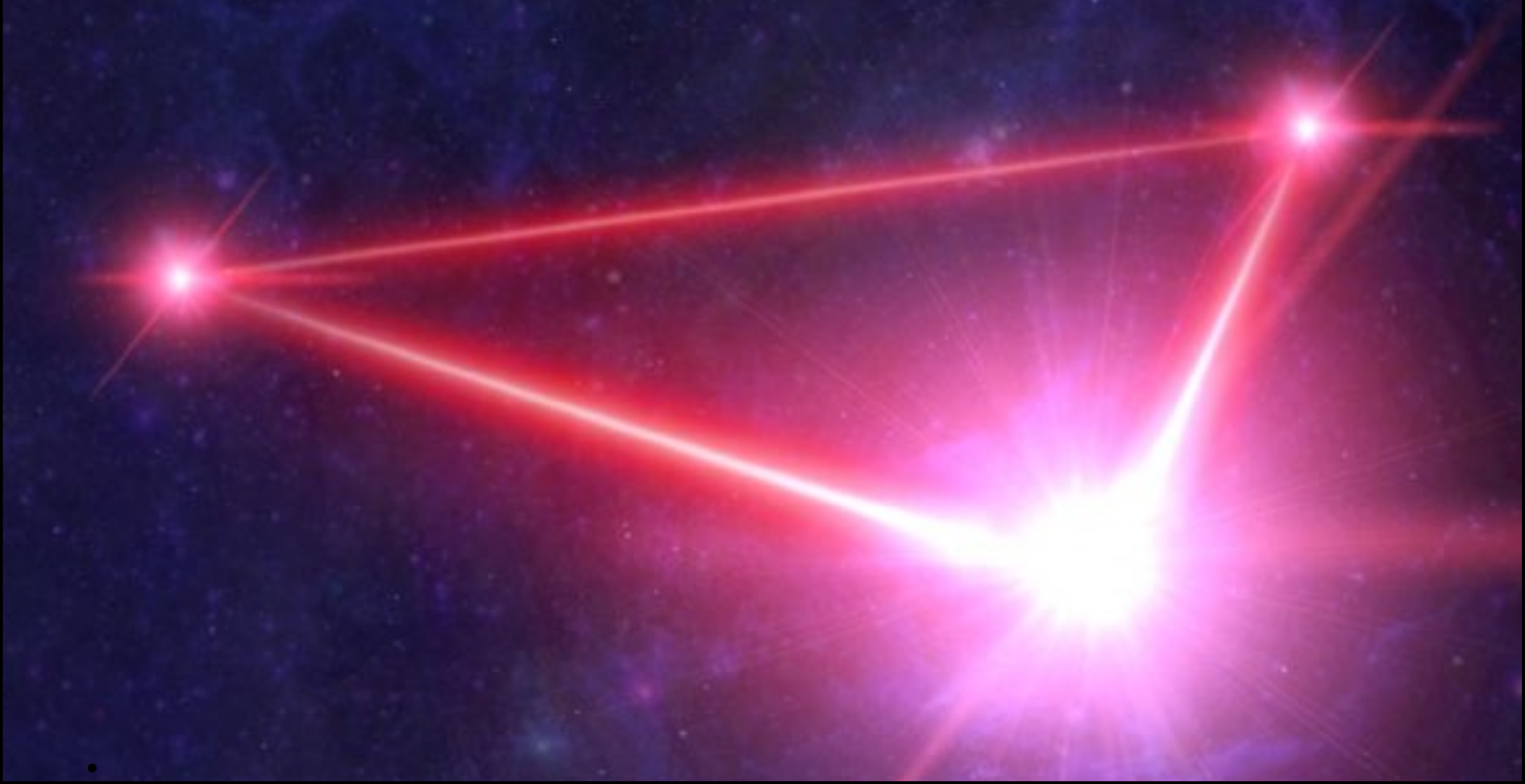


Space Interferometry: Deriving key principles of the LISA interferometry concepts (in one afternoon).

Lecture #10 (Gudrun Wanner*) at the PhD International School on Technologies in Gravitational Waves Detection 2026 (22.05.2026)

* Institute for Gravitational Physics of the Leibniz Universität Hannover

* Max Planck Institute for Gravitational Physics (Albert Einstein Institute)

https://www.lisamission.org/ https://www.quantumfrontiers.de/en/

A fairytale…

… a story…

… not a textbook.

→ Don't look for precise numbers

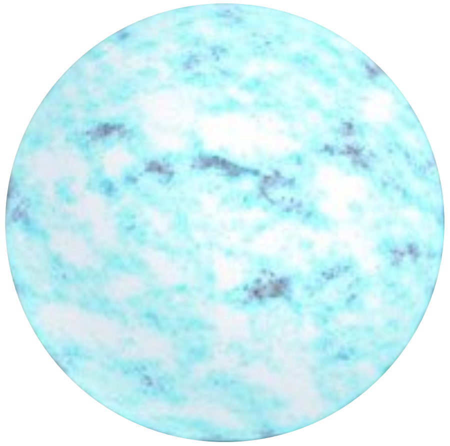
→ don't care for factors of 2

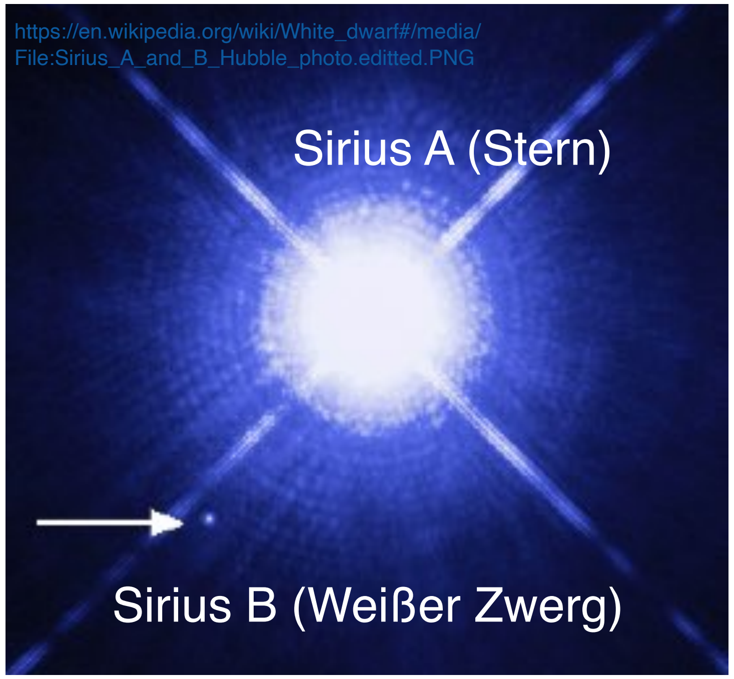
→ don't care if a sketch cannot be implemented as shown

→ simply don't care for details,

but care for the bigger picture

Black Holes	Neutron Stars	White Dwarfs
Radius ~3 km/$M_\odot$	Radius ~10-14 km	Radius ~7000 - 14000 km
	mass ~1-3 $M_\odot$	mass <1,44 $M_\odot$





($M_\odot$ = Sonnenmasse)

Ann. Phys. (Berlin) **529**, No. 1–2, 1600209 (2017) / DOI 10.1002/andp.201600209

annalen der physik

Original Paper

The basic physics of the binary black hole merger GW150914

LIGO Scientific and VIRGO Collaborations[*,**]

Received 5 August 2016, revised 21 September 2016, accepted 22 September 2016
Published online 4 October 2016

Ann. Phys. (Berlin) **529**, No. 1–2 (2017)

annalen der physik

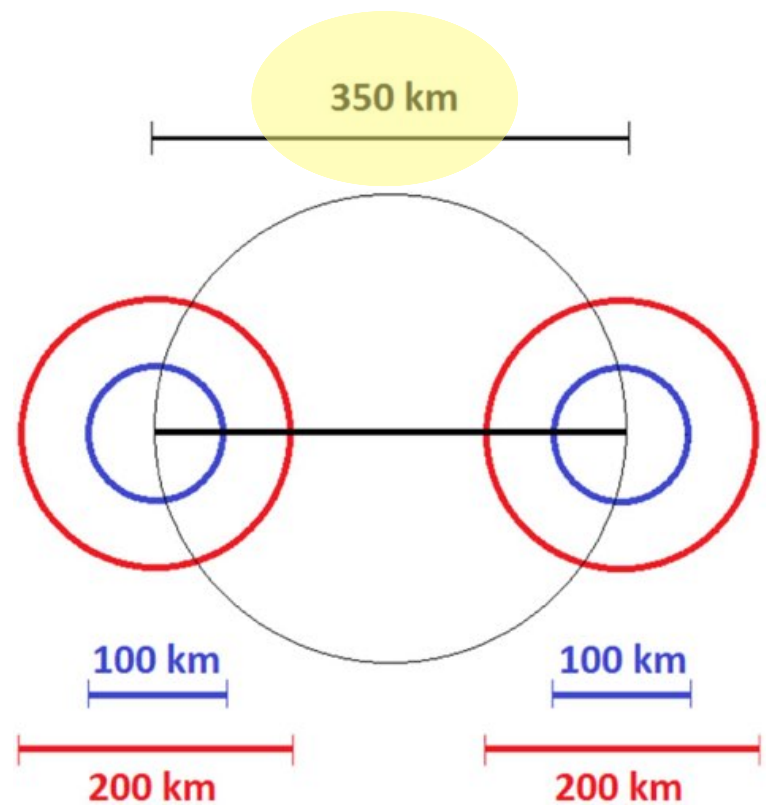
Original Paper

3 Evidence for compactness in the simplest case

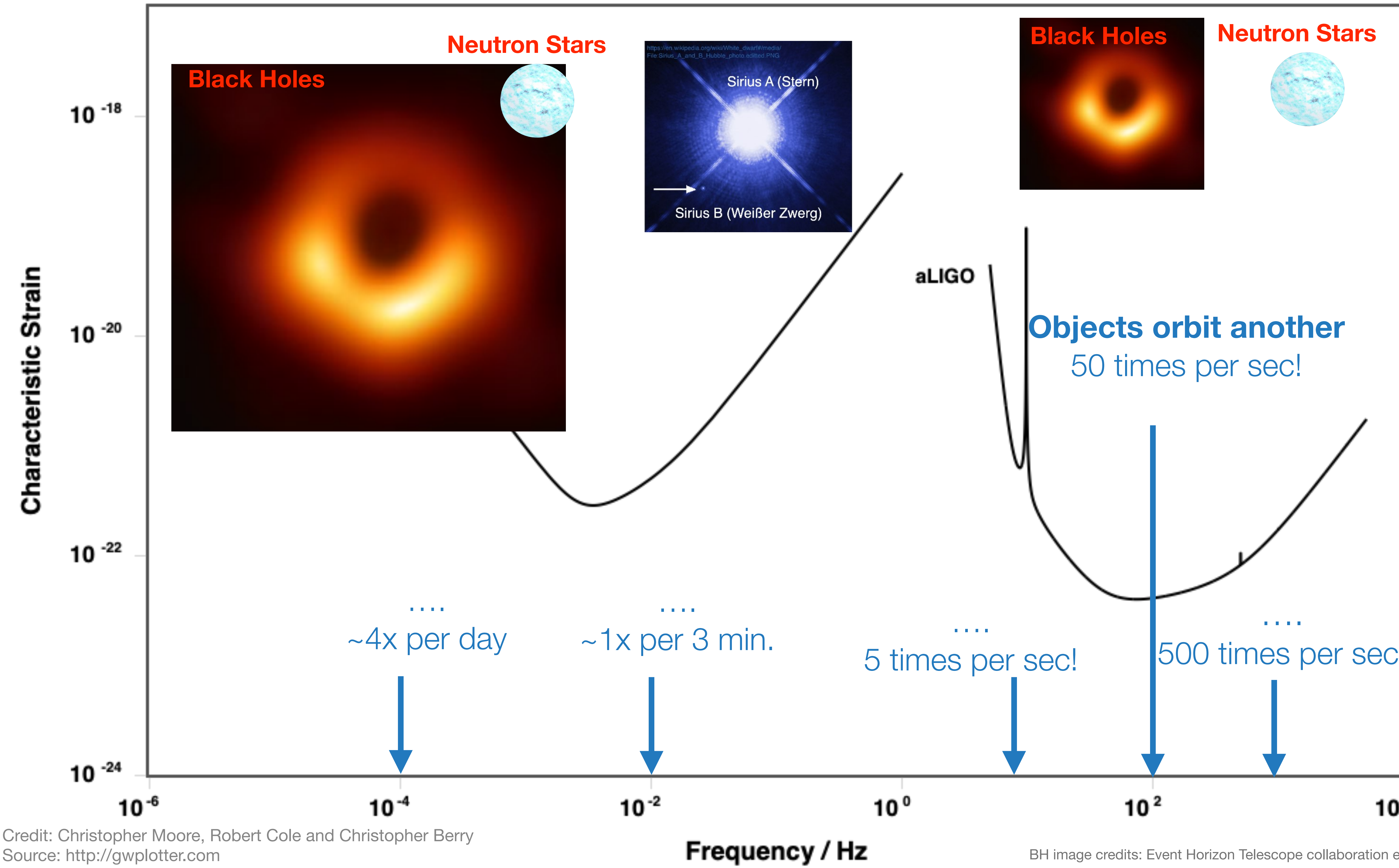
For simplicity, suppose that the two bodies have equal masses, $m_1 = m_2$. The value of the chirp mass then implies that $m_1 = m_2 = 2^{1/5}\mathcal{M} = 35\,\mathrm{M}_\odot$, so that the total mass would be $M = m_1 + m_2 = 70\,\mathrm{M}_\odot$. We also assume for now that the objects are not spinning, and that their orbits remain Keplerian and essentially circular until the point of peak amplitude.

Around the time of peak amplitude the bodies therefore had an orbital separation R given by

$$R = \left(\frac{GM}{\omega^2_{\mathrm{Kep}}\big|_{\max}} \right)^{1/3} = 350\,\mathrm{km}. \qquad (9)$$



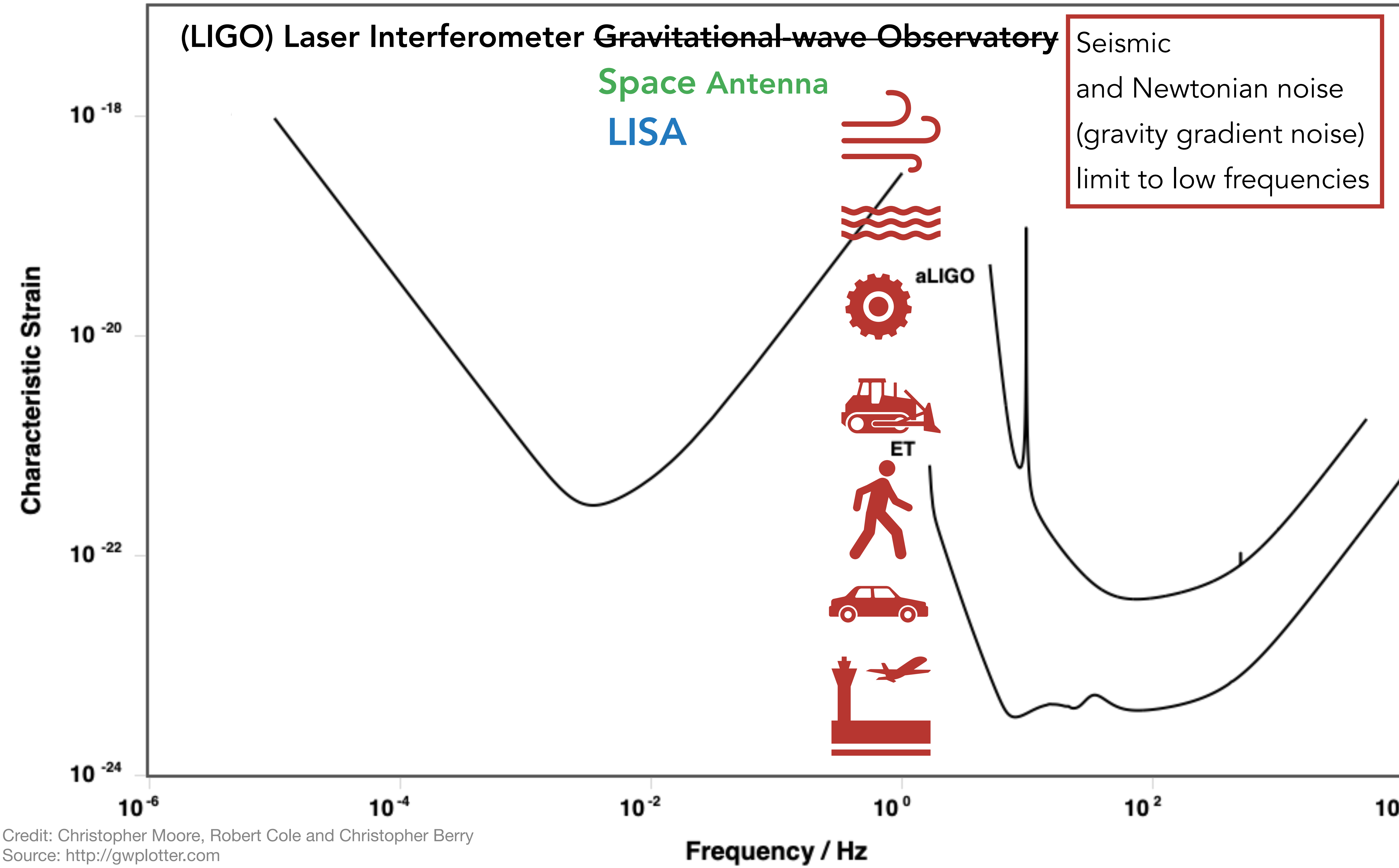
LISA vs ground-based detectors



Credit: Christopher Moore, Robert Cole and Christopher Berry
Source: http://gwplotter.com

BH image credits: Event Horizon Telescope collaboration et al.

LISA vs ground-based detectors



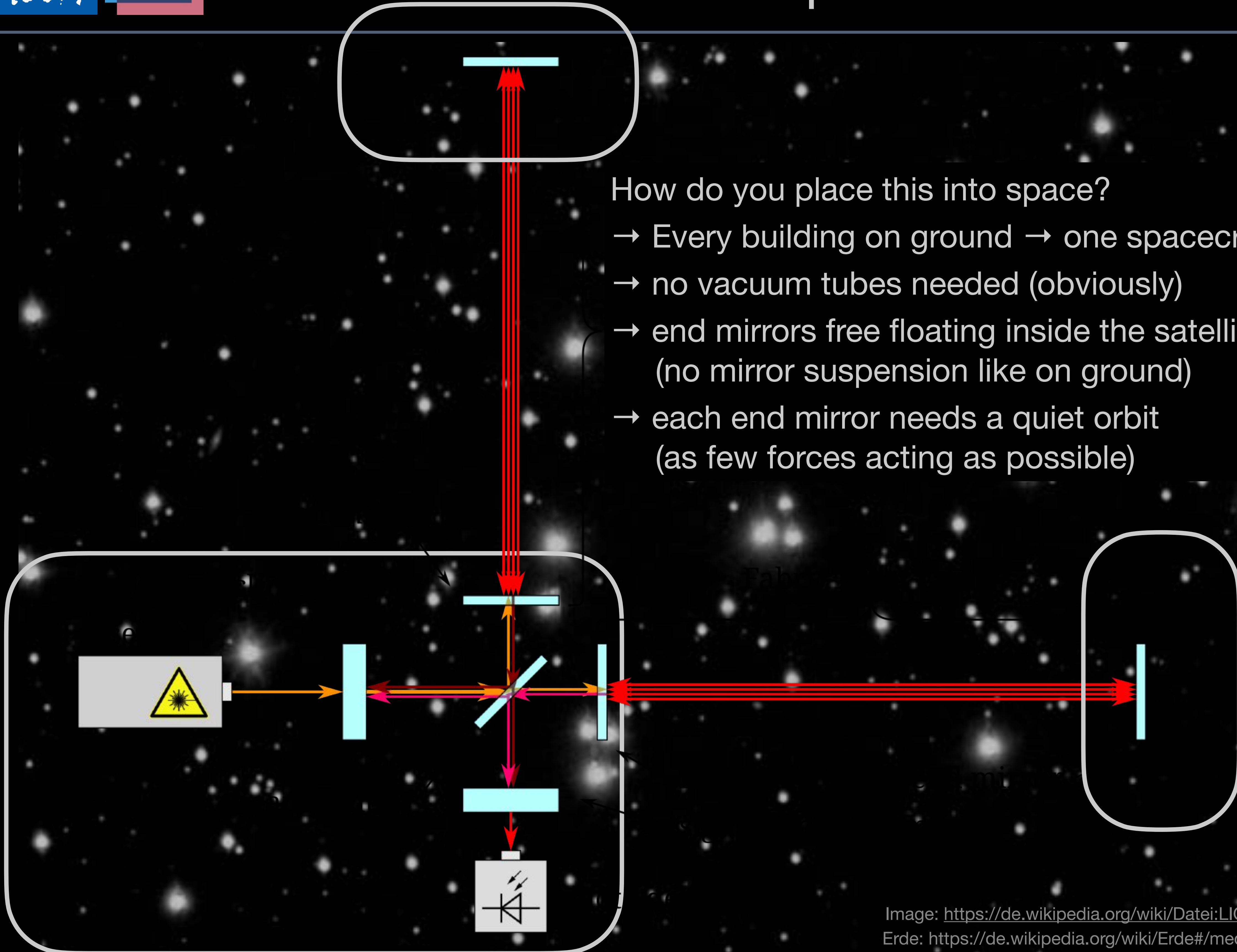
Seismic and Newtonian noise (gravity gradient noise) limit to low frequencies

Credit: Christopher Moore, Robert Cole and Christopher Berry
Source: http://gwplotter.com

How big would you make it?

- you ask your friend Bill Weber
  - He suggests: choose 2.5 mio km!
  - → See his slide "The right size for LISA: why L=2.5 million km?"

LISA - a LIGO in space?



How do you place this into space?
→ Every building on ground → one spacecraft!
→ no vacuum tubes needed (obviously)
→ end mirrors free floating inside the satellite (no mirror suspension like on ground)
→ each end mirror needs a quiet orbit (as few forces acting as possible)

Image: https://de.wikipedia.org/wiki/Datei:LIGO_simplified.svg
Erde: https://de.wikipedia.org/wiki/Erde#/media/Datei:Earth_From_the_Perspe
https://www.sternenpark-schwaebische-alb.de/einsteiger-astronomie/objekte-des-nachthimmels.html#sigplus_1001-3

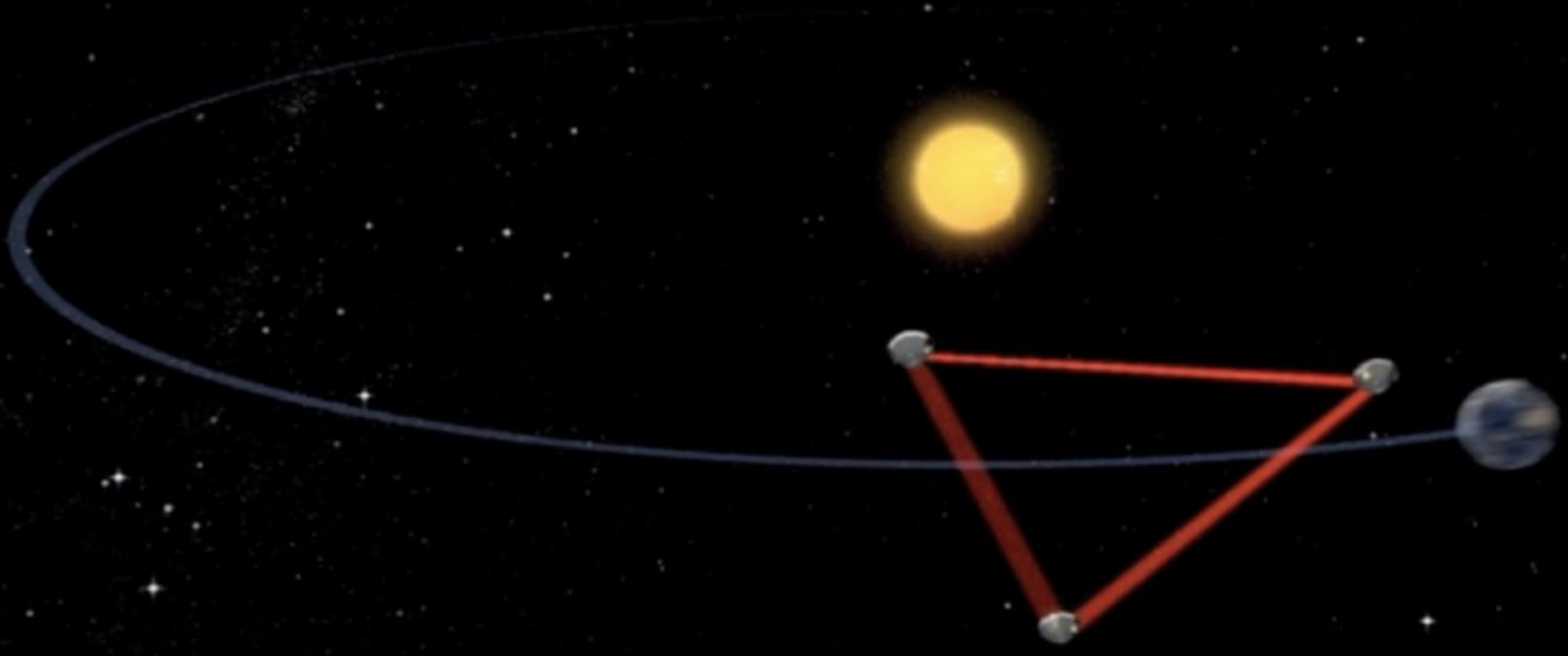
LISA - a LIGO in space?

