Optical Configurations

Lecture 1

Antonio Perreca
Gran Sasso Science Institute (GSSI)
INFN - Laboratori Nazionali del Gran Sasso

STGWD 2026 — PhD International School on Technologies in Gravitational Waves Detection

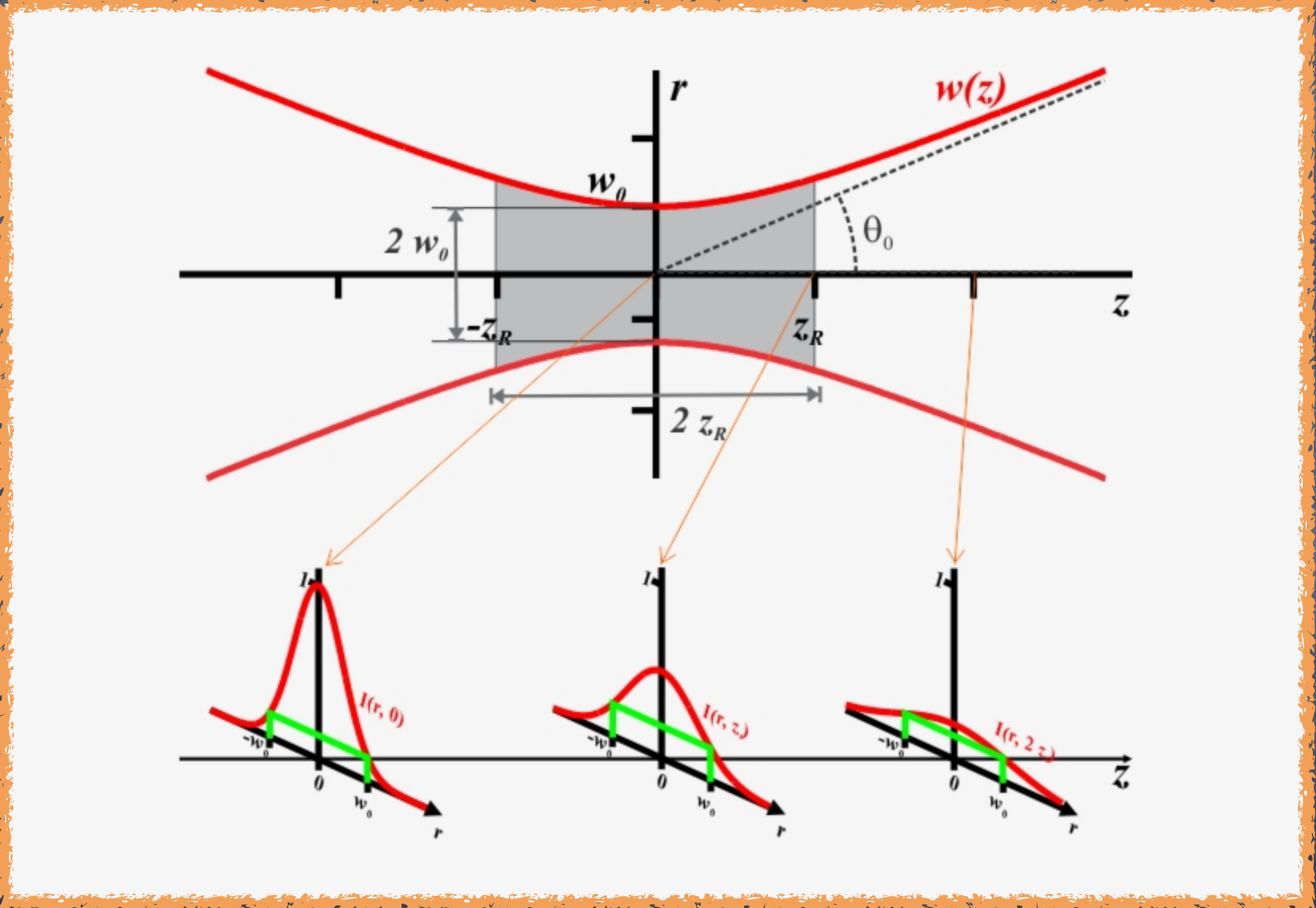
Erice (TP), Italy - May 2026

Lecture 1: Overview

- Gaussian beam (essential tools)
- Michelson interferometer
- Fabry-Perot cavities
- Power Recycling
- Signal Recycling
- Toward real detectors

Real laser beams are not plane waves

Longitudinal Profile



The minimum size is the waist w_0

Gaussian Beam
Description

$$\vec{E}(x,y,z,t) = \vec{E}_0 \, e^{-\frac{r^2}{w^2(z)}} \cdot e^{i\frac{kr^2}{2R(z)}} \cdot e^{i\arctan(\frac{z}{z_R})} \cdot e^{-i(kz-\omega t)}$$

Fundamental mode (or mode 00)

Gaussian Beam

Description

$$\vec{E}(x,y,z,t) = \vec{E}_0 \, e^{-\frac{r^2}{w^2(z)}} \, e^{i\frac{kr^2}{2R(z)}} \, e^{i\arctan(\frac{z}{z_R})} \, e^{-i(kz-\omega t)}$$

- $\vec{E}_0$: Amplitude
- $e^{-\frac{r^2}{w^2(z)}}$: Width
- $e^{i\frac{kr^2}{2R(z)}}$: Curvature
- $e^{i\arctan(\frac{z}{z_R})}$: Gouy phase
- $e^{-i(kz-\omega t)}$: Plane waves

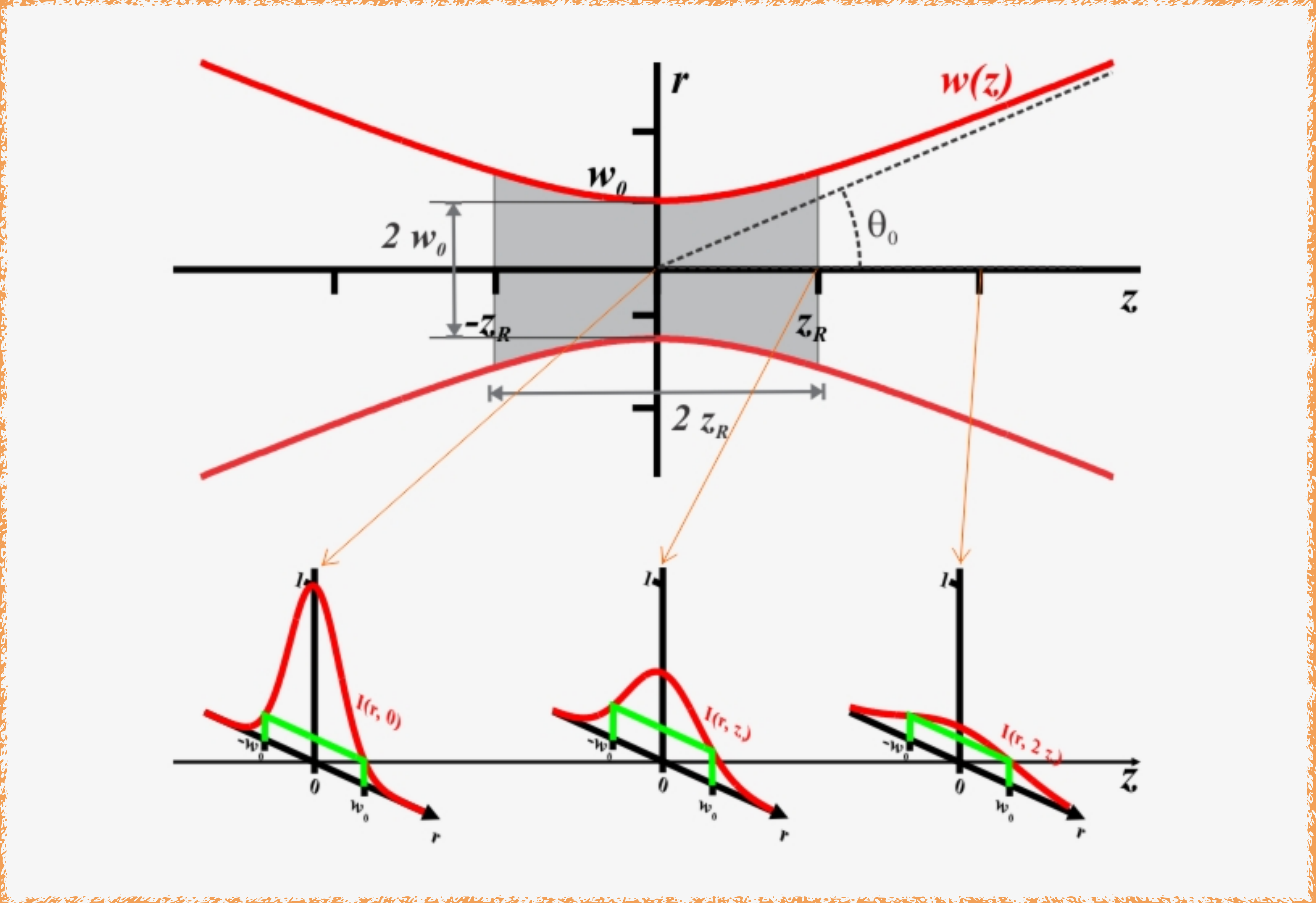
Fundamental mode (or mode 00)

Gaussian Beam
Beam size evolution

$$\overrightarrow{E}(x,y,z,t) = \overrightarrow{E}_0 \, e^{-\frac{r^2}{w^2(z)}} \cdot e^{\frac{ikr^2}{2R(z)}} \cdot e^{i\arctan(\frac{z}{z_R})} \cdot e^{-i(kz-\omega t)}$$

Beam size

$$w^2(z) = w_0^2 \left[1 + \left(\frac{z}{z_R}\right)^2\right]$$



- In z=0, w(z)=w0 is a minimum of the width, called waist
- z>>zR, FAR FIELD, w(z) is linear with z:
- In z=zR, w(z)=sqrt(2)w0

Fundamental mode (or mode 00)

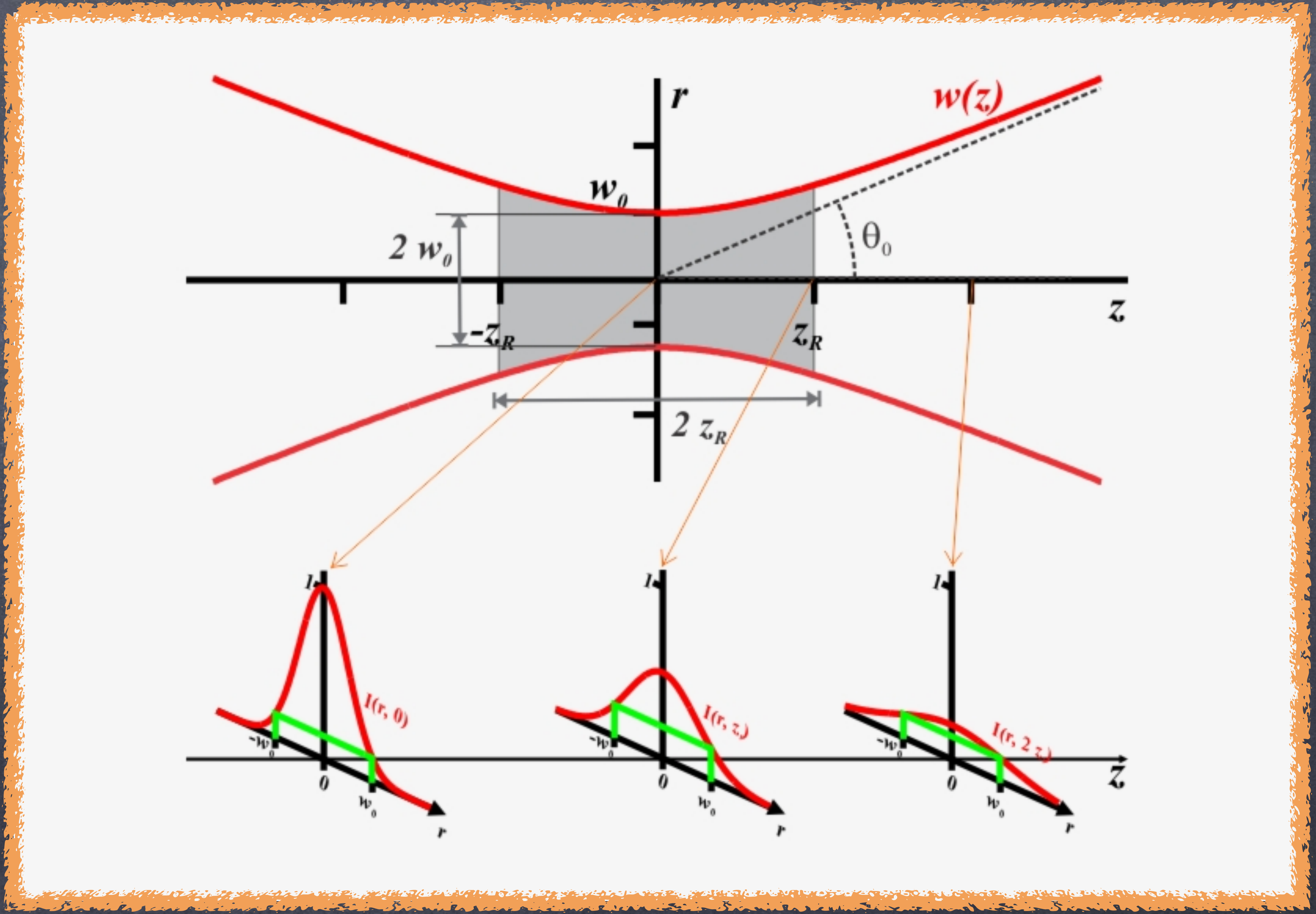
Gaussian Beam
Rayleigh range

$$\vec{E}(x,y,z,t) = \vec{E}_0\, e^{-\frac{r^2}{w^2(z)}} \cdot e^{\frac{ikr^2}{2R(z)}} \cdot e^{i\arctan(\frac{z}{z_R})} \cdot e^{-i(kz-\omega t)}$$

Rayleigh range

$$z_R = \frac{\pi w_0^2}{\lambda}$$

- Sets the natural scale of the beam
- Transition between near and far field



Fundamental mode (or mode 00)

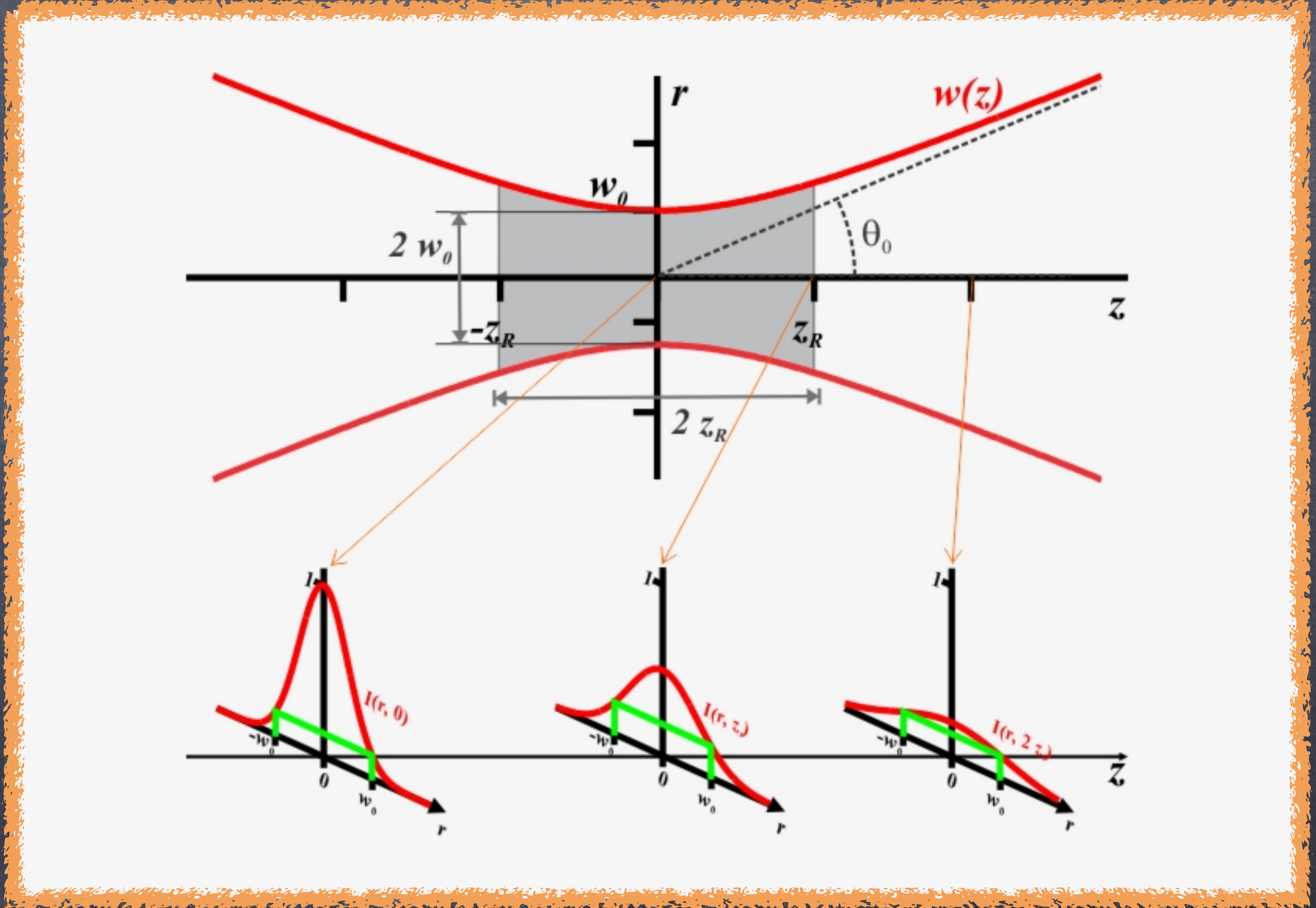
Gaussian Beam
Wavefront curvature

$$\vec{E}(x,y,z,t) = \vec{E}_0\, e^{-\frac{r^2}{w^2(z)}} \cdot e^{\frac{ikr^2}{2R(z)}} \cdot e^{i\arctan(\frac{z}{z_R})} \cdot e^{-i(kz-\omega t)}$$

Curvature

$$R(z) = z\left[1 + \left(\frac{z_R}{z}\right)^2\right]$$

- At the waist: R —-> inf —-> flat wavefront
- z>>zR, FAR FIELD, R(z)->z is linear with z



Fundamental mode (or mode 00)

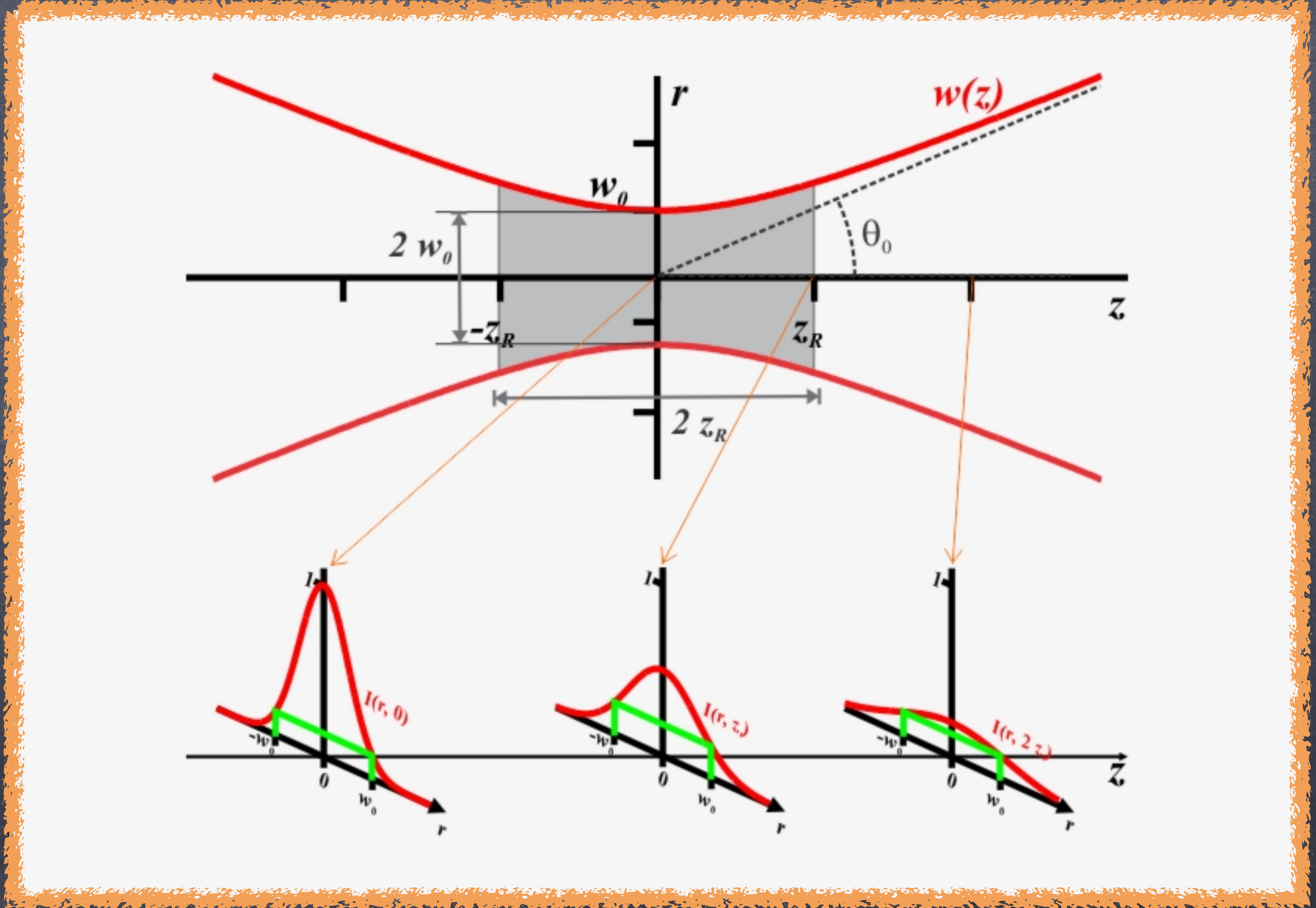
Gaussian Beam
Beam divergence

$$\vec{E}(x,y,z,t) = \vec{E}_0 \, e^{-\frac{r^2}{w^2(z)}} \cdot e^{\frac{ikr^2}{2R(z)}} \cdot e^{i\arctan(\frac{z}{z_R})} \cdot e^{-i(kz-\omega t)}$$

Divergence

$$\Theta_0 = \frac{\lambda}{\pi w_0}$$

- Smaller waist —-> larger divergence
- Larger waist —-> smaller divergence



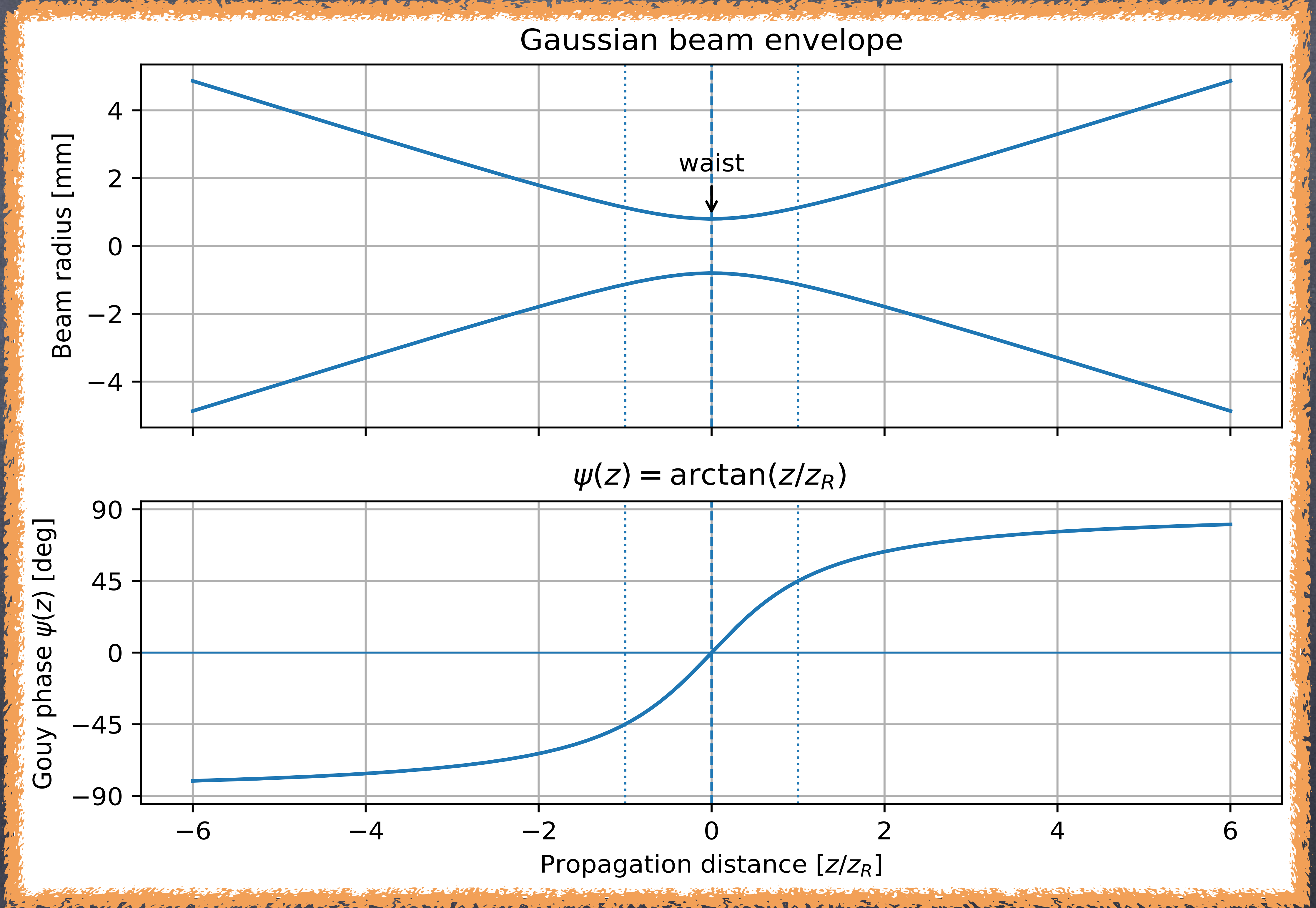
Fundamental mode (or mode 00)

