Fast likelihood evaluation of eccentric–precessing binaries using relative binning

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Why fast likelihood evaluation for eccentric signals

- 3G observatories expect to see many low-mass eccentric binaries
  - Eccentric capture in dense environment: globular clusters, young stellar clusters, and AGN disks
    - Results in a highly eccentric binary and merges quickly
  - NS and stellar mass BH are abundant in in dense astrophysical environments
  - Expected to detect hundreds to thousands of eccentric binaries with 3G

Kowalska et al. arXiv:1010.0511,
Baibhav et al. arXiv:1906.04197

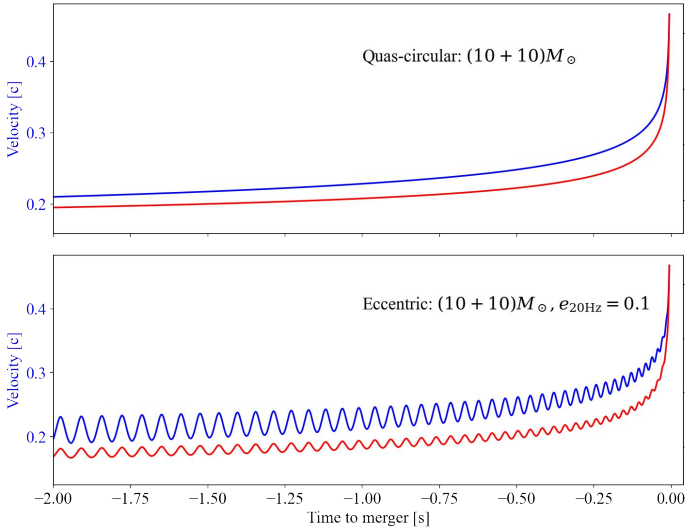
- Complex binary dynamics
  - Emitting GW at multiple harmonics
  - Longer signal
  - Generating waveform expensive

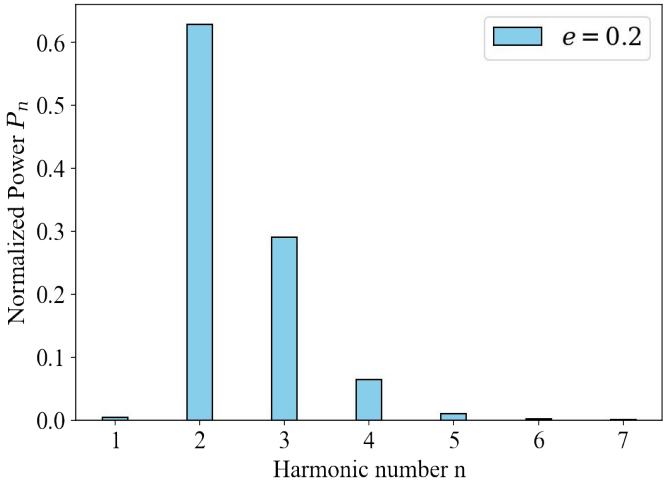
Complex binary dynamics: Emission at multiple harmonics

Turn on higher harmonics!

$$P_n \propto \frac{n^4}{32}\left[\left(J_{n-2}(ne) - 2eJ_{n-1}(ne) + \frac{2}{n}J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne)\right)^2 + (1-e^2)\left(J_n(ne)\right)^2\right]$$

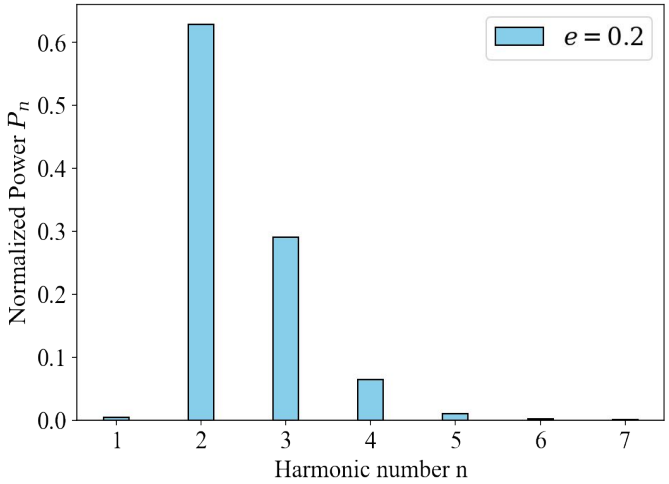
$$h_{+,\times}(t) = \sum_{n=1} A_n^{+,\times}(e)\cos(n\omega t + \phi_n)$$

