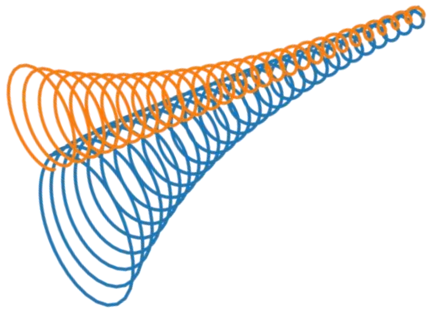
Alessandro Alberto Trani

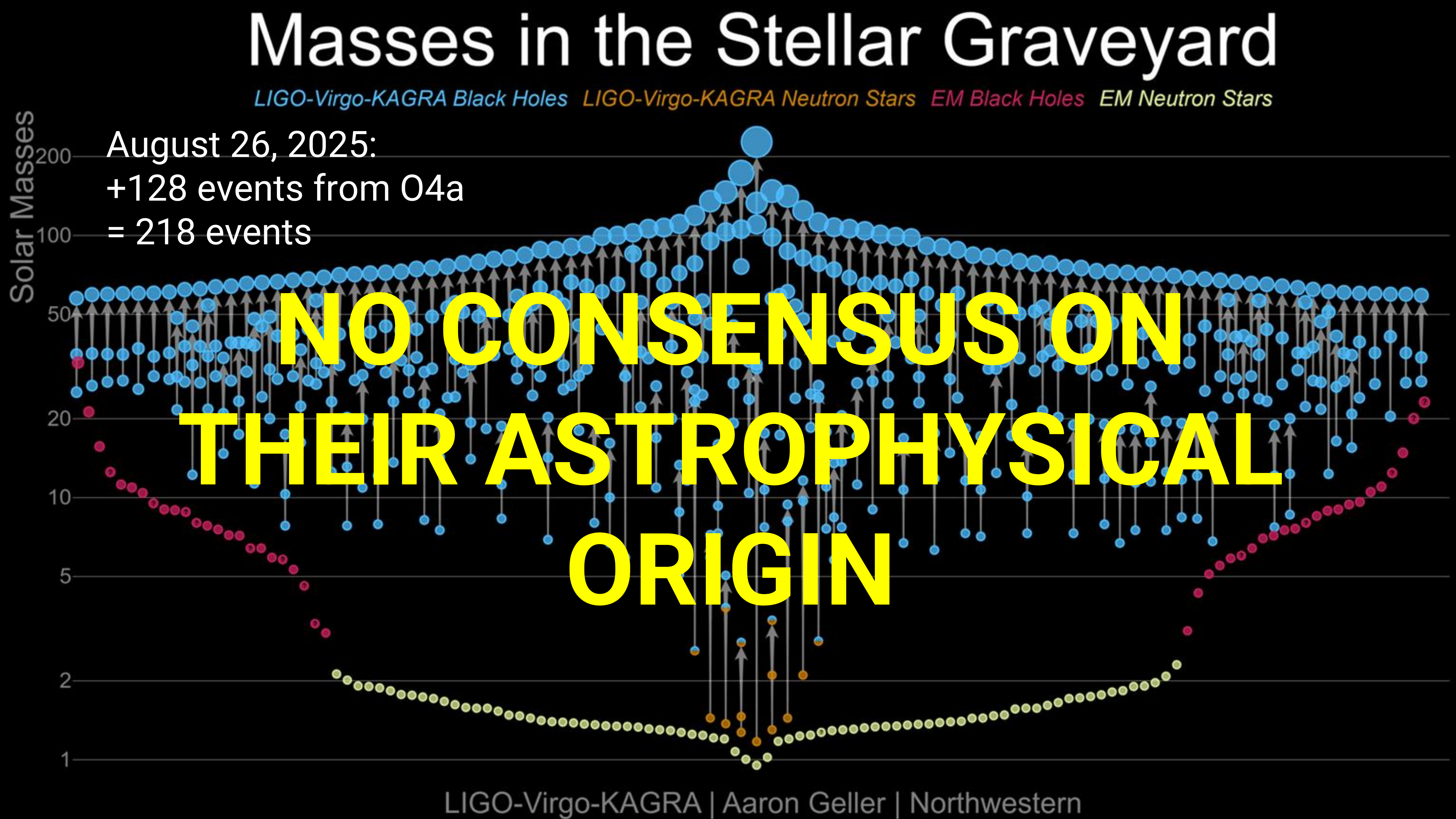
虎似　三努郎

On the formation channels of gravitational wave sources:

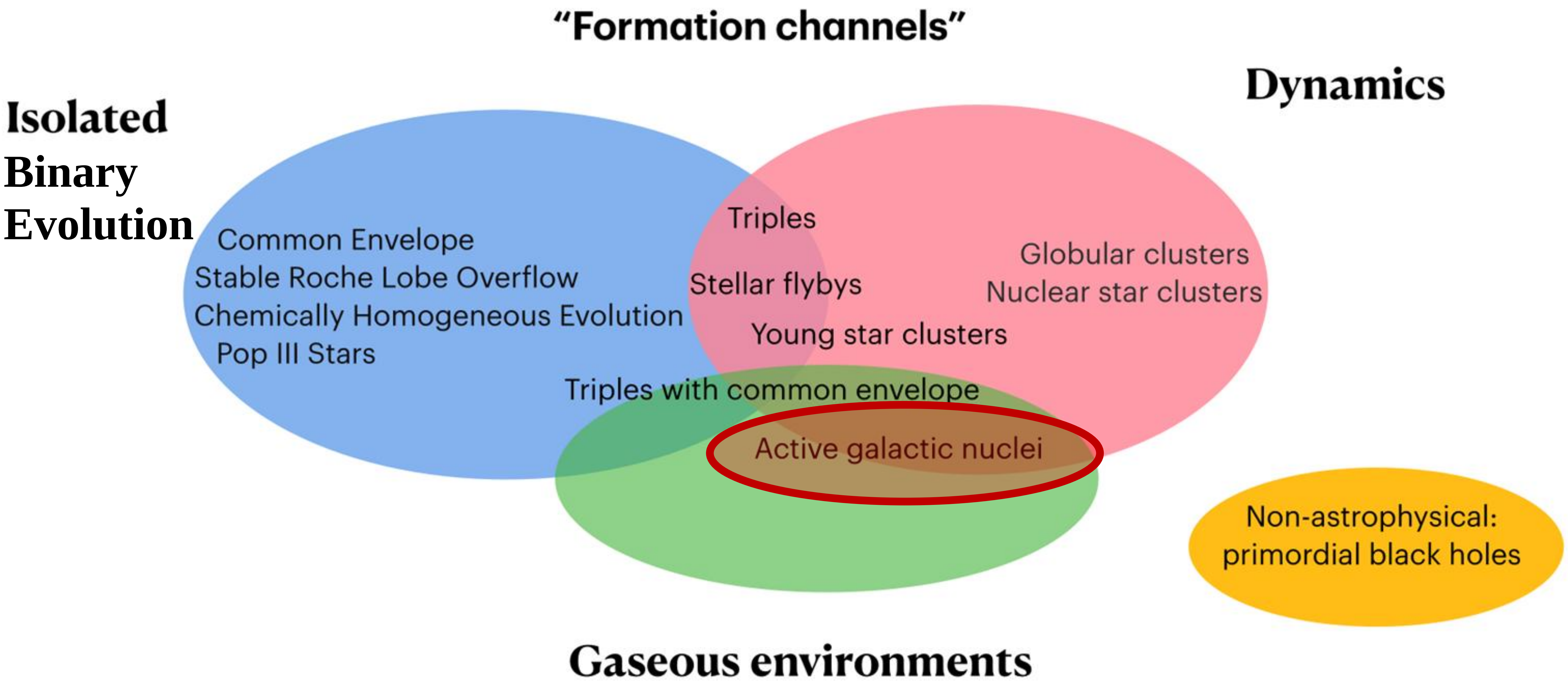
active galactic nuclei



Incoming associate professor,
University of Concepción


Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
EM Black Holes
EM Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1
August 26, 2025:
+128 events from O4a
= 218 events
NO CONSENSUS ON
THEIR ASTROPHYSICAL
ORIGIN
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Proposed evolutionary pathways to merging black holes



Scheme adapted from Maya Fishbach, adapted from Mike Zevin, adapted from Selma de Mink

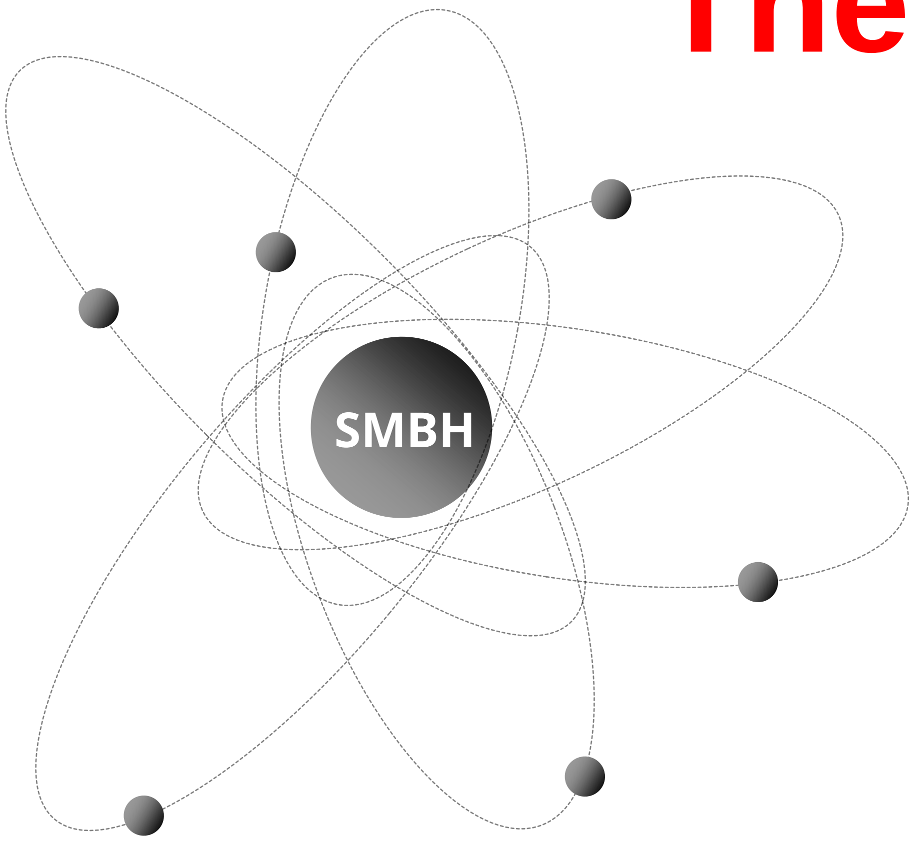
See review on astrophysical processes: Spera, **AAT** & Mencagli, 2022, Galax

FORMING BLACK HOLE BINARIES IN AGN DISKS

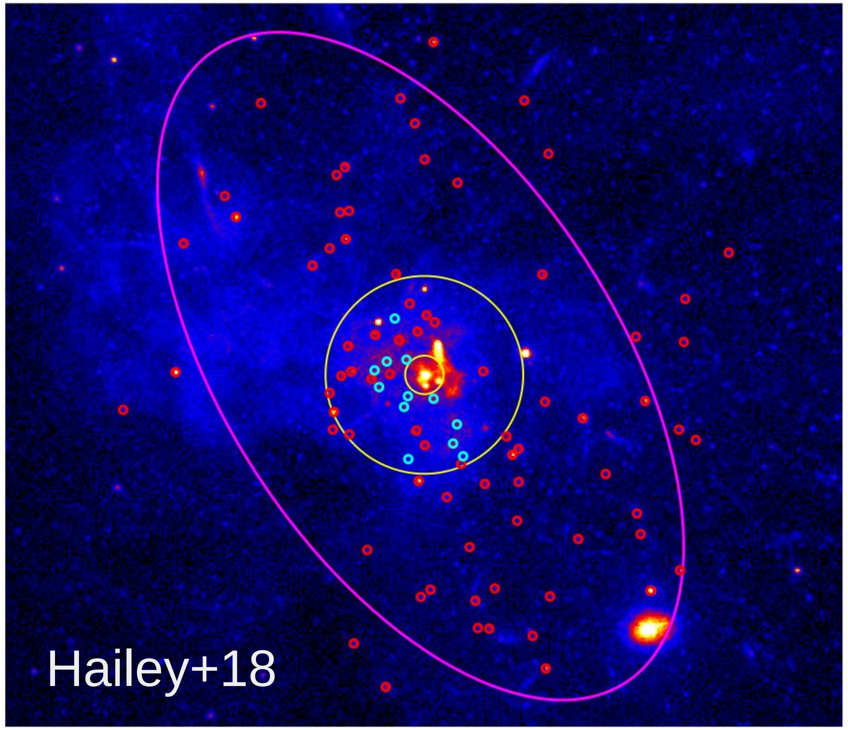
The AGN channel

The AGN channel

#0



- Dark cusp of stellar remnants predicted around SMBHs, due to dynamical friction (Bahcall & Wolf 1977, Alexander & Hopman 2009)



- In the Galactic centre: low-mass X-ray binaries trace dark cusp:
- may be black holes (Hailey+18)

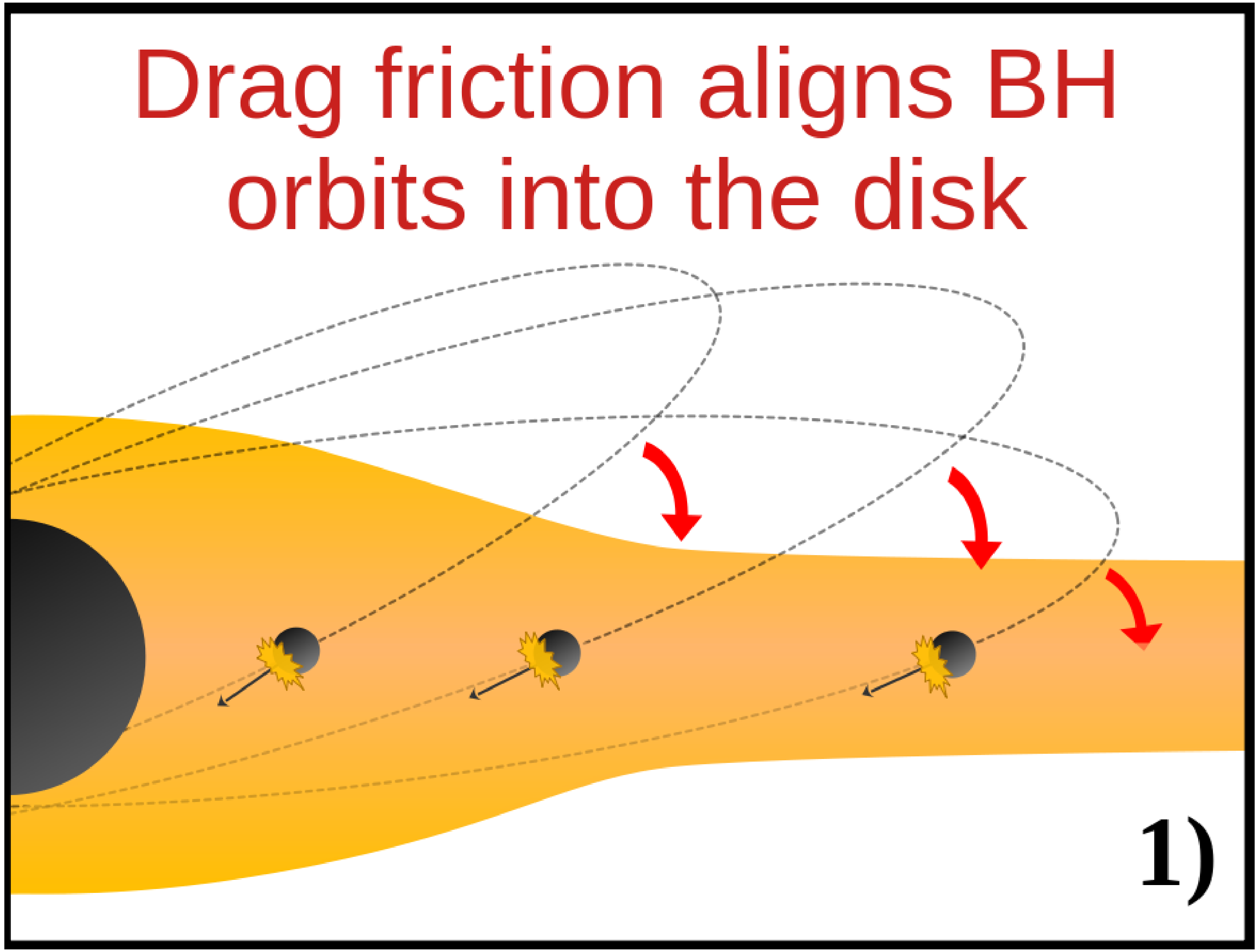
The AGN channel

#1

Once an AGN disk forms, BH orbits intersect it

Drag friction aligns the orbits within the disk:

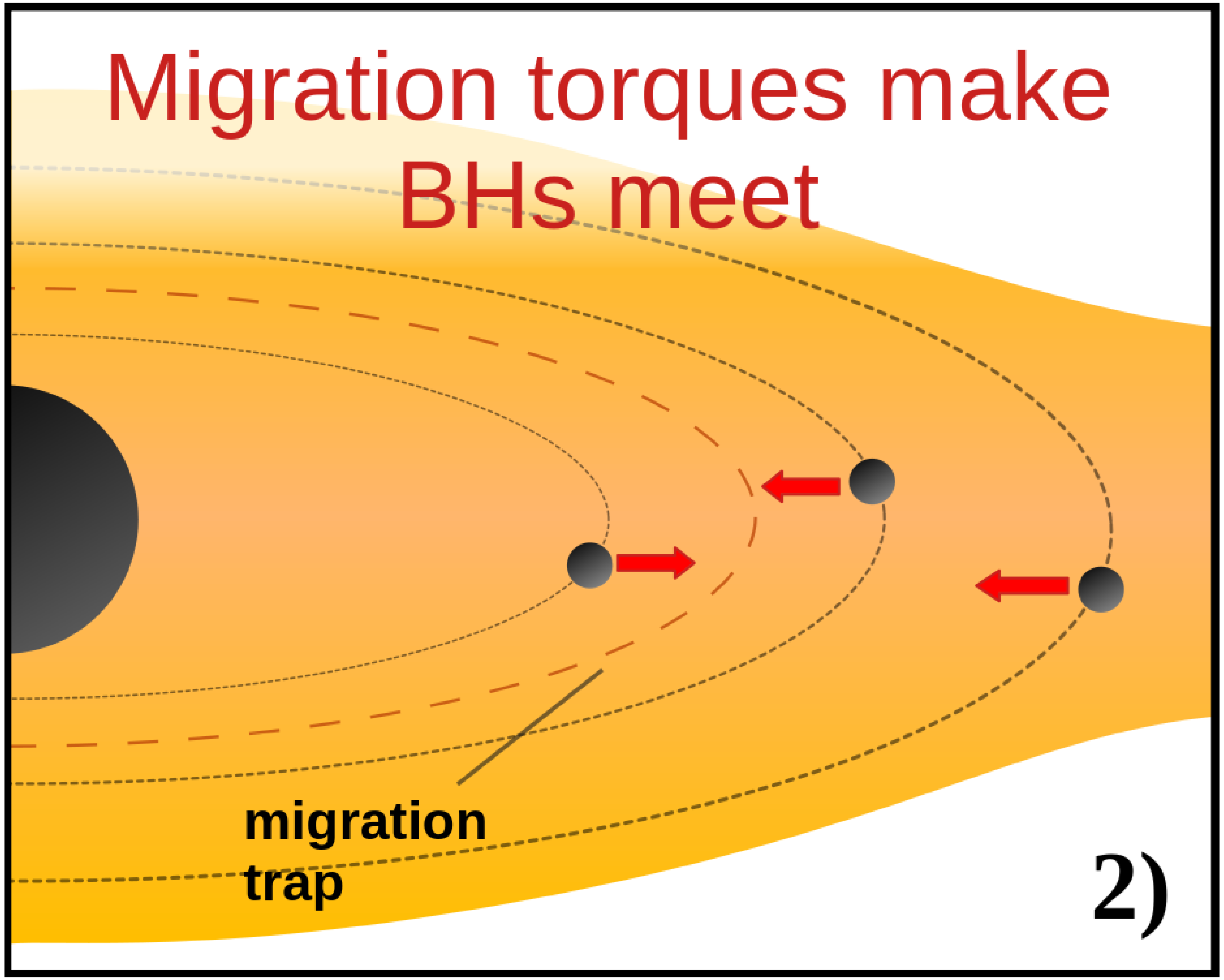
$$\tau_{\rm align} \sim \frac{v^3_{\rm Kep}(a)}{G^2 \rho_g m \frac{h}{r} \ln\Lambda} \sin^3\frac{i}{2}\sin i$$



The AGN channel

#2

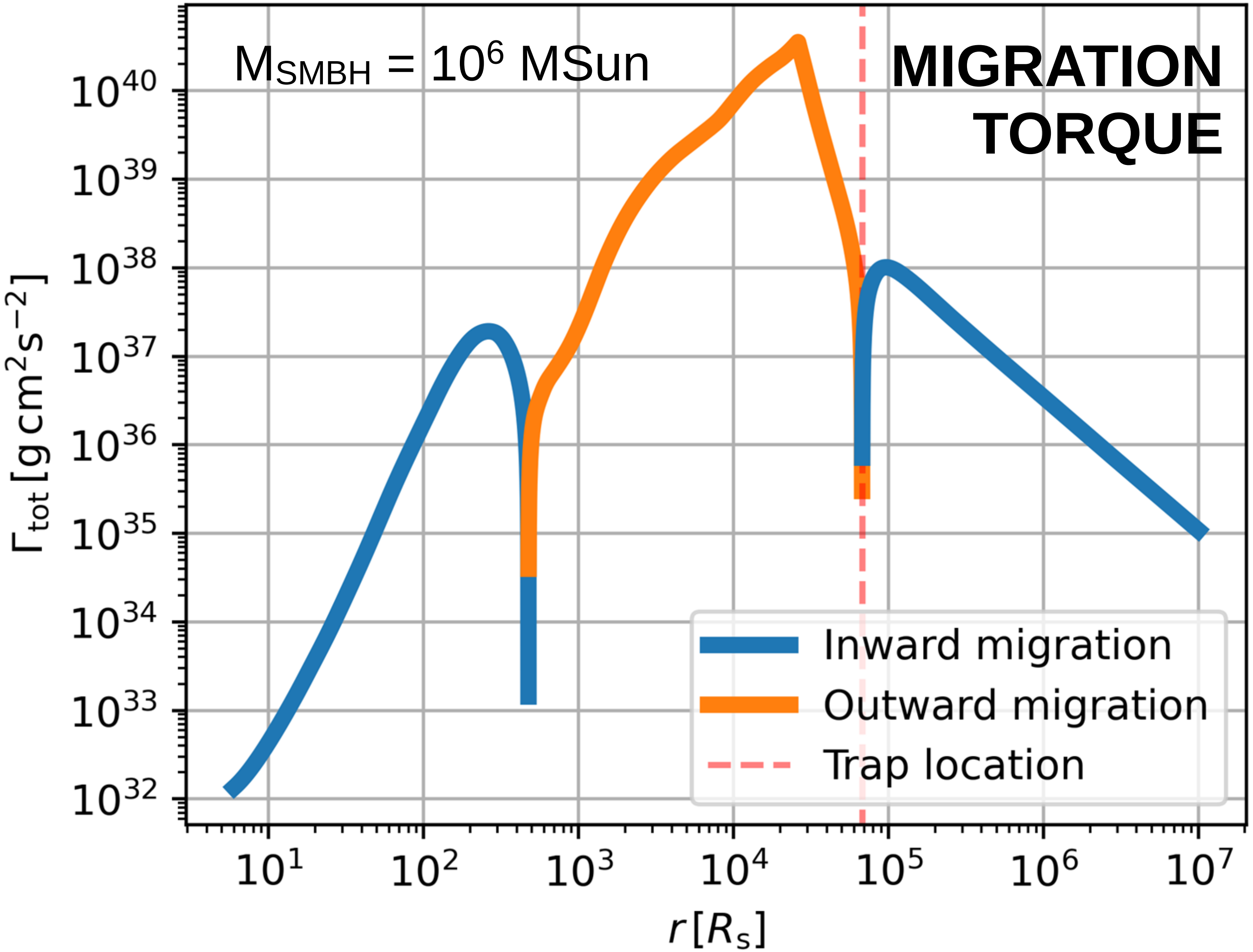
Convergent migration (Type I/Type II/Thermal torques; Grishin+2024) lets BH encounter within the diskc



The AGN channel

#2

Convergent migration (Type I/Type II/Thermal torques; Grishin+2024) lets BH encounter within the diskc



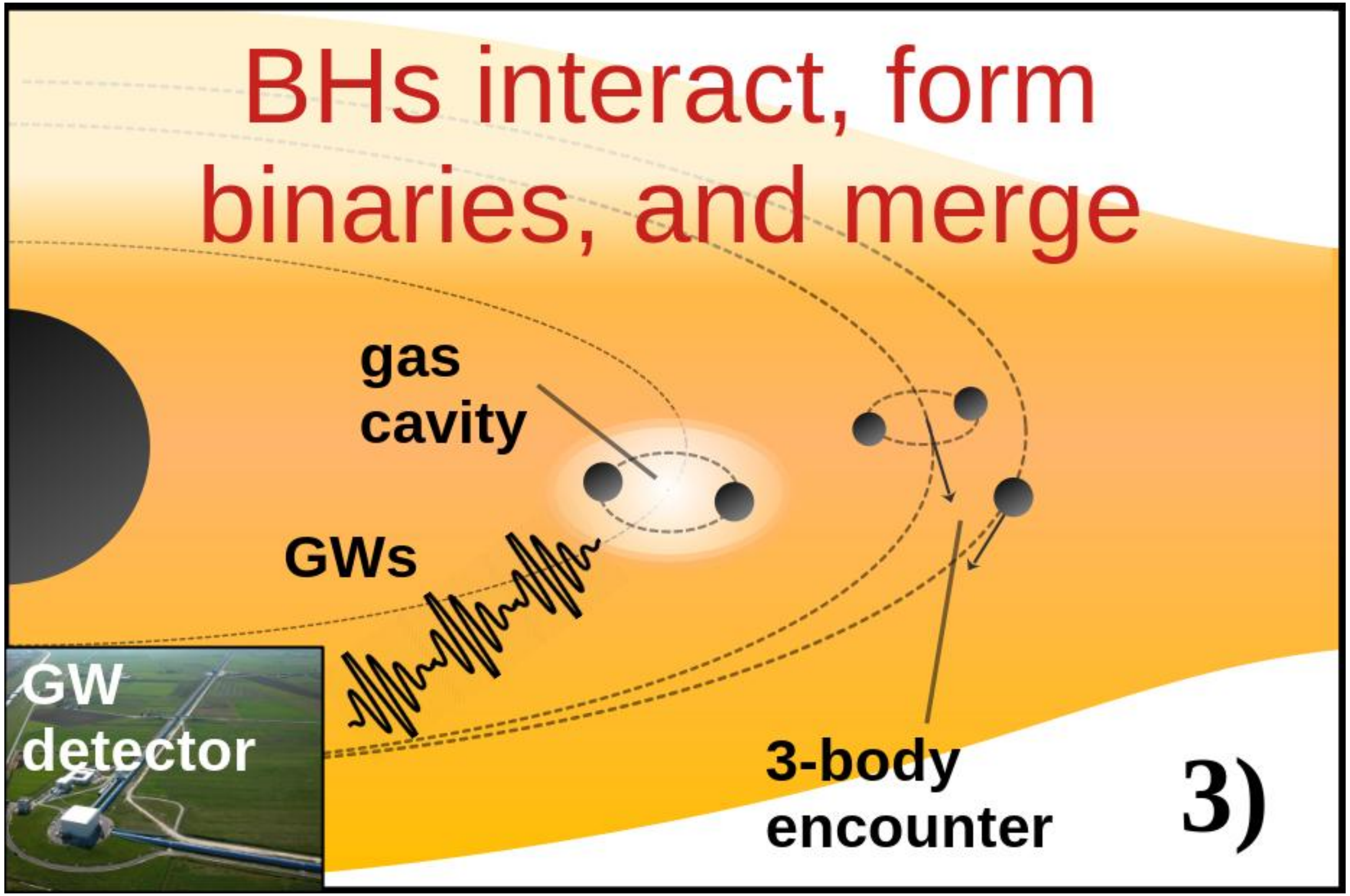
Gangardt, **AAT**, Bonnerot, Gerosa, 2403.00060

The AGN channel

#3

- 2-body interactions: may either merge via GW or form a binary

- 3-body interactions: can shrink the binaries until the merge via GW



State-of-the-art:

Fast population synthesis code modelling BH populations in AGNs

(Tagawa+2020, Vaccaro+2023,2025, McKernan+2025)

Pros:

- Fast

Cons:

