

Fast glitch removal method

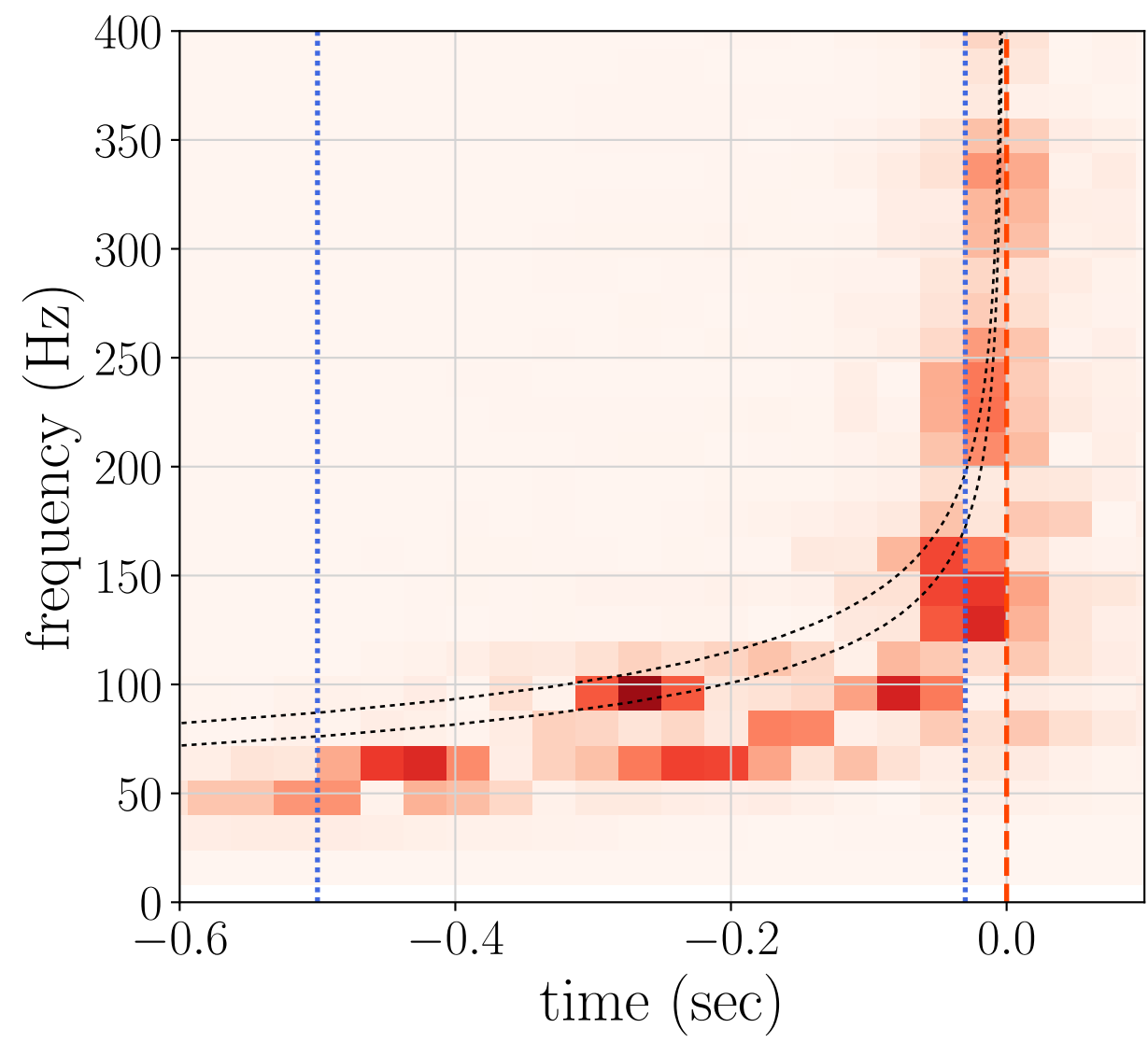
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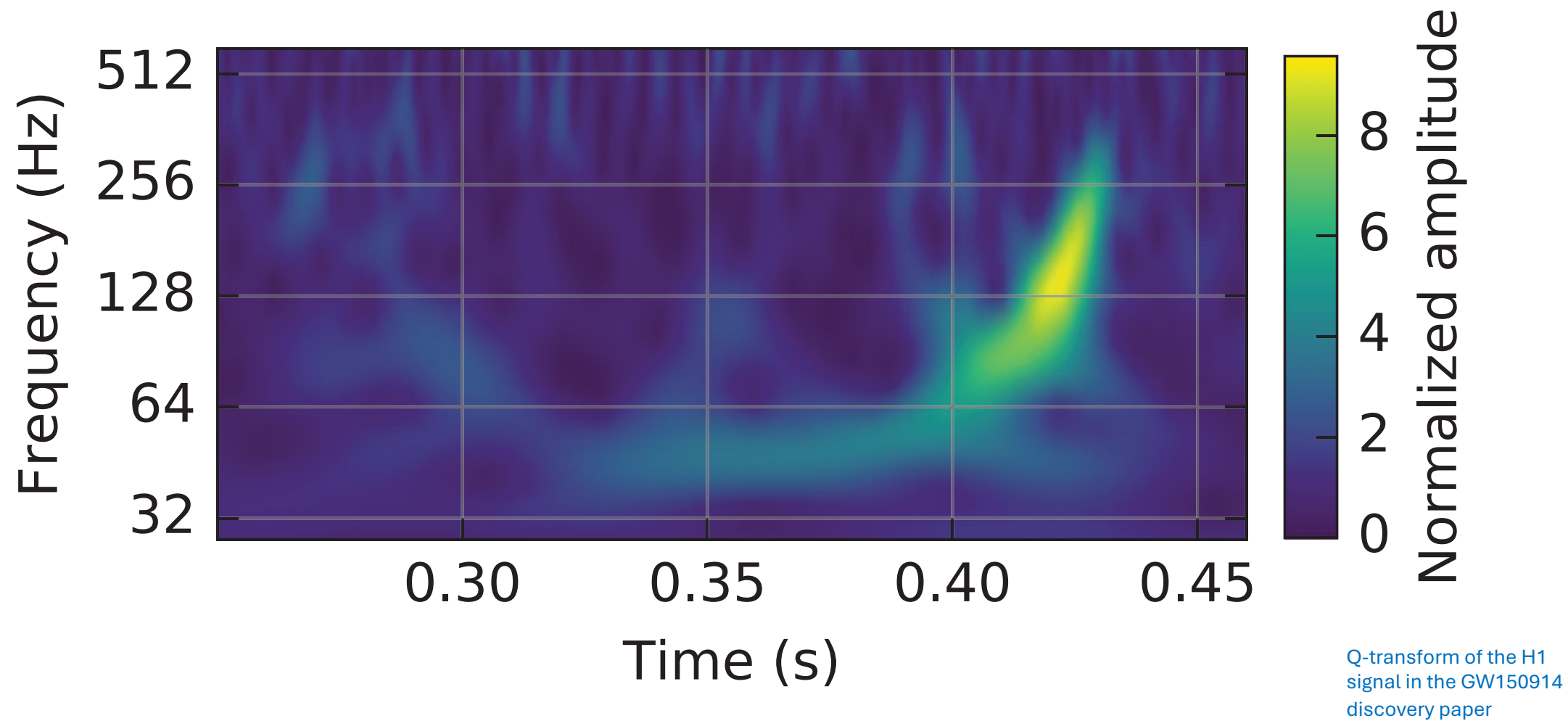
11-14 November 2025