Gaussian Beam
Gouy phase

$$\vec{E}(x,y,z,t) = \vec{E}_0\, e^{-\frac{r^2}{w^2(z)}} \cdot e^{\frac{ikr^2}{2R(z)}} \cdot e^{i\arctan(\frac{z}{z_R})} \cdot e^{-i(kz-\omega t)}$$

Gouy phase

$$GP = \arctan(\frac{z}{z_R})$$



- A Gaussian beam accumulates an extra phase while propagating through the waist

Fundamental mode (or mode 00)

Gaussian Beam
Complex beam parameter (q)

$$\vec{E}(x,y,z,t) = \vec{E}_0 \, e^{-\frac{r^2}{w^2(z)}} \cdot e^{\frac{ikr^2}{2R(z)}} \cdot e^{i\arctan(\frac{z}{z_R})} \cdot e^{-i(kz-\omega t)}$$

Complex beam parameter

$$\frac{1}{q} = \frac{1}{R(z)} - i\frac{\lambda}{\pi w^2(z)}$$

- Fully describe the beam
- Convenient for optical systems

Gaussian beam is fully described by
q or **w0,z0**

Fundamental mode (or mode 00)

Key Message
Gaussian beam parameters

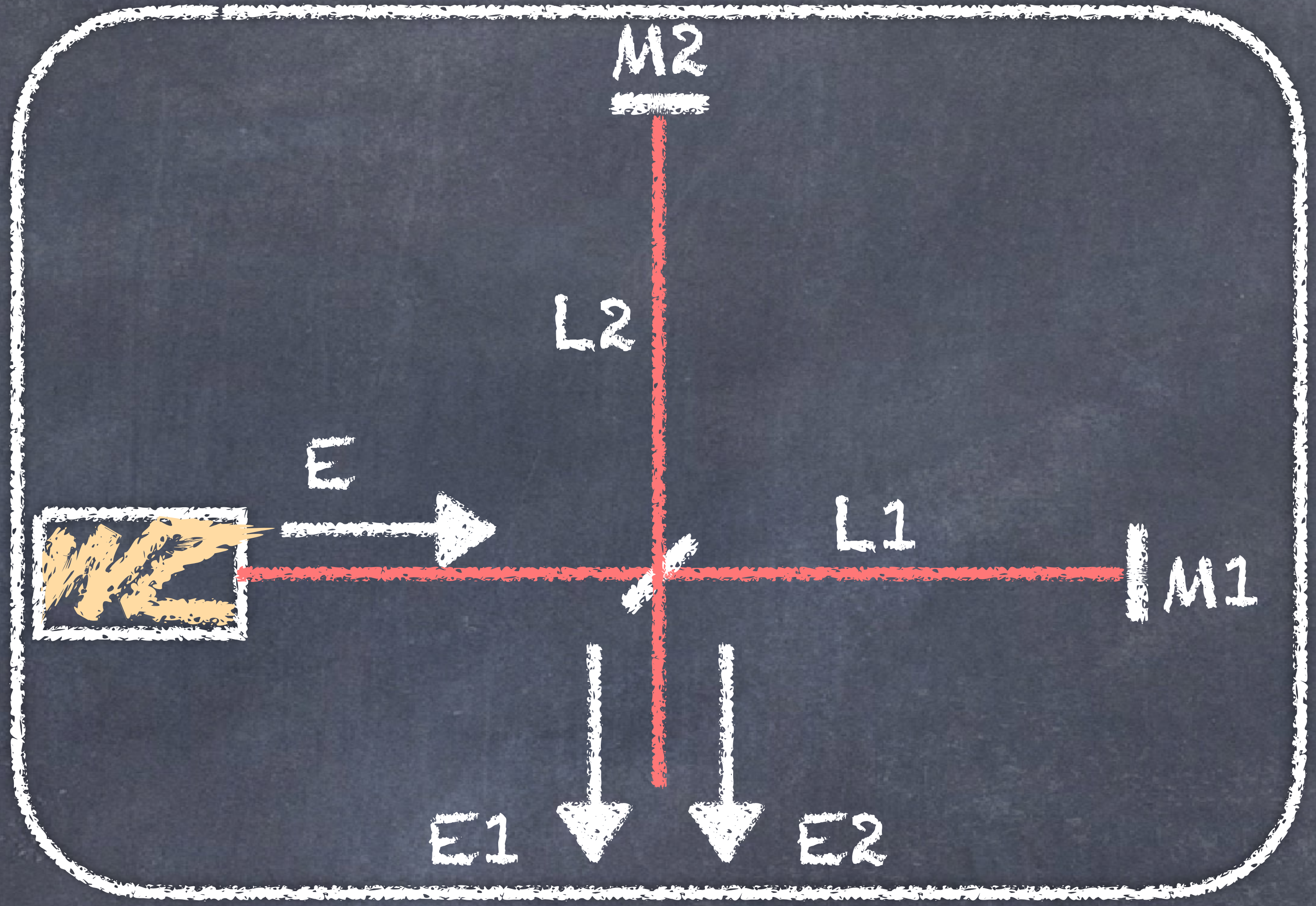
Gaussian beam is fully described
by its waist size **w0** and waist position **z0**

- Define the beam "everywhere"
- Must match the optical system

Michelson Interferometer

Michelson Interferometer
Generalities

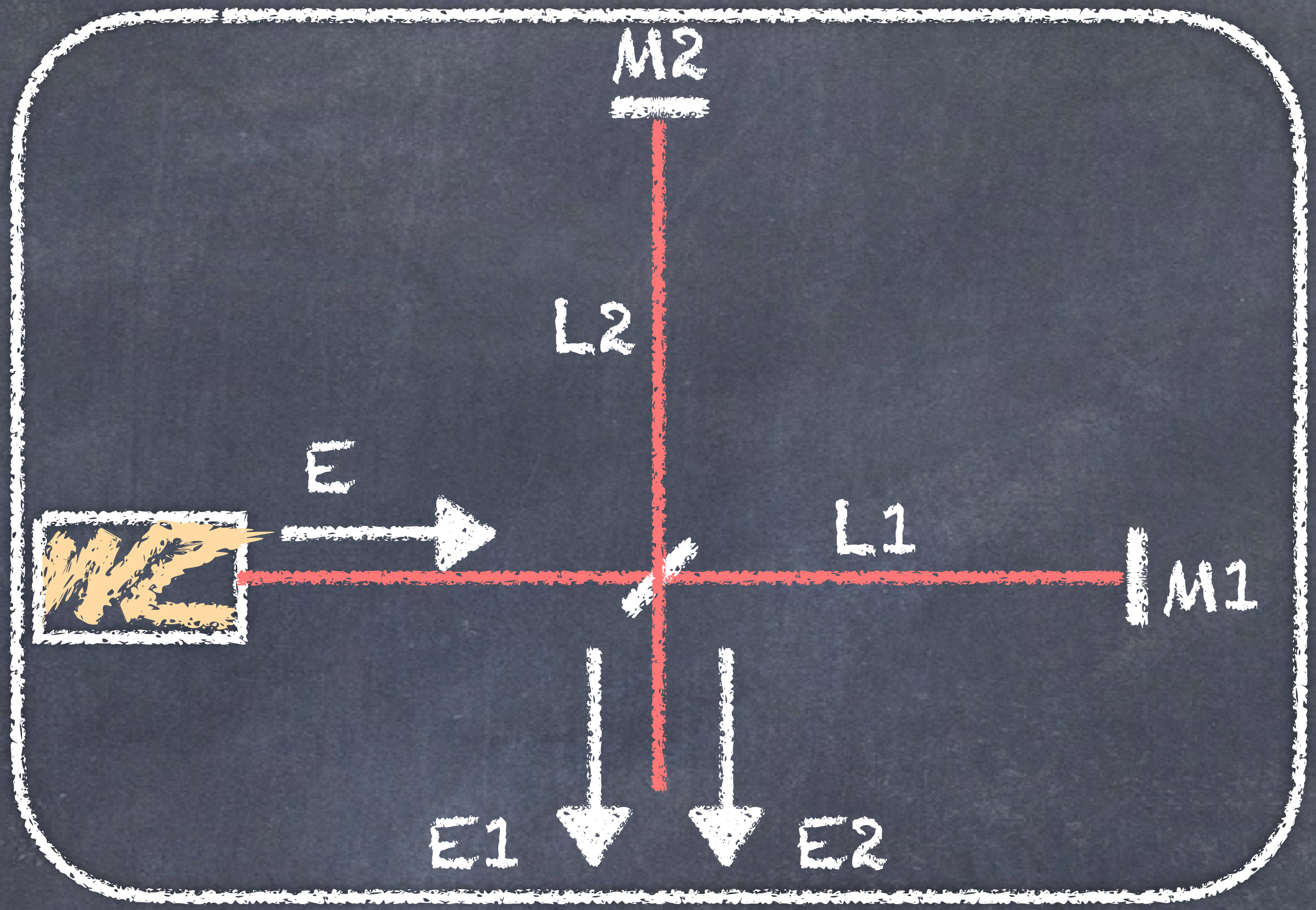
The Michelson Interferometer splits a single beam of light into two paths, bouncing the beams back and recombining them by using two mirrors and a beam splitter



- Measures differential arm lengths
- Converts length into phase

Michelson Interferometer
Generalities

The Michelson Interferometer splits a single beam of light into two paths, bouncing the beams back and recombining them by using two mirrors and a beam splitter



Reflectivity and Transmissivity

- R, r = Reflectivity coef.
- T, t = Transmittance coef.
- r, t = for field
- R, T = for power
- R = r^2, T = t^2

- Measures differential arm lengths
- Converts length into phase

Michelson Interferometer
Generalities

The Michelson Interferometer splits a single beam of light into two paths,
bouncing the beams back and recombining them by using two mirrors and a beam splitter

Light from laser

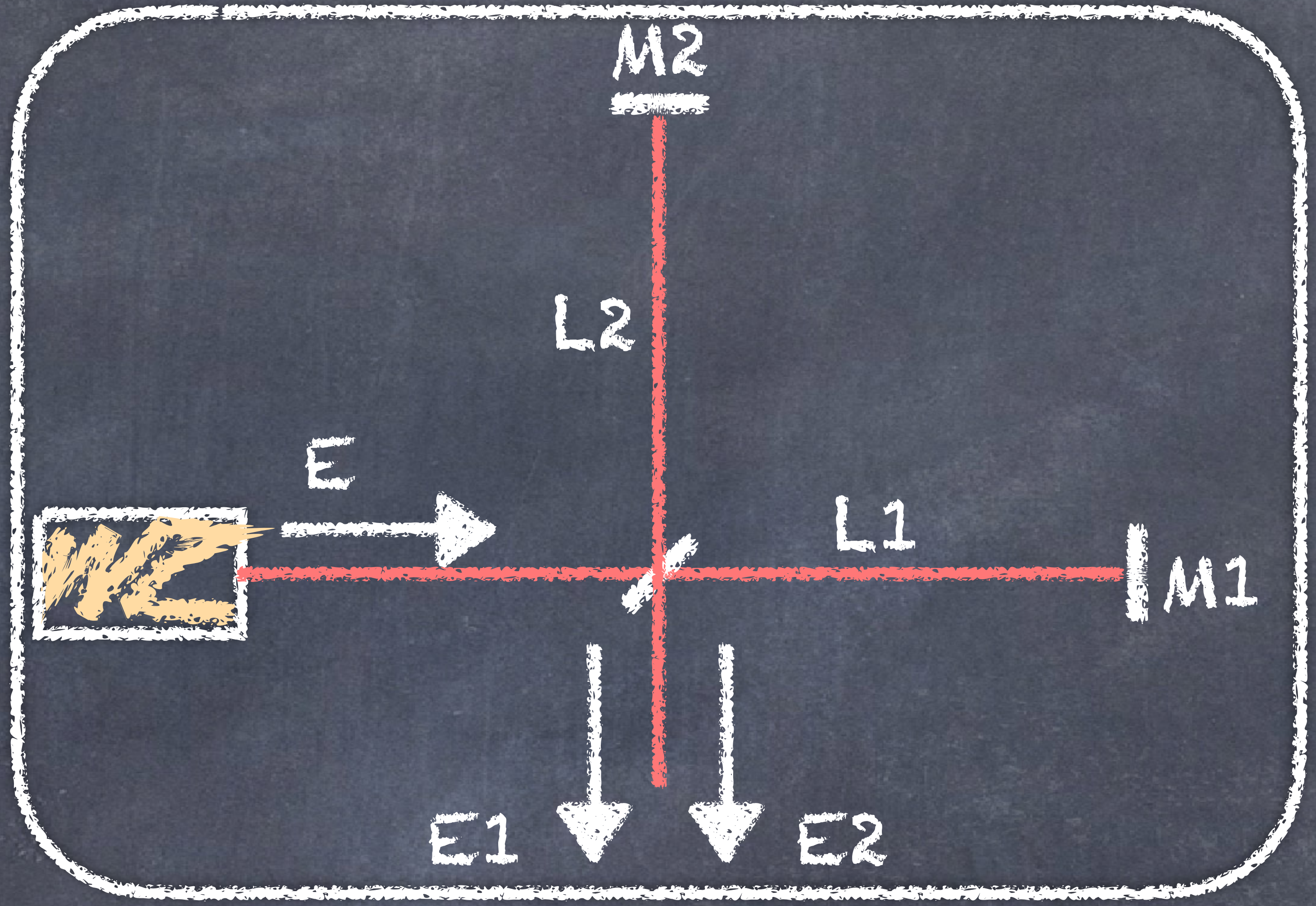
$$E(x, y, z, t) = E_0 e^{-i\omega t}$$

Plane wave

Superposition principle

$$E_1 + E_2 = \frac{i}{2}\, E\, (e^{2ikL_1} + e^{2ikL_2})$$

with r=t=1/sqrt(2)



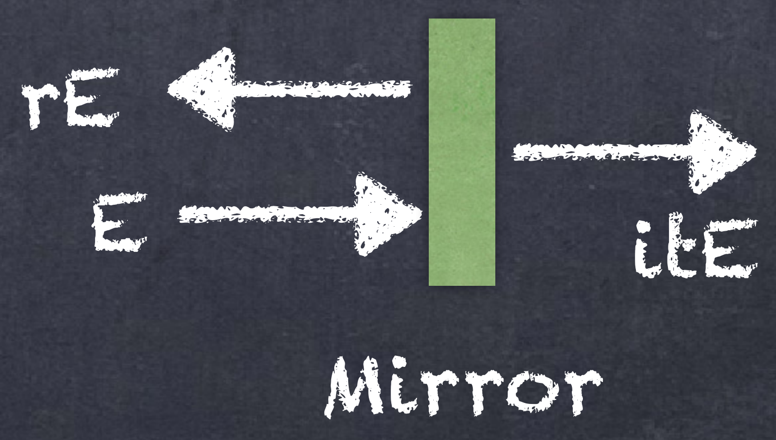
- Measures differential arm lengths
- Converts length into phase

Reflectivity and Transmissivity

- R, r = Reflectivity coef.
- T, t = Transmittance coef.
- r, t = for field
- R, T = for power
- R = r^2, T = t^2

Energy conservation

$$R + T + Loss = 1$$



Michelson Interferometer
Generalities

The Michelson Interferometer splits a single beam of light into two paths,
bouncing the beams back and recombining them by using two mirrors and a beam splitter

Light from laser

$$E(x, y, z, t) = E_0 e^{-i\omega t}$$

Plane wave

Superposition principle

$$E_1 + E_2 = \frac{i}{2} E \, (e^{2ikL_1} + e^{2ikL_2})$$

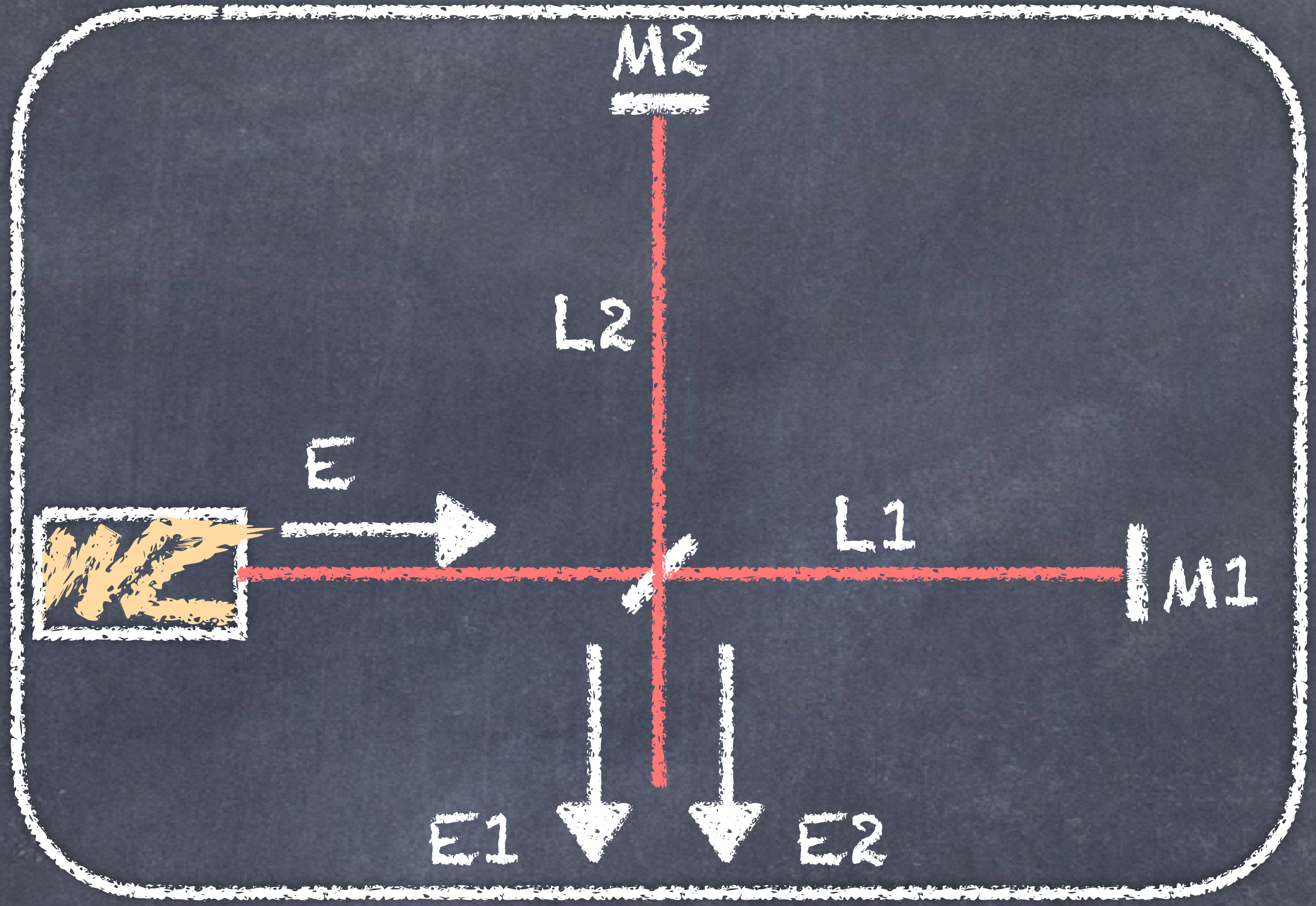
with r=t=1/sqrt(2)

Intensity

$$I = (E_1 + E_2) \cdot (E_1^* + E_2^*)$$

$$= E \cdot E^* \; \cos^2(k\Delta L)$$

with $\Delta L = L_1 - L_2$



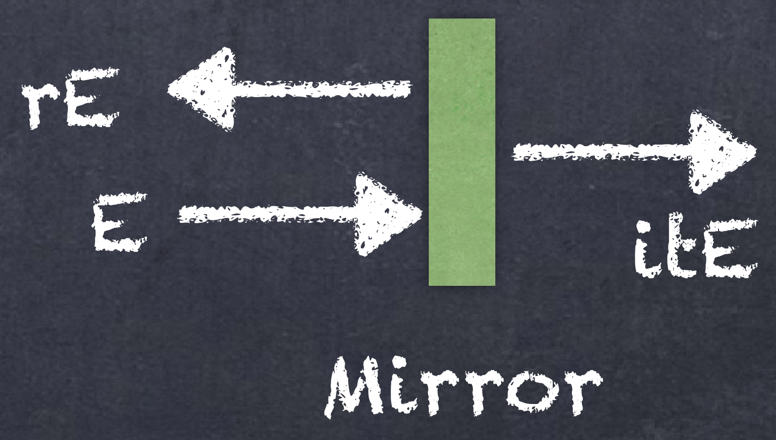
- Measures differential arm lengths
- Converts length into phase

Reflectivity and Transmissivity

- R, r = Reflectivity coef.
- T, t = Transmittance coef.
- r, t = for field
- R, T = for power
- R = r^2, T = t^2

Energy conservation

$$R + T + Loss = 1$$



Michelson Interferometer
Generalities

The Michelson Interferometer splits a single beam of light into two paths, bouncing the beams back and recombining them by using two mirrors and a beam splitter

Intensity

$$I = E \cdot E^* \ \cos^2(k\Delta L)$$

with $\Delta L = L_1 - L_2$

Maximum

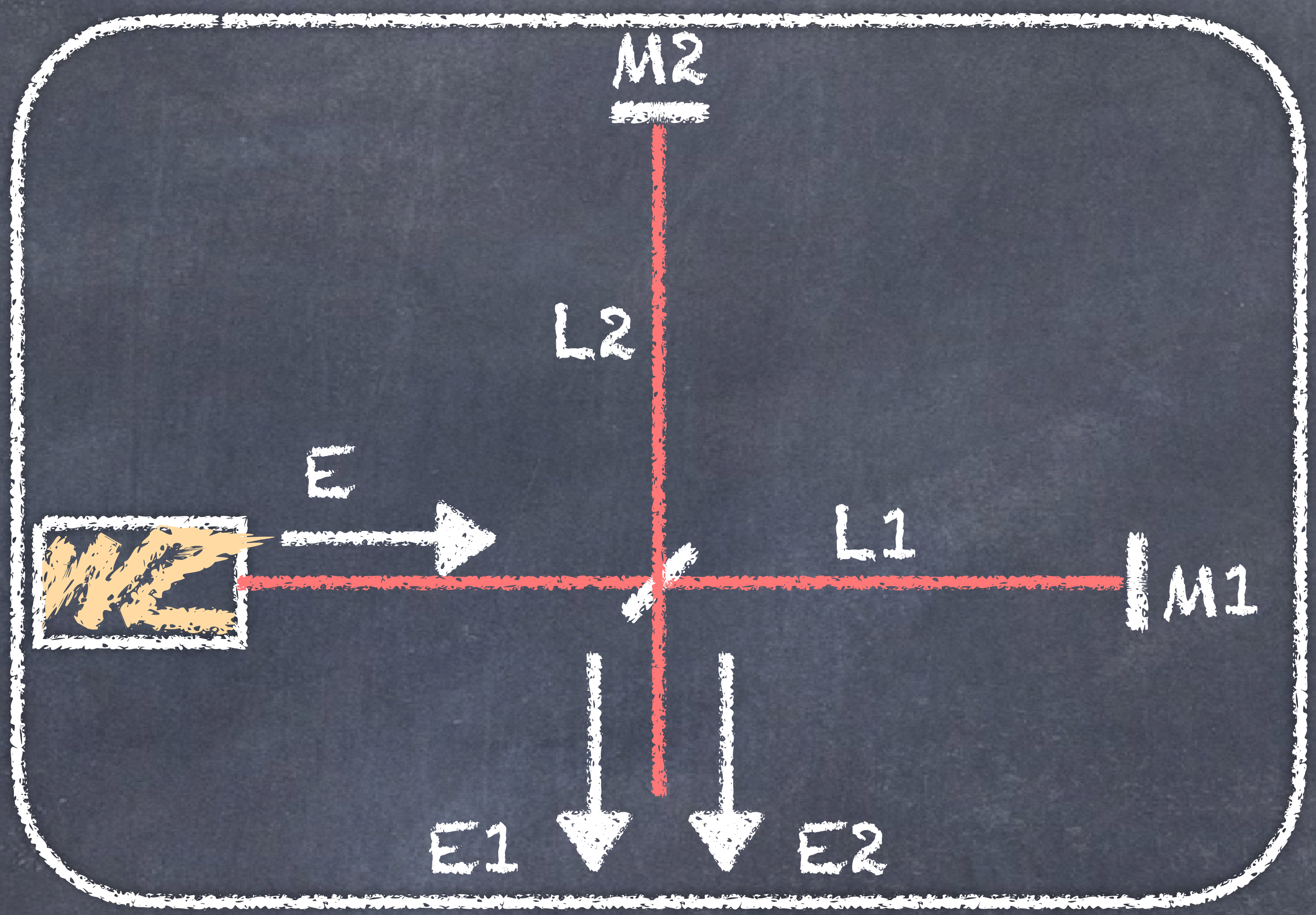
$$k\Delta L = \pi N \rightarrow \Delta L = N\frac{\lambda}{2}$$

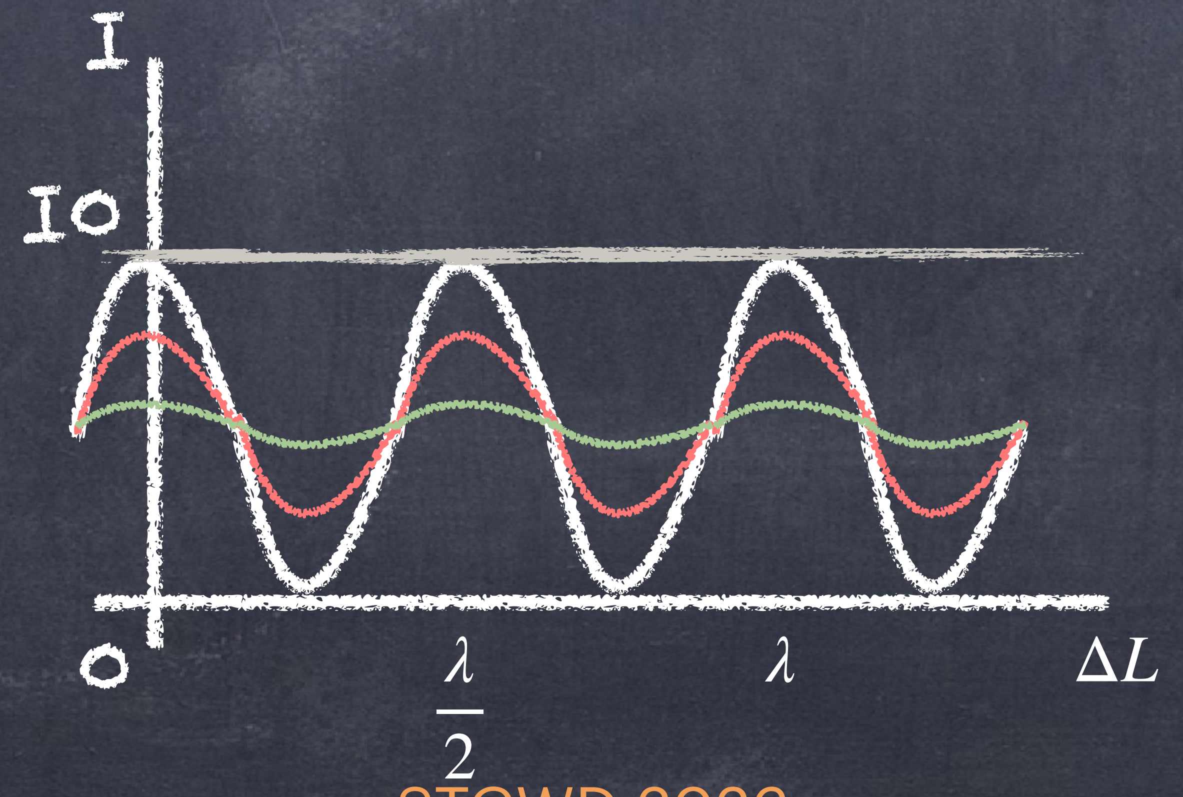
Minimum

$$k\Delta L = \frac{\pi}{2}(2N+1) \rightarrow \Delta L = \frac{2N+1}{4}\lambda$$

It does not matter how different the two arms are, Max and Min remain.