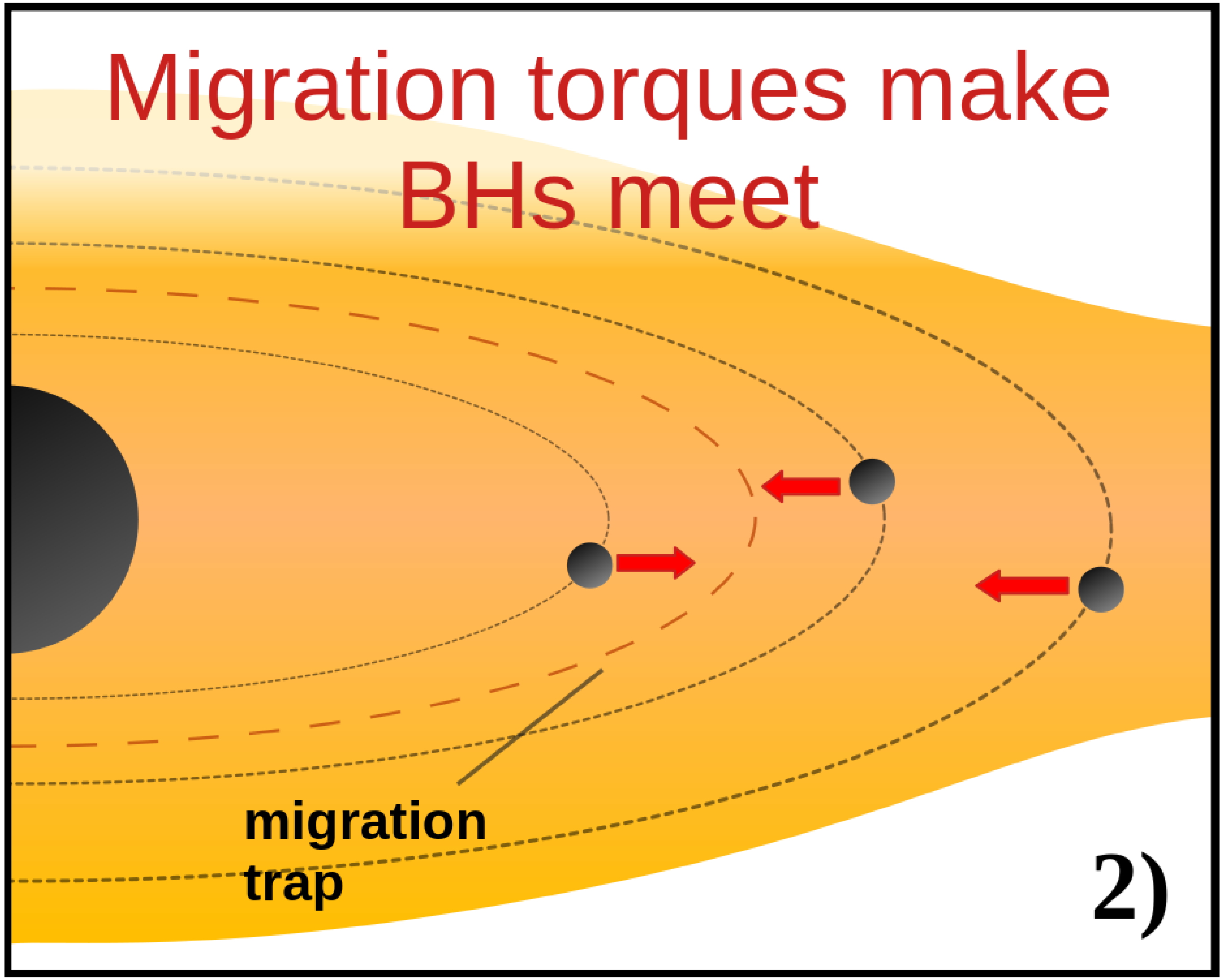
- Parametrized physics (gas capture, binary pairing, migration, 3-body encounters)
- Missing physics (BH-BH gravitational interactions)

PAIRING BLACK HOLES THROUGH MIGRATION

in collaboration with E.Grishin (Monash), J.Moncrieff (UWA), O.Pietrosanti (SISSA)

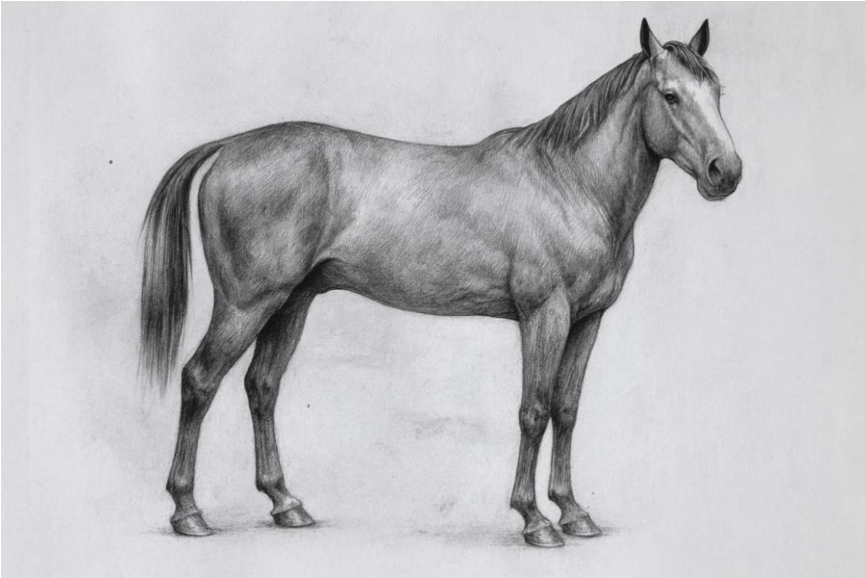
Migration in AGN disk

Black holes meet and pair thanks to migration torques



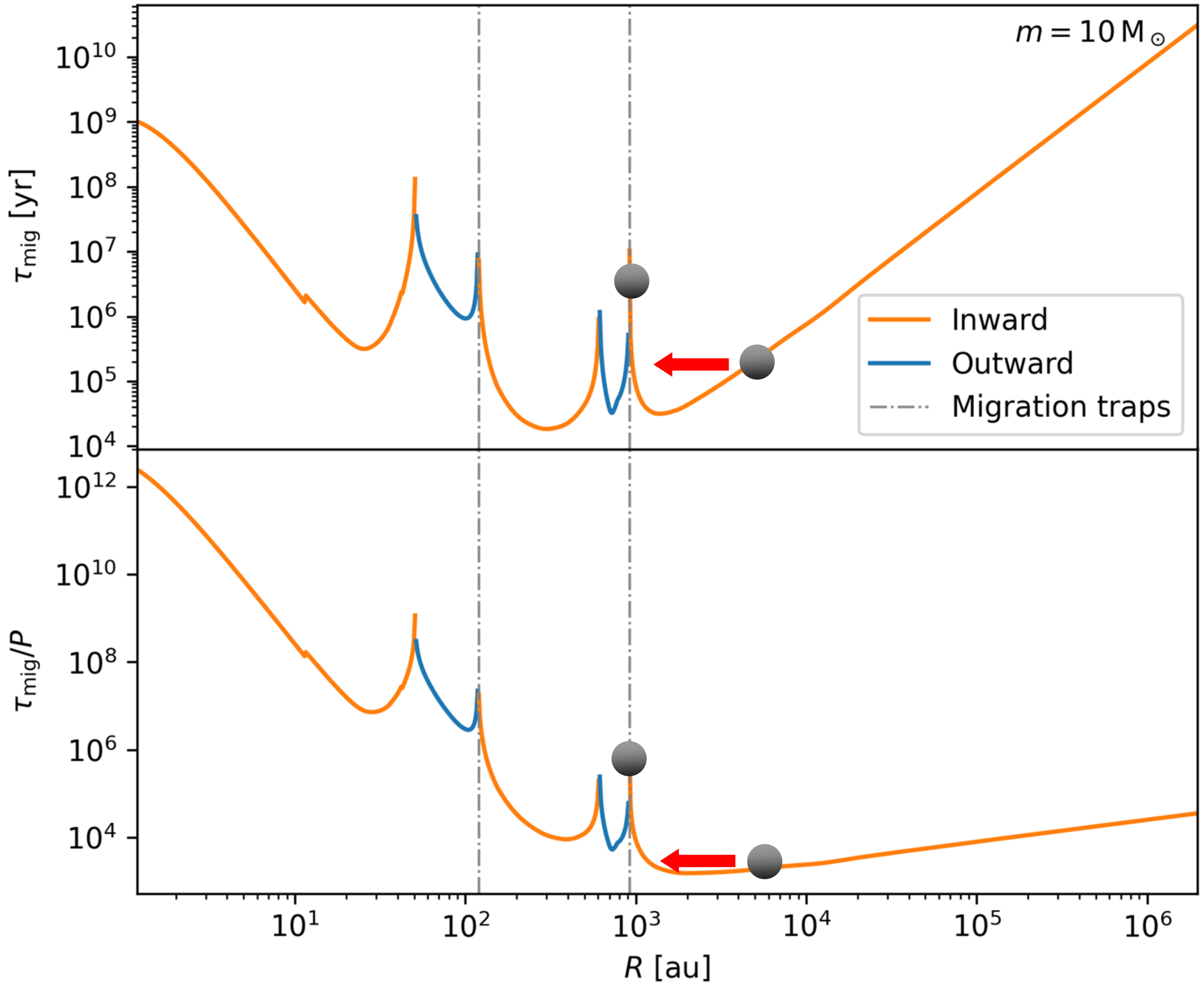
Expectations:

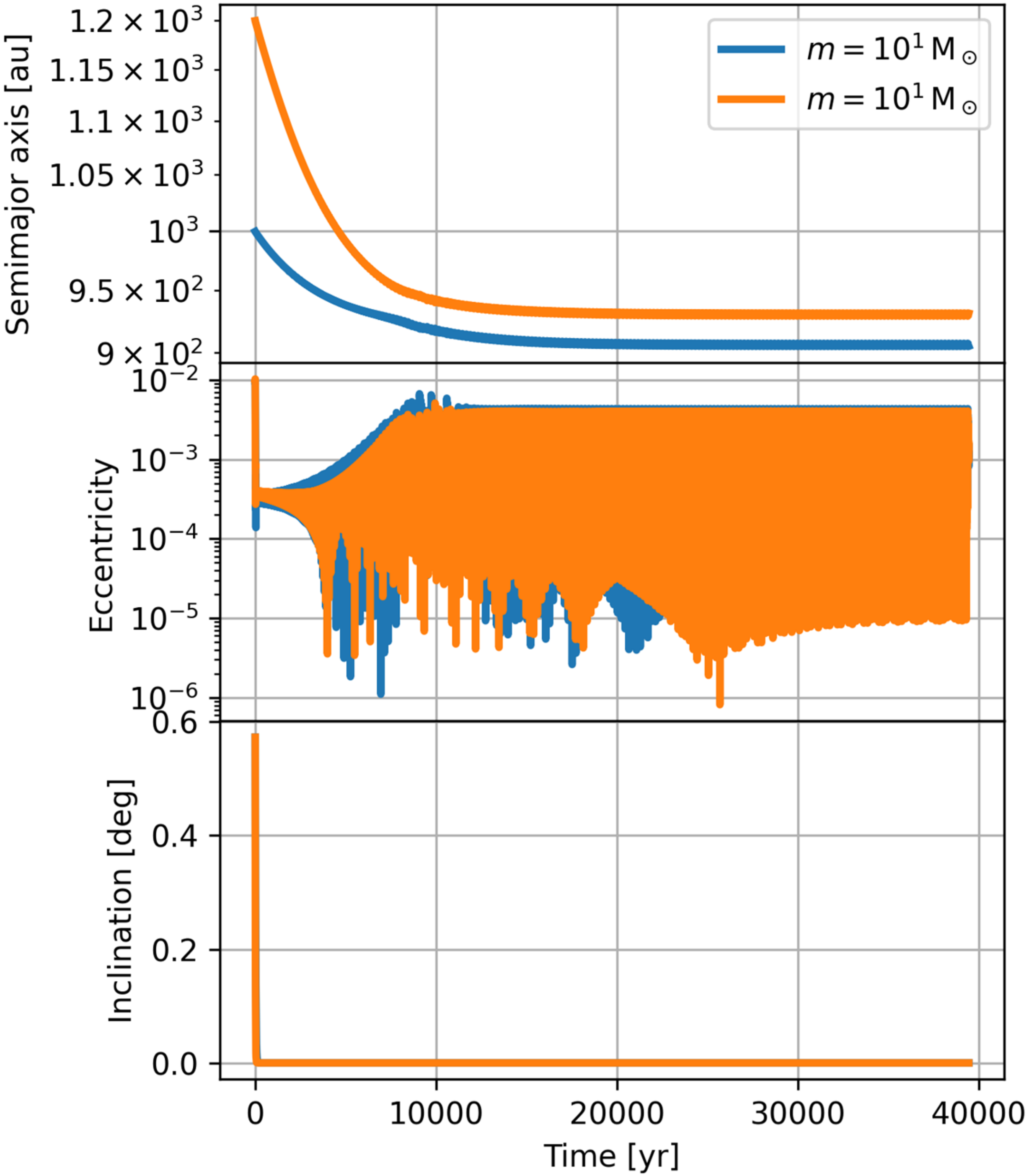
1. Inner black hole settles into the migration trap
2. Outer black hole reaches the inner one
3. Interaction! *Bam* GW merger



Simple test

Two 10 MSun BHs close to a migration trap

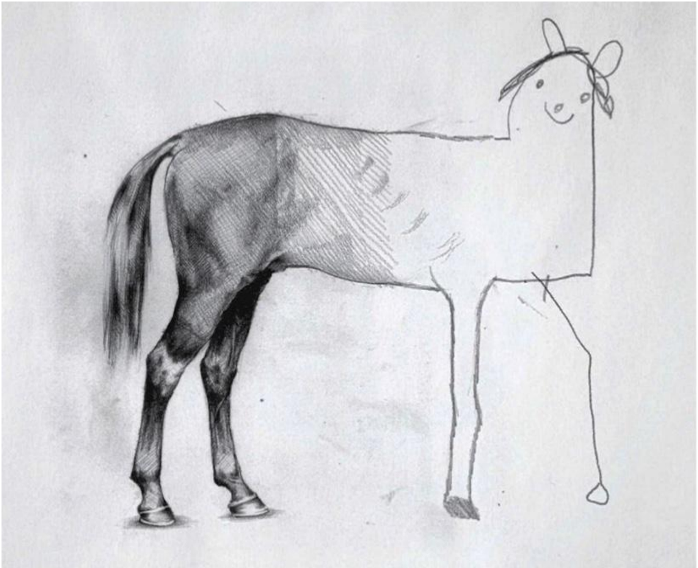


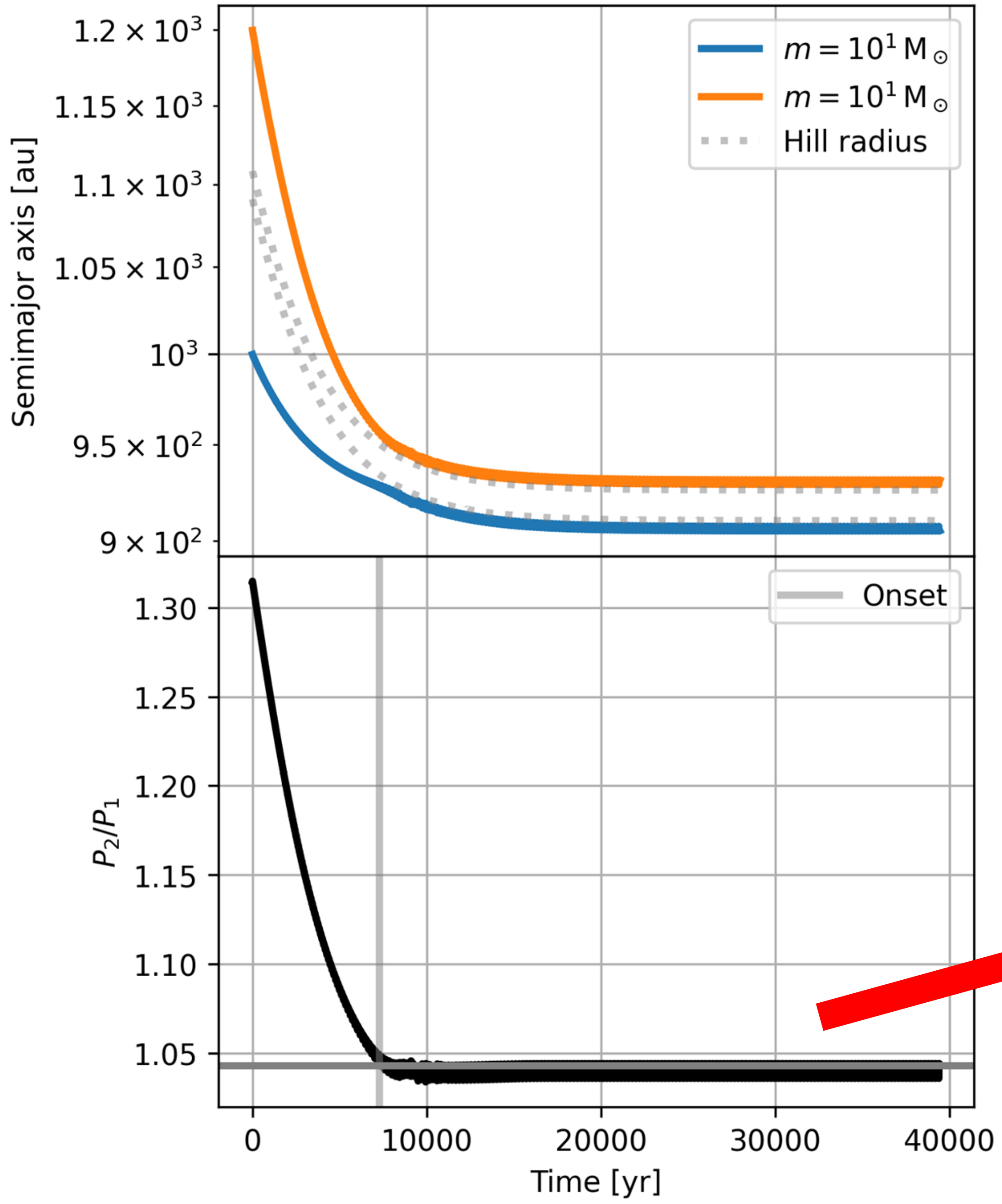


Reality:

1. Inner black hole settles into the migration trap
2. Outer black hole reaches the inner one
3. ~~3. Interaction! *Bam* GW merger~~

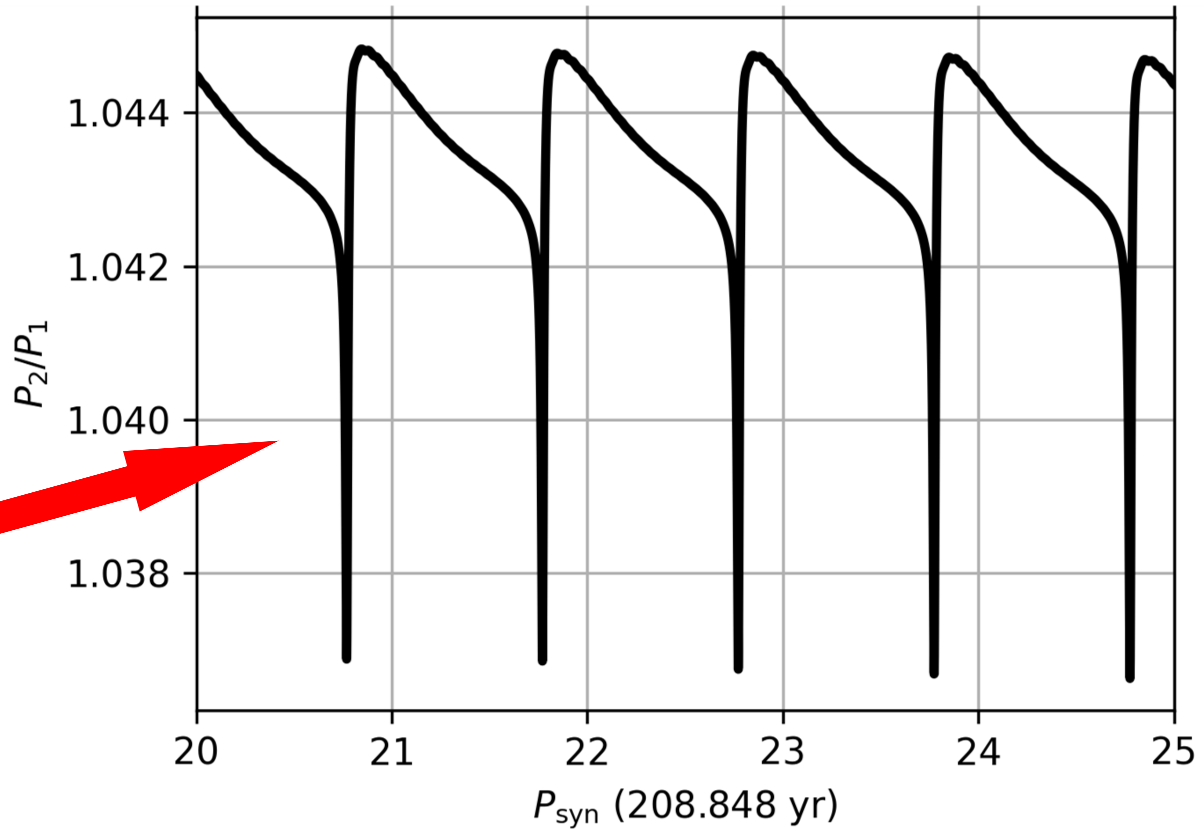
3. The two black holes enter into a mean motion resonannce

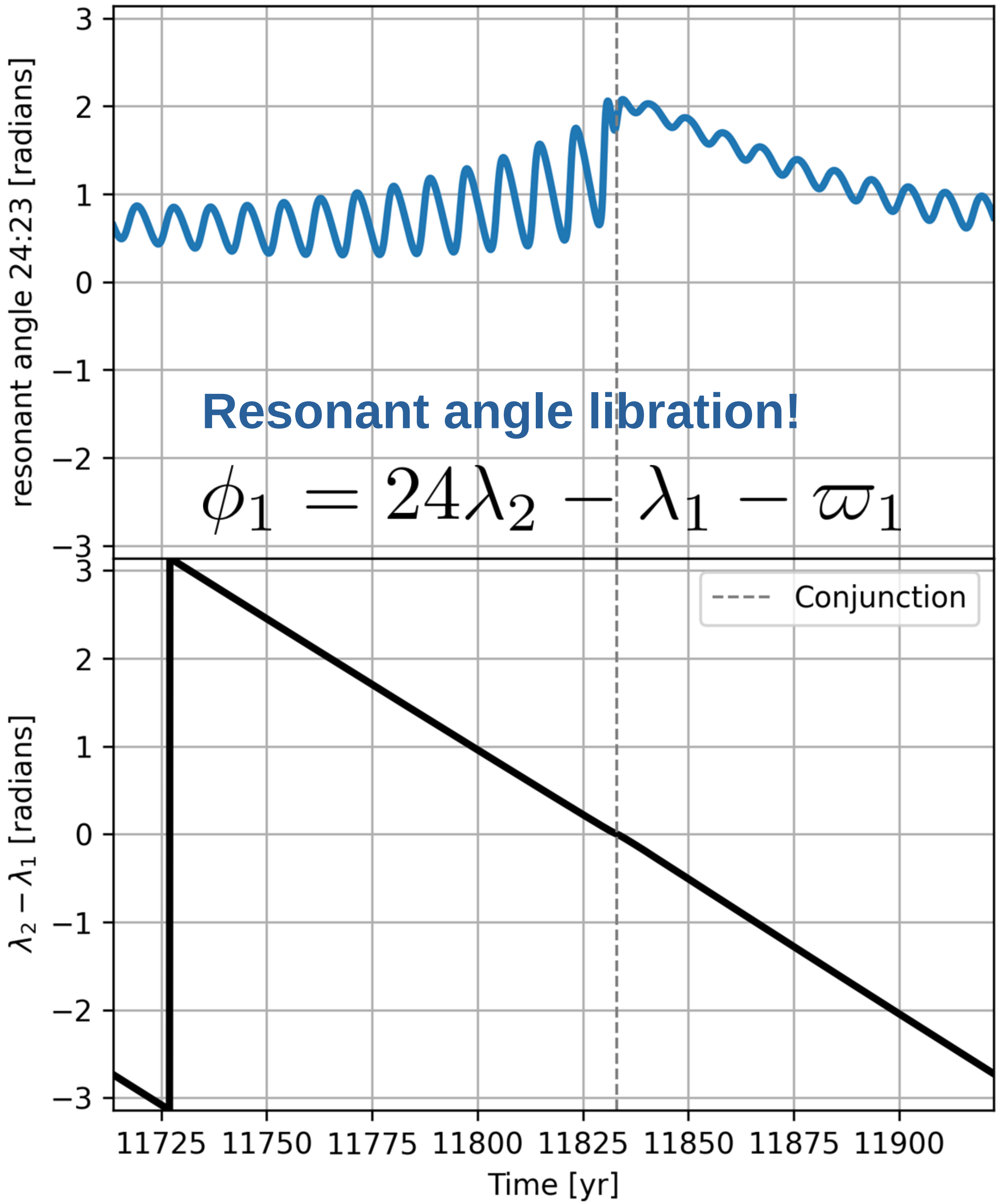




Nearly co-orbital resonance

1. Extremely tight mean-motion-resonance: period ratio ~24:23
2. Black holes settle barely outside their mutual Hill radius





Nearly co-orbital resonance

1. Extremely tight mean-motion-resonance: period ratio ~24:23
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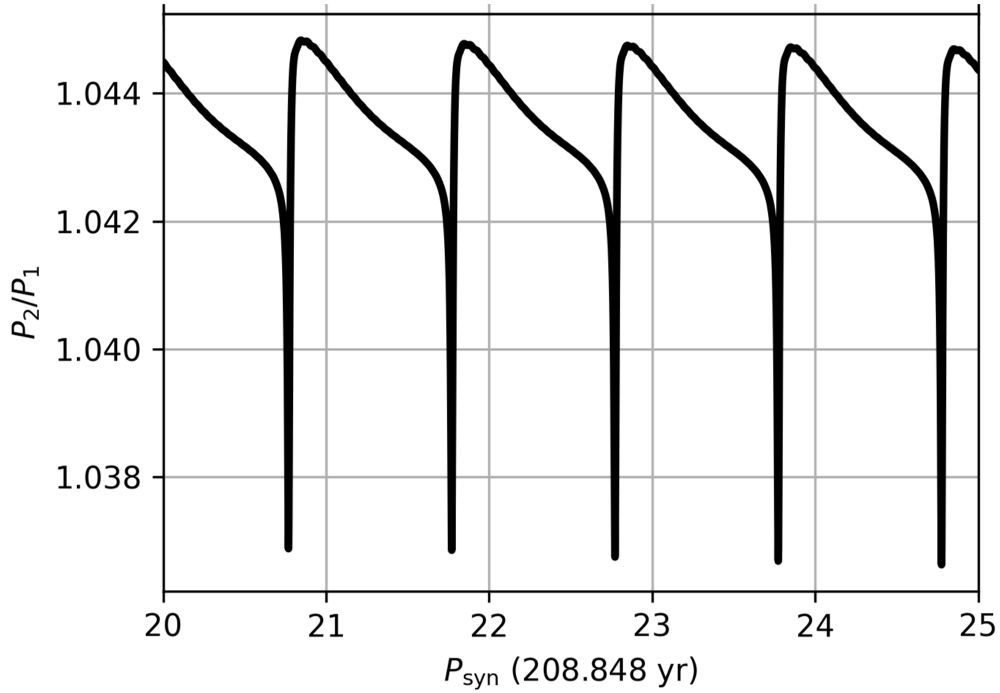
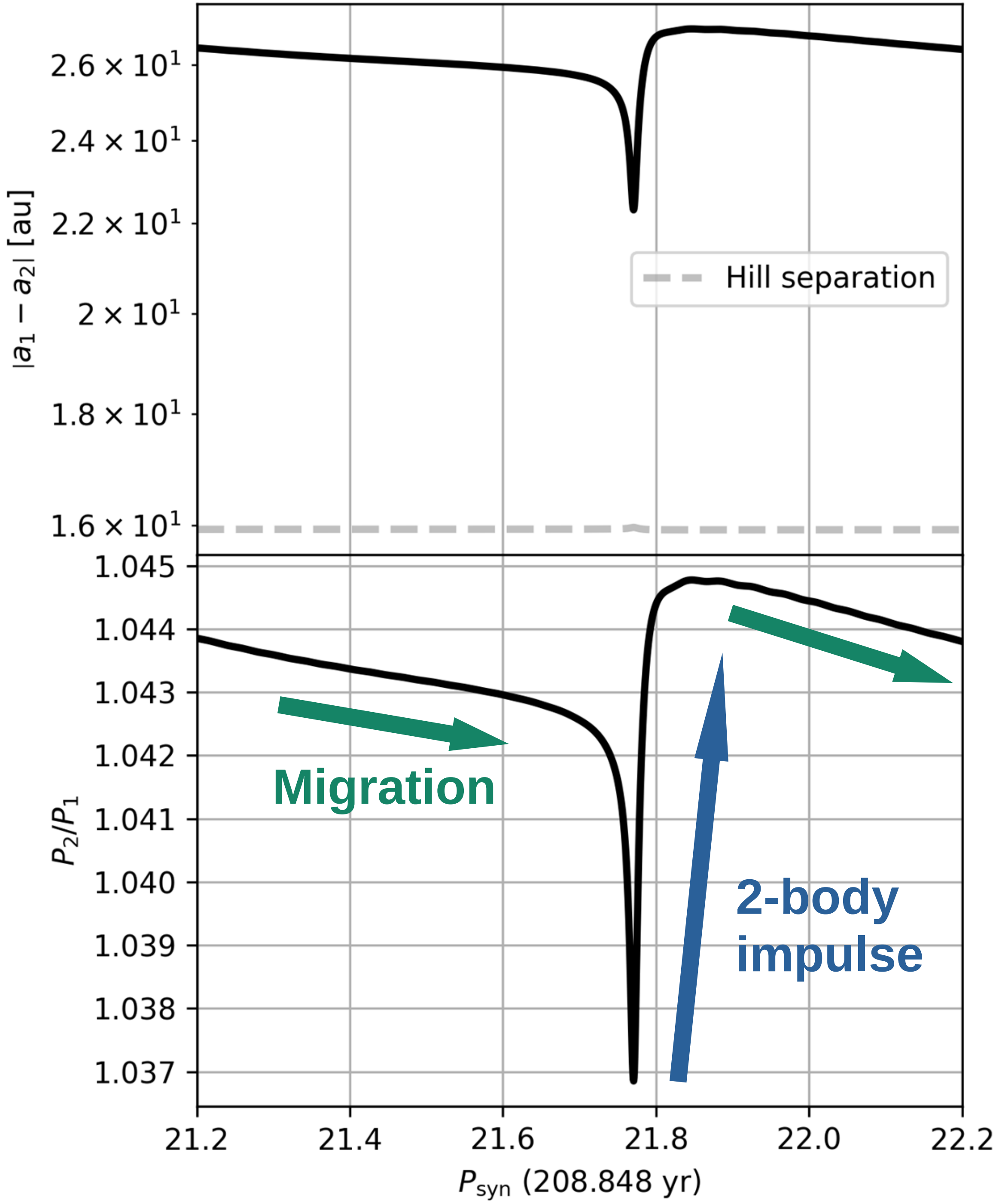
3. No 1:1 libration: not a horse-shoe/tadpole orbits (as in trojan co-orbital bodies)





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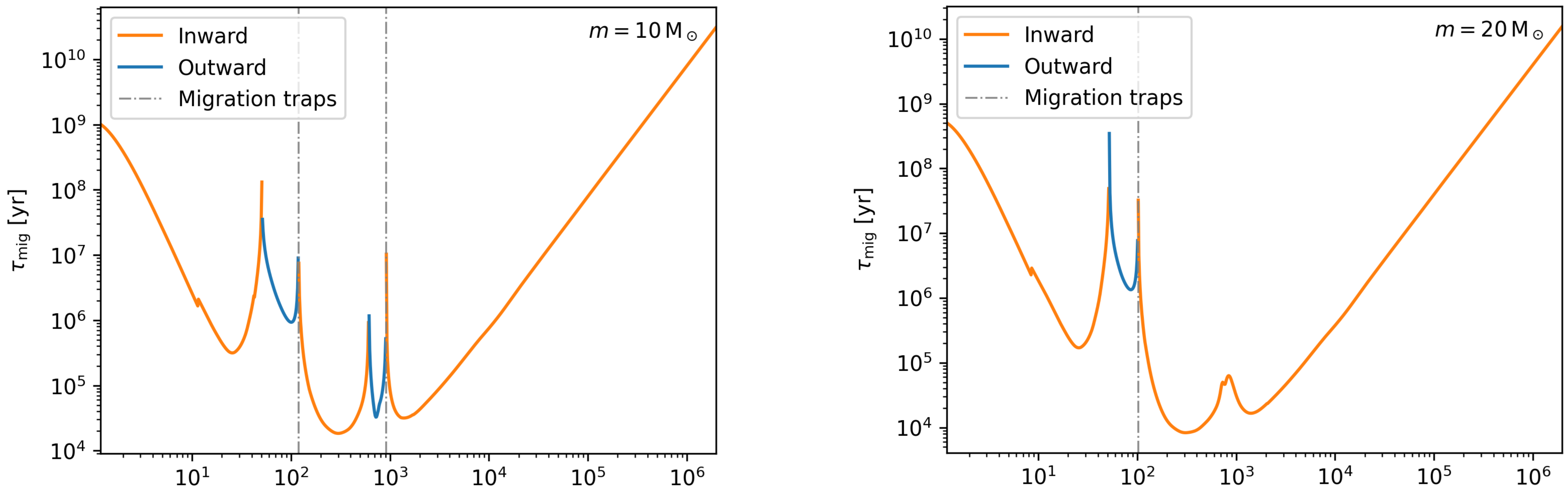
4. **Scatter/damping cycles**:
- 2-body impulse at conjunction repulse the bodies
- Migration brings back the bodies together

What about non-equal black hole masses?



