The passive rejection of seismic vibrations

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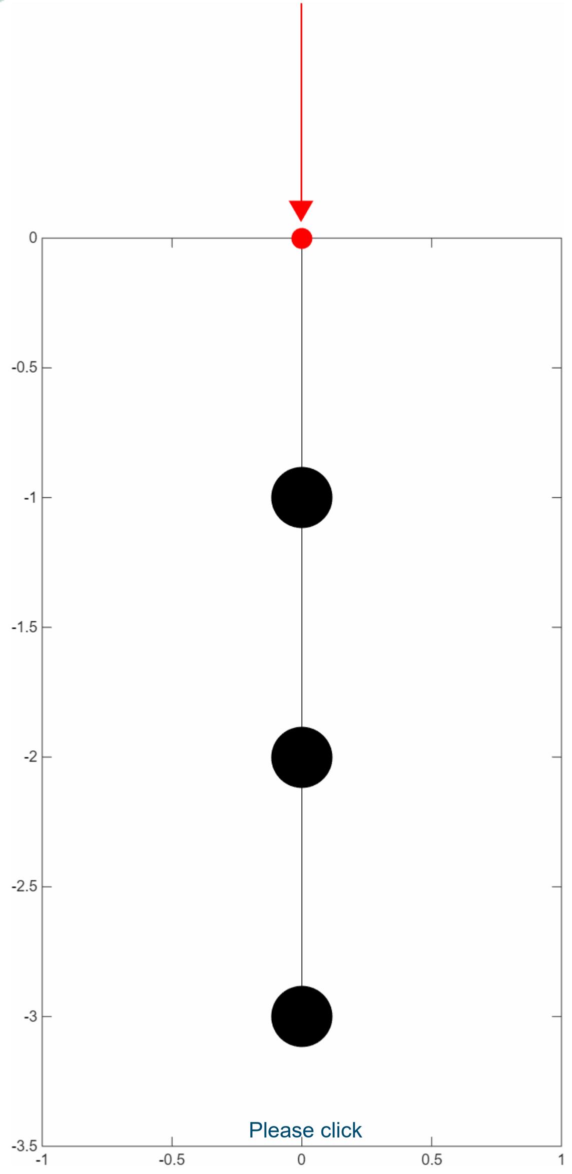

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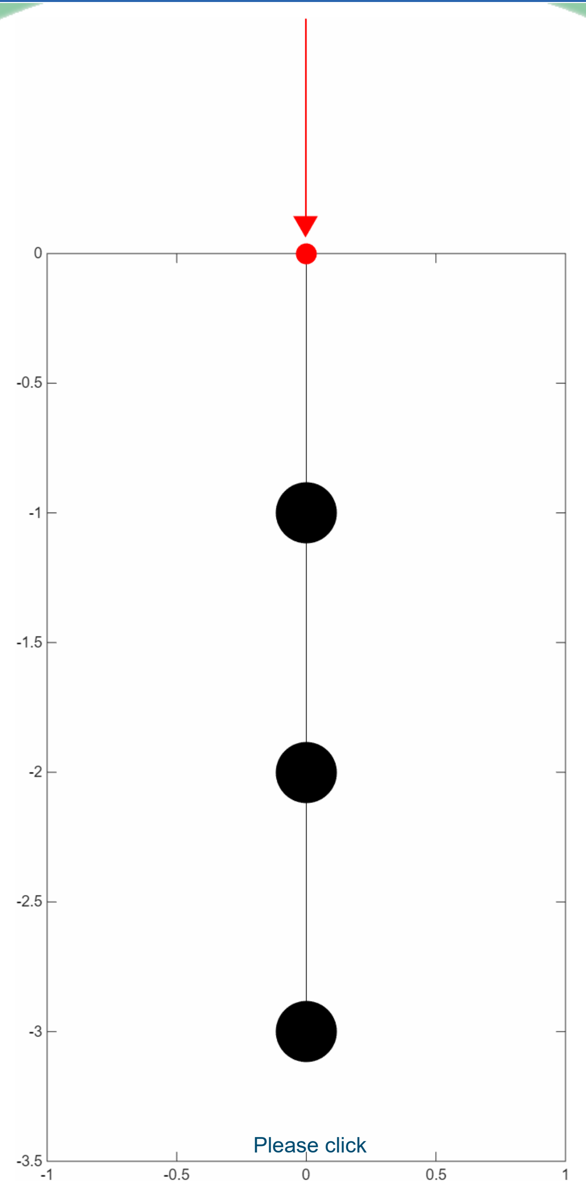
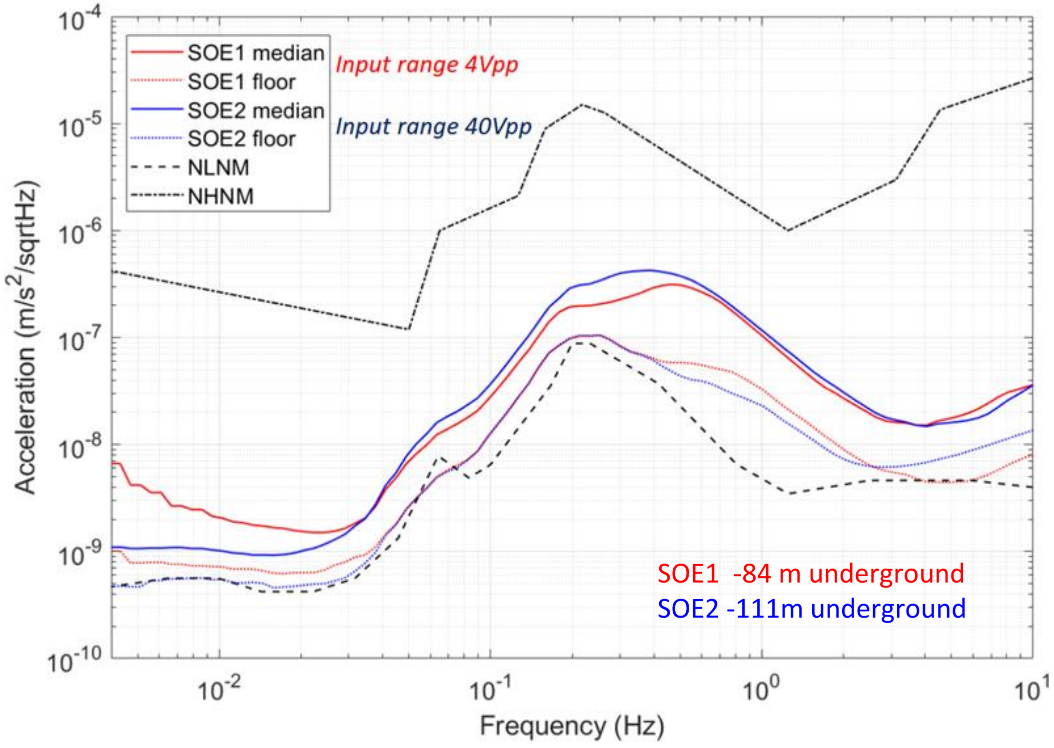


- Seism never sleeps

Sos Enattos

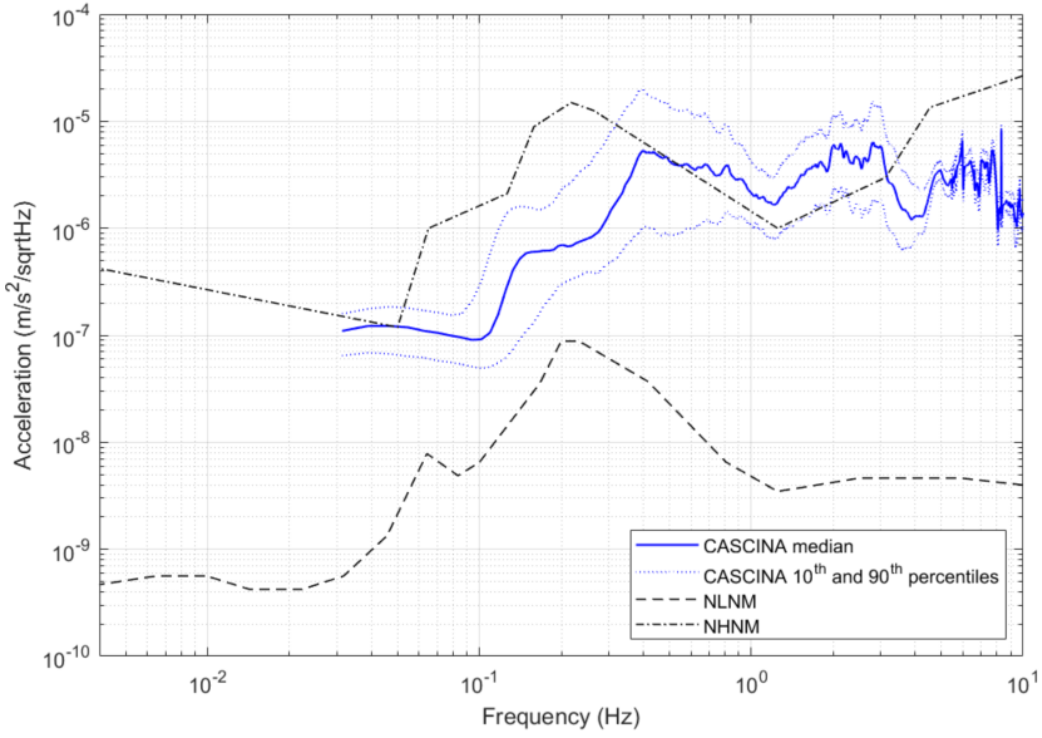
Vertical Acceleration ASD



L. Naticchioni – GWADW21 – May 17th – 21st 2021

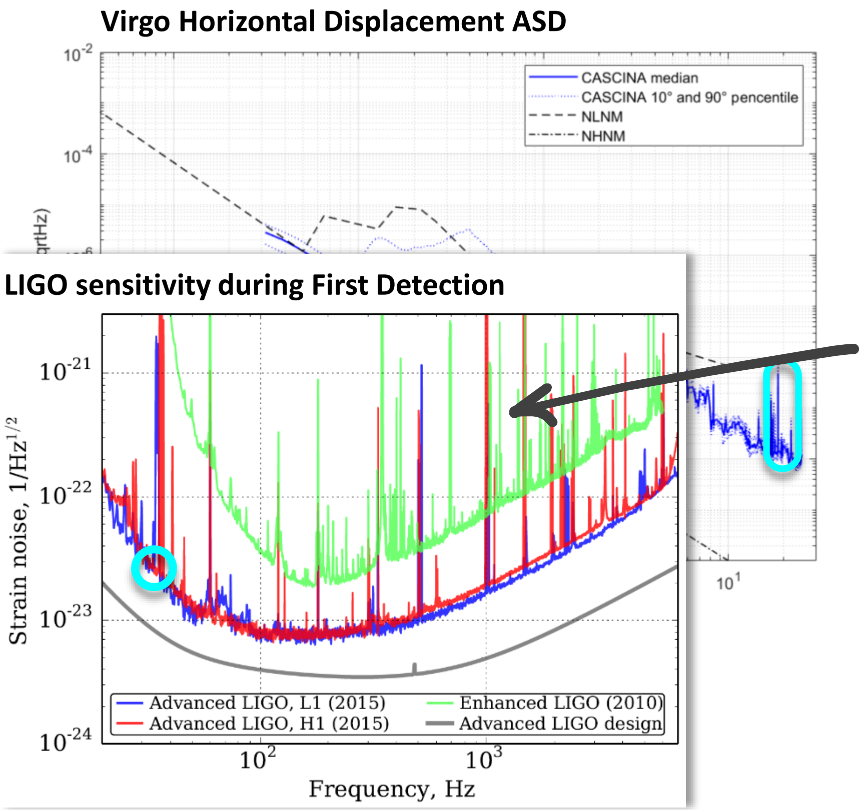
Virgo

Horizontal Acceleration ASD

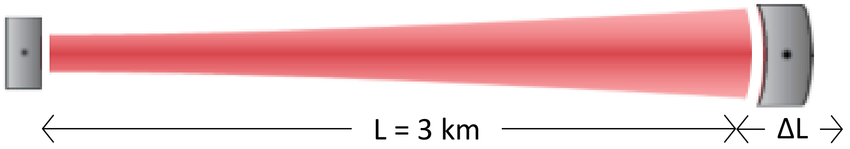


Courtesy of Irene Fiori (EGO-Cascina)

• Vibrations must be rejected



In Virgo, seismic displacements between $\mathbf{10^{-10}}$ and $\mathbf{10^{-8} m/\sqrt{Hz}}$ are recorded on ground **at 20 Hz**.



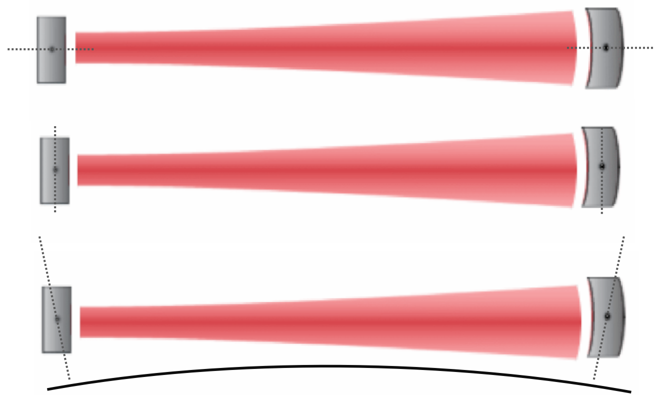
Without any filtering, this would mask strains of the $\mathbf{3\ km}$ arm like $h > \frac{10^{-10}\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}}{3\cdot 10^{3}\mathrm{m}} = 3\cdot 10^{-14}\frac{1}{\sqrt{\mathrm{Hz}}}$.

Horizontal vibrations act

- on the length of the optical paths;
- on the centering and alignment of the mirrors.

Vertical vibrations act

- on the centering and alignment of the mirrors;
- (slightly) on the length of the optical paths. This comes from the **convergence of the vertical directions** at a km-scale distance.