Your friend suggests: 60° Michelson, not 90°

and **close the triangle!**

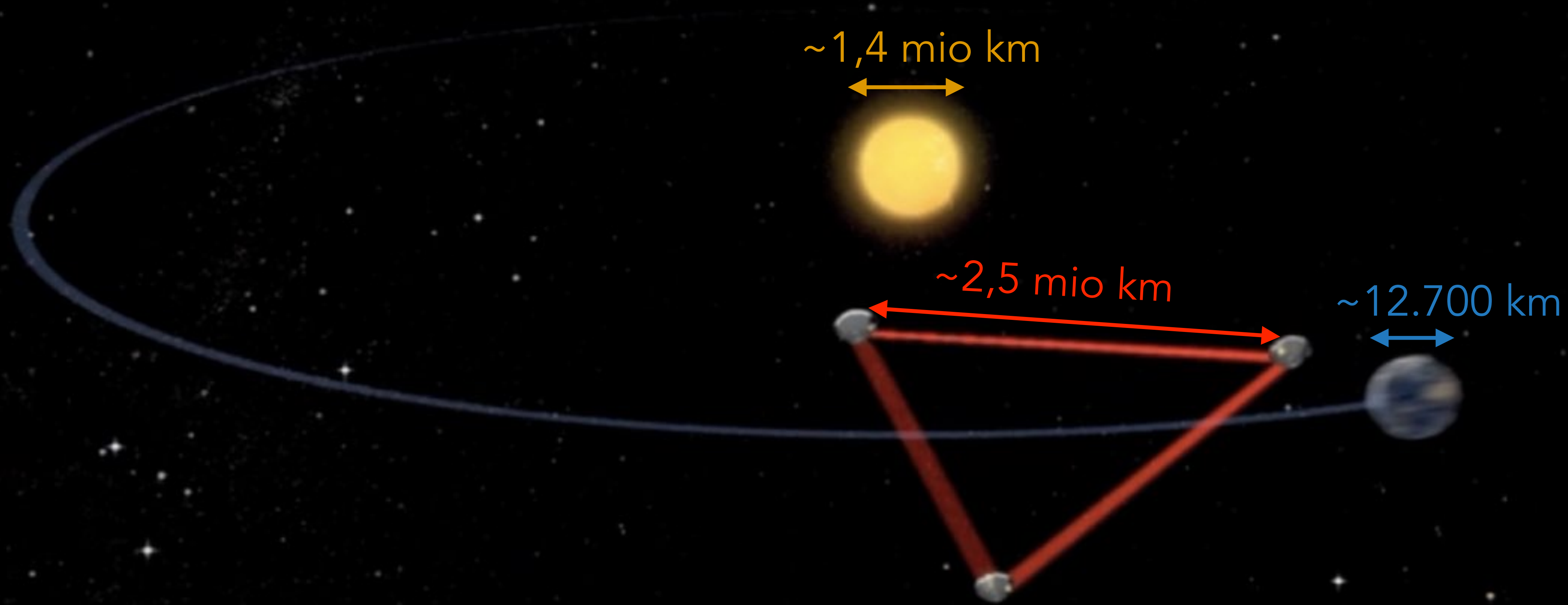
We need redundancy in space!!

and you gain physics

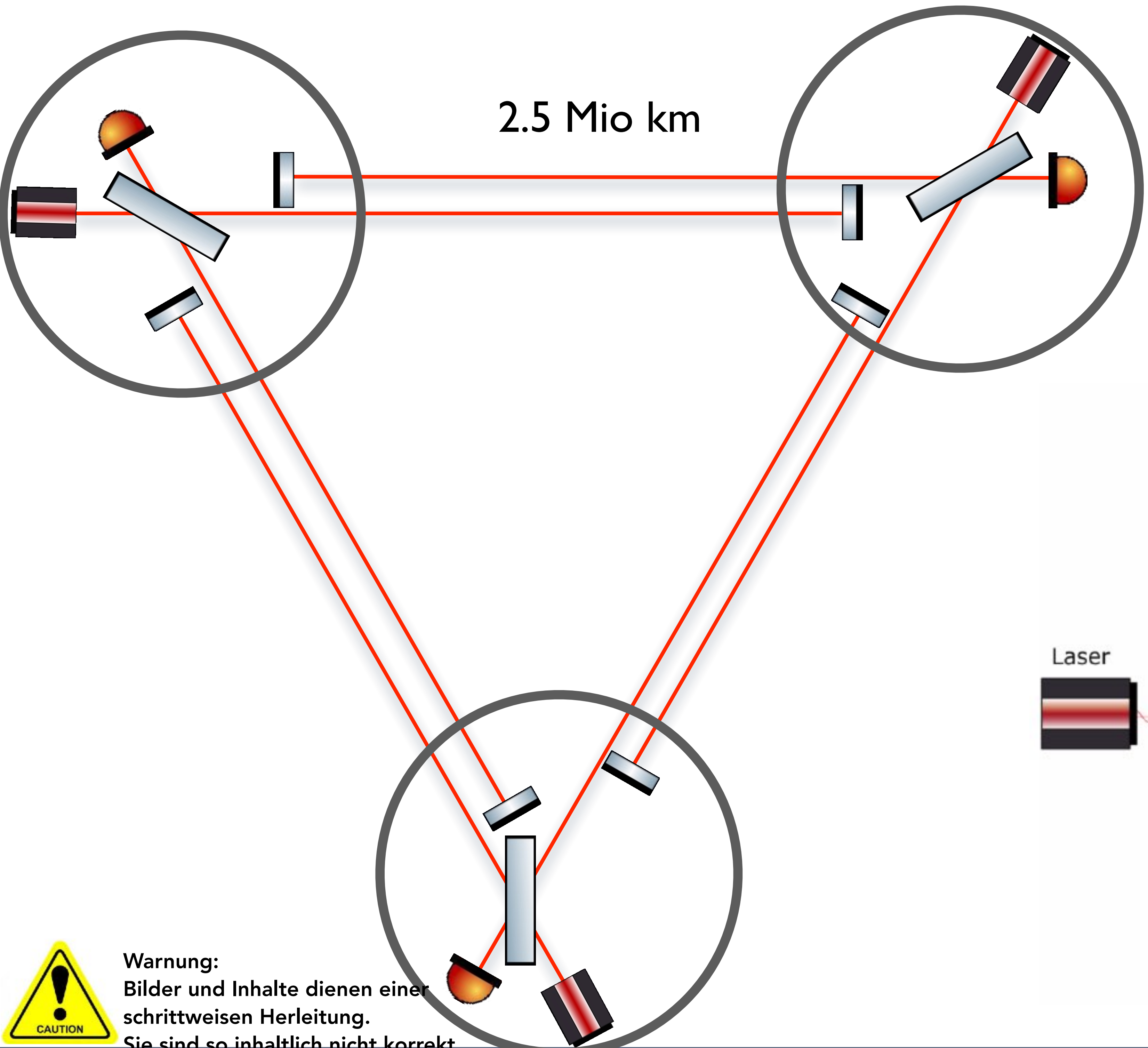


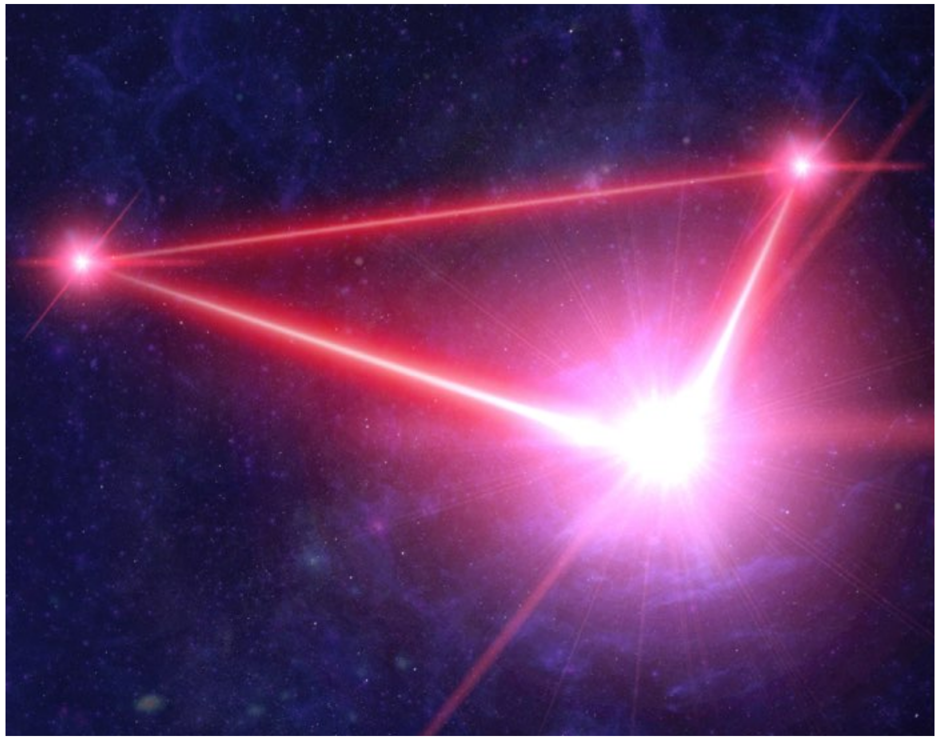
LISA - a LIGO in space?

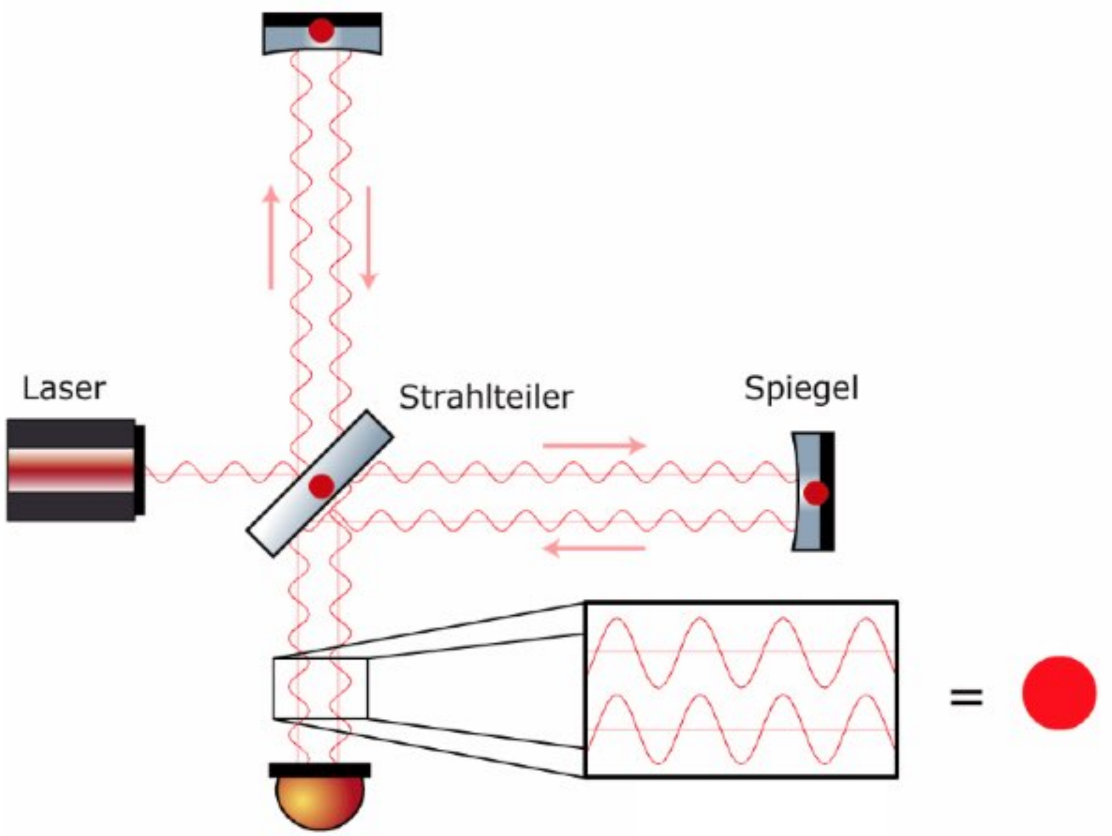
Let’s briefly look at the size of LISA



LISA - a LIGO in space?

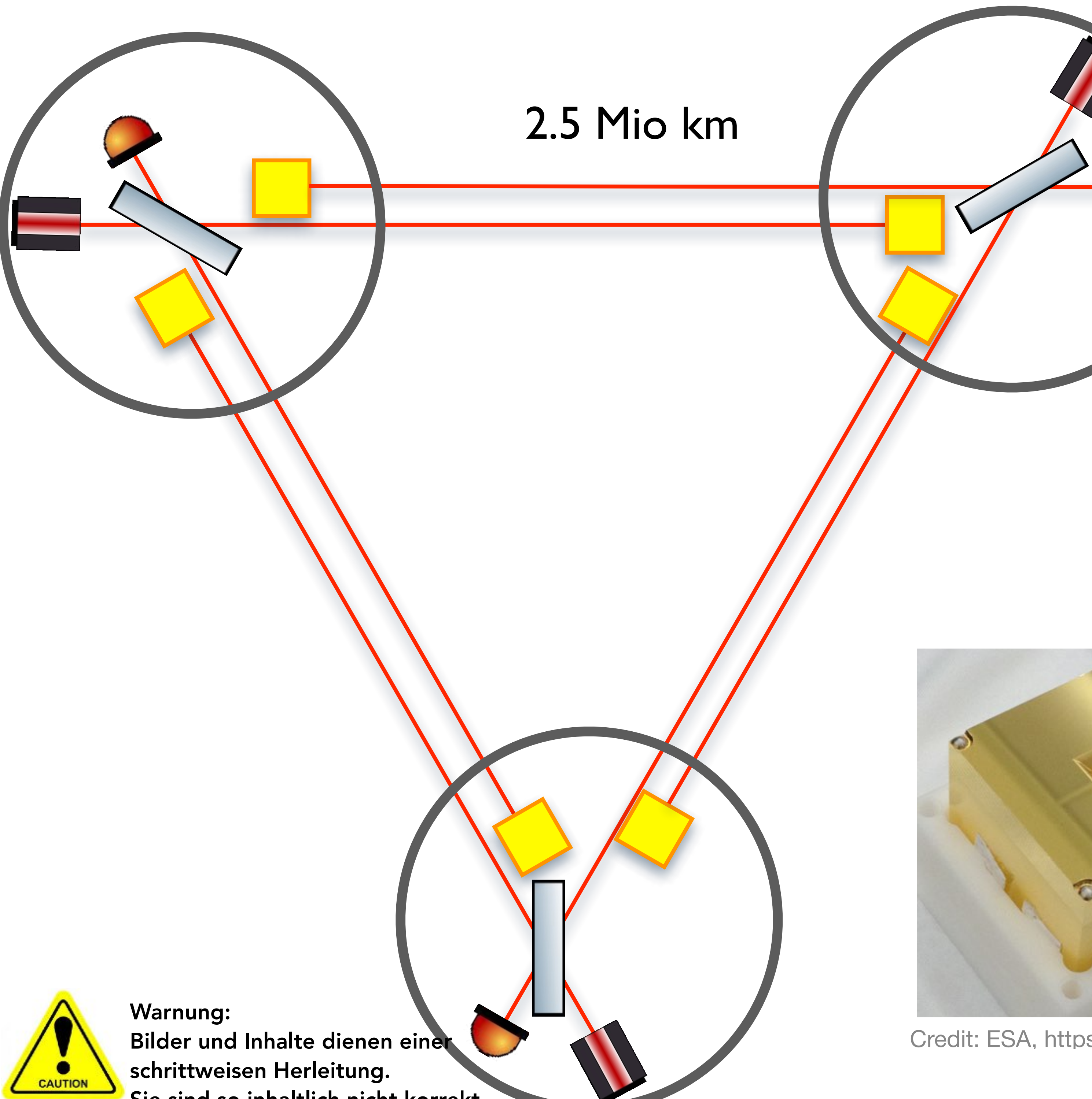


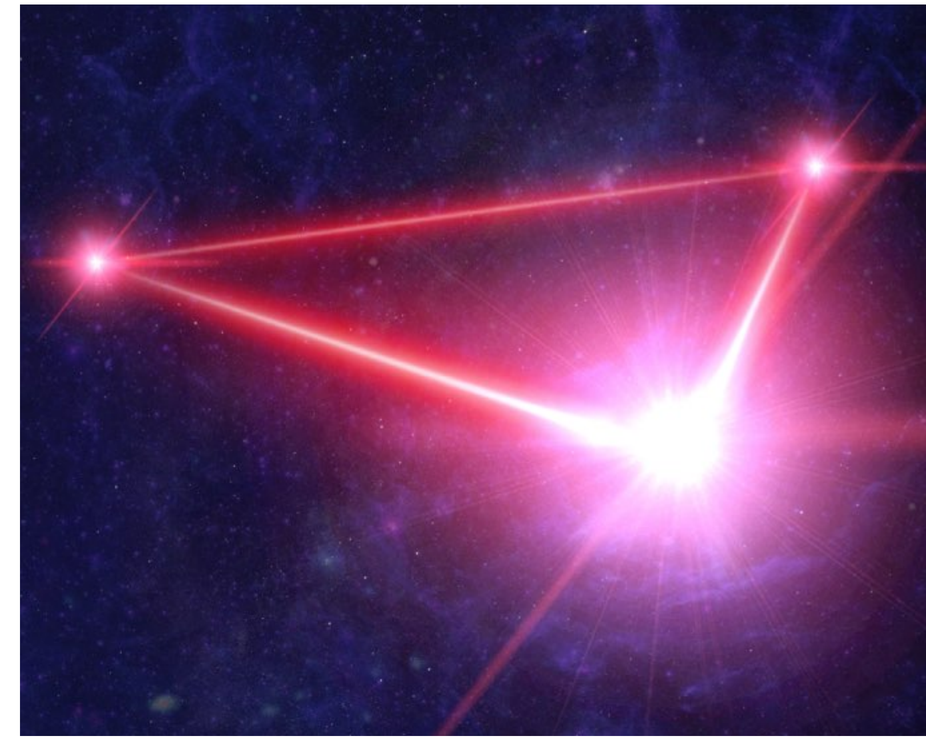


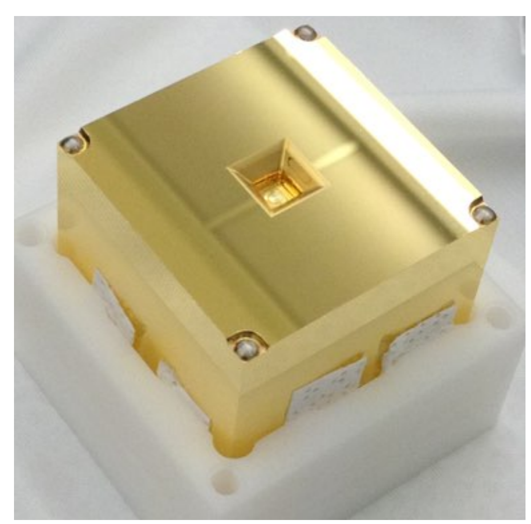


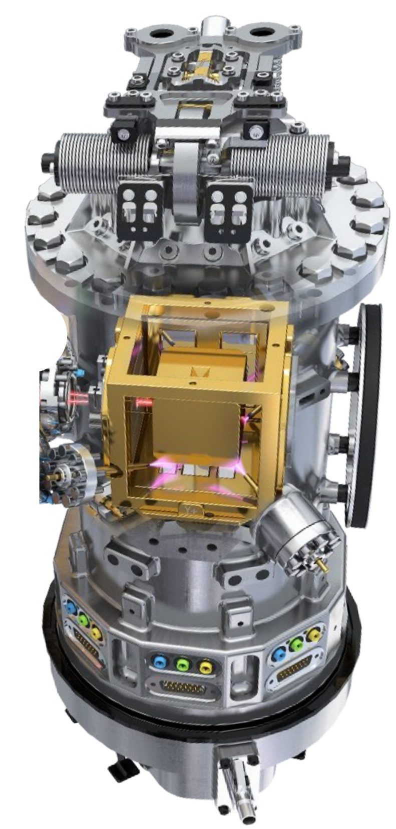
Warnung:
Bilder und Inhalte dienen einer schrittweisen Herleitung.
Sie sind so inhaltlich nicht korrekt.

LISA - a LIGO in space?






Credit: ESA, https://sci.esa.int/s/8qJbNGW


Credit ESA, https://sci.esa.int/s/wbpmPp8

Warnung:
Bilder und Inhalte dienen einer schrittweisen Herleitung.
Sie sind so inhaltlich nicht korrekt.

But you get worried….

… laser beams diverge!

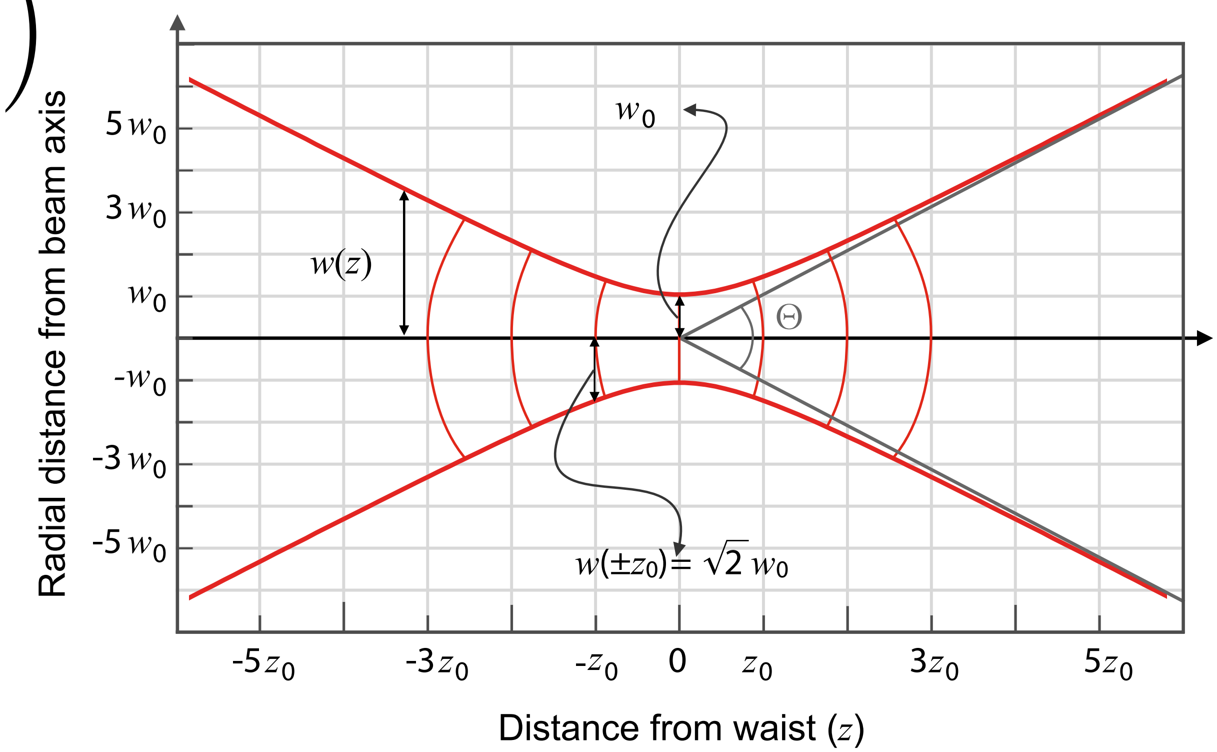
What is the size of the beam arriving at a remote spacecraft?