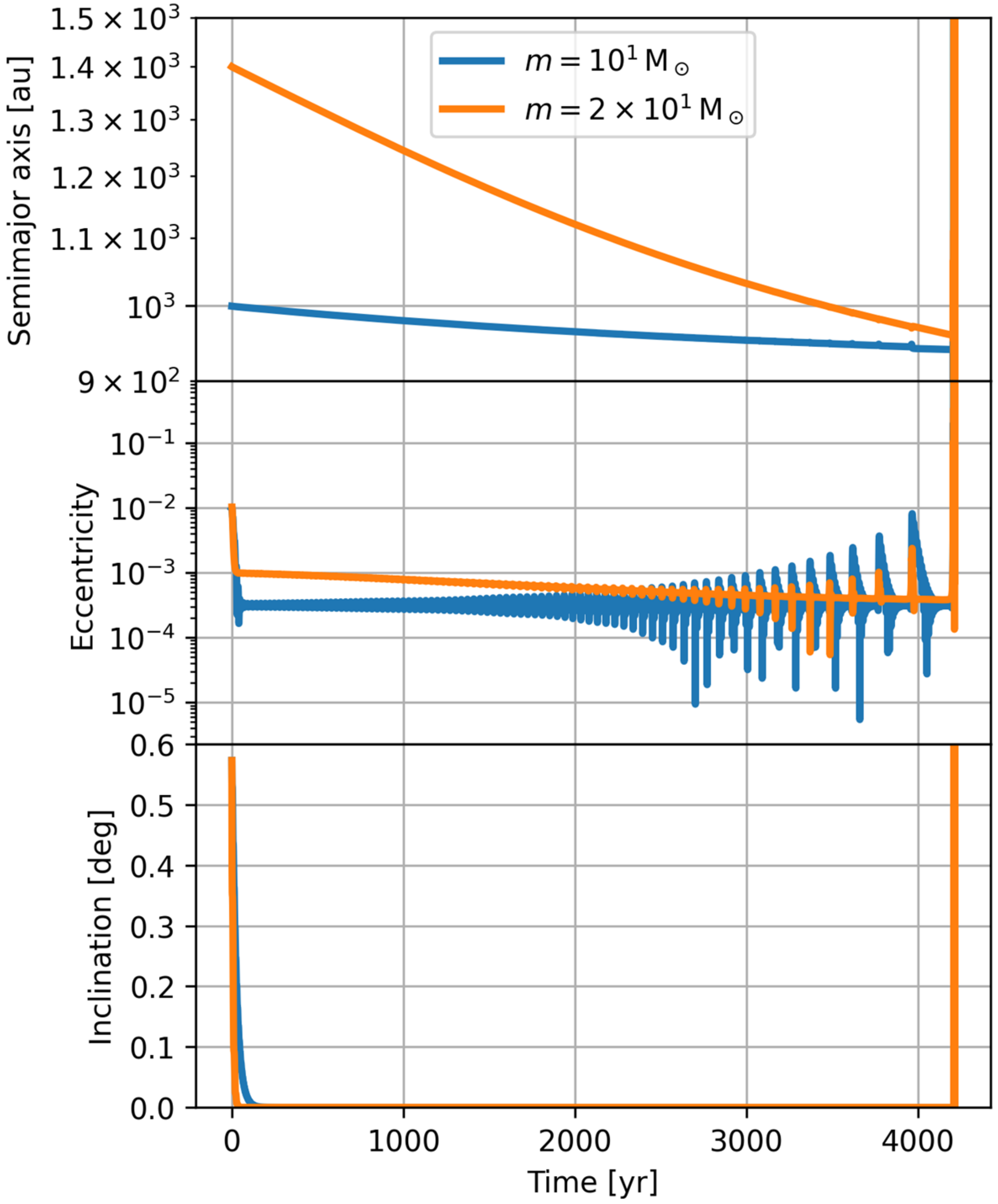
Different BH masses have different migration trap location

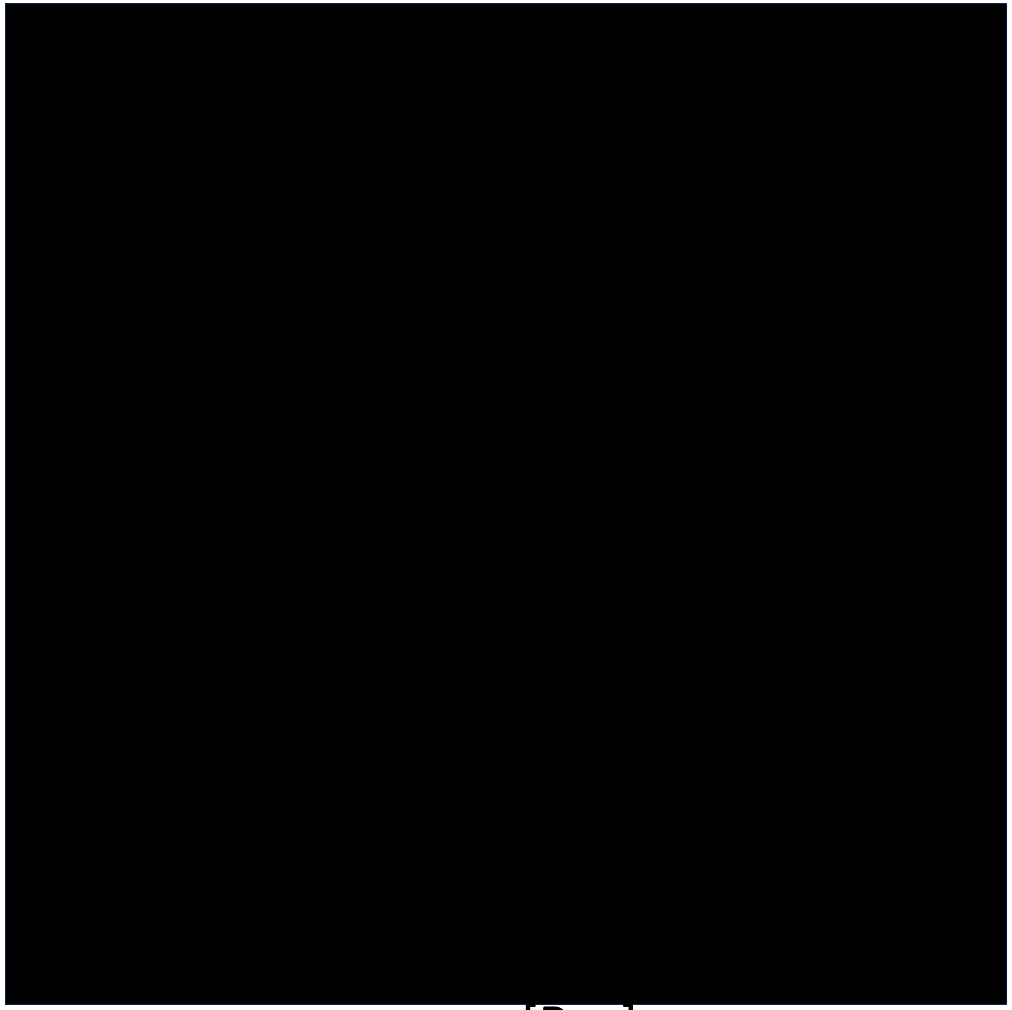
Rule of thumb: higher BH mass, trap location moves inward

(or disappears altogether)

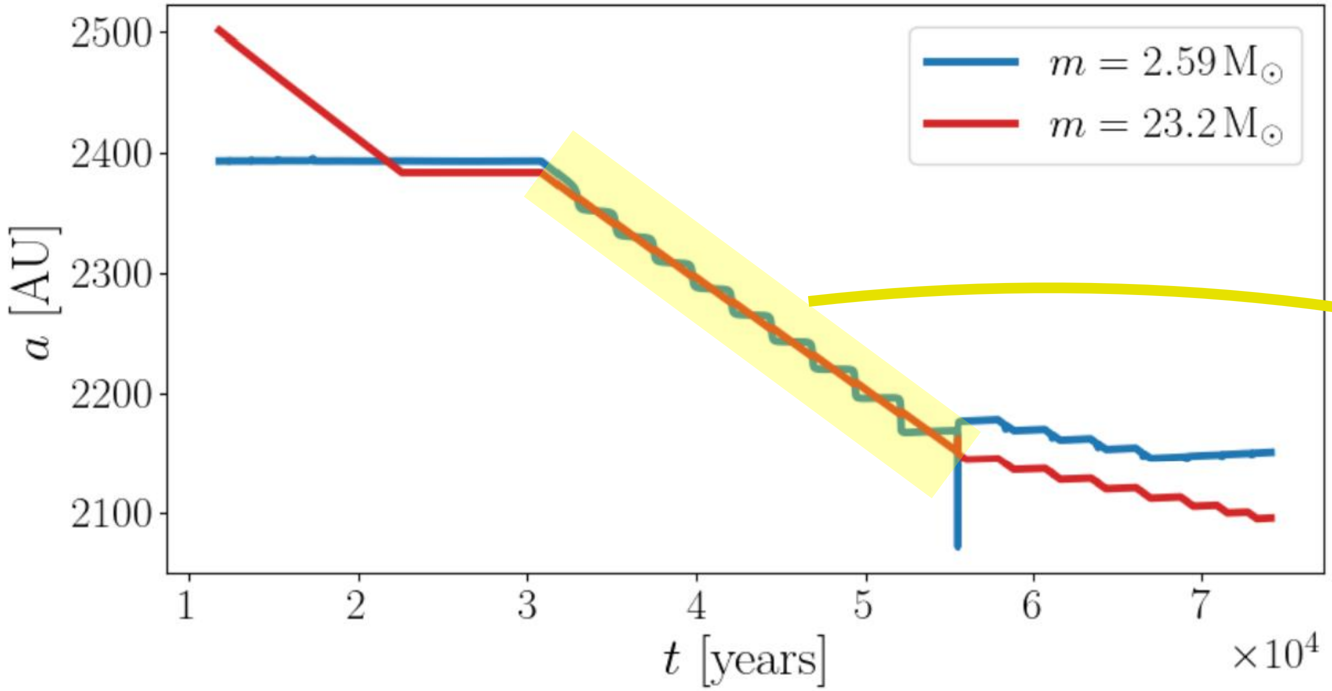
$10\ M_{Sun} + 20\ M_{Sun}$

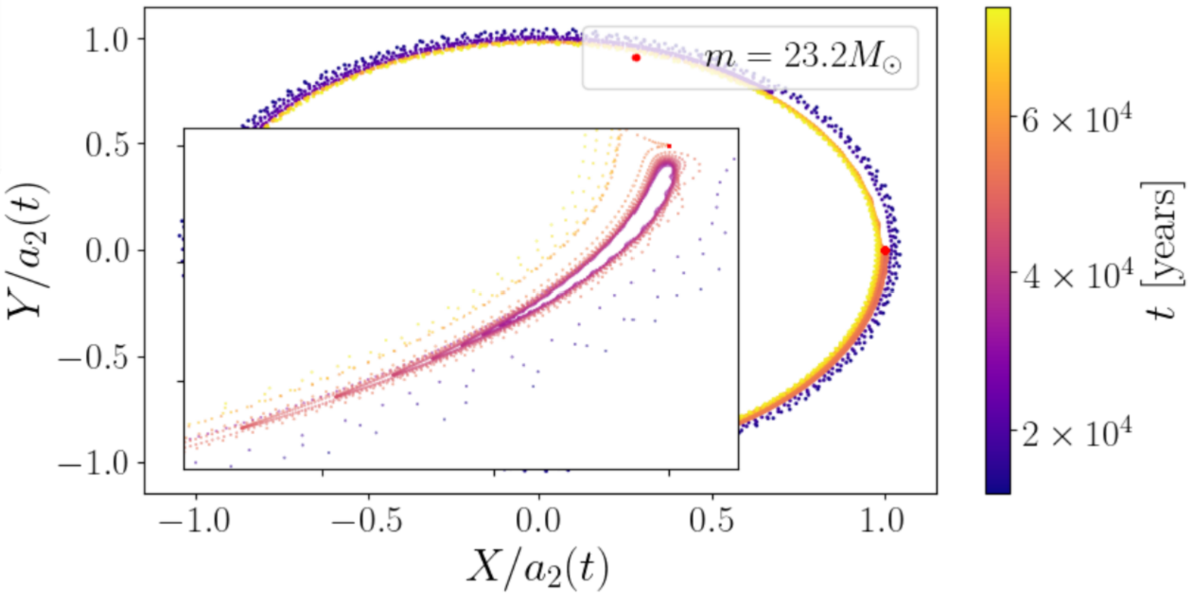
No co-orbital resonances,
merger occurs





Lower mass ratios: higher complexity



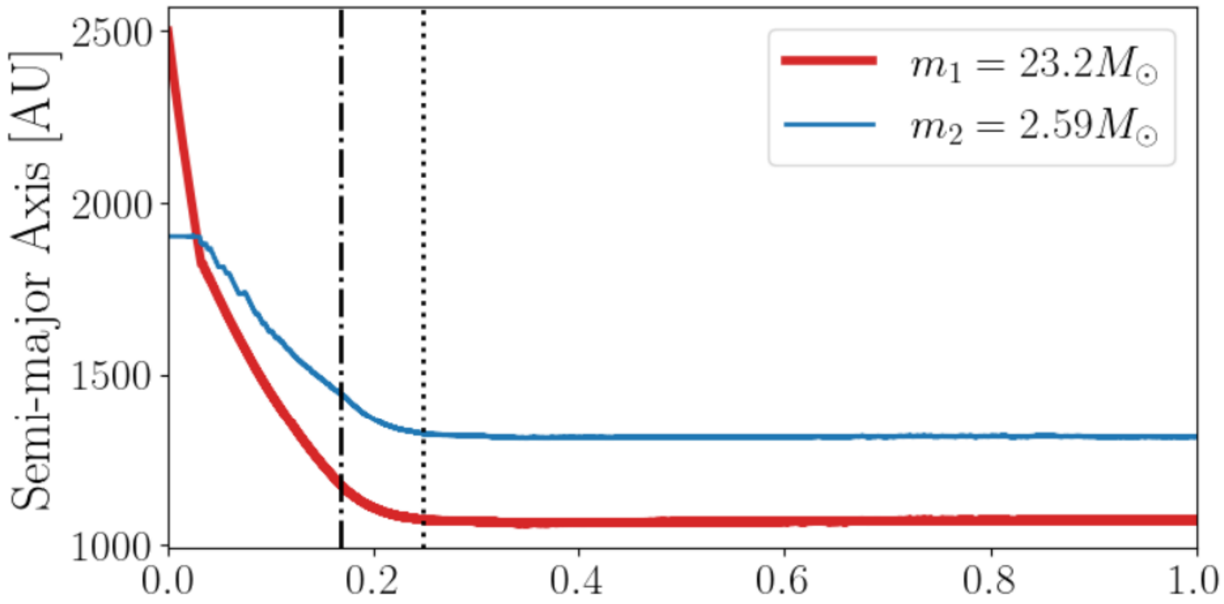


Capture into tadpole orbits:

- Smaller BHs are captured by higher mass ones, in trojan configuration

Moncrieff, Grishin, **Trani** et al., in prep.

Lower mass ratios: higher complexity



+ If resonance is not strong enough, BHs may just cross orbits and miss each other

+ No binary BH is formed

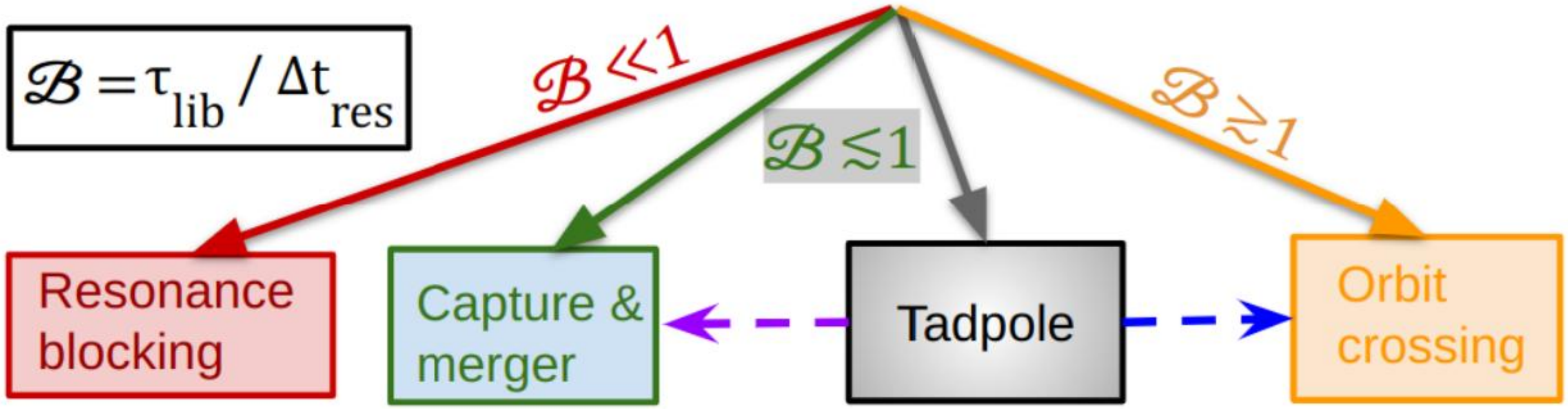
Moncrieff, Grishin, **Trani** et al., in prep.

Semi-analytic criterion to predict capture into mean-motion resonance (Batygin 2015)

+ Capture into mean motion resonance depending on relative migration rate

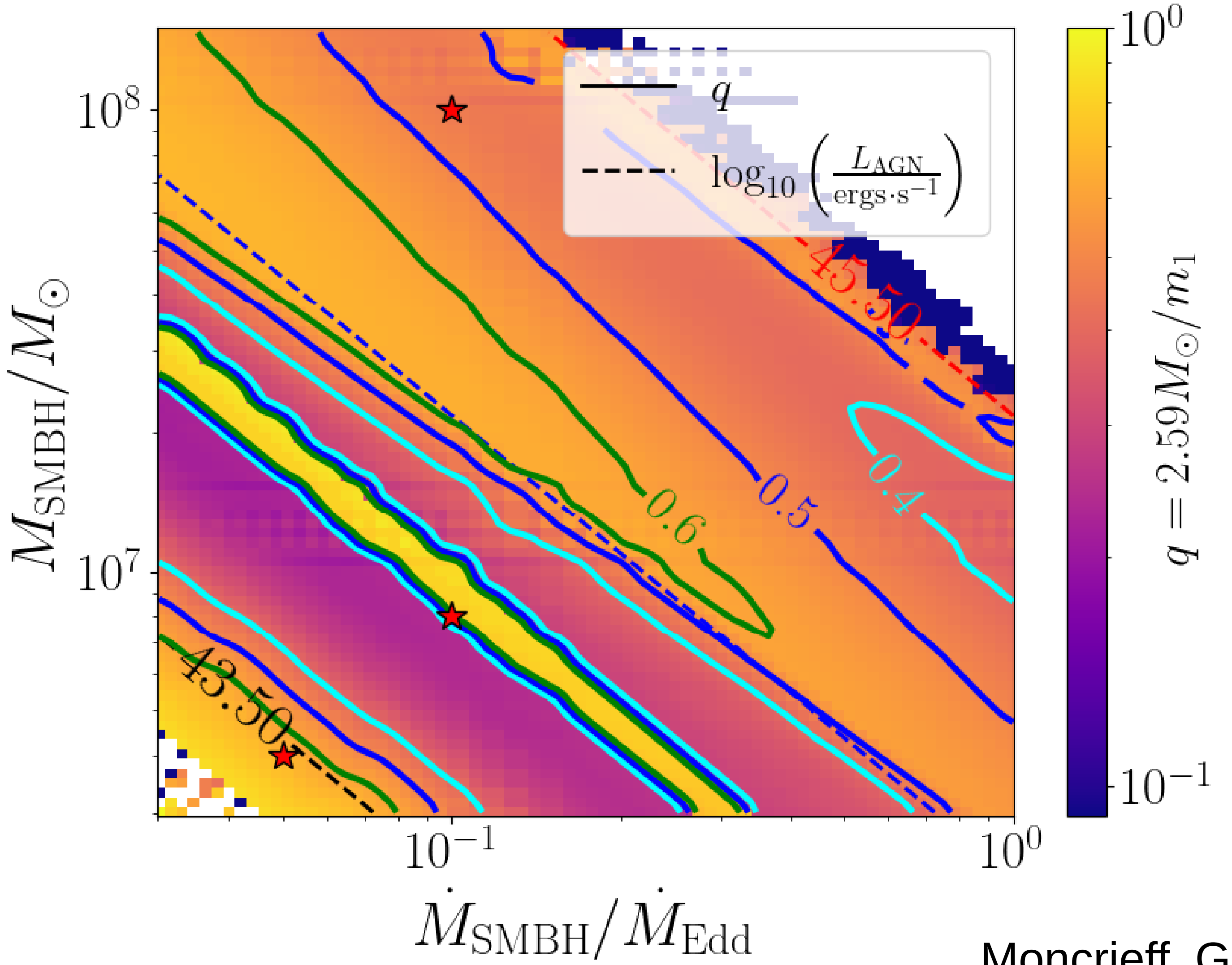
- Migration rate itself strongly dependent on **masses**, AGN **disk structure** and **location** in the disk

$$\mathcal{B} = \frac{\tau_{\rm lib}}{\Delta t} \approx \frac{P_2}{\tilde{\tau}_a}\left(\frac{M_*}{m_1+m_2}\right)^{4/3}\frac{1}{4k^{2/9}(\sqrt{3}|f^{(1)}_{\rm res}|)^{4/3}} < 1$$



Moncrieff, Grishin, **Trani** et al., in prep.

Different AGN disks favor specific merger mass ratios: maps to AGN luminosity



Moncrieff, Grishin, **Trani** et al., in prep.

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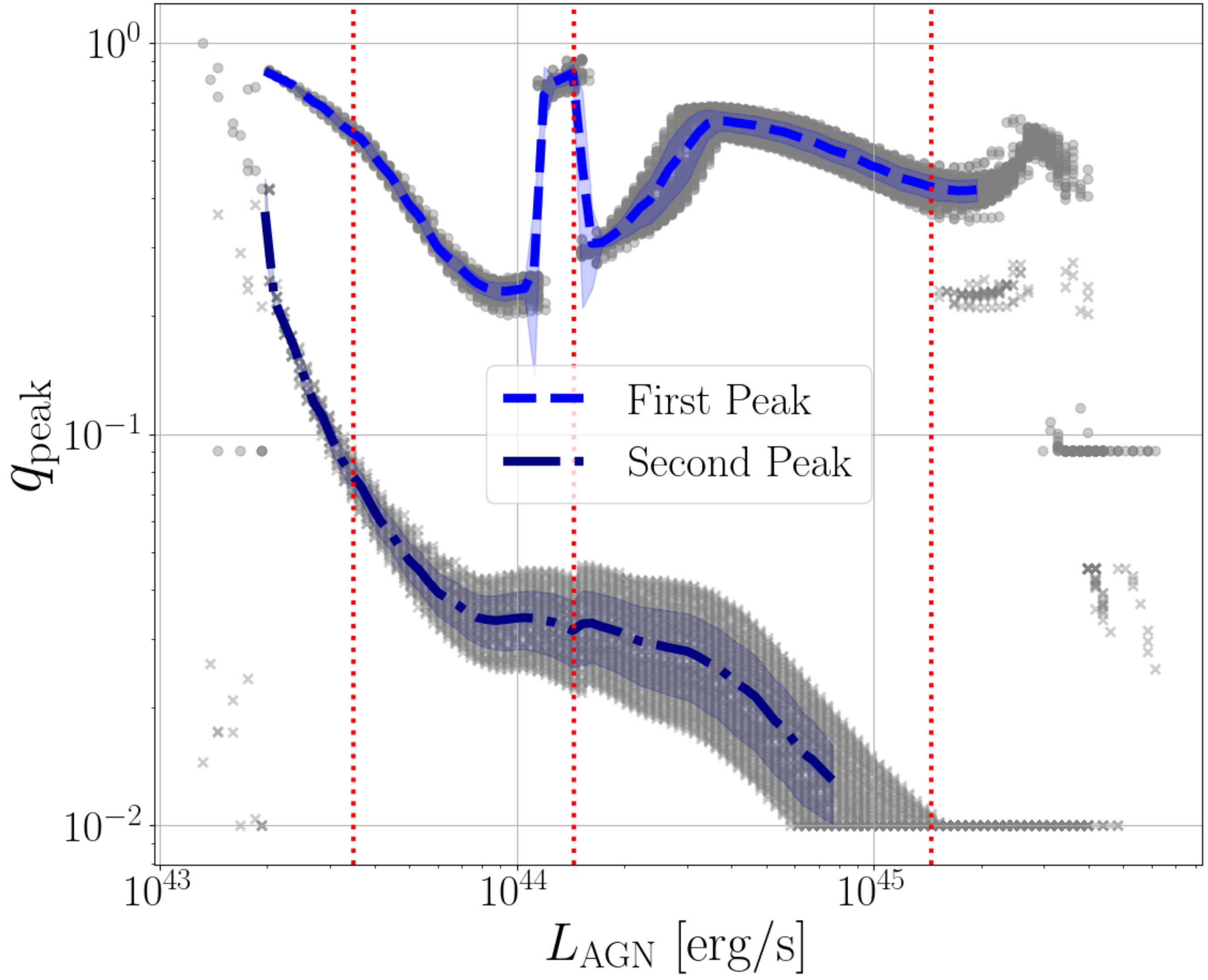


ieff, Grishin, **Trani** et al., in prep.