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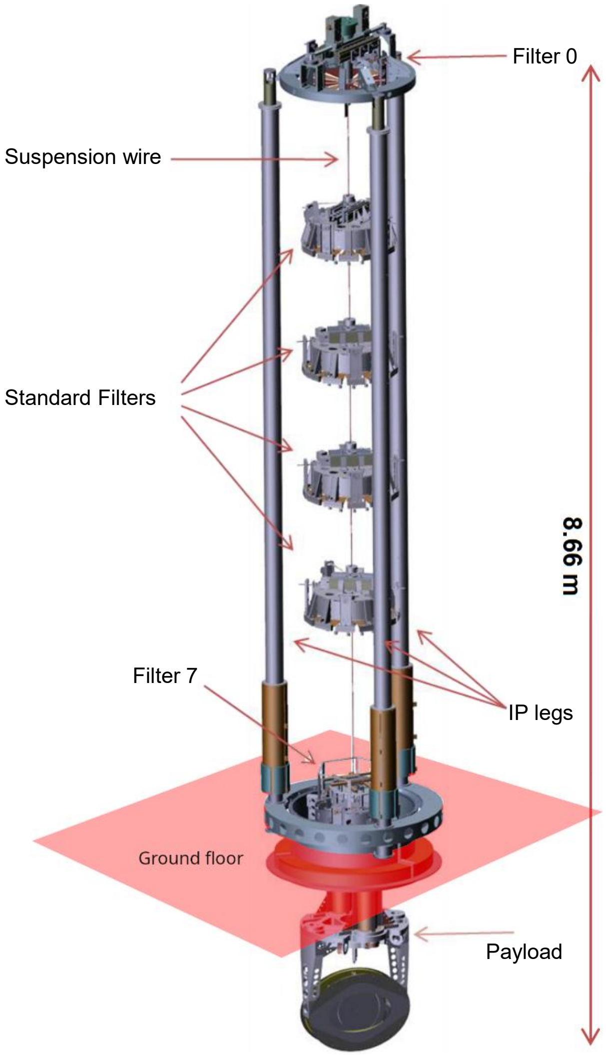


• The Superattenuator solution

Each cavity mirror is currently suspended in Advanced Virgo from a Superattenuator (SA).

The AdVirgo SA is a **passive** mechanical filter whose main components are

- an **Inverted Pendulun** (IP), made of three identical legs and a top ring;
- a top vertical oscillator called **Filter 0**;
- a chain of four **Standard Filters**, suspended to each other;
- a special final filter (**Filter 7**), suspended from the previous and connected to the Payload.

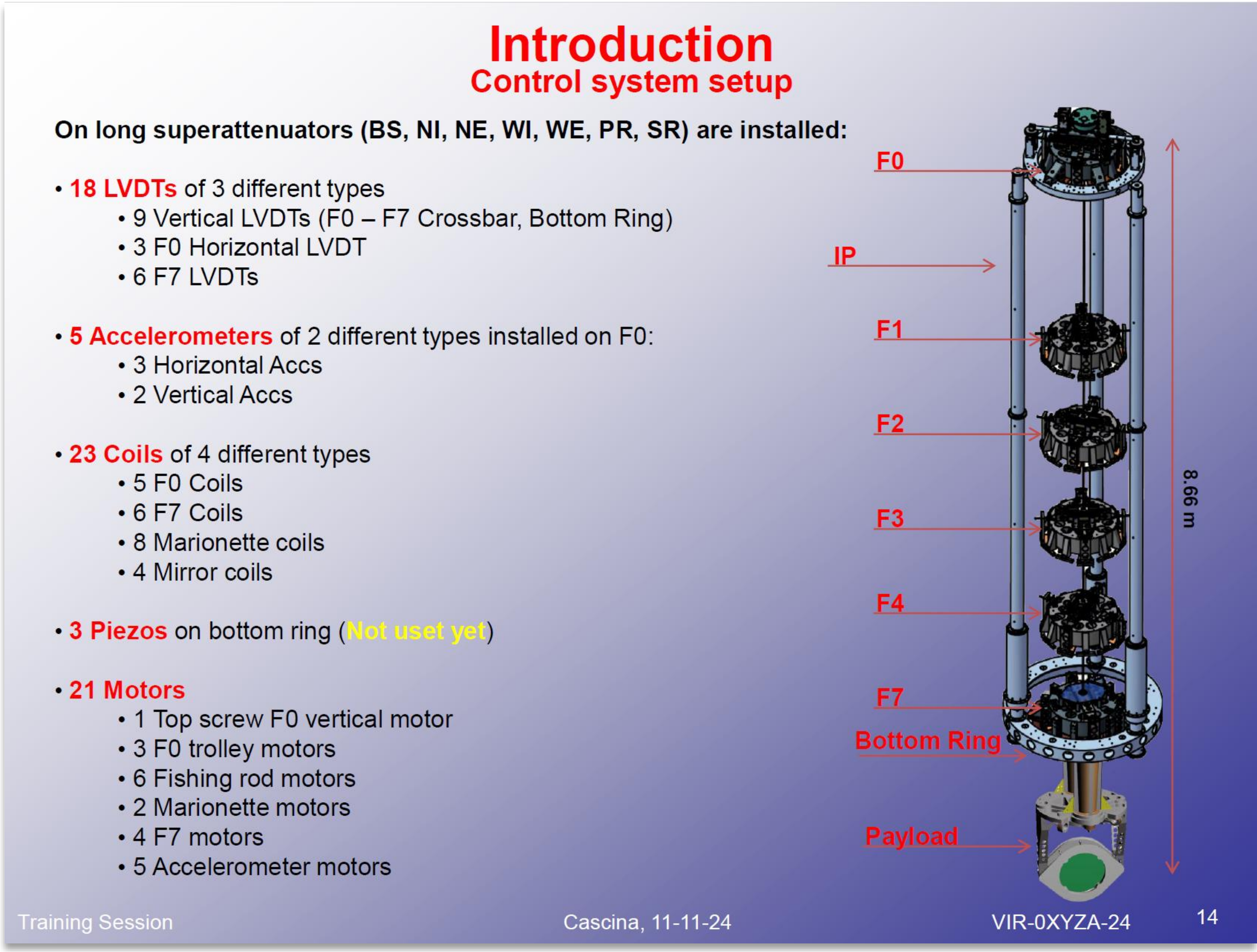
The **Payload** components are

- a stiff reference cage rigidly connected to Filter 7;
- a **Marionette**, suspended from Filter 7 and controlled from the cage;
- the **Mirror**, suspended from the Marionette and controlled from the cage.

See Valerio Boschi – SUSP Training session, https://tds.virgo-gw.eu/?r=16165

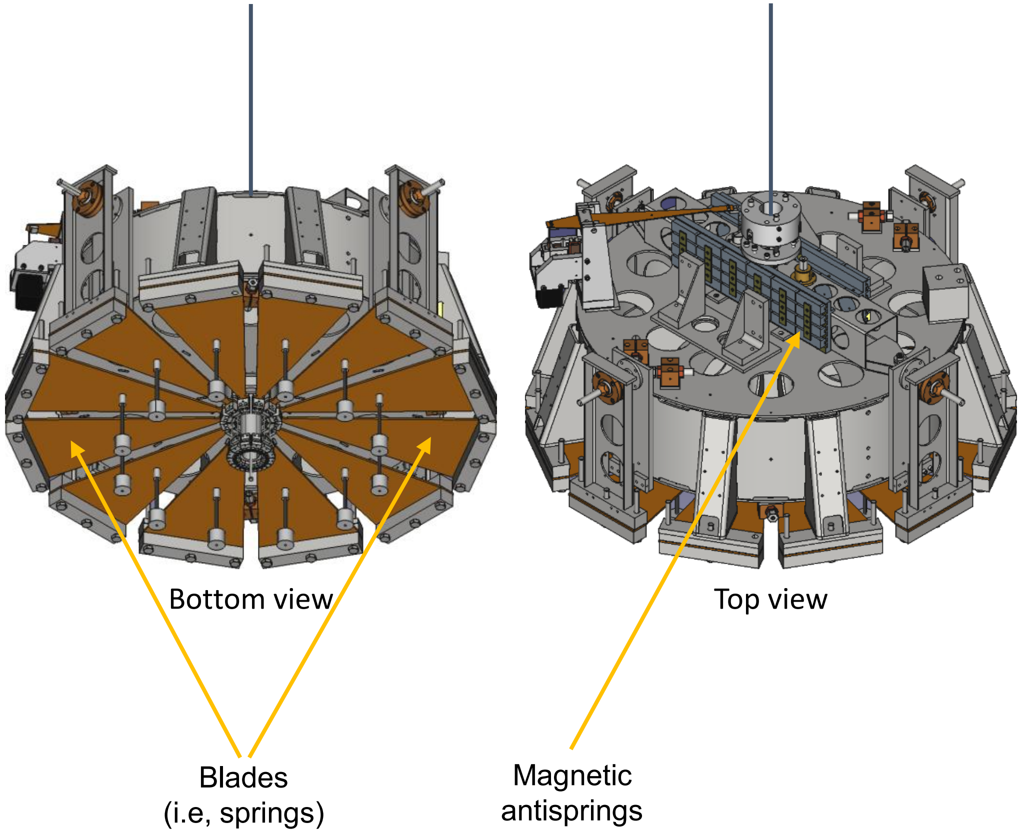
- The Superattenuator solution

Not so passive, indeed



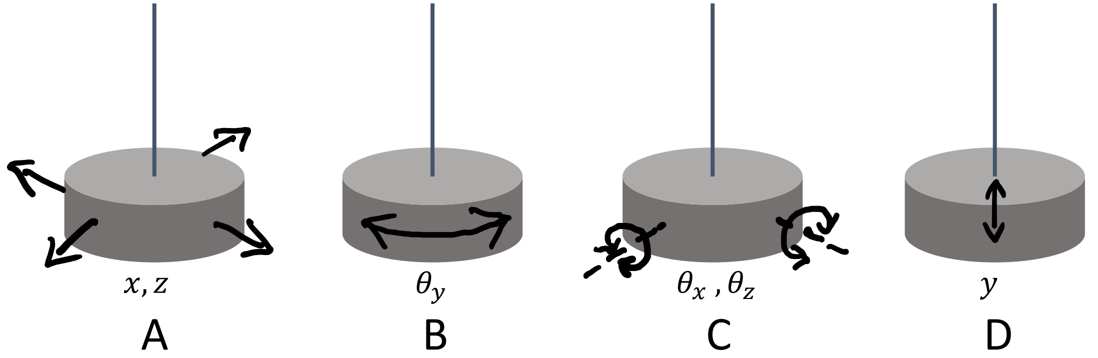
See Valerio Boschi – SUSP Training session, https://tds.virgo-gw.eu/?r=16165

- Standard Filter



The AdV Superattenuator has four suspended Standard Filters

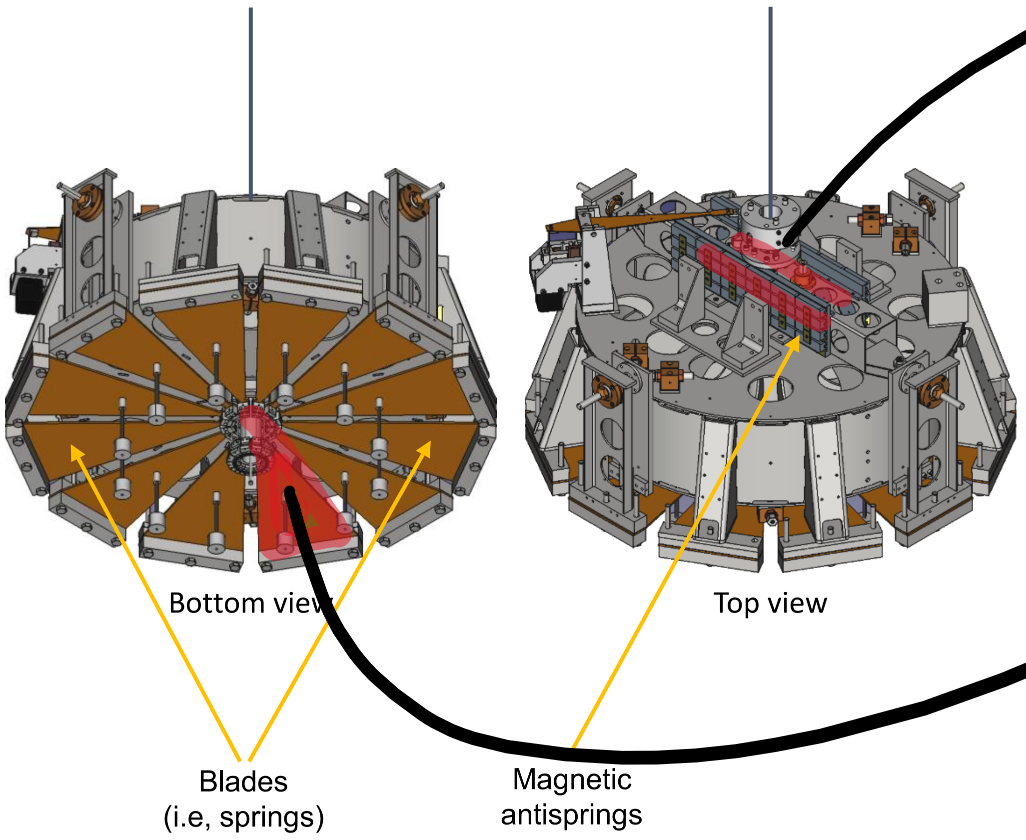
Each Standard Filter is a physical pendulum with several degrees of freedom.
Each DOF oscillates with its own frequency.