Max and Min depend on microscopic difference (~ lambda)





Contrast/Visibility

It is a number [0-1] to quantify the losses in the interference.

The contrast is max (=1, no losses) when the two interfering beams:

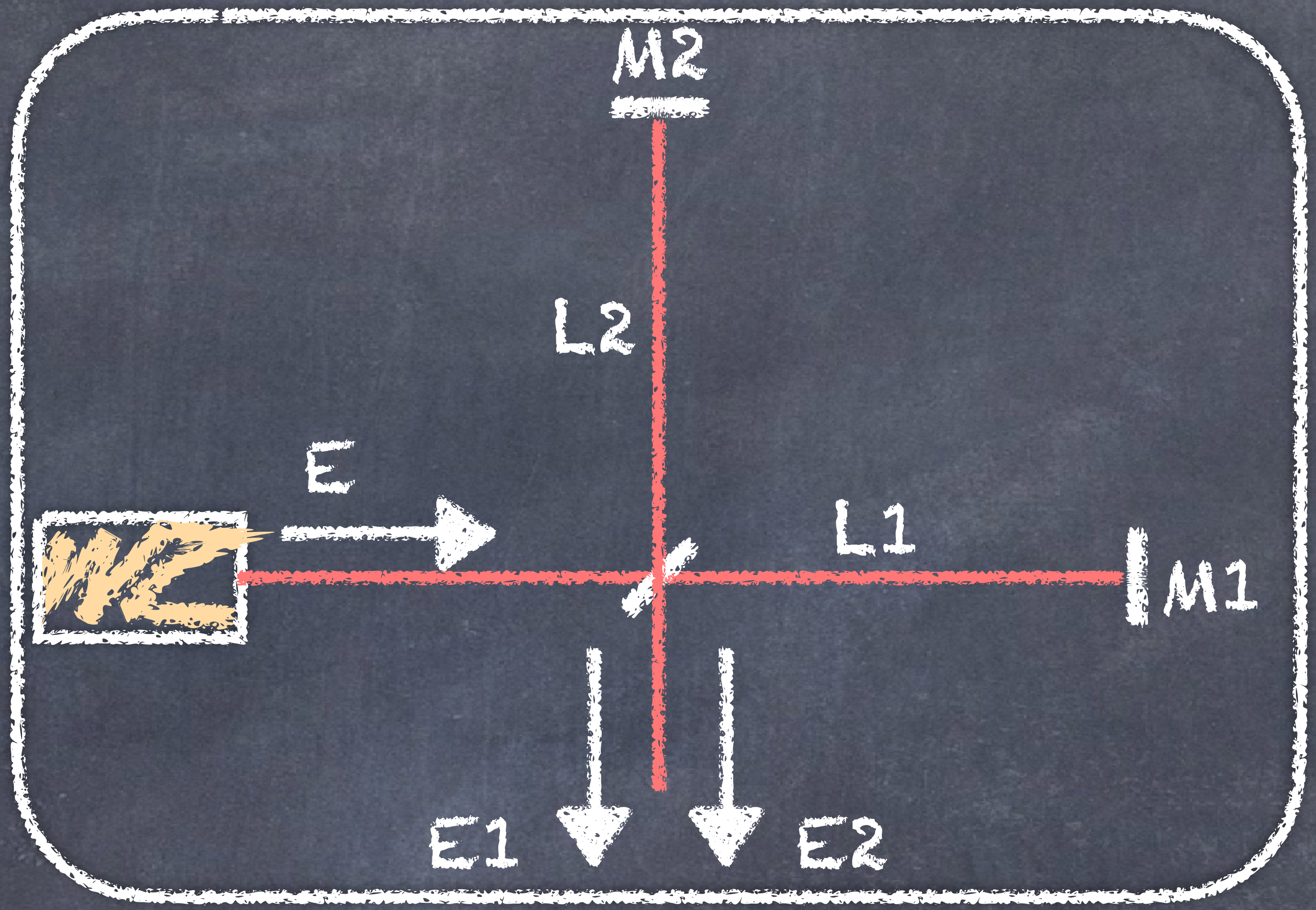
- Same intensity
- Perfectly overlapping

In general

$$I = (I_1 + I_2)\left[1 + \gamma \cos(\Delta\phi)\right]$$

γ —> Visibility

Michelson Interferometer
Limitations for gravitational waves



Limited sensitivity

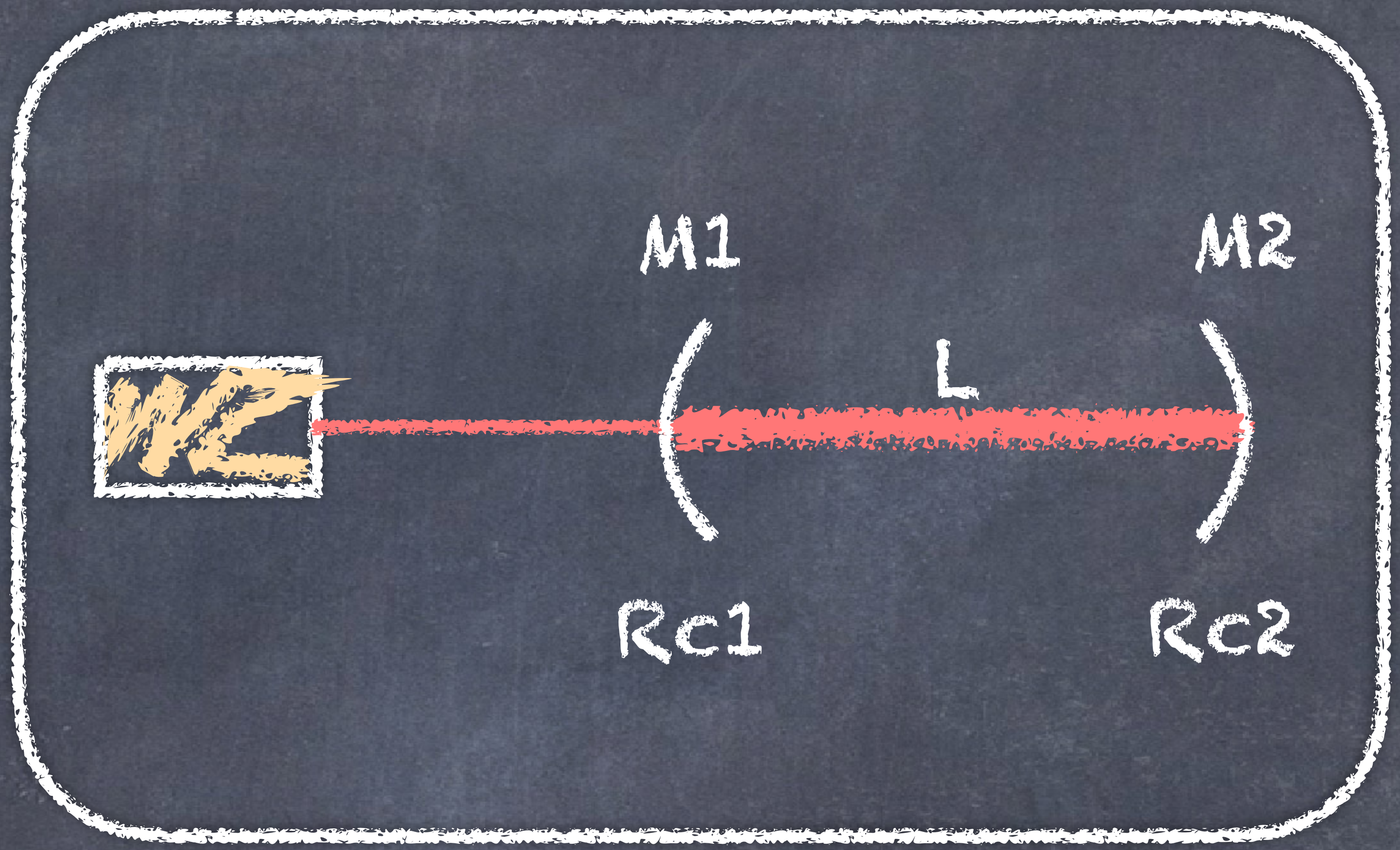
Need:

- Longer effective optical path
- Larger optical power in the arms

Fabry-Perot Cavity

Fabry-Perot cavity
Fields

The Fabry-Perot Interferometer uses two mirrors, M1 and M2, separated by a distance L to create interference among multiple beams, bouncing back and forward between the two mirrors



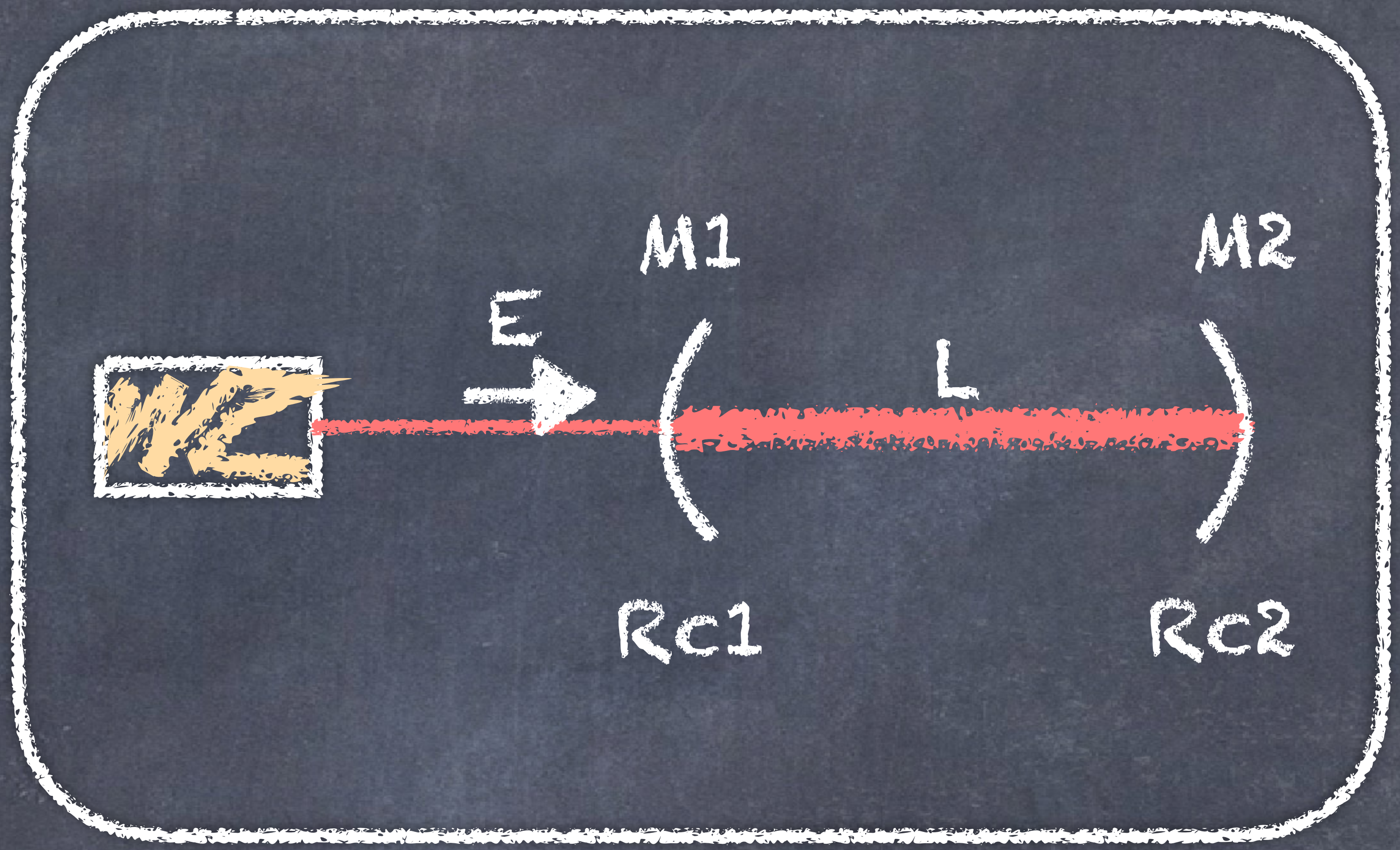
Fabry-Perot cavity
Fields

The Fabry-Perot Interferometer uses two mirrors, M1 and M2, separated by a distance L to create interference among multiple beams, bouncing back and forward between the two mirrors

Light from laser

$$E(x, y, z, t) = E_0 e^{-i\omega t}$$

Plane wave



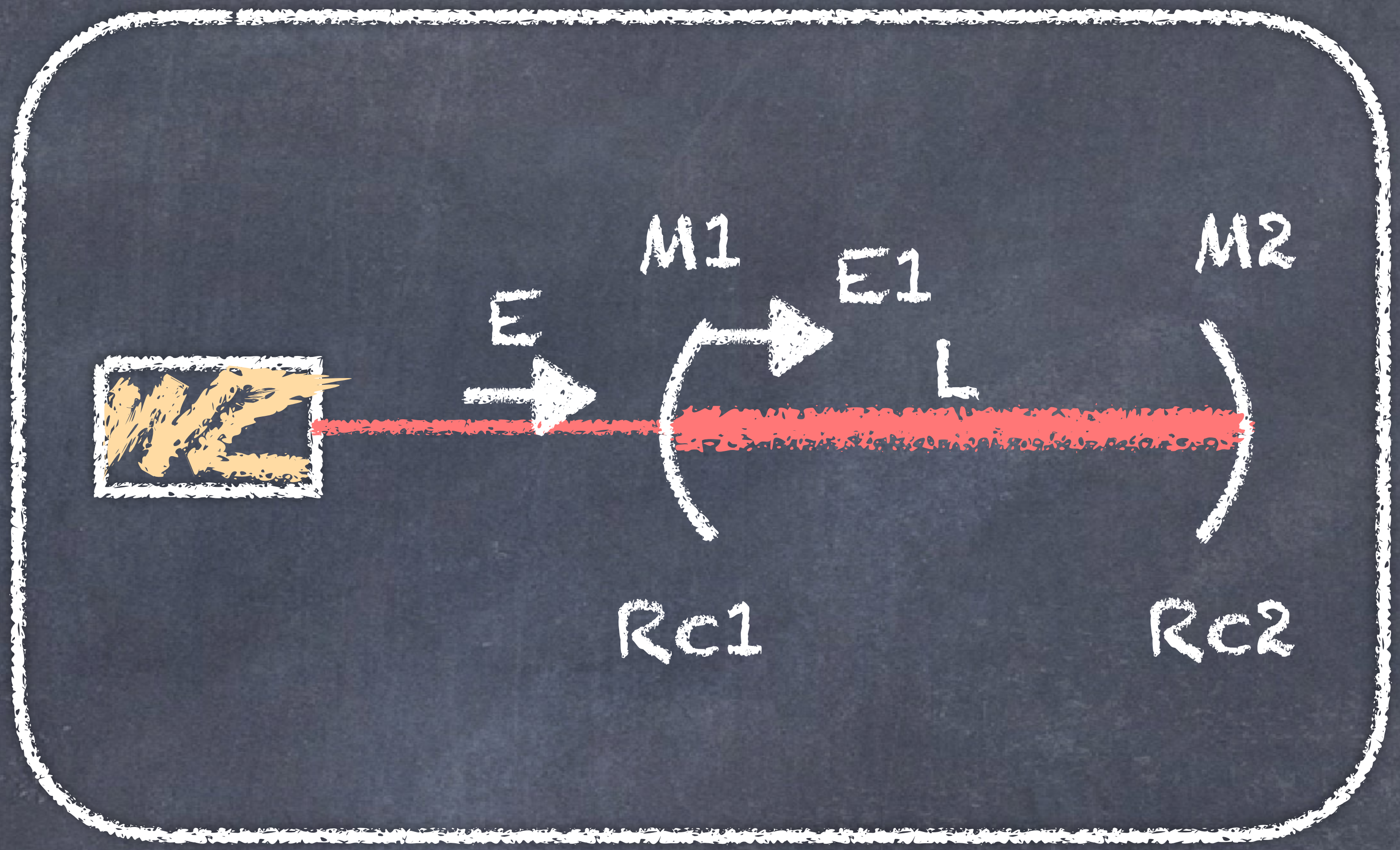
Fabry-Perot cavity
Fields

The Fabry-Perot Interferometer uses two mirrors, M1 and M2, separated by a distance L to create interference among multiple beams, bouncing back and forward between the two mirrors

Light from laser

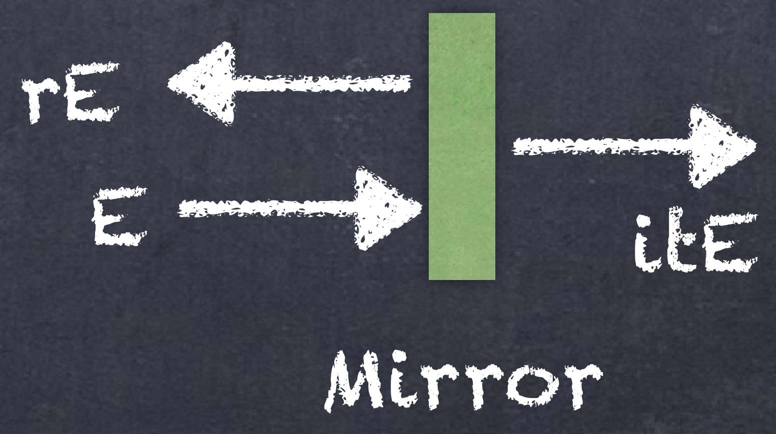
$$E(x, y, z, t) = E_0 e^{-i\omega t}$$

Plane wave



Building up fields

$$E_1 = i t_1 E$$



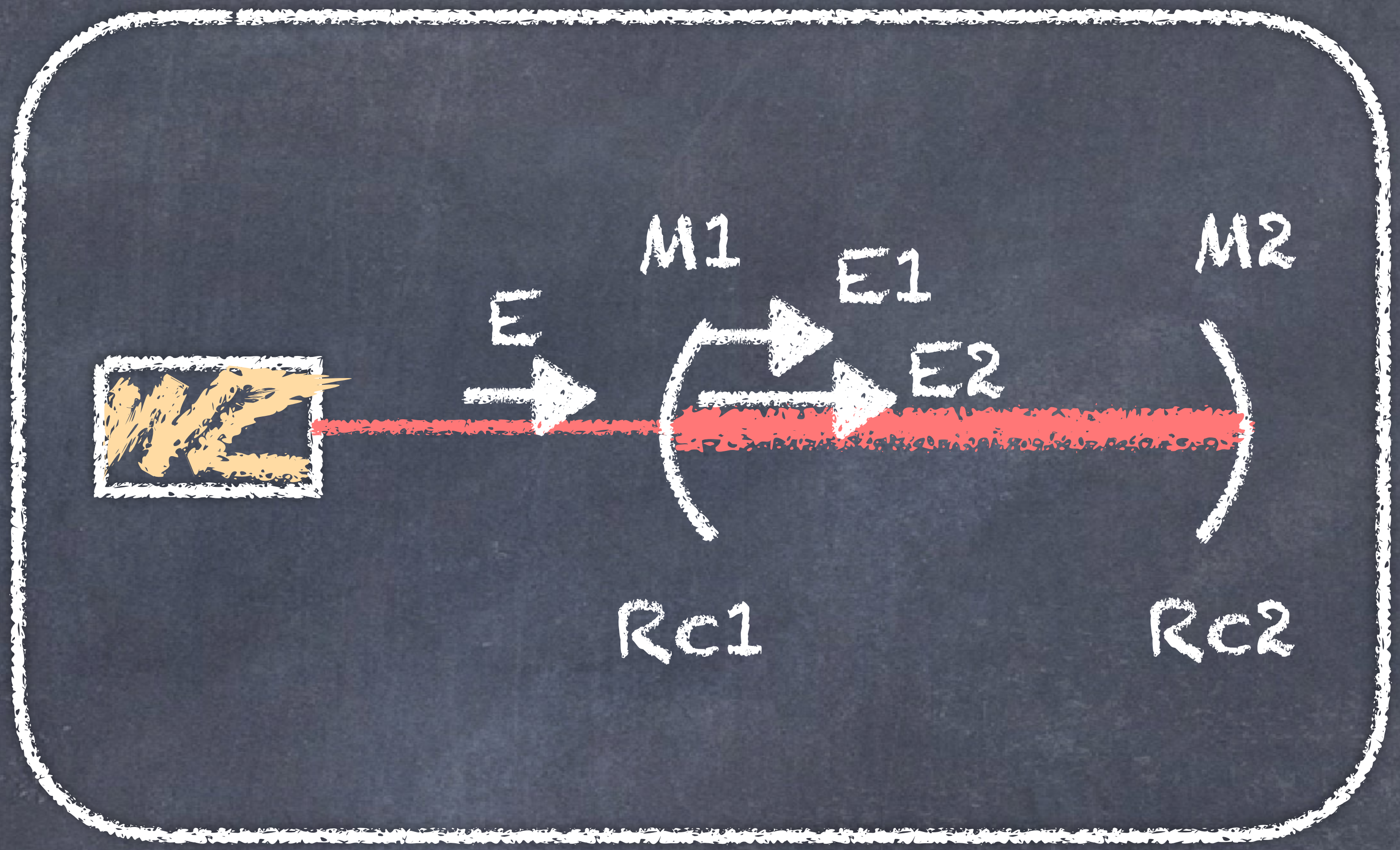
Fabry-Perot cavity
Fields

The Fabry-Perot Interferometer uses two mirrors, M1 and M2, separated by a distance L to create interference among multiple beams, bouncing back and forward between the two mirrors