Discrete wavelets



GW190814 scalogram
from [Vedovato et al, 2022](#)

Q-transforms



Q-transform: pro's and con's

- defining equations

$$X(\tau, \nu, Q) = \int_{-\infty}^{+\infty} dt s(t) g(t; \tau, \nu, Q) e^{-2\pi i \nu t}; \quad g(t; \tau, \nu, Q) = \left(\frac{8\pi \nu^2}{Q^2} \right)^{1/4} e^{-\left(\frac{2\pi \nu (t-\tau)}{Q} \right)^2}.$$

- the Q-transform is intermediate between the Short Time Fourier Transform and the continuous wavelet transform with the Morlet wavelets.
- optimal resolution reaching the Gabor limit, thanks to the Gaussian envelope
- the Q parameter lets us balance time and frequency resolution, given that

$$\sigma_{\tau}^{(Q)} = \frac{Q}{4\pi\nu}; \quad \sigma_{\nu}^{(Q)} = \frac{\nu}{Q}$$

- optimized Q parameter (based on **minimal mismatch between overlapping wavelets**)
- not invertible**, therefore no further processing of the transformed signals is possible

Invertible Q-transform

- defining equations

$$T(\tau, \nu, Q) = \int_{-\infty}^{+\infty} dt s(t) \psi^*(t; \tau, \nu); \quad \psi^*(t; \tau, \nu, Q) = \left(\frac{8\pi\nu^2}{Q^2} \right)^{1/4} e^{-\left(\frac{2\pi\nu(t-\tau)}{Q} \right)^2 - 2\pi i \nu(t-\tau)}.$$

which is a continuous Morlet wavelet transform

- optimal resolution reaching the Gabor limit, thanks to the Gaussian envelope
- the Q parameter lets us balance time and frequency resolution, given that

$$\sigma_{\tau}^{(Q)} = \frac{Q}{4\pi\nu}; \quad \sigma_{\nu}^{(Q)} = \frac{\nu}{Q}$$

- optimized Q parameter (based on **maximal sparsity of the discretized transform to maximize the energy density**)
- invertible**, therefore further processing of the transformed signals **is** possible

Invertibility

- continuous wavelets are not generally invertible; they are only if they satisfy the *admissibility condition*

$$C_\psi = \int_{-\infty}^{+\infty} \frac{|\tilde{\psi}(y)|^2}{|y|} dy < \infty$$

- the Morlet wavelets do not satisfy the admissibility condition, unless they are shifted by a small amount which depends on the central frequency of the wavelet: **this destroys some good properties of the wavelet**
- standard inversion formula

$$x(t) = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} d\nu \, d\tau \, T_x(\nu, \tau) \psi_{\nu, \tau}(t)$$

- non-standard inversion formula

$$s(\tau) = \frac{2}{(2\pi)^{3/2} \left(\frac{2}{\pi Q^2} \right)^{1/4} \operatorname{erf} \left(\frac{Q}{2} \right)} \operatorname{Re} \left[-i \int_0^{+\infty} \nu^{-3/2} \frac{\partial}{\partial \tau} T(\tau, \nu, Q) d\nu \right].$$

Invertibility (ctd.)

- **the non-standard inversion formula** does not require the admissibility condition, and **can be used directly with Morlet wavelets**
- using the Morlet wavelets has the added advantage that **one of the integrals can be calculated analytically: this produces a large speedup when computing the inverse transform**
- wavelets can be manipulated before applying the non-standard inversion formula, **to achieve any desired filtering in the TF domain**

$$s_D(\tau) = \operatorname{Re} \left[\int_{-\infty}^{+\infty} df \tilde{s}(f) e^{2\pi i f \tau} w(\tau, f, Q) \right]$$
$$w(\tau, f, Q) = \frac{1}{\operatorname{erf}\left(\frac{Q}{2}\right)} \sum_l \left\{ \operatorname{erf} \left[\frac{Q}{2} \left(\frac{f - \nu_l^{\text{low}}(\tau)}{\nu_l^{\text{low}}(\tau)} \right) \right] - \operatorname{erf} \left[\frac{Q}{2} \left(\frac{f - \nu_l^{\text{high}}(\tau)}{\nu_l^{\text{high}}(\tau)} \right) \right] \right\}.$$

Numerical implementation

The numerical implementation applies to whitened data samples taken with a well-defined sampling rate f_s

$$T_{\text{nd}}(\tau, \nu, Q) = \frac{\sqrt{f_s}}{N} \sum_m \tilde{s}(f_m) \tilde{\psi}^*(f_m; \tau, \nu, Q)$$

where:

$$\tilde{s}(f_m) = \sum_n s(t_n) e^{-2\pi i f_m t_n}$$

and where:

- we choose a dyadic tiling of the TF plane
- we select Q to produce a sparse signal representation in the TF plane

(see [Virtuoso and Milotti, 2024](#) for details)

Noise-induced fluctuations

In the case of Gaussian white noise background (the case of whitened signals), the energy of the Morlet wavelets used in the invertible Q-transform has well-defined statistical properties:

- pdf:

$$P(|T_{\text{nd}}(\tau, \nu, Q)|^2) = e^{-|T_{\text{nd}}(\tau, \nu, Q)|^2}$$

which is a chi-square distribution with 2 degrees of freedom

- two-point correlation function:

$$\langle T_{\text{nd}}^*(\tau_0, \nu_0, Q) T_{\text{nd}}(\tau, \nu, Q) \rangle = \sqrt{\frac{2\beta}{1 + \beta^2}} \exp \left[-\frac{\gamma^2 + \left(\frac{(1-\beta)Q}{2} \right)^2 + i(1 + \beta)\gamma Q}{1 + \beta^2} \right]$$

where:

$$\beta = \nu/\nu_0, \quad \text{and} \quad \gamma = [2\pi\nu(\tau_0 - \tau)]/Q$$

"Tile-wise" denoising

We associate an energy density to each tile of the discretized invertible Q-transform. We can remove excess noise (glitches) by discarding those tiles that have an above-threshold value, for some selected threshold (or that lie in between two given thresholds).

We have tested this denoising scheme both with real events detected in past LVK observing runs and publicly released, and with injections into data that include glitches. We used the catalog of glitches released by the GravitySpy project (<https://zenodo.org/records/5649212>).

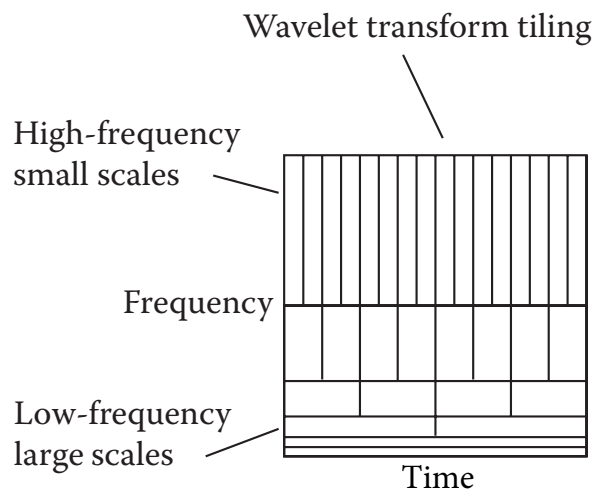
Here we show the results of deglitching, drawing random samples from the glitch catalog for Livingston, during O3b.