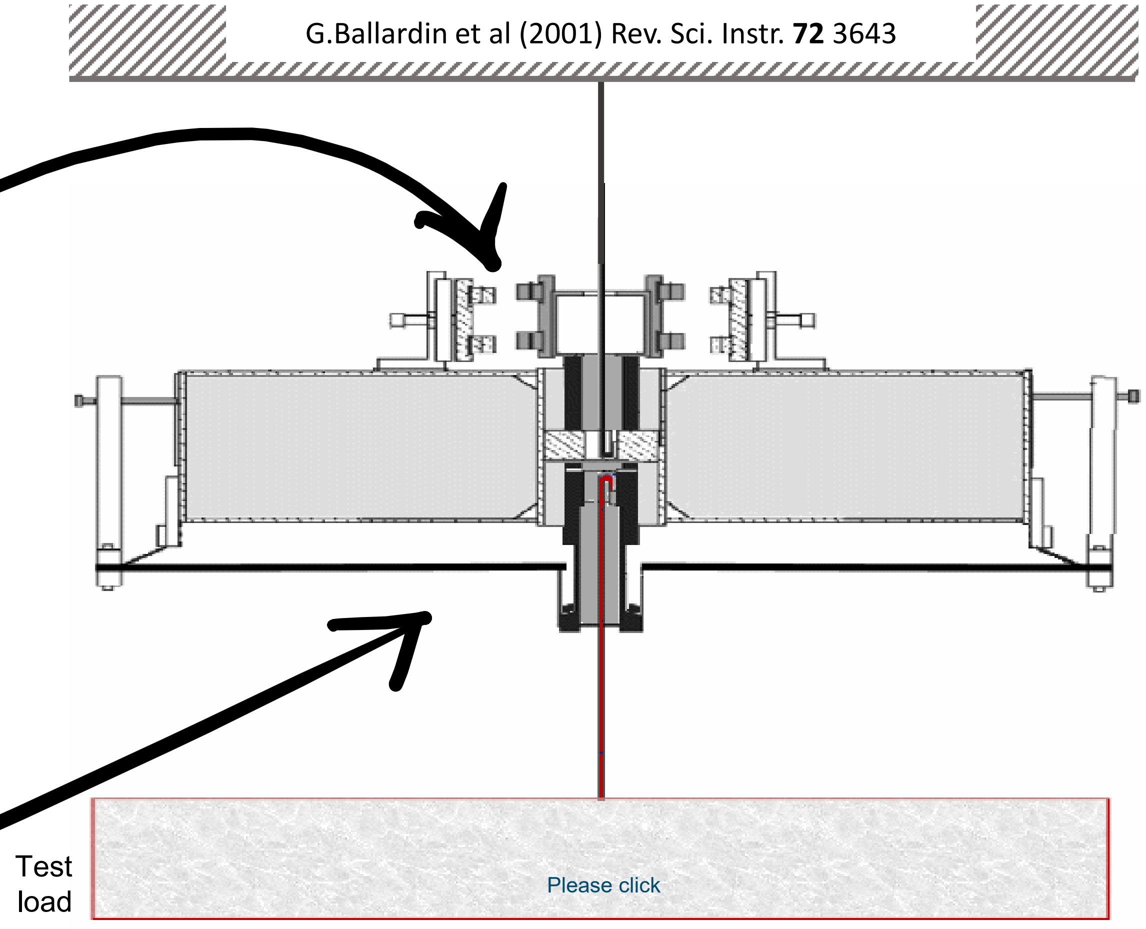


Oscillations A, B, C spontaneously occur depending on gravity, inertia of the massive body, wire properties.

Motion D requires a **dedicated vertical oscillator**.

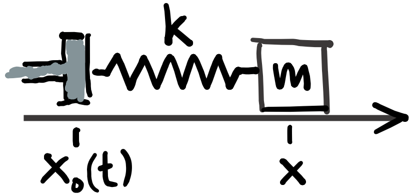
- Built-in vertical oscillator





Vertical oscillators are set in
Filter 0, Standard Filters, Filter 7

• A toy model of the oscillating filter



$$k = k_0 \cdot (1 + i\,\phi)$$

A mechanical filter can be understood in terms of **oscillators**.
Let's create a toy filter using an elastic oscillator as the building brick.

A metal spring is actuated on the left side by a piston whose position is $x_0(t)$.
At the right end of the spring a block can move over the spring axis.

The spring has a **loss angle $\boldsymbol{\phi}$**. This accounts for anelastic relaxations in the spring material.

$$m\,\ddot{x} = -k \cdot (x - x_0)$$

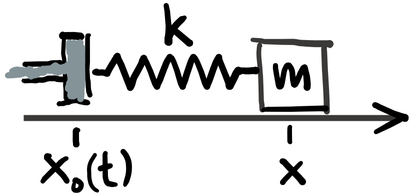
$$-m\omega^2 \tilde{x} = -k \cdot (\tilde{x} - \tilde{x}_0)$$

$$(k - m\omega^2)\tilde{x} = k\,\tilde{x}_0$$

$$\tilde{x} = -\frac{\omega_0^2 \cdot (1 + i\,\phi)}{\omega^2 - \omega_0^2 \cdot (1 + i\,\phi)}\tilde{x}_0$$

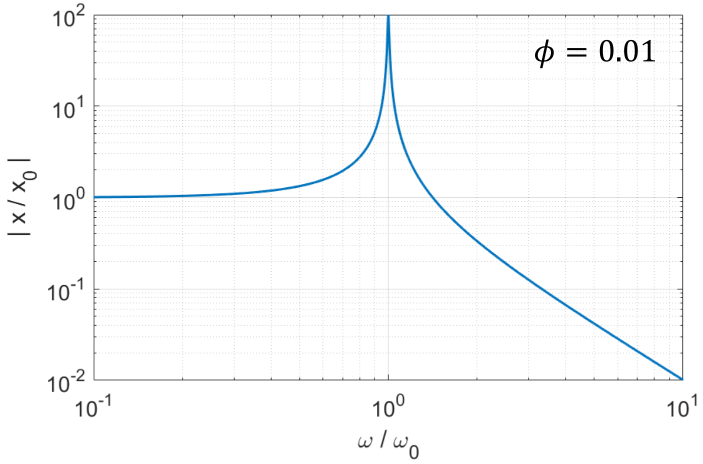
$$f_T(\omega) = -\frac{\omega_0^2 \cdot (1 + i\,\phi)}{\omega^2 - \omega_0^2 \cdot (1 + i\,\phi)}$$

- A toy model of the oscillating filter



$$k = k_0 \cdot (1 + i\,\phi)$$

$$Q \cong \phi^{-1}$$



Power transfer function $$|f_T(\omega)|^2 = \left|\frac{-\omega_0^2 \cdot (1 + i\,\phi)}{\omega^2 - \omega_0^2 \cdot (1 + i\,\phi)}\right|^2$$

Peak power transfer $$|f_T(\omega_0)|^2 = \left|\frac{1 + i\,\phi}{i\phi}\right|^2 \cong \frac{1}{\phi^2}$$

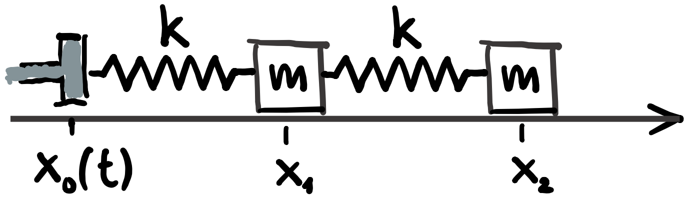
Pulsation **at half peak power**: $$\left|f_T(\omega_{1/2})\right|^2 = \frac{1}{2\phi^2}$$

$$\Rightarrow \left|\frac{{\omega_{1/2}}^2 - \omega_0^2 \cdot (1 + i\,\phi)}{\omega_0^2 \cdot (1 + i\,\phi)}\right|^2 = 2\phi^2 \qquad \Rightarrow \left|\frac{{\omega_{1/2}}^2}{\omega_0^2}(1 - i\,\phi) - 1\right|^2 = 2\phi^2$$

$\Rightarrow$ with $\omega_{1/2} = \omega_0 \cdot (1 + \delta)$, $\left|\frac{\omega_0^2(1+2\delta)}{\omega_0^2}(1 - i\,\phi) - 1\right|^2 = 2\phi^2$

$$\Rightarrow |2\delta - i\,\phi|^2 = 2\phi^2 \quad \Rightarrow 4\delta^2 = \phi^2 \quad \Rightarrow \delta = \pm\frac{\phi}{2} \quad \Rightarrow FWHM = \phi\omega_0$$

- A toy model of the oscillating filter



$$k = k_0 \cdot (1 + i\,\phi)$$

Let's complicate it a bit.
Two identical oscillators are connected in cascade.

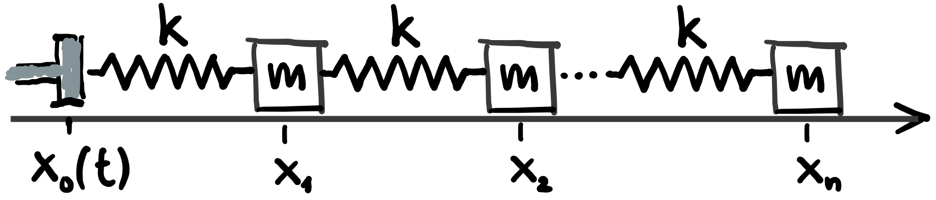
$$\begin{cases} -m\omega^2 \tilde{x}_1 = -k \cdot (\tilde{x}_1 - \tilde{x}_0) + k \cdot (\tilde{x}_2 - \tilde{x}_1) \\ -m\omega^2 \tilde{x}_2 = -k \cdot (\tilde{x}_2 - \tilde{x}_1) \end{cases}$$

$$\Rightarrow \begin{cases} (2k - m\omega^2)\tilde{x}_1 - k\tilde{x}_2 = k\tilde{x}_0 \\ -k\tilde{x}_1 + (k - m\omega^2)\tilde{x}_2 = 0 \end{cases} \Rightarrow \left[\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} - \frac{m\omega^2}{k} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} \tilde{x}_0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \mathbf{M} \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} \tilde{x}_0 \\ 0 \end{pmatrix}, \text{ with } \mathbf{M} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} - \frac{m\omega^2}{k} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \Rightarrow \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} \tilde{x}_0 \\ 0 \end{pmatrix}$$

- A toy model of the oscillating filter

Even more complicated.
Many identical oscillators connected to each other.



$$\begin{cases} -m\omega^2\tilde{x}_1 = -k\cdot(\tilde{x}_1 - \tilde{x}_0) + k\cdot(\tilde{x}_2 - \tilde{x}_1) \\ -m\omega^2\tilde{x}_2 = -k\cdot(\tilde{x}_2 - \tilde{x}_1) + k\cdot(\tilde{x}_3 - \tilde{x}_2) \\ \dots \\ -m\omega^2\tilde{x}_n = -k\cdot(\tilde{x}_n - \tilde{x}_{n-1}) \end{cases}$$

$$\Rightarrow \mathbf{M} = \begin{pmatrix} 2 & -1 & & & 0 \\ -1 & 2 & -1 & & \\ & -1 & \ddots & -1 & \\ & & -1 & 2 & -1 \\ 0 & & & -1 & 1 \end{pmatrix} - \frac{m\omega^2}{k}\begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ 0 & & & & 1 \end{pmatrix} = \begin{pmatrix} 2-\lambda & -1 & & & 0 \\ -1 & 2-\lambda & -1 & & \\ & -1 & \ddots & -1 & \\ & & -1 & 2-\lambda & -1 \\ 0 & & & -1 & 1-\lambda \end{pmatrix}$$

, with $\lambda = \frac{m\omega^2}{k}$

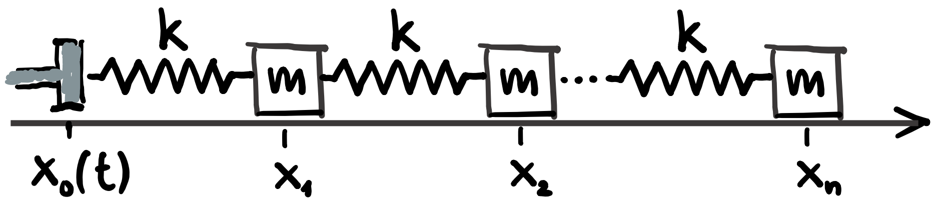
$$\Rightarrow \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_n \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} \tilde{x}_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$\Rightarrow$ displacement of the **last block**: $\tilde{x}_n = (\mathbf{M}^{-1})_{n,1}\,\tilde{x}_0$

$$(\mathbf{M}^{-1})_{n,1} = (-1)^{n+1}\frac{\mathrm{Det}(\mathbf{M}_{1,n})}{\mathrm{Det}(\mathbf{M})} = (-1)^{n+1}\frac{(-1)^{n-1}}{\mathrm{Det}(\mathbf{M})} = \frac{1}{\mathrm{Det}(\mathbf{M})}$$

$$\Rightarrow f_T(\omega) = \frac{1}{\mathrm{Det}(\mathbf{M})}$$

- A toy model of the oscillating filter



The transfer function from **input displacement** x_0 to **output displacement** x_n:

$$f_T(\omega) = \frac{1}{\mathrm{Det}\begin{pmatrix} 2-\lambda & -1 & & & 0 \\ -1 & 2-\lambda & -1 & & \\ & -1 & \ddots & -1 & \\ & & -1 & 2-\lambda & -1 \\ 0 & & & -1 & 1-\lambda \end{pmatrix}}, \quad \text{with } \lambda = \frac{m\omega^2}{k}$$

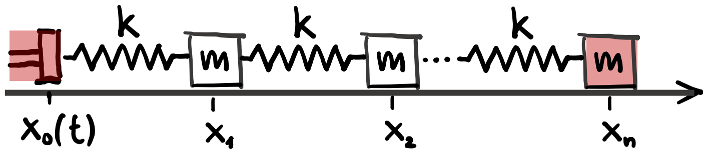
$$f_T(\omega) = \frac{1}{(-1)^n\lambda^n + c_{n-1}\lambda^{n-1} + \cdots + c_0} = \frac{1}{(-1)^n(\lambda-\lambda_1)(\lambda-\lambda_2)\dots(\lambda-\lambda_n)}$$

$$f_T(\omega) = \frac{[-\omega_0^2(1+i\phi)]^n}{[\omega^2-\lambda_1\omega_0^2(1+i\phi)]\,[\omega^2-\lambda_2\omega_0^2(1+i\phi)]\,\dots\,[\omega^2-\lambda_n\omega_0^2(1+i\phi)]}$$

$\lambda_1, \lambda_2, \dots \lambda_n$ are real numbers laying in the range (0,4).