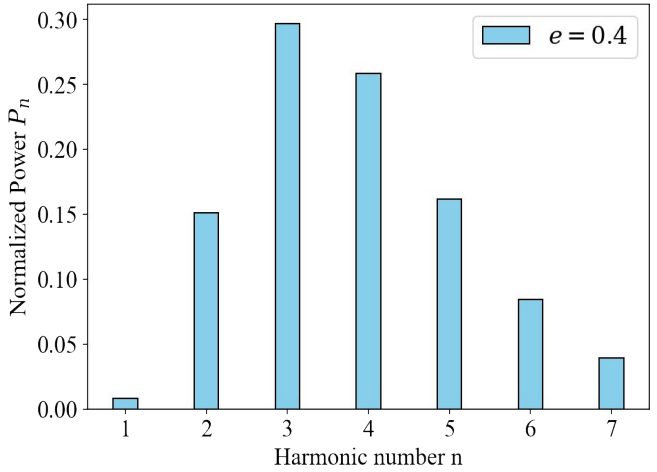




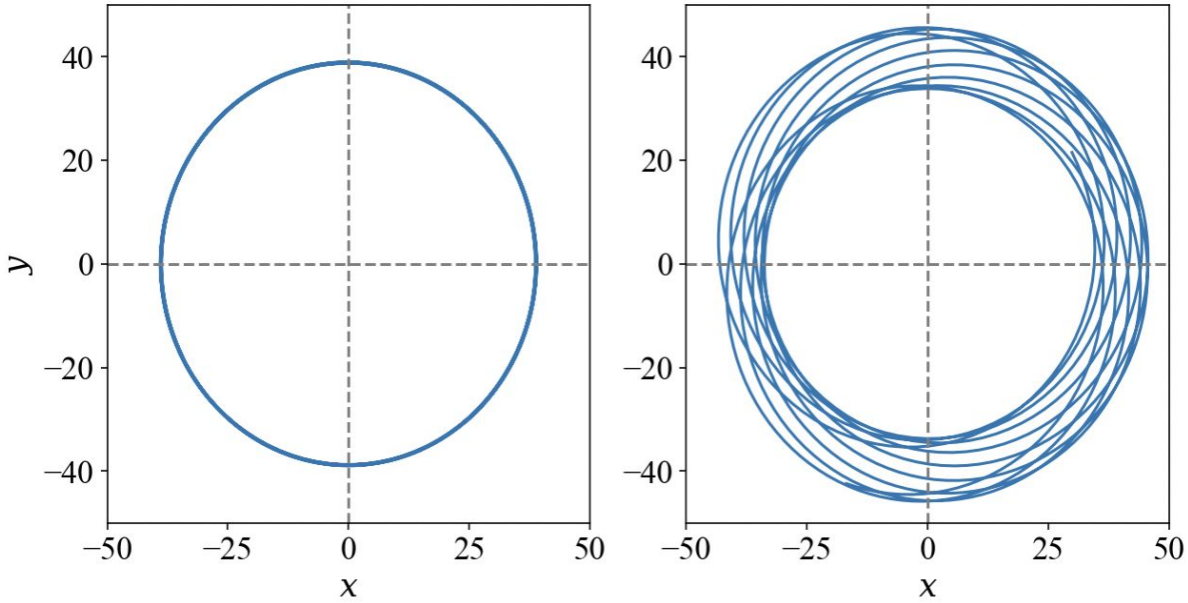
Complex binary dynamics: Emission at multiple harmonics

Stronger higher harmonics as eccentricity increases





Complex binary dynamics: Orbital precession



Circular

Eccentric

Eccentric binary dynamics

Dynamics in EFPE model:

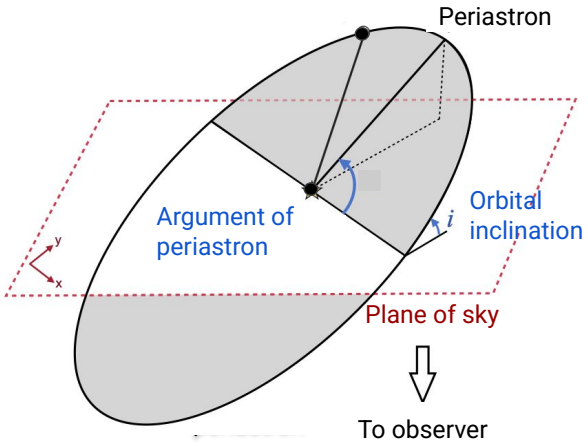
$$\mathcal{D}y = \eta y^9 \sum_{n\geq 0} a_n y^n,$$

$$\mathcal{D}e^2 = -\eta y^8 \sum_{n\geq 0} b_n y^n,$$

$$\mathcal{D}\lambda = y^3,$$

$$\mathcal{D}\delta\lambda = \frac{k y^3}{1+k}, \quad k = y^2 \sum_{n\geq 0} k_n y^n.$$