Take aways

- Merging black holes within AGN disks is not as simple as it sounds: **mean motion resonances and trojan configurations prevent binary formation**

- Different AGNs favor specific merger mass ratios

Not all roads lead to mergers: AGN discs mediate high mass-ratio binary black hole interactions

Jordan W. N. Moncrieff[1,2]★, Evgeni Grishin[2,3]†, Alessandro A. Trani[4,5], Fiona H. Panther[1,2], and Olga Pietrosanti[6]

Caveats – the assumptions so far:

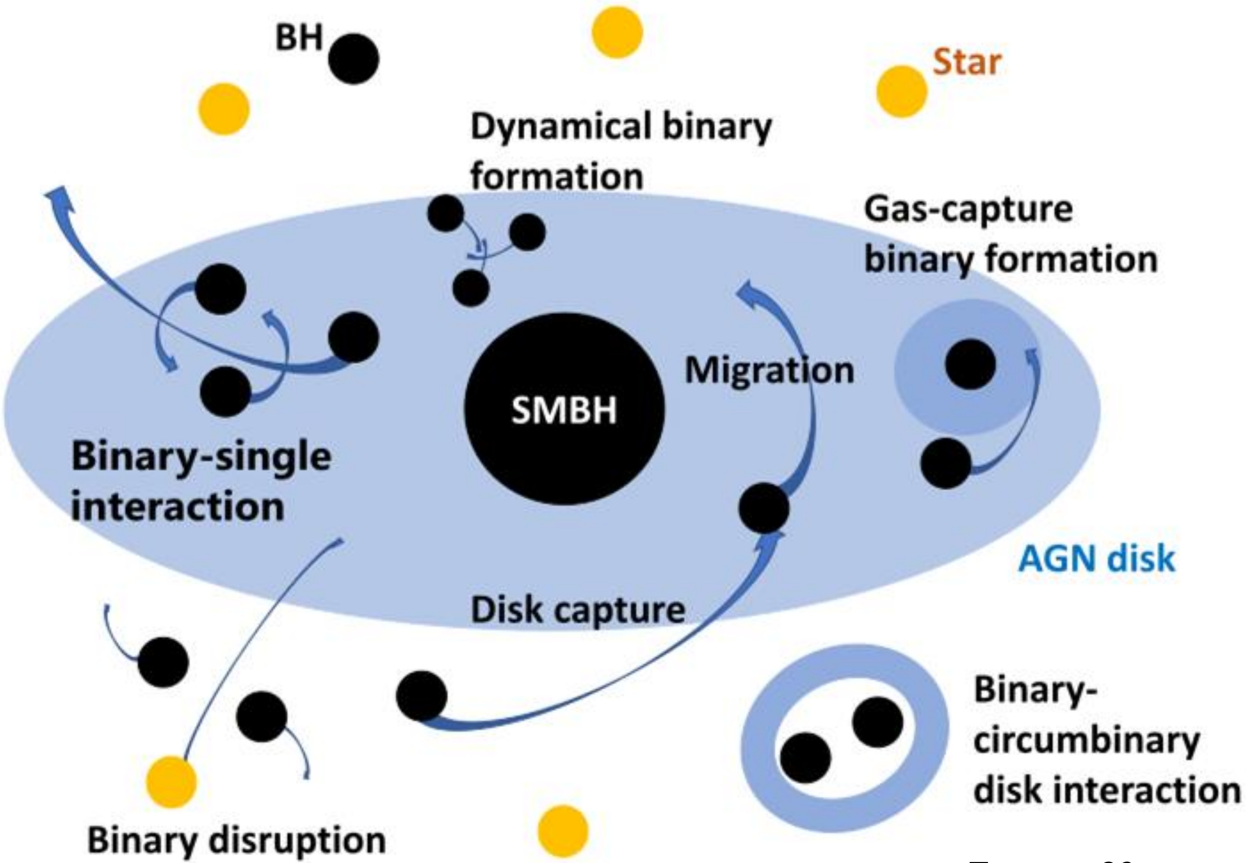
- Independent migration torques: interactions likely exist (Cui,Papaloizou,Szuszkiewicz 2021)
- Laminar gas flow: turbulence might disrupt migration and resonances (**Trani** & Di Cintio 2025; Wu,Chen,Lin 2024)
- Static AGN disk: black hole feedback will alter AGN structure (Epstein-Martin+2025)

Many pathways to merger in galactic nuclei with SMBH (+ AGN disk)

Interactions with the SMBH
- von Zeipel-Kozai-Lidov mechanism
- extreme mass ratio inspirals

Interactions with the AGN disk
- disk captures
- gas-capture binary formation
- disk migration
- binary-circumbinary gas interactions

Interactions among stellar black holes
- single-single (2-body) encounters
- binary-single (3-body) encounters



Tagawa+20

Black holes may form also via starformation within the disk

Migration types

The AGN channel

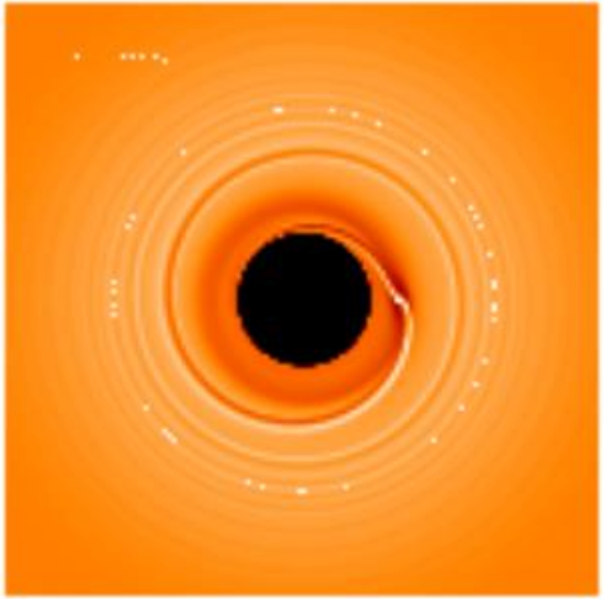
#2

$$\Gamma_{\rm I} = C_{\rm I}\Gamma_0 = C_{\rm I} q^2 r^4 \Omega_{\rm K}^2 \Sigma_{\rm I} \left(\frac{h}{r}\right)^{-2}$$

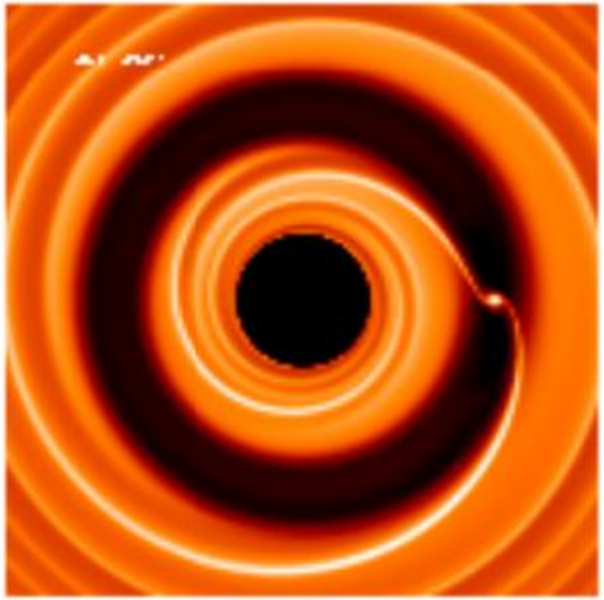
- Small mass, does not open gap
- Net effect from differential Lindblad + corotation torques
- Typically inward, but depends on disk profile

$$\Gamma_{\rm II} = \frac{C_{\rm II}}{C_{\rm I}} \frac{1}{1+K/25} \Gamma_{\rm I}$$

- High mass, opens a gap
- Coupled to viscous evolution
- Always inward



Type I



Type II

$$\Sigma_{\rm II} = \frac{\Sigma_{\rm I}}{1+K/25},$$

$$K = q^2 \left(\frac{h}{r}\right)^{-5} \alpha^{-1}.$$

Migration types

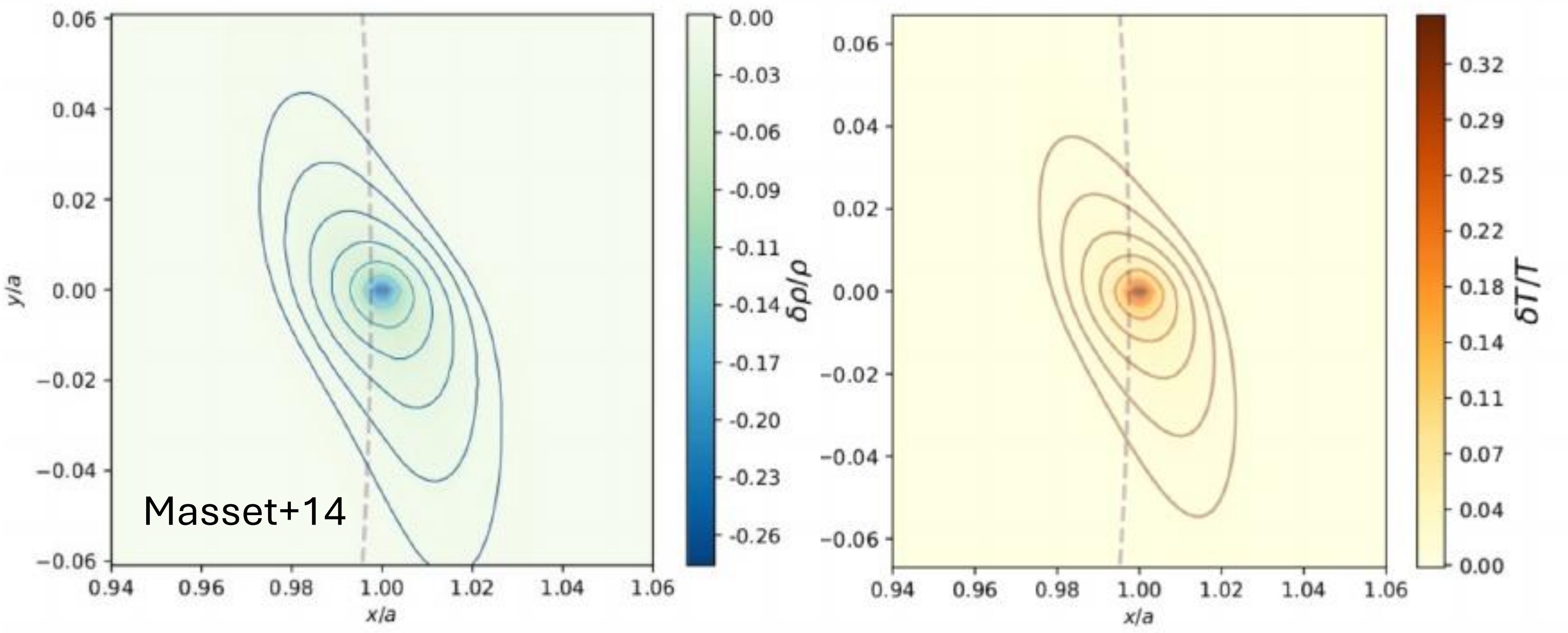
The AGN channel

#2

Thermal torque

$$\Gamma_{\rm th} = \Gamma_{\rm hot} + \Gamma_{\rm cold} = 1.61\frac{\gamma-1}{\gamma}\frac{x_c}{\lambda_c}\left(\frac{L_{\rm BH}}{L_{\rm c}} - 1\right)\left(\frac{h}{r}\right)^{1}\Gamma_0$$

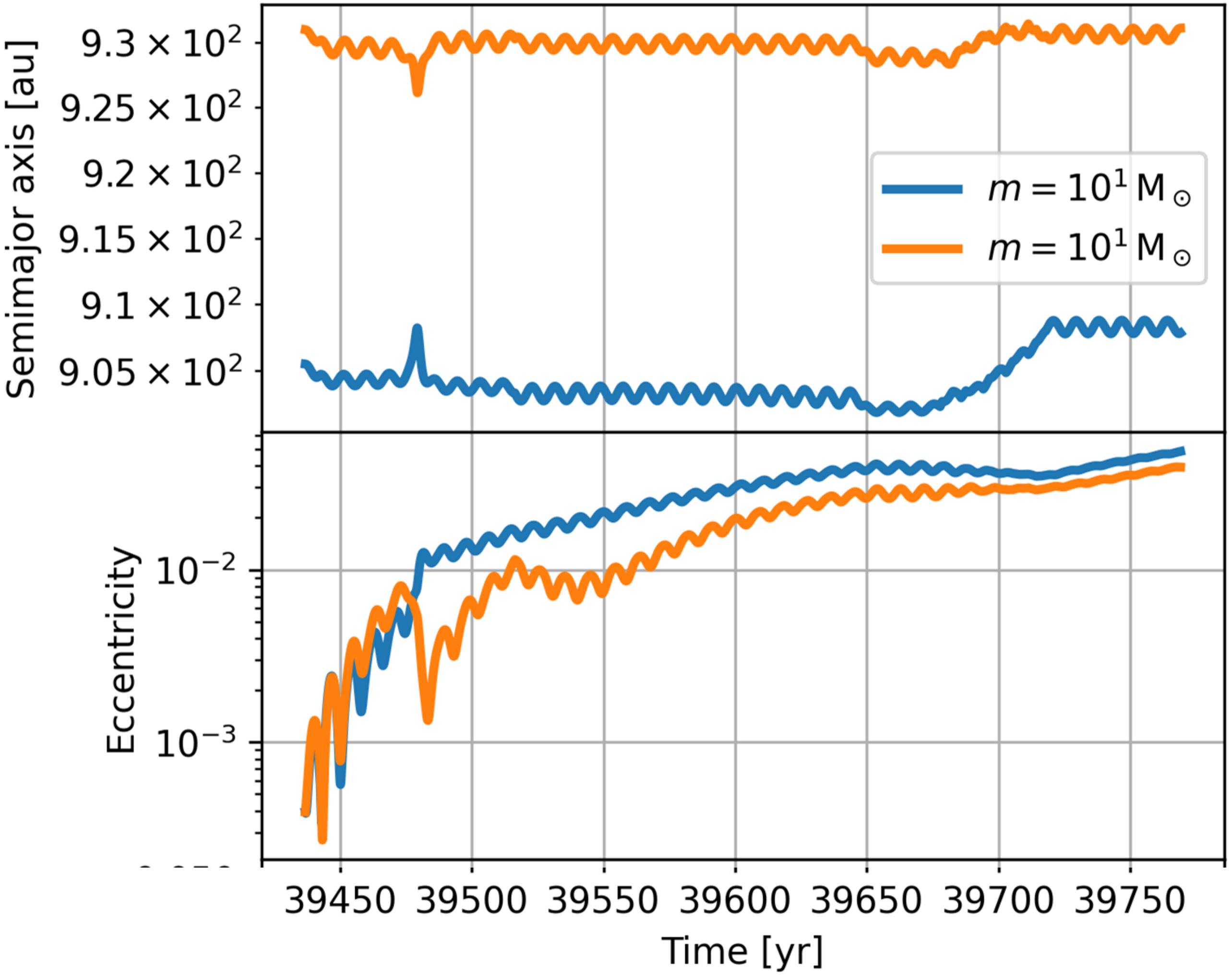
- Caused by heating from embedded body
- Heated, underdense trailing lobe induces a positive torque
- Can reverse inward migration



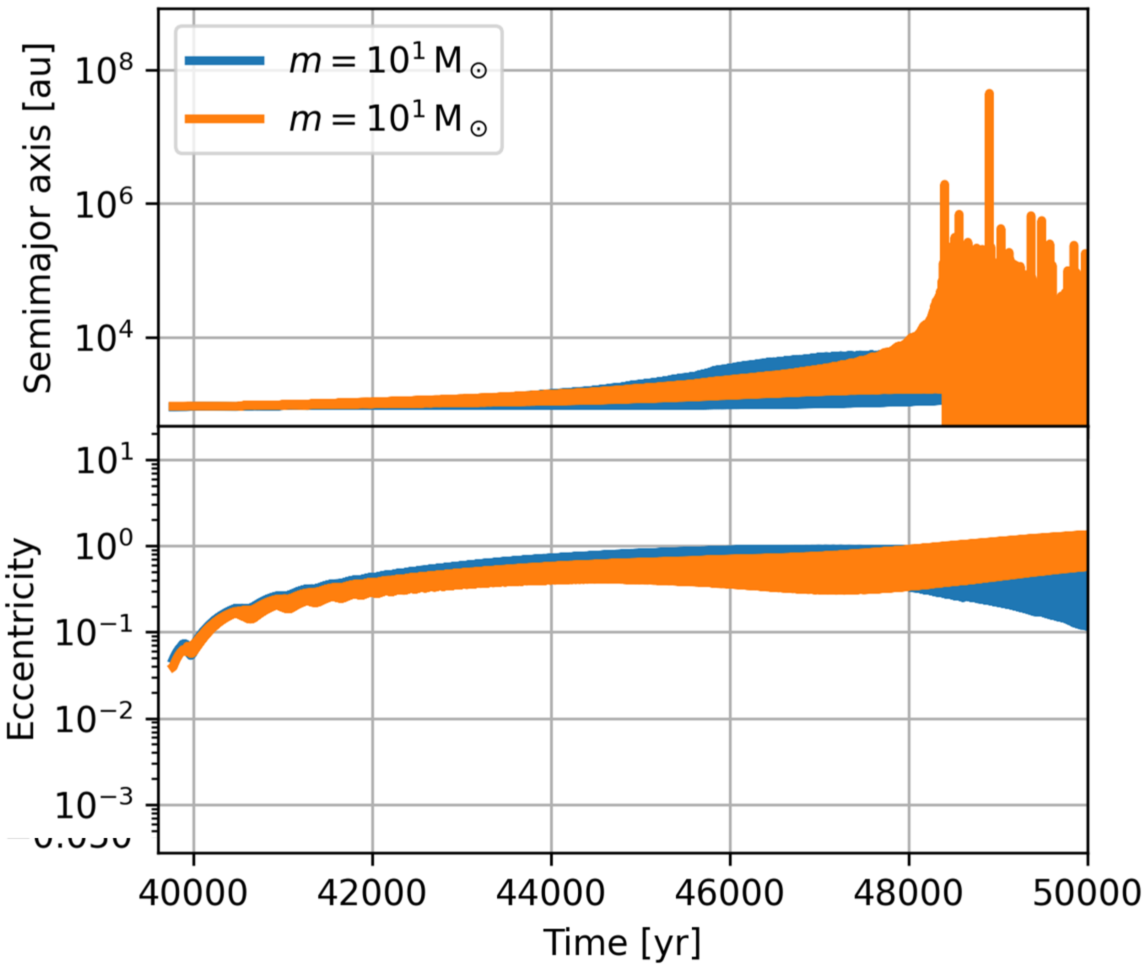
Not a stable resonance:

migration and eccentricity damping keep it in check

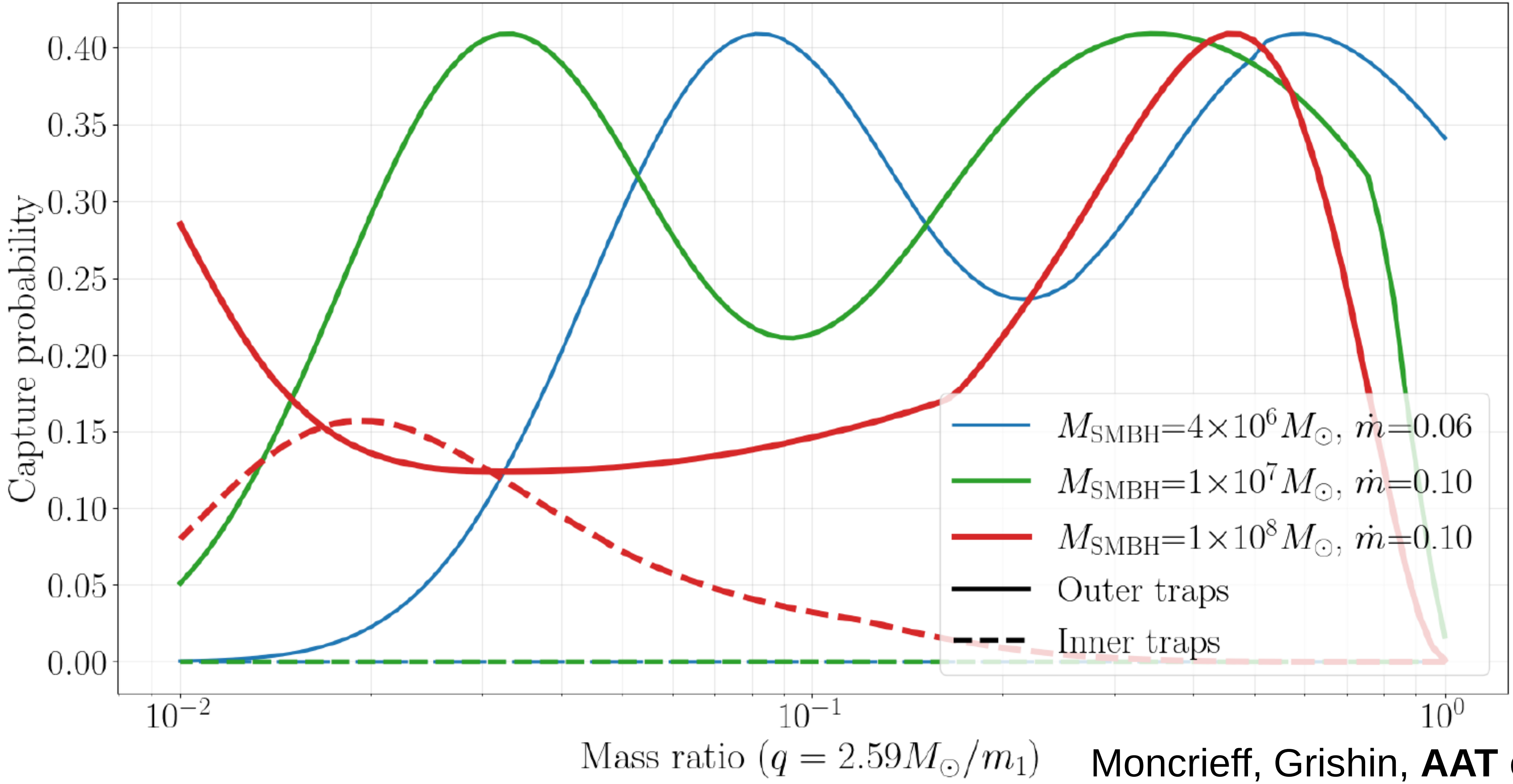
After turning off migration:



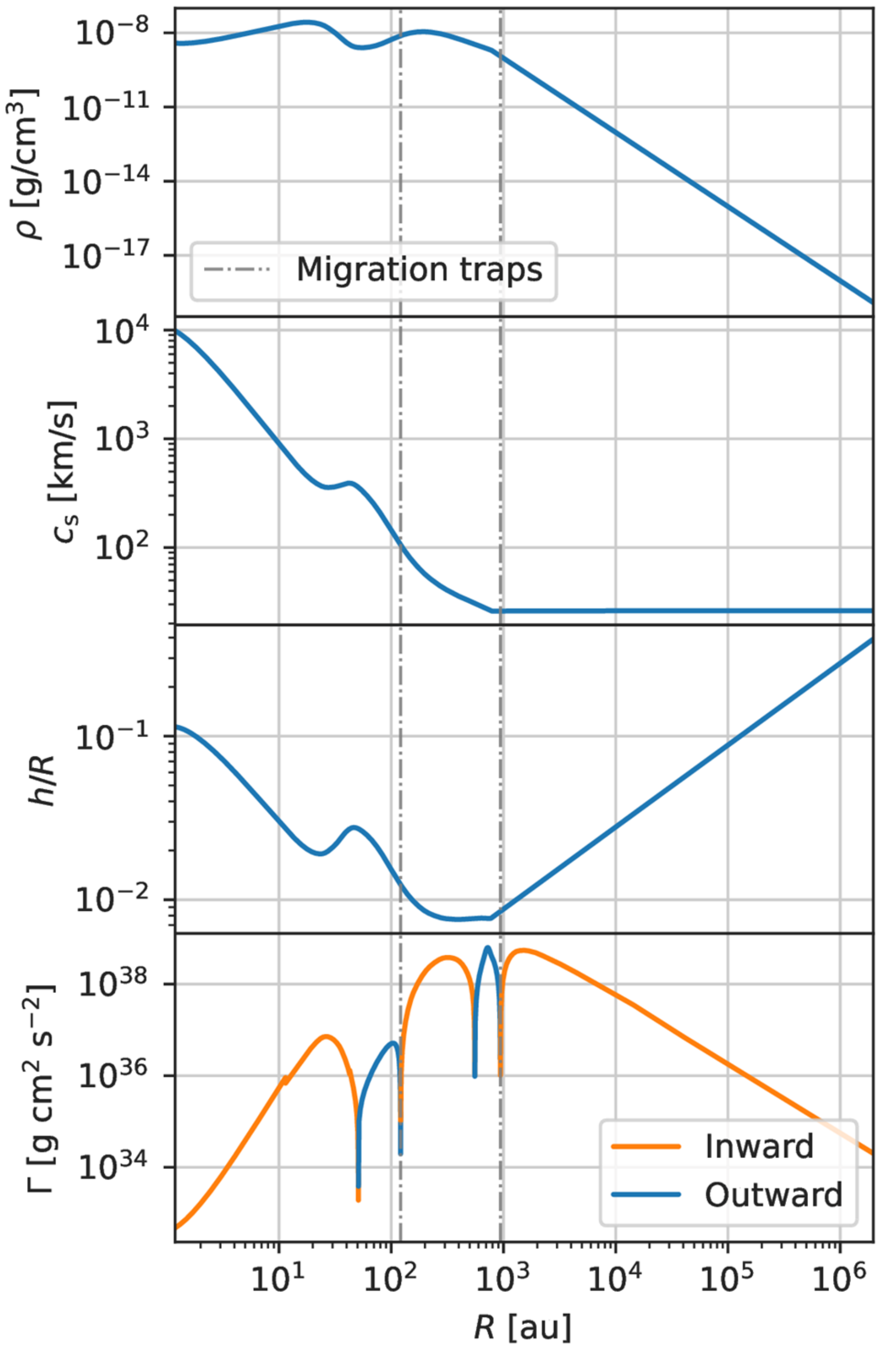
Final outcome: merger



Different AGN disks favor mergers of specific mass ratios



Moncrieff, Grishin, **AAT** et al., in prep.



DOI 10.5281/zenodo.10723301

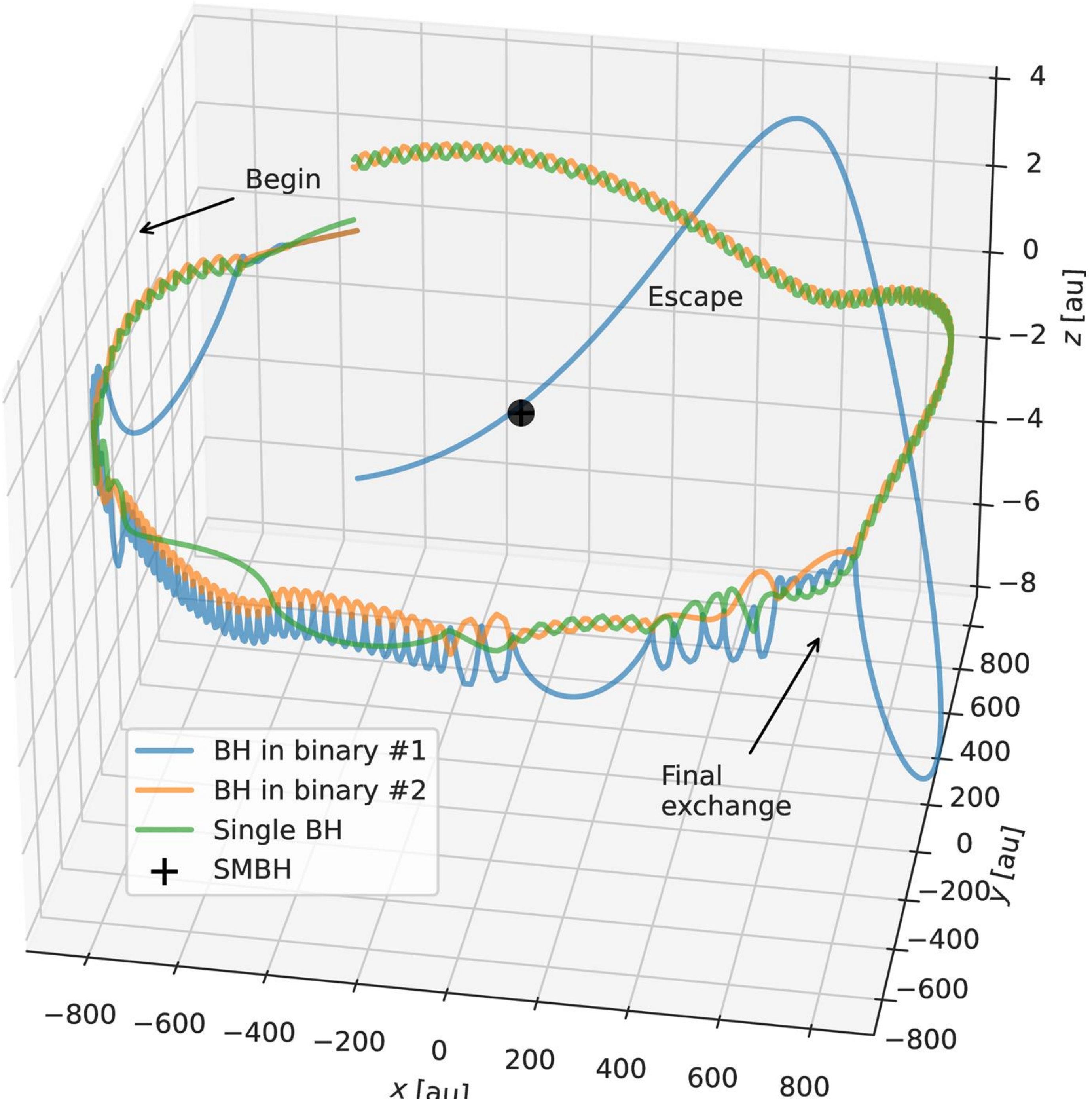
pagn

Gangardt, **AAT**, Bonnerot, Gerosa 2024

pagn is a Python module to solve the equations from Sirko and Goodman 2003 and Thompson et al. 2005 in a way that is simple, clear and highly customisable. Our aim is to provide a code that with the correct inputs provides a fully evolved AGN disk model through parametric 1D curves for key disk parameters such as temperature and density. We expect key applications of this code to be studies of migration torques in AGN disks, simulations of compact object formation inside gas disks, and comparisons with new, more complex models of AGN disks.