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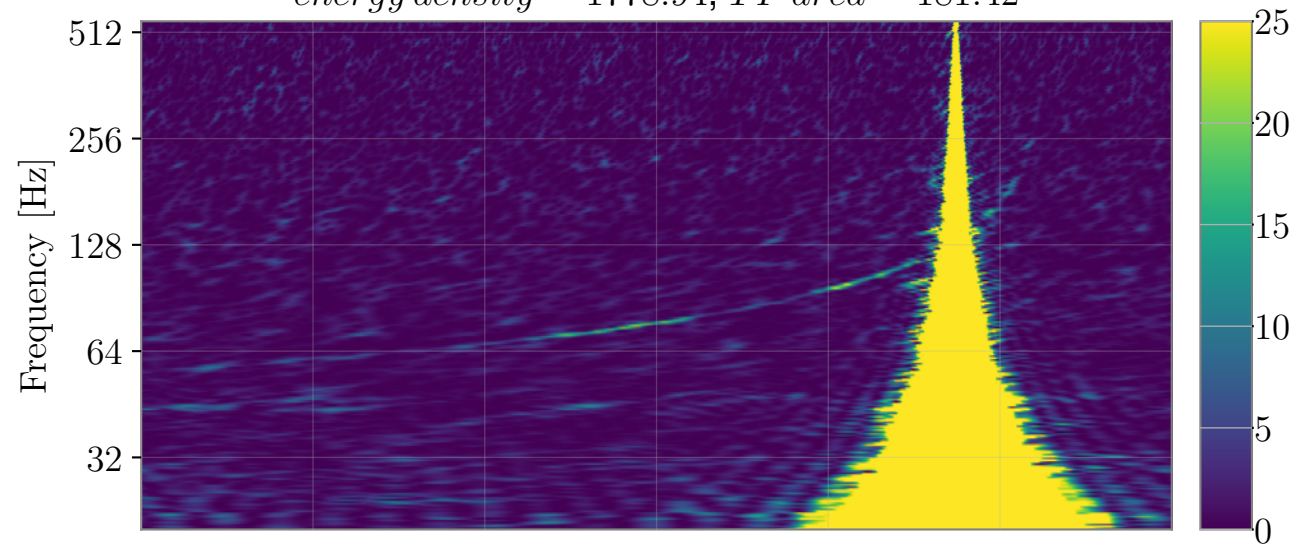
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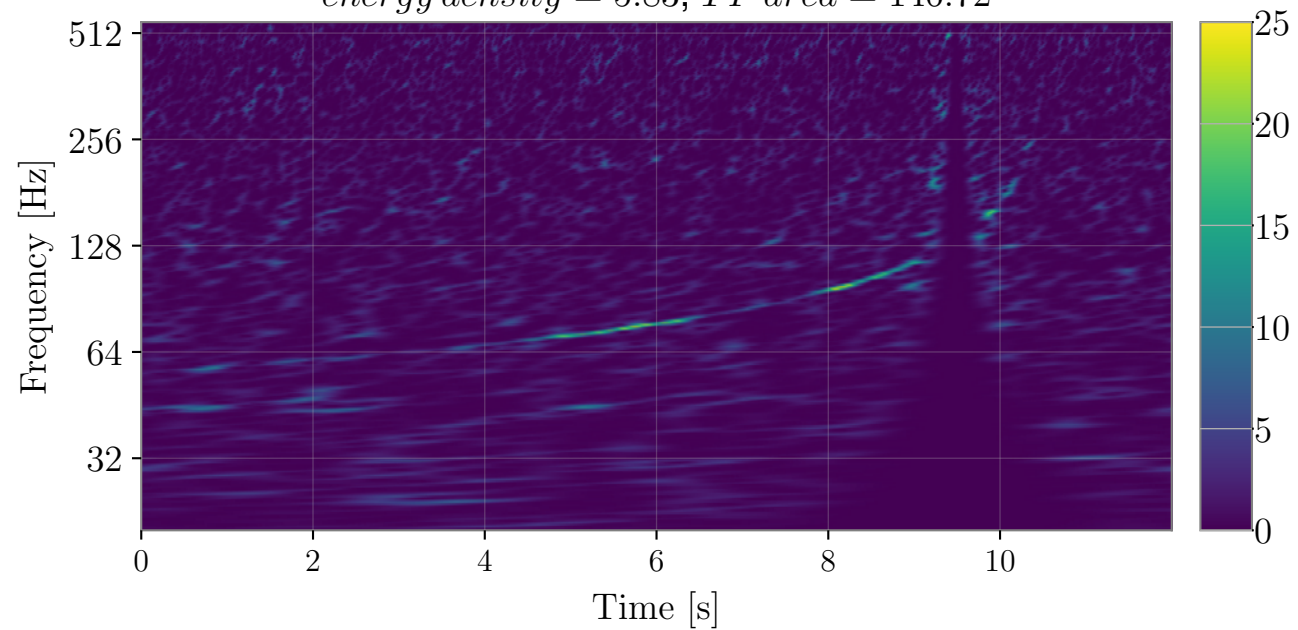


(again, for details on the tiling, see [Virtuoso and Milotti, 2024](#))

$Q = 108.69, p = 0.004, \text{energy peak} = 26422.24,$
 $\text{energy density} = 1778.94, TF \text{ area} = 181.42$



$Q = 108.69, p = 0.004, \text{energy peak} = 23.35,$
 $\text{energy density} = 5.83, TF \text{ area} = 146.72$