A rough estimate using

Gaussian Beams

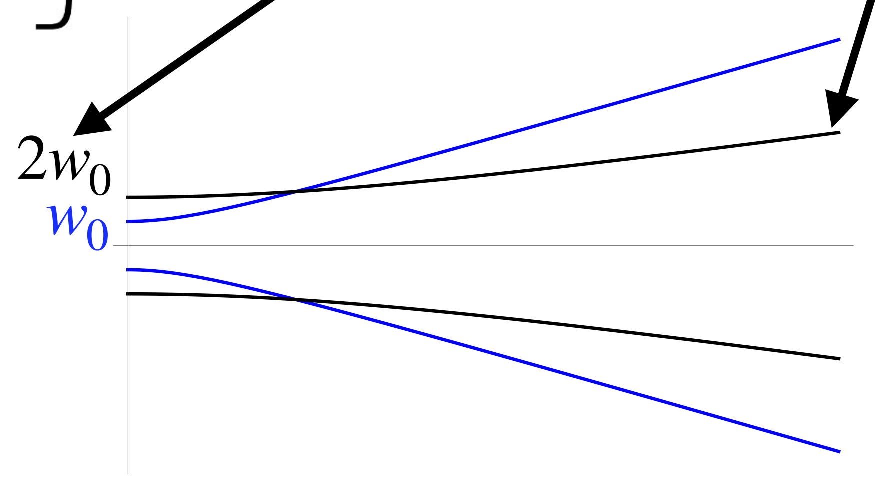
Gaussian beams in a nutshell

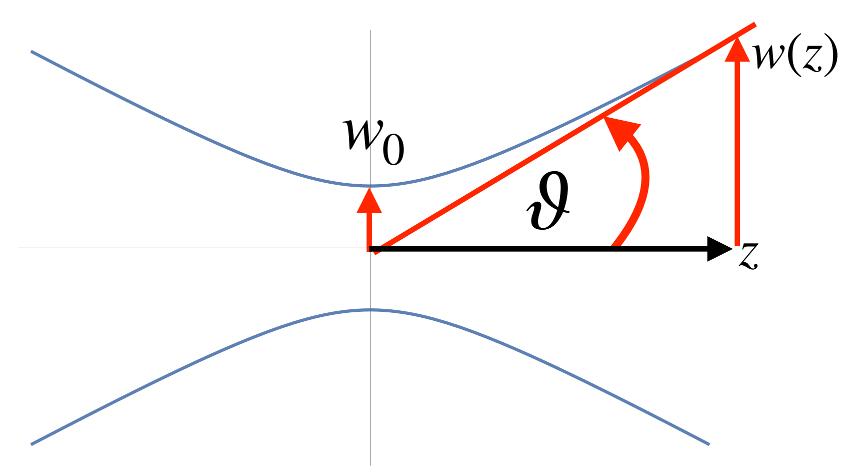
- $E = \sqrt{\frac{2P_b}{\pi w^2(z)}} \exp\left(-\frac{x^2+y^2}{w^2(z)}\right) \exp\left[-ikz - i\left(k\frac{x^2+y^2}{2R(z)} - \eta(z)\right)\right] e^{i\omega t}$
- Spot radius (i.e. radius at which the intensity dropped by $1/e^2$ compared to the axial value) : $w(z) = w_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}$
- Waist size = smallest radius along z: $w_0 = \sqrt{\frac{z_0\lambda}{\pi}} = w(z=0)$
- Rayleigh range: distance by which the spot radius increased by $\sqrt{2}$:
  $$w(z_0) = \sqrt{2}w_0$$
- Radius of curvature (wavefront property): $R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right]$
- Gouy phase $\eta(z) = \arctan(z/z_0)$
- Divergence (half-angle): $\vartheta = \frac{\lambda}{\pi w_0}$
- in the far field: $w(z) = \vartheta \cdot z$



→ The smaller the waist, the more the beam diverges!

→ The larger the waist, the smaller the beam in the far field!

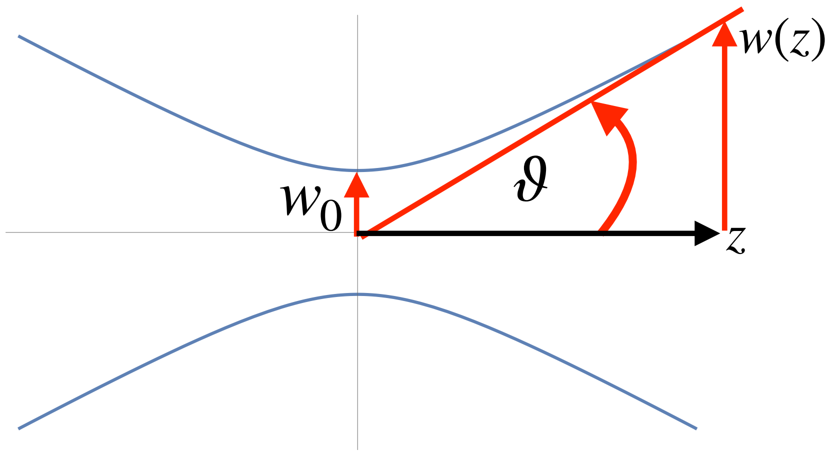




Spot size and beam power received at a remote spacecraft

- $$E = \sqrt{\frac{2P_b}{\pi w^2(z)}} \exp\left(-\frac{x^2+y^2}{w^2(z)}\right) \exp\left(-ikz - i\left(k\frac{x^2+y^2}{2R(z)} - \eta(z)\right)\right) e^{i\omega t}$$

- **Spot size estimate:**
  What's the beam radius arriving at the remote spacecraft?



  - Divergence (half-angle): $\vartheta = \frac{\lambda}{\pi w_0}$
  - in the far field: $w(z) = \vartheta \cdot z = \frac{\lambda}{\pi w_0} \cdot z$
  - wavelength $\lambda = 1064\,\text{nm}$
  - propagation distance? Armlength → $z = 2.5 \cdot 10^9\,\text{m}$

  → $w(z) = \frac{\lambda}{\pi w_0} \cdot z = \frac{1064\,\text{nm}}{\pi w_0} \cdot 2.5 \cdot 10^9\,\text{m}$

  - Let's guess some waist sizes: $w_0 = 1$ cm, $w_0 = 15$ cm, $w_0 = 25$ cm

    → $w(z) \approx 85\,\text{km}$, $w(z) \approx 5.6\,\text{km}$, $w(z) \approx 3.4\,\text{km}$

→ **You find again: it's favorable to have a large waist size.**

→ **You expect your satellites to need telescopes!**

→ **You quickly prove this…**

→ **what's the received power, if the receiving aperture is as large as the transmitting one?**

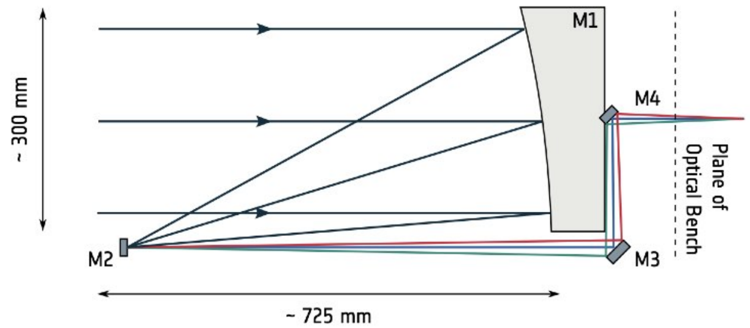


Image: https://www.cosmos.esa.int/web/lisa/lisa-redbook

Bill Weber's slide from yesterday

due to clipping at the telescope → rather work with diffracted light, not Gaussian beams…

see slide “LISA constellation and <<fundamental >> shot noise limit” by Bill Weber

Received power??

- $$E = \sqrt{\frac{2P_b}{\pi w^2(z)}} \exp\left(-\frac{x^2+y^2}{w^2(z)}\right) \exp\left[-ikz - i\left(k\frac{x^2+y^2}{2R(z)} - \eta(z)\right)\right] e^{i\omega t}$$
- **What's the power received at the remote spacecraft?**
  - **You assume: the receiving aperture radius matches the transmitted radius.**
  - **We assume a 1cm (or 15cm, or 25 cm) receiving aperture radius**
  - $$P = \int_S dS\, I(x,y,z,t)$$
    $$= \int_0^{0.15\,\mathrm{m}} 2\pi r dr \frac{2P_b}{\pi w^2(z)} \exp\left(-\frac{2r^2}{w^2(z)}\right)$$
  - you test these numbers and find:

$$\int_0^{0.01\,\mathrm{m}} 2\pi r \frac{2P}{\pi w^2} \mathrm{Exp}\left[-2\frac{r^2}{w^2}\right] dr \;/.\; w \to 84670.4\,\mathrm{m}$$

$$\int_0^{0.15\,\mathrm{m}} 2\pi r \frac{2P}{\pi w^2} \mathrm{Exp}\left[-2\frac{r^2}{w^2}\right] dr \;/.\; w \to 5644.7\,\mathrm{m}$$

$$\int_0^{0.25\,\mathrm{m}} 2\pi r \frac{2P}{\pi w^2} \mathrm{Exp}\left[-2\frac{r^2}{w^2}\right] dr \;/.\; w \to 3386.8\,\mathrm{m}$$

2.78666×10^{-14} P

1.41231×10^{-9} P

1.08976×10^{-8} P

- Amplitude of a Gaussian beam
  $$A = A(x,y,z,t) = \sqrt{\frac{2P_b}{\pi w^2(z)}} \exp\left(-\frac{x^2+y^2}{w^2(z)}\right)$$
- Intensity $I = |E|^2 = A(x,y,z,t)^2$
- Power $P = \int_S dS\, I(x,y,z,t)$

- Our assumed waist sizes:
- $w_0 = 1$ cm, $w_0 = 15$ cm, $w_0 = 25$ cm

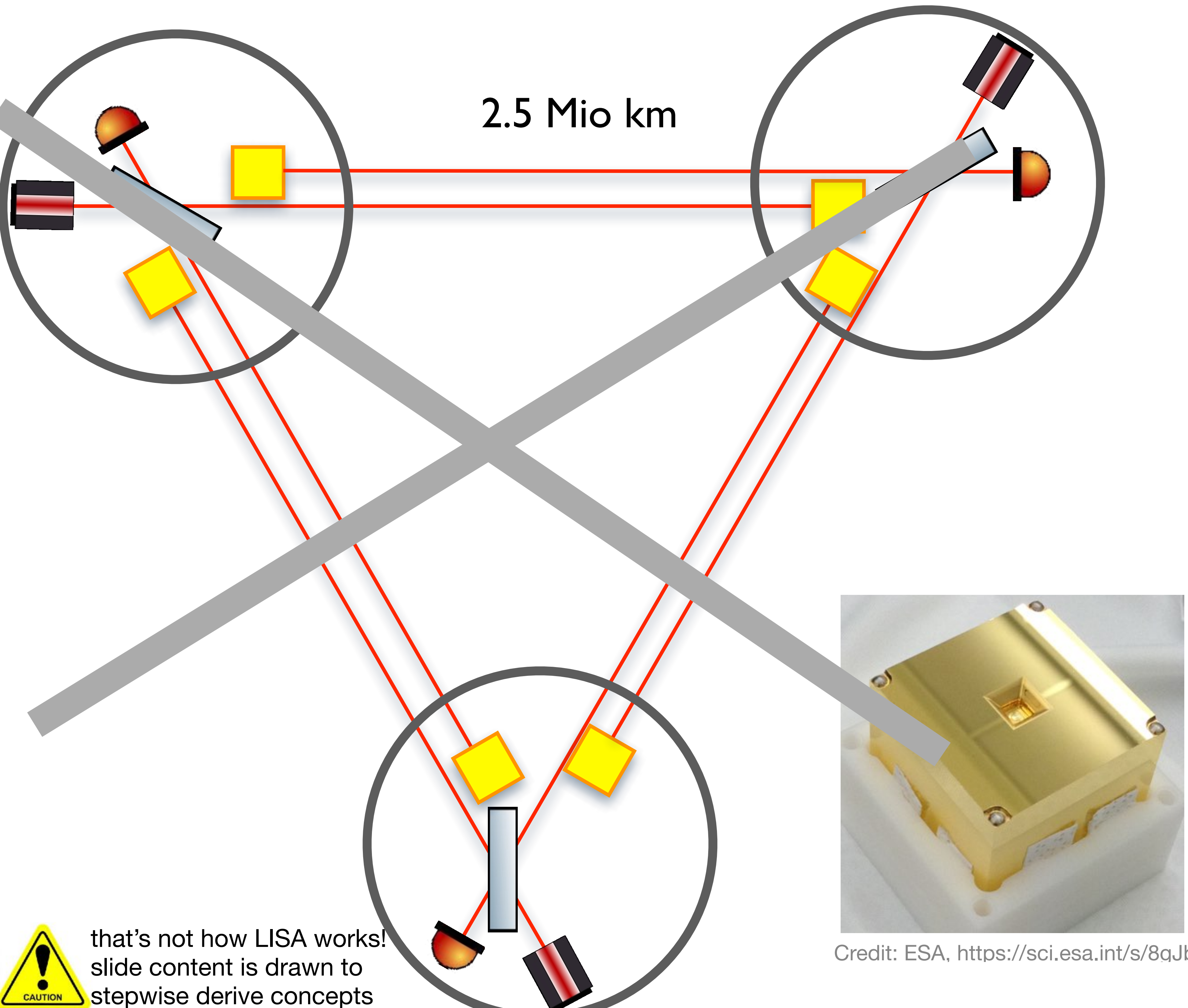
→ $w(z) \approx 85\,\mathrm{km}$, $w(z) \approx 5.6\,\mathrm{km}$, $w(z) \approx 3.4\,\mathrm{km}$

→ All numbers seem tiny!
→ You ask ESA for a 50 cm (25cm radius) telescope,
  → ESA rejects: too expensive.
→ You double-check your numbers and decide: a 30 cm telescope will do - with some creativity…
→ You also note: each telescope is used in transmit and receive direction.
  → A 30cm-diameter telescope does not fit a 15 cm waist-radius Gaussian beam!
  → So your estimate was only very rough.
  → In reality: even less received light, which is split onto several photodiodes.

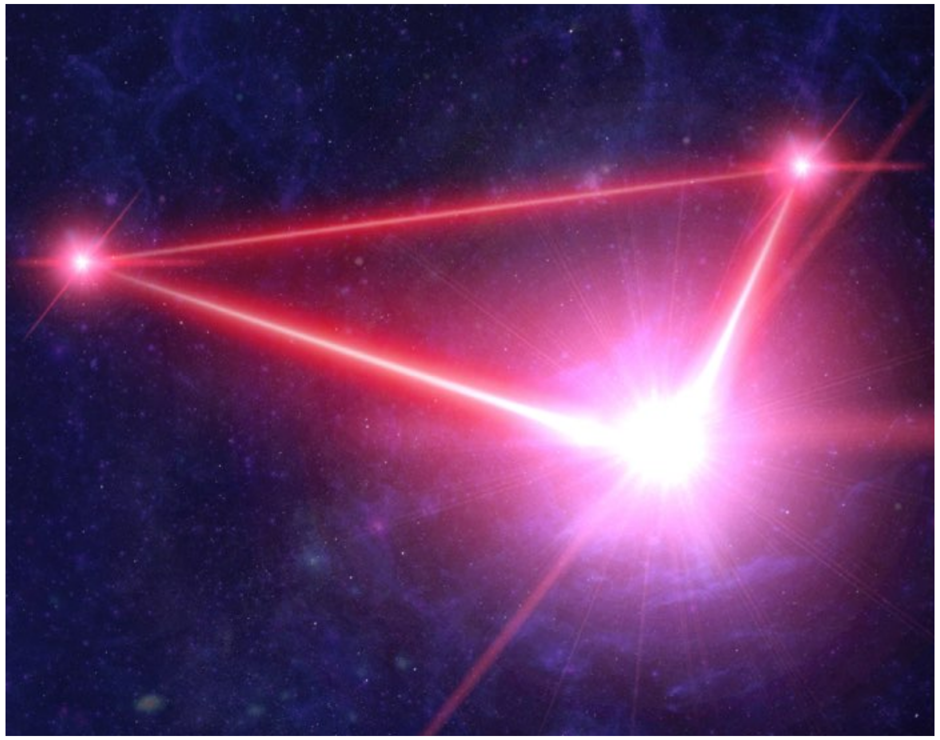
LISA - a LIGO in space?

LISA - completely over-simplified.

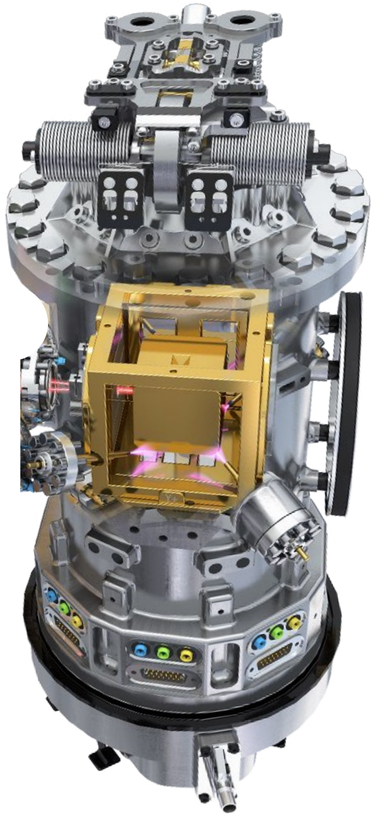
This is not how LISA actually works.



that's not how LISA works!
slide content is drawn to
stepwise derive concepts



Credit: ESA, https://sci.esa.int/s/8qJbNGW

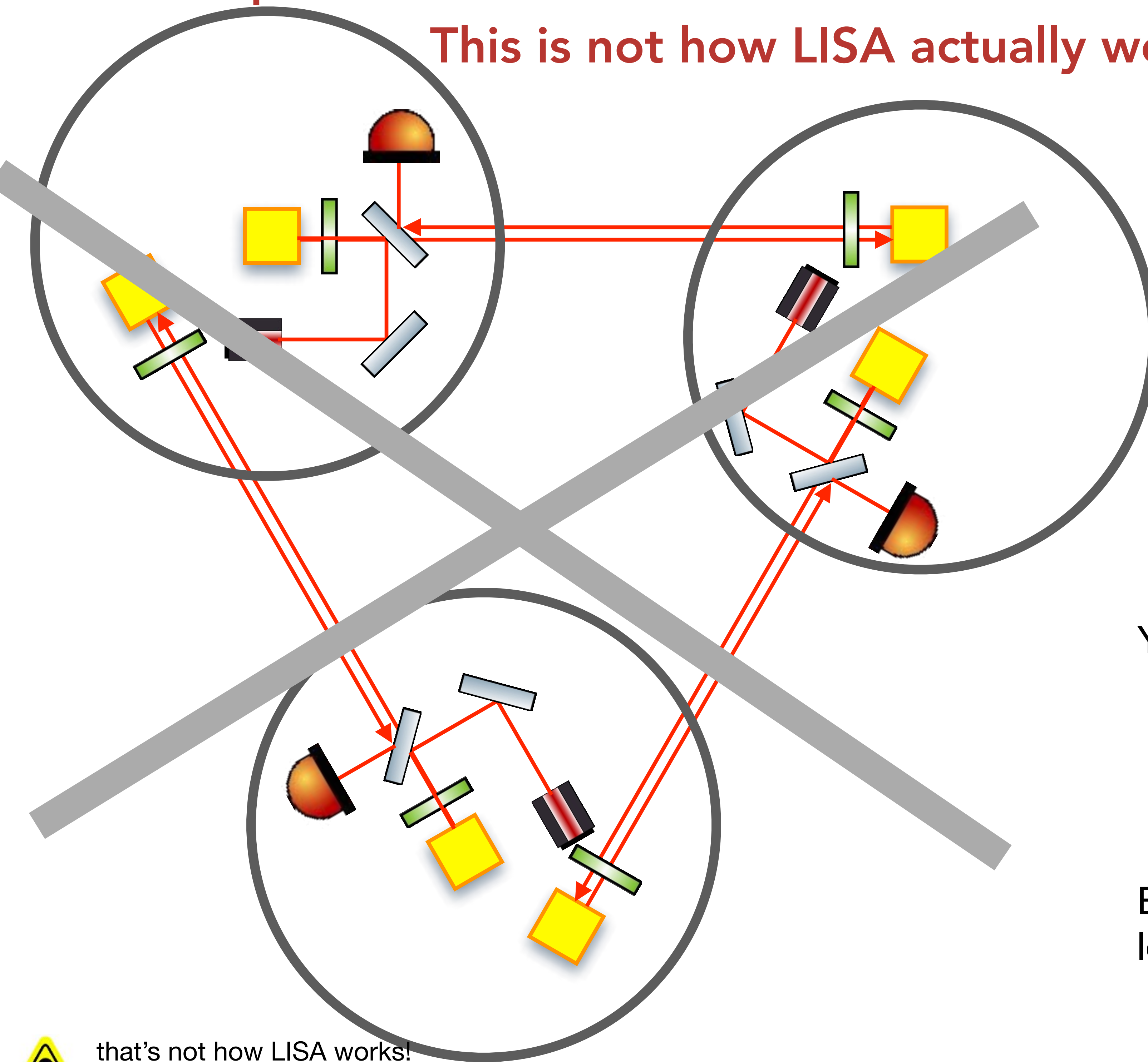


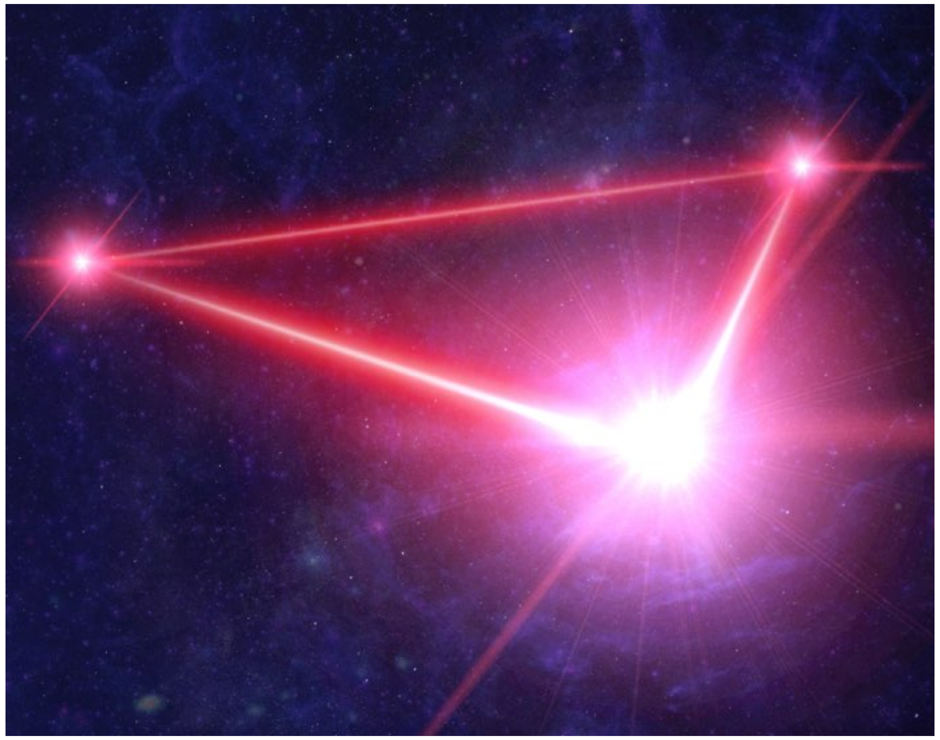
Credit ESA, https://sci.esa.int/s/wbpmPp8

LISA - a LIGO in space?

LISA - simplified.

This is not how LISA actually works.