Working on the next code update

(new opacity tables with varying metallicity, updated migration torque calculations, and more)



Numerical methods

TSUNAMI

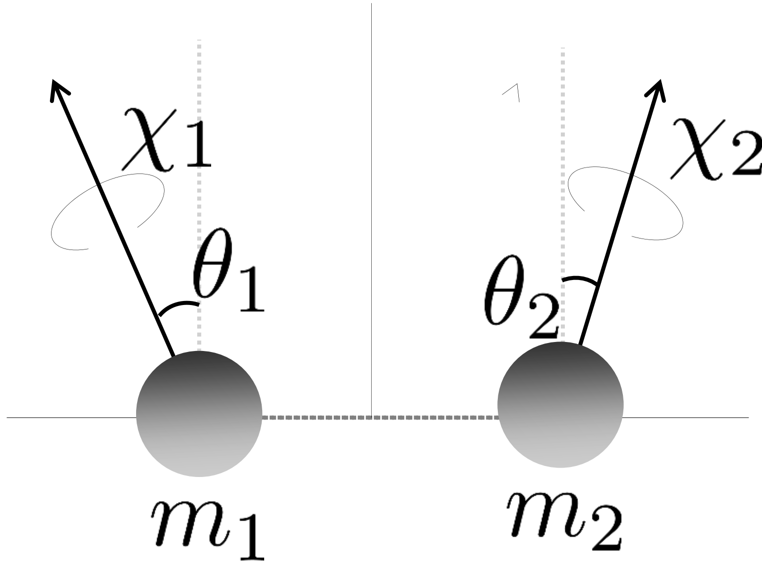
Trani & Spera 2023

- Accurate due to Bulirsh-Stoer extrapolation to zero timestep
- Regularized Hamiltonian: avoids the singularity at $r \to 0$
- Post-Newtonian corrections up to 3.5PN
- Tidal forces (dynamical + equilibrium tides)
- Spin-orbit coupling (tidal + PN)
- Python interface

What do gravitational waves tell us?

Main observables:

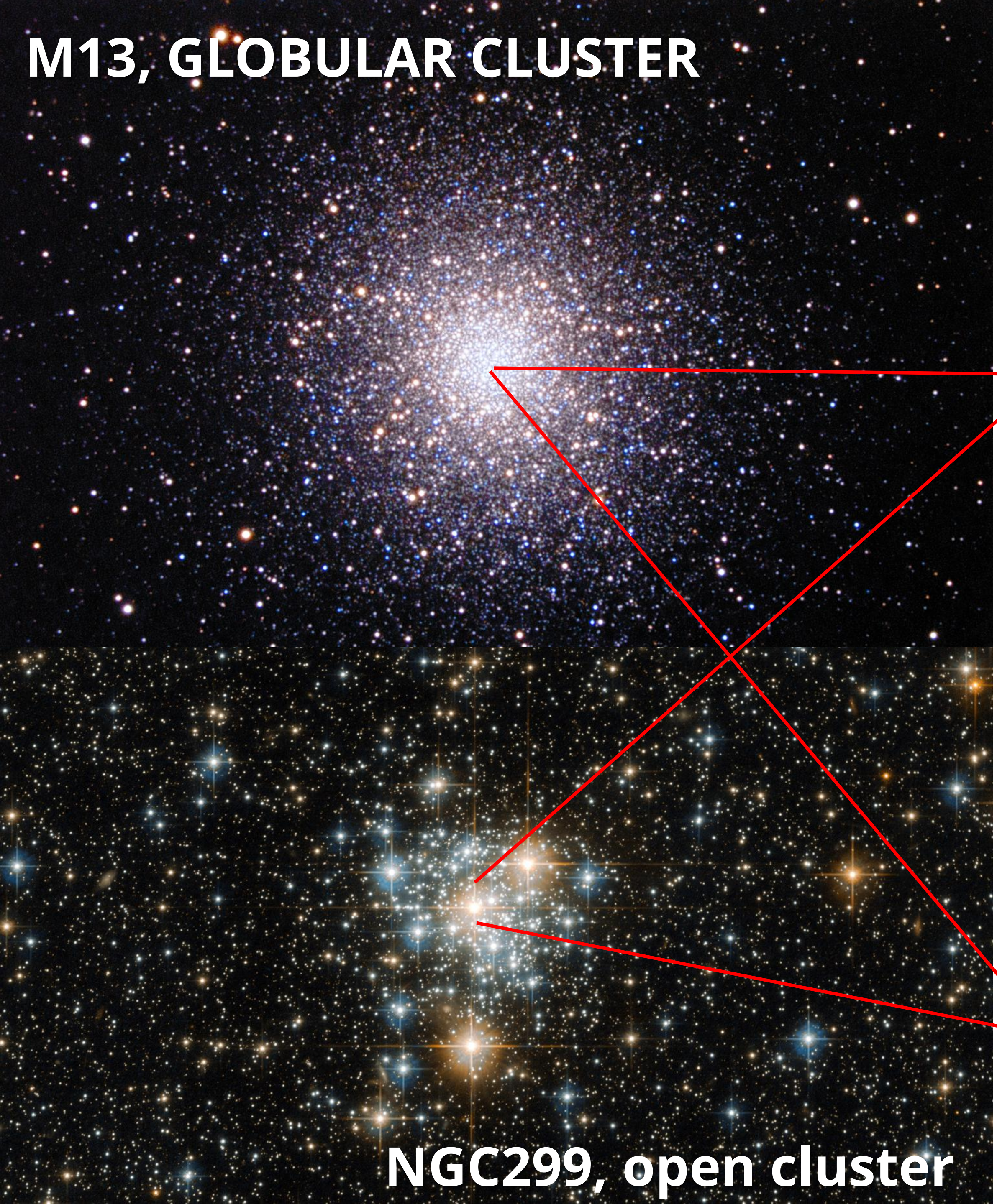
- masses and their ratio $q = \frac{m_2}{m_1} < 1$
- spins and their orientation
- merger rate ~14-26 yr^{-1} Gpc^{-3} (BH-BH)
- eccentricity
- sky localization and distance

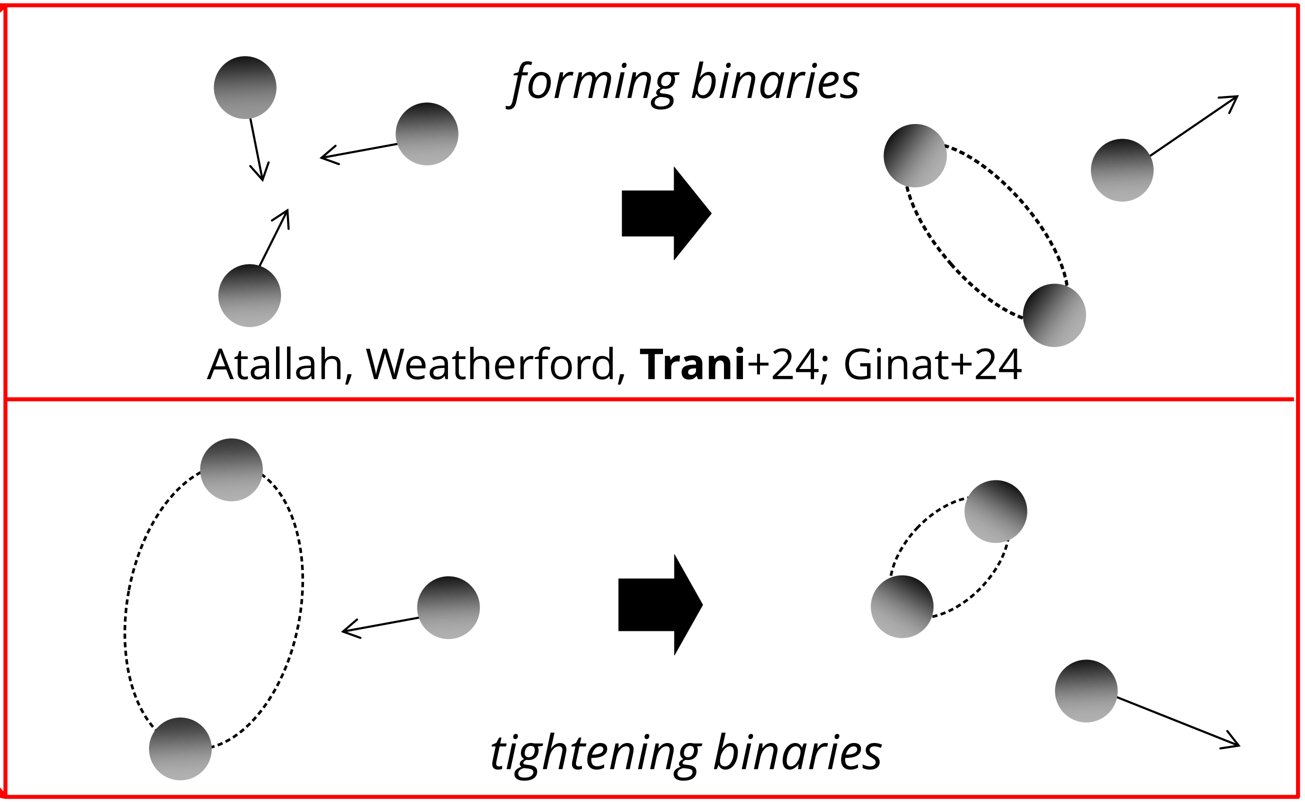


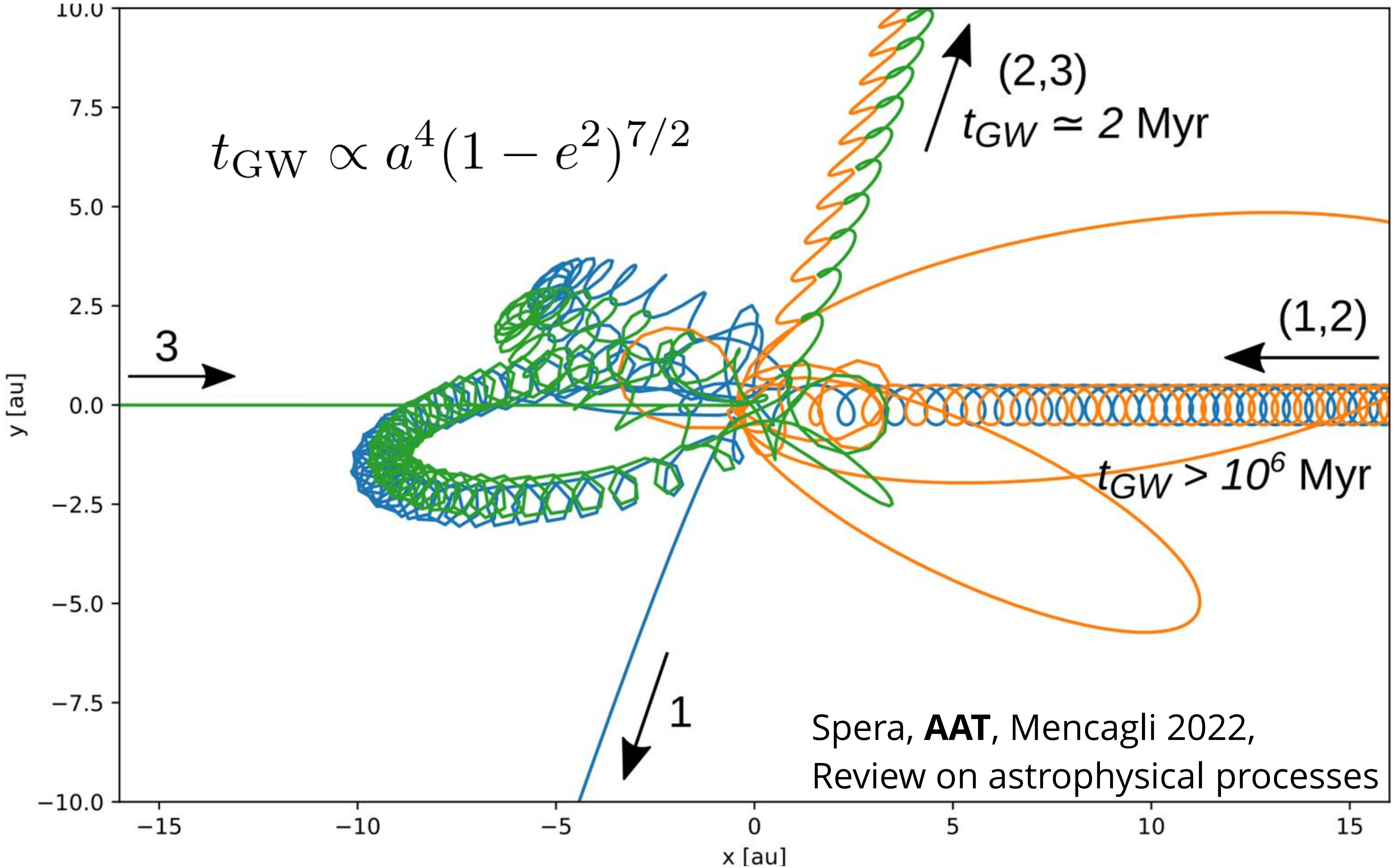
$$\chi_{\rm eff} = \frac{m_1\chi_1\cos\theta_1 + m_2\chi_2\cos\theta_2}{m_1 + m_2}$$

Three-body encounters:

key process for producing GW events from compact object binaries







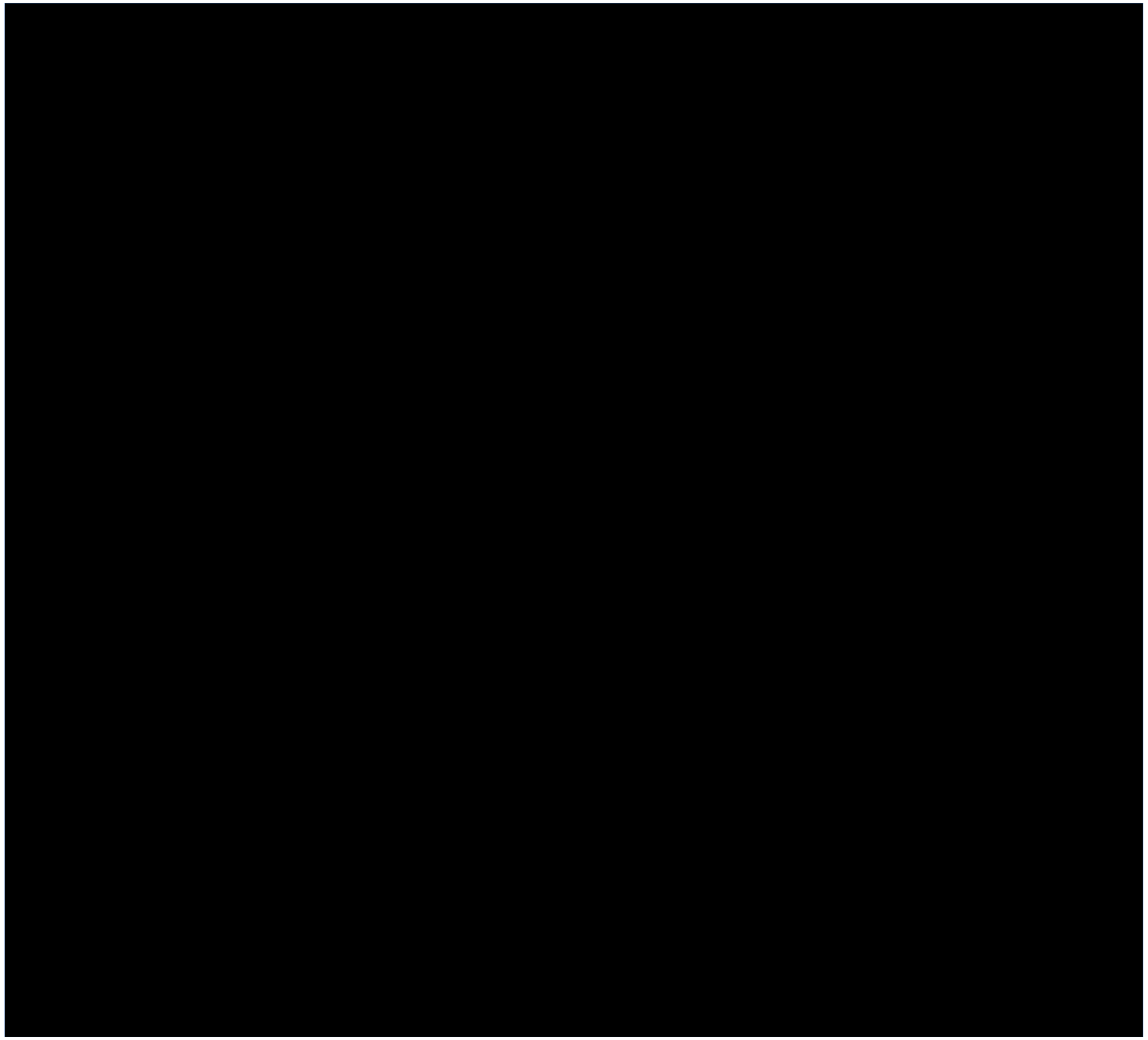
Spera, **AAT**, Mencagli 2022,
Review on astrophysical processes

TAKE AWAY MESSAGE #1

Key features of GW sources from three-body encounters in star clusters:

1. ECCENTRICITY

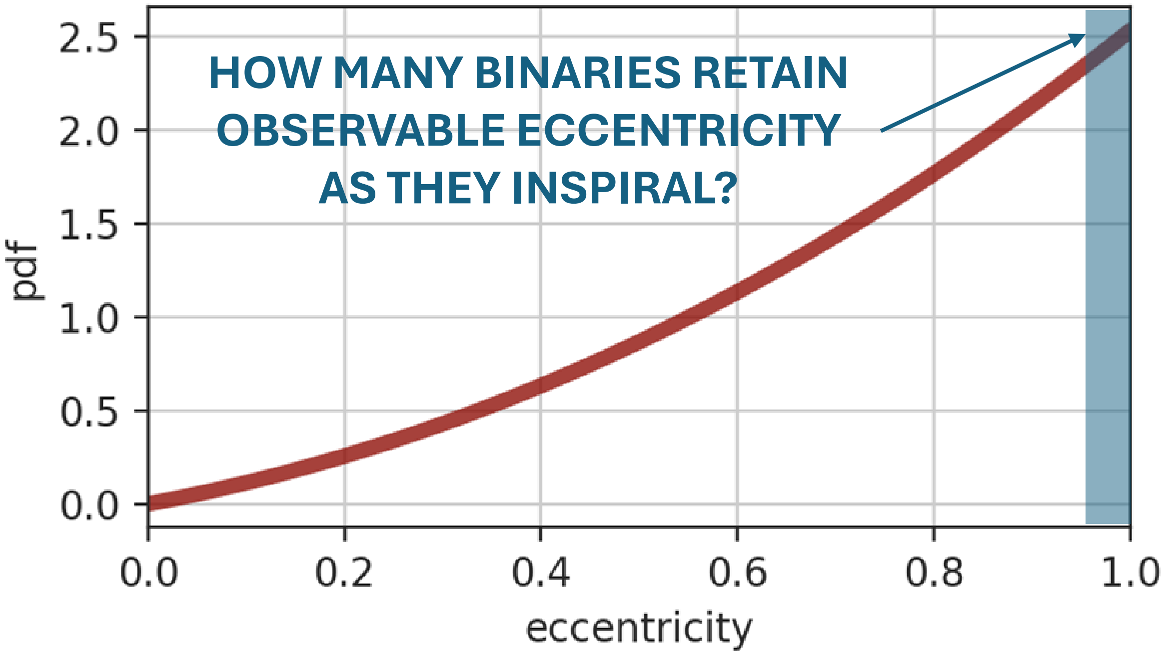
HOW TO PREDICT THE ECCENTRICITY OF GW SOURCES?



Chaotic 3-body interactions justify the use of statistical approaches

Monaghan 1976a,b; Valtonen & Karrtunen 2006;
Stone & Leigh 2019; Kol 2022; Ginat & Perets 2023

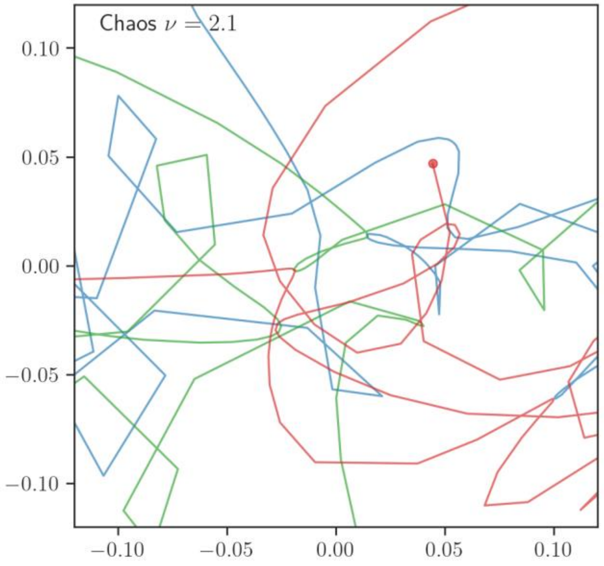
Eccentricity distribution of post-encounter binaries



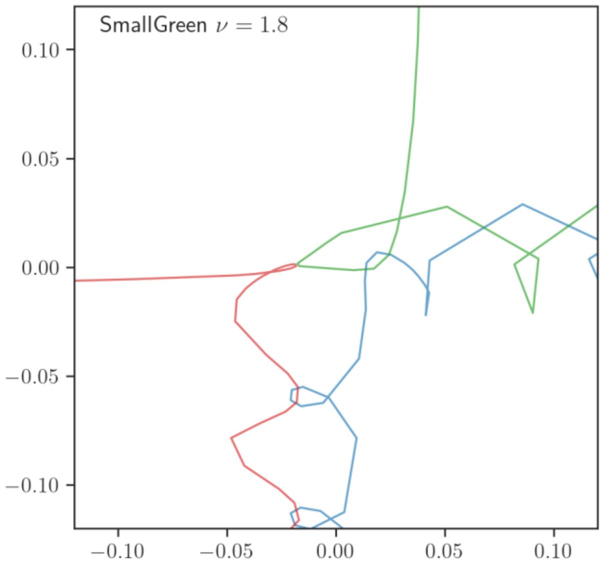
WHEN STATISTICS FAIL: REGULAR THREE-BODY INTERACTIONS

~1/3rd of interactions are regular (non-chaotic):
statistical theories fail to predict the most eccentric binaries

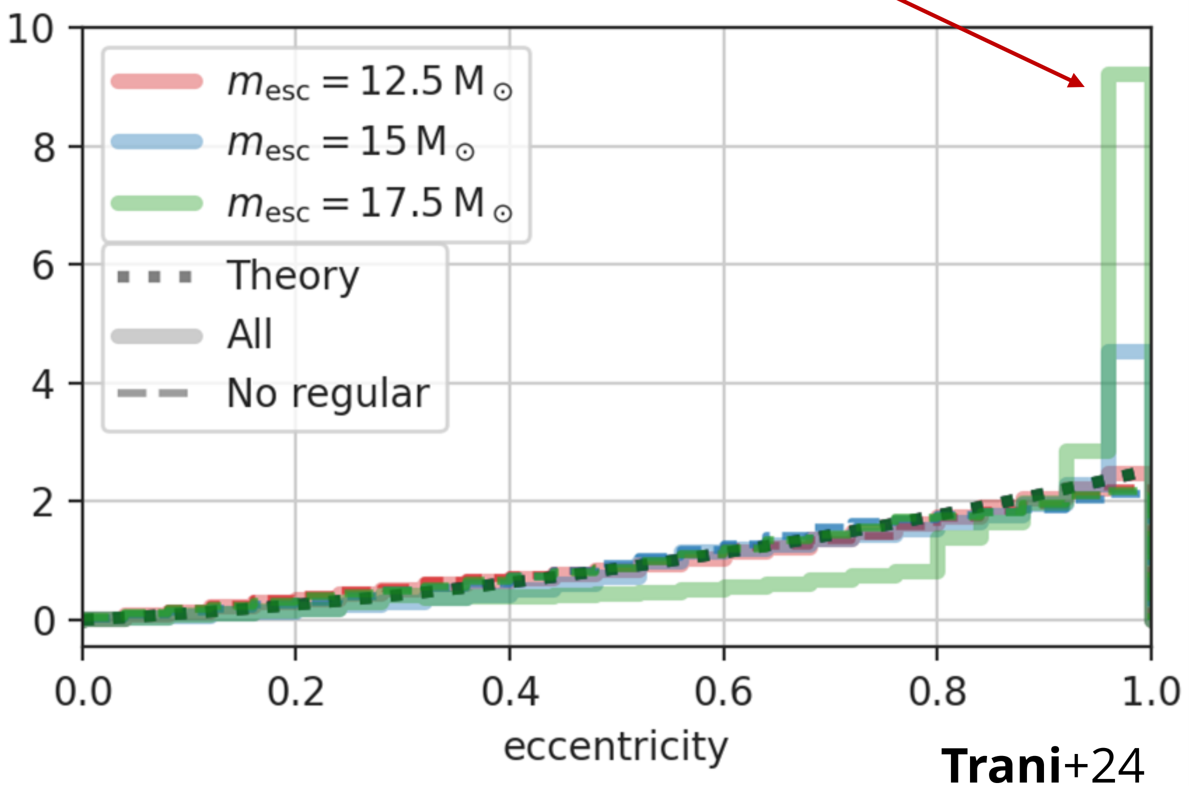
“Messy” chaotic interaction



“Clean” regular interaction

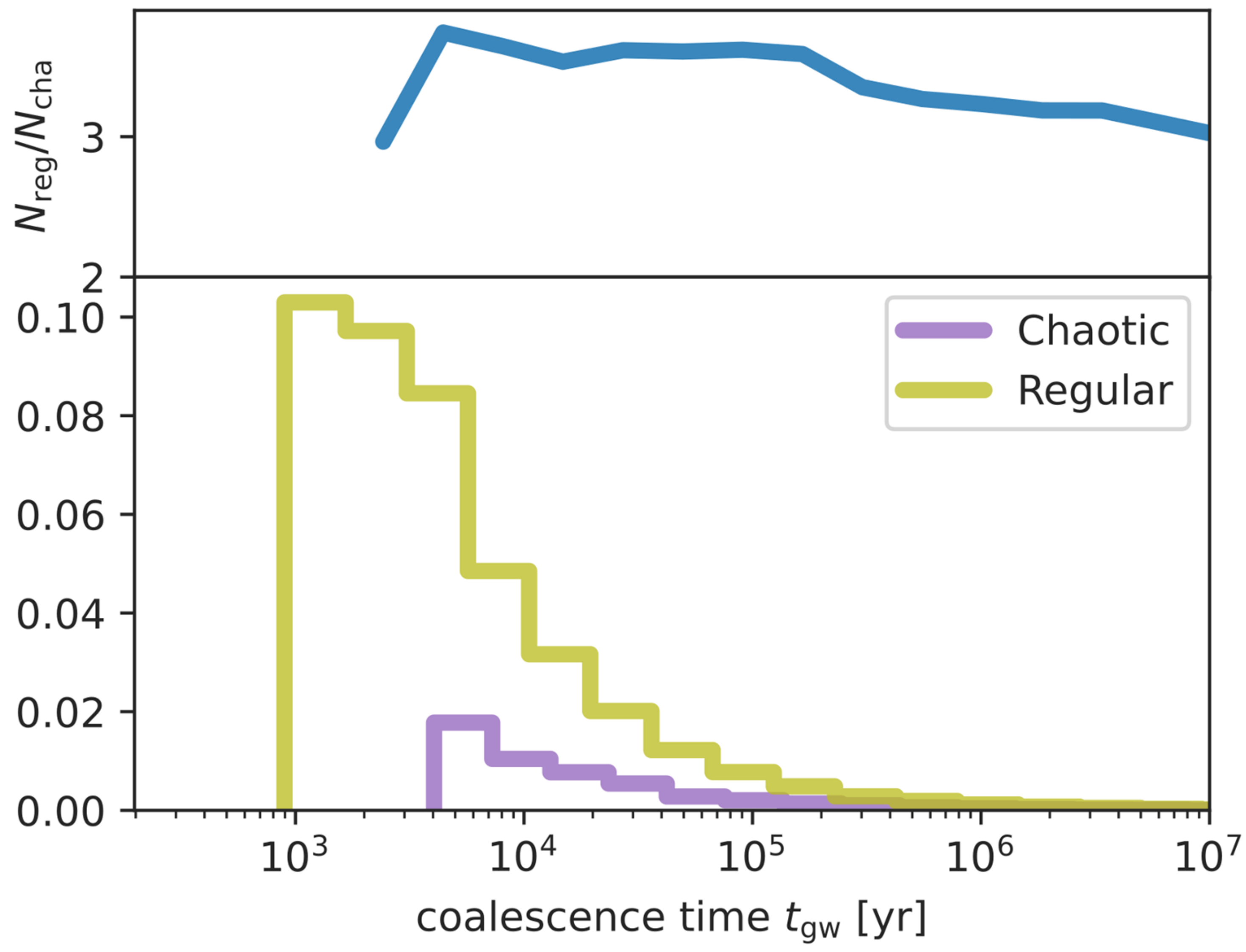


Binaries from regular interactions: highly eccentric



Trani+24

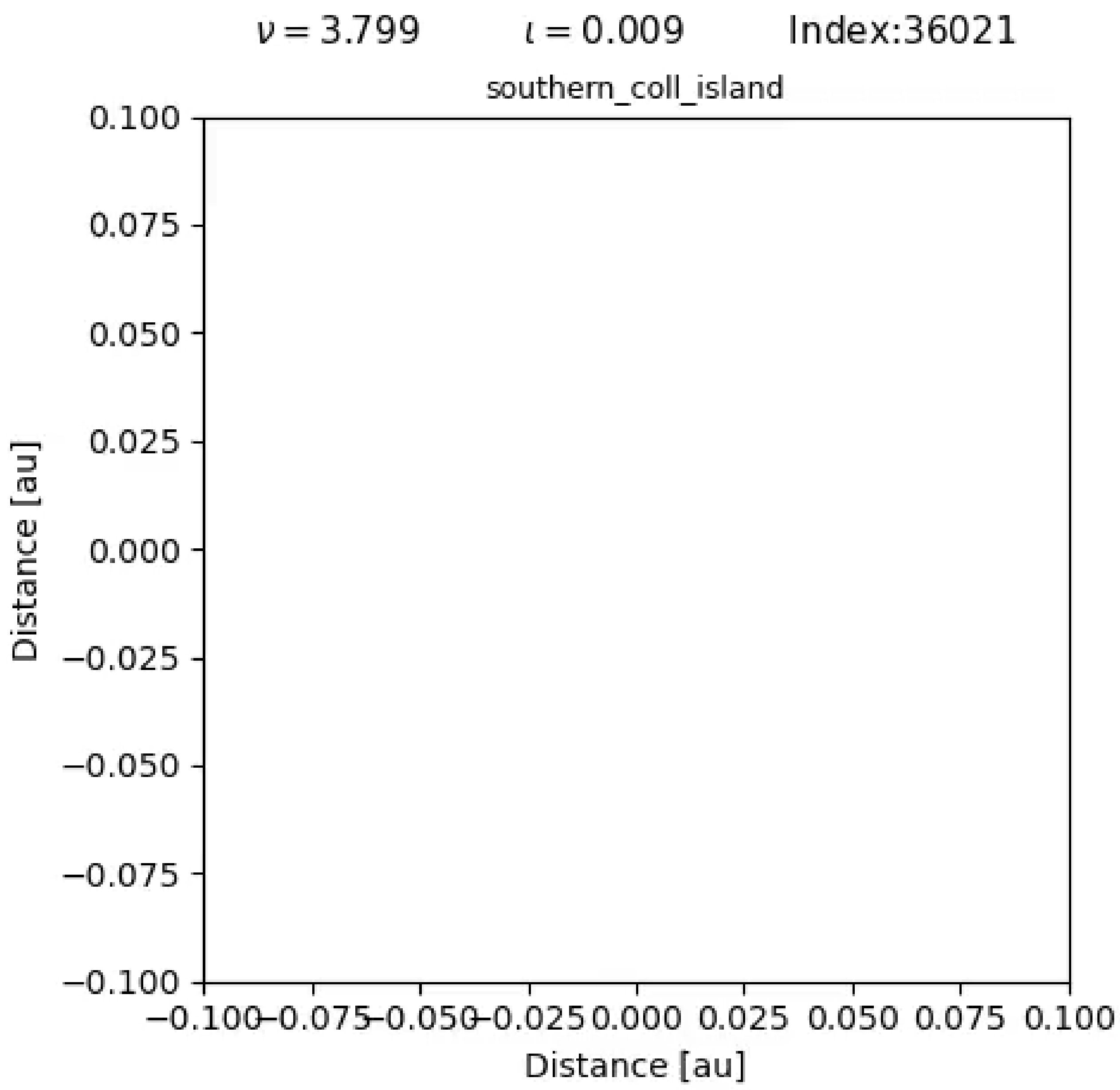
BINARIES FROM REGULAR INTERACTIONS MERGE QUICKER