For $\omega \gg 2\omega_0$, $$f_T(\omega) = [-(1+i\phi)]^n \left(\frac{\omega_0}{\omega}\right)^{2n}$$ (Low pass filter)

- A toy model of the oscillating filter

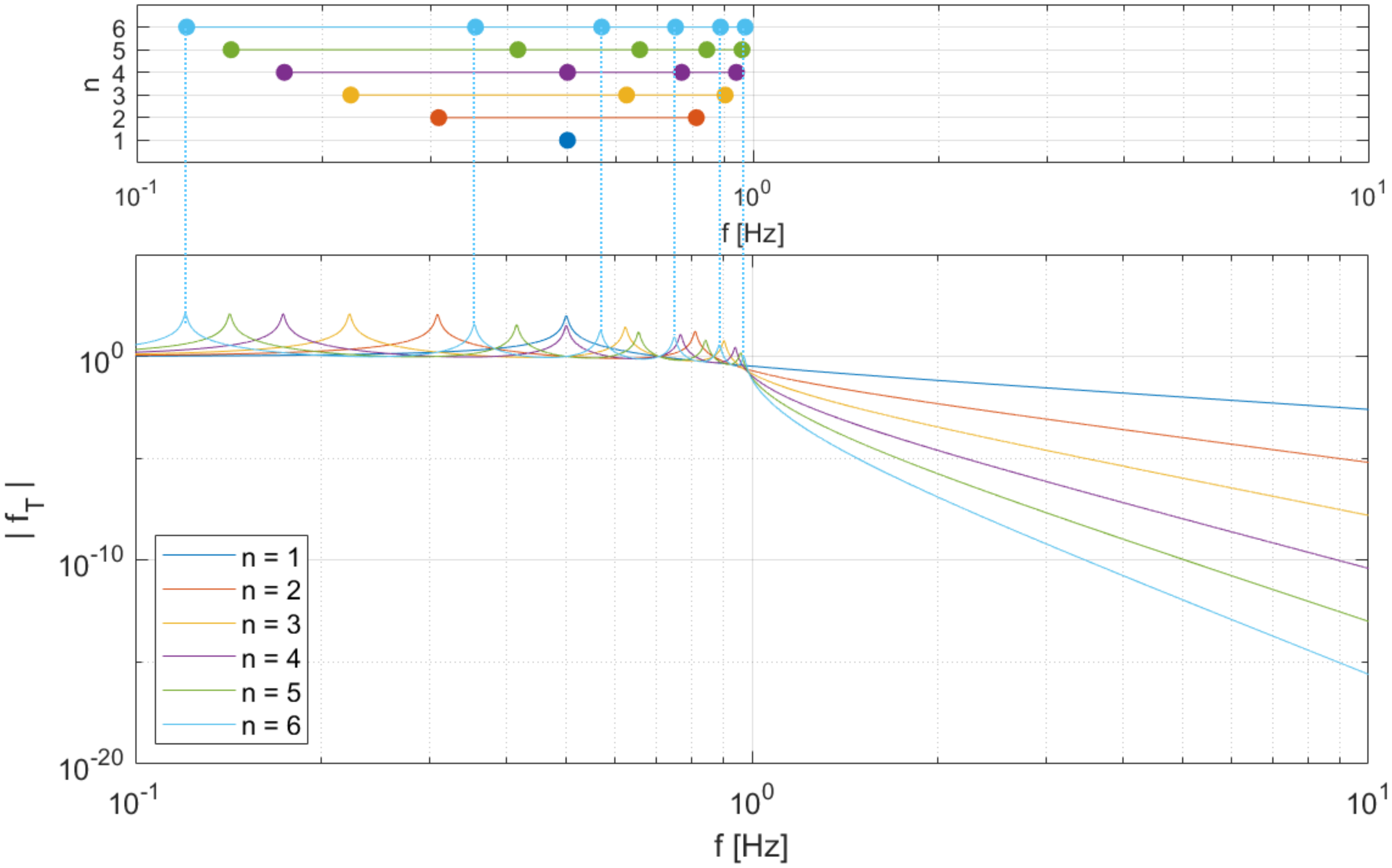


$k = k_0 \cdot (1 + i\,\phi)$

$f_0 = 0.5$ Hz
$\phi = 0.01$
Growing n

Low n start filtering at lower frequencies.
High n have better rejections.
Above $2f_0$ all configurations filter anyway.

The transfer function from **input displacement** x_0 to **output displacement** x_n



• Suspended oscillators

The linearization is left as an exercise.

Suggestion (work out the tension forces bottom-up):

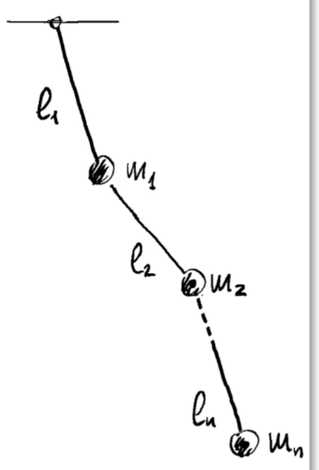
$$\vec{t}_n = [-p_n\theta_n,\ p_n]$$
$$\vec{t}_{n-1} = [-(p_{n-1} + t_n)\theta_{n-1},\ p_{n-1} + t_n]$$
…

where the angles amount to $\theta_i = \frac{x_i - x_{i-1}}{l_i}$.

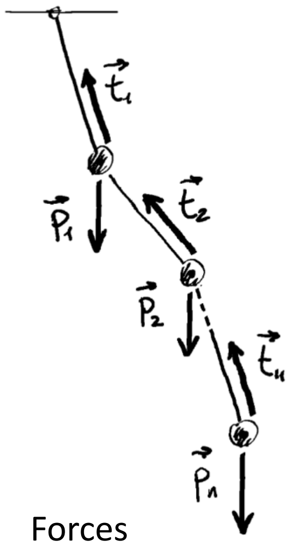
You'll get $\boxed{\tilde{y}_i = -(l_1 + l_2 + \dots l_i)}$ and $\boxed{\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\\ \vdots\\ \tilde{x}_n\end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix}\tilde{x}_0\\ 0\\ \vdots\\ 0\end{pmatrix}}$

with
$$\mathbf{M} = g\begin{pmatrix} \frac{1}{m_1}\left(\frac{M_1}{l_1}+\frac{M_2}{l_2}\right) & -\frac{M_2}{m_1 l_2} & & & 0 \\ -\frac{M_2}{m_2 l_2} & \frac{1}{m_2}\left(\frac{M_2}{l_2}+\frac{M_3}{l_3}\right) & -\frac{M_3}{m_2 l_3} & & \\ & & \ddots & & \\ & & -\frac{M_{n-1}}{m_{n-1} l_{n-1}} & \frac{1}{m_{n-1}}\left(\frac{M_{n-1}}{l_{n-1}}+\frac{M_n}{l_n}\right) & -\frac{M_n}{m_{n-1} l_n} \\ 0 & & & -\frac{M_n}{m_n l_n} & \frac{M_n}{m_n l_n} \end{pmatrix} - \omega^2\begin{pmatrix}1 & & & 0\\ & 1 & & \\ & & \ddots & \\ 0 & & & 1\end{pmatrix}$$

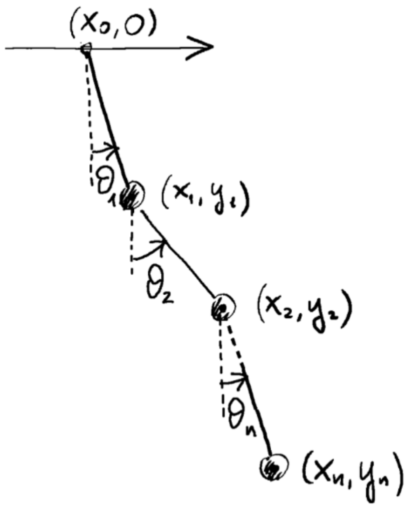
and $\boxed{M_i = m_i + m_{i+1} + \dots m_n}$ (the loads).



Parameters



Forces



Coordinates

Same tridiagonal structu[re]
and signs
as for the toy model.

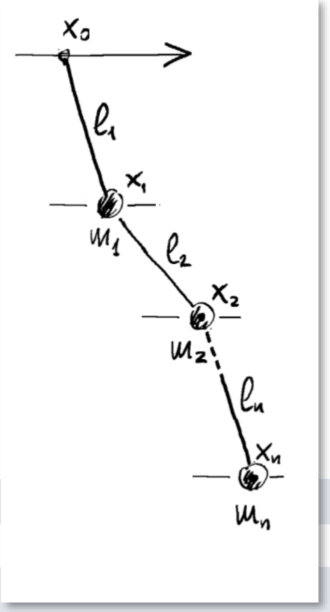
Different rows contain
different values.

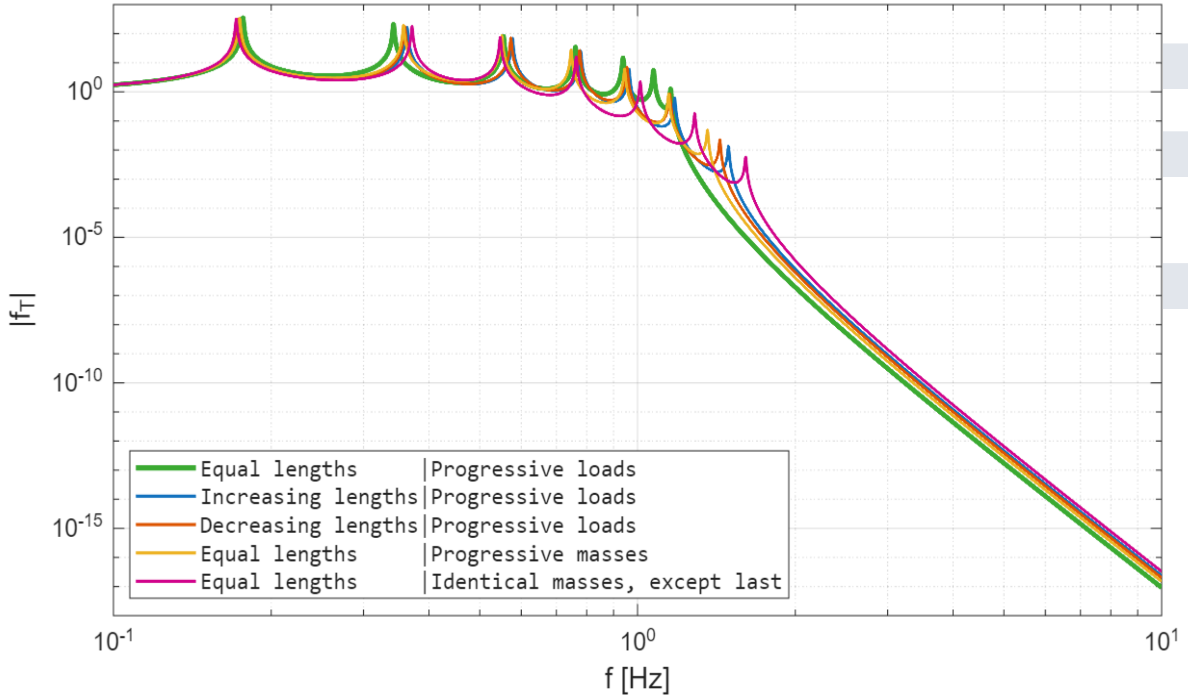
• Suspended oscillators

$$\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\\ \vdots\\ \tilde{x}_n\end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix}\tilde{x}_0\\ 0\\ \vdots\\ 0\end{pmatrix}$$

⇒ displacement of the **last mass**: $\tilde{x}_n = (\mathbf{M}^{-1})_{n,1}\,\tilde{x}_0$

⇒ transfer function: $f_T(\omega) = (\mathbf{M}^{-1})_{n,1}$





An example

$n = 7$	[7 masses, including test mass]
$l_1 + l_2 + \cdots + l_n = 12$ m	[total length = 12 m]
$M_1 = 12\, m_n$	[total load = 12 x test mass]

Within the upper constraints, we can implement, for instance,

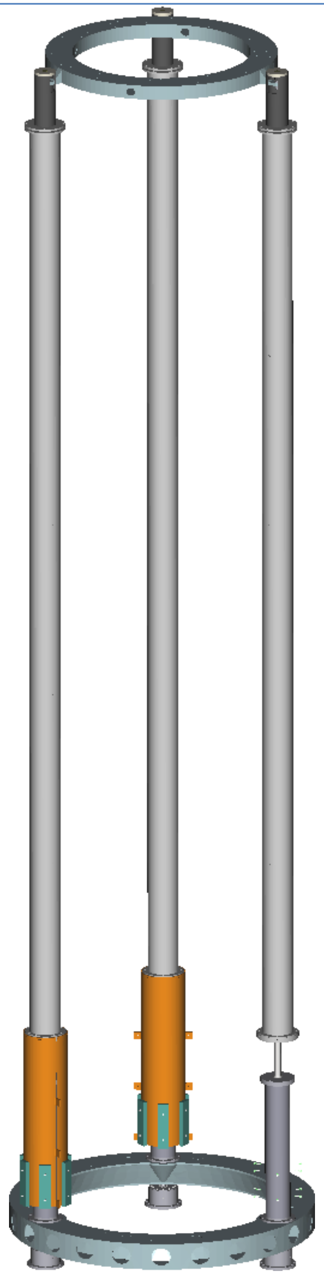
$l_i = l_{i-1}$	[Equal lengths]
$M_i = \frac{1}{\sqrt[7]{12}} M_{i-1}$	[Loads in geom. progression]

NOTE: the progression rule applies to loads, not to masses

When you try different distributions of LENGTHS
or different distribution of LOADS
you always get **higher f_T-s**, i.e. **worse filtering**.