Light from laser

$$E(x, y, z, t) = E_0 e^{-i\omega t}$$

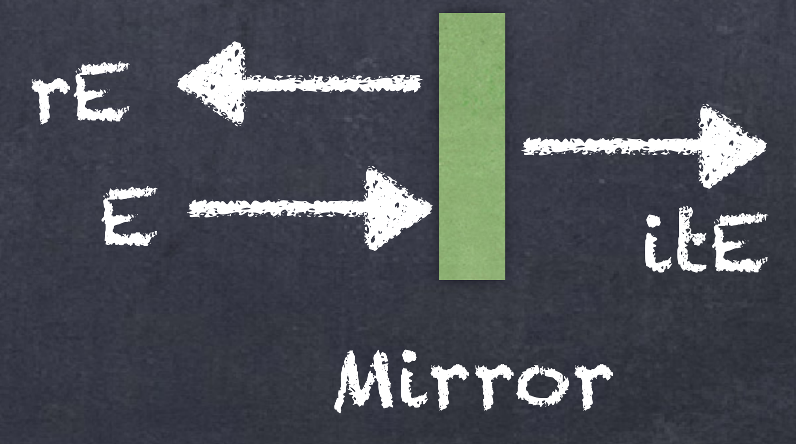
Plane wave



Building up fields

$$E_1 = it_1 E$$

$$E_2 = r_1 r_2 e^{2ikL} E_1 = it_1 r_1 r_2 e^{2ikL} E$$



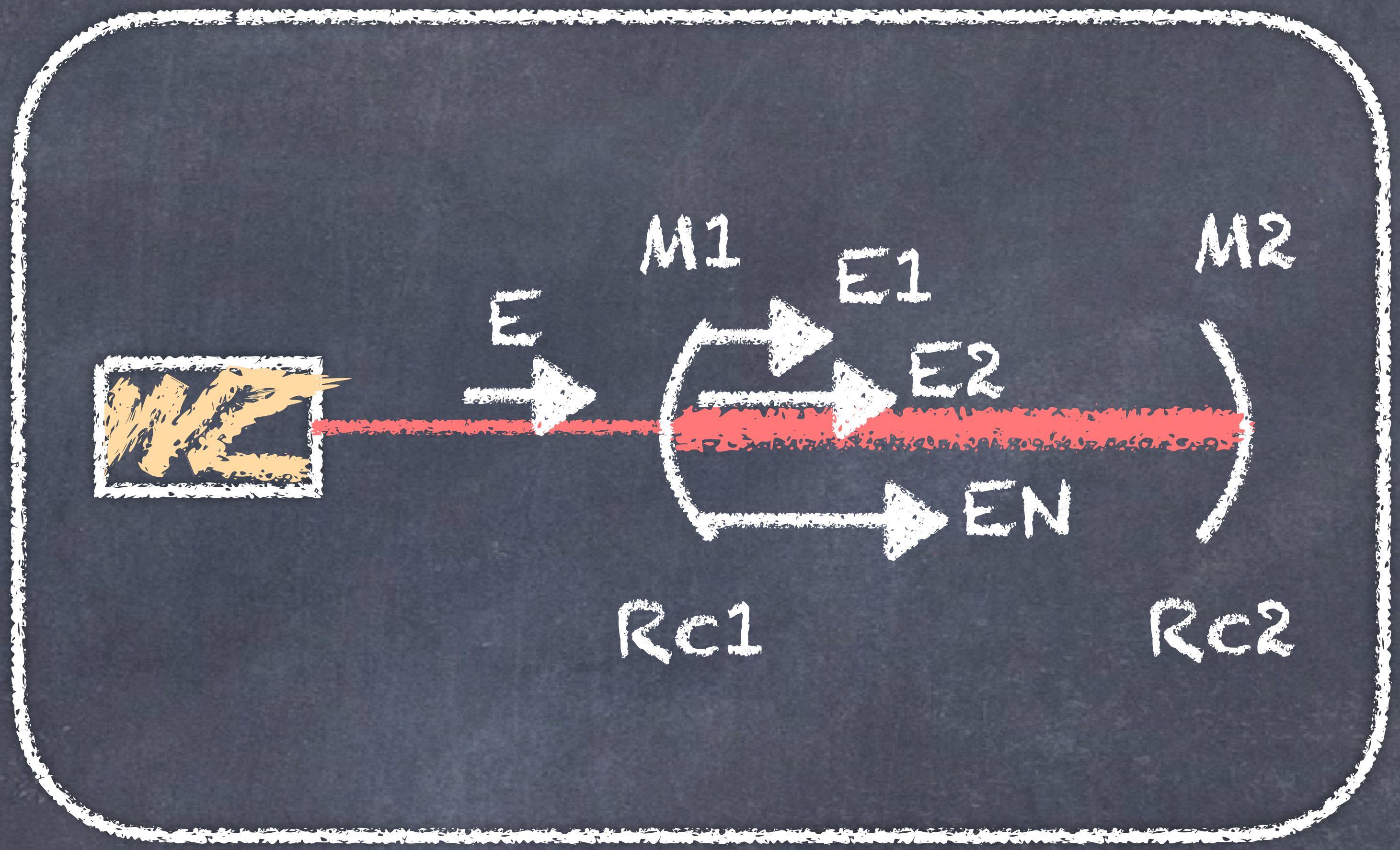
Fabry-Perot cavity
Fields

The Fabry-Perot Interferometer uses two mirrors, M1 and M2, separated by a distance L to create interference among multiple beams, bouncing back and forward between the two mirrors

Light from laser

$$E(x, y, z, t) = E_0 e^{-i\omega t}$$

Plane wave



Building up fields

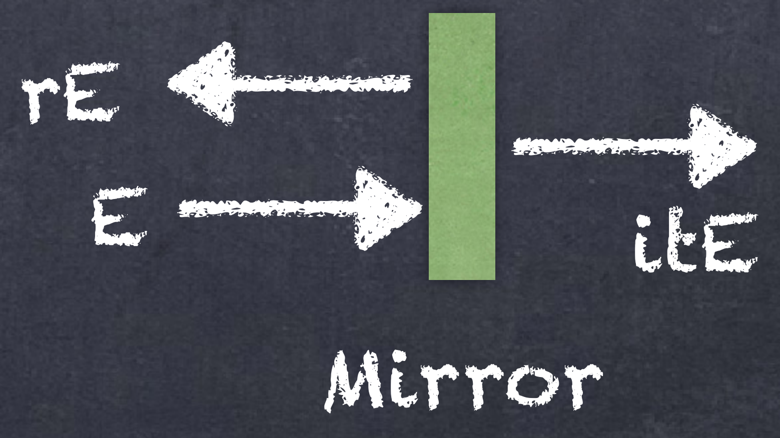
$$E_1 = it_1 E$$

$$E_2 = r_1 r_2 e^{2ikL} E_1 = it_1 r_1 r_2 e^{2ikL} E$$

$$\dots$$

$$E_N = r_1 r_2 e^{2ikL} E_{N-1} = it_1 (r_1 r_2)^{N-1} e^{2(N-1)ikL} E$$

Transitory



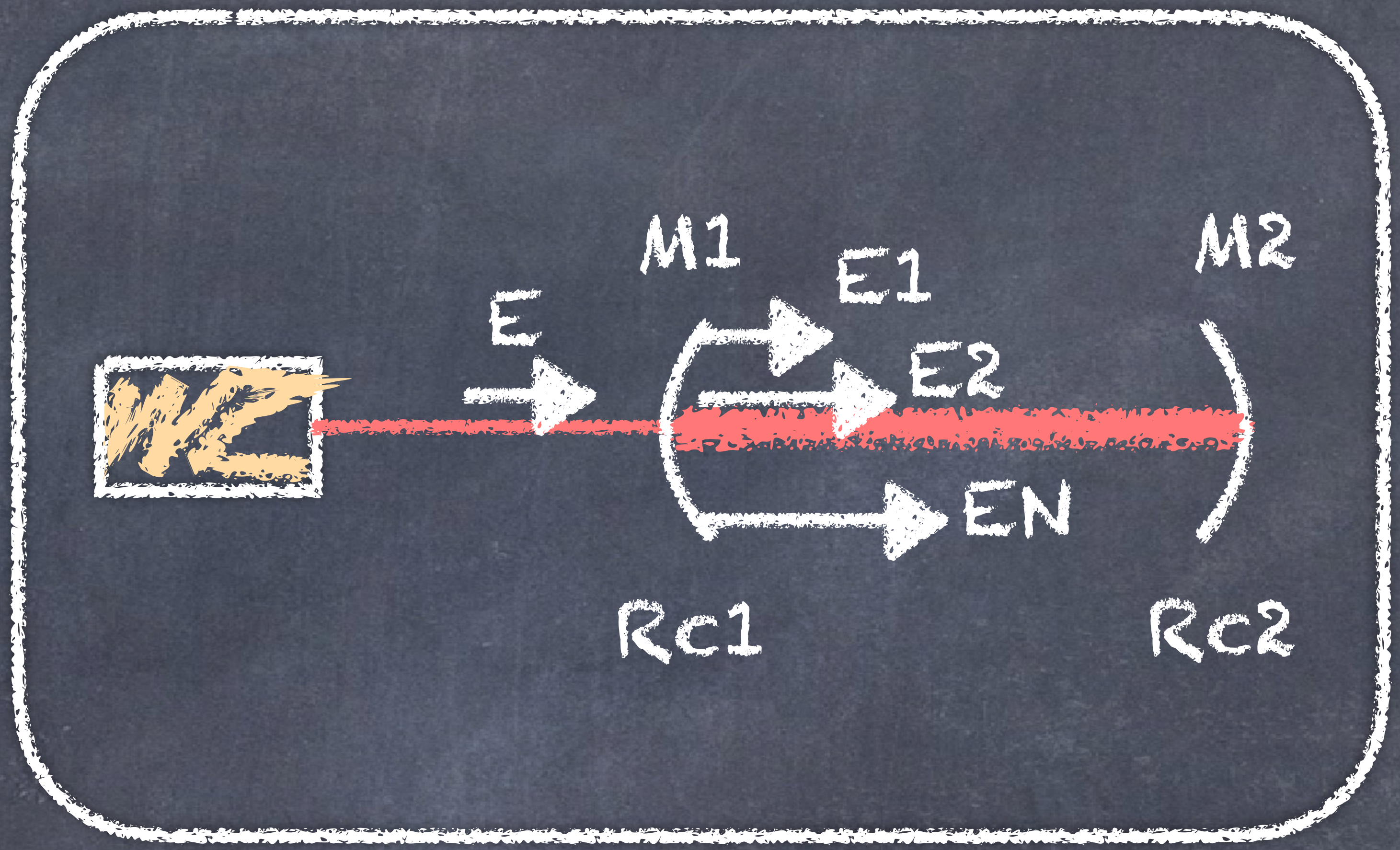
Fabry-Perot cavity
Fields

The Fabry-Perot Interferometer uses two mirrors, M1 and M2, separated by a distance L to create interference among multiple beams, bouncing back and forward between the two mirrors

Light from laser

$$E(x, y, z, t) = E_0 e^{-i\omega t}$$

Plane wave



Building up fields

$$E_1 = it_1 E$$

$$E_2 = r_1 r_2 e^{2ikL} E_1 = it_1 r_1 r_2 e^{2ikL} E$$

$$\dots$$

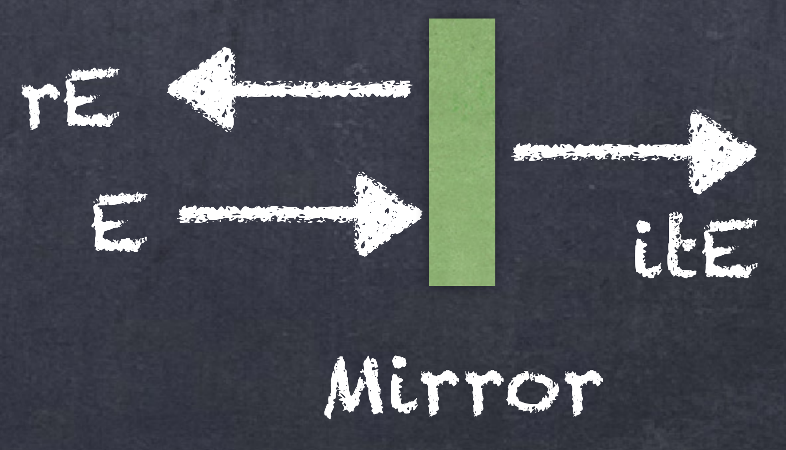
$$E_N = r_1 r_2 e^{2ikL} E_{N-1} = it_1 (r_1 r_2)^{N-1} e^{2(N-1)ikL} E$$

Transitory

Internal field (Steady State)

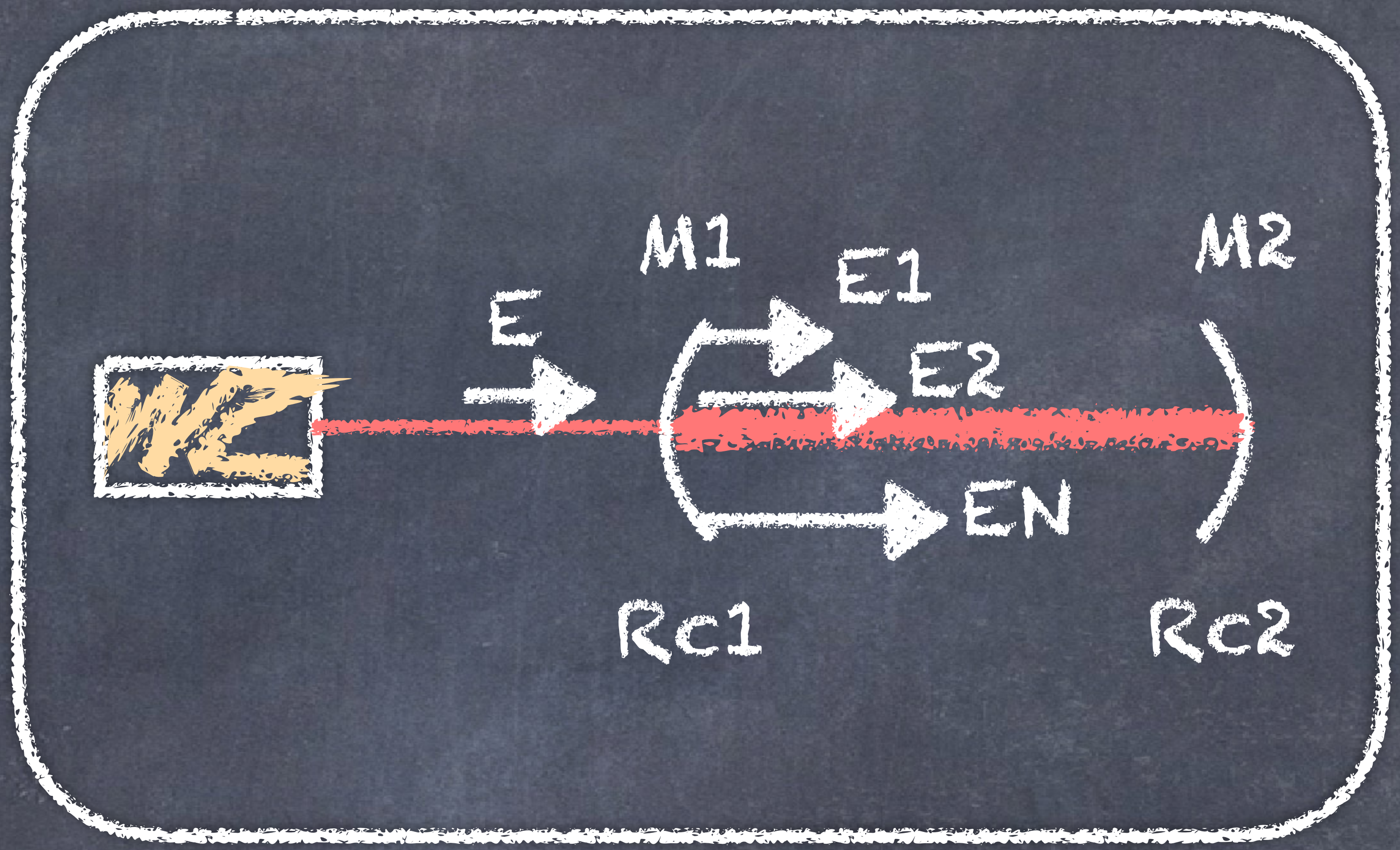
$$E_{Int} = E_1 + E_2 + E_3 + \dots + E_N$$

$$= \frac{1}{1 - r_1 r_2 e^{2ikL}} it_1 E$$



Fabry-Perot cavity
Fields

The Fabry-Perot Interferometer uses two mirrors, M1 and M2, separated by a distance L to create interference among multiple beams, bouncing back and forward between the two mirrors



Reflected field (Steady State)

$$E_{Ref} = \frac{r_1 - r_2 e^{2ikL}}{1 - r_1 r_2 e^{2ikL}} E$$

Internal field (Steady State)

$$E_{Int} = \frac{1}{1 - r_1 r_2 e^{2ikL}} i t_1 E$$

Transmitted field (Steady State)

$$E_{Trans} = \frac{e^{ikL}}{1 - r_1 r_2 e^{2ikL}} i t_1 i t_2 E$$

It is the sum of two components

- Prompt reflection
- Leakage

Fabry-Perot cavity
Power

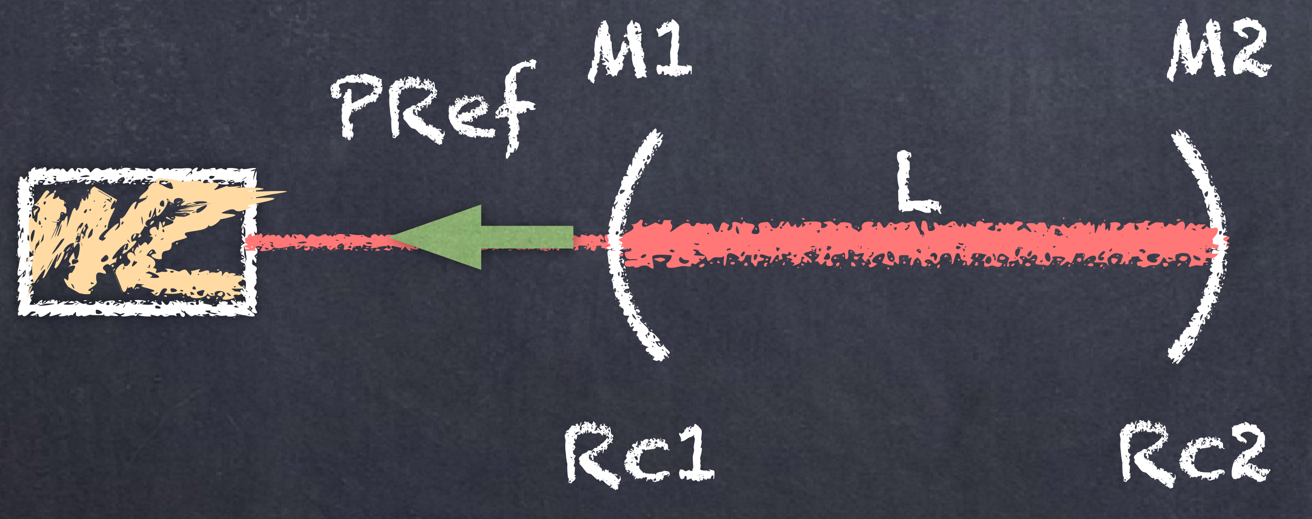
Reflected field (Steady State)

$$E_{Ref} = \frac{r_1 - r_2 e^{2ikL}}{1 - r_1 r_2 e^{2ikL}} E$$

Reflected Power

$$P_{Ref} = E_{Ref} \cdot E^*_{Ref}$$

$$= \frac{r_1^2 + r_2^2 - 2r_1 r_2 \cos(2kL)}{1 + (r_1 r_2)^2 - 2r_1 r_2 \cos(2kL)} t_1^2 E_0^2$$



Internal field (Steady State)

$$E_{Int} = \frac{1}{1 - r_1 r_2 e^{2ikL}} i t_1 E$$

Internal Power

$$P_{Int} = E_{Int} \cdot E^*_{Int}$$

$$= \frac{1}{1 + (r_1 r_2)^2 - 2r_1 r_2 \cos(2kL)} t_1^2 E_0^2$$

Max: cos(2kL)=1

$$2kL = 2N\pi$$

$$\rightarrow L = N\frac{\lambda}{2}$$

$$\rightarrow f = N\frac{c}{2L}$$

FSR

Resonance

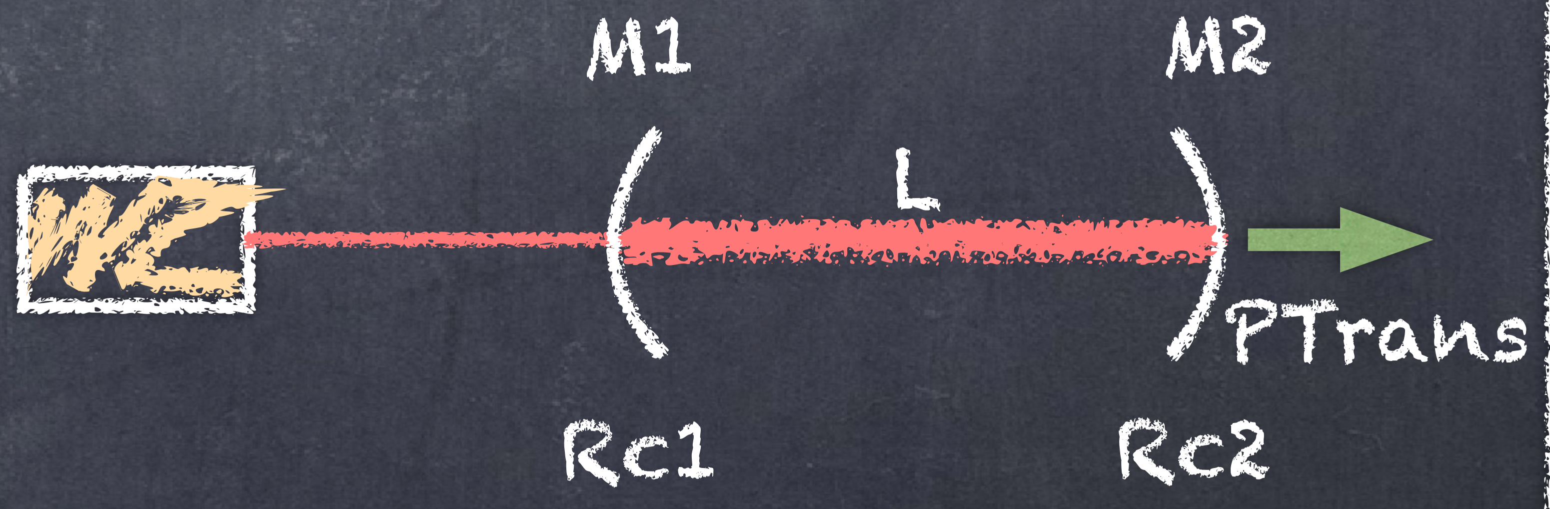
Transmitted field (Steady State)

$$E_{Trans} = \frac{e^{ikL}}{1 - r_1 r_2 e^{2ikL}} i t_1 i t_2 E$$

Transmitted Power

$$P_{Trans} = E_{Trans} \cdot E^*_{Trans}$$

$$= \frac{t_1^2 t_2^2}{1 + (r_1 r_2)^2 - 2r_1 r_2 \cos(2kL)} E_0^2$$



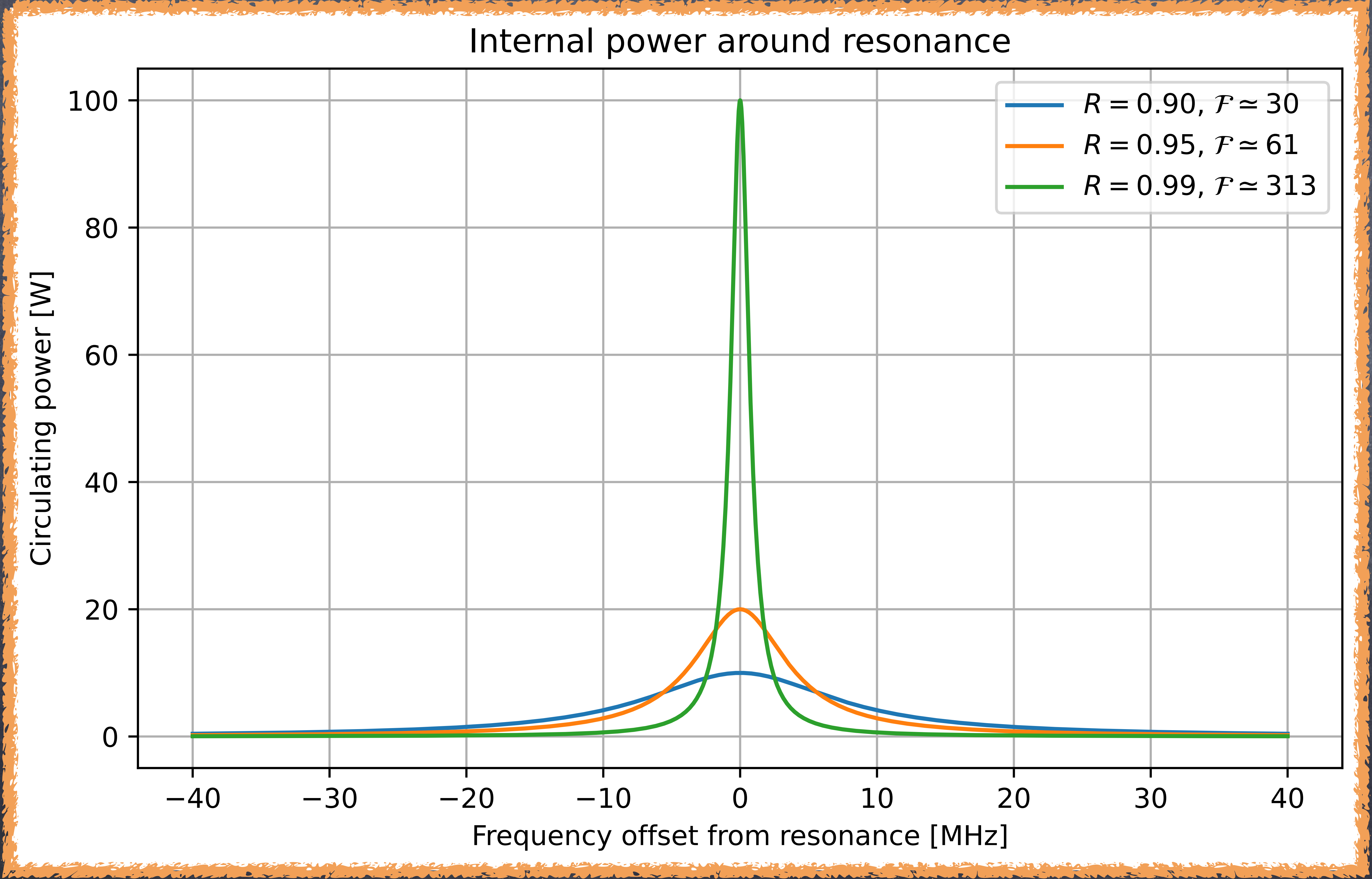
Fabry-Perot cavity
Power buildup and bandwidth

