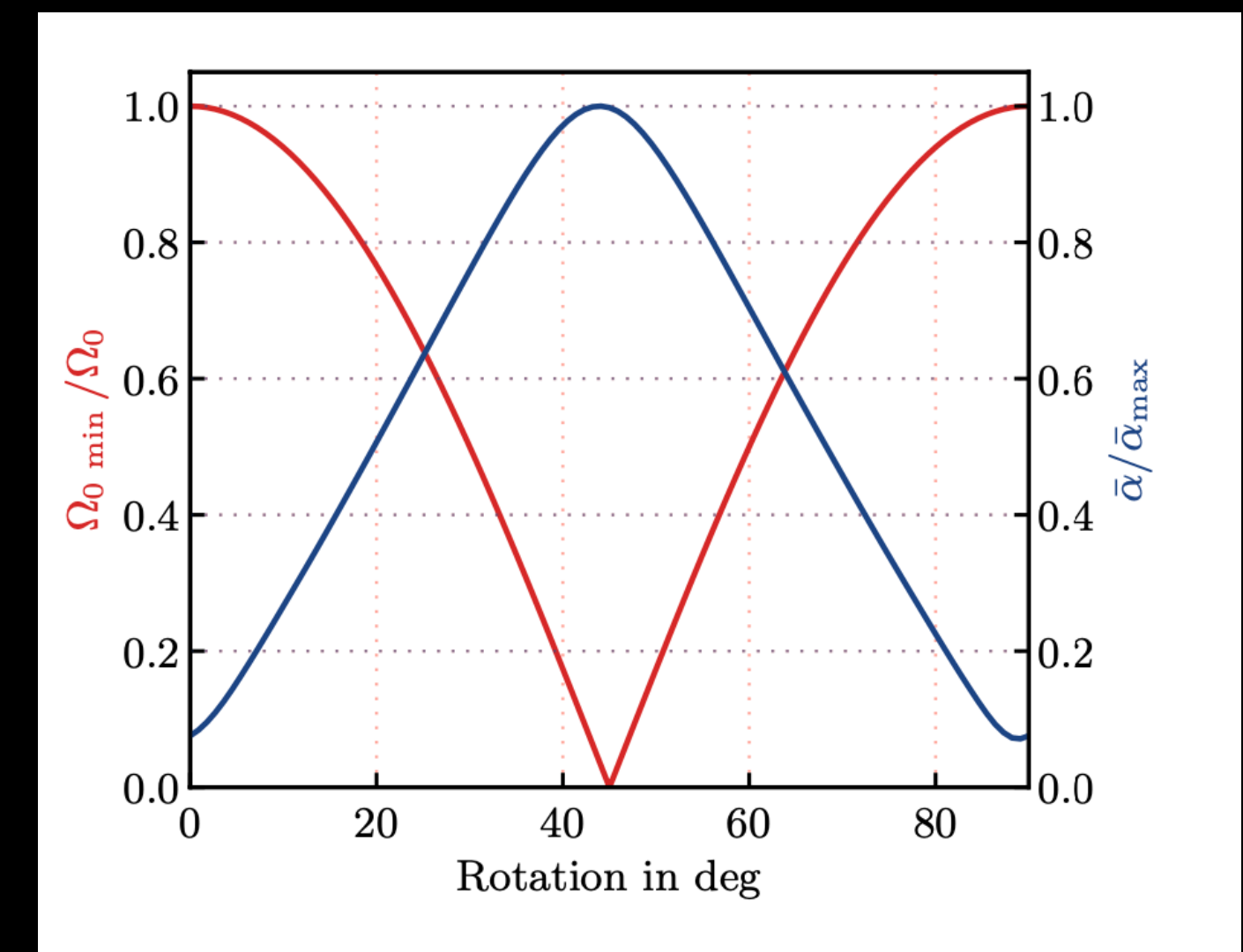
S = L ET in Sardinia
10 km arms

Y = CE in Atlantic Ocean
40 km arms