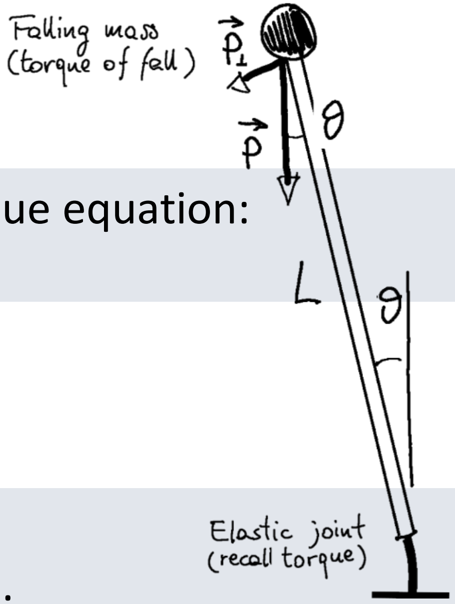
⇒ The perfect SA has **equal lengths** and **loads in progression**.

Find the explanation in: A.Bove et al. 1997 Europhys. Lett. **40**(6) 601.



- # The Inverted Pendulum

Three legs set on elastic joints.



If we ignore any other masses except the load M on top of the IP, we get the torque equation:

$$M\,L^2\ddot{\theta} \cong -\kappa\theta + MgL\,\theta$$

$$\Rightarrow\ \omega_0{}^2 = \frac{\kappa - MgL}{ML^2}\ \Rightarrow\ \omega_0 = \sqrt{\frac{\kappa}{ML^2} - \frac{g}{L}} = \sqrt{\frac{k}{M} - \frac{g}{L}}$$

The frequency is set to about $50\ \mathrm{mHz}$.
The Inverted Pendulum is tuned to a (much) smaller frequency than the SA filters.
It stops the frequencies that would resonate with the filters. ⇒ It acts as a PRE-ISOLATOR.

For the transfer function, we need a more accurate model (not explained here).
Top mass **horizontal displacements** depends on **displacements of the joint** by

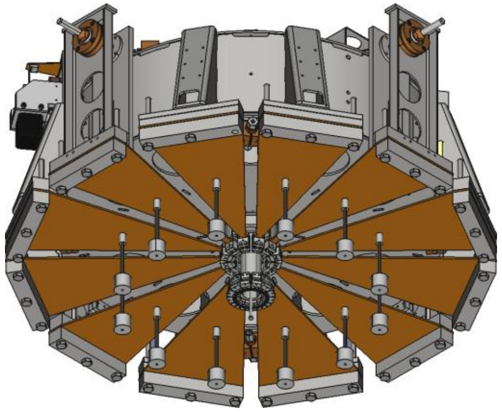
$$f_T = -\frac{\kappa - (m_{IP}\ l_{CM} + M\ L)\ g}{I_{IP}\ \omega^2 - [\kappa - (m_{IP}\,l_{CM} + M\ L)\ g]}$$

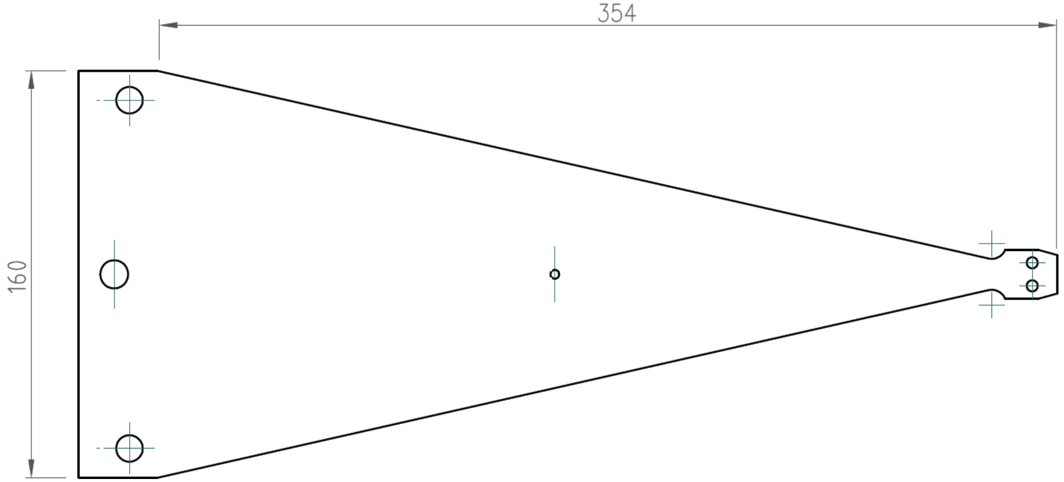
for a **tuned IP** with $I_{IP} = m_{IP}\,l_{CM}L$ (I_{IP} referred to the joint) [same algebraic shape as previous f_T -s].

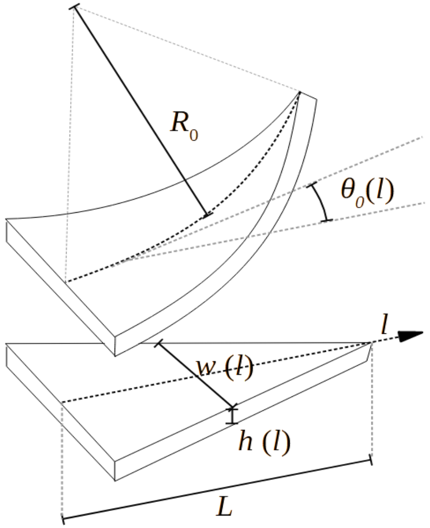
The orange bells are meant to tune the IP momentum of inertia.

G.Losurdo et al. 1999 Rev. Sci. Instrum. **70**(5) 2508

• Elastic blades







SA blades are **curved at rest** and designed to be flattened by a load suspended from their tip.

They are made in **maraging steel** (maraging 250), a low carbon Fe alloy
Ni (18%), Co (8%), Mo (5%),
Ti (0.5%), C (≤0.03%) + Fe

Relevant parameters

$E = 187$ GPa (at 20°C)

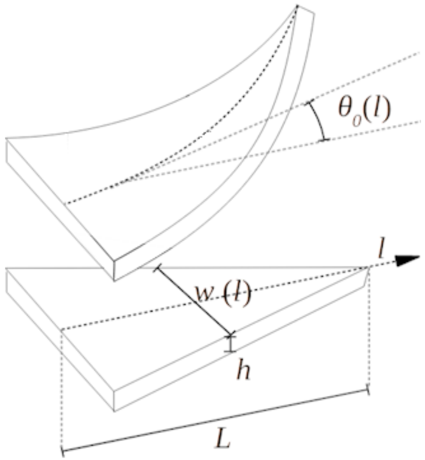
$UTS = 1.85$ GPa

$\phi = 3 \cdot 10^{-5}$

S.Braccini et al 2000 *Meas. Sci. Technol.* **11** 467

• Blade mechanics

G.Cella, Personal communication - Write me for a copy



- $U = \frac{1}{2}\int_0^L E\,\frac{w(l)h^3}{12}\left(\frac{d\theta}{dl} - \frac{d\theta_0}{dl}\right)^2 dl + F\int_0^L \sin\theta\, dl$

valid for a **very thin blade** with **any cut profile $w(l)$** and **any rest bending $\theta_0(l)$**

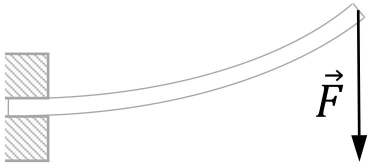
By minimizing this energy with respect to the possible profiles $\theta(l)$ we get a **condition** on the rest bending $\theta_0(l)$ for the blade **to lay flat** under the load $\bar{F}$

$$\frac{d\theta_0}{dl} = \frac{12\bar{F}}{Eh^3}\frac{(L-l)}{w(l)}$$

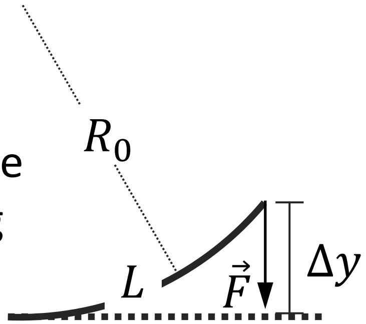
We discover that a **triangular profile** $w(l) = w_0\frac{(L-l)}{L}$ have a constant $\frac{d\theta_0}{dl}$.

This means an arc of circle profile with radius

$$R_0 = E\frac{w_0 h^3}{12L\bar{F}}$$



The **spring constant** k can be retrieved from the flattening effect of the load

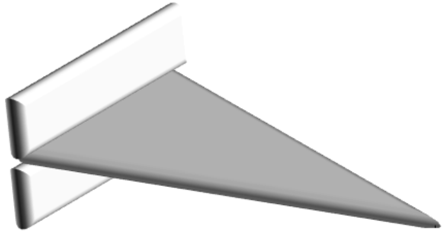


$$\Delta y = R_0\left(1 - \cos\frac{L}{R_0}\right) \cong R_0\frac{1}{2}\left(\frac{L}{R_0}\right)^2 = \frac{L^2}{2R_0}$$

$$k = \bar{F}/\Delta y \qquad \Rightarrow \quad k = E\frac{w_0 h^3}{12LR_0}\frac{2R_0}{L^2}$$

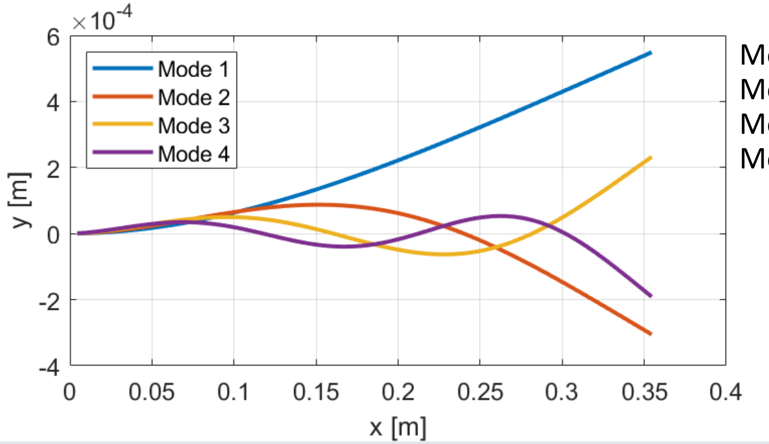
$$k = E\frac{w_0 h^3}{6L^3}$$

- Internal modes

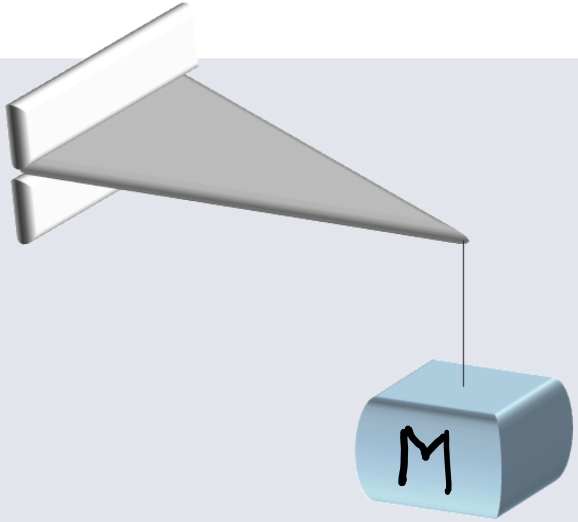


Flat unloaded triangular blade

$E = 187$ GPa
$\rho = 8000$ kg/m^3
$L = 354$ mm
$w_0 = 110$ mm
$h = 3.5$ mm
$R_0 = \infty$
$F = 0$

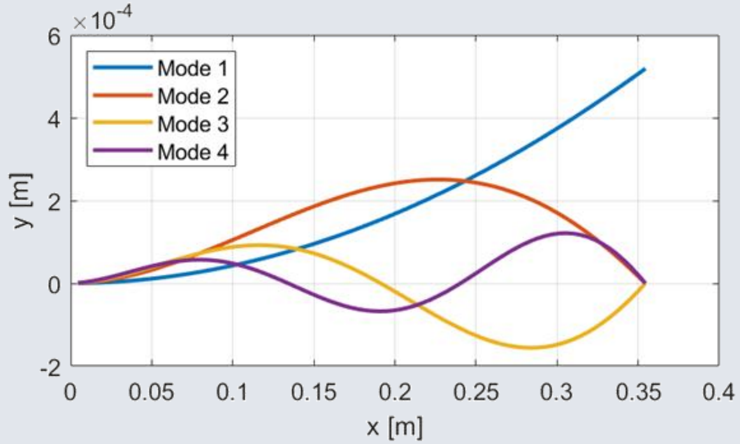


Mode 1 42 Hz
Mode 2 185 Hz
Mode 3 450 Hz
Mode 4 832 Hz



Pre-bent triangular blade
with adjusted (i.e. flattening) load

$E = 187$ GPa
$\rho = 8000$ kg/m^3
$L = 354$ mm
$w_0 = 110$ mm
$h = 3.5$ mm
$R_0 = 450$ mm
$F = 46.7\ \mathrm{kg} \cdot g = 458$ N