- Binaries from **regular interactions** coalesce in shorter time
- **3 times more numerous** than chaotic binaries

Chaotic vs regular channels for binary formation may produce distinct GW populations

Trani+24



Excursion inspiral

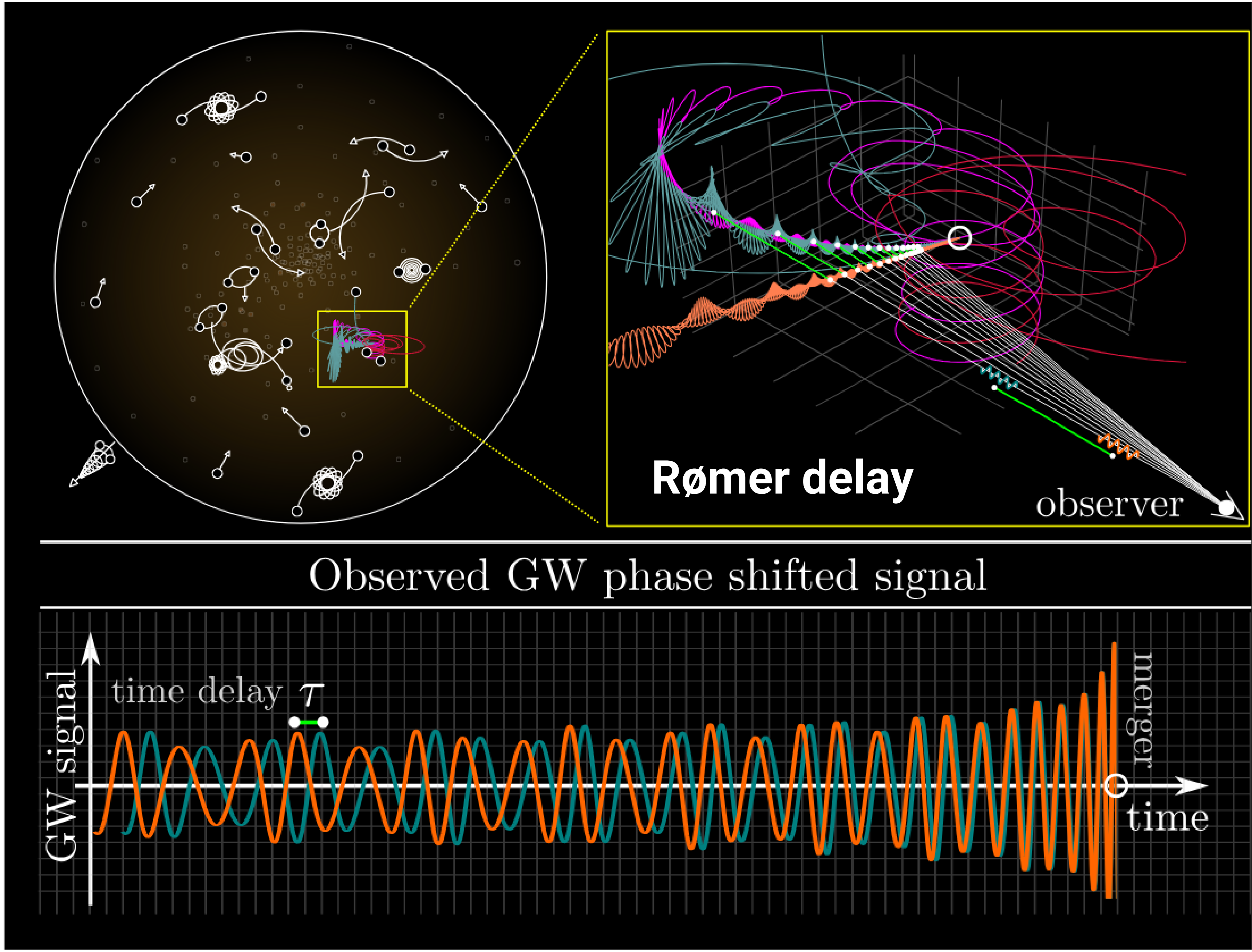
TAKE AWAY MESSAGE #2

Key features of GW sources from three-body encounters in star clusters:

1. ECCENTRICITY
2. **PERTURBED**

Phase shift as smoking gun for individual events in dense environments

Samsing+25;Zwick+25;
Hendriks,incl.**AAT+**,submitted



Phase shift as smoking gun for individual events in dense environments

Samsing+25;Zwick+25;
Hendriks,incl.**AAT+**,submitted

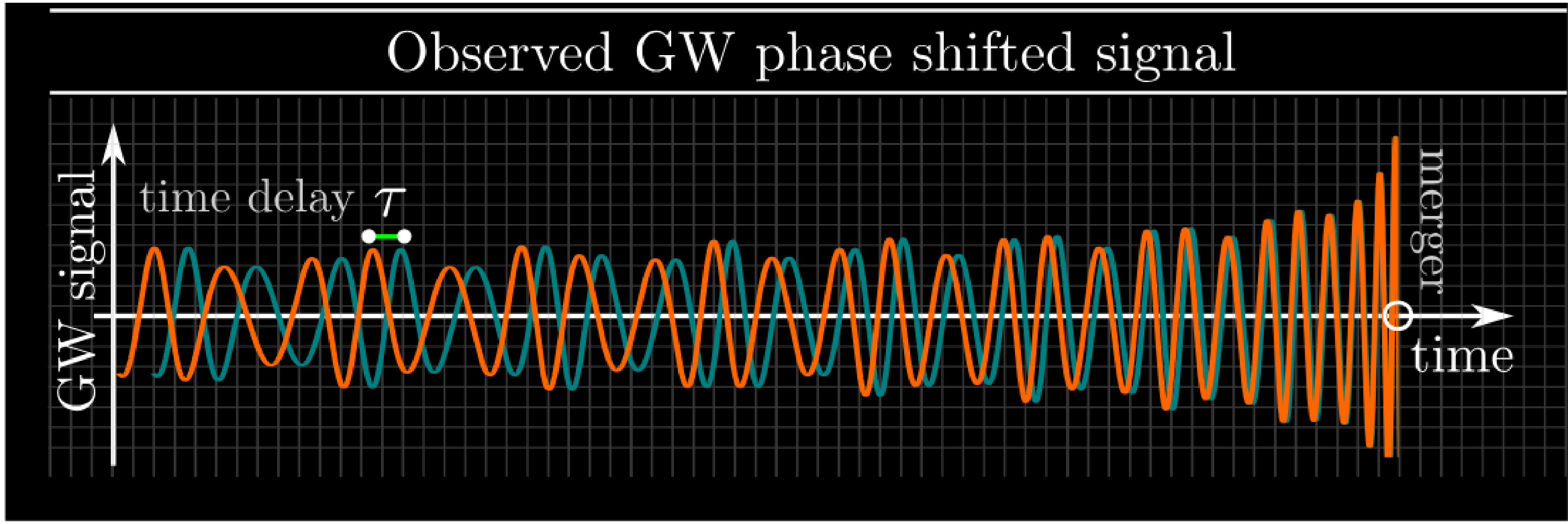
Possibly detectable with ET?

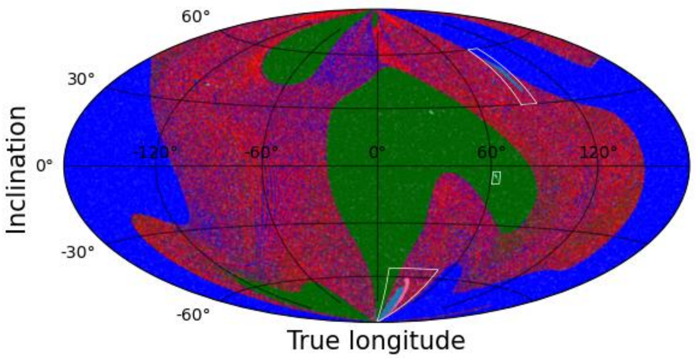
Phase shift maps directly to mass and distance of the perturber

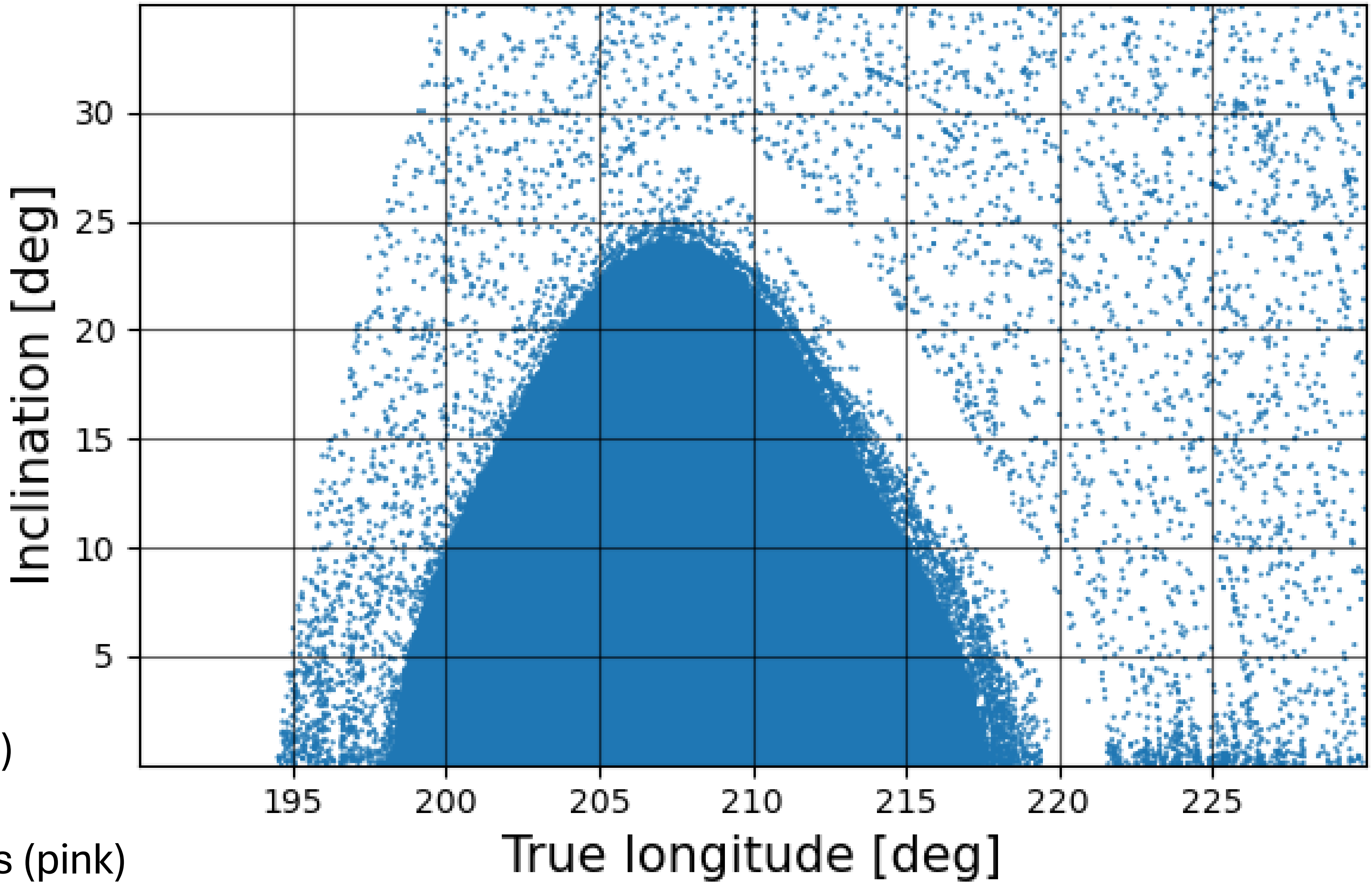
one finds that $\Delta\phi$ can be written as,

$$\Delta\phi \approx \frac{1}{2}\frac{G^{3/2}}{c}\frac{m_3 m_{12}^{1/2}}{R^2} \times \frac{t^2}{a(t)^{3/2}}, \tag{9}$$

where $a(t)$ is the SMA of the BBH at time t, R is the



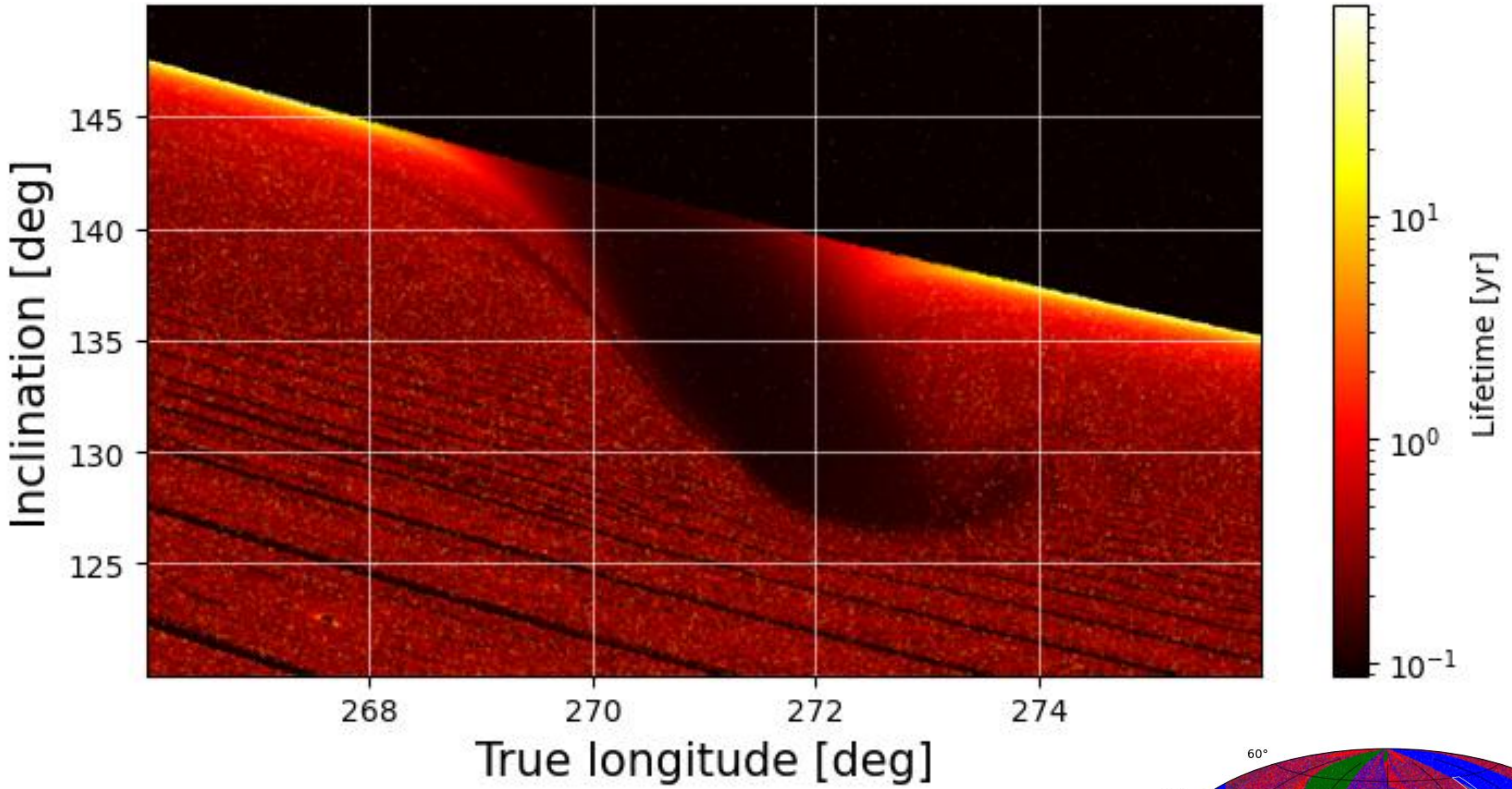




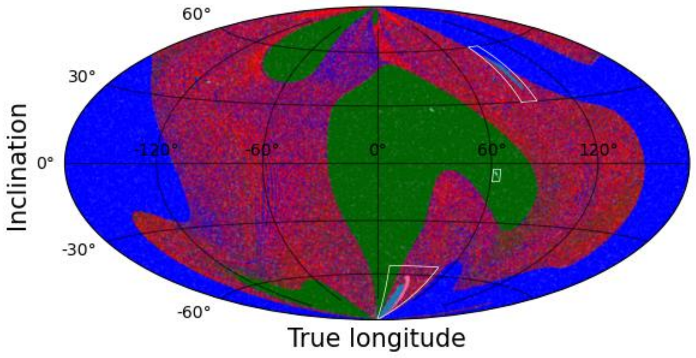
Head-on collision island (blue)

Inspiral and head-on collisions (pink)

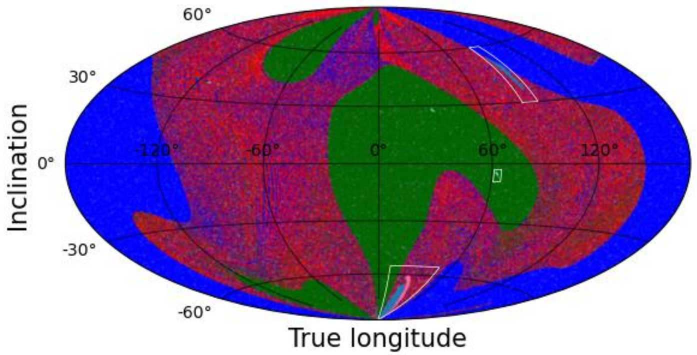
Halo of collisions



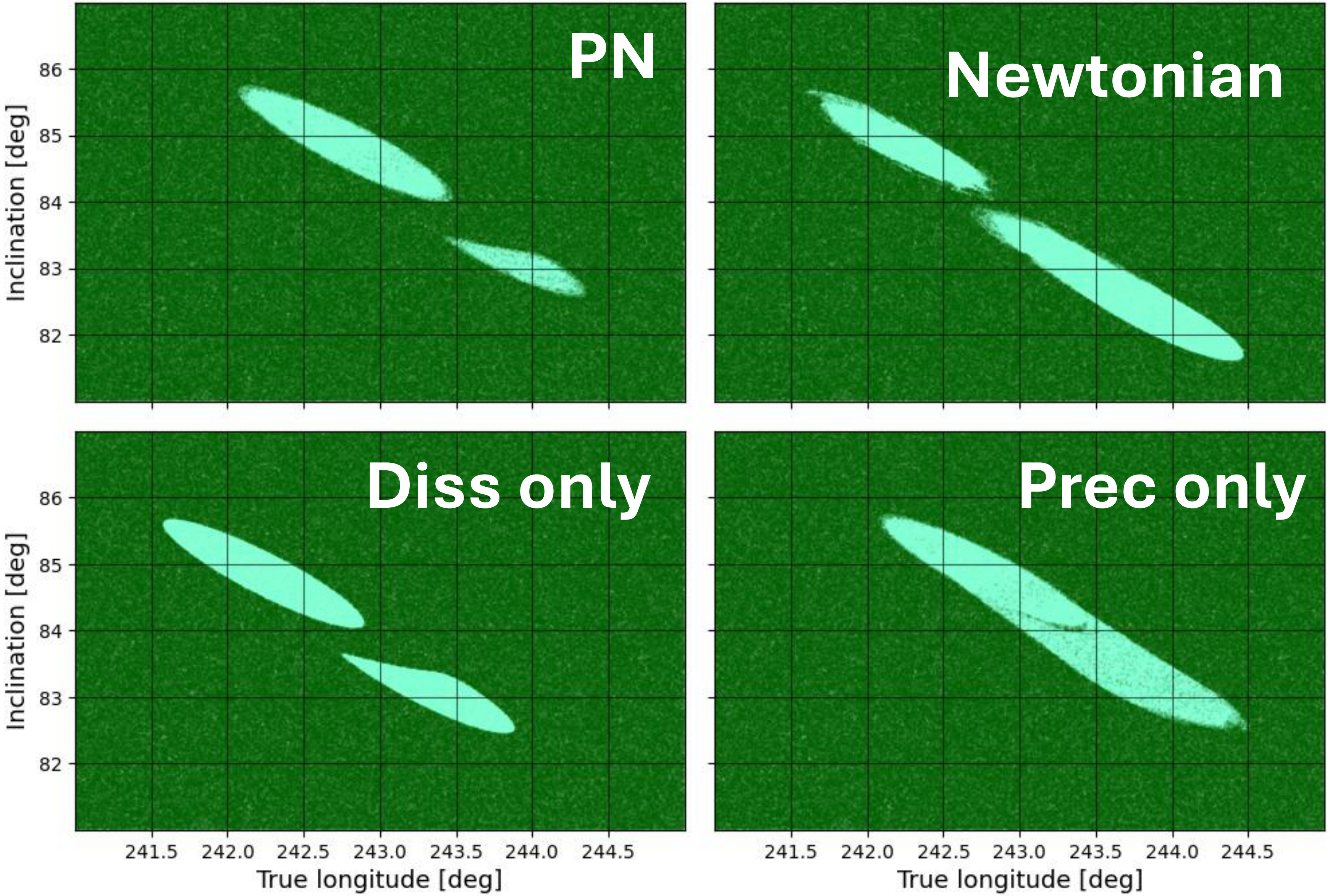
Inspiral collisions island
Long lived regular interactions
Regular structures in chaotic region



PN role in regular vs chaotic collisions

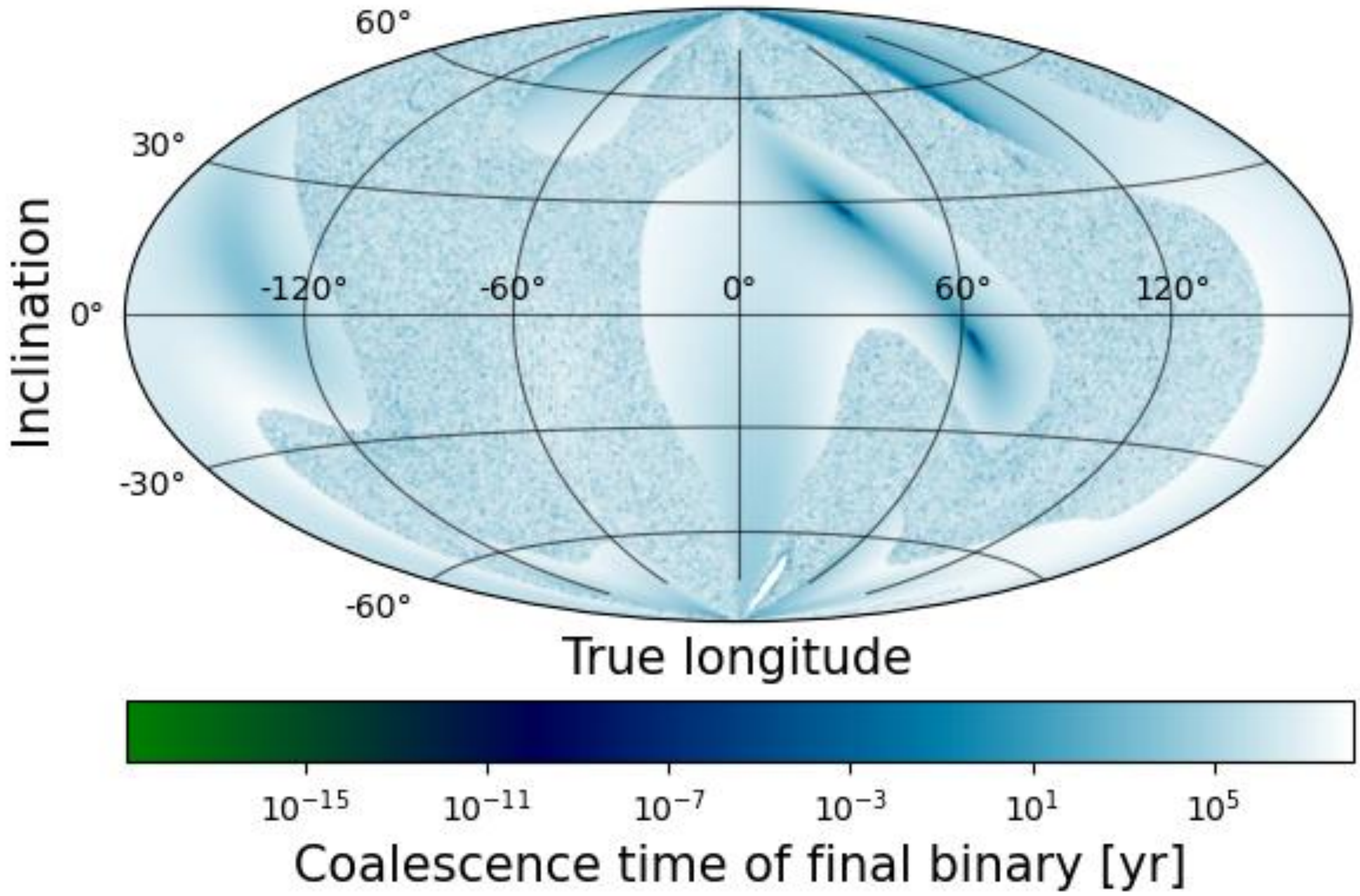


Embedded in a regular island
Consists of inspiral collisions
Larger in the Newtonian case