Your first idea:

- Save a factor of 2 in power by **splitting up each Michelson**

→ Measure the distance variation between two test masses

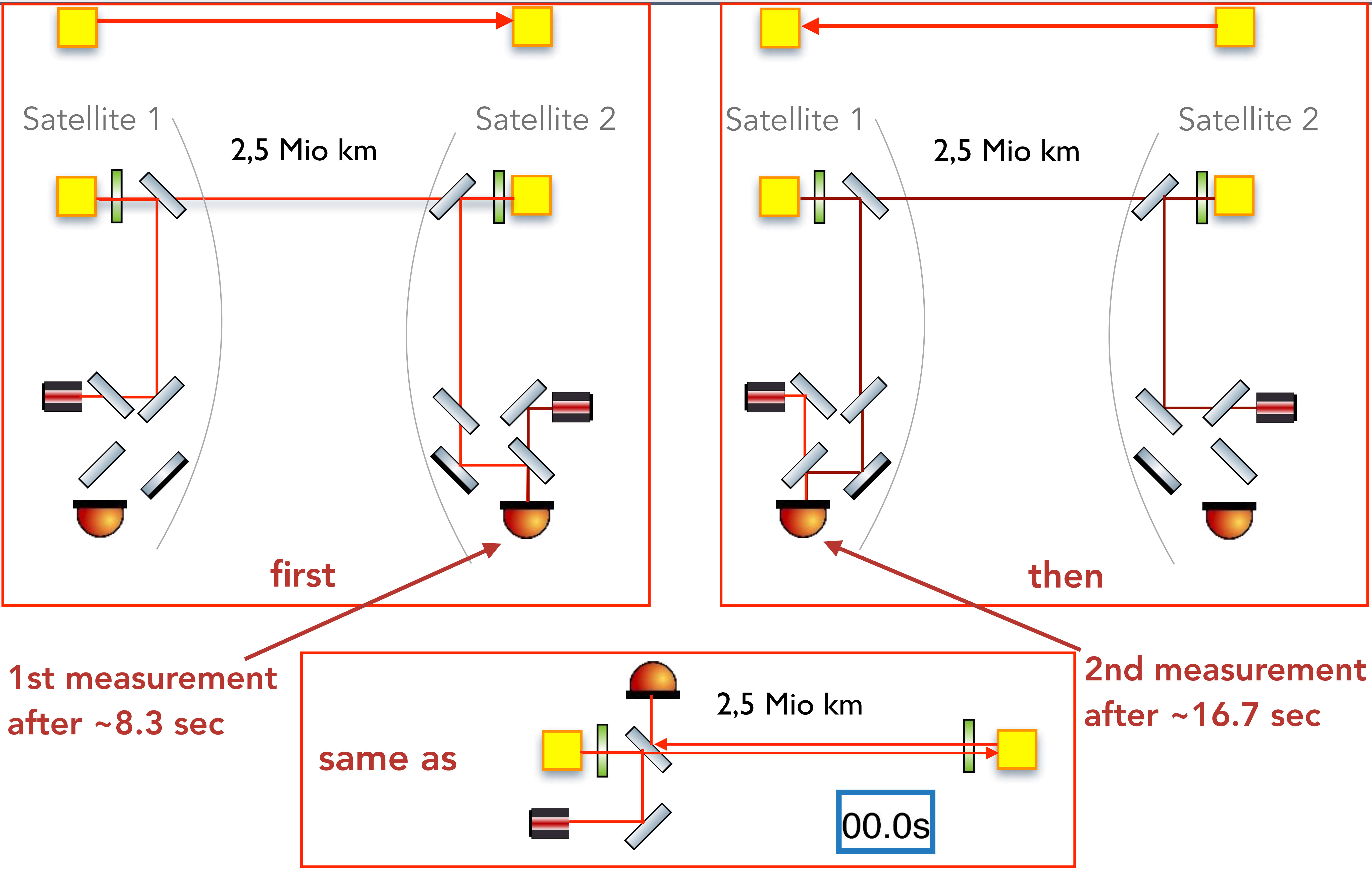
But: sending light from one SC to another, we lose 9 orders of magnitude in light power!

→ **reflecting back means** 10^{-18}!

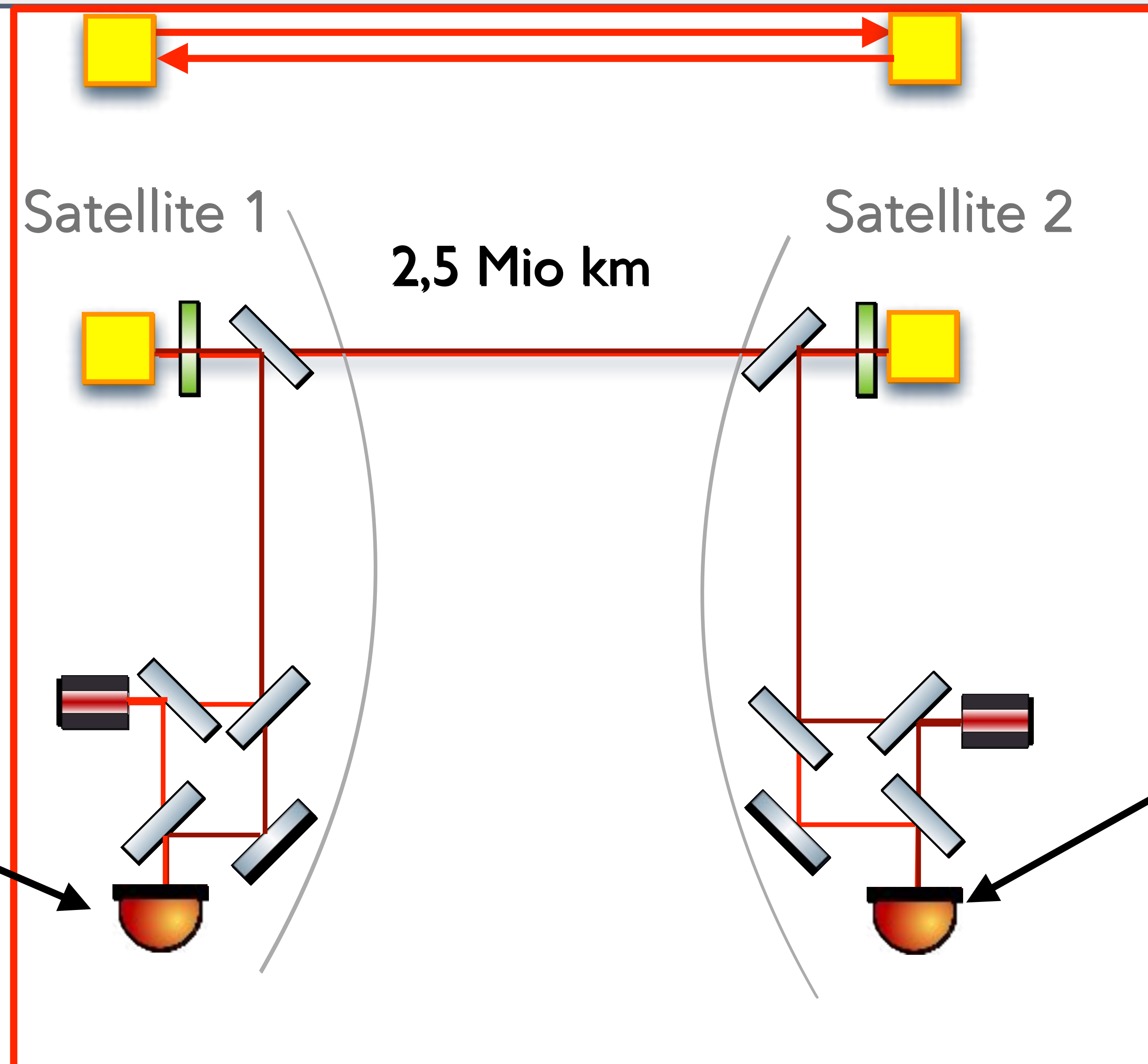
→ that's not solved with a factor of 2

that's not how LISA works! slide content is drawn to stepwise derive concepts

LISA interferometry concepts



LISA interferometry concepts



16.5	0.003
16.7	0.003
16.9	0.002
17.1	0.004
17.3	0.003
17.5	0.003
17.7	0.005
17.9	0.004
18.1	0.005

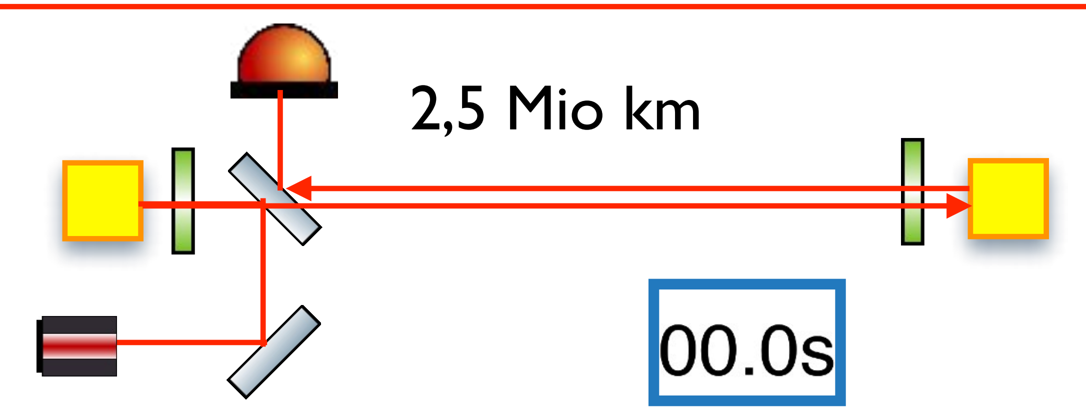
8.3	0.001
8.5	0.002
8.7	0.001
8.9	0.003
9.1	0.002
9.3	0.002
9.5	0.003
9.7	0.003
9.9	0.004

Lasers run continuously.

We subtract phases measured at different locations and different times.

Subtraction done on ground (Earth)!

same as



Time Delay Interferometry

- How can we measure the distance variation between two test masses?

  - Let's reduce the drawings before we actually get to a functioning LISA

this does not work yet!

$$\tilde{f}_1(t + 16s) - \tilde{f}_1(t)$$

- Laser frequency noise $\tilde{f}$ couples to length readout. Coupling is proportional to arm length mismatch ΔL:

$$\tilde{x} \approx \frac{\lambda \Delta L}{c} \tilde{f}$$

$$\approx \frac{10^{-6}\,\text{m}}{3 \cdot 10^{8}\,\text{m/s}} 5.000.000\,\text{km} \cdot 30\,\text{Hz}/\sqrt{\text{Hz}}$$

$$\approx 500\,\mu\text{m}/\sqrt{\text{Hz}}$$

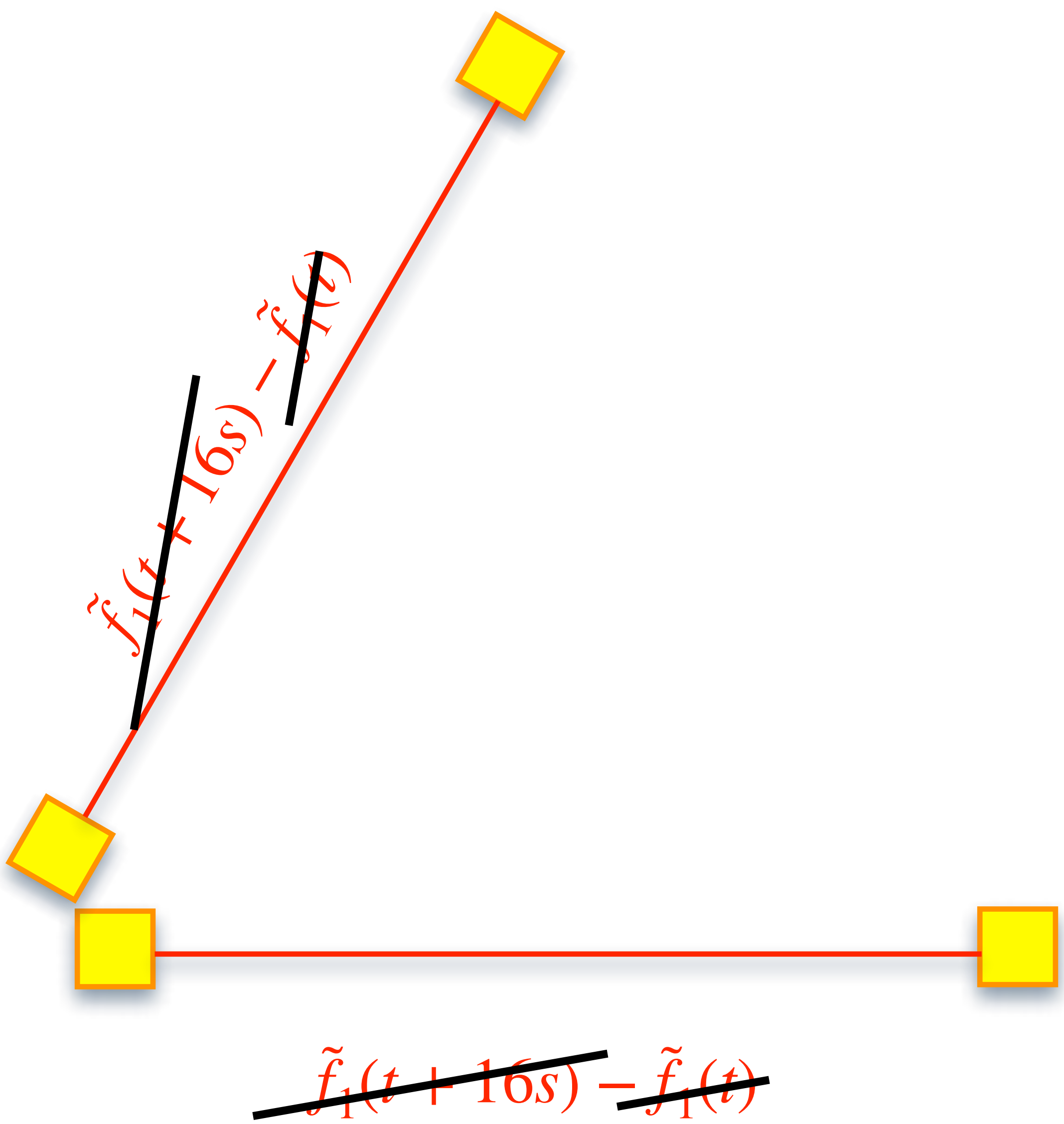
- That's way too high.
  We need picometer-levels!

CAUTION

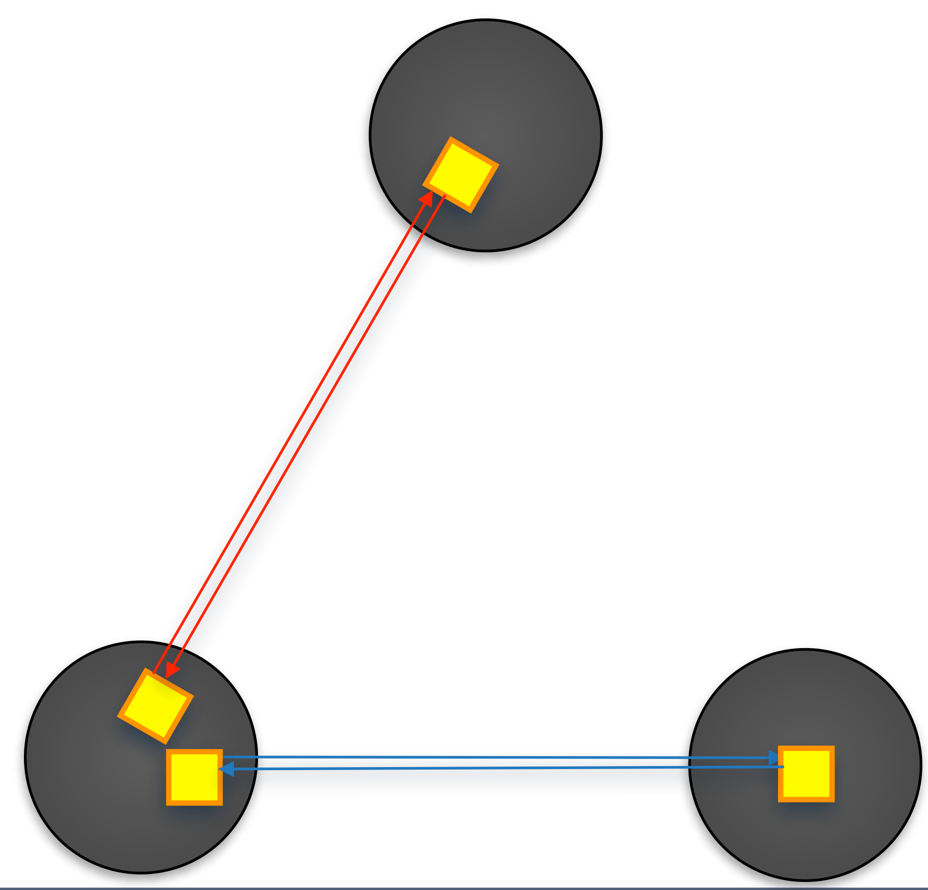
oversimplified and just a thought experiment, that's not how LISA works!!

Time Delay Interferometry

- This is “**T**ime **D**elay **I**nterferometry”
  - means: by combining measurements from different locations at different times, we synthesize an effective equal arm length interferometer

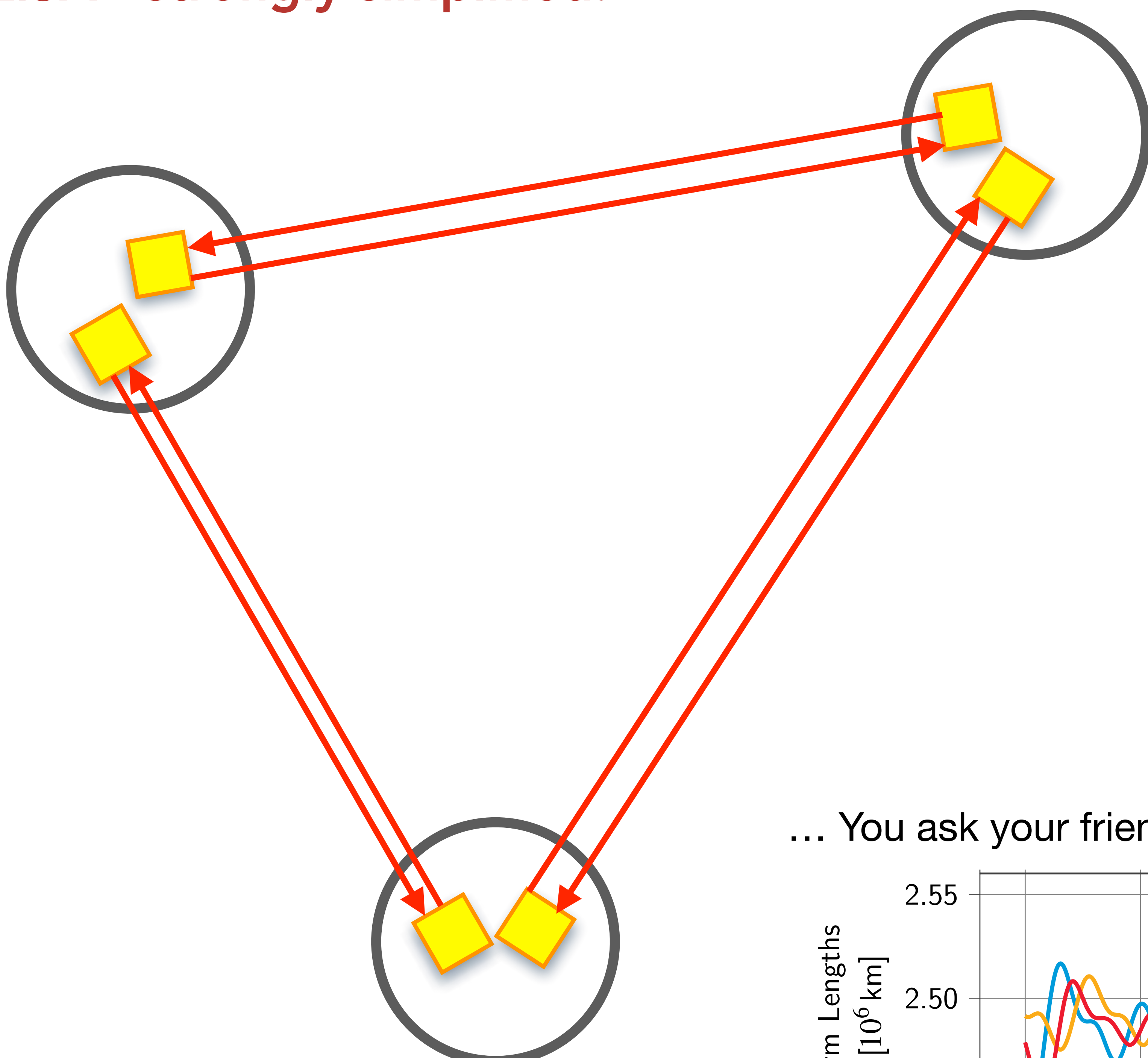


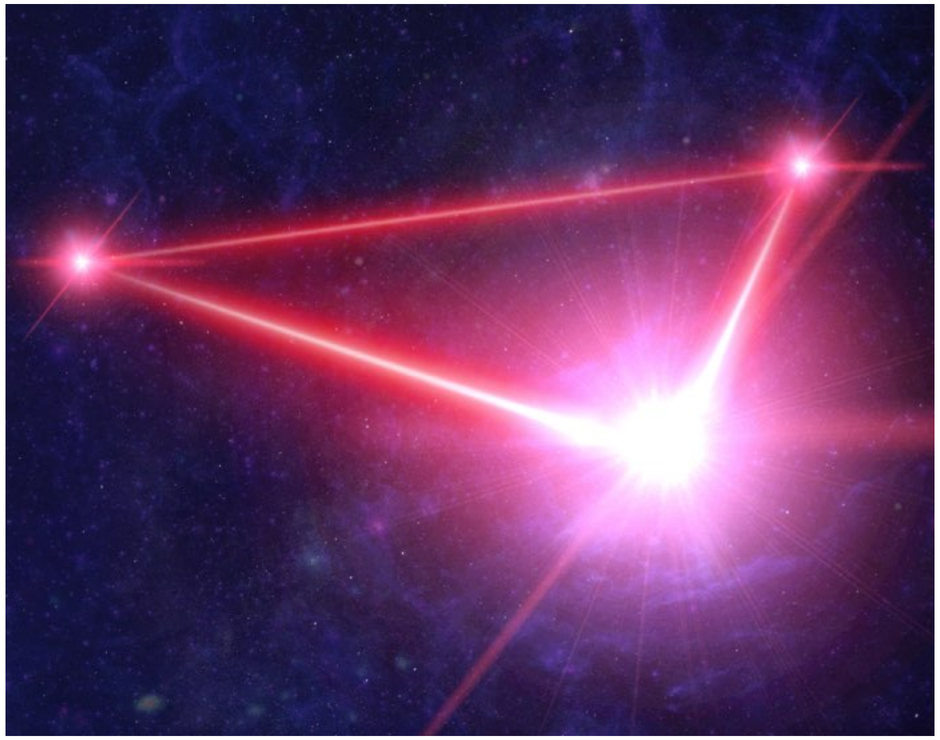
- This is **TDI generation 0:**
  - Assumptions:
    - ***equal and constant arm lengths!***



oversimplified and just a thought experiment, that’s not how LISA works!!

LISA - strongly simplified.



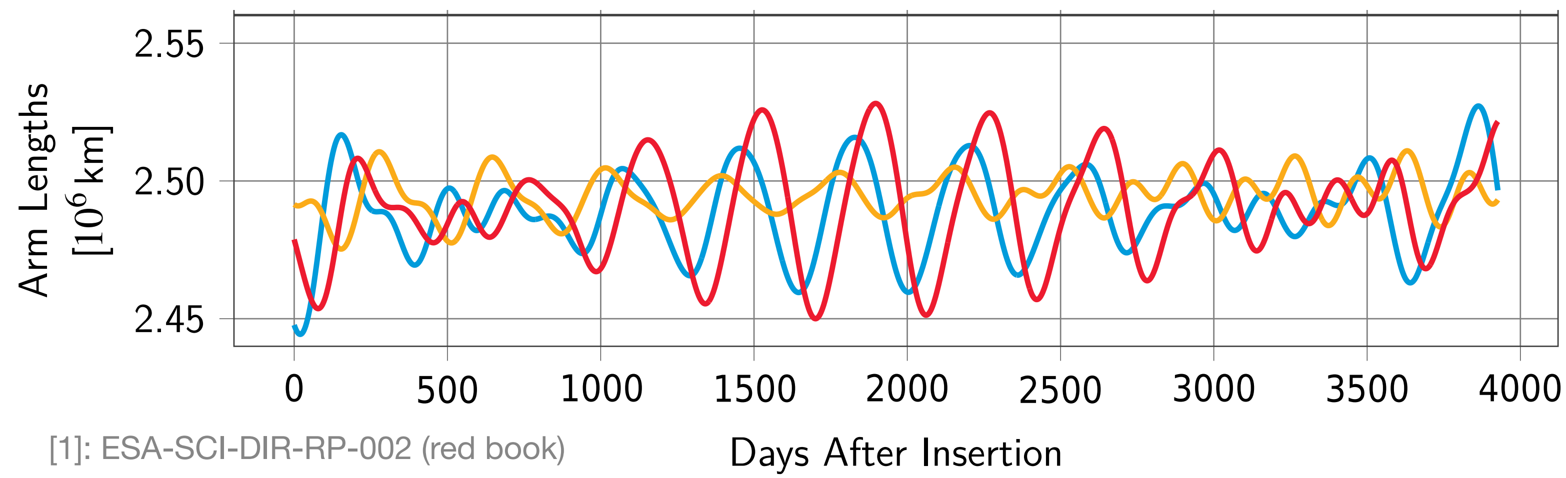


This should work! - you think…

… but satellites are not buildings on Earth

… how stable are the arm lengths?