Mode 1 1.35 Hz
Mode 2 101 Hz
Mode 3 311 Hz
Mode 4 642 Hz

$\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

• Triangular blade summary

Preset curvature radius for a load $\bar{F}$

$$R_0 = E\frac{w_0 h^3}{12\, L\, \bar{F}}$$

Single blade spring constant

$$k = E\frac{w_0 h^3}{6L^3}$$

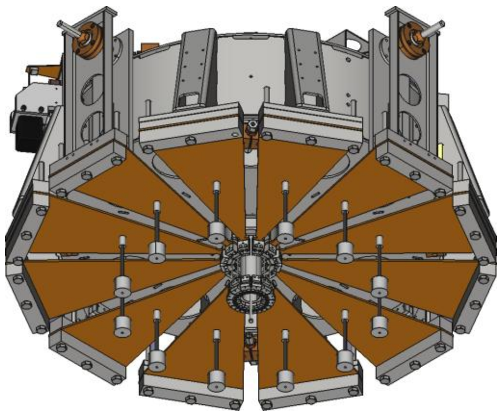
Single blade resonant frequency (1st)

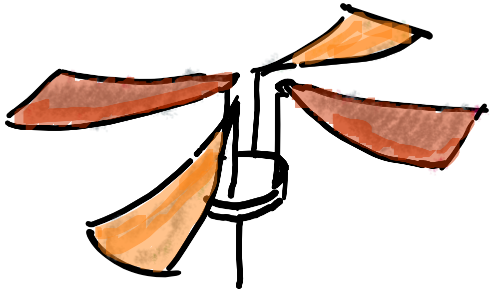
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

Surface stress of the flattened blade

NEW

$$s = \frac{Eh}{2R_0}$$





Filter summary

$$\bar{F}_{tot} = \sum_i \bar{F}_i$$

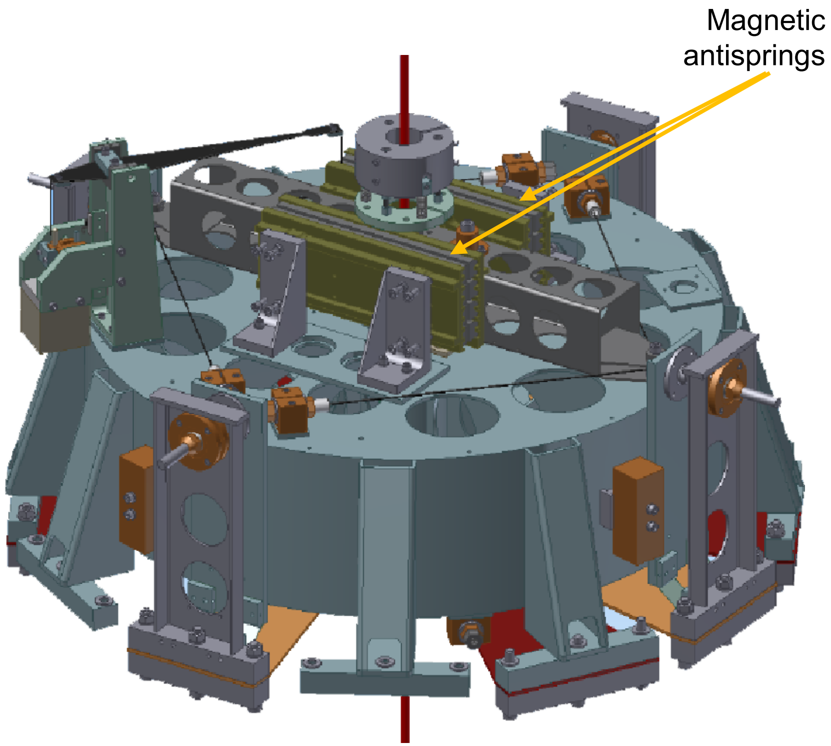
$$k_{tot} = \sum_i k_i$$

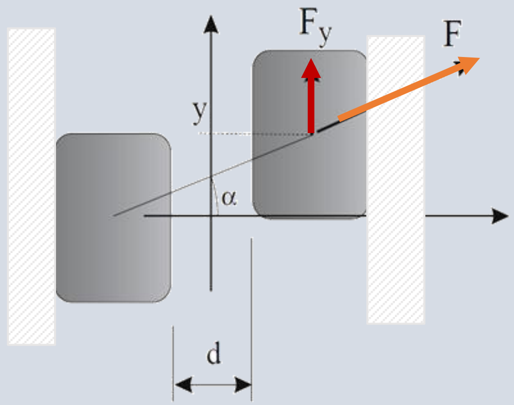
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_{tot}}{M_{tot}}}$$

$$s_i < 0.5\, UTS \;\; \forall i$$

Blades work **in symmetric couples** of identical specimens.
Couples can differ from each other.
If the **total load** of the filter is the **sum of the individual flattening loads** all blades work flat.

- Magnetic Anti-Springs





MAS working principle

$$F_y = -(k_{as}y + a_{as}y^3)$$

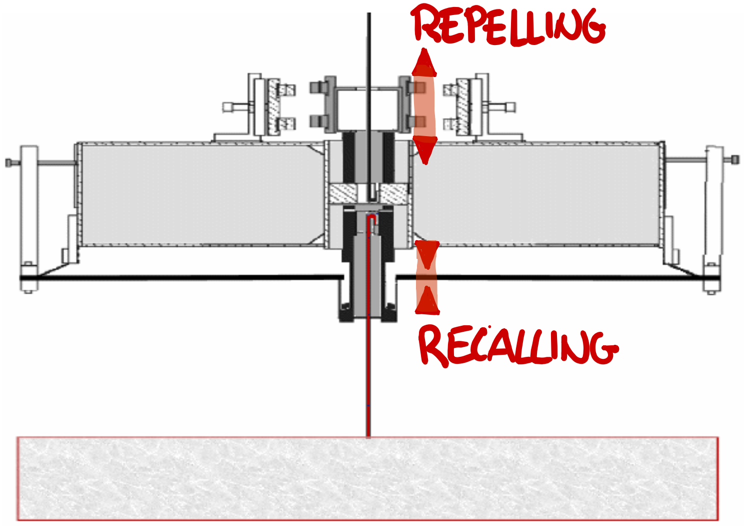
with

$$k_{as} < 0$$
$$a_{as} > 0$$

(unavoidable nonlinearity)

G. Curci et al, Effects of non-linearities..., https://tds.virgo-gw.eu/?r=7616

Two forces are exerted on the central column:



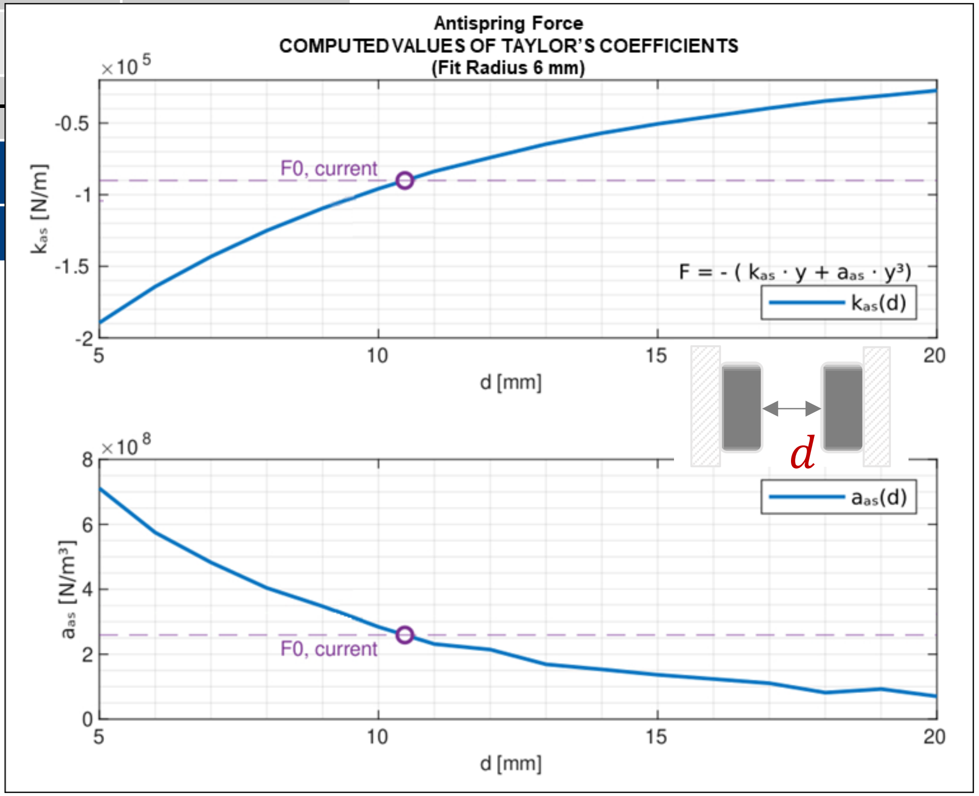
$$F_{tot} = -(k_{el} + k_{as})\, y + O_3(y)$$

stable as long as it is $k_{el} + k_{as} > 0$

- Sum of k-s

Current configuration		Filter 0	Filter 1	Filter 2	Filter 3	Filter 4	Filter 7
M	Suspended mass [kg]	1057	884	719	579	461	146
k_{el}	Elastic stiffness [N/m]	93863	78496	63840	51404		
k_{as}	Antispring stiffness [N/m]	- 90108	- 75356	- 61286	- 49348	-	
a_{as}	Nonlinearity [10^8 N/m^3]	2.6	2.3	1.8	1.4		
d	Antispring tuning [mm]	10.5	8.5	10.5	12		

Some figures in Virgo



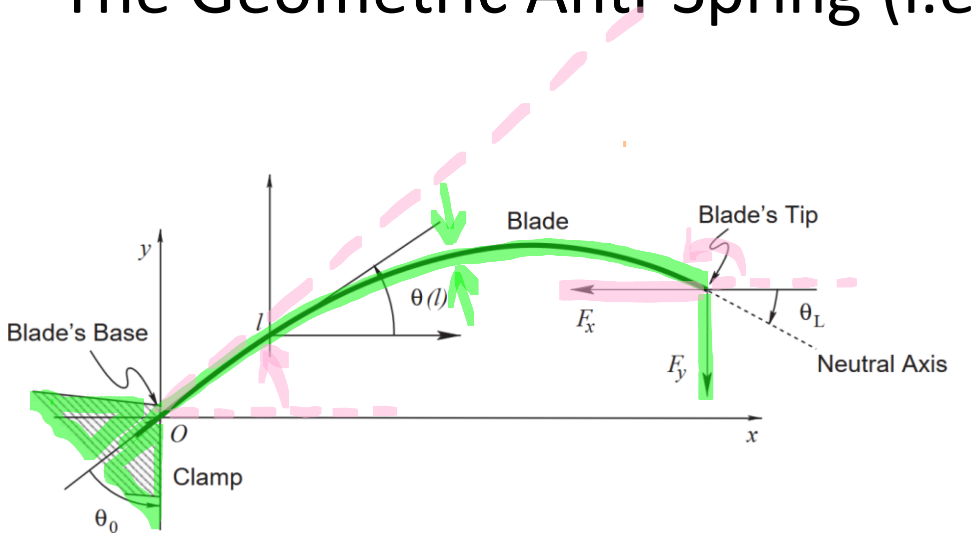
Filter 0
Magnetic
Anti-Spring
simulation

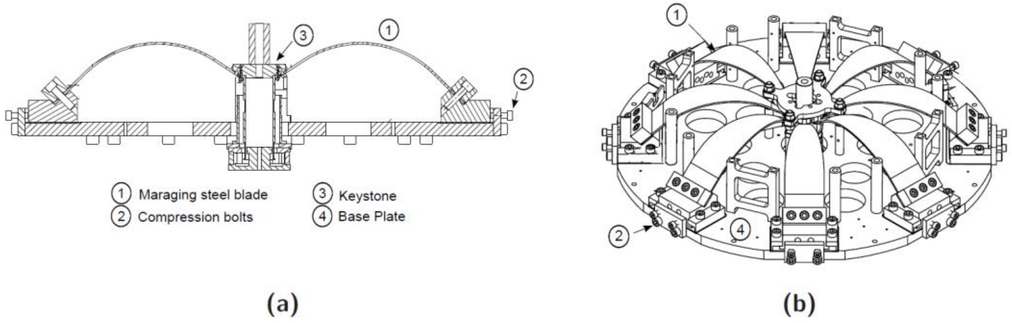
For **Filter 0**, $k = (93863 - 90108)\,\frac{\mathrm{N}}{\mathrm{m}} = 3755\,\frac{\mathrm{N}}{\mathrm{m}}$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{3755\frac{N}{m}}{1057\ \mathrm{kg}}} = 0.3\ \mathrm{Hz}$$

Without MAS $$f_{el} = \frac{1}{2\pi}\sqrt{\frac{93863\,\frac{N}{m}}{1057\ \mathrm{kg}}} = 1.5\ \mathrm{Hz}$$

• The Geometric Anti-Spring (i.e. a tunable blade)





(a) (b)

G.Cella et al. 2005 Nucl. Instr. and Meth. A **540** 502
M.R. Blom et al. 2015 Physics Procedia **61** 641
(non-exhaustive list!)