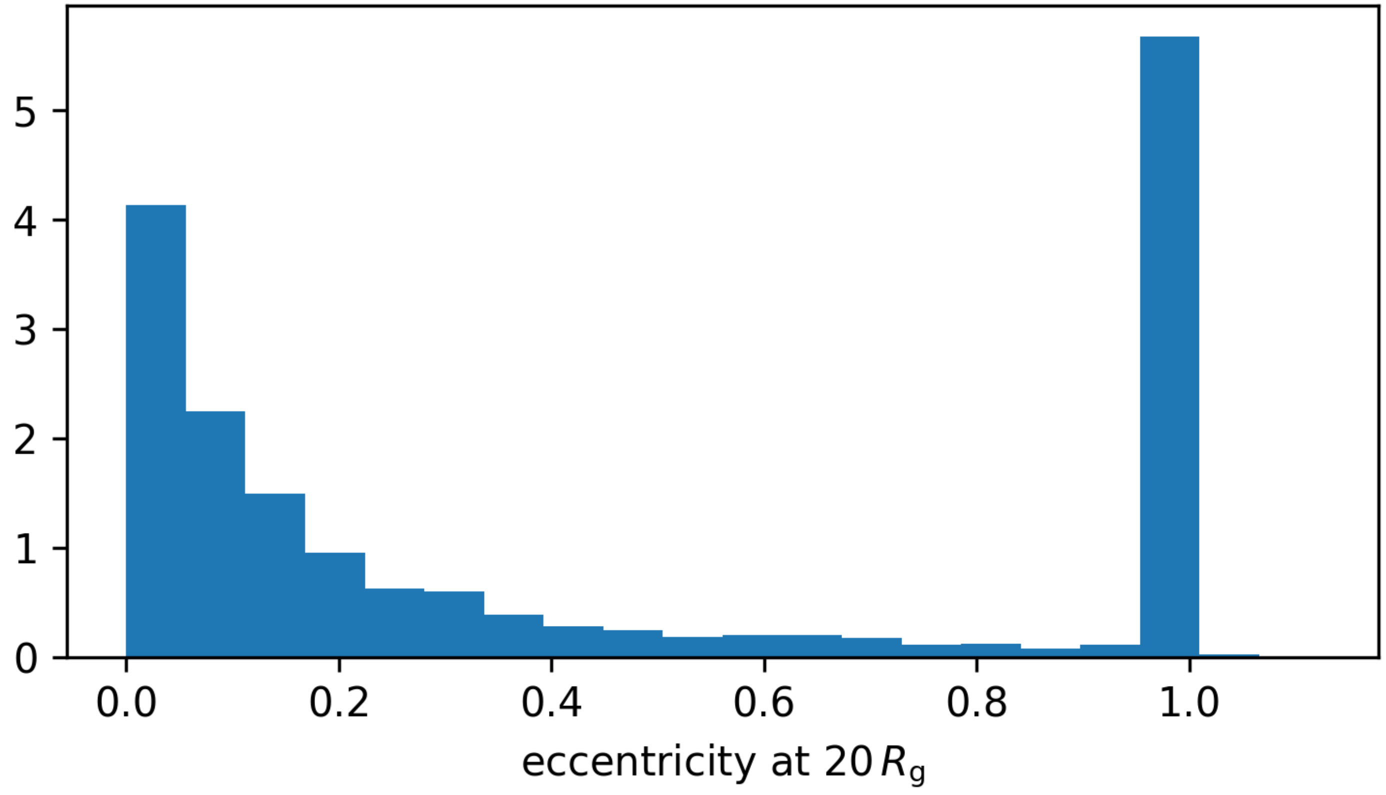
$v = 0.2v_c, b = 0.5a$	$\frac{N_{reg}}{N}$	$\frac{N_{col}}{N}$	$\frac{N_{reg,col}}{N}$	$\frac{N_{reg,col}}{N_{col}}$
Post-Newtonian	0.49	0.017	0.0035	0.21
Newtonian	0.49	0.0052	0.00062	0.12
$v = 0.5v_c, b = a$	$\frac{N_{reg}}{N}$	$\frac{N_{col}}{N}$	$\frac{N_{reg,col}}{N}$	$\frac{N_{reg,col}}{N_{col}}$
Post-Newtonian	0.53	0.0089	0.0028	0.32
Newtonian	0.53	0.0032	0.0011	0.35
$v = 0.8v_c, b = 1.5a$	$\frac{N_{reg}}{N}$	$\frac{N_{col}}{N}$	$\frac{N_{reg,col}}{N}$	$\frac{N_{reg,col}}{N_{col}}$
Post-Newtonian	0.84	0.0022	0.00055	0.26
Newtonian	0.84	0.00091	0.00041	0.46

- Frequency of collisions rises due to dissipation
- Stronger effect at low v and b
- $N_{reg,col}/N_{col}$ is proportionally smaller at high v and b

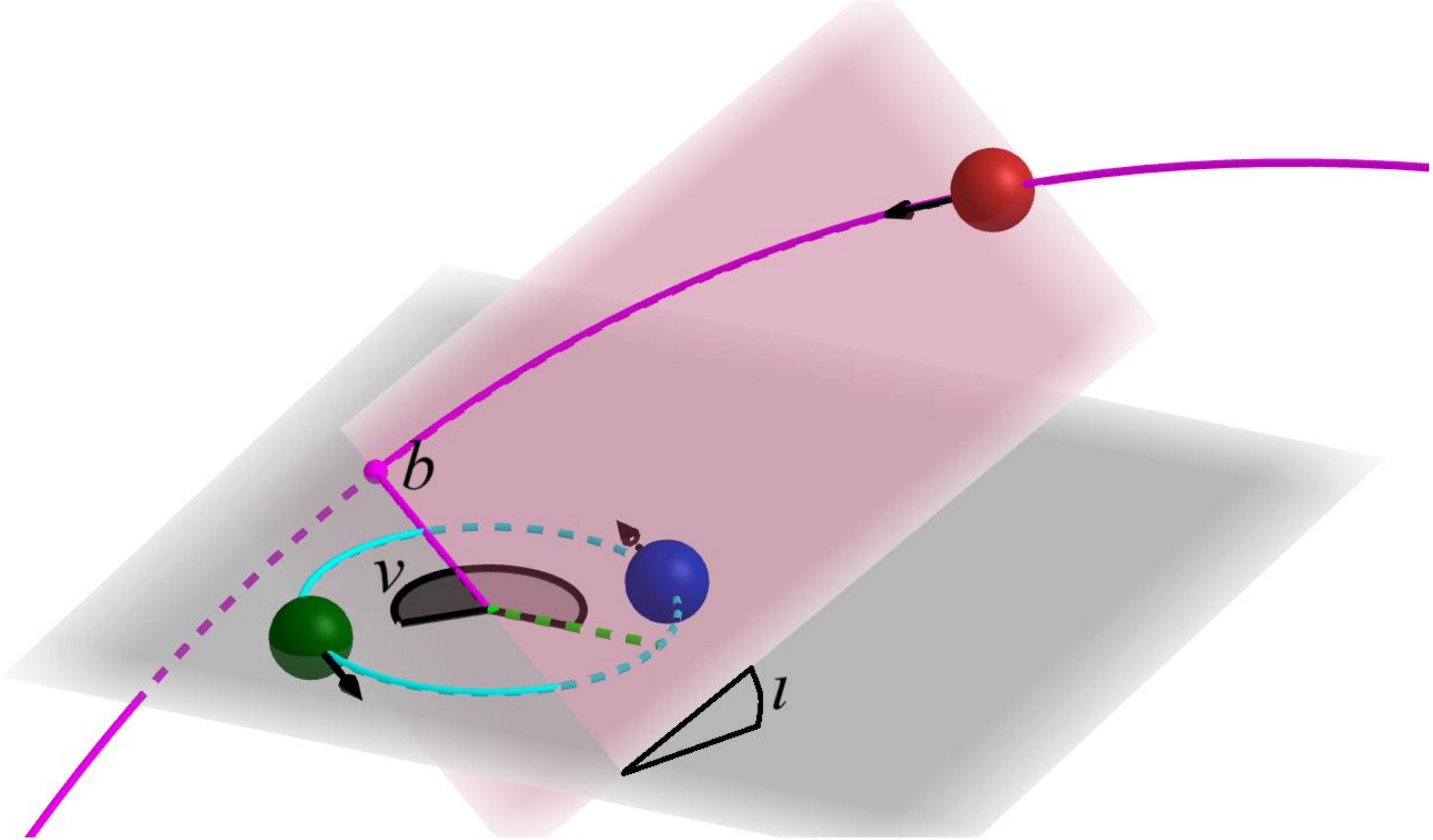

60°
30°
0°
-30°
-60°
-120°
-60°
0°
60°
120°
Inclination
True longitude
10^{-15}
10^{-11}
10^{-7}
10^{-3}
10^{1}
10^{5}
Coalescence time of final binary [yr]

Eccentric mergers. Open issues:

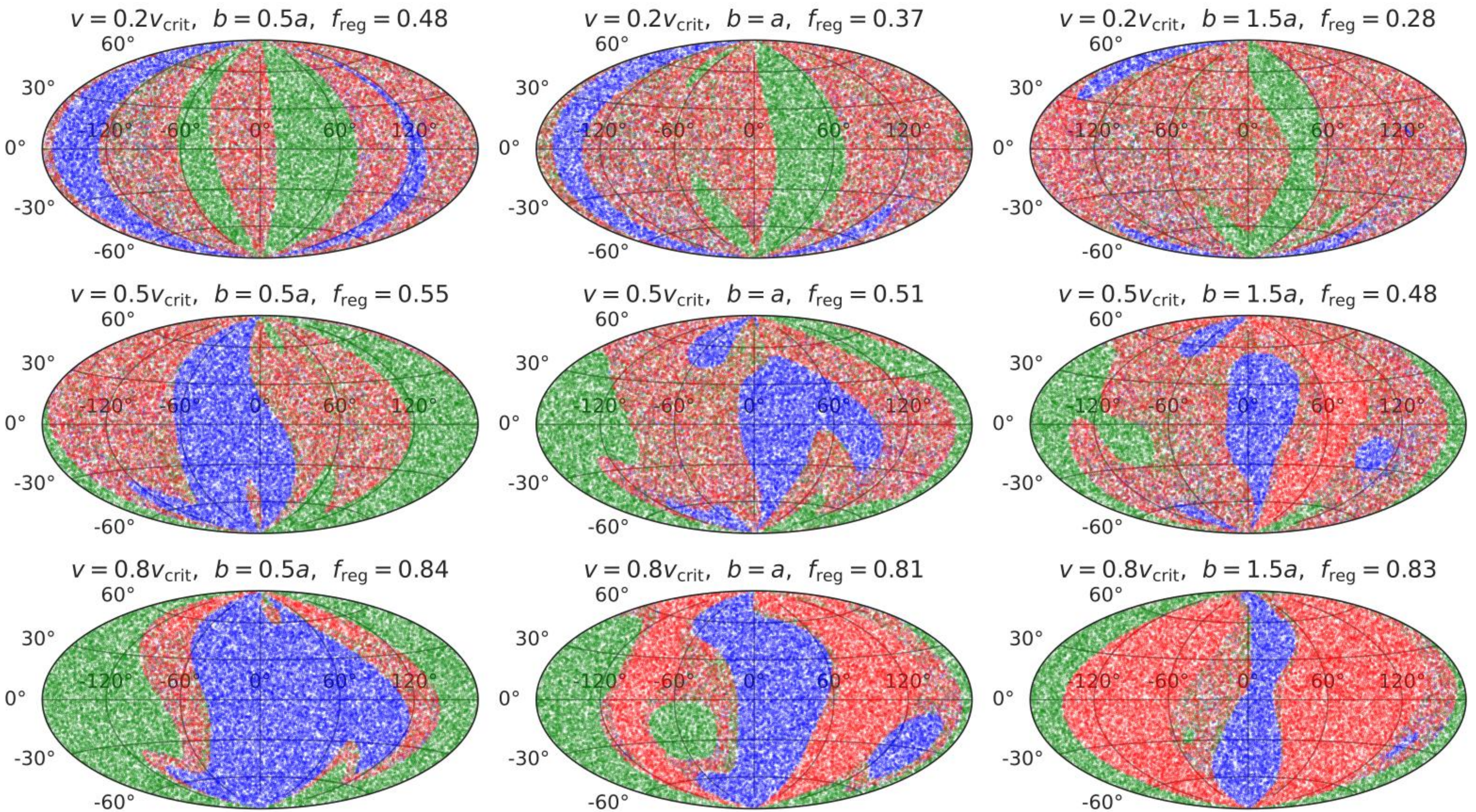
- Expected number of eccentric mergers in LVK band and astrophysical implications (tip-of-the-iceberg argument)
- Eccentricity thresholds and dependency on BH masses
- Detectability: burst at pericenter passage versus eccentric waveform
- Proper definition of eccentricity. Eccentric waveform approximant's eccentricity may be different from post-Newtonian definition



Shape of the phase-space depends on initial conditions – but mixing between regular and chaotic regions is consistent



From hyperbolic-binary single encounters simulations…



$v=0.2v_{\mathrm{crit}},\ b=0.5a,\ f_{\mathrm{reg}}=0.48$
$v=0.2v_{\mathrm{crit}},\ b=a,\ f_{\mathrm{reg}}=0.37$
$v=0.2v_{\mathrm{crit}},\ b=1.5a,\ f_{\mathrm{reg}}=0.28$
$v=0.5v_{\mathrm{crit}},\ b=0.5a,\ f_{\mathrm{reg}}=0.55$
$v=0.5v_{\mathrm{crit}},\ b=a,\ f_{\mathrm{reg}}=0.51$
$v=0.5v_{\mathrm{crit}},\ b=1.5a,\ f_{\mathrm{reg}}=0.48$
$v=0.8v_{\mathrm{crit}},\ b=0.5a,\ f_{\mathrm{reg}}=0.84$
$v=0.8v_{\mathrm{crit}},\ b=a,\ f_{\mathrm{reg}}=0.81$
$v=0.8v_{\mathrm{crit}},\ b=1.5a,\ f_{\mathrm{reg}}=0.83$
60°
30°
0°
-30°
-60°
-120°
-60°
0°
60°
120°

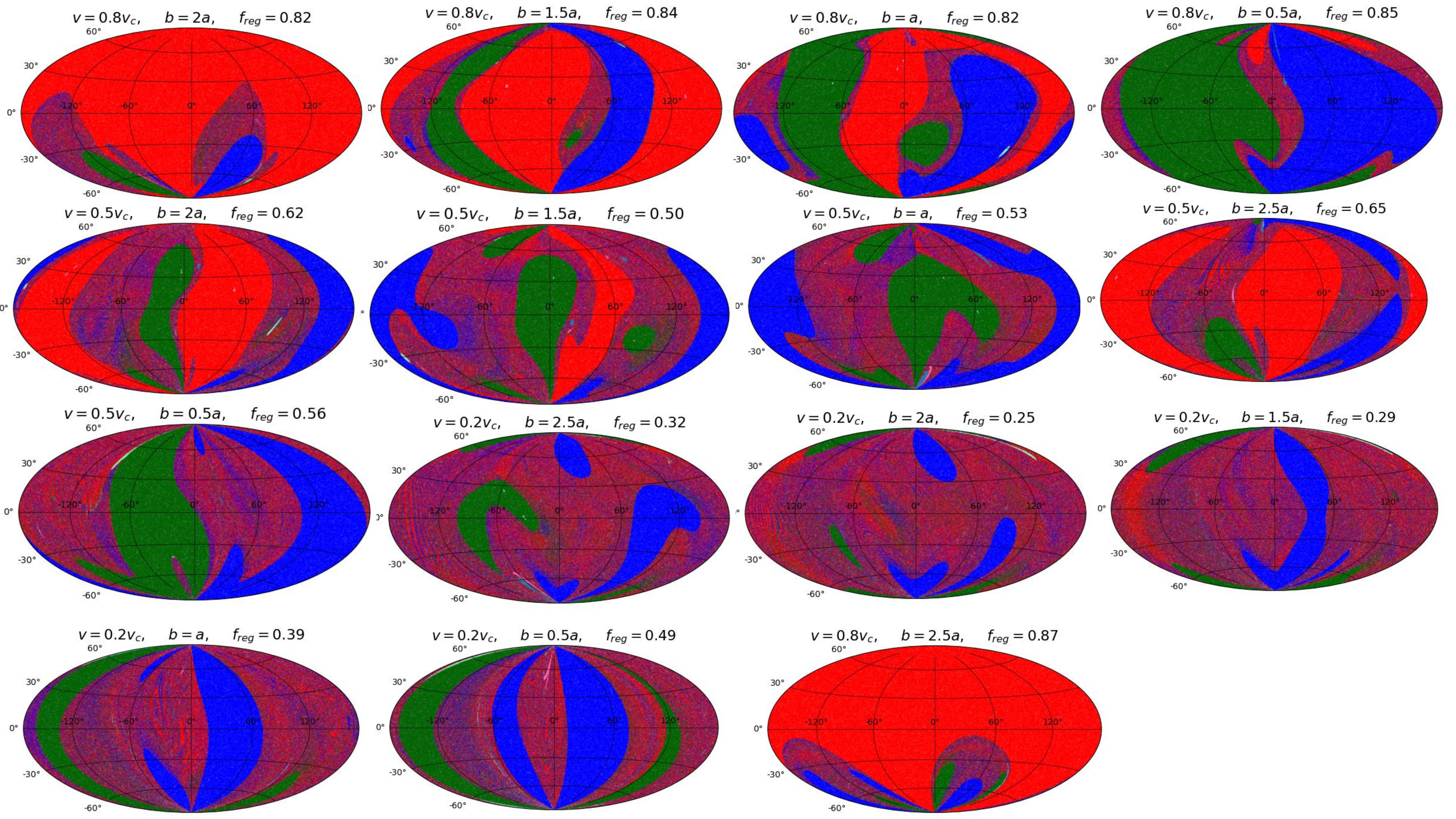
Questions #2

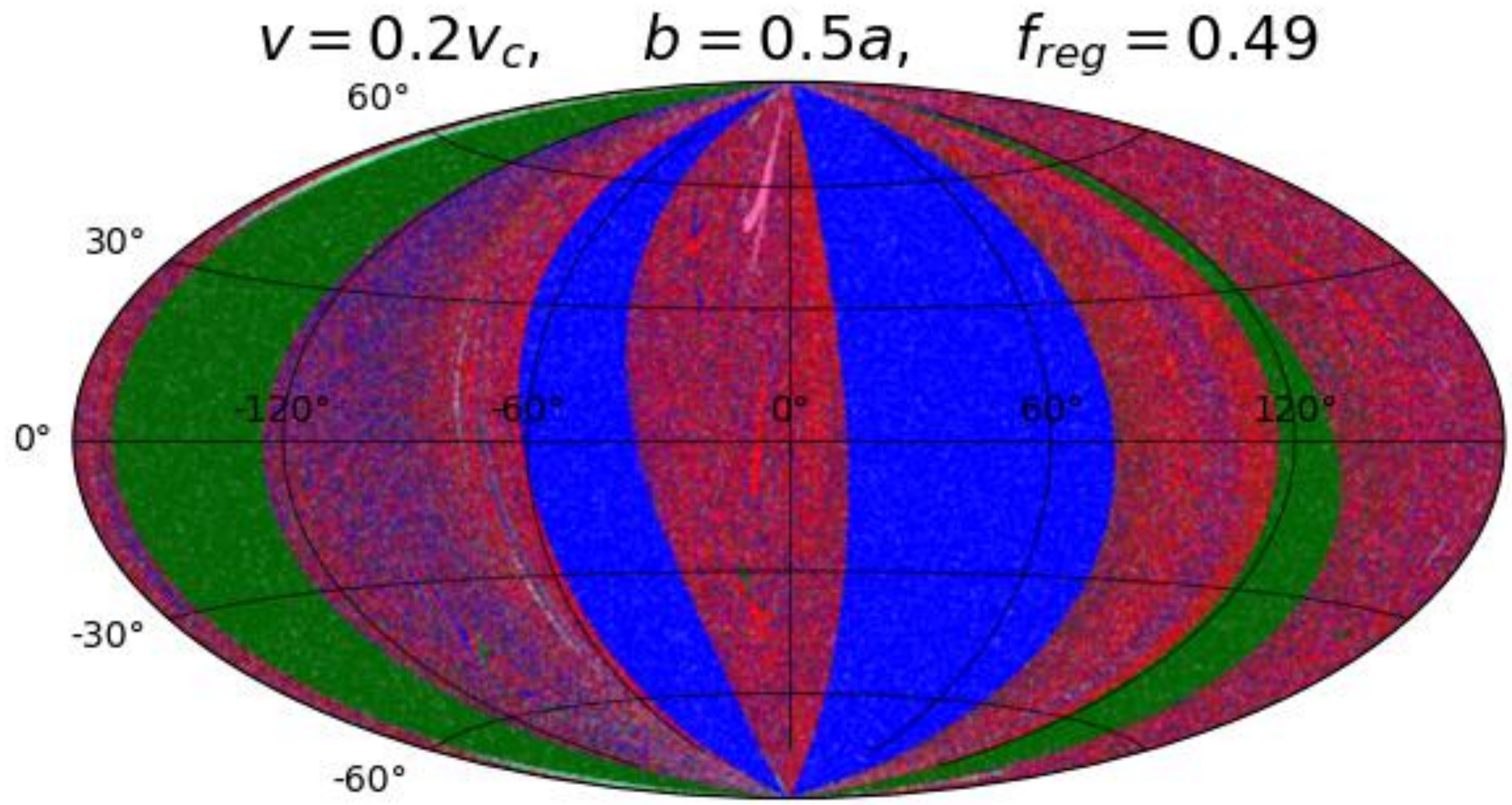
1. How do regular regions emerge?

2. How to describe them?

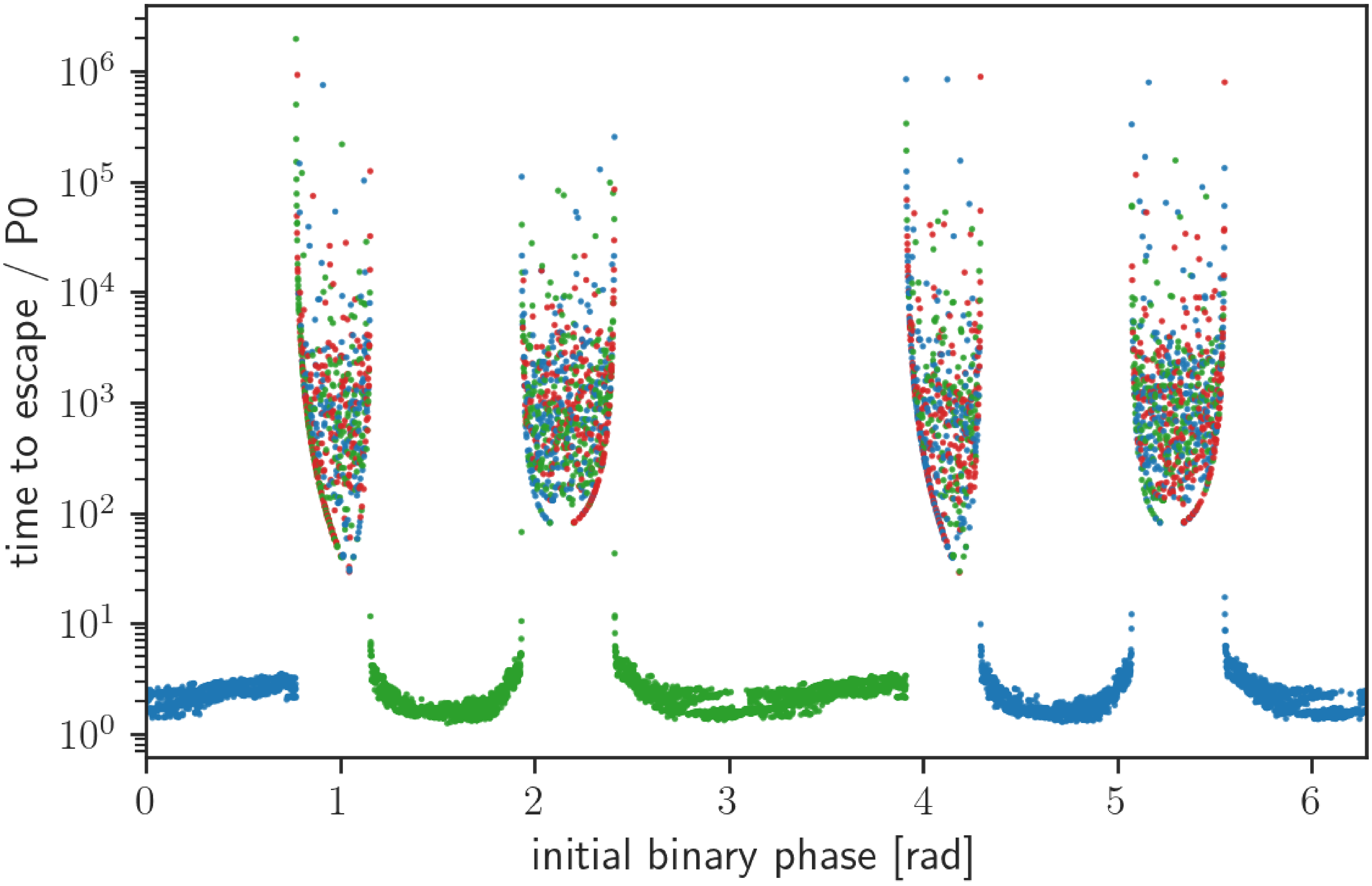
Towards a complete solution for the Newtonian three-body problem:

- Statistical escape theory for the chaotic phase space
- Deterministic theory for the regular phase space

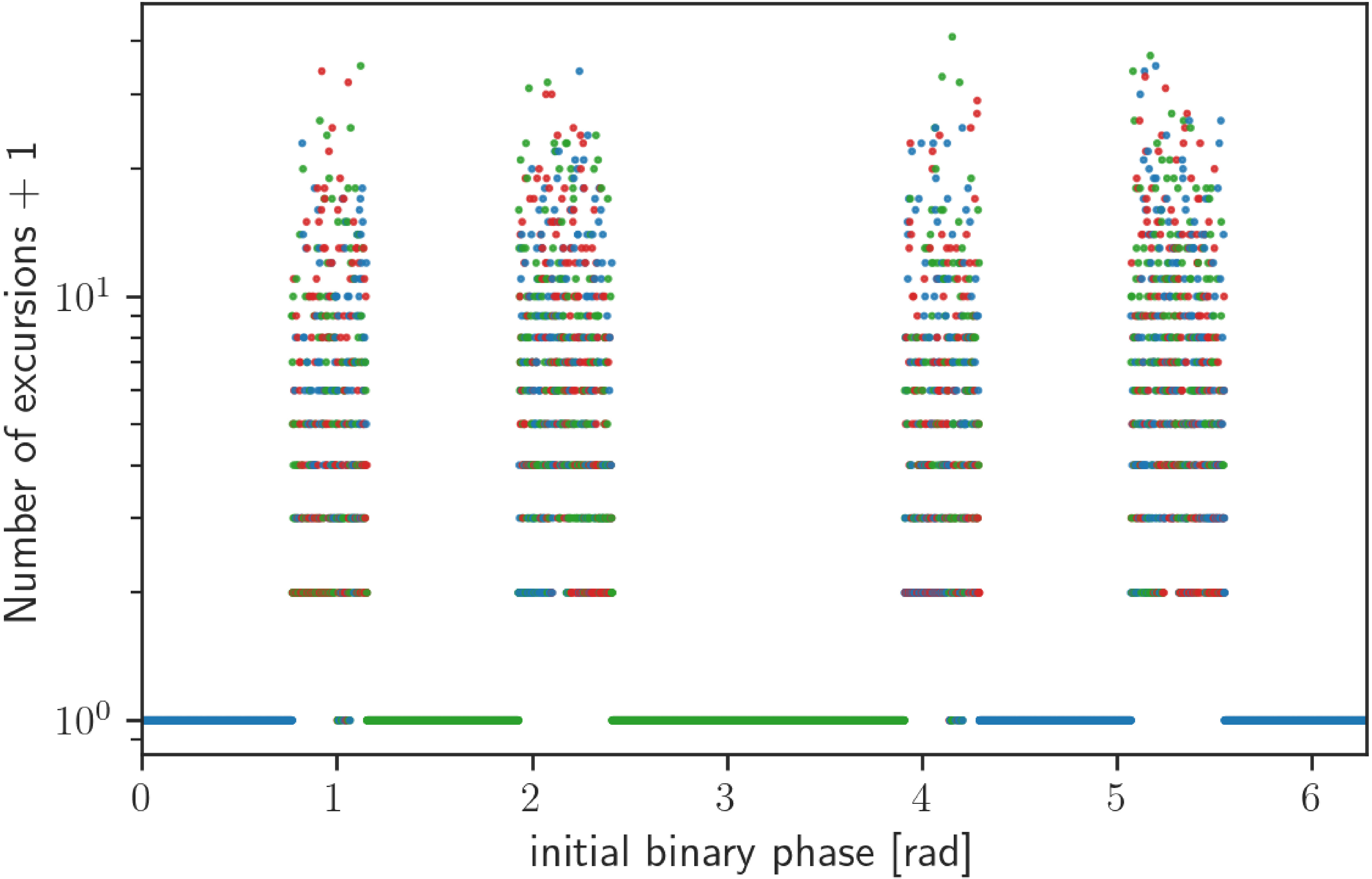

$v = 0.8v_c$, $b = 2a$, $f_{reg} = 0.82$
$v = 0.8v_c$, $b = 1.5a$, $f_{reg} = 0.84$
$v = 0.8v_c$, $b = a$, $f_{reg} = 0.82$
$v = 0.8v_c$, $b = 0.5a$, $f_{reg} = 0.85$
$v = 0.5v_c$, $b = 2a$, $f_{reg} = 0.62$
$v = 0.5v_c$, $b = 1.5a$, $f_{reg} = 0.50$
$v = 0.5v_c$, $b = a$, $f_{reg} = 0.53$
$v = 0.5v_c$, $b = 2.5a$, $f_{reg} = 0.65$
$v = 0.5v_c$, $b = 0.5a$, $f_{reg} = 0.56$
$v = 0.2v_c$, $b = 2.5a$, $f_{reg} = 0.32$
$v = 0.2v_c$, $b = 2a$, $f_{reg} = 0.25$
$v = 0.2v_c$, $b = 1.5a$, $f_{reg} = 0.29$
$v = 0.2v_c$, $b = a$, $f_{reg} = 0.39$
$v = 0.2v_c$, $b = 0.5a$, $f_{reg} = 0.49$
$v = 0.8v_c$, $b = 2.5a$, $f_{reg} = 0.87$