- PN expansion parameter $y = (M\omega)^{1/3}/\sqrt{(1-e^2)}$
- Orbital eccentricity e
- Mean orbital phase λ
- Argument of periastron $\delta\lambda$
- Differential operator $\mathcal{D} = M/(1-e^2)^{3/2}\frac{d}{dt}$



A. Klein et al. arXiv:1801.08542
A. Klein, arXiv:2106.10291
J. N. Arredondo et al, arXiv:2402.06804

Eccentric waveform in frequency domain

$$\tilde{h}_{+,\times}(f) = \underbrace{\sum_i \sqrt{2\pi}\, T_i^{\mathrm{SPA}}\, e^{i\left(2\pi f t_i^{\mathrm{SPA}} - \phi(t_i^{\mathrm{SPA}}) - \pi/4\right)}}_{\text{Fourier modes}} \underbrace{\sum_{k=-k_{\max}}^{k_{\max}} a_{k,k_{\max}}\, \mathcal{A}^{+,\times}_{2,m_i,n_i}\left(t_i^{\mathrm{SPA}} + kT_i^{\mathrm{SPA}}\right)}_{\text{SUA correcttion to amplitude}}$$

$$\left.\frac{d\phi(t)}{dt}\right|_{t=t^{\mathrm{SPA}}} = 2\pi f$$

$$T^{\mathrm{SPA}} = \left|\frac{d^2\phi(t)}{dt^2}\right|^{-1/2}_{t=t^{\mathrm{SPA}}}$$

Effectively, $\tilde{h}(f) = \sum_i \tilde{h}_i(f)$

SUA keeps the SPA's phase accuracy but adds a smarter treatment of the amplitude

Waveform generator package pyEFPE Morras et al. arXiv:2502.03929

➔ Includes eccentricity and spin precession

Relative binning scheme for eccentric signal

$$\ln \mathcal{L}(\vec{\lambda}) = -\frac{1}{2}\langle d - h(\vec{\lambda}) \,|\, d - h(\vec{\lambda})\rangle$$

$$\Rightarrow \ln \mathcal{L} = \langle d \,|\, h\rangle - \frac{1}{2}\langle h \,|\, h\rangle - \frac{1}{2}\langle d \,|\, d\rangle$$

Integral involves complex multiplication of $N = f_{max} \times T$

➔ *Becomes expensive for long duration signal*

Relative binning scheme: *approximate likelihood without recomputing the waveform at all frequencies*

$$r(f) = \frac{h(f)}{h_0(f)} = r_0(h,b) + r_1(h,b)\,(f - f_c(b)) + \cdots$$

Must be slowly varying over frequency

Previous studies: Zackay et al. arXiv:1806.08792
Krishna et al. arXiv:2312.06009

With higher-order modes:
mode-by-mode treatment

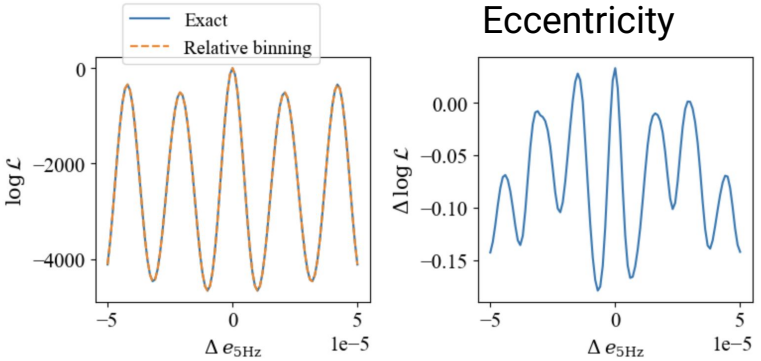
Leslie et al. arXiv:2109.09872
Narola et al. arXiv:2308.12140

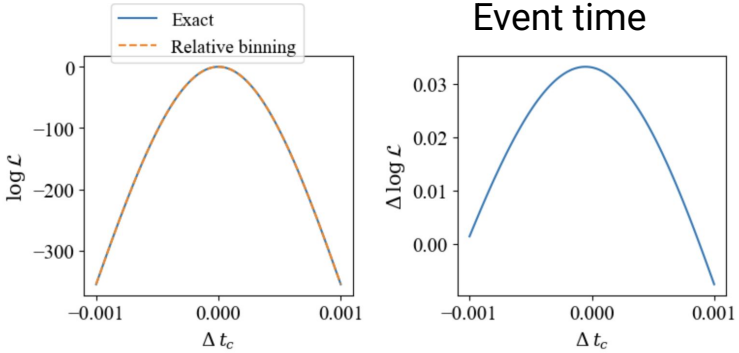
Apply mod-by-mode treatment for Fourier modes