Internal Power

$$P_{int} = \frac{1}{1 + (r_1 r_2)^2 - 2 r_1 r_2 \cos(2kL)} t_1^2 E_0^2$$

Higher mirror reflectivity

→ larger circulating power
→ narrower bandwidth
(higher finesse)



Parameters

Frequency pole

$$f_p = \frac{FSR}{\pi} \arcsin \frac{1 - r_1 r_2}{2\sqrt{r_1 r_2}}$$

Full Width Half Maximum

$$FWHM = 2 f_p$$

Finesse

$$F \sim \frac{\pi \sqrt{r_1 r_2}}{1 - r_1 r_2}$$

Fabry-Perot cavity
Power

Reflected Power

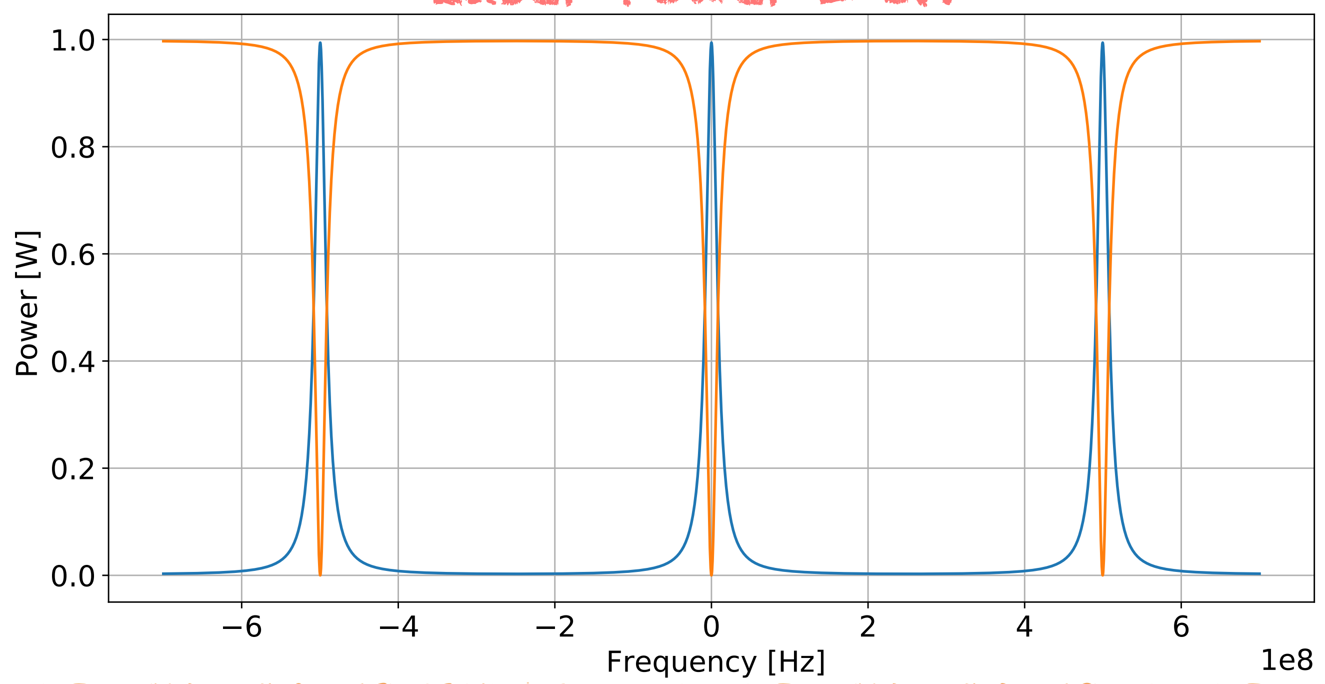
$$P_{Ref} = E_{Ref} \cdot E^*_{Ref}$$
$$= \frac{r_1^2 + r_2^2 - 2r_1r_2\cos(2kL)}{1 + (r_1r_2)^2 - 2r_1r_2\cos(2kL)} t_1^2 E_0^2$$

Transmitted Power

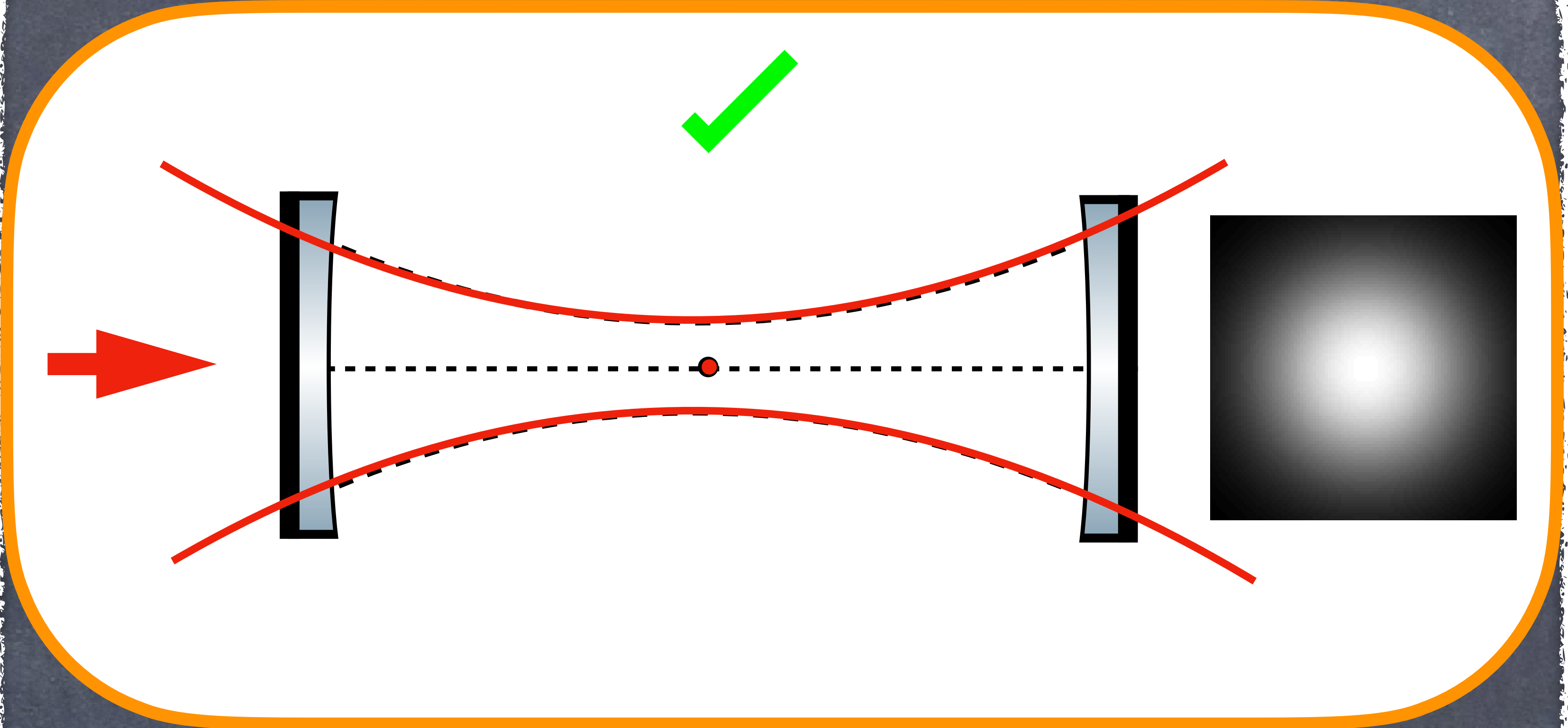
$$P_{Trans} = E_{Trans} \cdot E^*_{Trans}$$
$$= \frac{t_1^2 t_2^2}{1 + (r_1r_2)^2 - 2r_1r_2\cos(2kL)} E_0^2$$