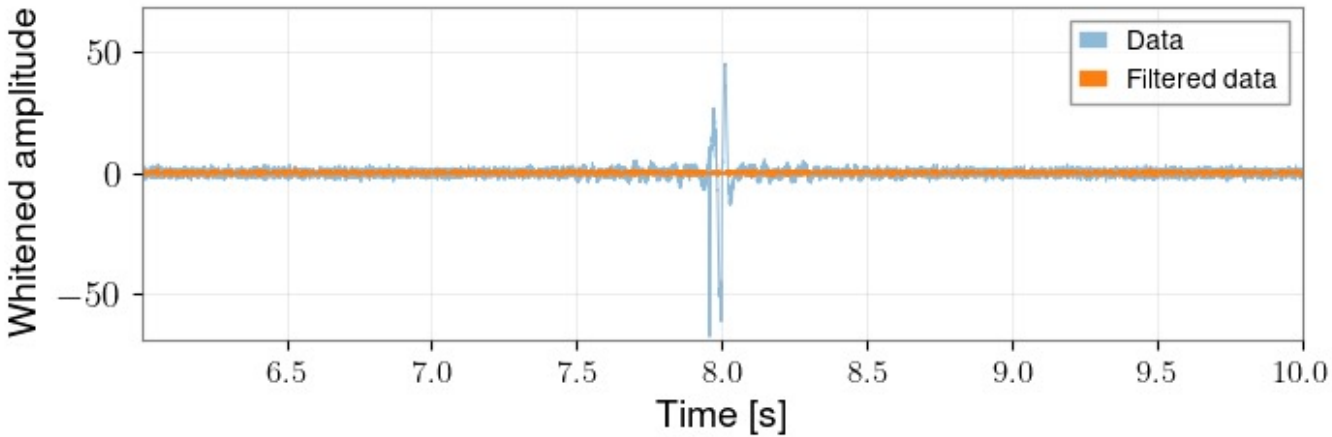
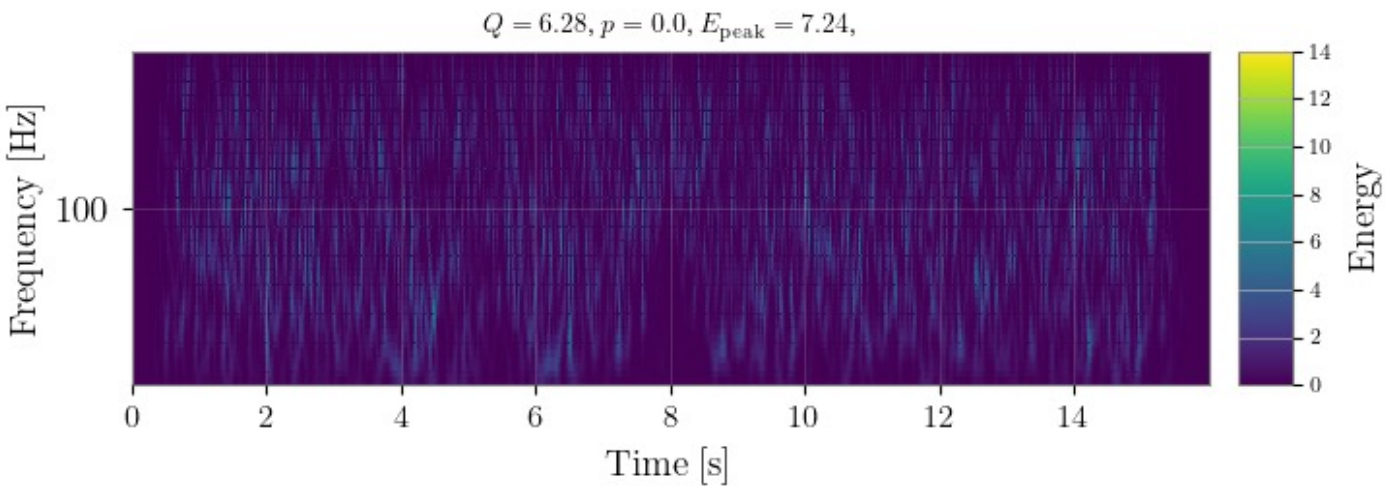
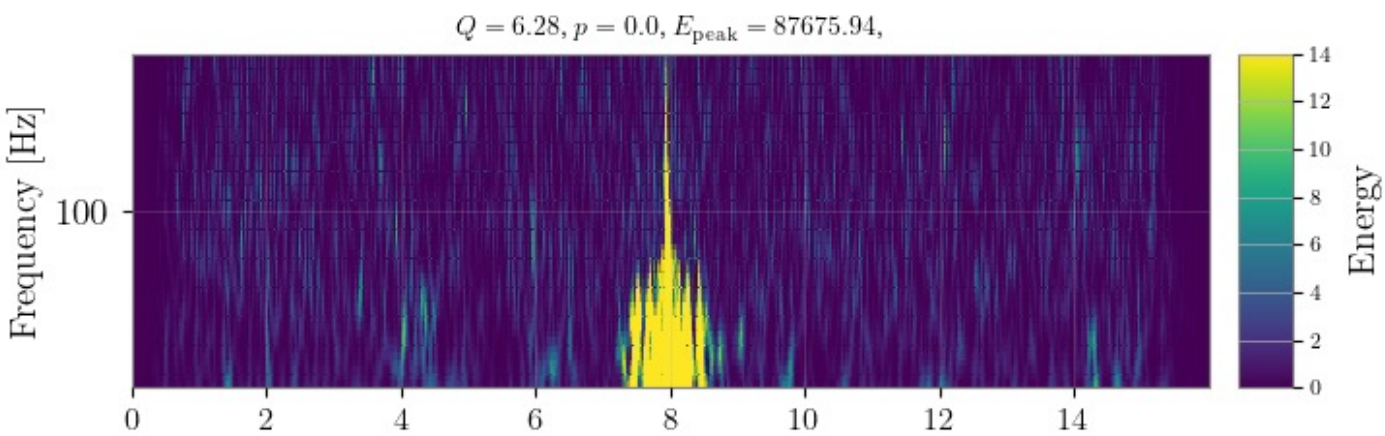


Virtuoso and Milotti, 2024

denoising_threshold = 7.0

Glitch 142301 in file "L1_O3b.csv"

row: 142301
GPS time: 1268351513.19922
SNR: 387.094360351562
Duration: 4.0
Peak frequency: 26.0149402618408
Label: Light_Modulation
Confidence: 0.97454571723938



denoising_threshold = 7.0

Glitch 145123 in file "L1_O3b.csv"

row: 145123

GPS time: 1256888726.84766

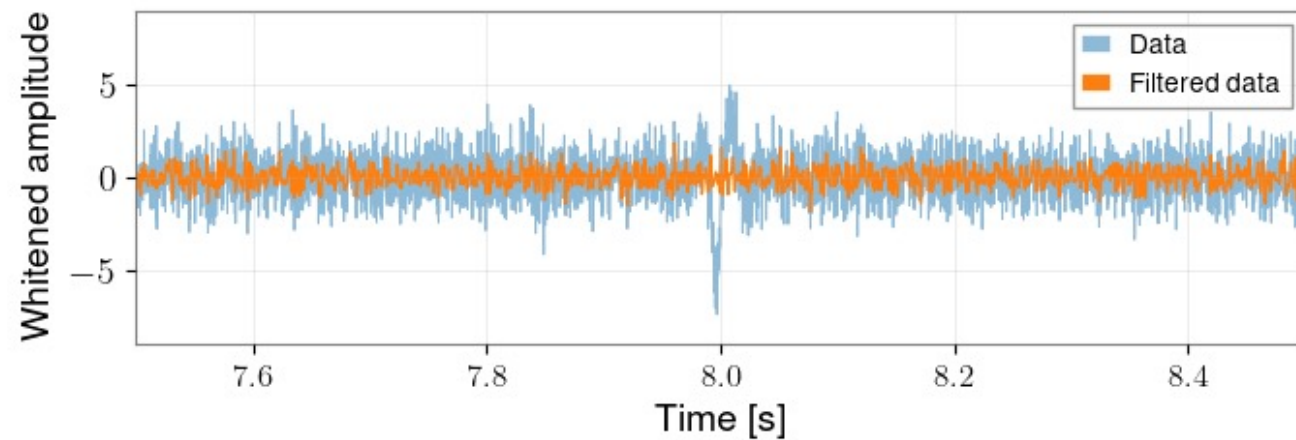
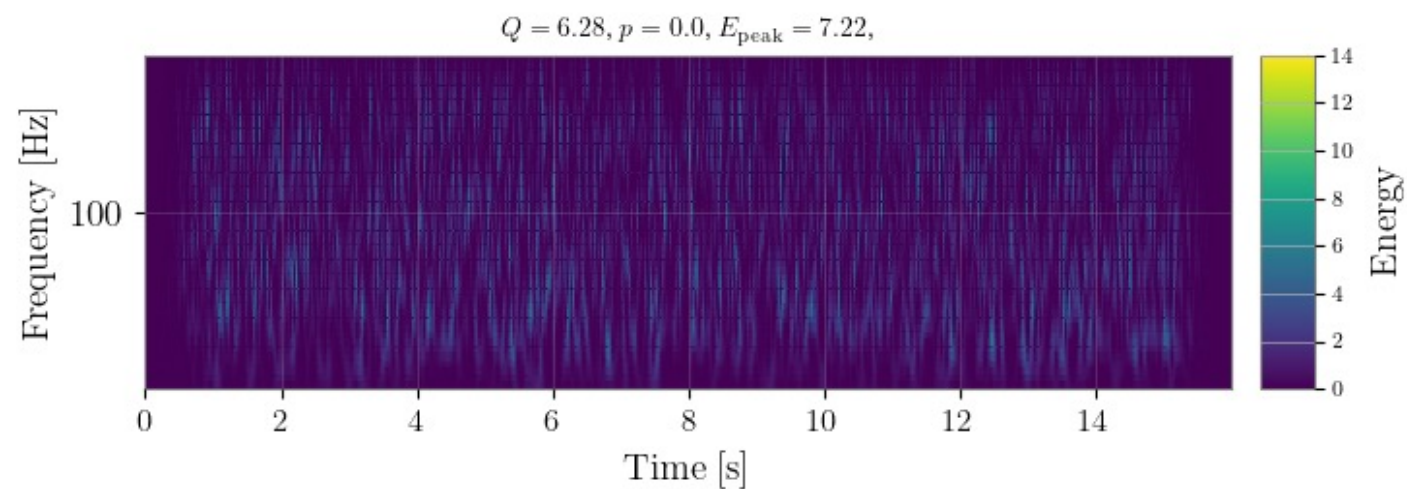
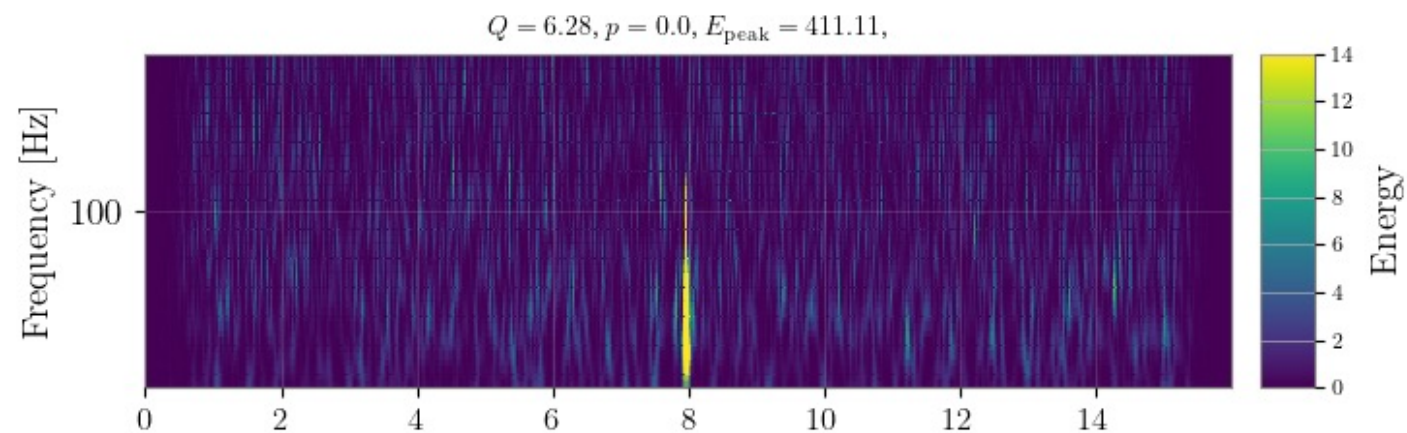
SNR: 25.5926609039307