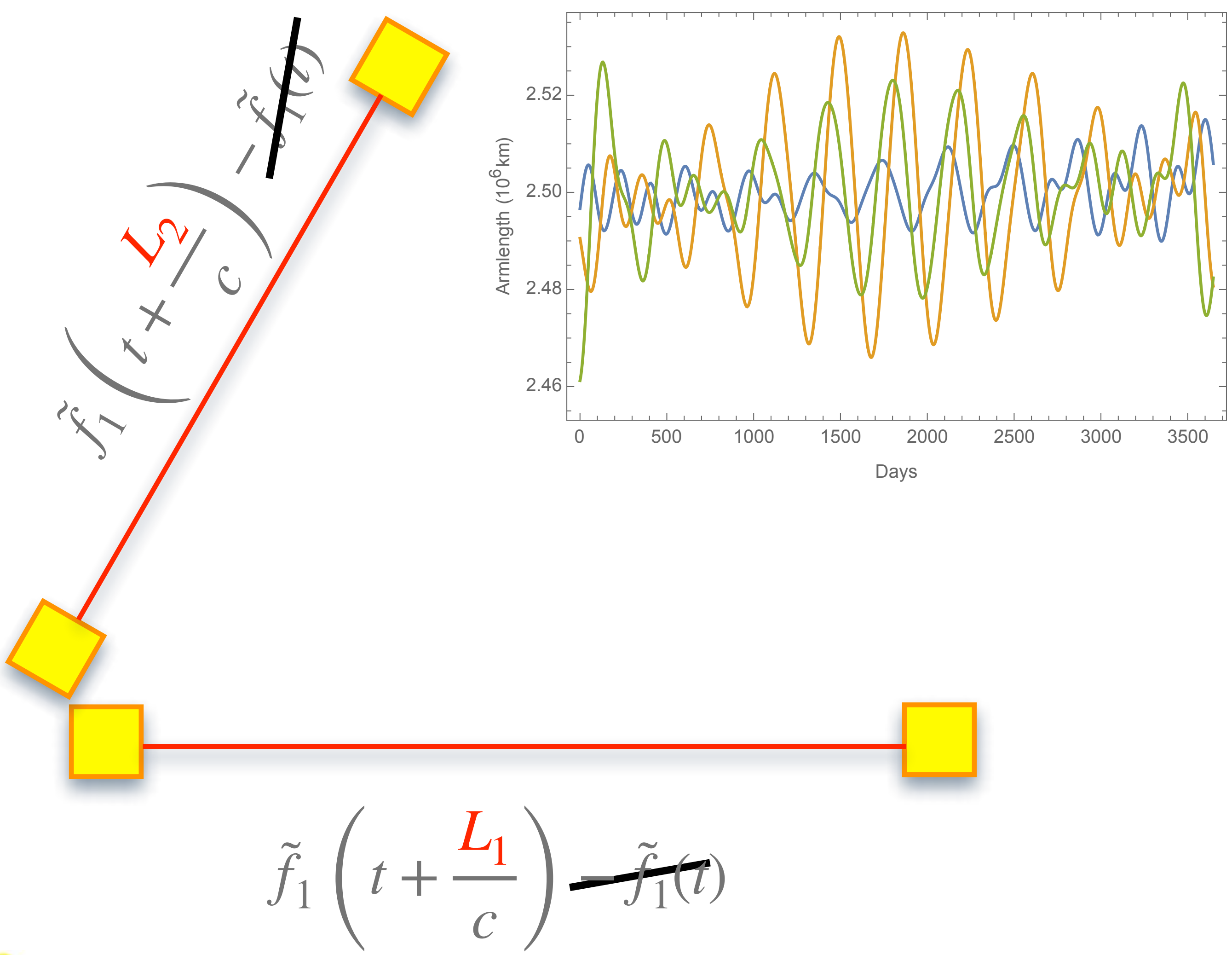
… You ask your friend [1] again, who replies



[1]: ESA-SCI-DIR-RP-002 (red book)

that's not how LISA works! slide content is drawn to stepwise derive concepts

- This is "**T**ime **D**elay **I**nterferometry"
  - means: by combining measurements from different locations at different times, we synthesize an effective equal arm length interferometer



→ This is a simple Michelson interferometer
→ Would work, if the arms were equal.
→ Arm lengths vary by ~1% (~25.000 km) during a year

- Laser frequency noise $\tilde{f}$ couples to length readout. Coupling is proportional to arm length mismatch ΔL:

$$\tilde{x} \approx \frac{\lambda \Delta L}{c} \tilde{f}$$
$$\approx \frac{10^{-6}\,\mathrm{m}}{3 \cdot 10^{8}\,\mathrm{m/s}} 2 \cdot 25.000\,\mathrm{km} \cdot 30\,\mathrm{Hz}/\sqrt{\mathrm{Hz}}$$
$$\approx 5\,\mu\mathrm{m}/\sqrt{\mathrm{Hz}}$$

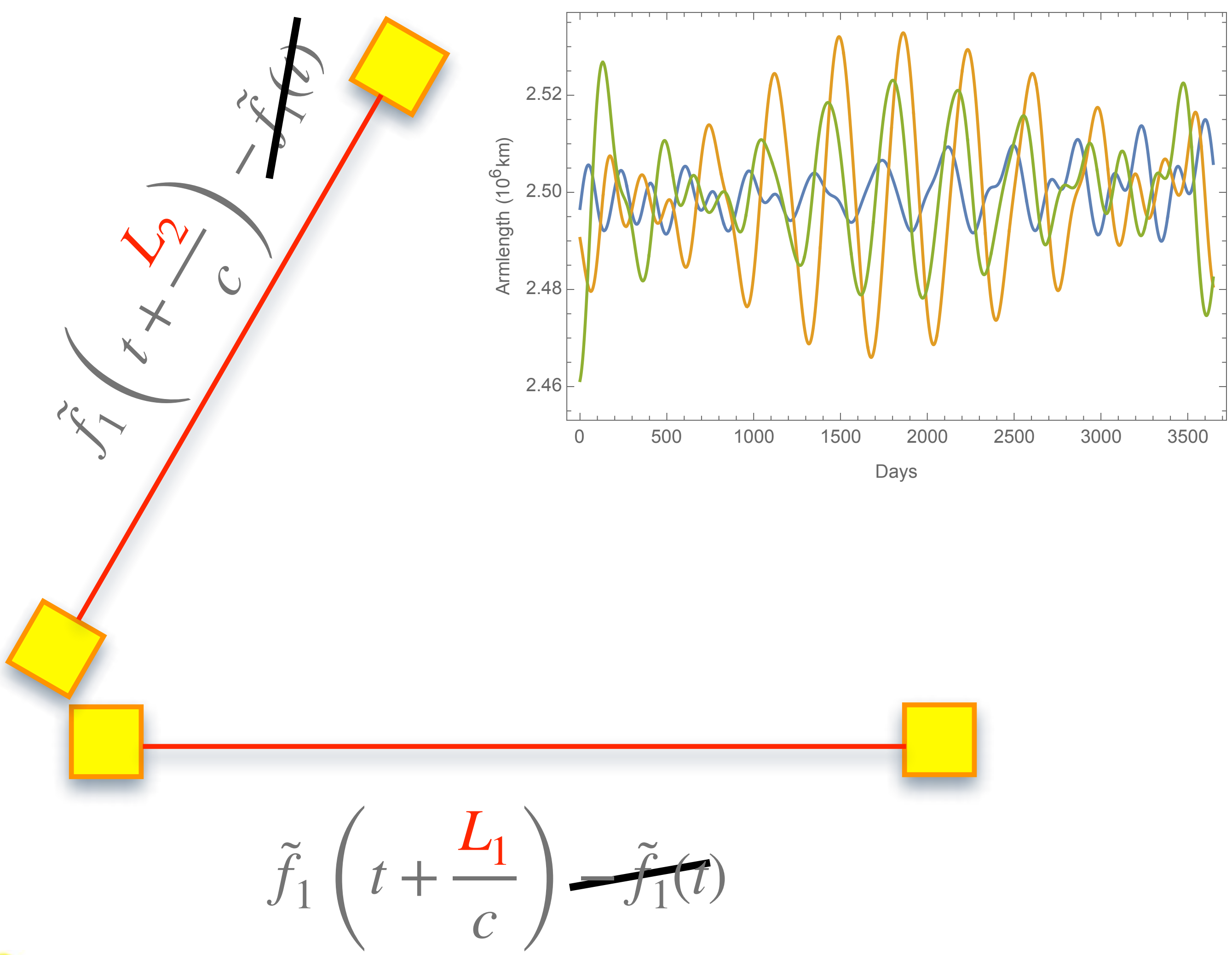
- That's too high, we need picometer-levels!

oversimplified and just a thought experiment, that's not how LISA works!!

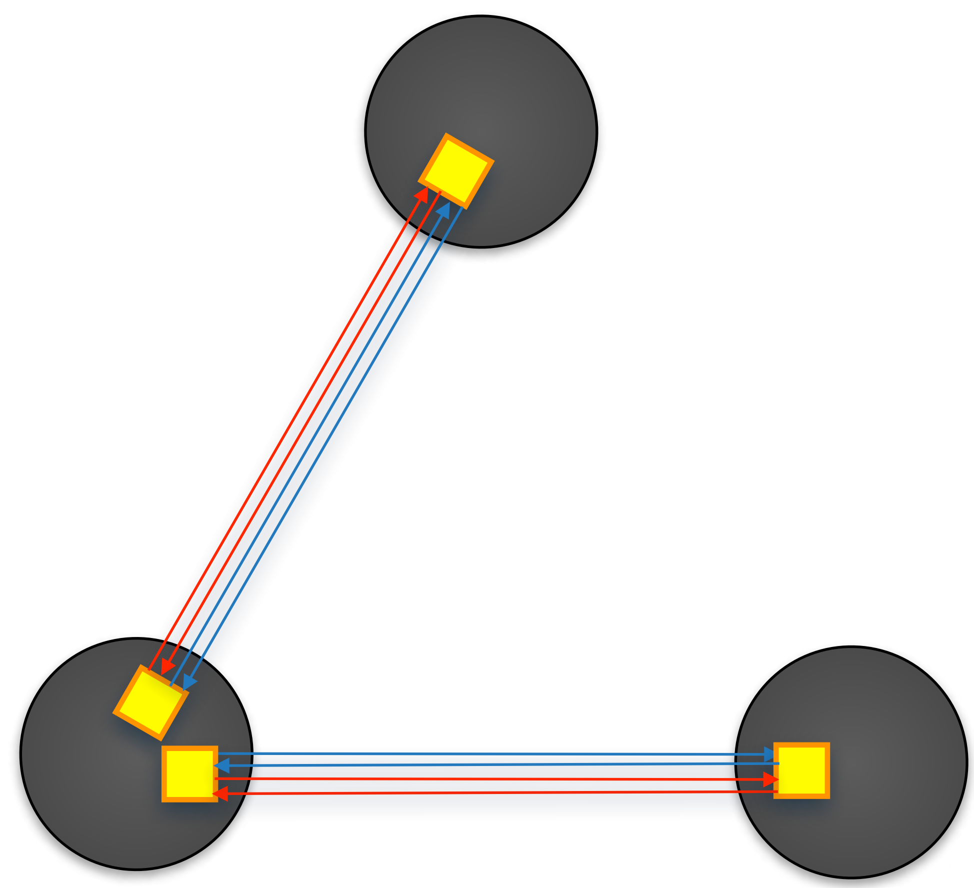
Credit for figure armlength variation: ESOC & Oliver Jennrich

Time Delay Interferometry

- This is "**T**ime **D**elay **I**nterferometry"
  - means: by combining measurements from different locations at different times, we synthesize an effective equal arm length interferometer



- This is **TDI generation 1:**
  - Assumptions:
    - **unequal and constant arm lengths!**



oversimplified and just a thought experiment, that's not how LISA works!!

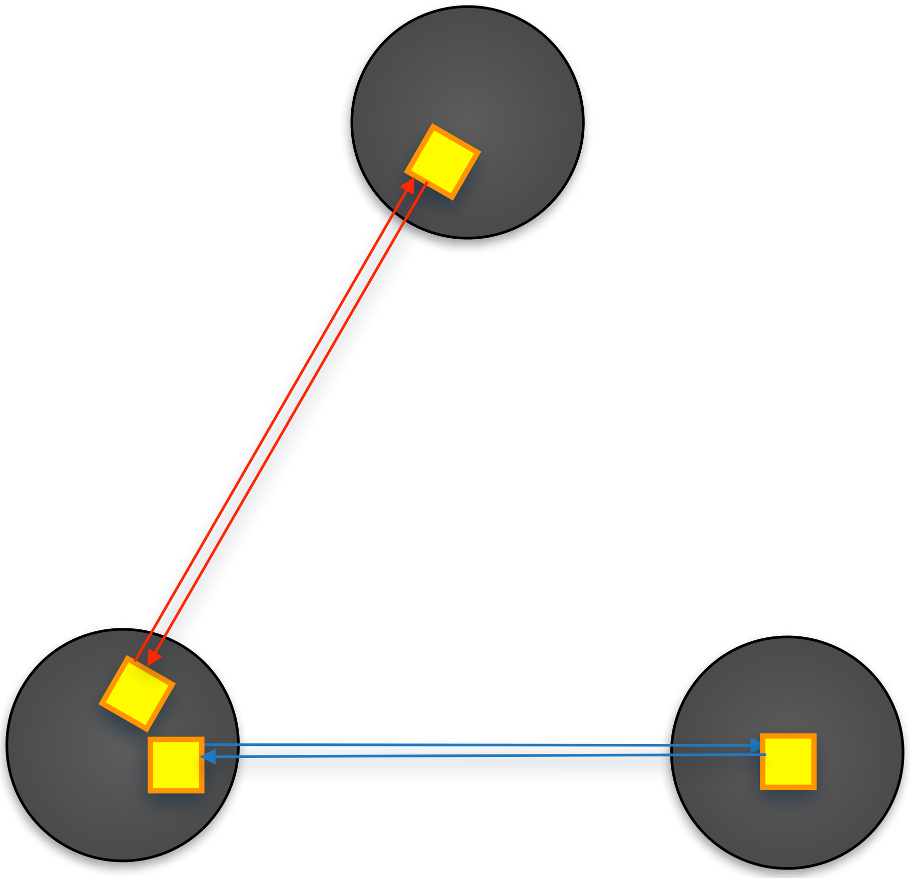
Credit for figure armlength variation: ESOC & Oliver Jennrich

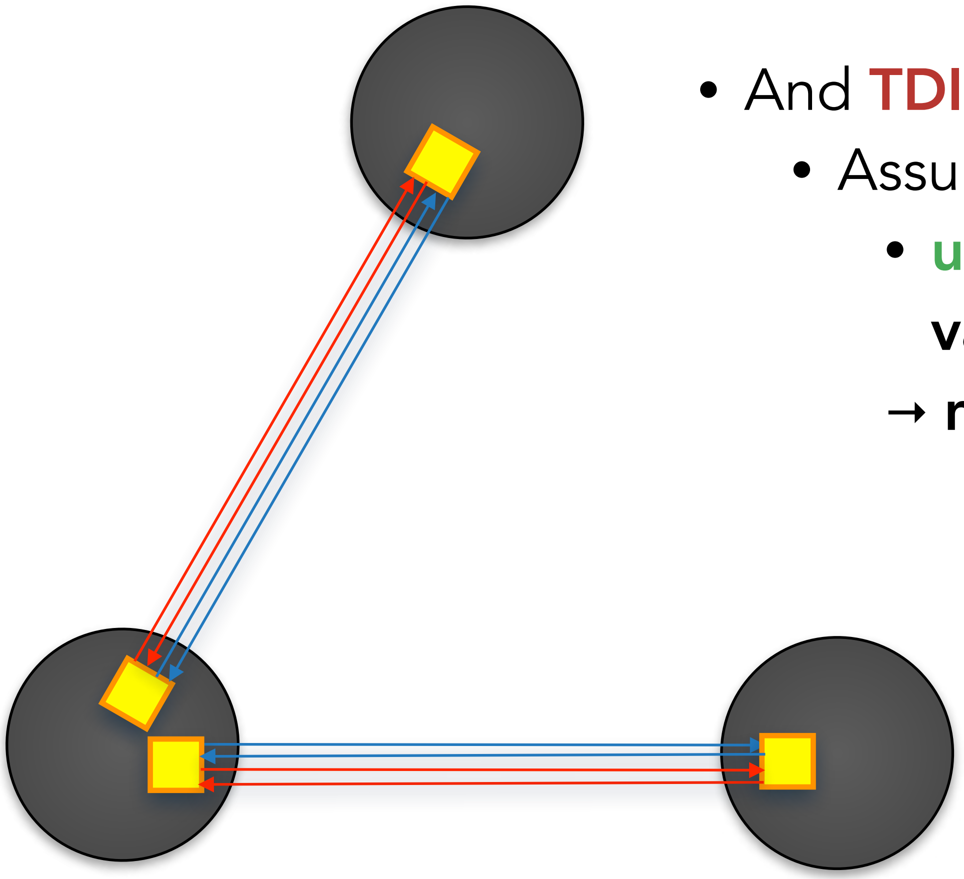
Time Delay Interferometry

- This is "Time Delay Interferometry"
  - means: by combining measurements from different locations at different times, we synthesize an effective equal arm length interferometer

- This is **TDI generation 0:**
  - Assumptions:
    - **equal and constant arm lengths!**

- This is **TDI generation 1:**
  - Assumptions:
    - **unequal and constant arm lengths!**

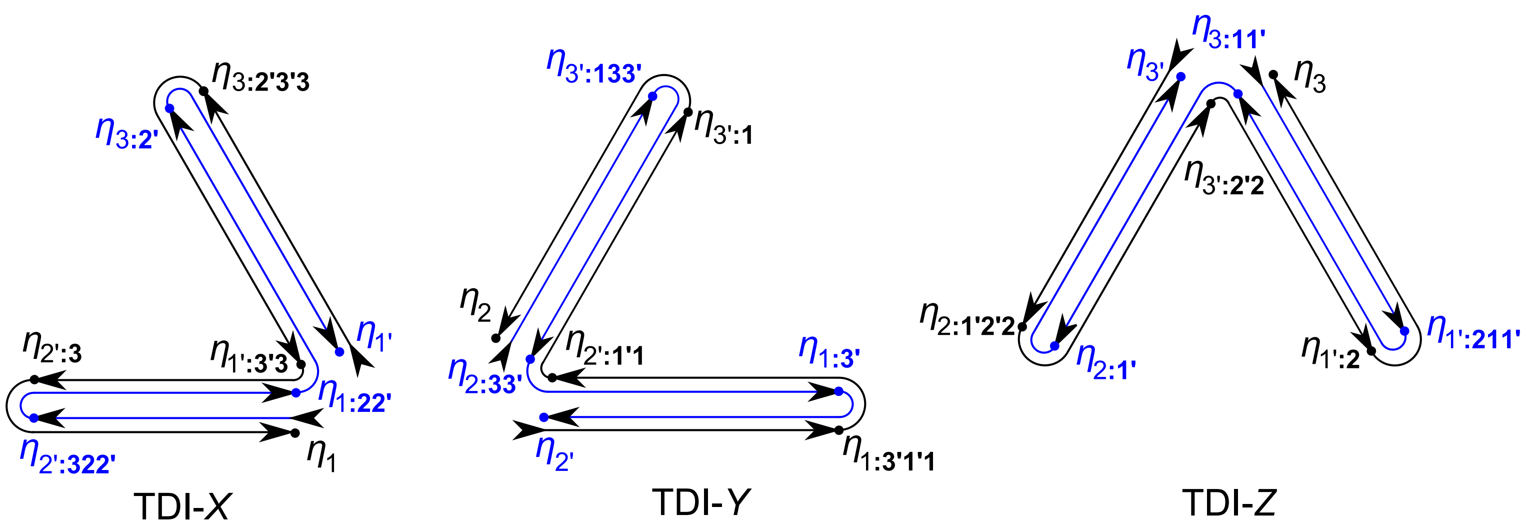


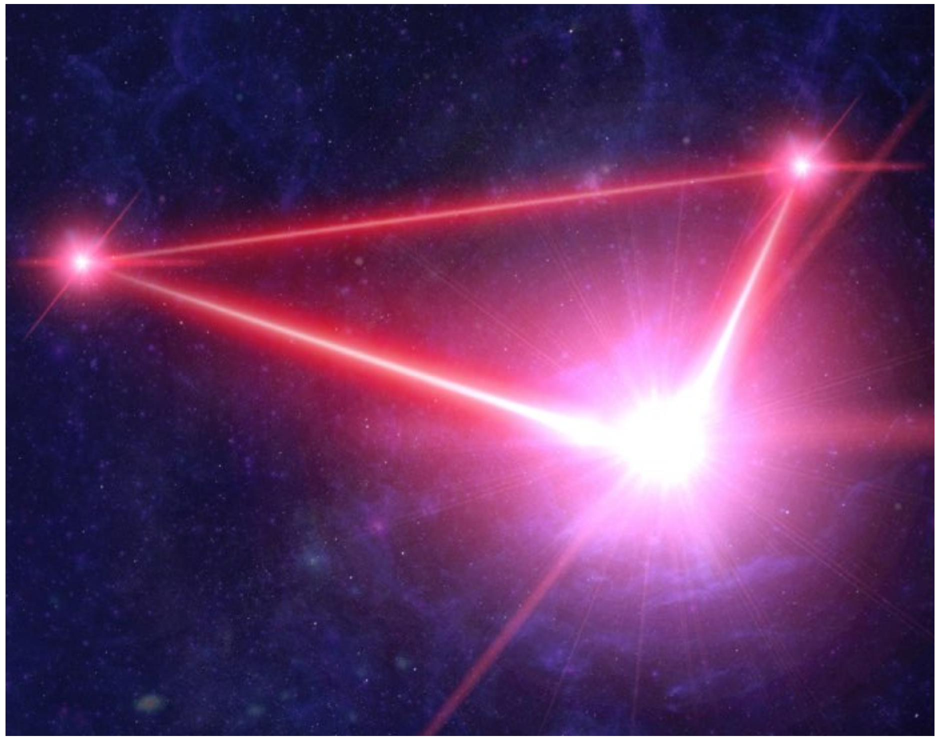


- And **TDI generation 2?**
  - Assumptions:
    - **unequal and time-varying arm lengths!**