Reflected Power Transmitted Power

Case R1=R2

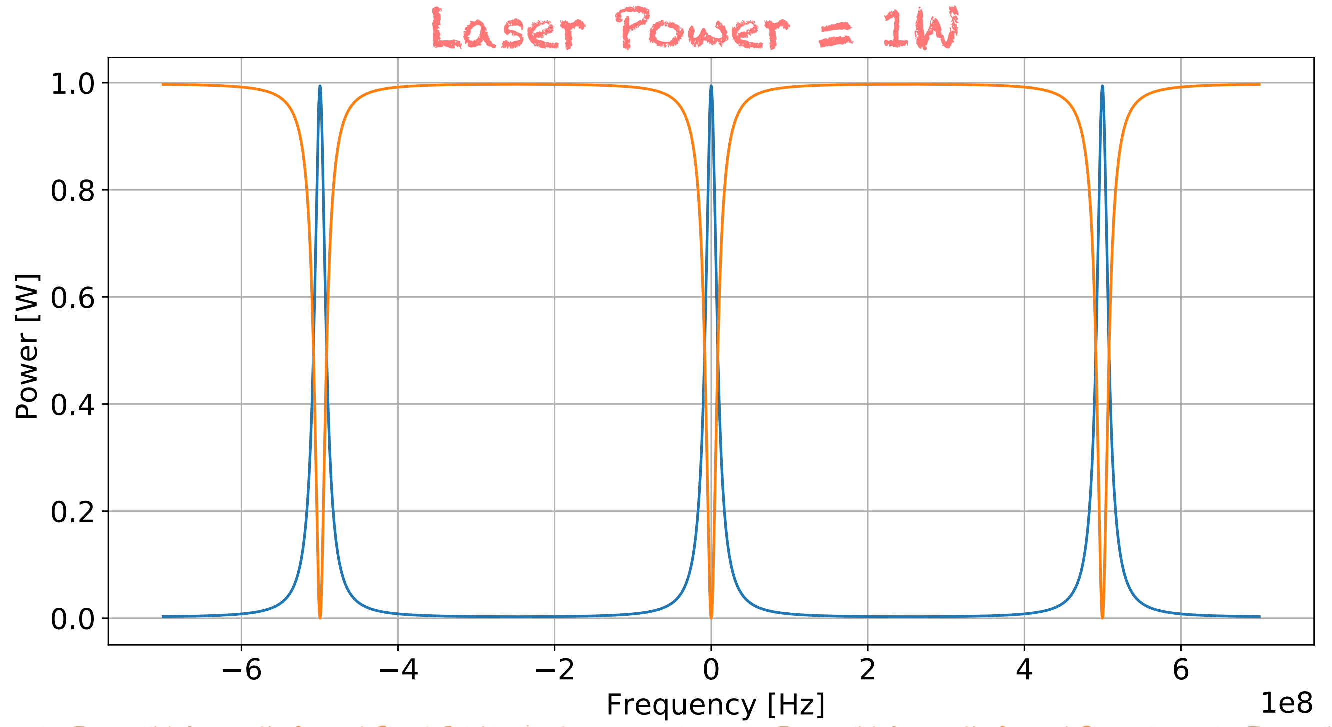


Fabry-Perot cavity
Power



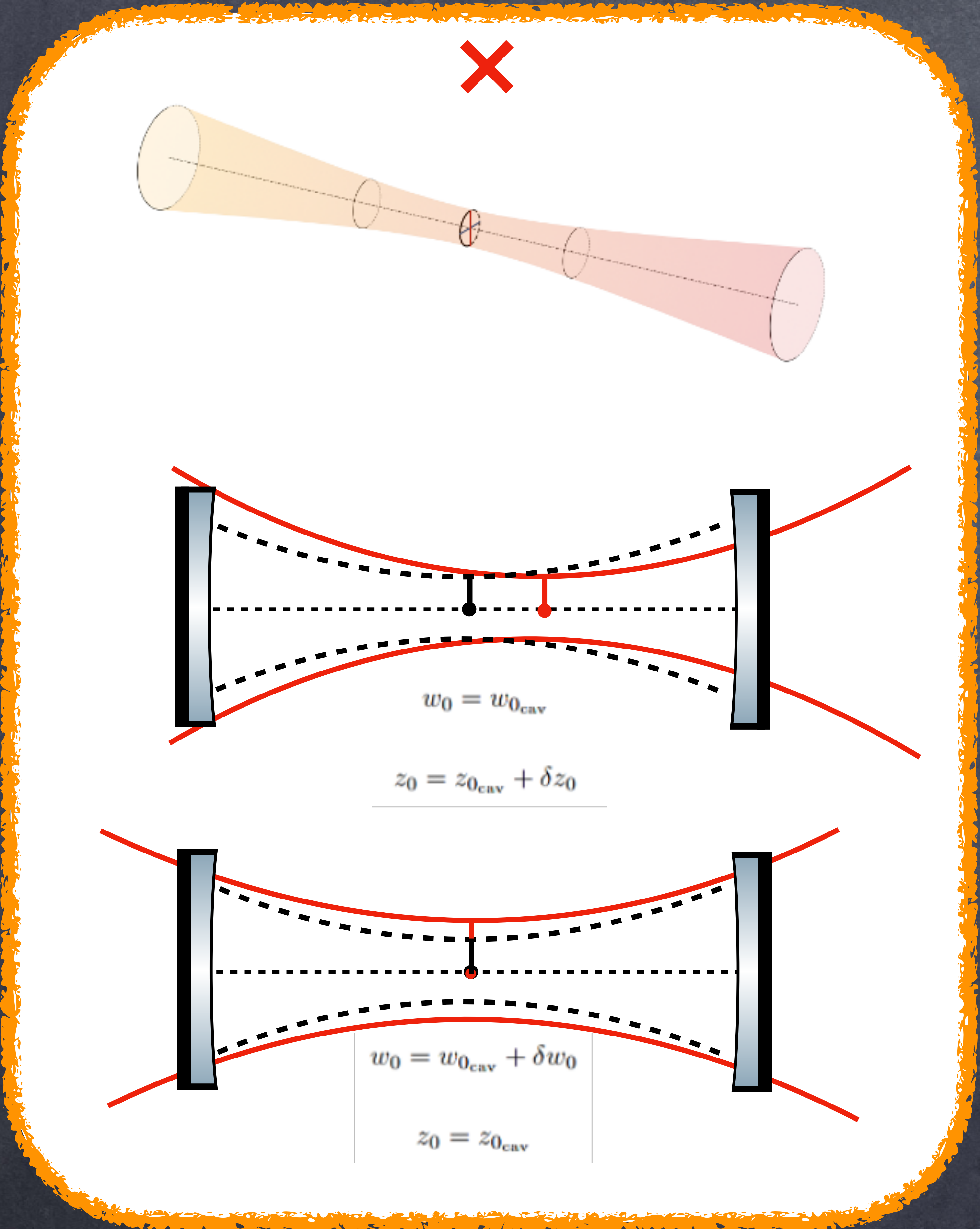
Perfect coupling

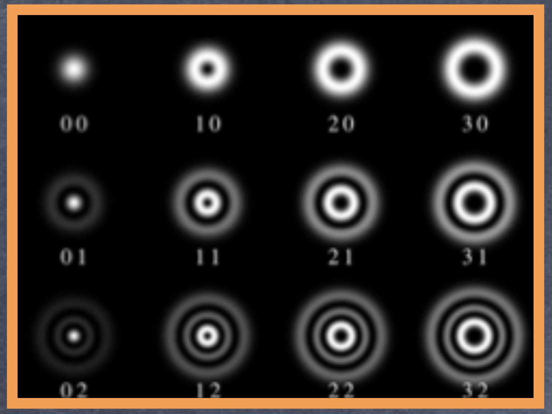
Reflected Power   Transmitted Power

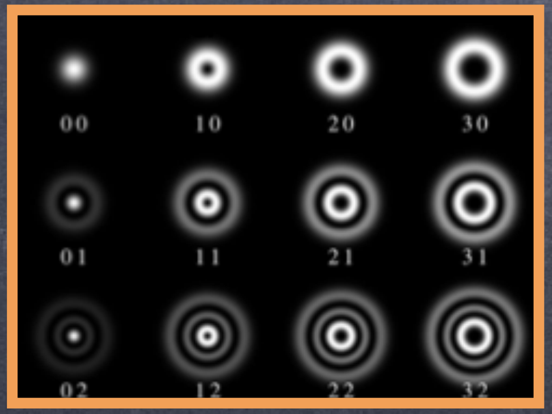


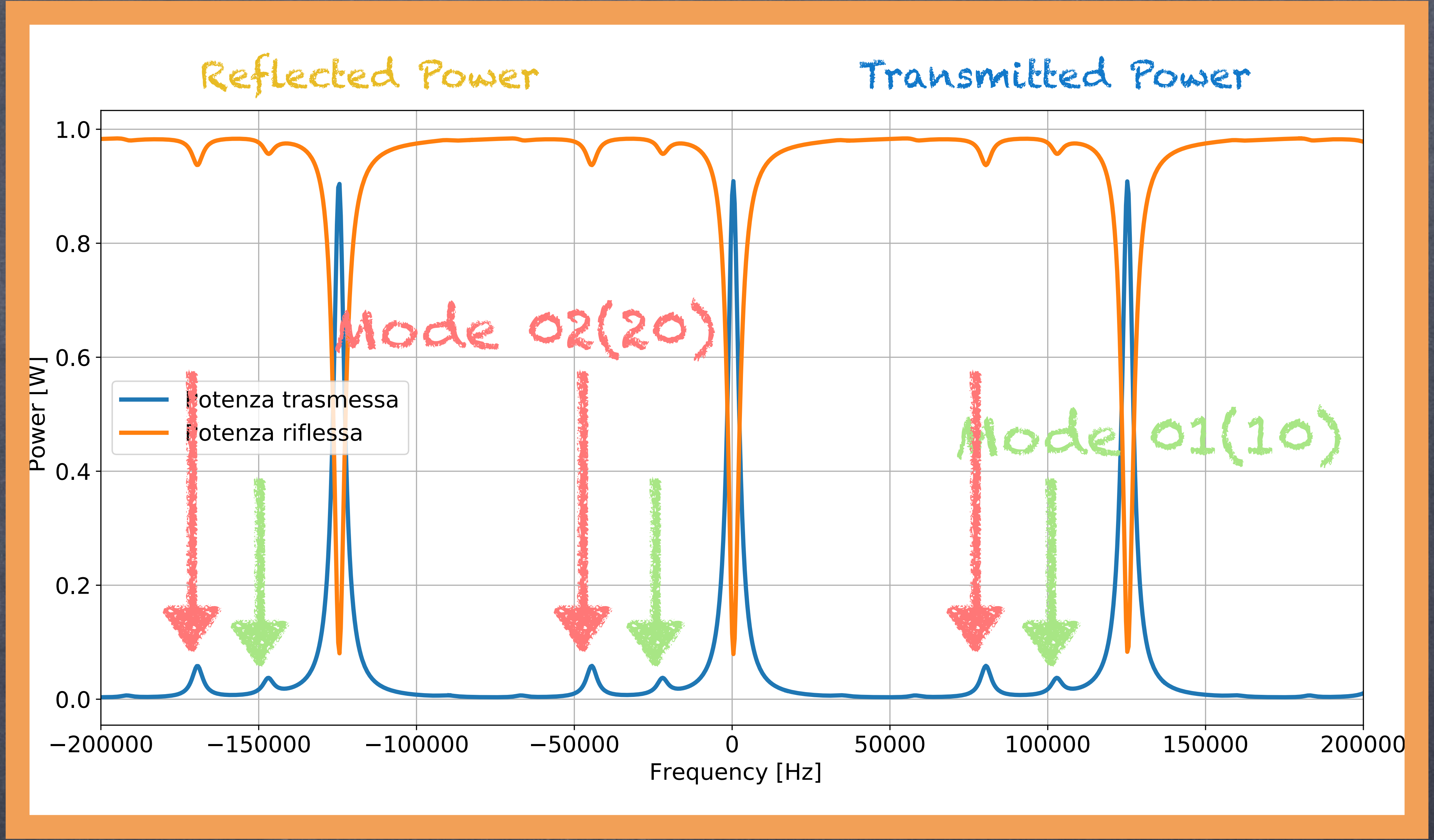
Case R1=R2

Fabry-Perot cavity
Beam inside the cavity









Optical losses due to unperfected coupling

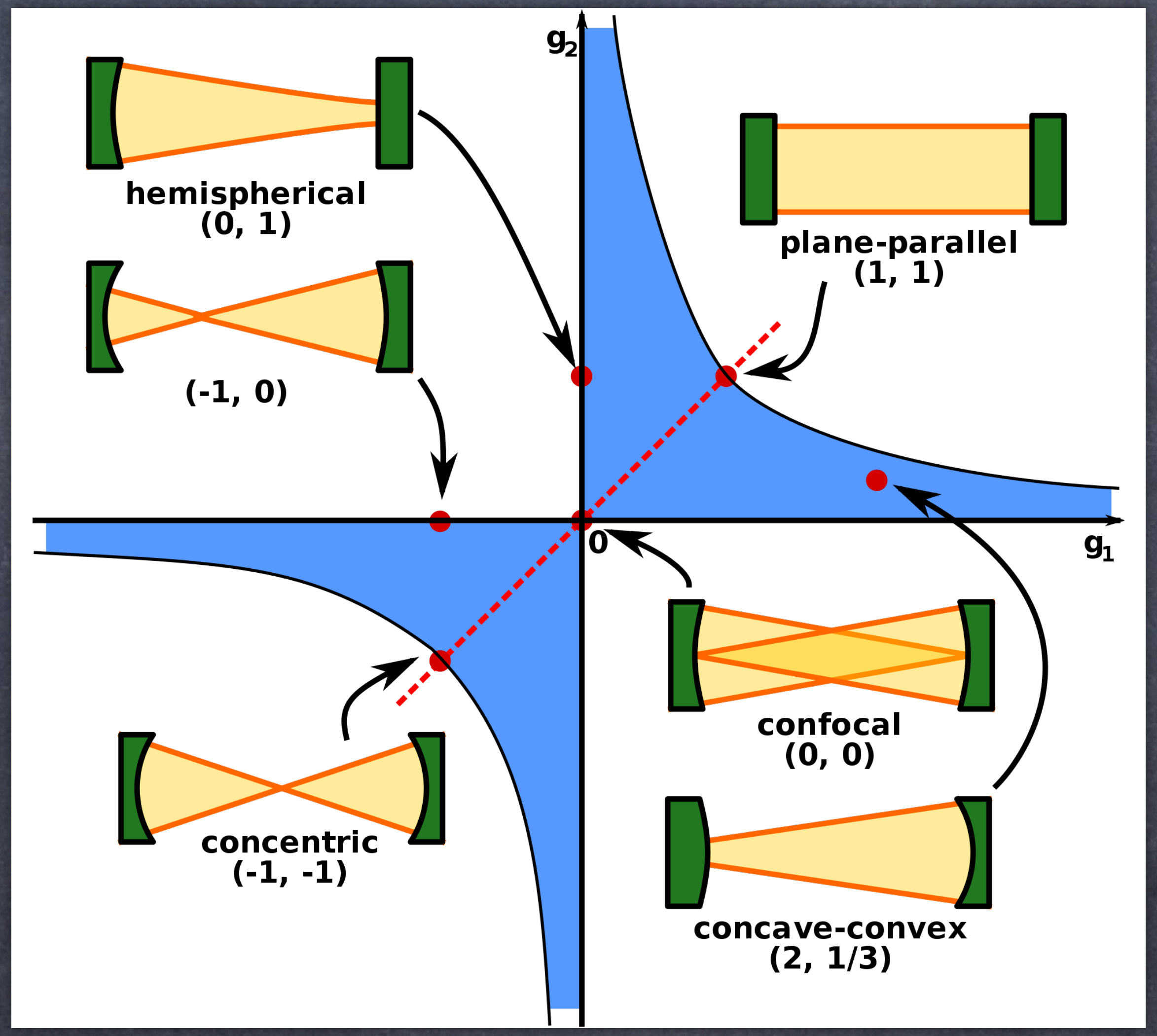
Fabry-Perot cavity
Stability condition

Depends on Mirror curvature Rci and on cavity length L

Stability condition

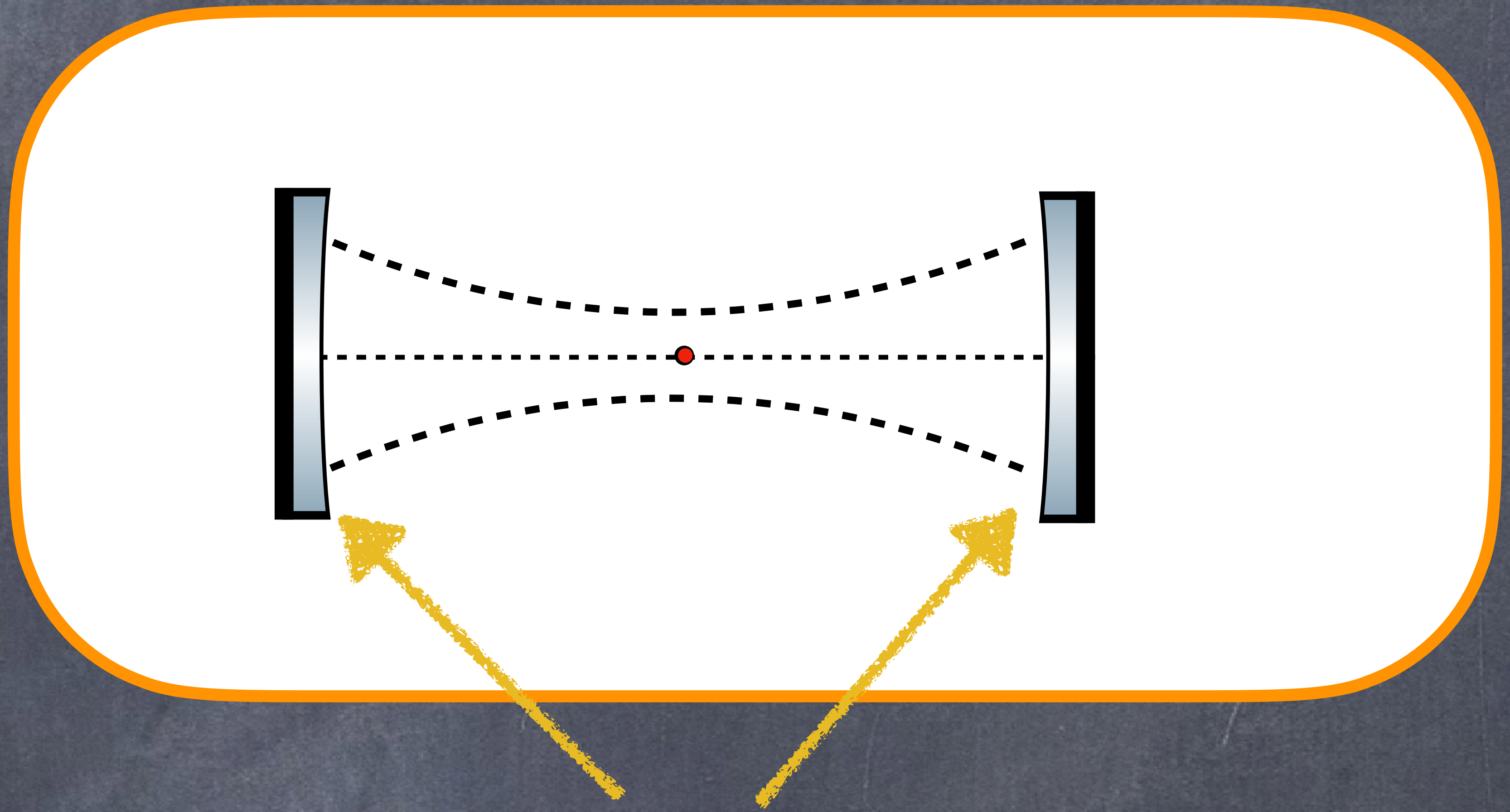
$0 < g1\,g2 < 1$

with $gi = 1 - L/Rci$



- Beam size on mirrors fixed by L and mirror curvature
  - Waist and its position are also fixed

- Not arbitrary anymore

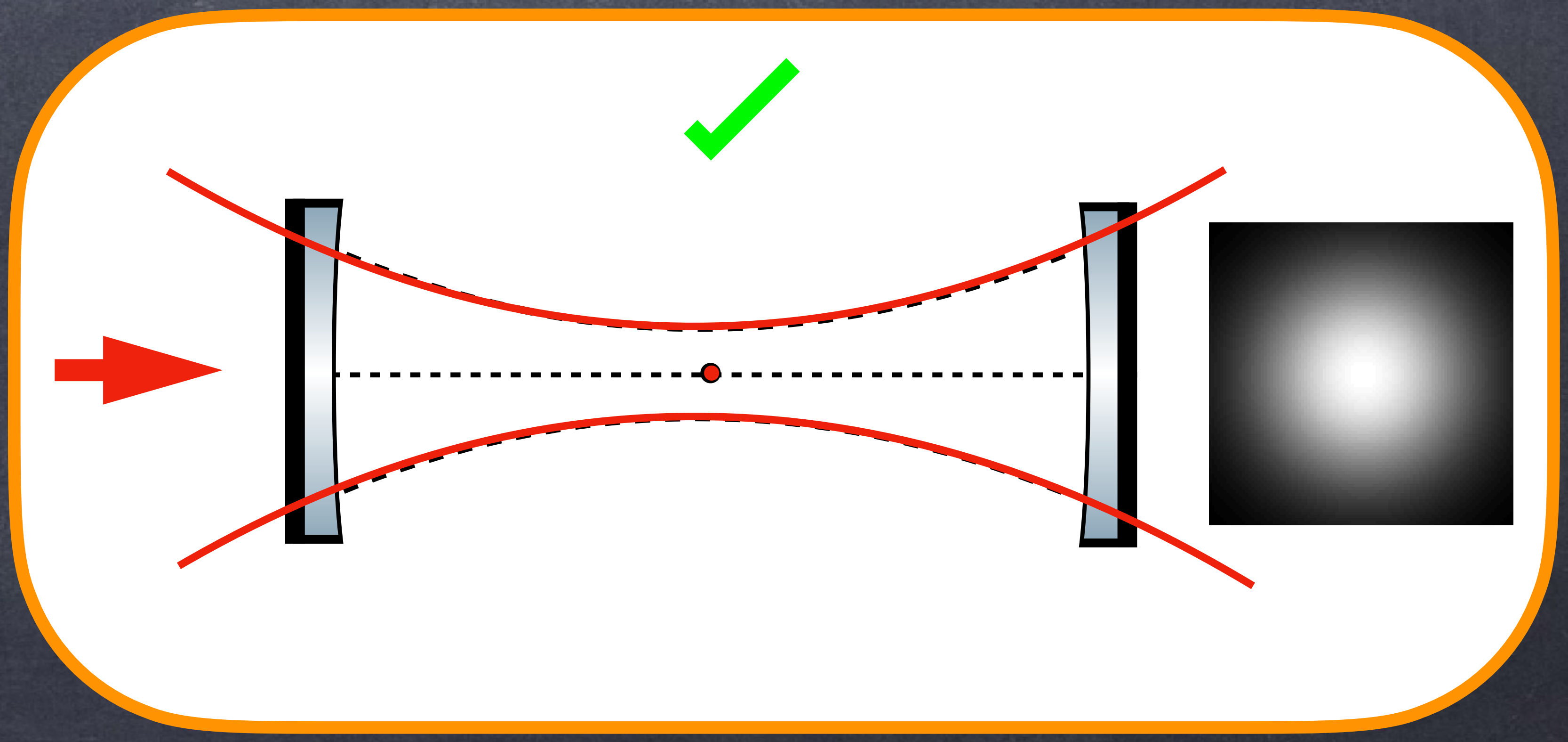


Beam size

Symmetric cavity

$$w_m^2 = \frac{\lambda L}{\pi} \frac{1}{\sqrt{1 - g_1 g_2}}$$

First real design constraint



Fabry-Perot cavity
Long cavities

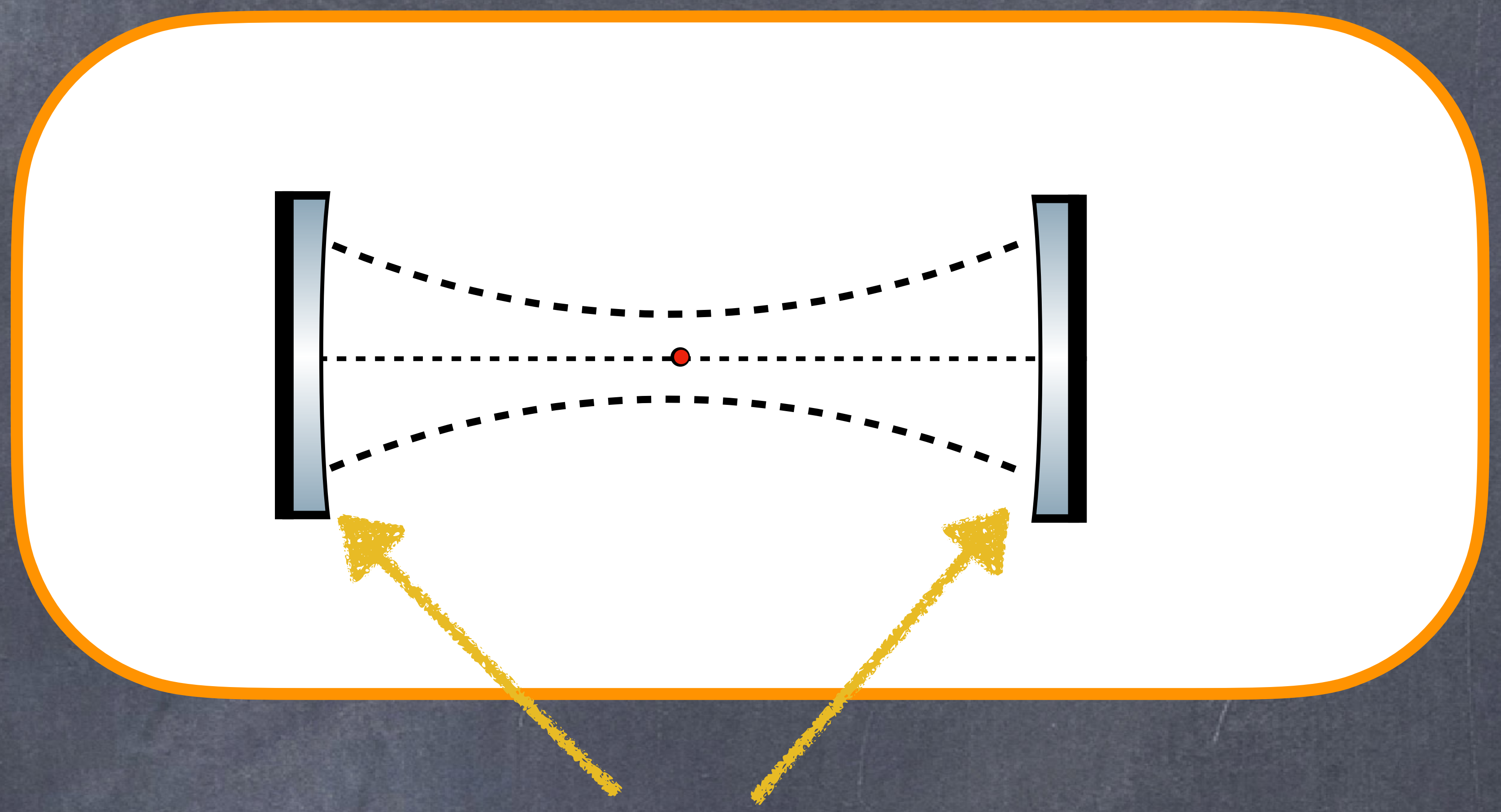
- For fixed g1g2:

Beam size

$$w_m^2 \propto L$$

- Long cavities require larger beam spots unless one moves closer to the edge of stability

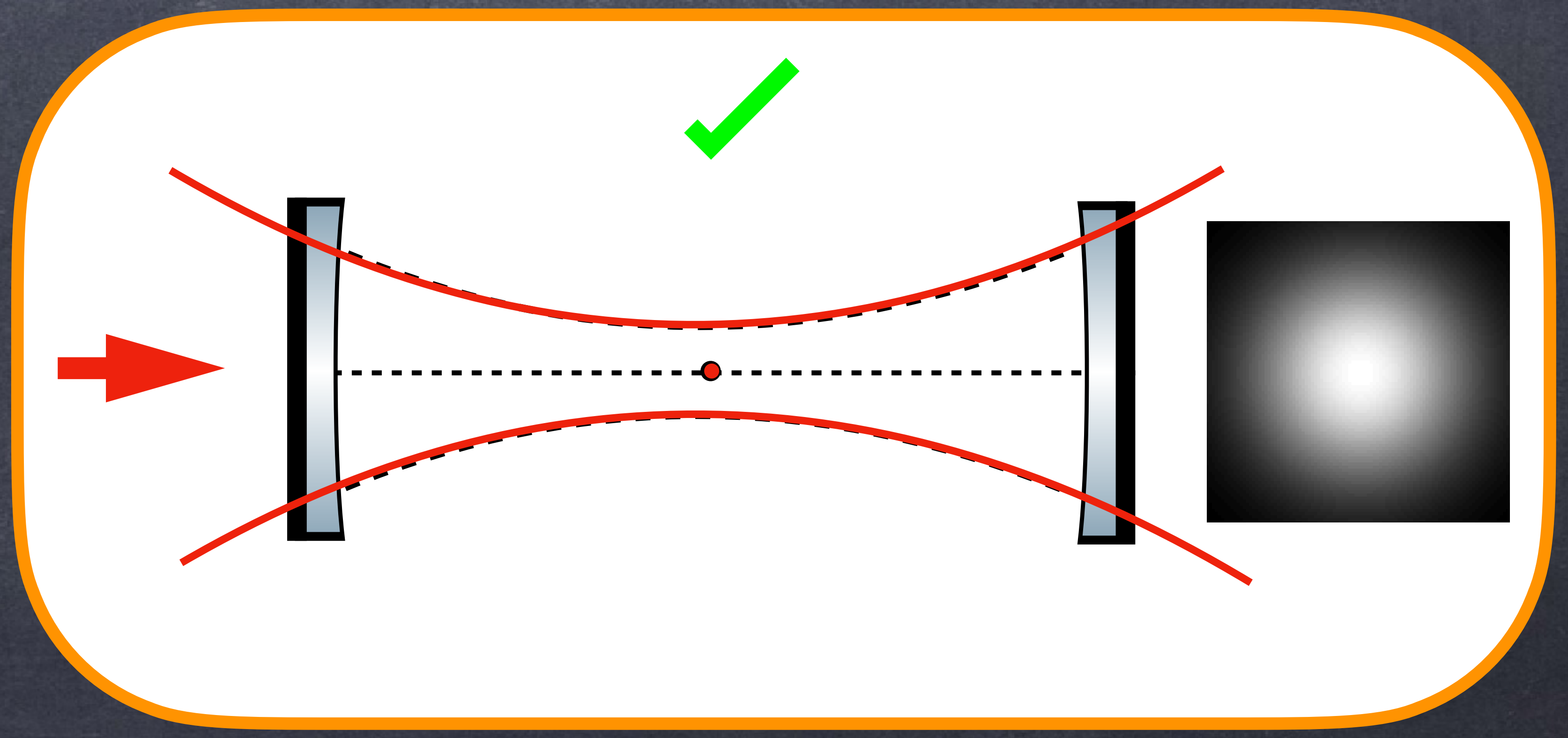
First real design constraint



Beam size

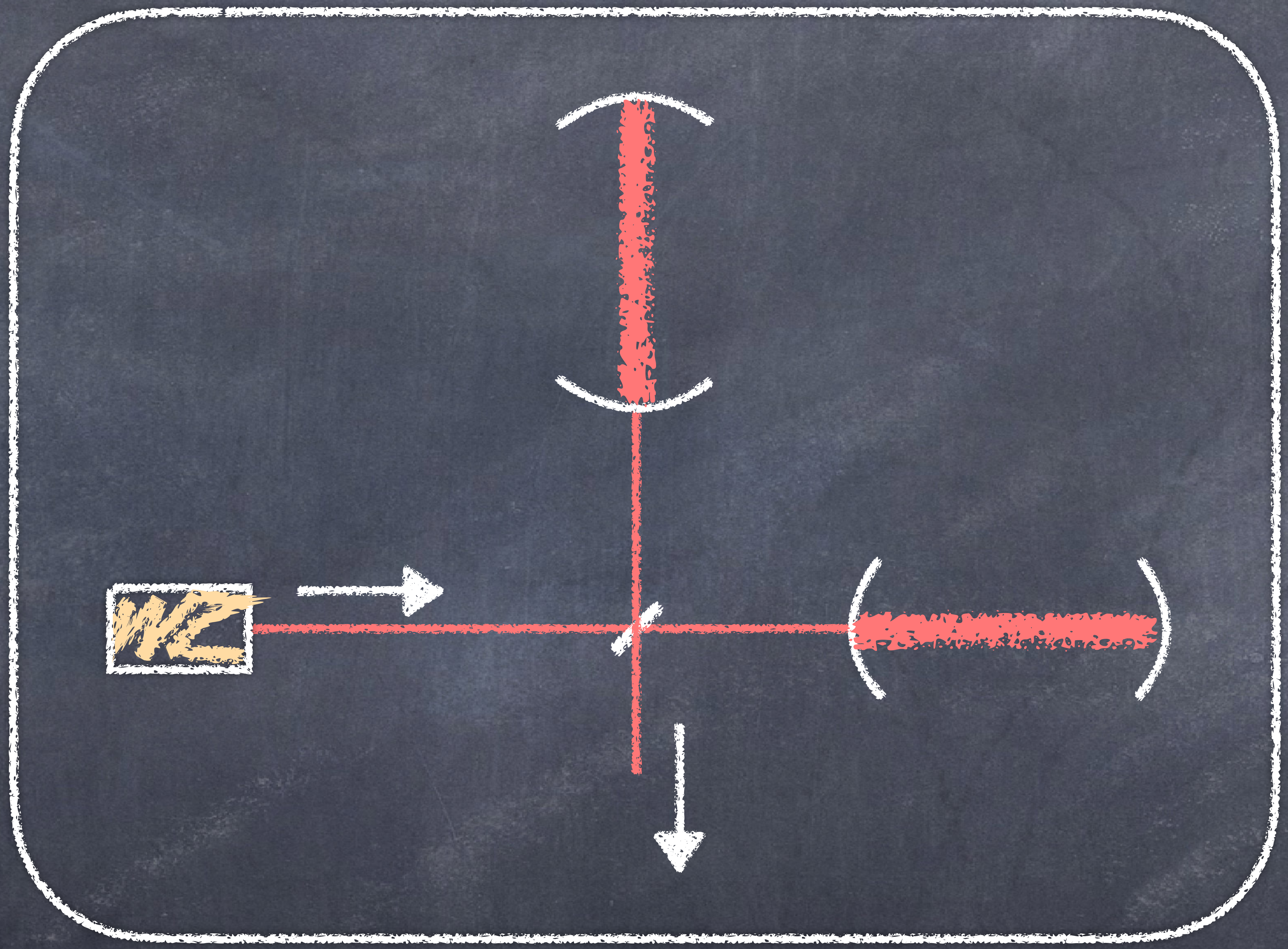
Symmetric cavity →

$$w_m^2 = \frac{\lambda L}{\pi} \frac{1}{\sqrt{1 - g_1 g_2}}$$



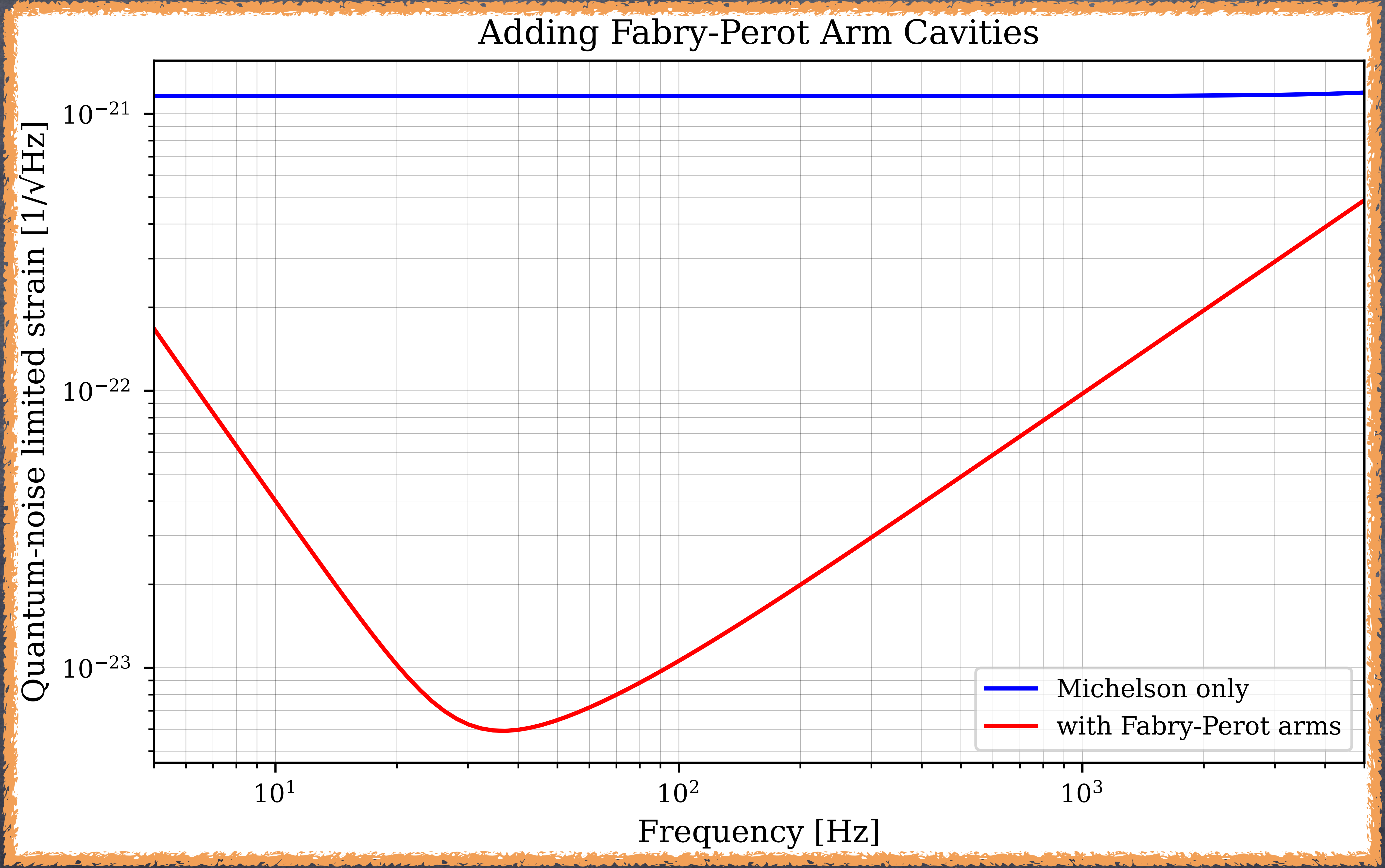
Michelson with arm cavities

Key idea



- Increases effective optical path
- Increases stored optical power

Michelson with arm cavities
Effect on the sensitivity

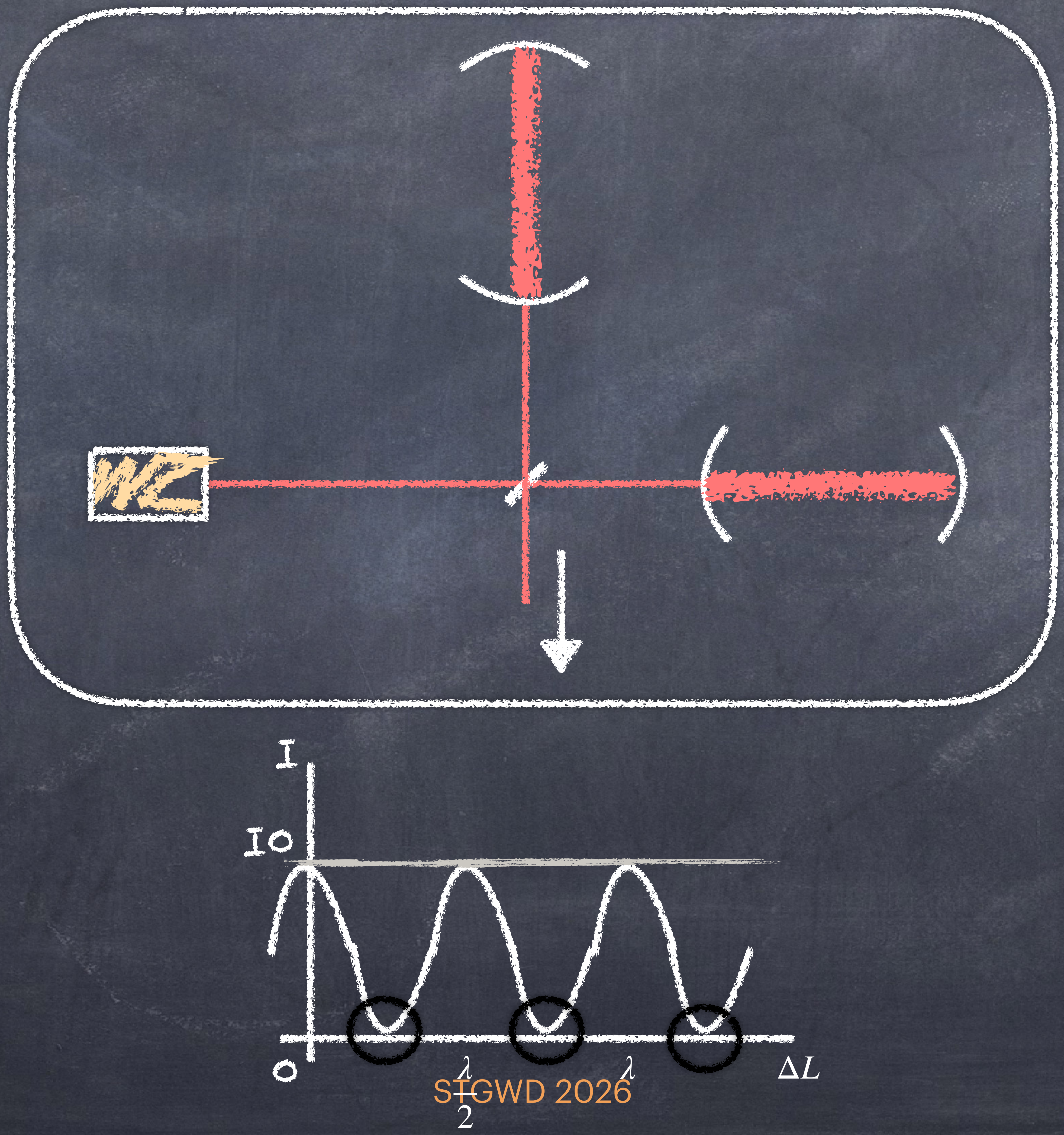


Illustrative comparison: same model, with/without resonant arm cavities.