Energy of a compressed blade

- $U = \frac{1}{2}\int_0^L E\,\frac{w(l)h^3}{12}\left(\frac{d\theta}{dl}\right)^2 dl - F_y\int_0^L \sin\theta\, dl - F_x\int_0^L \cos\theta\, dl$

Let's compare this to slide 19 (*Blade mechanics*)

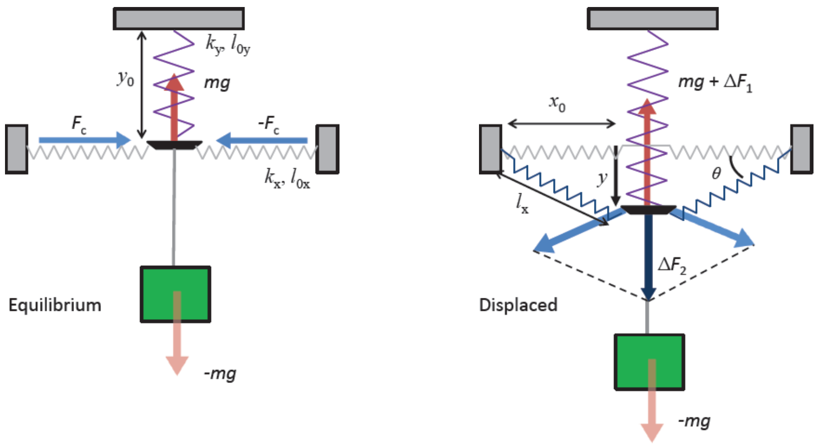
Similarities:

- Metal blade
- Constrained base angle
- Vertical load
- Constant and small thickness

Differences:

- No pre-curvature
- Different base angle ($\theta_O > 0$ instead of $\theta_O = 0$)
- Constrained tip angle ($\theta_L < 0$)
- Horizontal compression (Constrained length)

• The Geometric Anti-Spring



(a) (b)

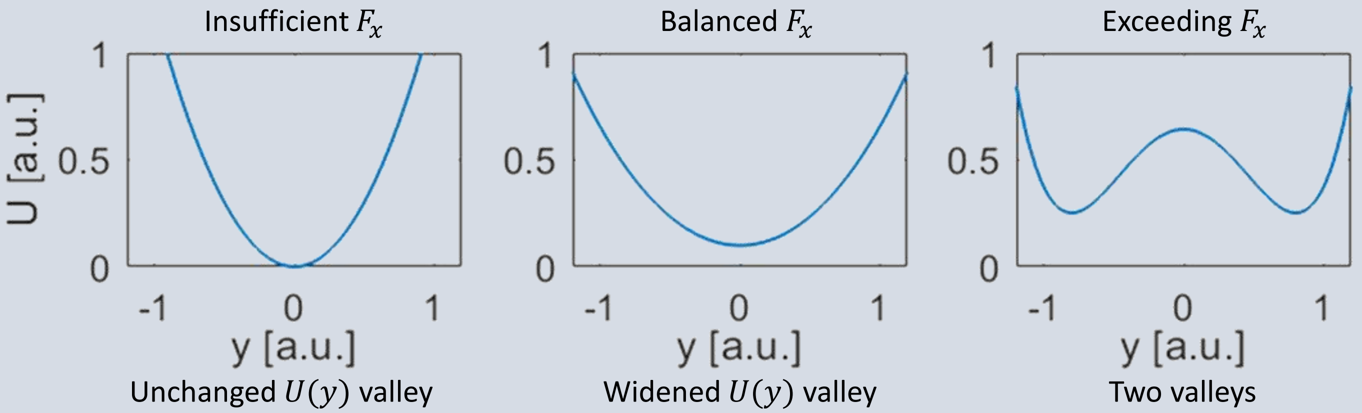
Note: if **the repelling modulus exceeds the recalling one**, the compressed springs win and expand themselves until a stable equilibrium point. The system is **bistable**.

Intuitive model (not for real life design)

(a) A vertical spring is in equilibrium with the weight of a load. Two horizontal **counteracting compressed springs** are in an **unstable equilibrium**.

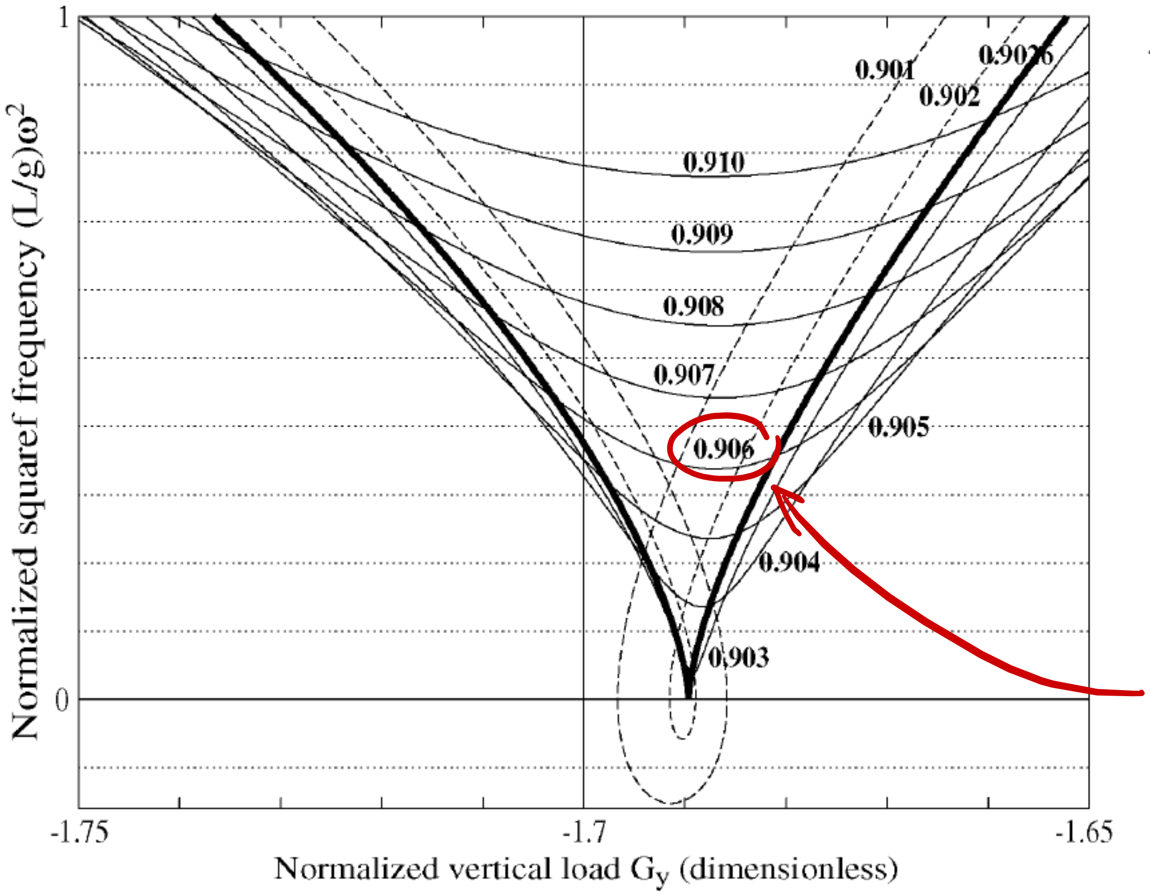
(b) When the load leaves the equilibrium position, the vertical spring exerts a recall, while the vertical component of the compressed spring force become an expelling force.

Less intuitive (and still insufficient) **model**

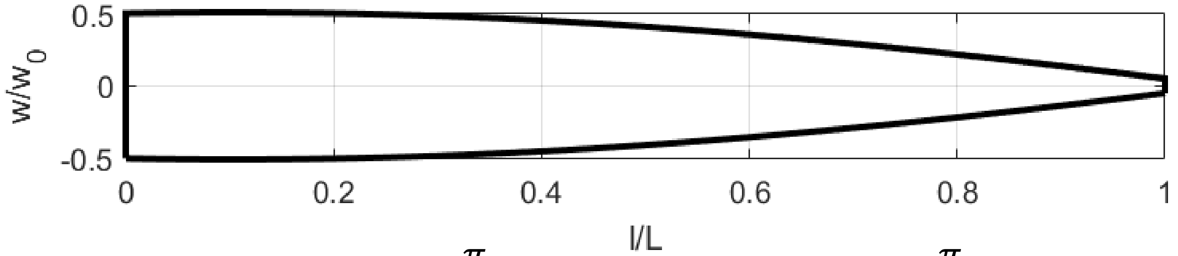


• GAS how-to

G.Cella et al. 2005 Nucl. Instr. and Meth. A **540** 502



Computed general solution for a preset blade shape (TAMA)

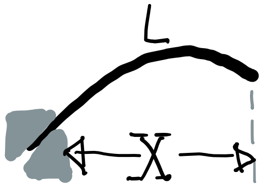


with base angle $\theta_O = \frac{\pi}{4}$ and tip angle $\theta_L = -\frac{\pi}{6}$.

Dimensionless variables

$$G_y = 12\frac{L^2}{Ew_0h^3}F_y \text{ (load)}$$

$$x = X/L \text{ (horizontal constrain)}$$



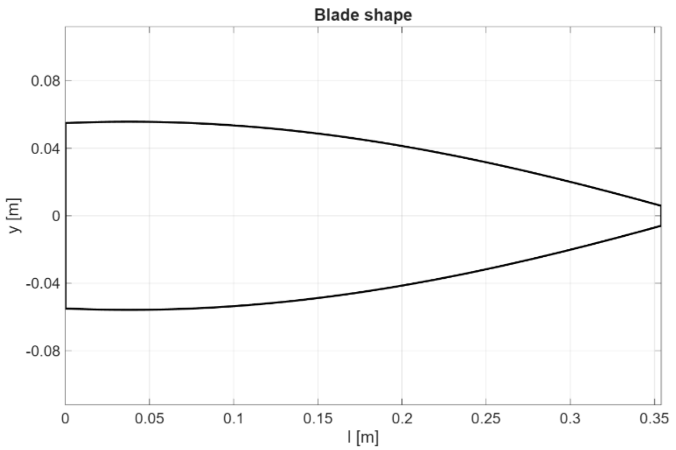
Legend

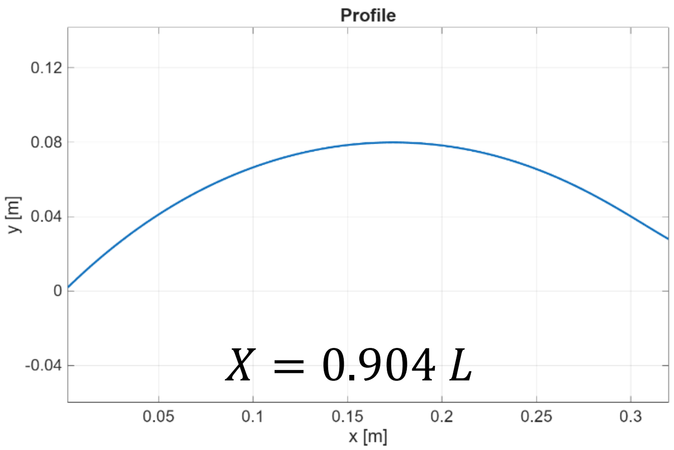
Curves in full lines belong to stable states ($x > 0.9026$), dashed to bistable ($x < 0.9026$)

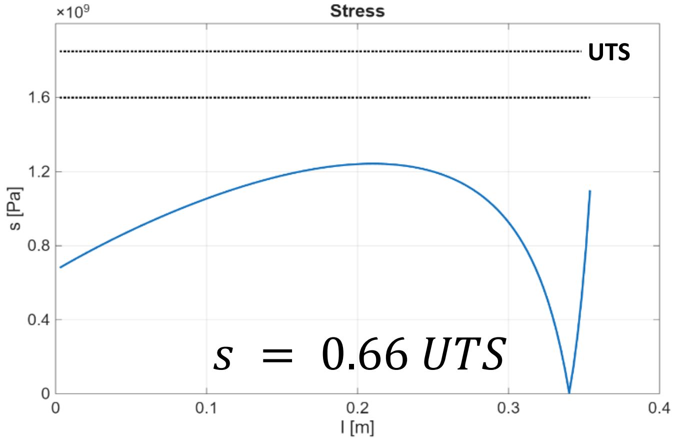
Stable states have minimum frequency for $G_y \cong 1.69$.

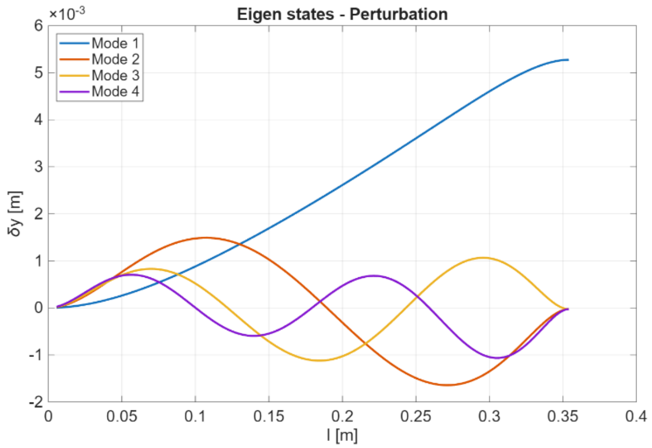
- Some (numerical) outcomes

Simulated GAS
TAMA shape
$L = 354$ mm
$w_0 = 110$ mm
$h = 2.74$ mm
$E = 187$ GPa
$UTS = 1.85$ GPa
$M = 48.4$ kg