  → more paths needed

Michelson combinations





Sagnac combinations

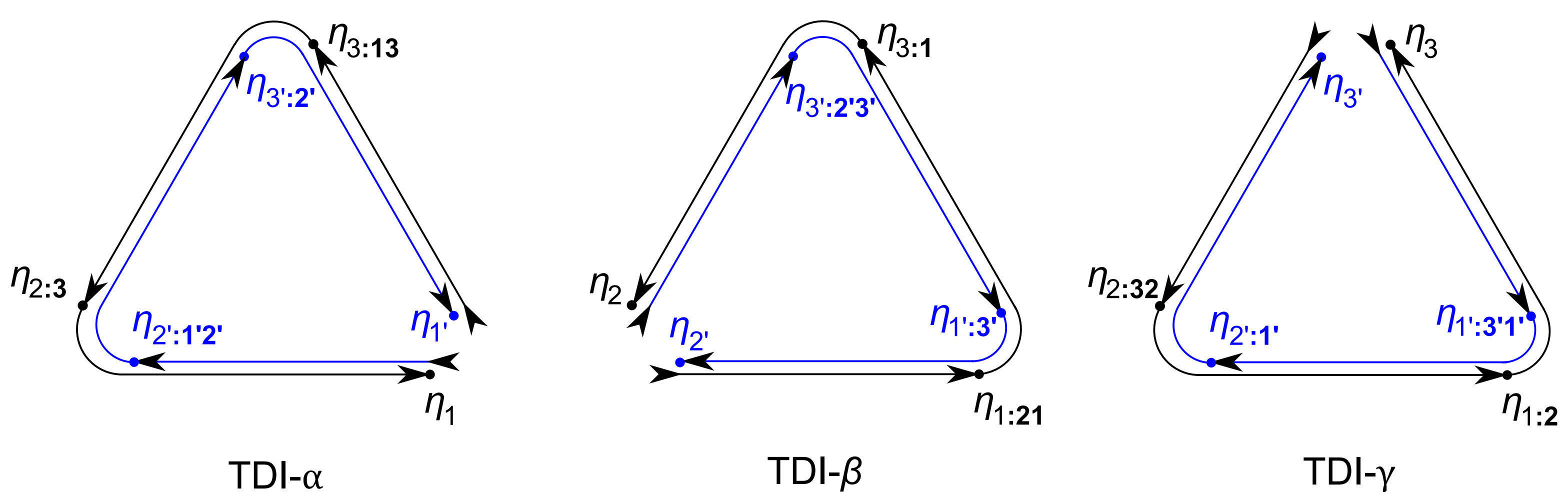


image credit: Markus Otto, thesis
https://doi.org/10.15488/8545

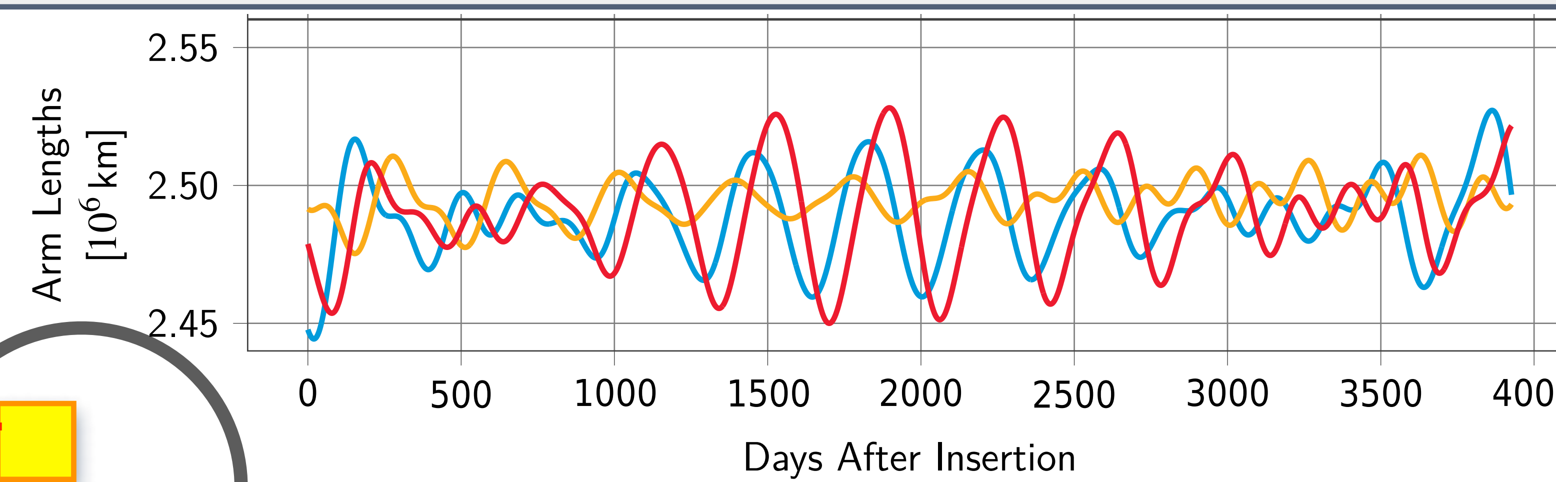
More complete list of combinations:
https://doi.org/10.1088/1361-6382/ab9d5b

Time Delay Interferometry (TDI)

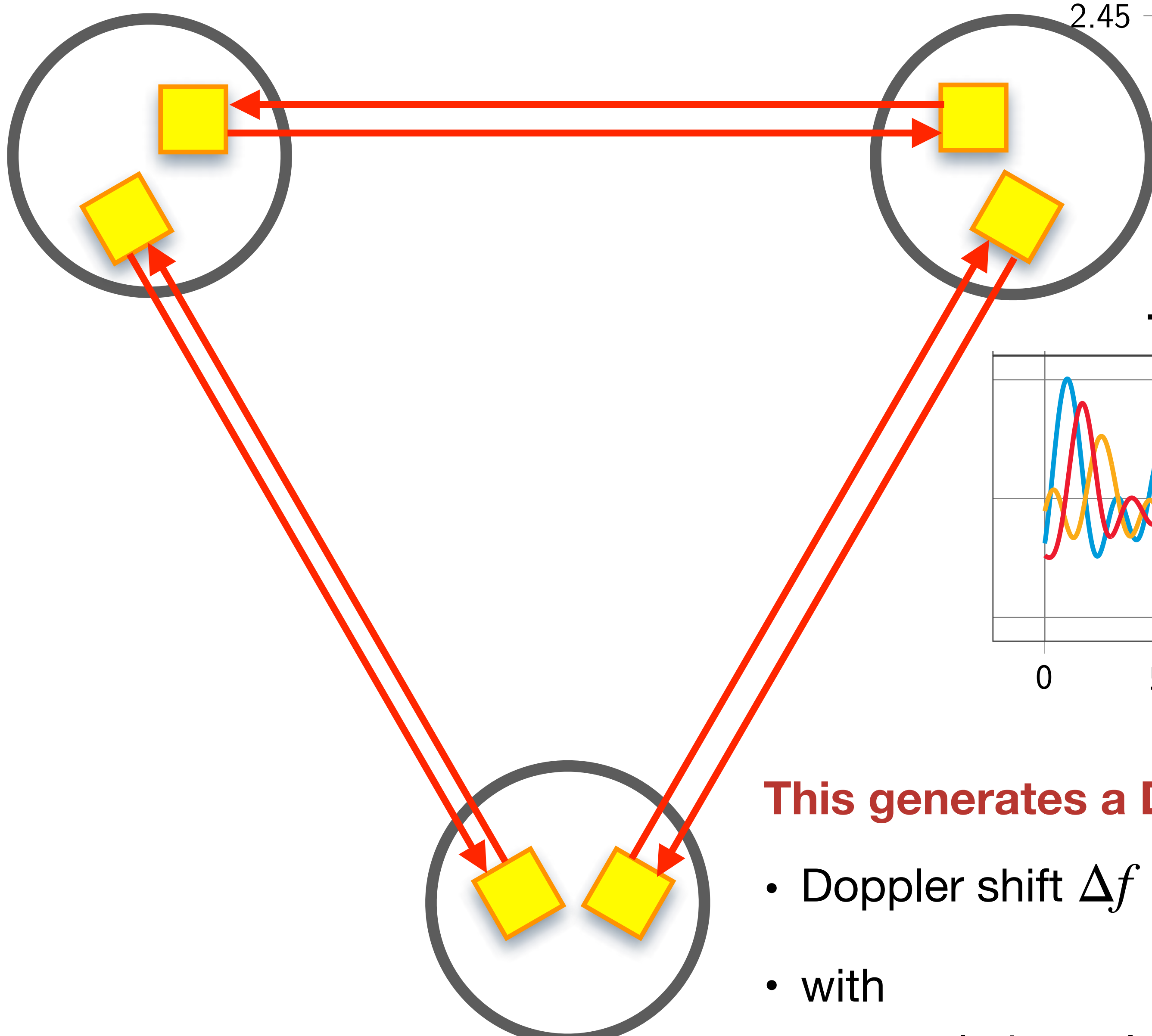
- combinations generate virtual **equal arm length** interferometers
- TDI suppresses ~8 orders of magnitude of laser frequency noise
- there are **many** different combinations (see Muratore et al 2020 Class. Quantum Grav. **37** 185019)
- TDI combinations are **formed on ground**

LISA's changing arm lengths…

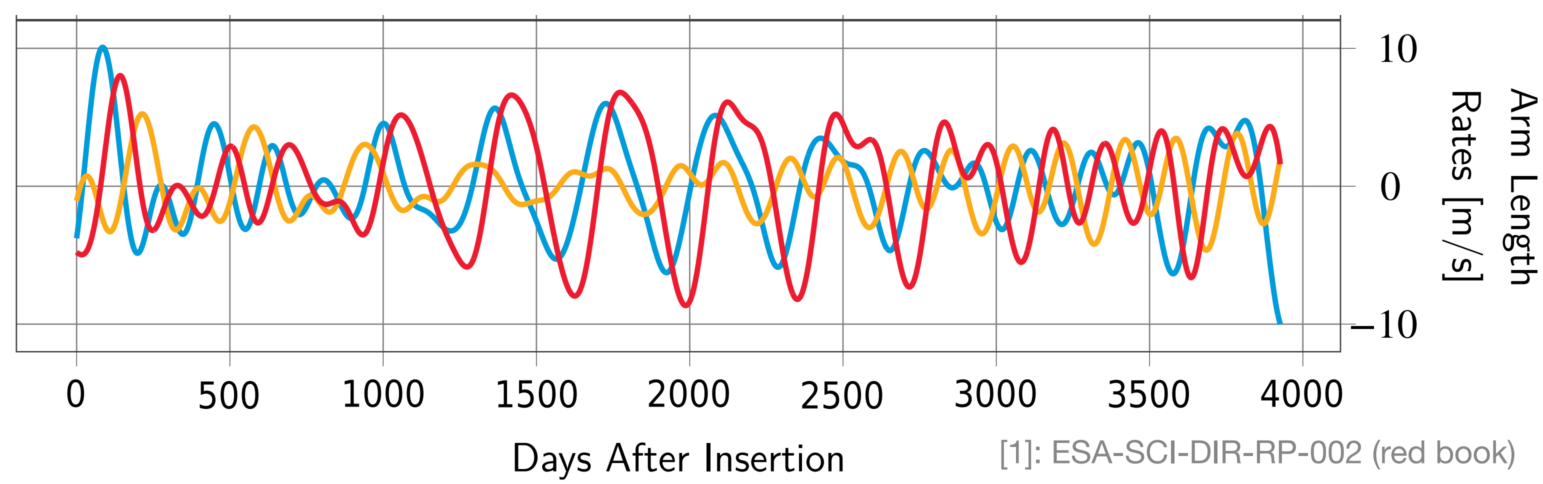
you continue thinking…



… The arm lengths change significantly!



… with arm length rates of m/s-level!



[1]: ESA-SCI-DIR-RP-002 (red book)

This generates a Doppler effect!

- Doppler shift $\Delta f \approx f \cdot \frac{v}{c} = \frac{c}{\lambda} \cdot \frac{v}{c} = \frac{v}{\lambda}$
- with
  - a relative velocity of 10 m/s
  - a waveleght of 1064 nm ≈ 1µm

$$\Delta f = \frac{v}{\lambda} = \frac{10\,\mathrm{m/s}}{10^{-6}\,\mathrm{m}} = 10^7\,\mathrm{Hz} = 10\,\mathrm{MHz}$$

→ The superimposing laser beams have different frequencies!!

CAUTION that's not how LISA works! slide content is drawn to stepwise derive concepts

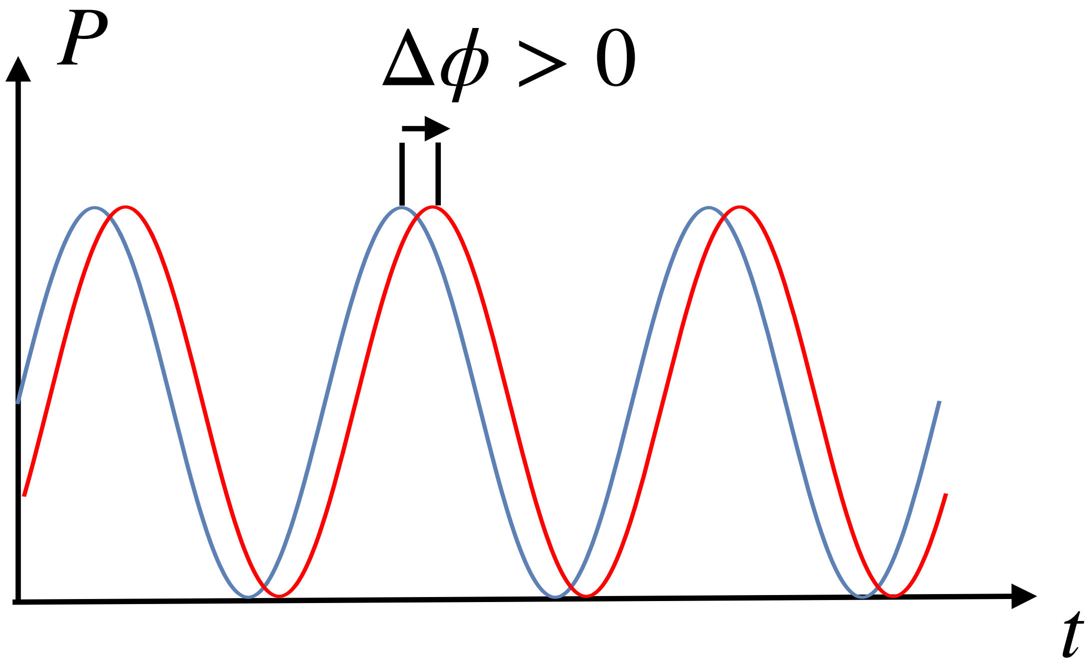
Heterodyne Interferometry!

- **How can one recover the phase in a heterodyne interferometer?**
  - Quick computation with simplified plain wave assumption:
    - amplitudes $a_1, a_2 \in$ Reals
    - powers P_1, P_2 per beam
    - the total detected power P is given by:

$$P = \left| a_1 e^{i\omega_1 t} + a_2 e^{i(\omega_2 t + \Delta\phi)} \right|^2$$
$$= \left(a_1^2 + a_2^2 + 2\Re\left(a_1 a_2 e^{i[(\omega_1 - \omega_2)t - \Delta\phi]} \right) \right)$$
$$= P_1 + P_2 + 2\sqrt{P_1 P_2}\cos[(\omega_2 - \omega_1)t - \Delta\phi]$$

→ Detected power varies harmonically with time!

→ Observation of the phase shift $\Delta\phi$ requires a reference oscillation!



Principles of I/Q phase demodulation

- **How can one recover the phase in a heterodyne interferometer?**
  - Optical power on a photodiode oscillates with time:
    $P = \bar{P}\left(1 + c\cos(\omega_h t - \phi)\right)$
  - How can we extract the phase ϕ?

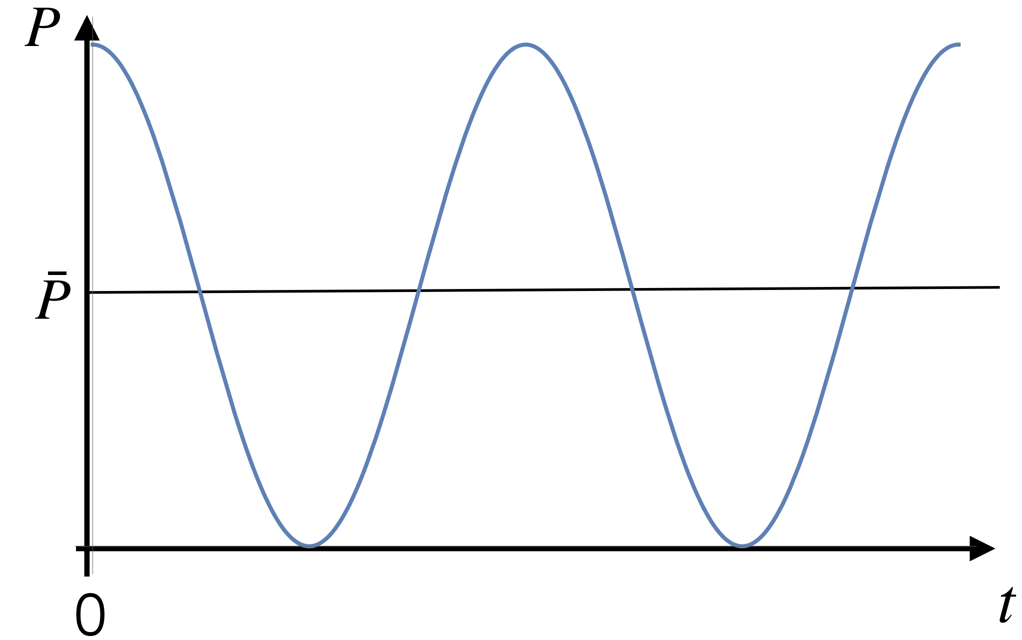
- If we know the heterodyne frequency ω_h:
  - multiply P with either $\cos(\omega_h t)$ or $\sin(\omega_h t)$
    - and integrate over one (or several) full cycle

$$\rightarrow I := \int_{\varphi=0}^{\varphi=2\pi} d\varphi \cos(\varphi)\left(\bar{P}(1 + c\cos(\varphi - \phi))\right)$$
$$= c\bar{P}\pi\cos(\phi)$$

$$\rightarrow Q := \int_{\varphi=0}^{\varphi=2\pi} d\varphi \sin(\varphi)\left(\bar{P}(1 + c\cos(\varphi - \phi))\right)$$
$$= c\bar{P}\pi\sin(\phi)$$

$$\rightarrow \phi = \arctan\left(\frac{Q}{I}\right)$$

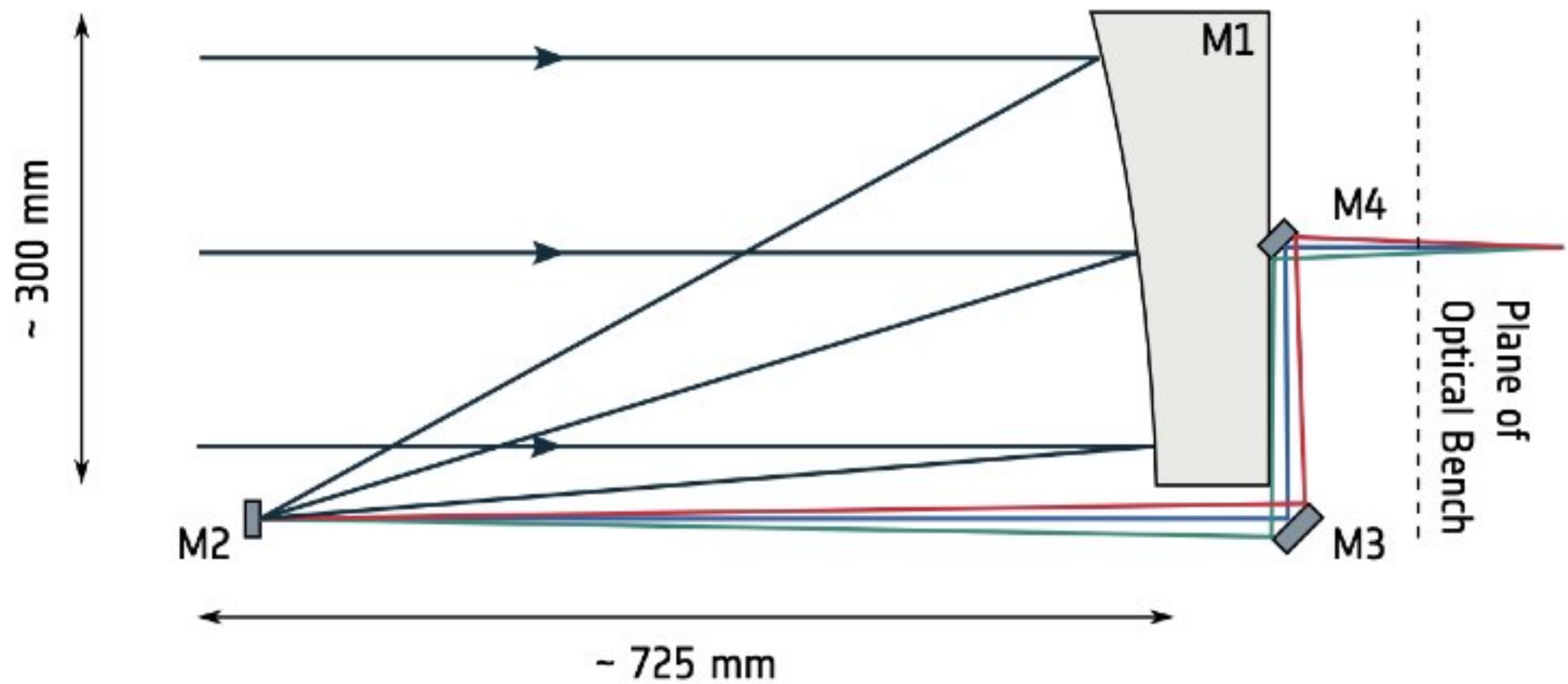
- In experiments:
  - numerical correspondent
  - instead of integral, a sum over a few points is sufficient
- If the heterodyne frequency is not constant and known (like in LISA):
  → find the heterodyne frequency first… (a detail for another day…)



Using: $\varphi = \omega_h t$
I : “in-phase”
Q: “quadrature” = 90° out of phase

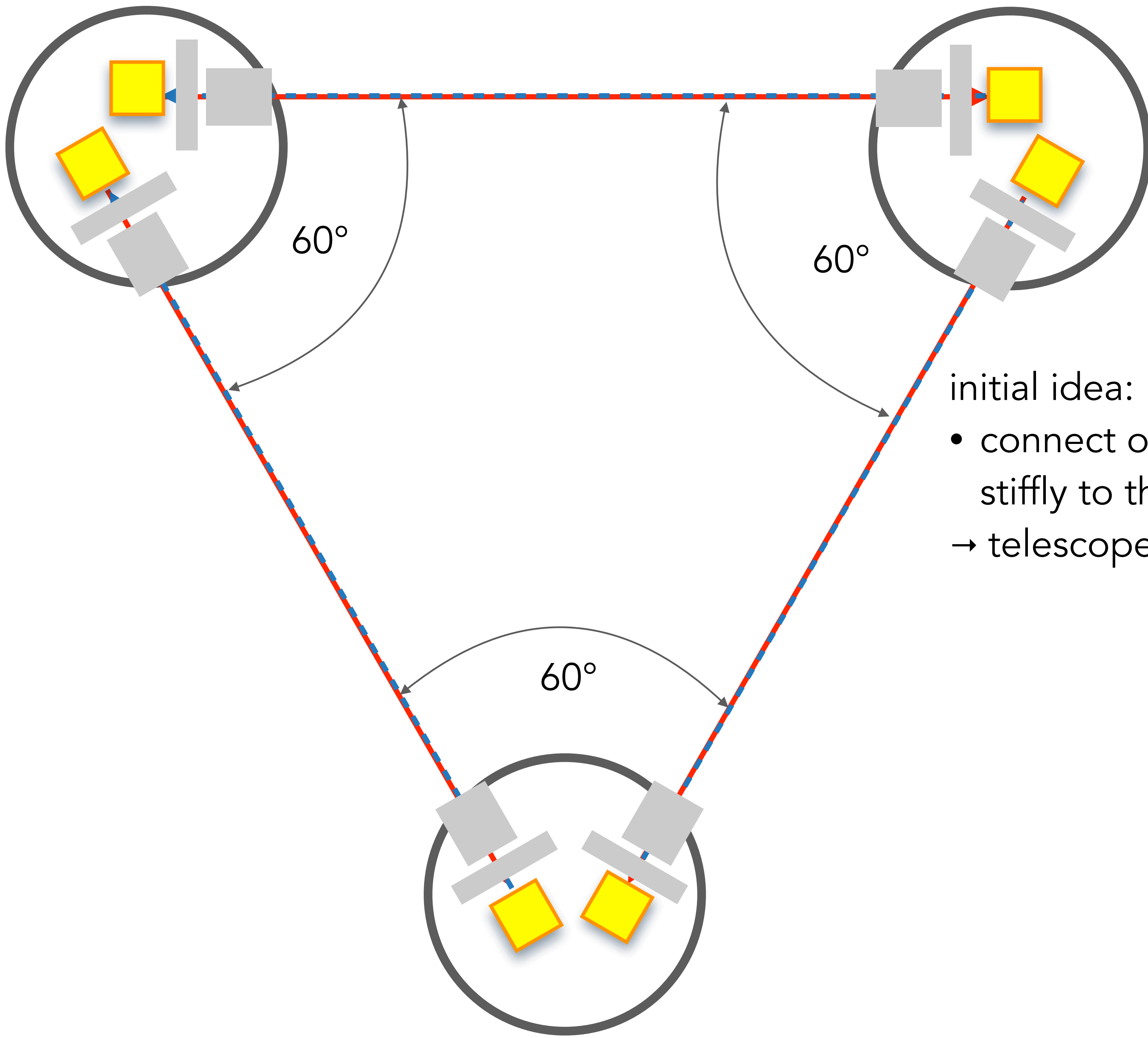
Photodiodes and Telescopes

- ok, so LISA is heterodyne with beat notes in the tens of MHz.
  → we need photodiodes that can be read out that fast!
    → 10 MHz = 1/(100 ns)
  → We assume 1064 nm as wavelength (like current ground-based detectors have)
  → **You find InGaAs photodiodes - but no larger than 1mm diameter!**
- Your telescope diameter is 30 cm
  → the telescope needs to **compress the beam size by about a factor of 300**!
  - In LISA, this is split into two parts:
    - telescope (compression factor 134 [1])
    - an additional imaging system does the rest.