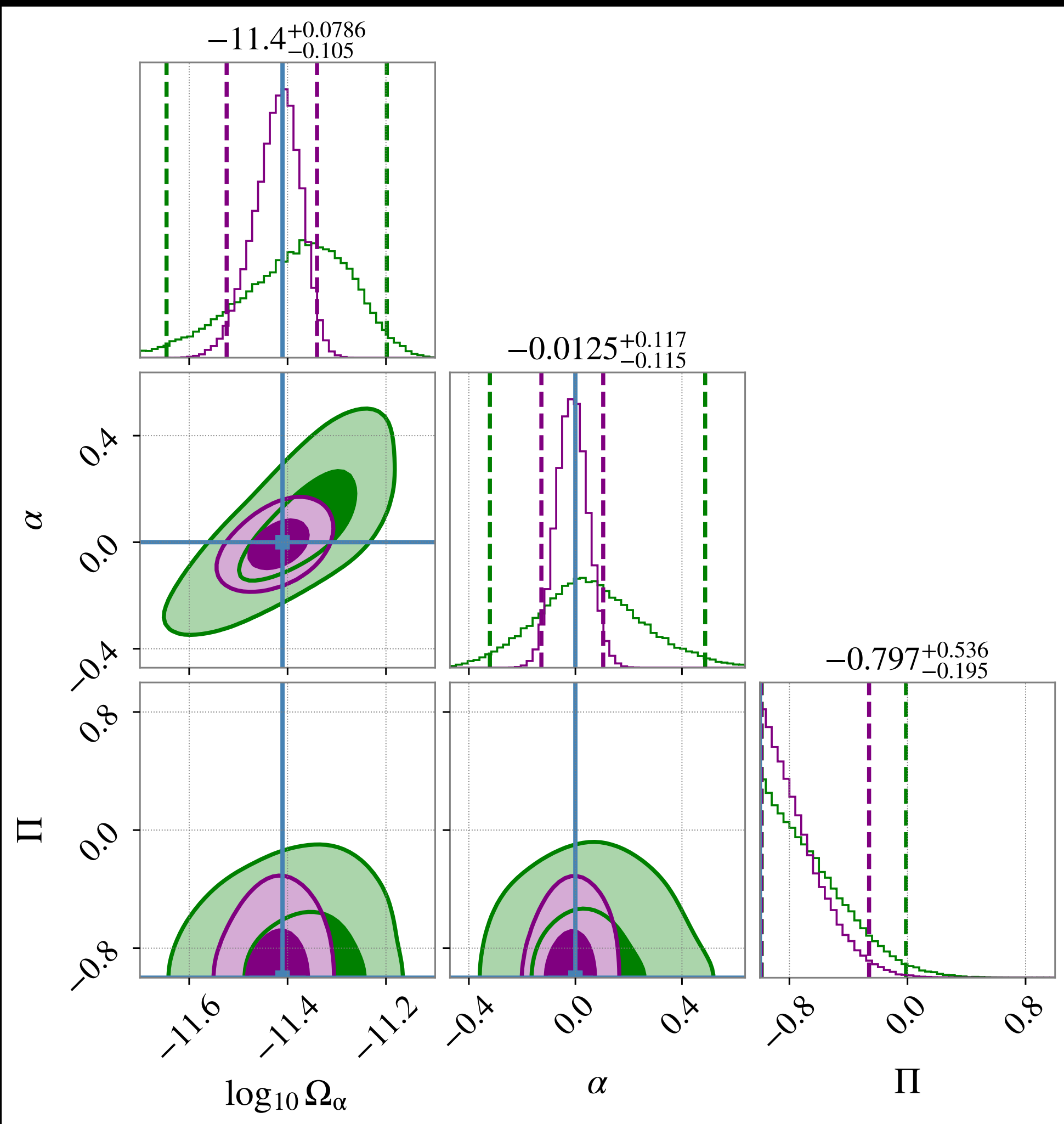
R = L ET in three-border region
Belgium, the Netherlands,
Germany
15 km arms