Duration: 1.0

Peak frequency: 39.7892990112305

Label: Tomte

Confidence: 0.9999997615814208



denoising_threshold = 7.0

Glitch 185667 in file "L1_O3b.csv"

row: 185667

GPS time: 1261053843.0625

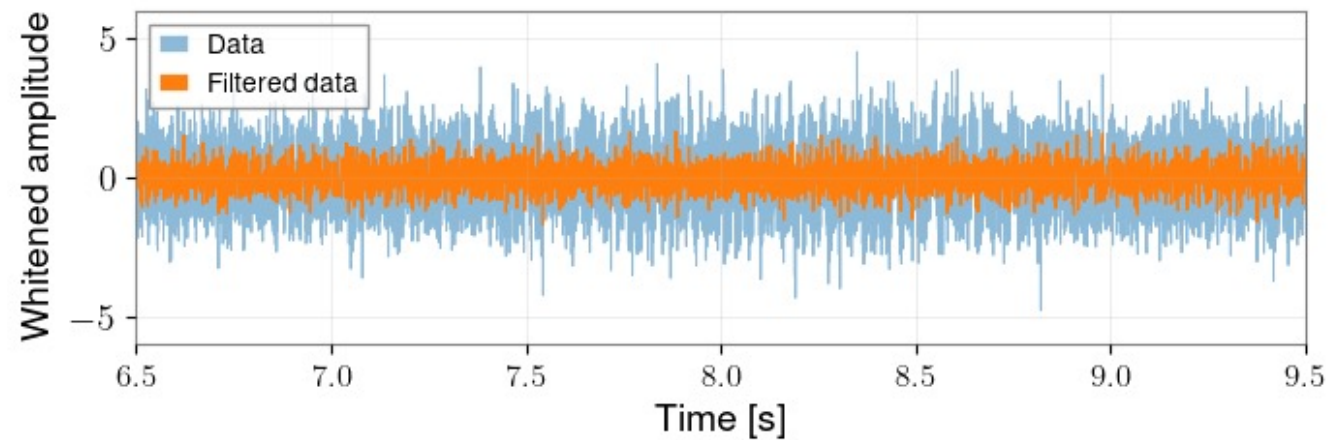
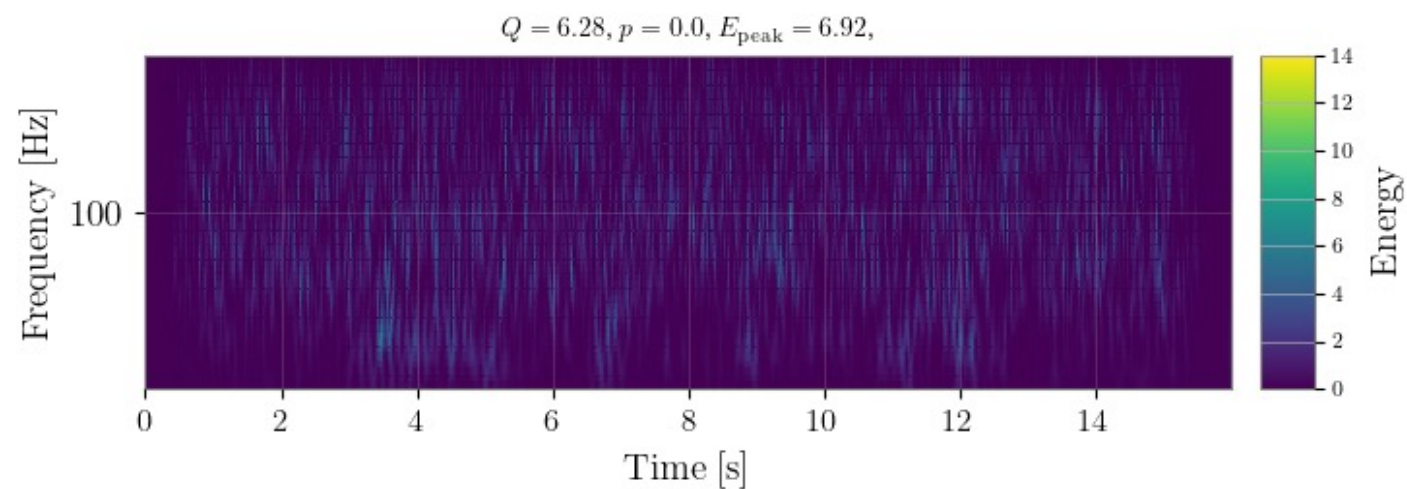
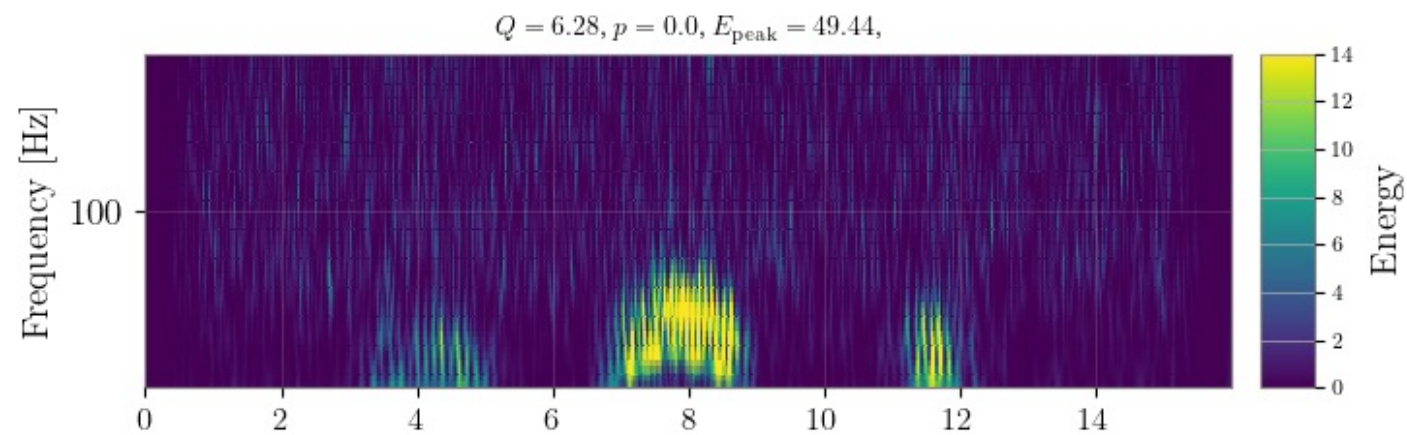
SNR: 25.5363903045654