[1] and image: https://www.cosmos.esa.int/web/lisa/lisa-redbook

What you designed so far…

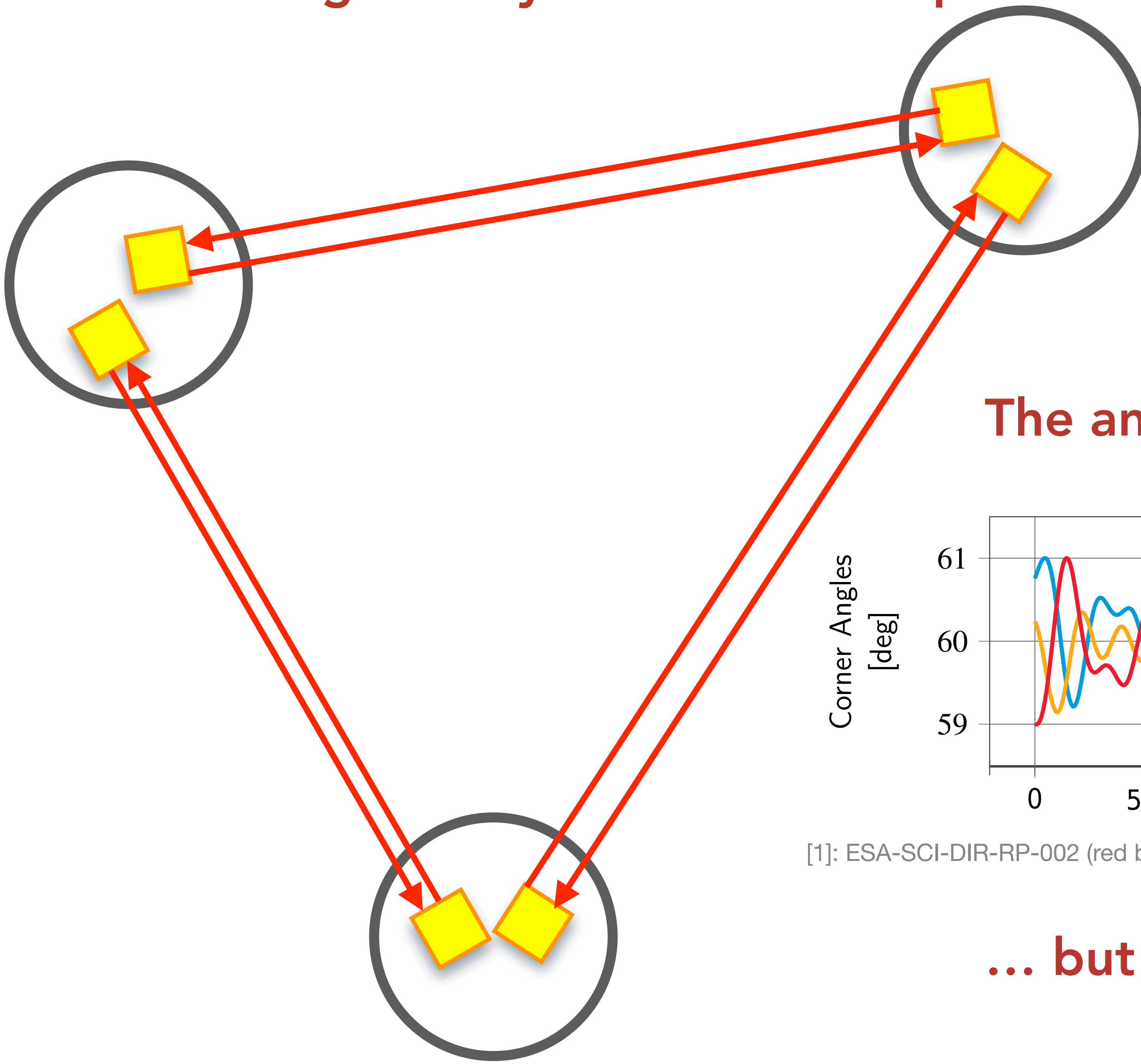


initial idea:

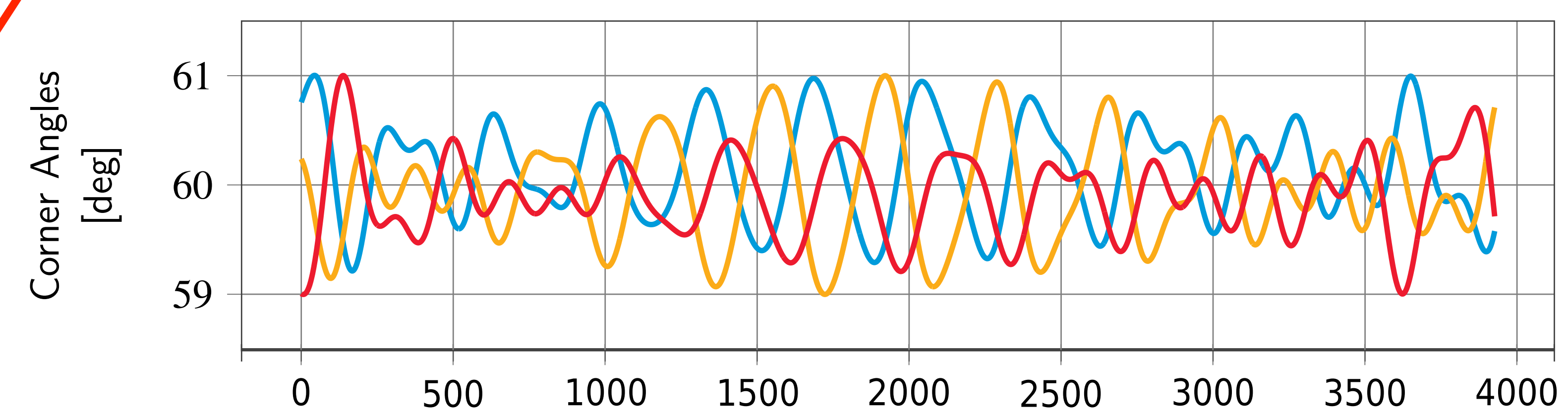
- connect optical bench + telescope stiffly to the telescope

→ telescopes pointing direction is fixed

LISA's arm lengths vary → it's not an equilateral triangle!



The angles vary... (!)



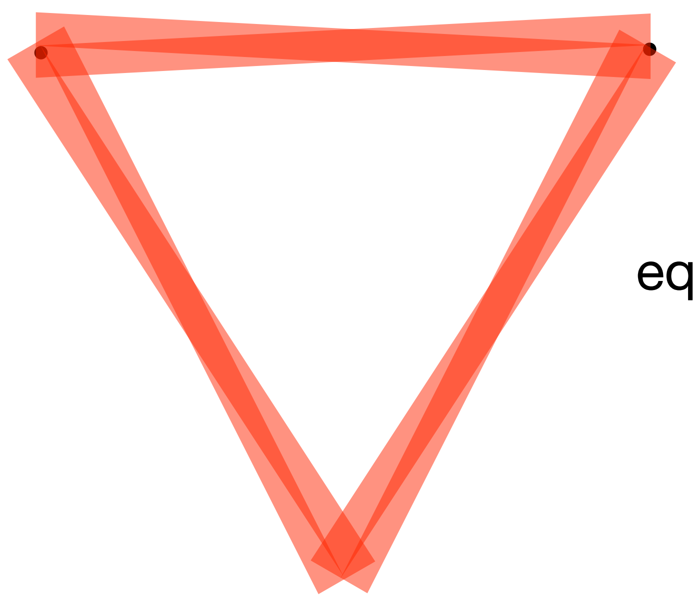
[1]: ESA-SCI-DIR-RP-002 (red book)

... but slowly...

that's not how LISA works!
slide content is drawn to
stepwise derive concepts

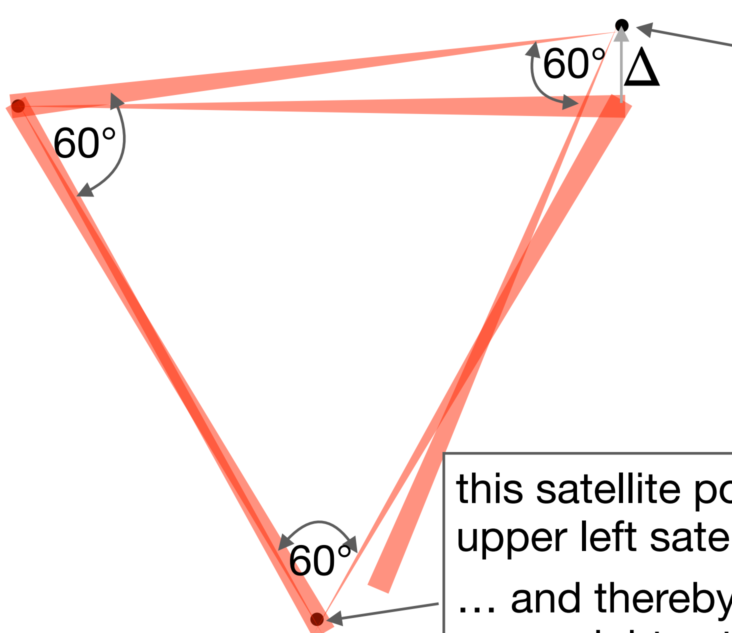
LISA interferometry basics

nominal constellation: all angles are 60°



equilateral LISA

breathing + assumption of stiff pointing directions



satellite shifted against position in an equilateral triangle

it points towards the LHS satellite…

… and thereby misses the lower satellite

this satellite points to the upper left satellite

… and thereby misses the upper right satellite

- The image is exaggerated!
  - Does the beam indeed miss the remote spacecraft?

- quick estimate of the lateral displacement Δ:

$$\Delta = \frac{1^\circ \pi}{180^\circ} \cdot L \approx 0.0174L$$

$$\approx 0.0174 \cdot 2.5 \cdot 10^9\,\mathrm{m} \approx 44\,000\,\mathrm{km}$$

- we roughly estimated the radius of the received beam to be $5\,\mathrm{km}$

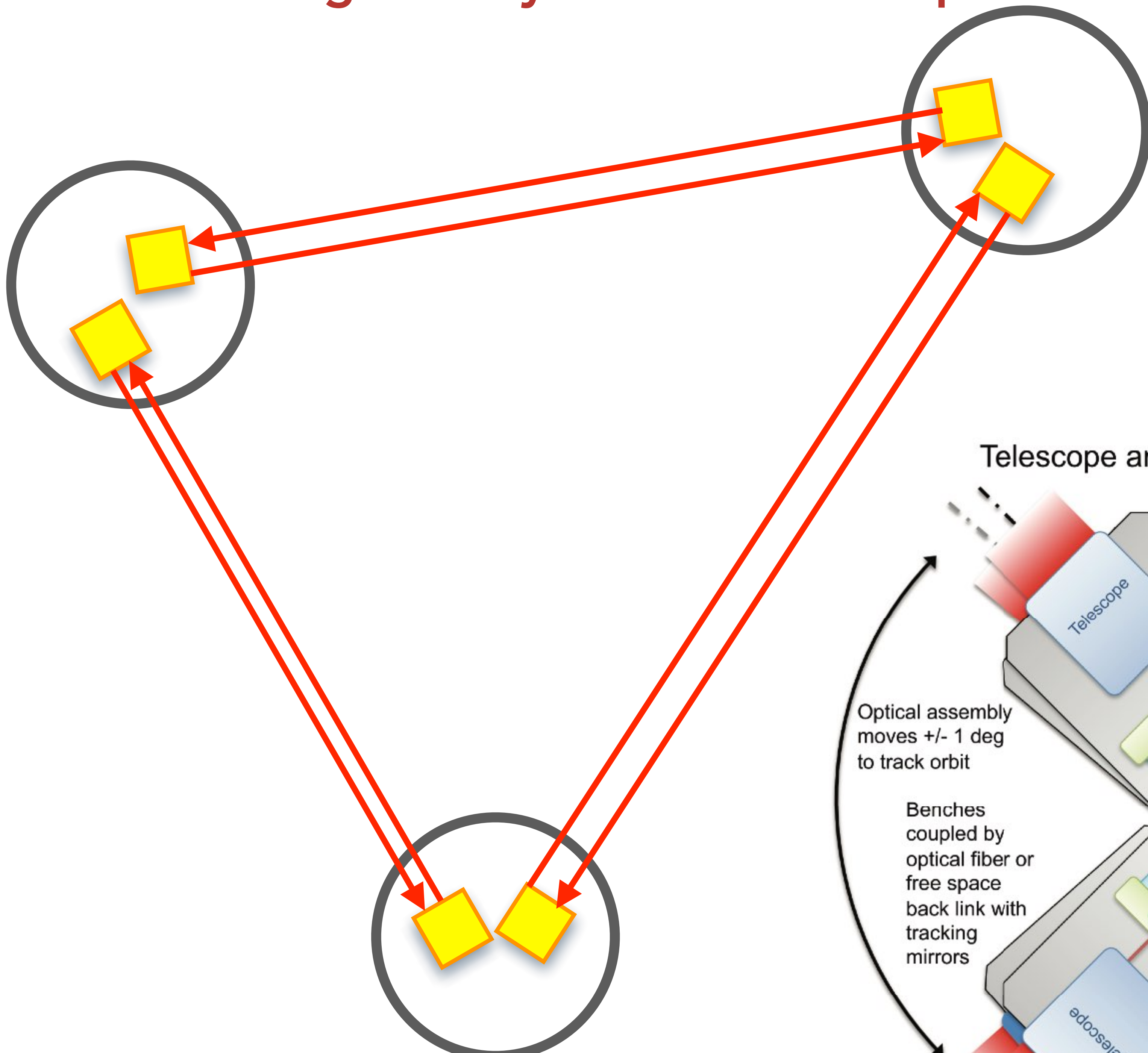
→ beam completely misses the receiver!
By about 9000 beam radii (for our assumptions) …

→ The transmitters need to point towards both remote spacecraft

  → the angle between the transmit-directions needs to adjust to the breathing angle

- Quick note: All telescopes transmit and receive simultaneously
  → but it's the transmit-direction that needs to point
  → the receiver cannot compensate for the beam missing it

LISA's arm lengths vary → it's not an equilateral triangle!



- The triangle's opening angles vary over time
- The laser pointing needs to adapt to the current angle
  - How do you achieve that?

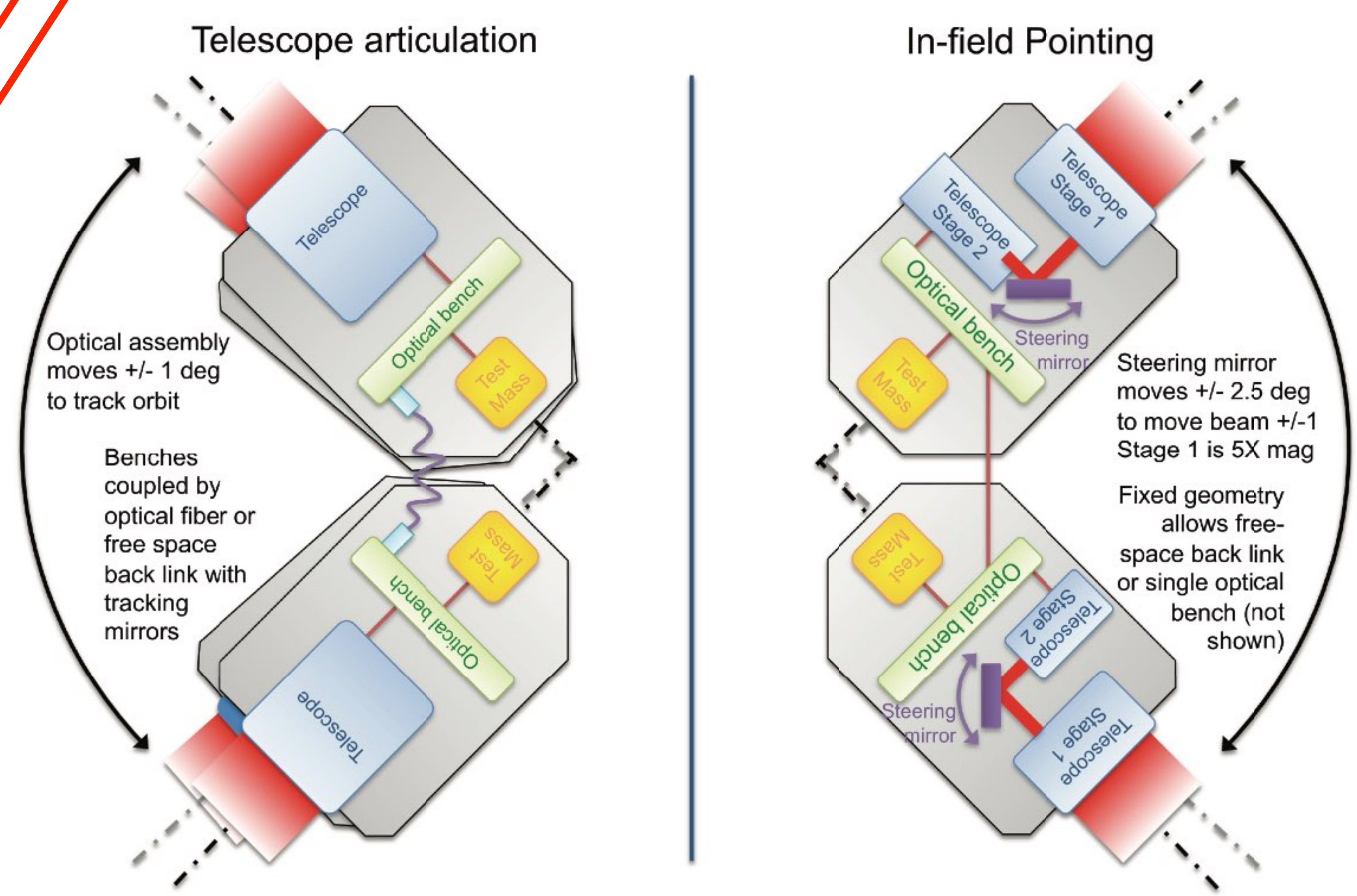


Image from Livas2017, DOI :10.1088/1742-6596/840/1/012015

that's not how LISA works!
slide content is drawn to
stepwise derive concepts

Breathing → In-field pointing (?)

LISA's arm lengths vary → it's not an equilateral triangle!

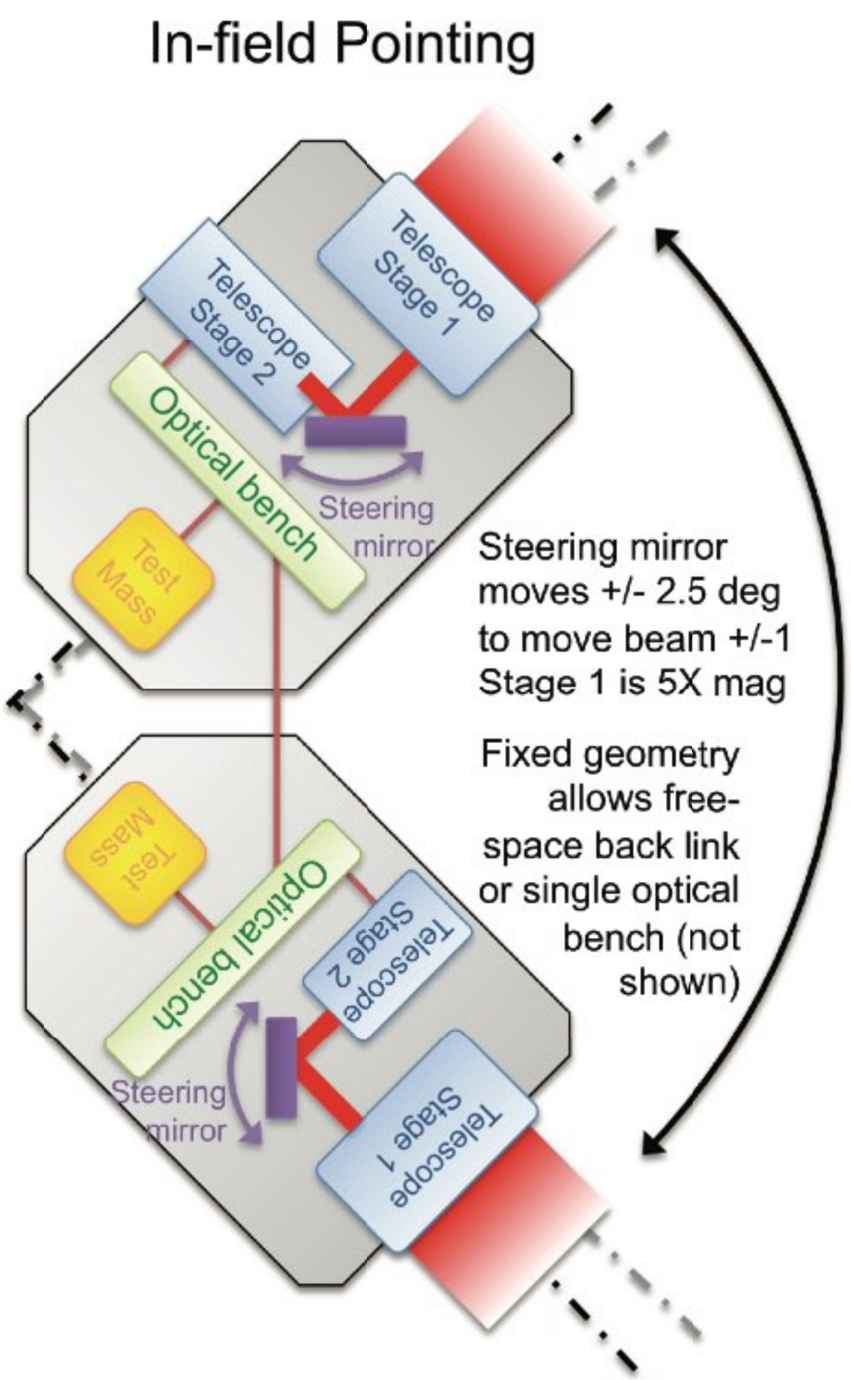


Image from Livas2017, DOI :10.1088/1742-6596/840/1/012015

- **"In-field pointing"**:
  - place a rotatable mirror ("steering mirror") in the beam path
  - consider angular magnification!
    - 1° in free space = 134° behind the telescope!
      = 300° on the PDs!
  - mirror would need to be inside the telescope, before angles are that strongly magnified!
- Some **disadvantages** of in-field pointing
  - rotating component in the beam path
    - → its motion could cause optical pathlength changes
      - → phase noise
    - → tilting beam inside the telescope probes different mirror parts.
      - → aberration / wavefront errors / … → phase noise

LISA's arm lengths vary → it's not an equilateral triangle!