Different configurations

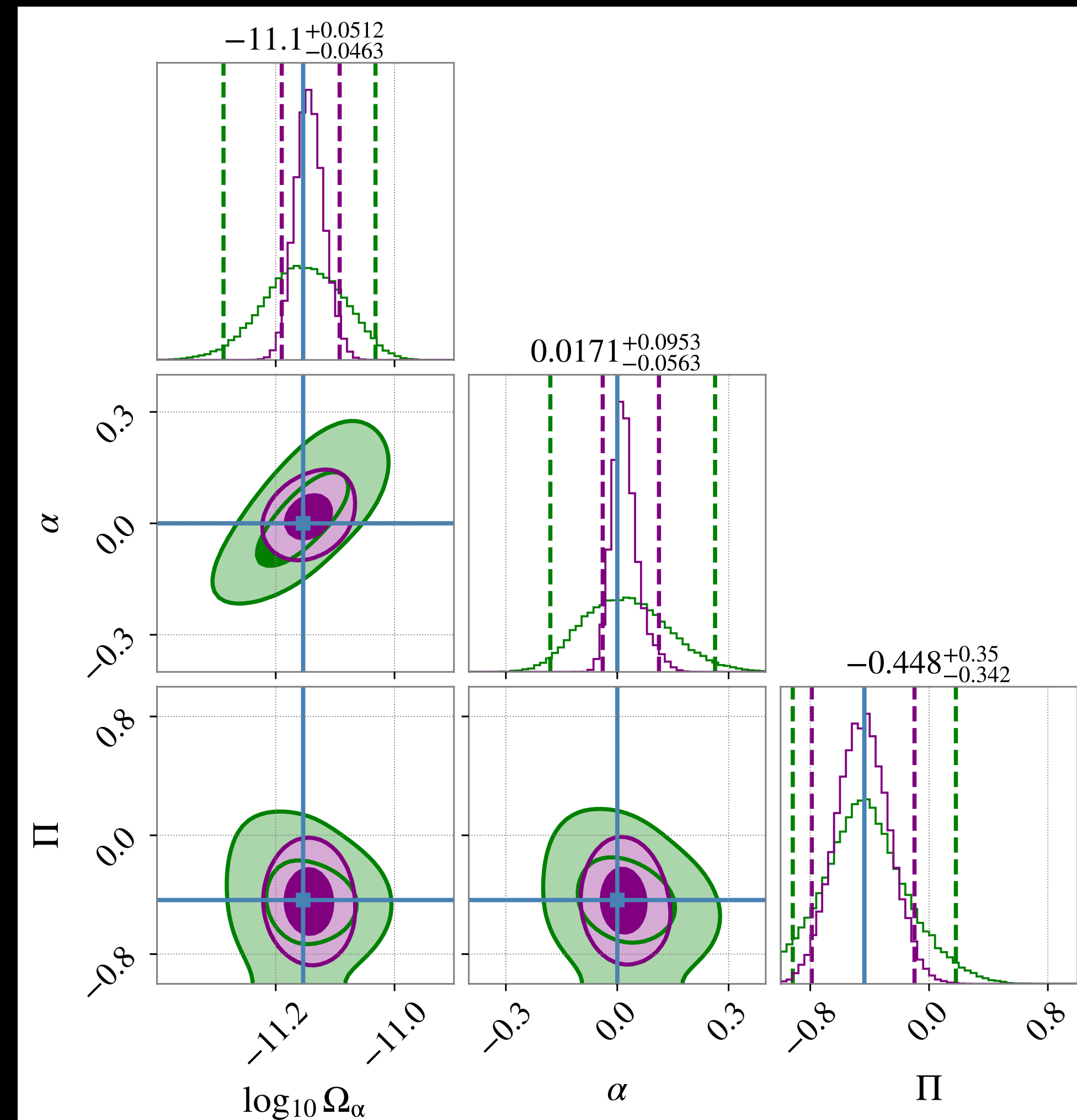
- There are a lot of possibilities, so we start with the 5 configurations of ET + 2 CE's (2 L's: one with 20 km arms and one with 40 km arms), defined in [this paper](#)
 - “Optimal” Δ_{10} +2 CE
 - “Not optimal” Δ_{10} +2 CE
 - “Optimal” 2L +2 CE
 - “Not optimal” 2L +2 CE
 - “Hybrid” 2L +2 CE
- With “optimal”= a 3G network optimized for **stochastic background** searches and “Not optimal”= a 3G network optimized for localizing CBC searches “Hybrid” = a 3G network that makes a trade-off between **stochastic** and sky-localization searches



Parameter Estimation



We are able to recover parameters within 1 or 2 σ



Each injection run searched for a PV ($\Pi \neq 0$) and a non-PV ($\Pi = 0$) model. From each the repeated injections, we should have 10 sets of data of each model $\mathcal{M}$ that has associated posteriors, log Bayes $\log \mathcal{B}_{\text{noise}}^{\mathcal{M}}$, and log Bayes standard deviation (error) $\sigma_{\mathcal{M},\text{noise}}$.

For a given injection in set i , one can find the model preference with:

$$\log \mathcal{B}_{\Pi=0}^{\Pi \neq 0} = \log \mathcal{B}_{\text{noise}}^{\Pi \neq 0} - \log \mathcal{B}_{\text{noise}}^{\Pi=0}$$
$$\sigma_{\Pi \neq 0, \Pi=0}^2 = \sigma_{\Pi \neq 0, \text{noise}}^2 + \sigma_{\Pi=0, \text{noise}}^2$$

Finally, for whichever set of data ($\Pi \neq 0$ vs. noise, $\Pi \neq 0$ vs. $\Pi = 0$) you can combine the results with:

$$\log \mathcal{B}_{\text{comb}} = \frac{\sum_i \log \mathcal{B}_i \cdot w_i}{\sum_i w_i}$$
$$\sigma_{\text{comb}} = \left(\sum_i w_i \right)^{-1/2}$$

where $w_i = 1/\sigma_i^2$. (The code accounts for non-runs by only adding an entry when $\sigma_i > 0$ i.e. when the PE run actually completed)

Note that when all $w_i = \text{const.}$ are the same value, the above combined values become your usual average and spread over N sets via Central Limit Theorem.