Duration: 3.0

Peak frequency: 35.1623115539551

Label: Scattered_Light

Confidence: 0.9692816138267516

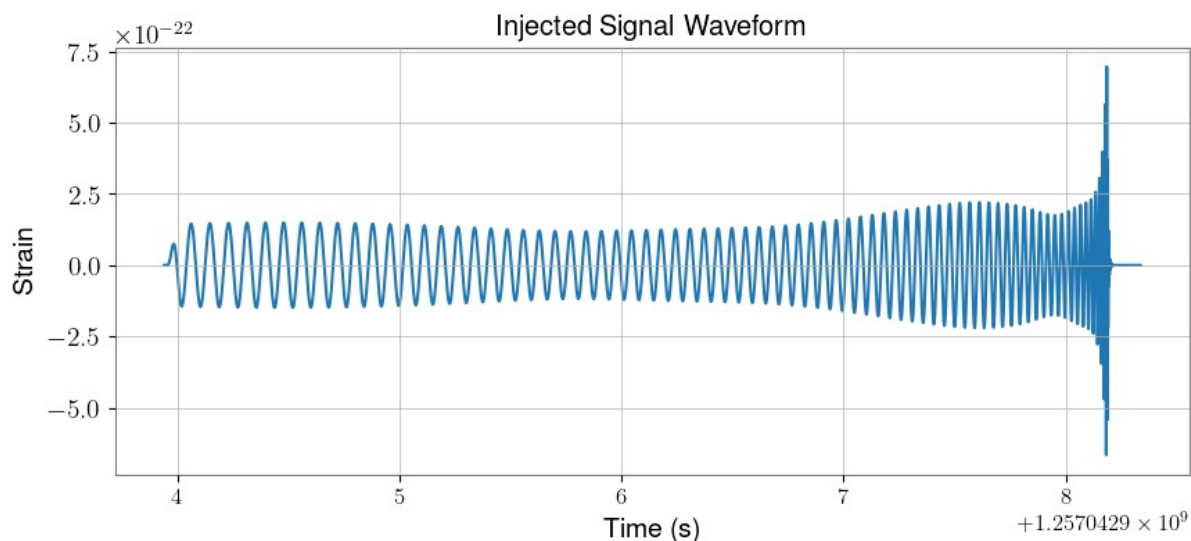


"Tile-wise" denoising, with injections

We associate an energy density to each tile of the discretized invertible Q-transform. We can remove excess noise (glitches) by discarding those tiles that have an above-threshold value, for some selected threshold.

We have tested this denoising scheme both with real events detected in past LVK observing runs and publicly released, and with injections into data that include glitches. We used the catalog of glitches released by the GravitySpy project (<https://zenodo.org/records/5649212>).

Here, we show a few results obtained with the injection of a waveform for [GW200129_065458](#), obtained from the IMRPhenomTPHM approximant and MaxL parameters obtained with Bilby (Prod 5).