Power Recycling

Power Recycling
Key idea

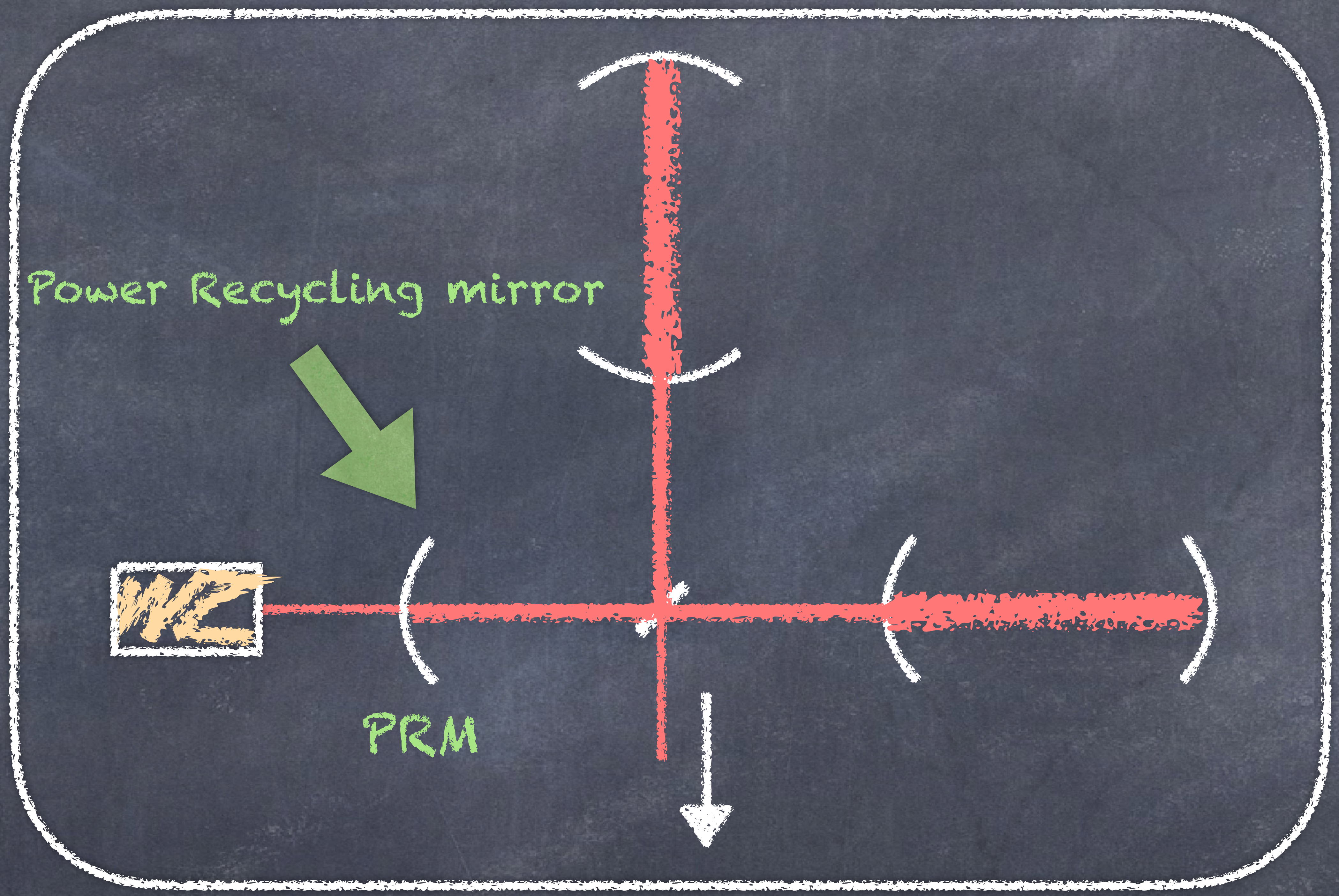
- At dark fringe, light is reflected back

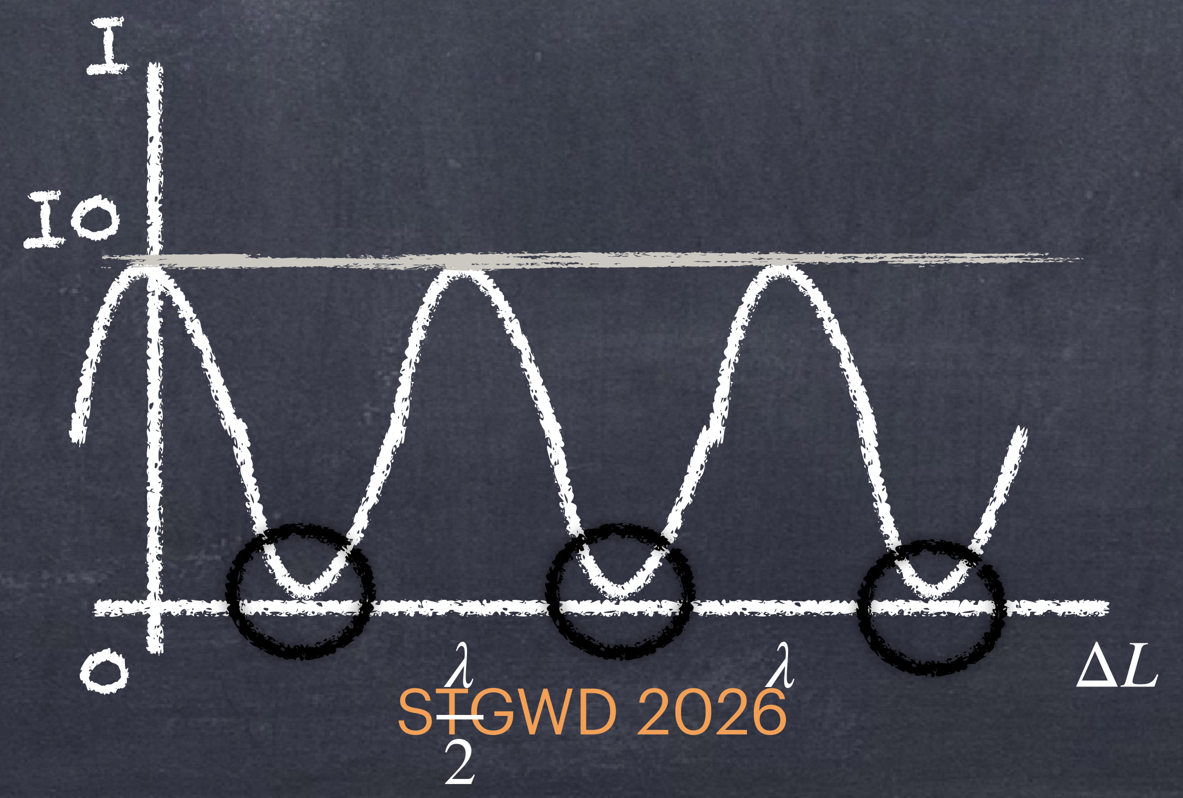


Can we reuse this light?

Power Recycling
Key idea

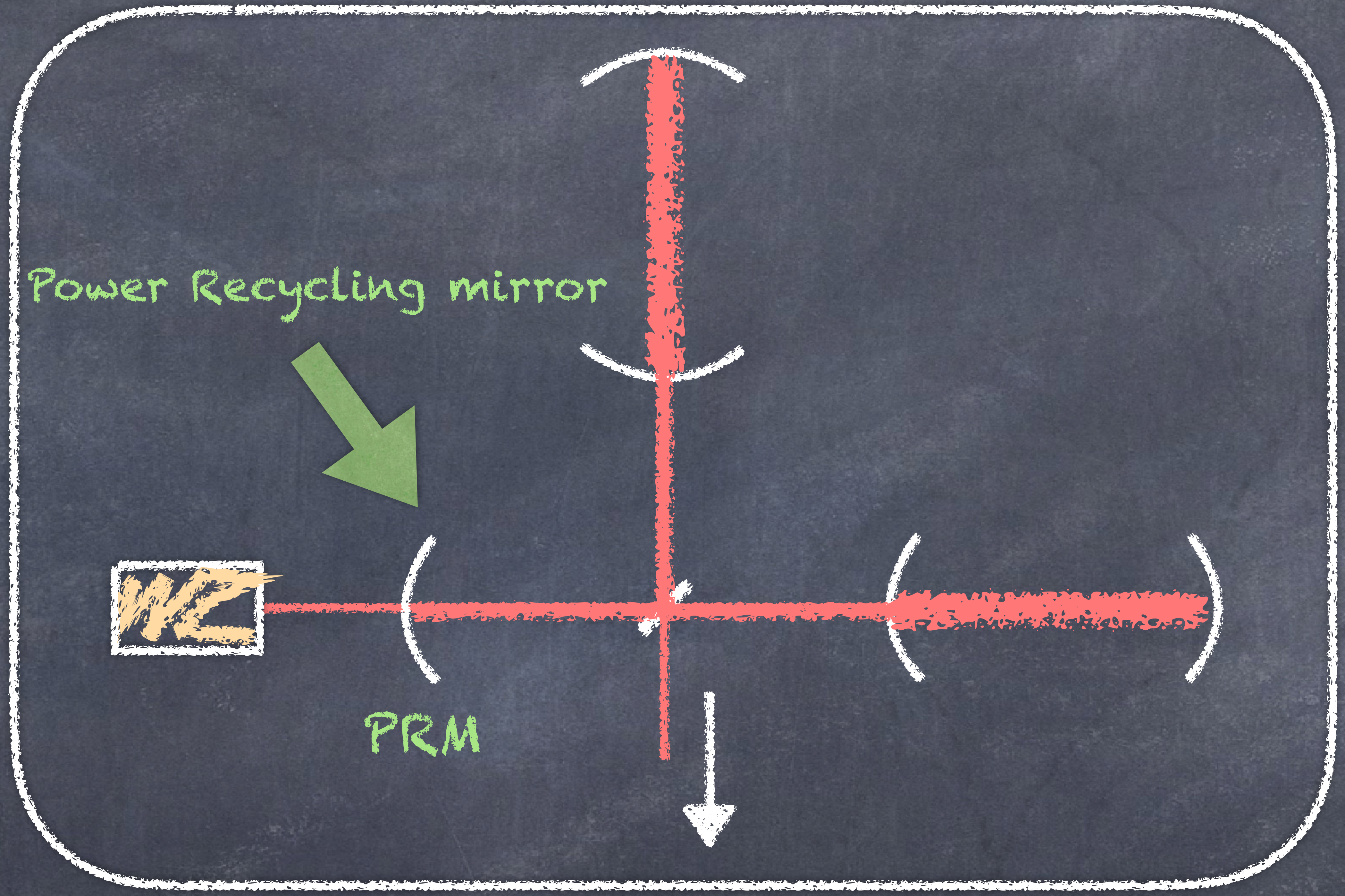
- At dark fringe, light is reflected back





Power Recycling
Key idea

- At dark fringe, light is reflected back

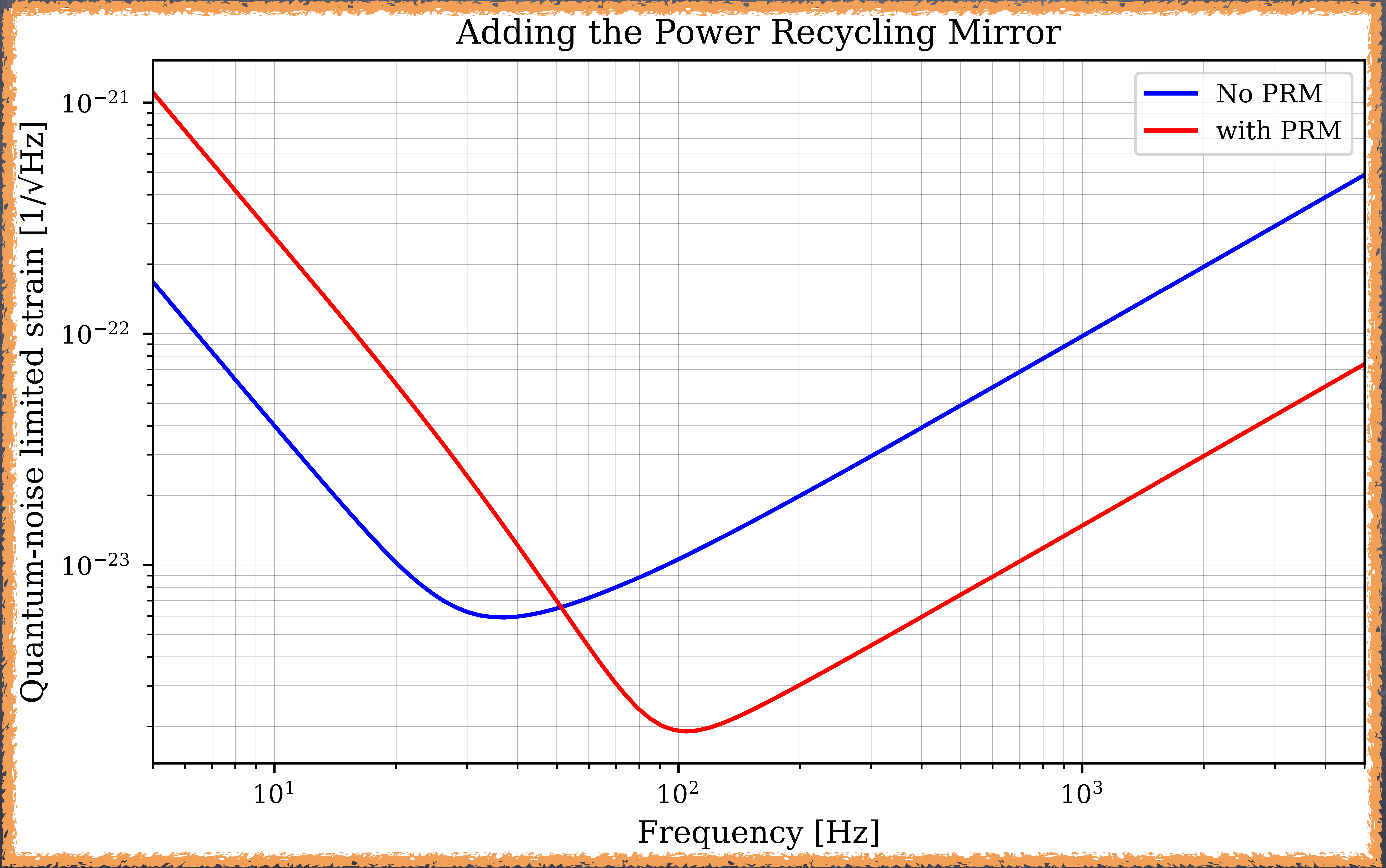


Power Recycling = Cavity for the carrier

- Increased circulating power
- “No need” to increase laser power

Power Recycling
Effect on the Sensitivity

- At dark fringe, light is reflected back

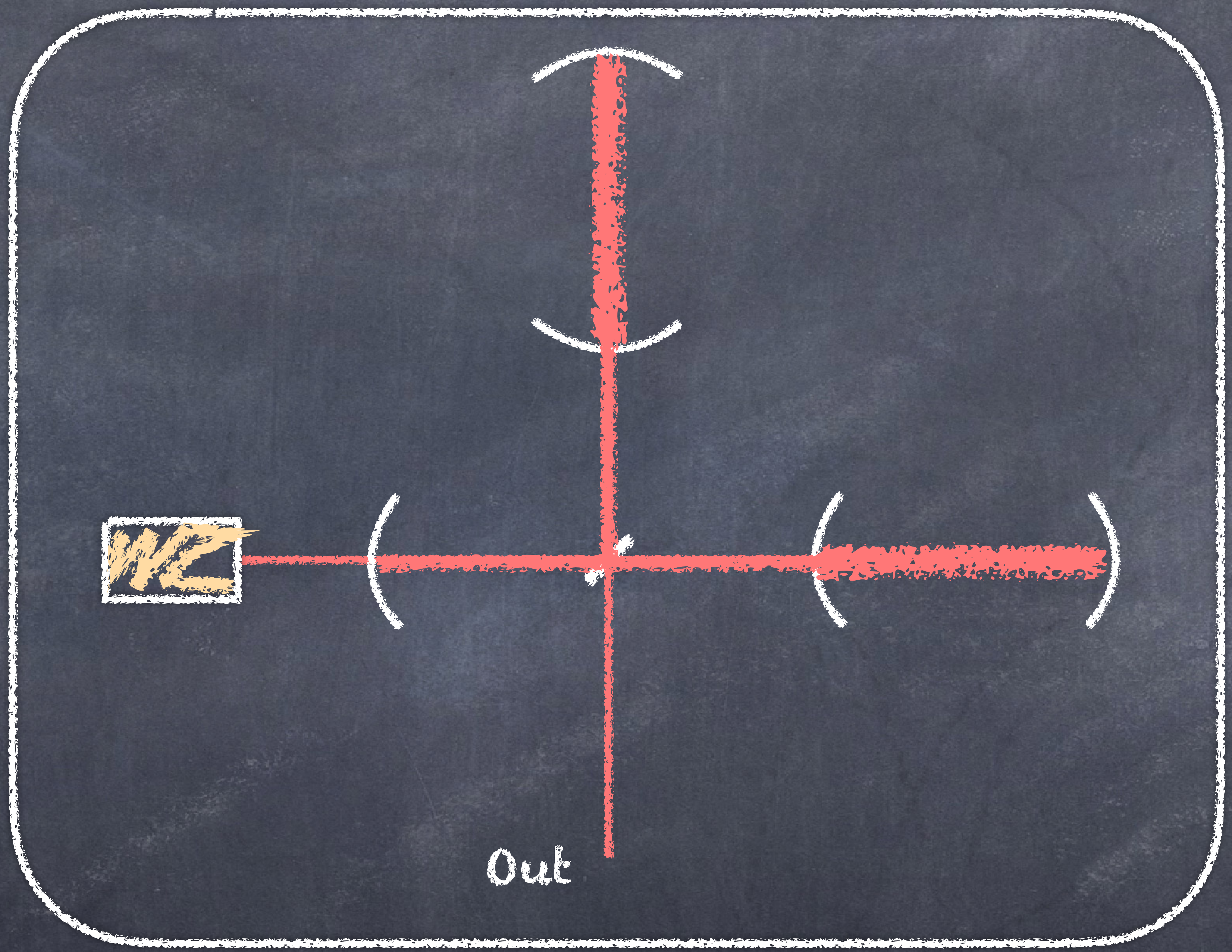


Increasing the circulating power improves Shot Noise, but Radiation Pressure increases at low frequency.

Signal Recycling

GW effect on laser light
Key idea

- GW phase modulates the light —-> GW signal is in the sidebands



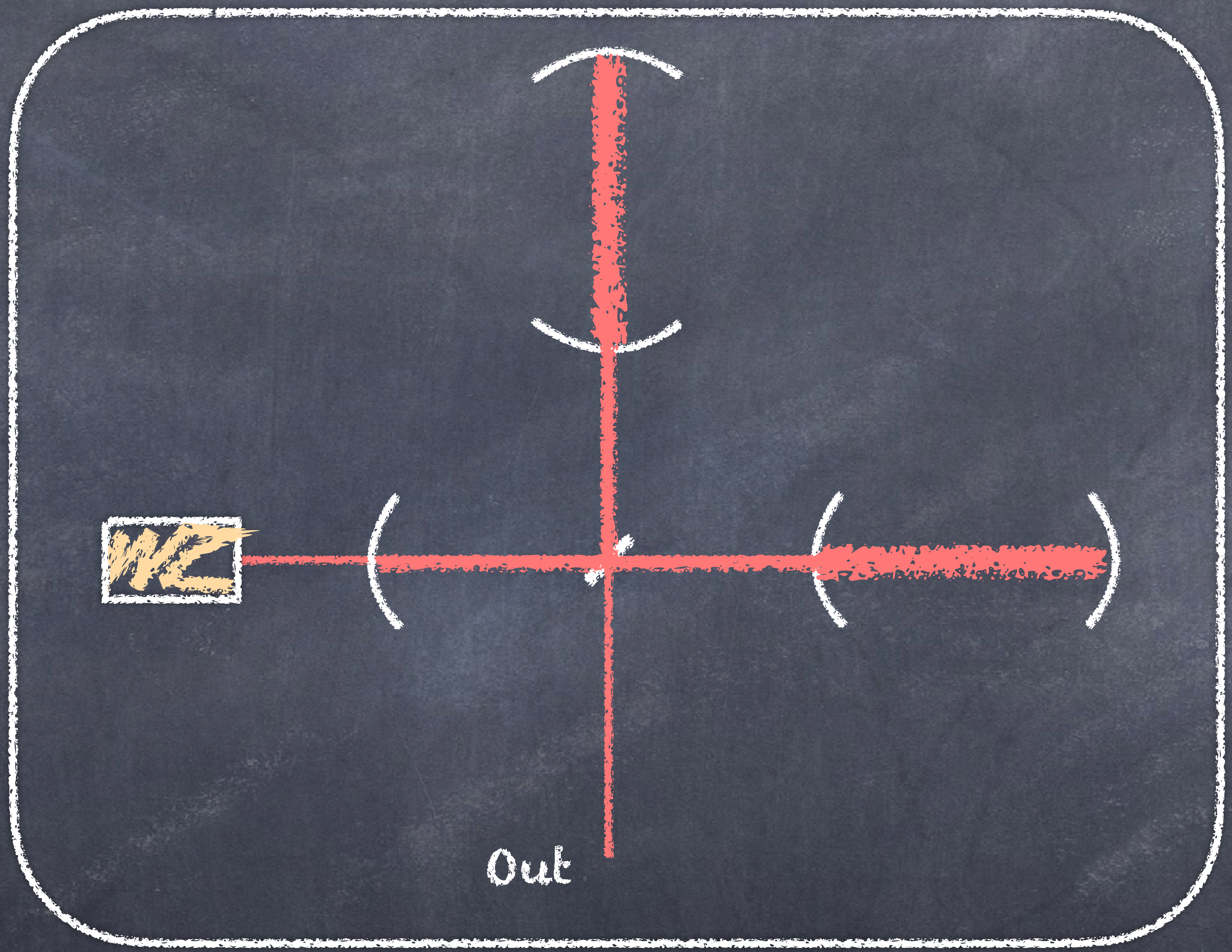
$\omega_l \pm \omega_{gw}$

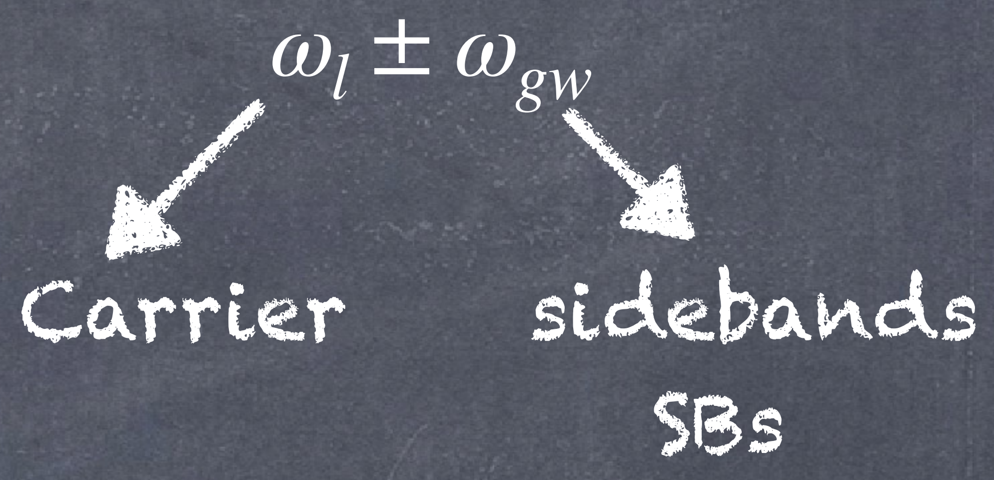
Carrier sidebands SBs

- Where do they go?
- Can we control them?