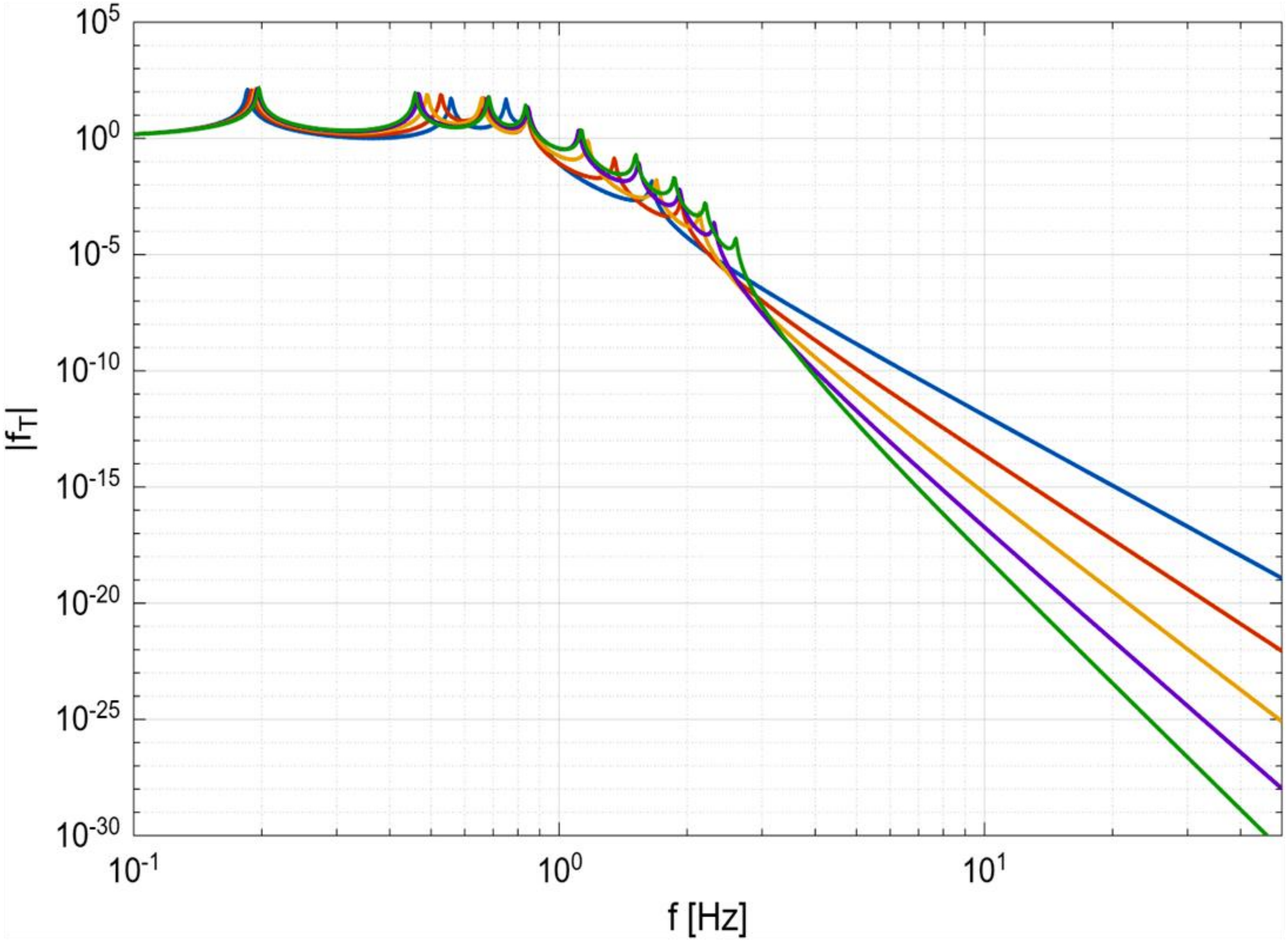


$\Rightarrow f = 0.38$ Hz

Same blade without GAS constrains

$\Rightarrow f = 1.10$ Hz

- And now, some problems
  - Problem #1 (easy one)



A SuperAttenuator with total length $9\ \mathrm{m}$ have been configurated with 5, 6, 7, 8 or 9 suspended masses (including MAR and MIR).

The figure represents the absolute value of the transfer function from suspension point to mirror.

Assign
- A **number of suspended masses** to each plot.

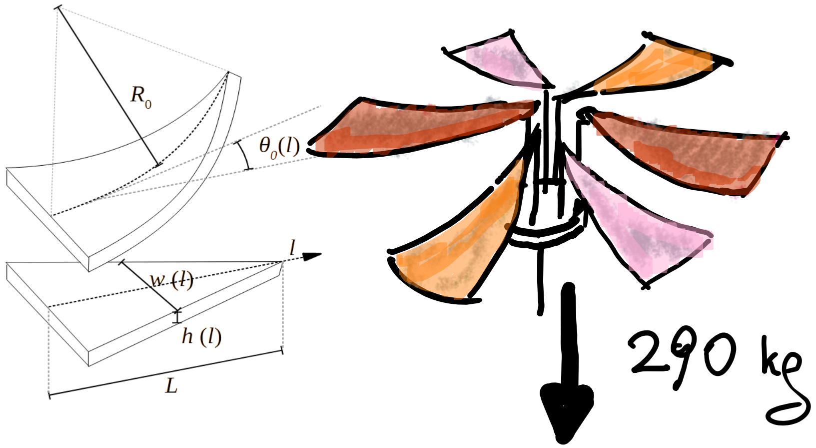
Choose
- The simplest configuration with **rejection factor better than 10^{14} from 10 Hz**

• Problem #2

Set up the maraging 250 blades of a Virgo-like lowest filter.

$L = 354$ mm
$w_0 = 110$ mm
$h = 3.5$ mm
6 blades

$M = 290$ kg



Find
- correct rest **curvature R_0**,
- **frequency** (assume blade mass << M),
- **stress**.

Compare stress with **UTS**.

Look around for necessary equations and data!

- **Triangular blade summary**

Preset curvature radius for a load $\bar{F}$	$R_0 = E \frac{w_0 h^3}{12\, L\, \bar{F}}$
Single blade spring constant	$k = E \frac{w_0 h^3}{6L^3}$
Single blade resonant frequency (1st)	$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$
Surface stress of the flattened blade	$s = \frac{Eh}{2R_0}$

Filter summary

$$\bar{F}_{tot} = \sum_i \bar{F}_i$$

$$k_{tot} = \sum_i k_i$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_{tot}}{M_{tot}}}$$

$$s_i < 0.5\, UTS \;\; \forall i$$

(applied in Virgo Superattenuator filters)

Maraging 250 figures

$$E = 187 \text{ GPa}$$

(at 20°C)

$$UTS = 1.85 \text{ GPa}$$

$$\phi = 3 \cdot 10^{-5}$$

GAS variables

Normalized load	$G_y = 12 \frac{L^2}{E w_0 h^3} F_y$	Normalized frequency	$\Omega = \sqrt{\frac{L}{g}\omega^2}$
Normalized horizontal constrain	$x = X/L$		

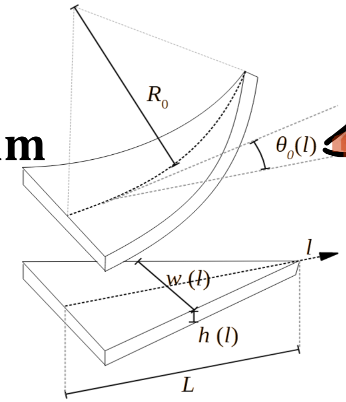
• Problem #3

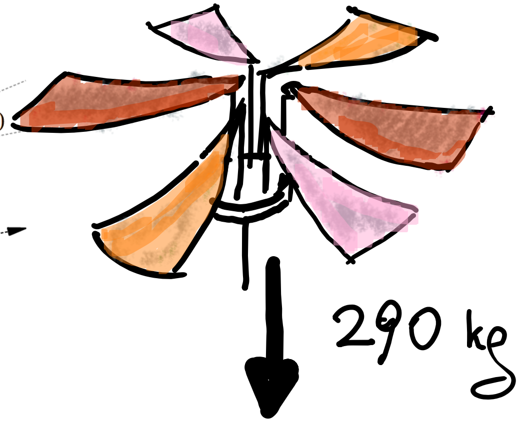
Choose between two possible blade bases w_0 and set thickness h to get the same f as in Problem #1.
Which base is the best solution?

$L = 354$ mm
$w_0 = \mathbf{55\ mm}$ or $\mathbf{110\ mm}$
6 blades

$M = 290$ kg

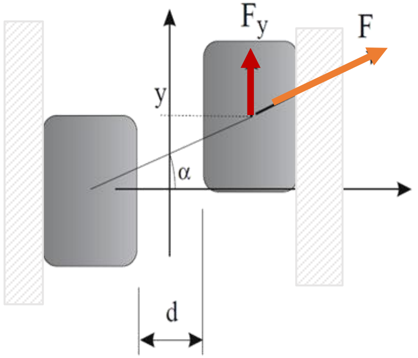
$[f = 1.318$ Hz$]$





Target
- find correct **curvature R_0** for both bases,
- find correct **thickness h** for both bases,
- find **stress** for both bases,
- choose the **best solution**.

- Problem #4



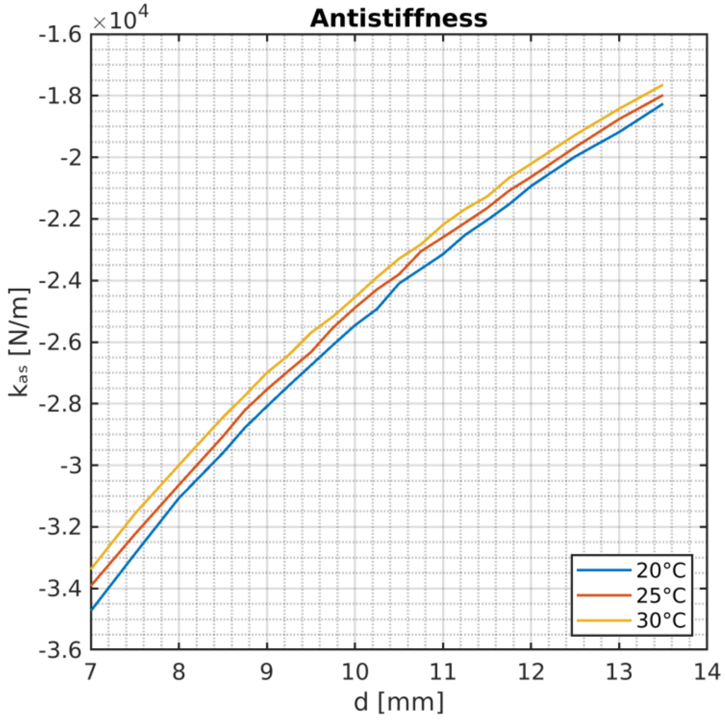
Add ferrite Magnetic Anti-Springs to the same Vigo filter as in Problems #1 and 2.
Tune the MAS to the design frequency f, by choosing the correct distance d.

$M = 290\ \mathrm{kg}$

$f_{in} = 1.5\ \mathrm{Hz}$ * (before MAS)

* Filter frequency is usually higher than the pure blade frequency

$f = \mathbf{0.50\ Hz}$



Negative stiffness of the installed MAS at some different temperatures

Find
- the correct **distance** $\boldsymbol{d}$.

Estimate the **frequency variation** with 5°C temperature increase.

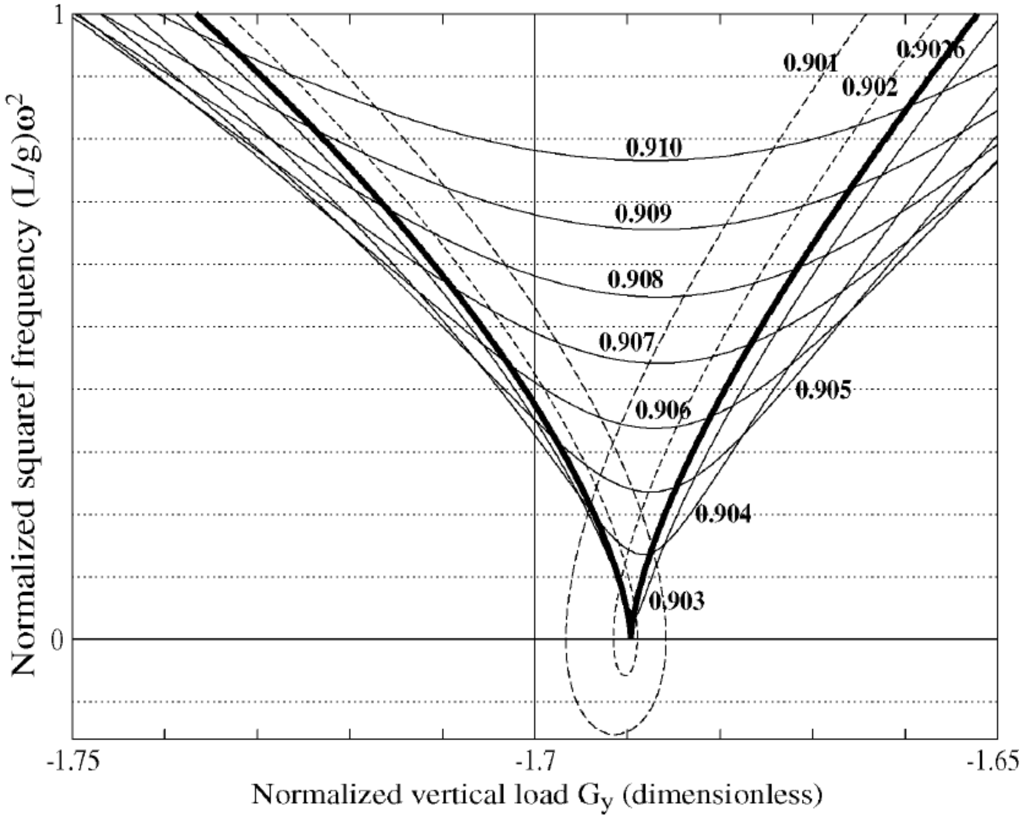
• Problem #5

Let's switch to Geometric Anti-Springs.
Provide the same filter of the previous problems with GAS vertical oscillators instead of blades+MAS.
Tune the filter to the design frequency f.

TAMA shape
$L = 354\ \mathrm{mm}$
$w_0 = 110\ \mathrm{mm}$
6 blades

$M = 290\ \mathrm{kg}$

$f = \mathbf{0.50\ Hz}$



Find

- the correct **thickness h** that matches the optimum condition $G_y = -1.69$.
- the correct **horizontal compression factor x** that tunes the filter.

- Solutions

Problem #1	Blu: 5 masses, Red: 6 masses, Orange: 7 masses Purple: 8 masses, Green: 8 masses The simplest configuration with asked rejection from 10 Hz is the 7 masses configuration
Problem #2	$R_0 = 0.438\text{ m},\ f = 1.318\text{ Hz},\ s = 0.404\,UTS$
Problem #3	$R_0 = 0.438\text{ m}$ both, 110 mm base: same h and s as in Problem #1 55 mm base: $h = 4.4\text{ mm},\ s = 0.508\,UTS$ (discarded!)
Problem #4	$d = 10.8\text{ mm},\ \Delta f = +0.04\text{ Hz}$
Problem #5	$h = 2.74\text{ mm},\ x = 0.906$