MaxL parameters

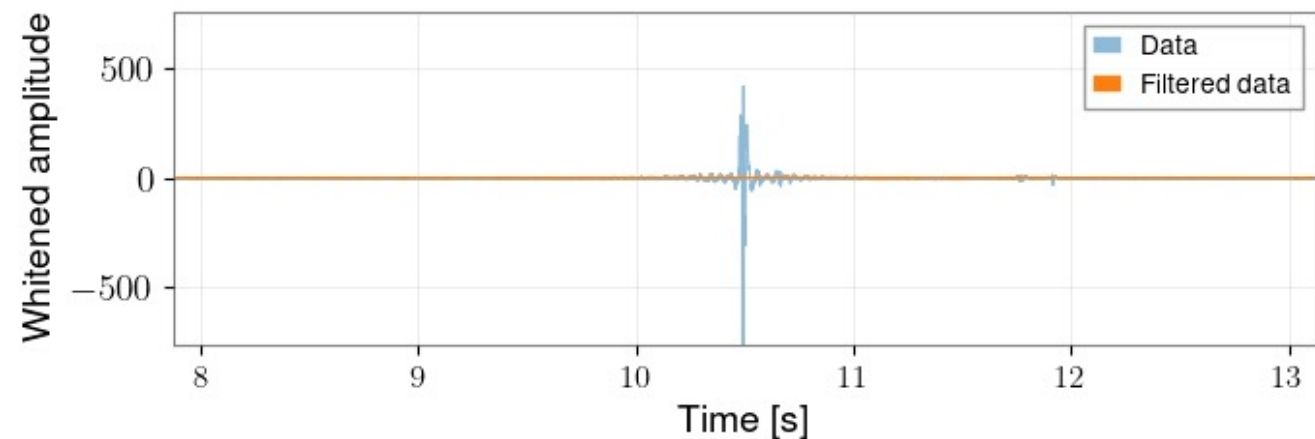
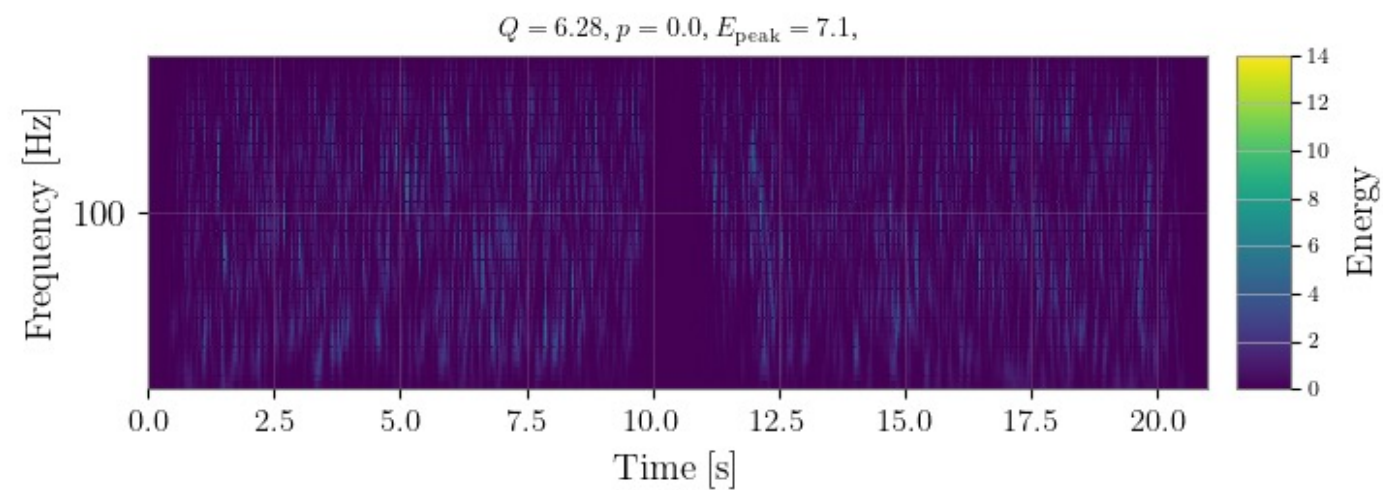
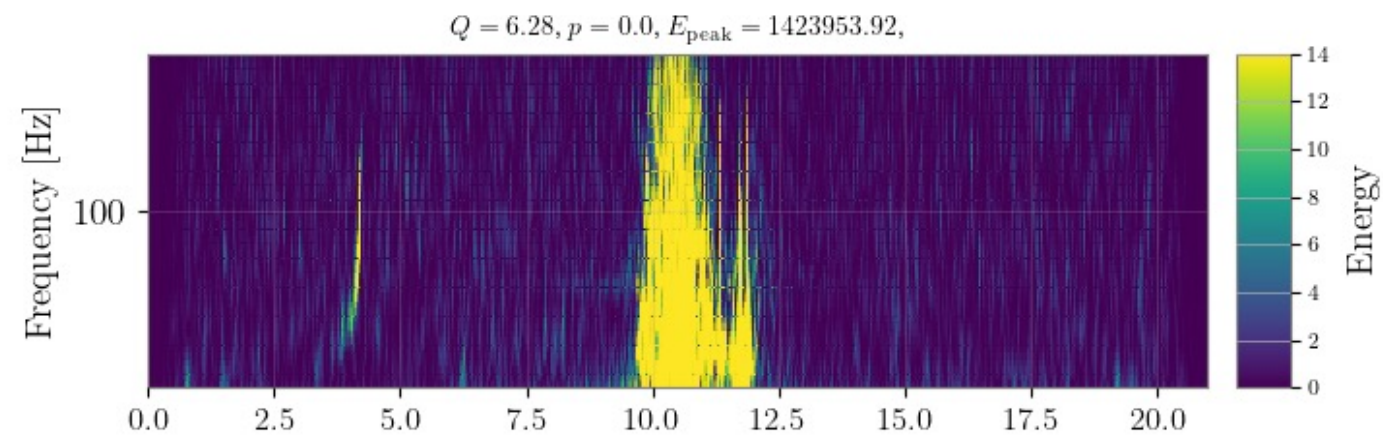
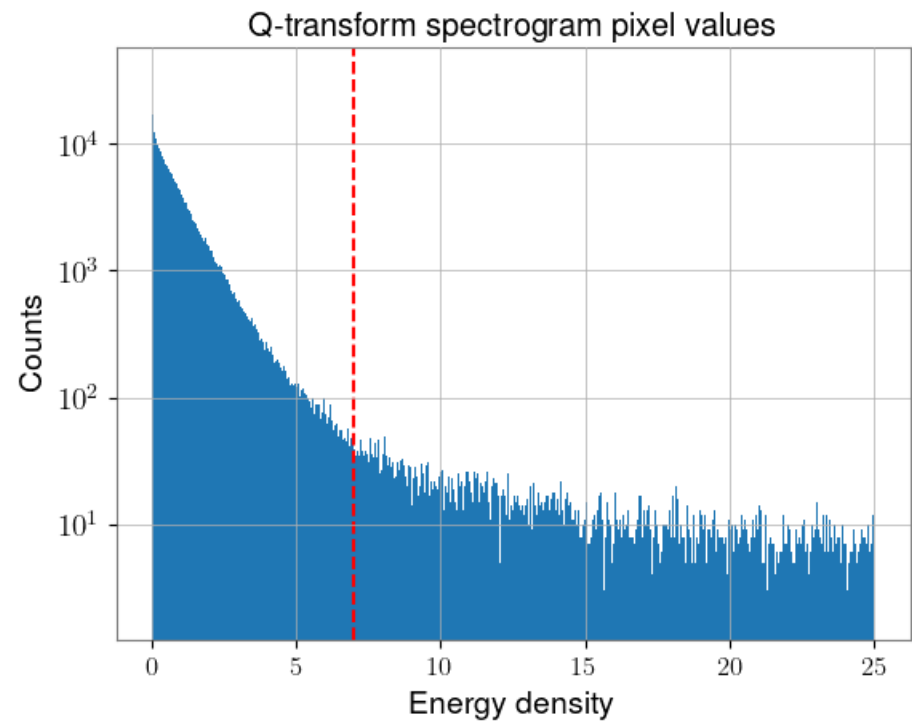
SNR=26.79

$D_L=791.31$ Mpc

denoising_threshold = 7.0

Glitch 145673 in file "L1_O3b.csv"

row: 145673
GPS time: 1257042914.44141 SNR:
1673.49304199219
Duration: 5.25
Peak frequency: 32.1732215881348
Label: Extremely_Loud
Confidence: 0.999979734420776



denoising_threshold = 20.0

Glitch 145673 in file "L1_O3b.csv"

row: 145673

GPS time: 1257042914.44141 SNR:

1673.49304199219

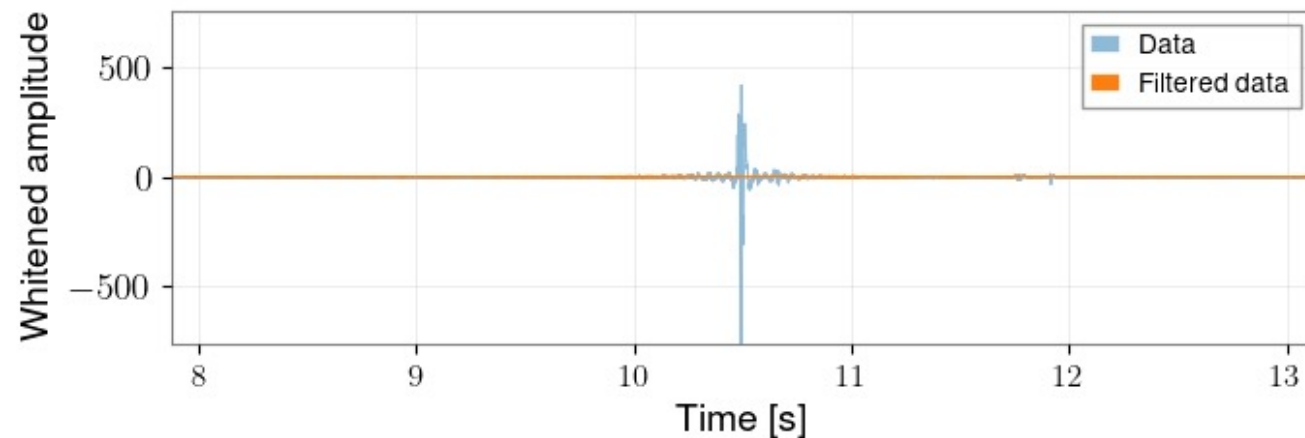
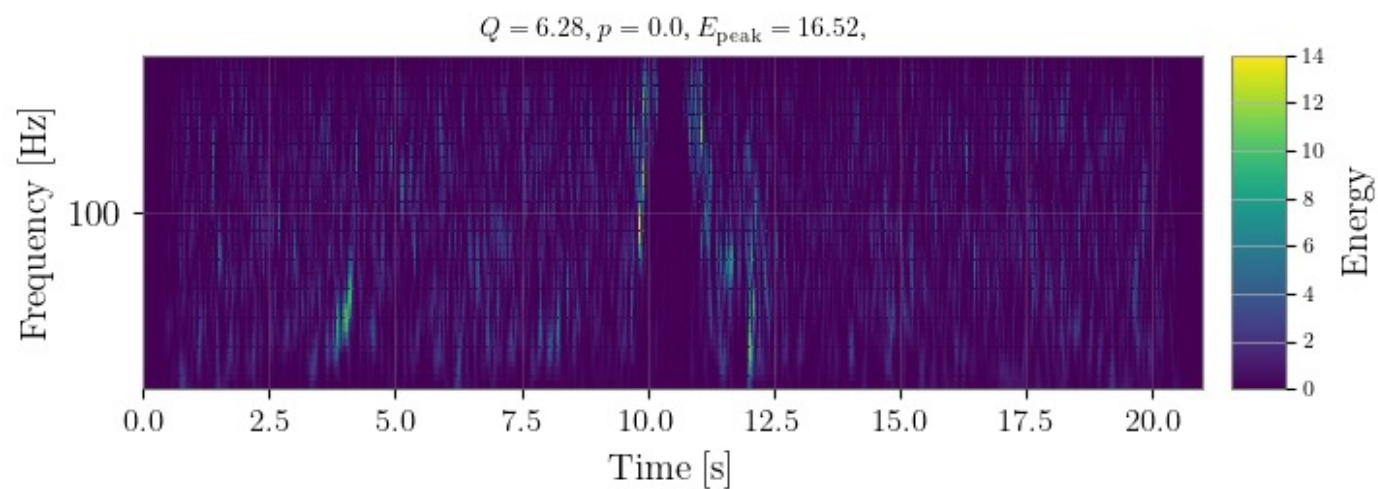
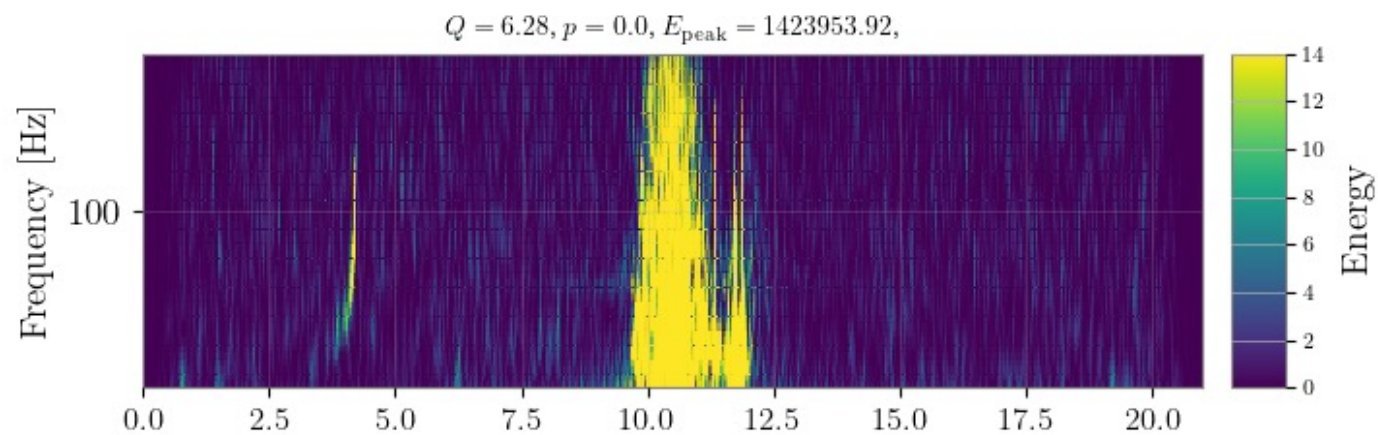
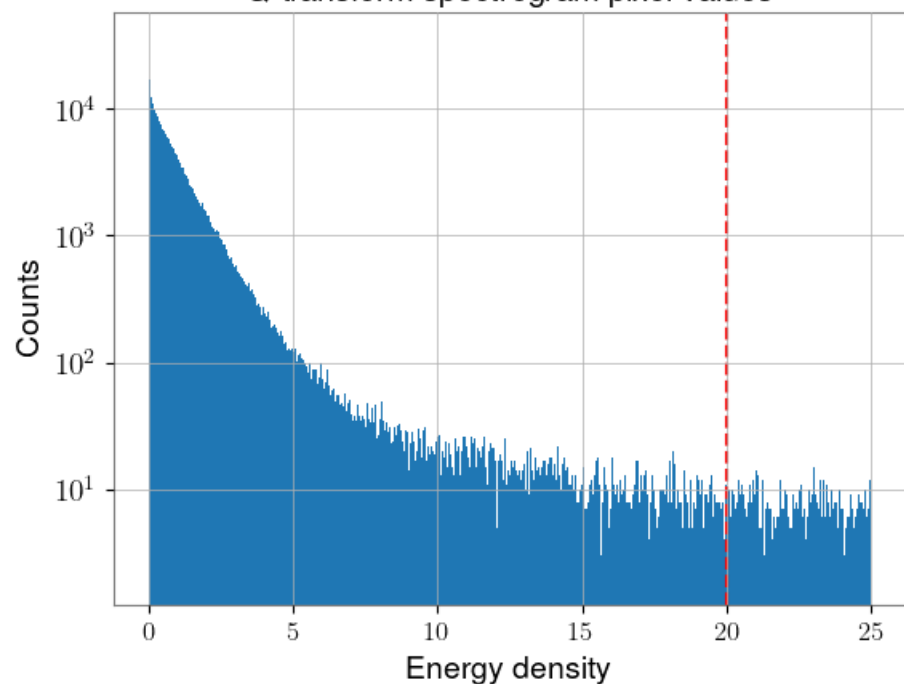
Duration: 5.25

Peak frequency: 32.1732215881348

Label: Extremely_Loud

Confidence: 0.9999979734420776

Q-transform spectrogram pixel values



Software

- The tests reported here used the software developed alongside the paper describing the invertible Q-transform and the invertible Qp-transform ([Virtuoso and Milotti, 2024](#)), which is available on Zenodo (<https://zenodo.org/records/10649073>)
- The library distributed with the SW in Zenodo is not optimized. Simple tests indicate that in this implementation the computational complexity is proportional to $O(N^2)$, where N is the number of samples.
- A PyTorch implementation of the invertible Qp-transform exists (<https://github.com/Unoaccaso/timeseries/tree/main>), and it was shown to produce a very large speedup (around 100). This means that we can expect the deglitching of a 16 s stretch of data sampled at 4.096 kHz to take ~ 1 s.

Conclusions

- The deglitching method presented here is mathematically exact and statistically well-characterized.
- The method can be used as-is, with existing data.
- The test implementation shows that glitches can be removed very effectively, but also that a simple thresholding method cannot distinguish between signals and glitches, requiring a specification of the TF region with the glitch.
- A future non-supervised implementation will need to be able to recognize the TF region with the glitch and limit the glitch removal procedure to that specific region.