GW effect on laser light
Key idea

- GW phase modulates the light —-> GW signal is in the sidebands

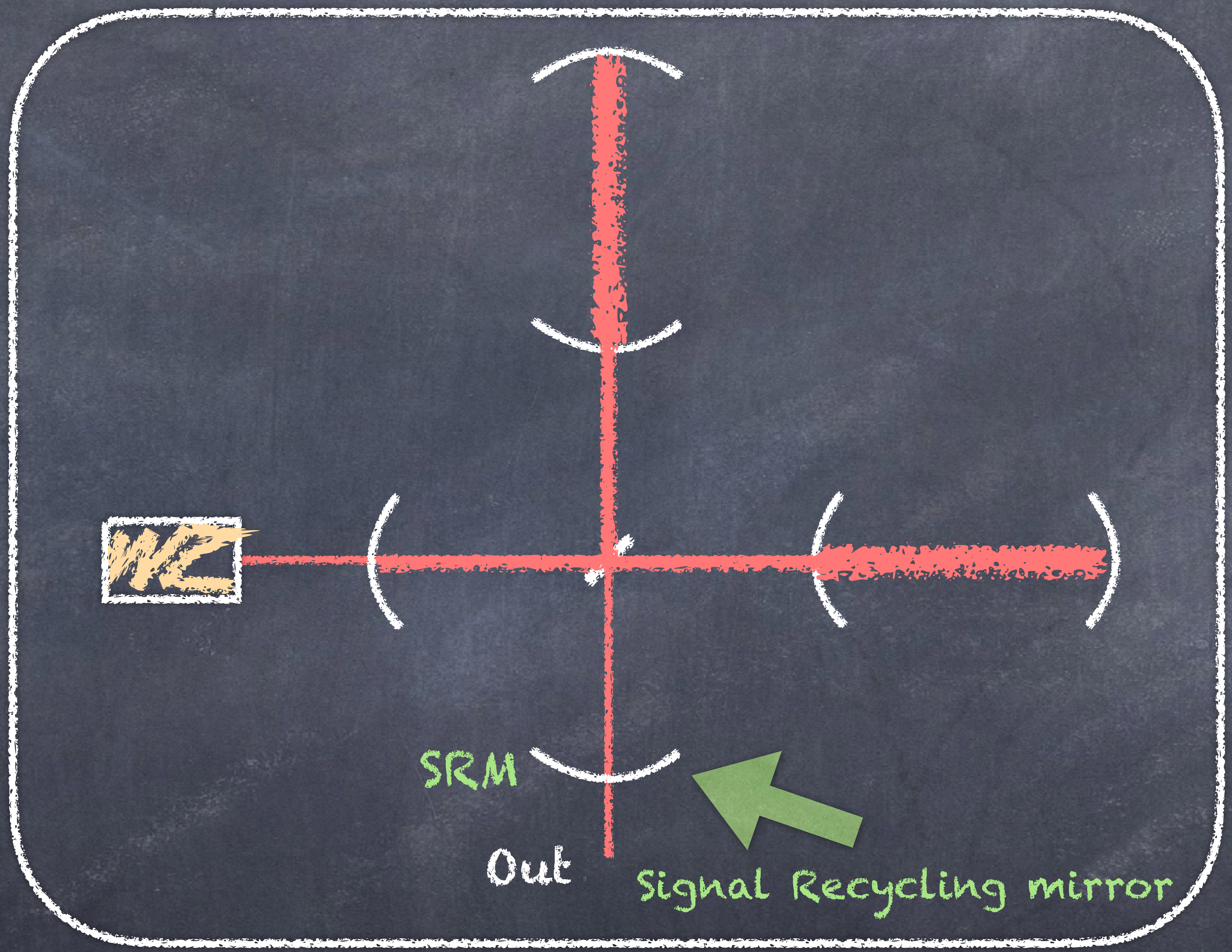


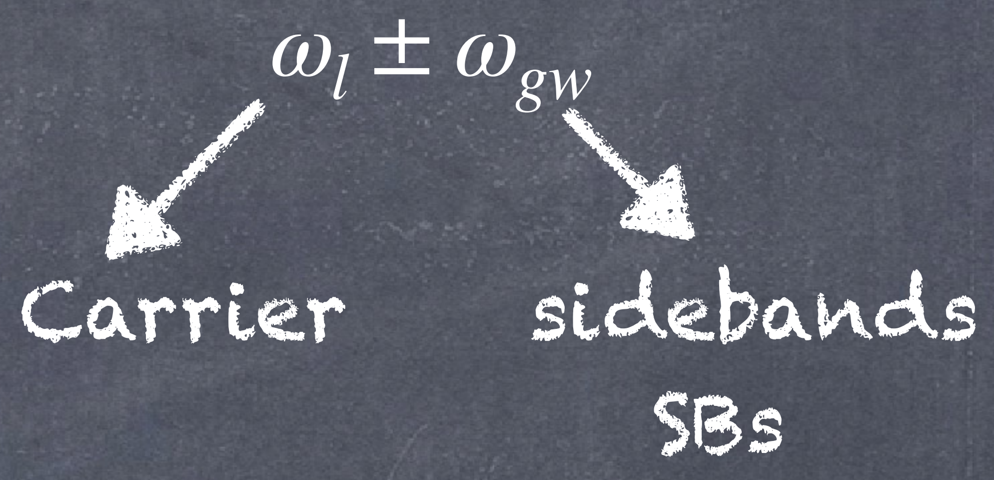


- Where do they go? —-> At the dark port: **Carrier is rejected**, **SBs are transmitted**
- Can we control them? —->

Signal Recycling
Key idea

- GW phase modulates the light —-> GW signal is in the sidebands





Signal Recycling (SRC) = Cavity for the SB

- Where do they go? —-> At the dark port: **Carrier is rejected**, **SBs are transmitted**
- Can we control them? —-> We place a mirror (SRM) at the output port

GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

Signal Recycling
Two regimes

Tuned / Broadband

- Signal Recycling cavity is resonant for the carrier
- Broadband response
- Symmetric sidebands response

Microscopic displacement of the Signal Recycling mirror sets the cavity tuning

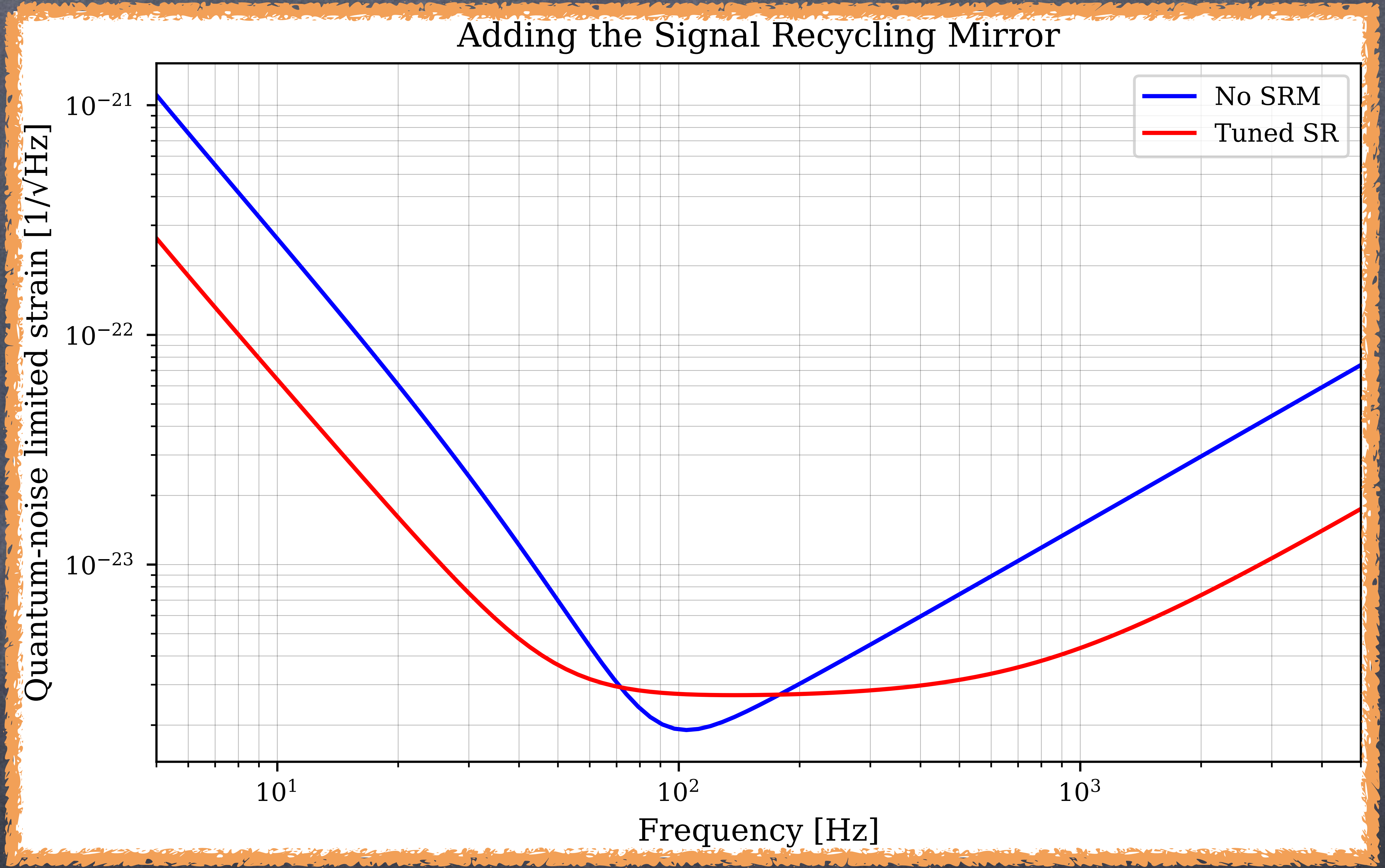
Detuned / Peaked

- Signal Recycling cavity is NOT resonant for the carrier
- Enhanced sensitivity in a selected frequency band
- Asymmetric sidebands response

Signal Recycling
Shaping the detector response

Tuned / Broadband

- Signal Recycling cavity is resonant for the carrier
- Broadband response
- Symmetric sidebands response



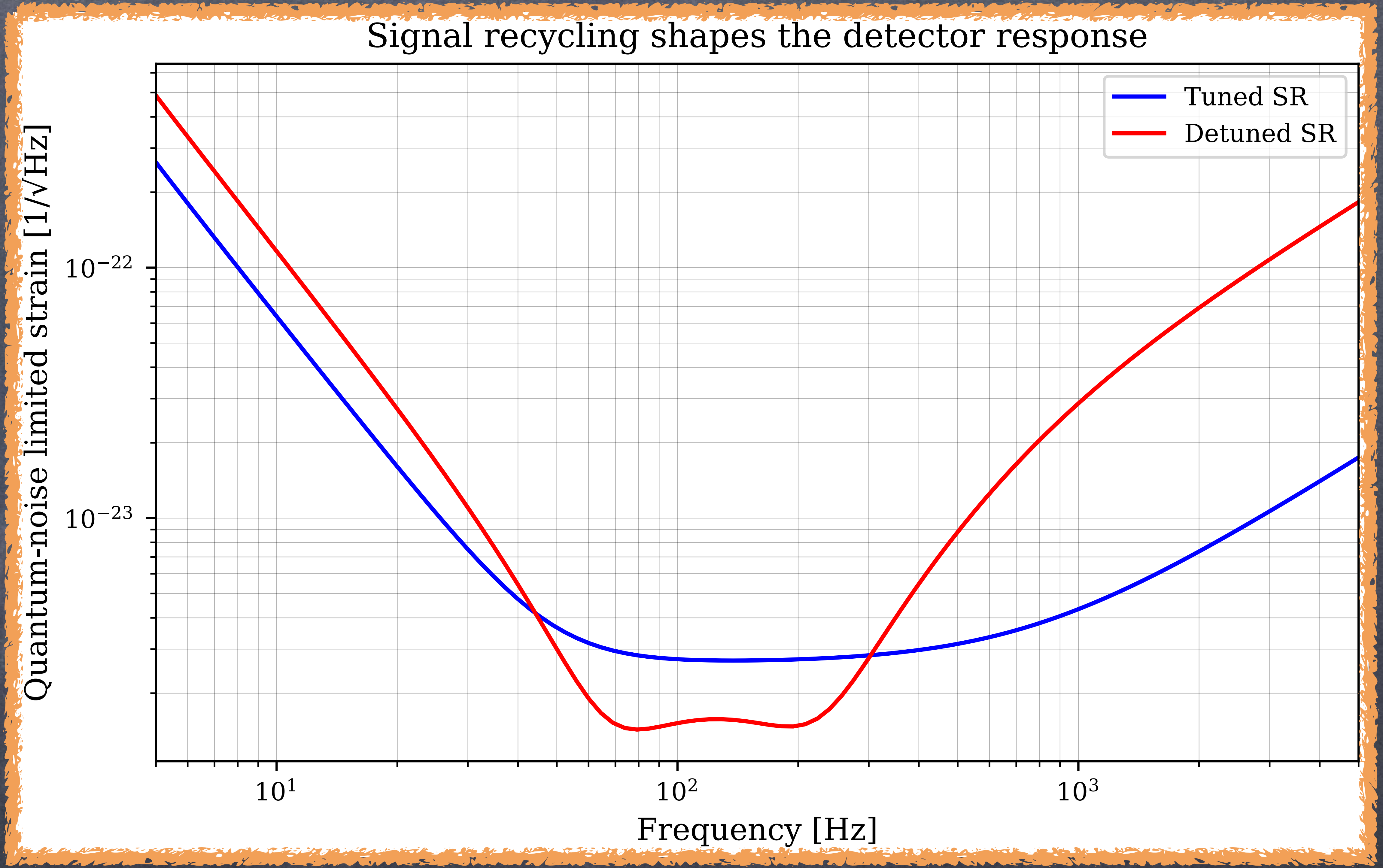
Signal Recycling
Shaping the detector response

Tuned / Broadband

- Signal Recycling cavity is resonant for the carrier
- Broadband response
- Symmetric sidebands response

Detuned / Peaked

- Signal Recycling cavity is NOT resonant for the carrier
- Enhanced sensitivity in a selected frequency band
- Asymmetric sidebands response



Key Messages

- Real laser beams are **Gaussian beams**
→ beam size and waist position must match the optical system

- A **Michelson interferometer** senses differential length
→ but a simple Michelson is not sensitive enough

- **Fabry-Perot** arm cavities store light
→ larger effective optical path
→ larger circulating power
→ finite bandwidth: the cavity pole

- **Power recycling** increases the carrier power buildup
→ we reuse the light reflected at the dark fringe

- **Signal recycling** acts on the signal sidebands
→ the detector response can be shaped

Further reading

- P. R. Saulson, Fundamentals of Interferometric Gravitational Wave Detectors, World Scientific, 1994 / 2017
- A. E. Siegman, Lasers, University Science Books, 1986.
- A. Freise and K. A. Strain, “Interferometer techniques for gravitational-wave detection”, Living Reviews in Relativity 13, 1 (2010); updated version, Living Reviews in Relativity 19, 1 (2016)
- B. J. Meers, “Recycling in laser-interferometric gravitational-wave detectors”, Physical Review D 38, 2317–2326 (1988)
- J. Mizuno, Comparison of optical configurations for laser-interferometric gravitational-wave detectors, PhD thesis, Universität Hannover / Max-Planck-Institut für Quantenoptik, 1995
- S. Hild et al., “Demonstration and comparison of tuned and detuned signal recycling in a large-scale gravitational wave detector”, Classical and Quantum Gravity 24, 1513–1523 (2007)
- Finesse 3 documentation and examples.

...THANK YOU


Key Message
Gaussian beam parameters
Gaussian beam is univocally described by w0,z0
Define the beam "everywhere"
Must match the optical system
Antonio Perreca
STGWD 2026
Signal Recycling
Key idea
GW phase modulates the light —> GW signal is in the sidebands
Carrier
sidebands
SRM
Out
Signal Recycling mirror
Signal Recycling = Cavity for the SB
Where do they go? —> Carrier is still resonating, SB is transmitted out
Can we control them? —> We place a mirror (SRM) at the output port
40
Michelson Int
Limita
M2
L2
E
L1
M1
E1
E2
Limited sensitivity
Need more effective length
17
Fabry-Perot
Lon
xed stability parameter g:
Beam size
Large L —> large beam size
Stability becomes critical
First real design constraint
31
Power Recycling
Key idea
At dark fringe, light is reflected back
Power Recycling mirror
PRM
Power Recycling = Cavity for the carrier
Increased circulating power
"No need" to increase laser power
35

Fabry-Perot cavity
Power

- Large circulating power depends on mirror reflectivity

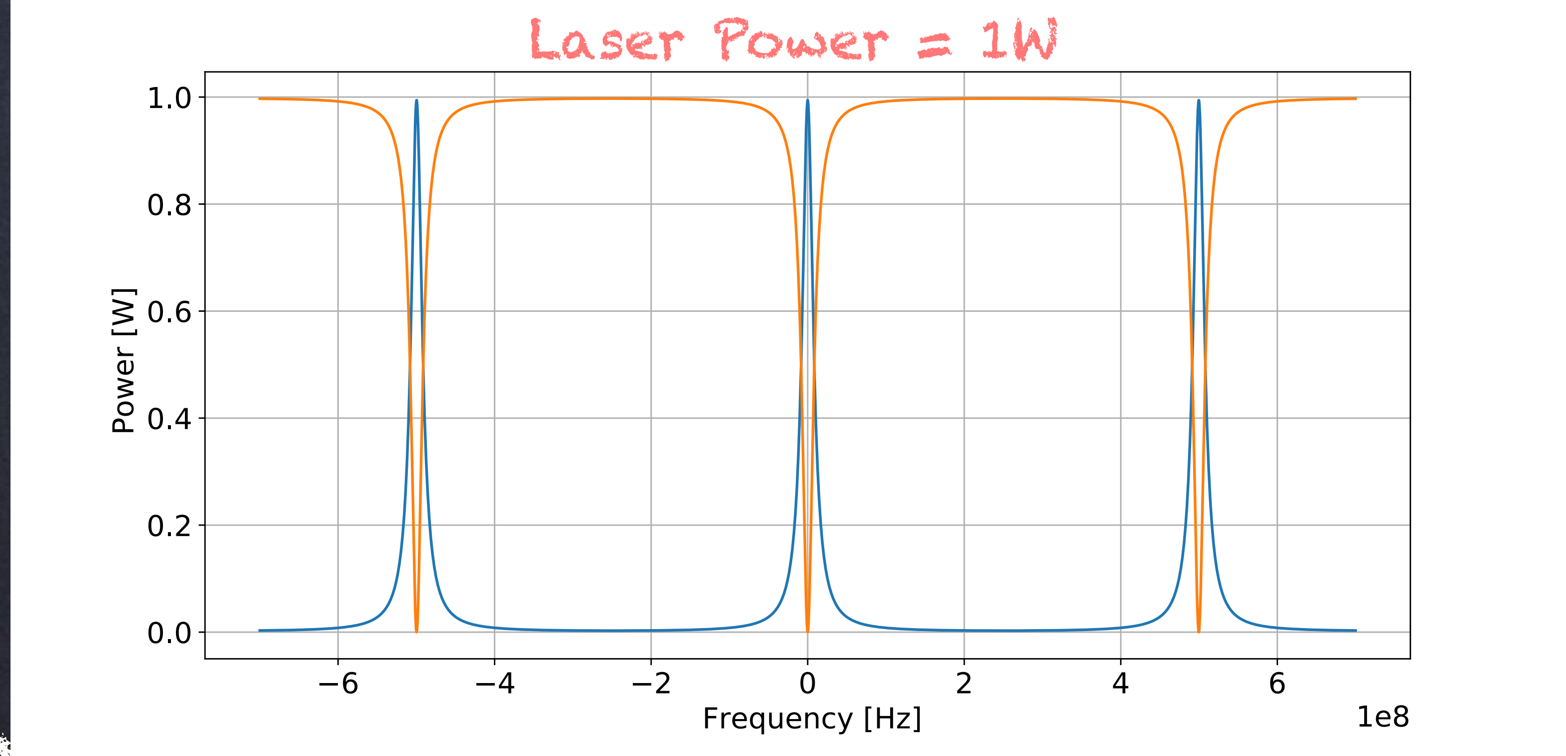
Reflected Power

$$P_{Ref} = E_{Ref} \cdot E^*_{Ref} = \frac{r_1^2 + r_2^2 - 2r_1r_2\cos(2kL)}{1 + (r_1r_2)^2 - 2r_1r_2\cos(2kL)} t_1^2 E_0^2$$

Transmitted Power

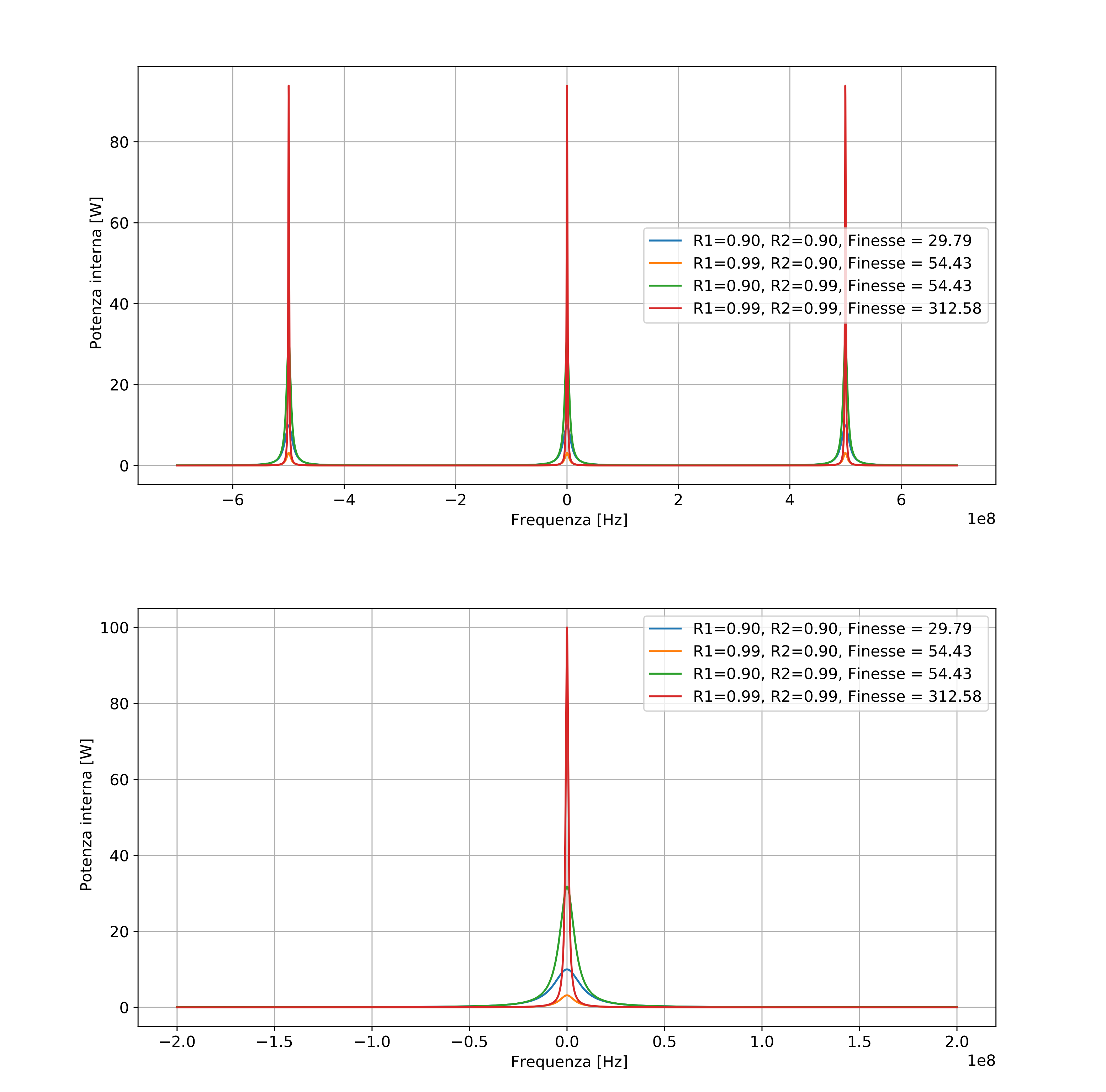
$$P_{Trans} = E_{Trans} \cdot E^*_{Trans} = \frac{t_1^2 t_2^2}{1 + (r_1r_2)^2 - 2r_1r_2\cos(2kL)} E_0^2$$

Reflected Power — Transmitted Power



Internal Power

$$P_{Int} = E_{Int} \cdot E^*_{Int} = \frac{1}{1 + (r_1r_2)^2 - 2r_1r_2\cos(2kL)} t_1^2 E_0^2$$



Parameters

Frequency pole

$$f_p = \frac{FSR}{\pi} \arcsin \frac{1 - r_1r_2}{2\sqrt{r_1r_2}}$$

Full Width Half Maximum

$$FWHM = 2f_p$$

Finesse

$$F = \frac{FSR}{FWHM} = \frac{\pi}{2\ \arcsin \frac{1 - r_1r_2}{2\sqrt{r_1r_2}}} \sim \frac{\pi\sqrt{r_1r_2}}{1 - r_1r_2}$$