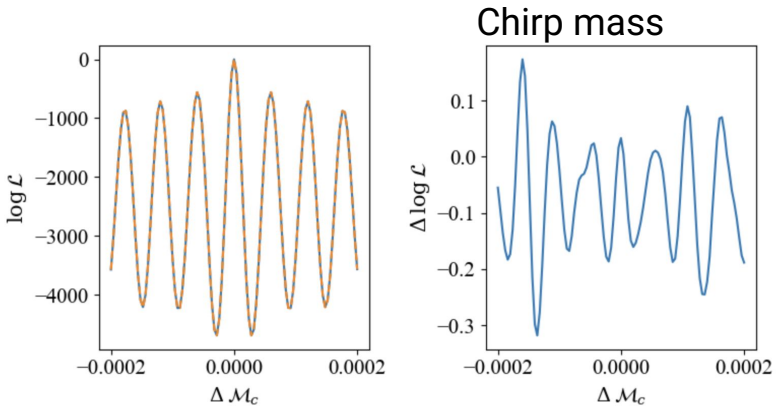
$$\tilde{h}(f) = \sum_i \tilde{h}_i(f)$$

Accuracy of relative binning likelihood

Varying single parameter at a time



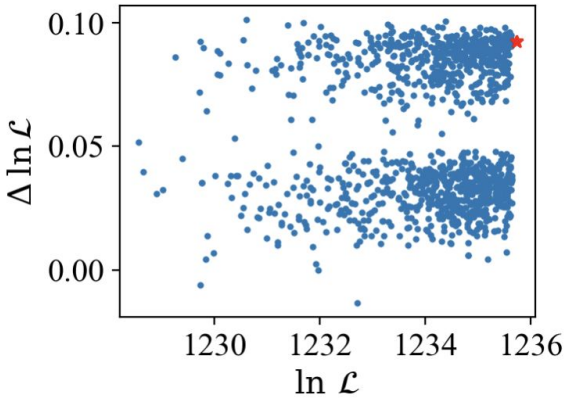


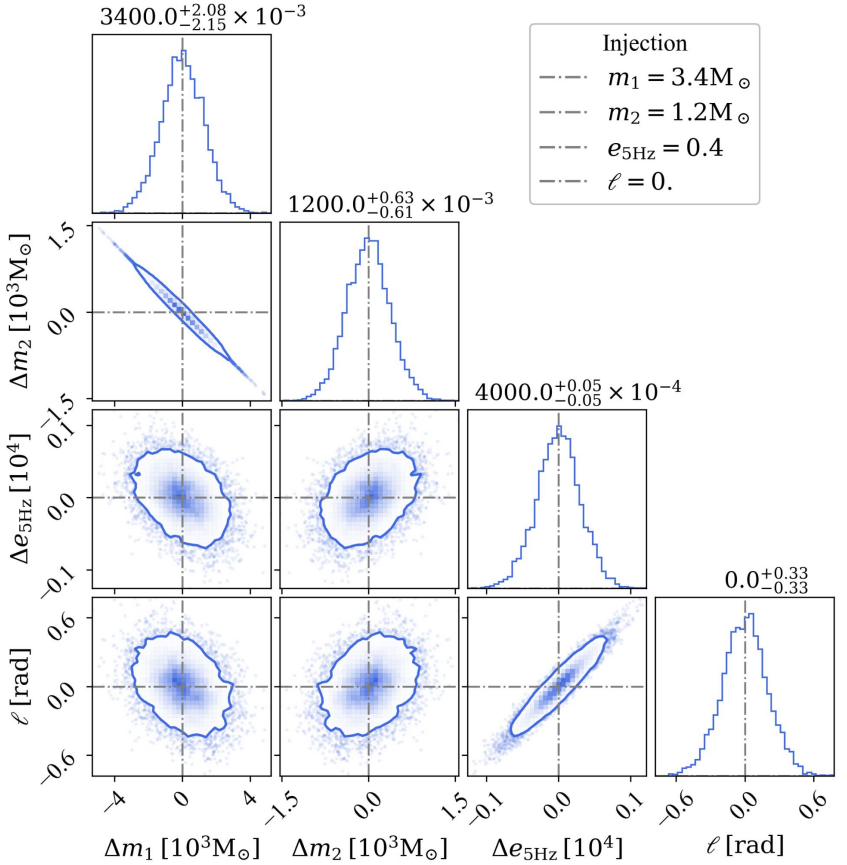


Results for NSBH injection

Eccentric NSBH injection with ET sensitivity

- Lower cutoff frequency 5 Hz, injection SNR ~50, Signal length 4096 sec
- Standard Bilby & Dynesty setup
  - ~ 3 day over 64 cpus
  - Standard PE will take > 100 days





Conclusion

- Develop relative binning scheme for eccentric waveform using the Fourier modes
- We can achieve substantial speedup for 3rd generation observatories
  - For BNS systems, improvement factor ~ 1000
    - Substantial for lower cutoff frequency 2 Hz

Thank you for listening!