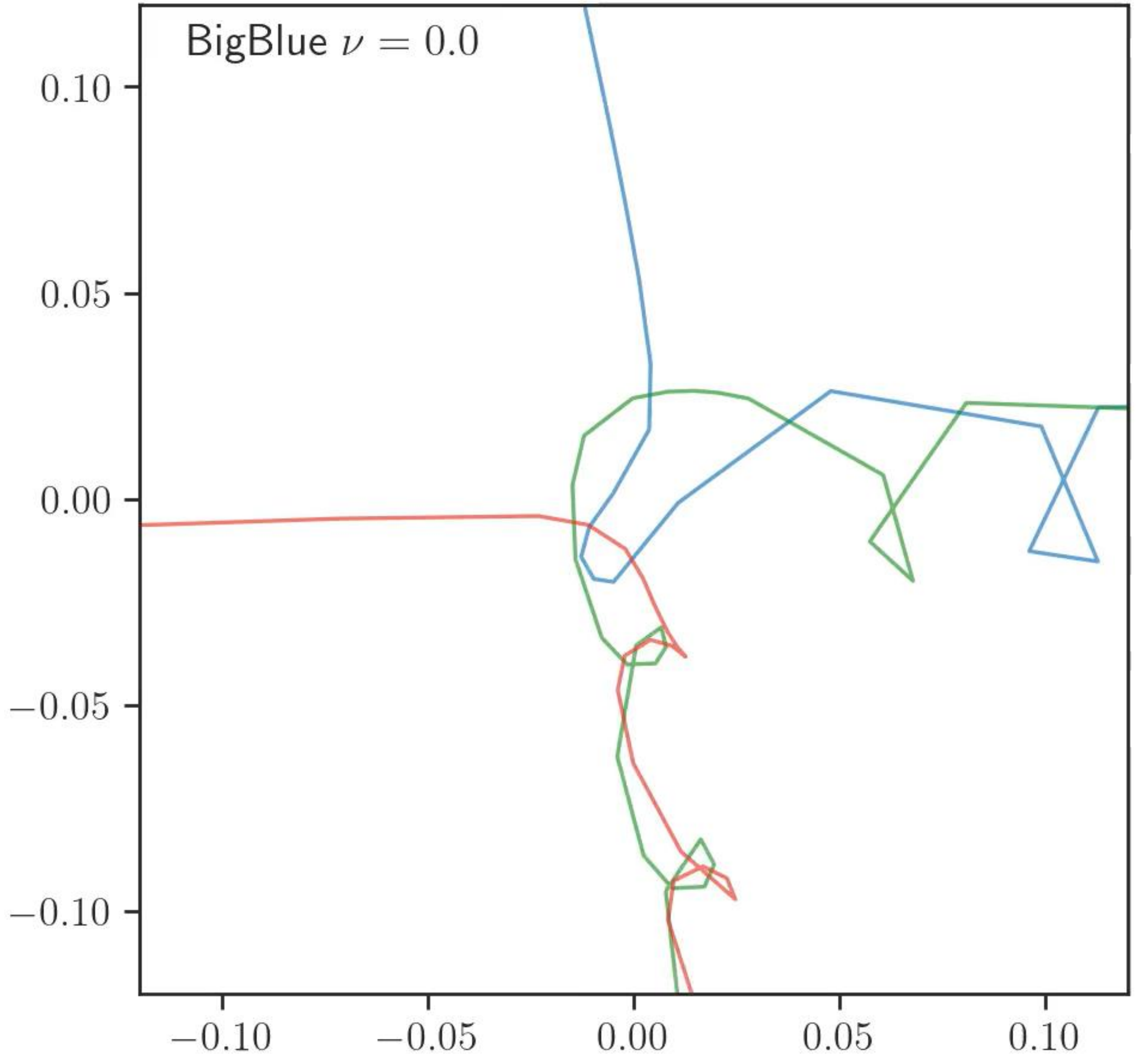
- Back to Newtonian
- Hyperbolic binary-single
- Let's cut at phi=0

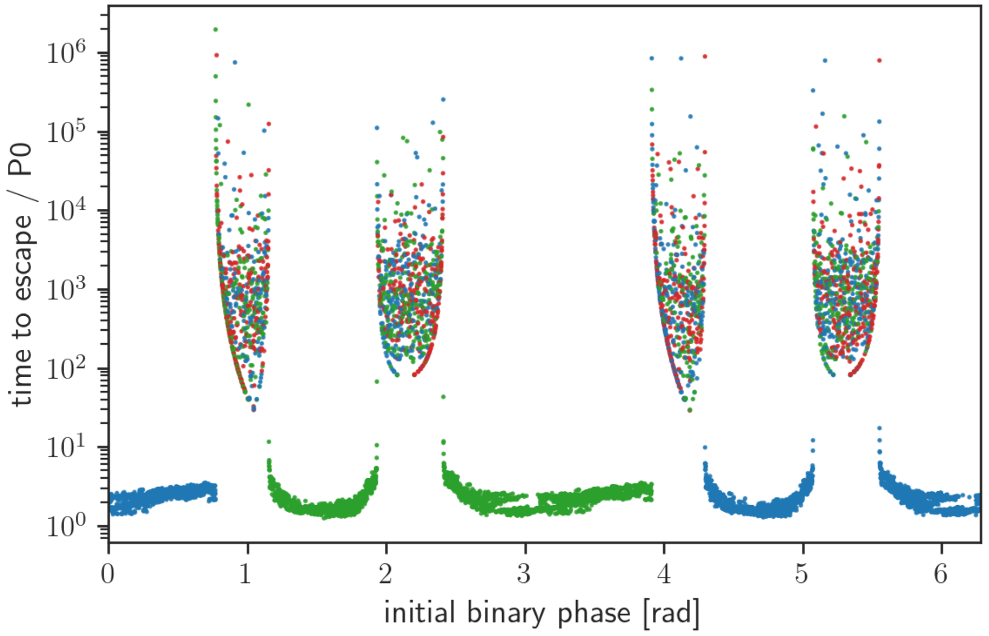


time to escape / P0
initial binary phase [rad]
10^0
10^1
10^2
10^3
10^4
10^5
10^6
0
1
2
3
4
5
6



Number of excursions + 1
10^1
10^0
0
1
2
3
4
5
6
initial binary phase [rad]


BigBlue $\nu = 0.0$
0.10
0.05
0.00
−0.05
−0.10
−0.10
−0.05
0.00
0.05
0.10


time to escape / P0
initial binary phase [rad]
10^6
10^5
10^4
10^3
10^2
10^1
10^0
0
1
2
3
4
5
6

SDEs for eccentricity and inclination are (almost) a 2-dimensional Ornstein-Uhlenbeck process

deterministic damping

stochastic forcing

$$\mathrm{d}X_t = -\theta X_t \mathrm{d}t + \sigma \mathrm{d}W_t,$$

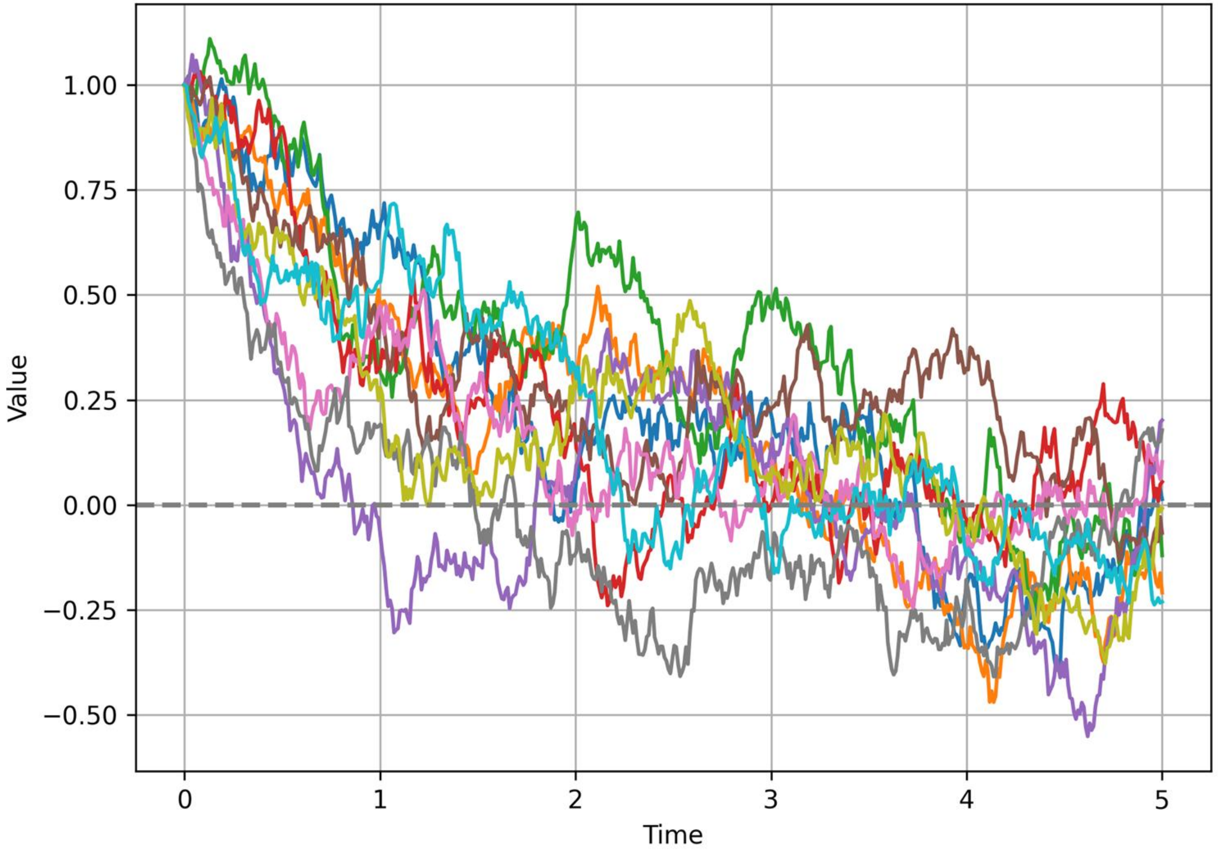
a.k.a. damped random walk

Widely used in finance, biology, and neuroscience, but also to model *AGN lightcurve variability*!

Main difference:

W is a Wiener process (white noise, delta-function autocorrelation)

but we have exponentially-decaying autocorrelation



SDEs for eccentricity and inclination are (almost) a 2-dimensional Ornstein-Uhlenbeck process

$$\frac{de}{dt} = -\eta_* + \eta_* \sqrt{\frac{a}{\mu}} \left(v_{\mathrm{turb},r} \sin \nu + 2 v_{\mathrm{turb},\phi} \cos \nu \right)$$

+ equation for longitude of pericenter $\frac{d\varpi}{dt}$

After a change of variables

$$\begin{cases} g_x = e \cos \varpi, \\ g_y = e \sin \varpi. \end{cases}$$

$$\frac{\mathrm{d}g_x}{\mathrm{d}t} = \underline{-\eta_* g_x} + \eta_* \sqrt{\frac{a}{\mu}} \underline{\left(2 v_{\mathrm{turb},\phi} \cos \ell + v_{\mathrm{turb},r} \sin \ell \right)},$$

$$\frac{\mathrm{d}g_y}{\mathrm{d}t} = \underline{-\eta_* g_y} + \eta_* \sqrt{\frac{a}{\mu}} \underline{\left(2 v_{\mathrm{turb},\phi} \sin \ell - v_{\mathrm{turb},r} \cos \ell \right)},$$

deterministic damping

stochastic forcing

Which is exactly of the form of the Ornstein–Uhlenbeck process:

$$\mathrm{d}X_t = -\theta X_t \mathrm{d}t + \sigma \mathrm{d}W_t,$$

$$\langle g_x^2 \rangle = D^2 \int_0^\infty \int_0^\infty \exp\big(-\eta_*(t_1+t_2)\big) \cdot \langle \Sigma_x(t-t_1), \Sigma_x(t-t_2) \rangle \, \mathrm{d}t_1 \, \mathrm{d}t_2 =$$

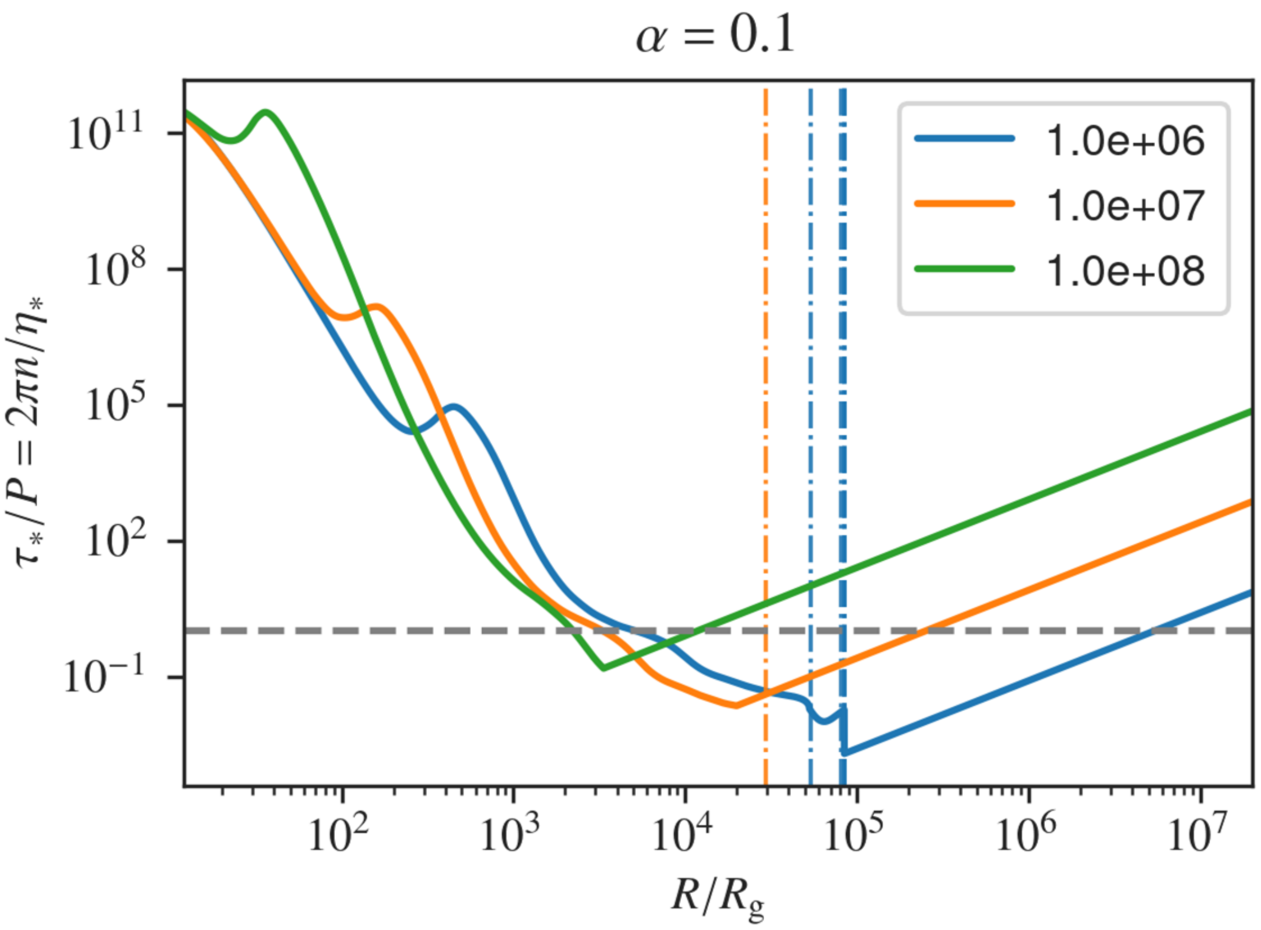
$$= \frac{2\eta_* \sigma_{\rm turb}^2}{v_{\rm circ}^2} \int_0^\infty \exp(-\eta_* t) S(t) \, \mathrm{d}t =$$

$$= \frac{2\eta_* \sigma_{\rm turb}^2}{v_{\rm circ}^2} \frac{A(\eta_* + \Omega_{\rm circ}) - B n}{(\eta_* + \Omega_{\rm circ})^2 + n^2}$$

Analytic expression for the variance of eccentricity and inclination

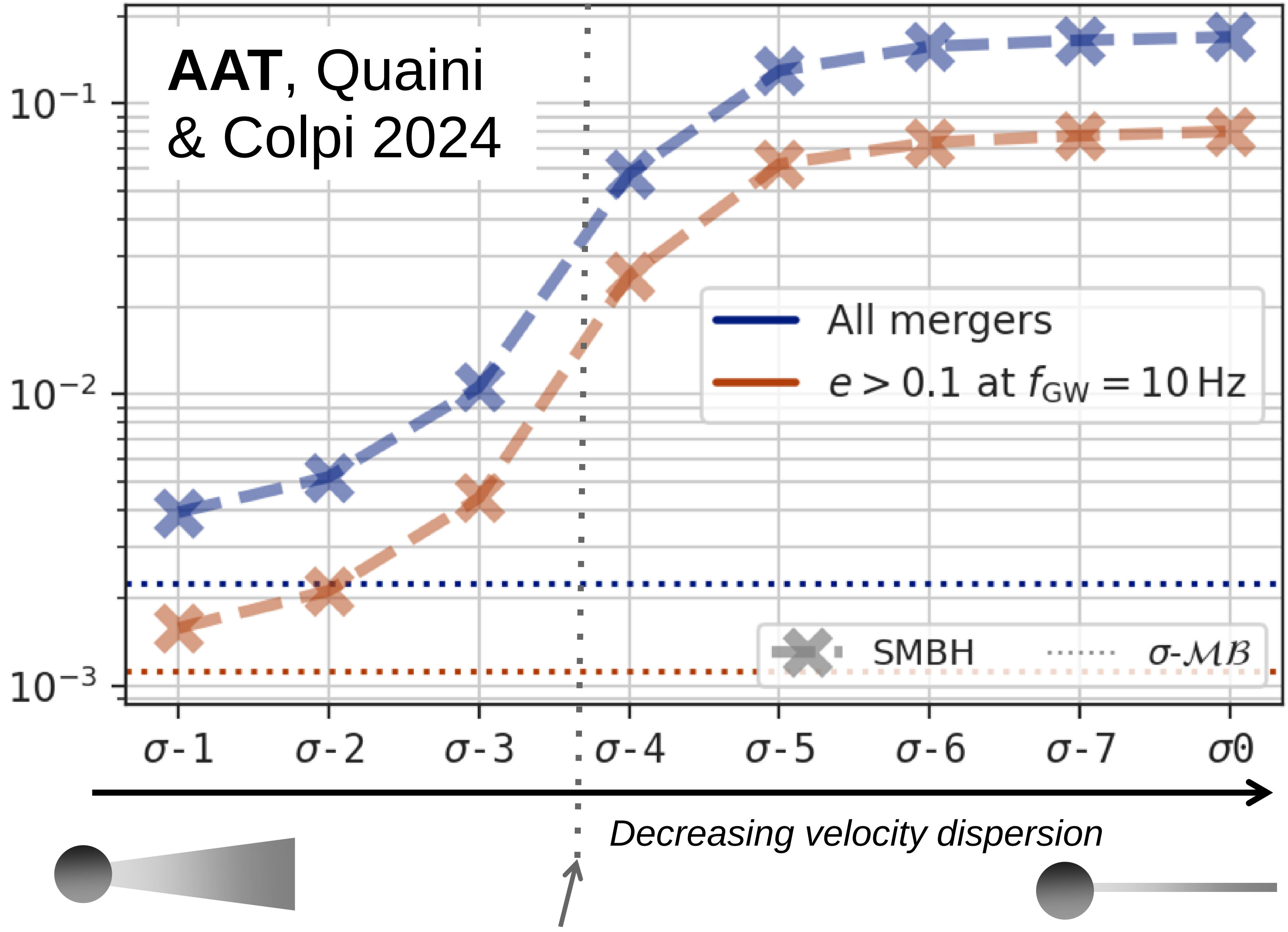
$$\sigma_e^2 = 15\,\sigma_\iota^2 = 10\,\alpha \left(\frac{H}{R}\right)^2, \quad \text{(fast damping, } \eta_* \gg n\text{)},$$

$$\sigma_e^2 = \frac{5}{2}\,\sigma_\iota^2 = 5\,\alpha \frac{\eta_*}{n} \left(\frac{H}{R}\right)^2, \quad \text{(slow damping, } \eta_* \ll n\text{)}.$$



Damping rate vs orbital frequency

Migration traps lie in the $\eta_* \sim n$ regime



transition from random-motion dominated to shear-dominated

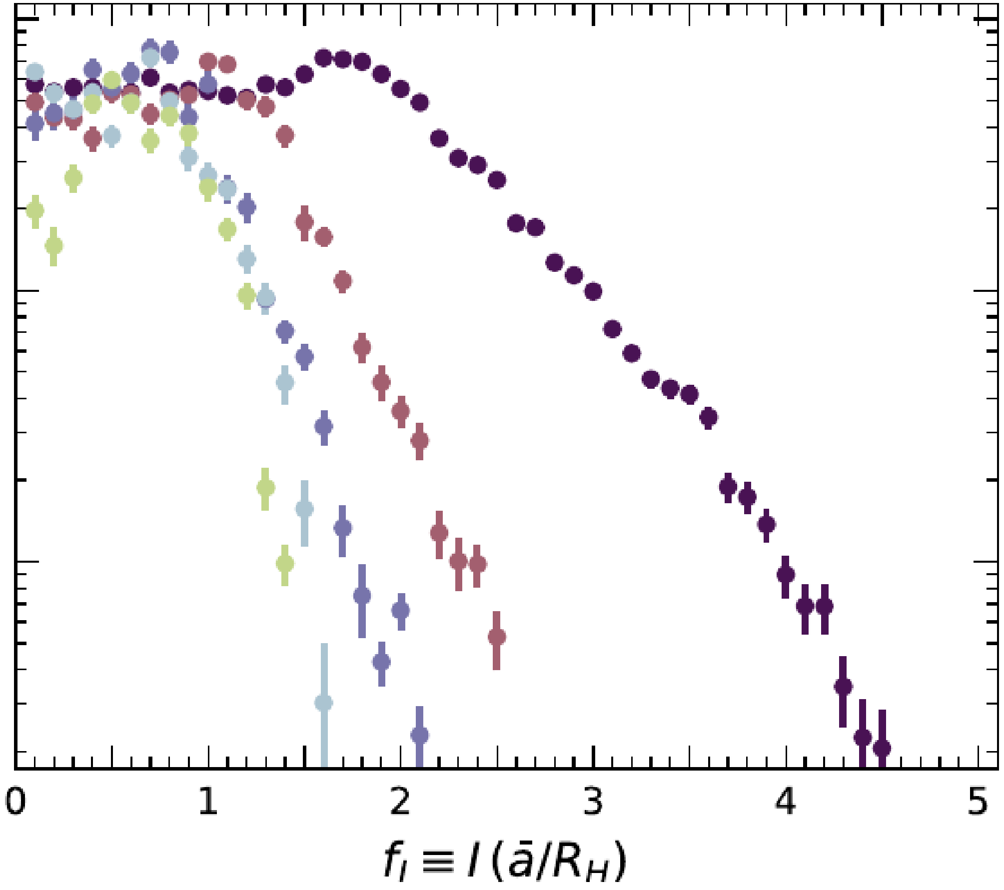
Merger efficiency from 3-body encounters VS "disk" eccentricity and inclination

Encounters between eccentric and inclined orbits suppress merger rates

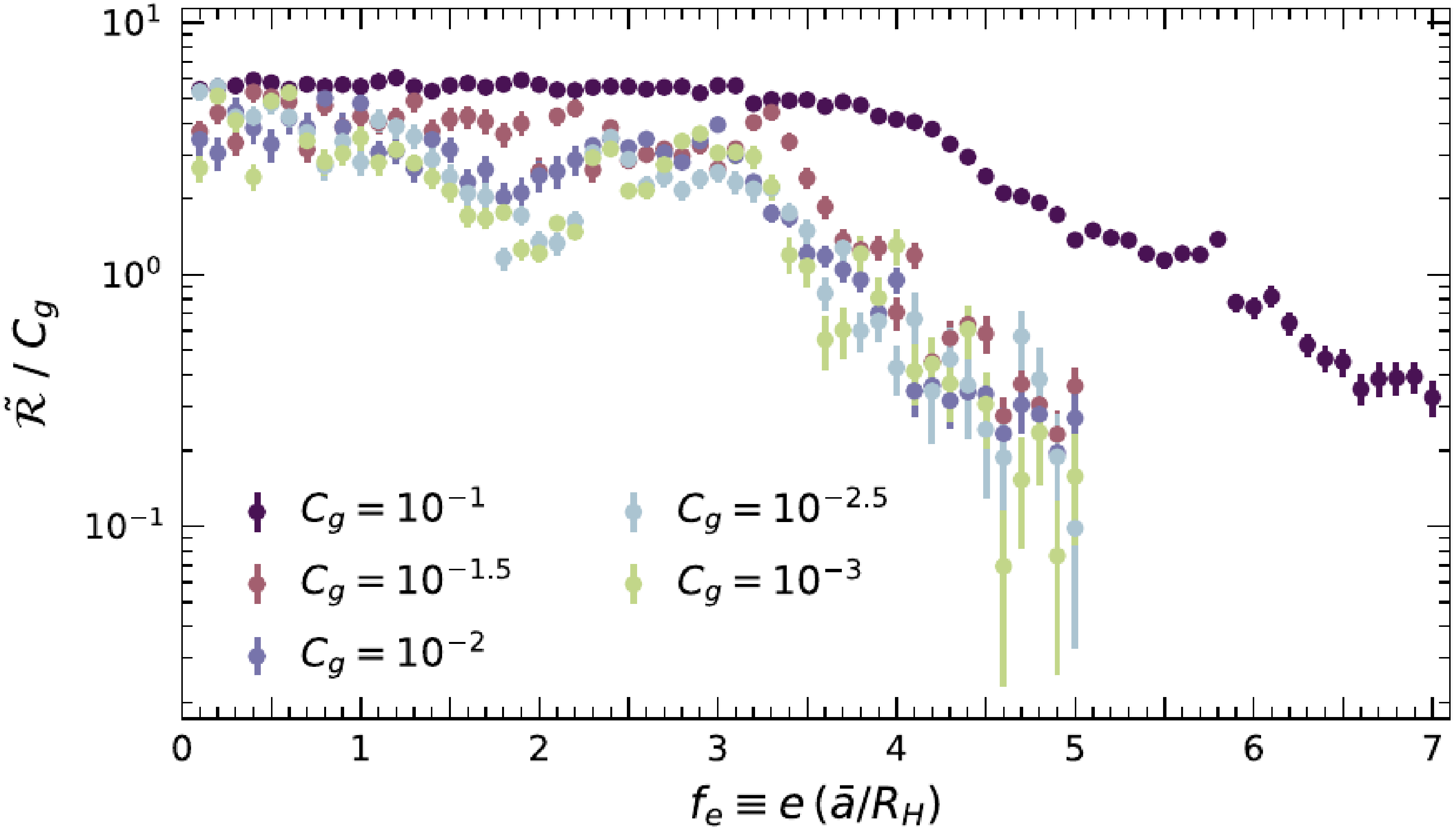
See also Gaia Fabj's talk

Single-single to binary capture rates

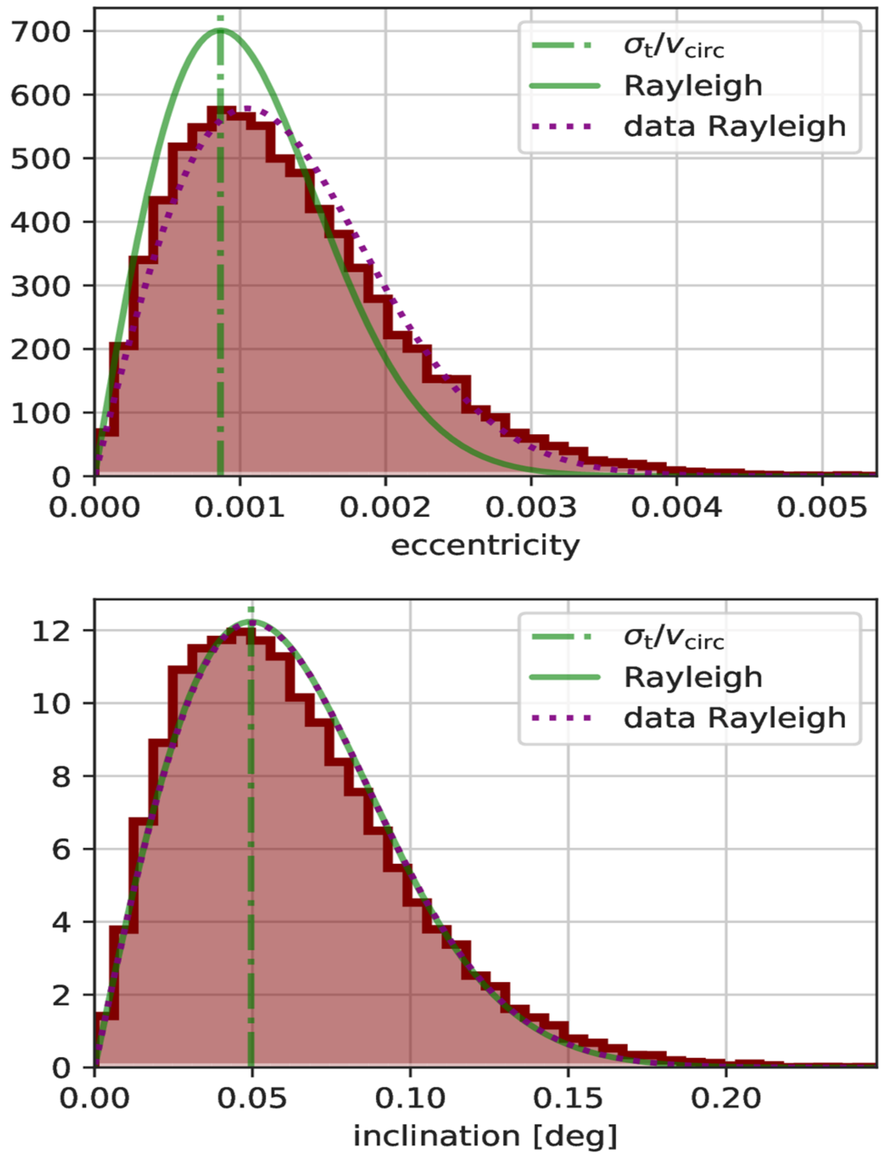
VS inclination



VS eccentricity



Dodici & Tremaine 2024



Takeaways

- Black holes embedded in AGNs are *not* aligned and circular

- Typical inclinations and eccentricities can be large $e, i \approx \sqrt{\alpha}\, H/R$

- Velocity dispersion of encountering BHs is *random-motion dominated,* rather than *shear dominated*

- **Likely has large impact on binary-formation and merger rates**

AAT & Di Cintio, 2506.02173