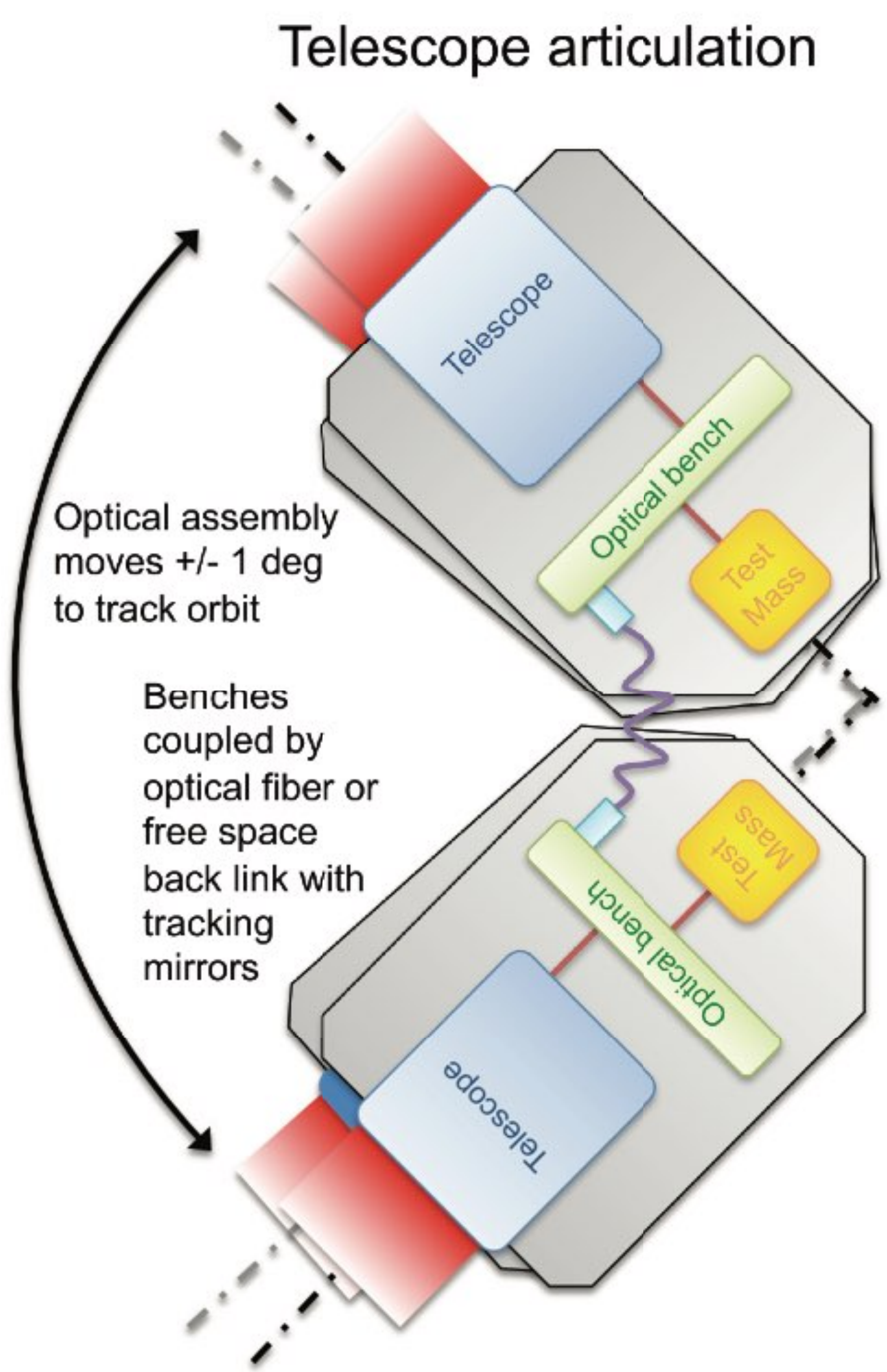


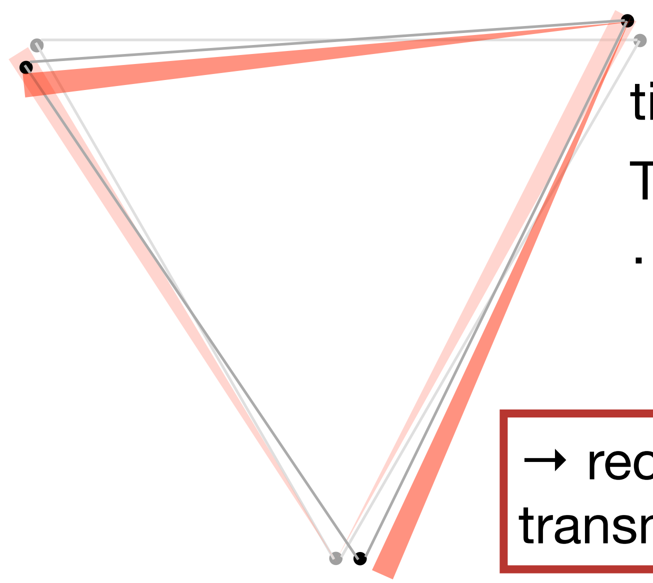
Image from Livas2017, DOI :10.1088/1742-6596/840/1/012015

- Alternatively: rotate all optically relevant components together!
  - 1 test mass (+surrounding structure),
  - 1 optical bench (mirrors, beamsplitters, photodiodes, to perform the interferometry)
  - 1 telescope

  } **MOSA**
- **MOSA** = **M**oving **O**ptical **S**ub**A**ssembly
  - 2 MOSAs per spacecraft
  - rotate relative to one-another
- Advantage
  - no moving component in the beam path
  - rotation is common-mode for all optics

    → **no beam walk over mirror surfaces**
- Disadvantage
  - rotation of a large, heavy, inert structure

→ in this trade-off (in-field pointing vs MOSAs), you decide for telescope pointing (MOSAs)



time $t = 8.3\,\mathrm{s}$:

The remote spacecraft receive the light
… and need to transmit in the direction
of the other remote spacecraft at $t = 16.7\,\mathrm{s}$

even if the arm lengths were constant…
… the constellation moves!

→ receiving direction and transmission direction do not match!!

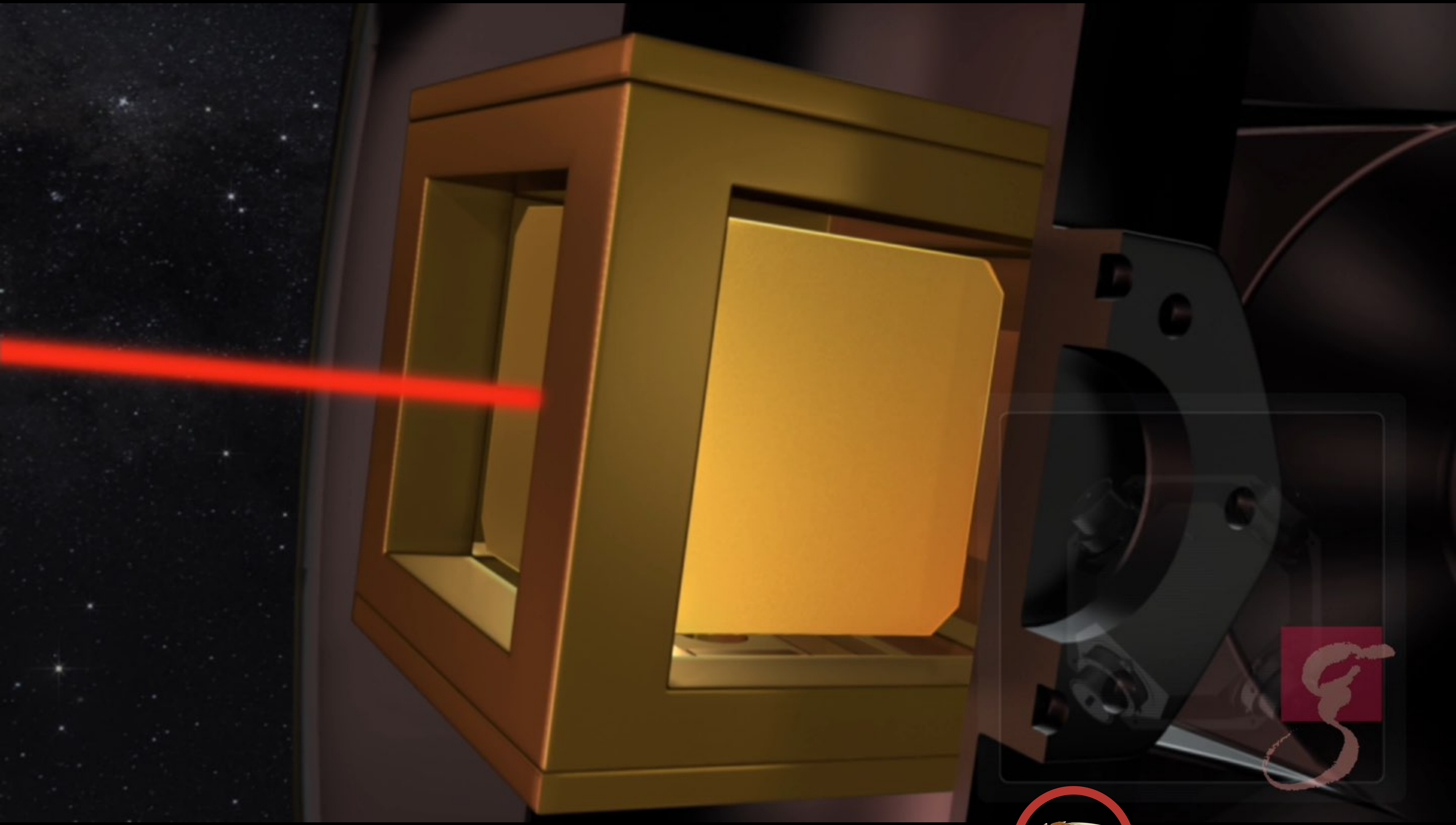
time $t = 0$:

SC_1 transmits light **towards** remote SC-**position in 8.3 s**!

→ You decide:

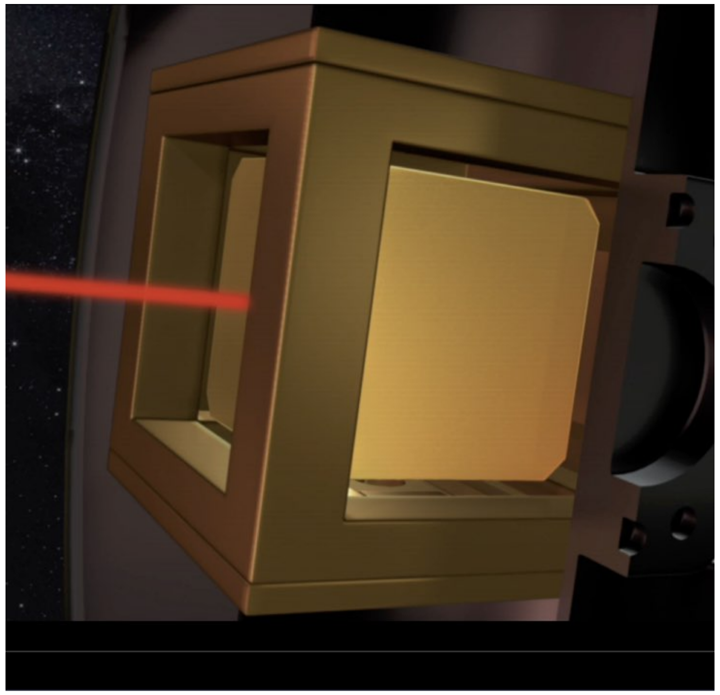
- LISA needs a mechanism that offsets the transmit axis from the received axis.
- You name it: “Point Ahead Angle Mechanism” (PAAM)

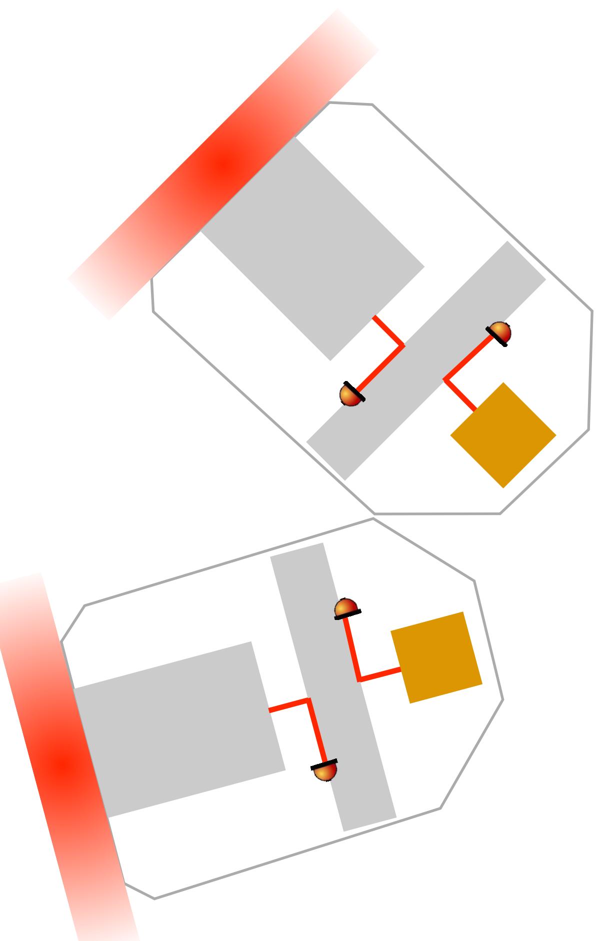
… Drag-free control…

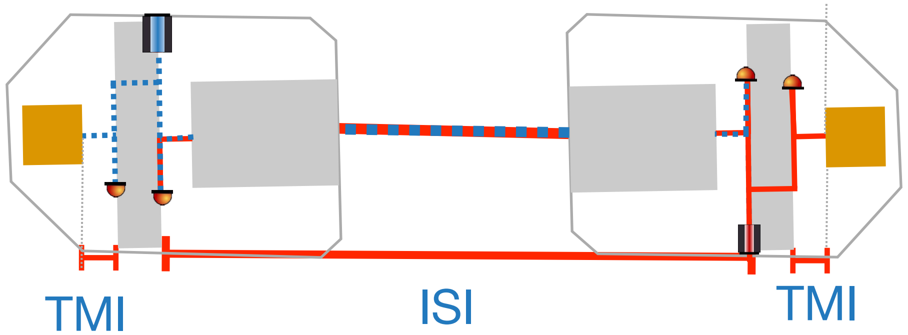


Drag-free control and the TMI

- **Drag-free control** = commanding the spacecraft to follow the test mass along the “sensitive axis”
- sensitive axis: line connecting a local with a remote test mass
- Requires a control loop
  - what’s the input signal?
    → electrostatic readout?
      → too noisy
    → **optical readout**!
      → we need an interferometer to read out the test mass displacement relative to the electrode housing
      → “**Test mass interferometer** (TMI)”
        → optical bench and electrode housing: stiffly connected
          → displacement TM vs electrode housing = displacement TM vs optical bench
          → Reflect the received beam, the transmit beam **and** the TMI-beam from the test mass?
            → sounds complicated…
          → instead: split each single link into three measurements!
            → TMI measures local TM relative to local optical bench
            → **Inter-Satallite Interferometer (ISI)** measures local opical bench relative to a remote optical bench
            → remote TMI measures remote TM relative to the remote optical bench







Those are not interferometers yet!

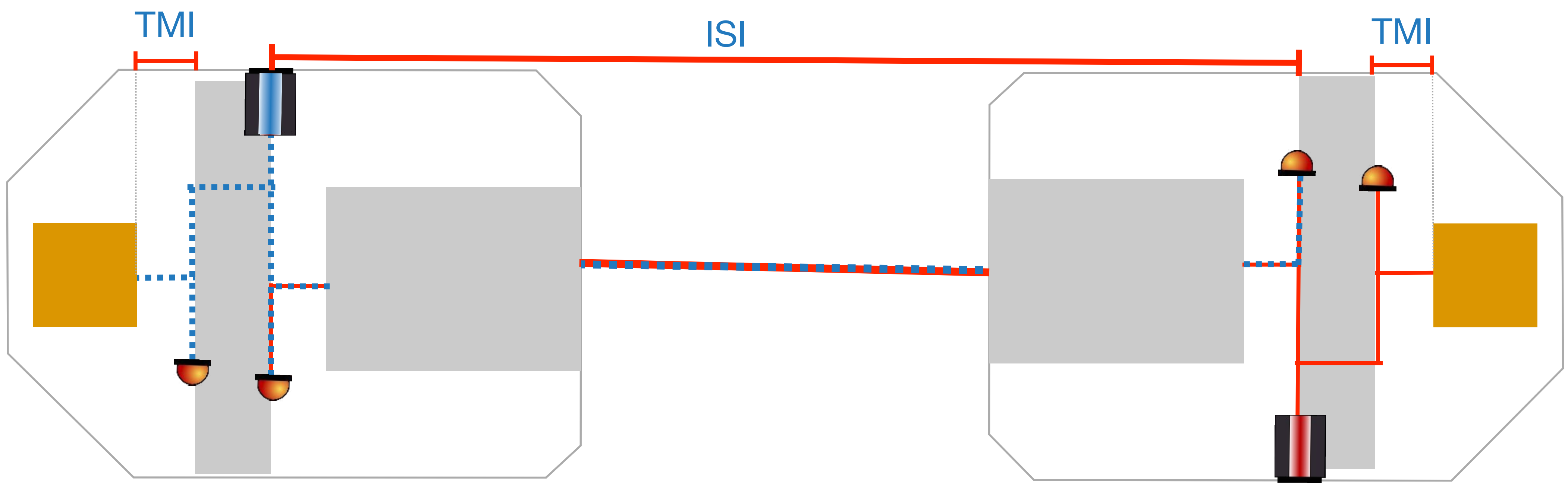
Each shown beam needs to interfere with a reference beam.

→ What light should we use?

ISI: received beam
+ small part of the transmit beam

TMI: small part of transmit beam + ???

- **Test mass interferometer**



TMI:

- **interfere beam with itself?**
  - → Would make the TMI homodyne!
  - → not ideal:
    - → ISI heterodyne, but TMI homodyne?
    - → would require completely different phase readout
  - → NO!
  - → Better use a frequency-shifted reference beam!

→ What light interferes where?

ISI: received beam
+ small part of the transmit beam

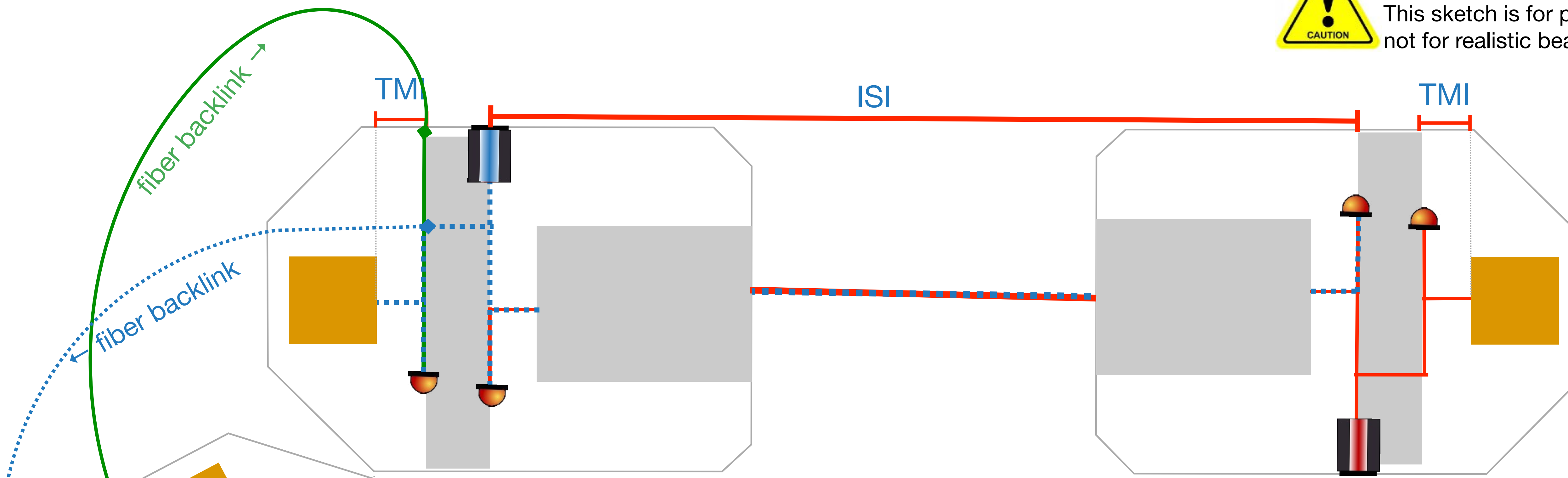
TMI: small part of transmit beam
+ ???

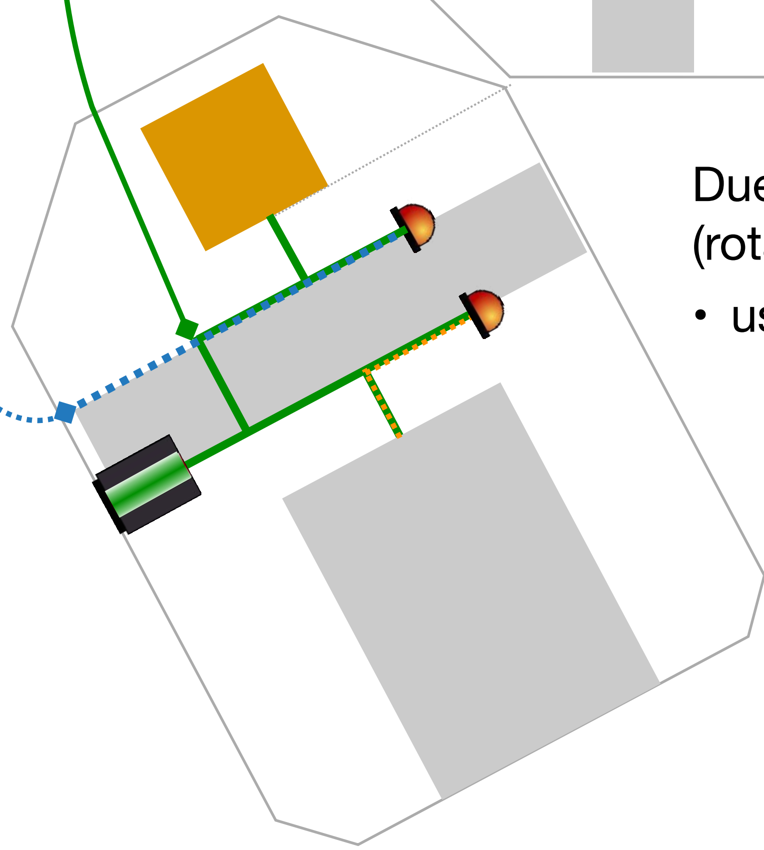
Mirrors are suppressed in this reduced sketch.
This sketch is for principal ideas, not for realistic beam paths.

Test mass interferometer (TMI)

- **Test mass interferometer**

Mirrors are suppressed in this reduced sketch.
This sketch is for principal ideas, not for realistic beam paths.





Due to MOSA's
(rotating optical benches):

- use optical fibers for beam delivery
  - "Fiber Backlink"

→ What light interferes where?

ISI: received beam
+ small part of the transmit beam

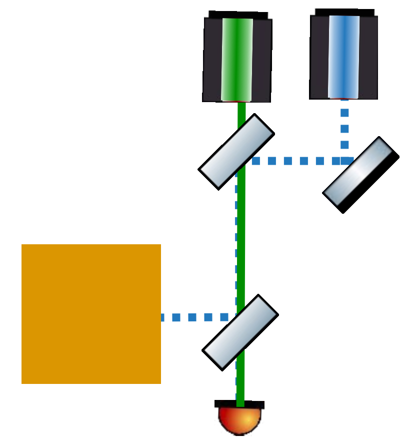
TMI: small part of transmit beam
+ beam from adjacent bench!

→ Interference between different lasers?

→ Apples & Pairs?

Test mass interferometer (TMI)

- **How can one recover the phase in a heterodyne interferometer?**
  - Optical power on a photodiode oscillates with time:
    $P = \bar{P}\left(1 + c\cos(\omega_h t - \phi)\right)$
  - How can we extract the phase ϕ?

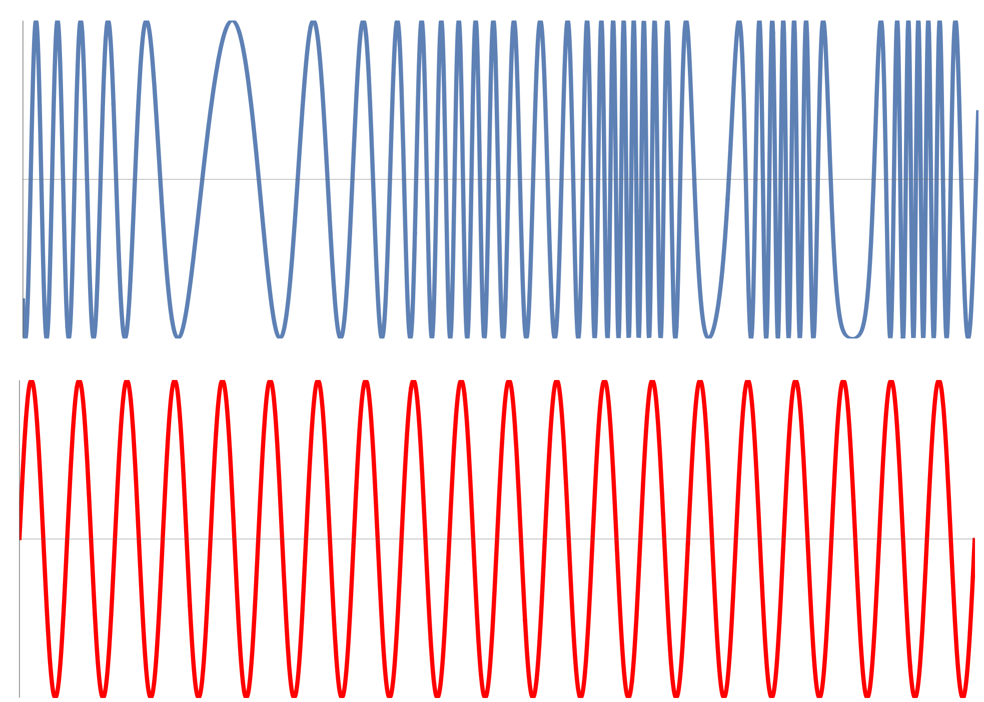


- If we know the heterodyne frequency ω_h:
  - define $I := \int_{\varphi=0}^{\varphi=2\pi} d\varphi \cos(\varphi)\left(\bar{P}(1 + c\cos(\varphi - \phi)\right)$
    $= c\bar{P}\pi\cos(\phi)$
  - define $Q := \int_{\varphi=0}^{\varphi=2\pi} d\varphi \sin(\varphi)\left(\bar{P}(1 + c\cos(\varphi - \phi)\right)$
    $= c\bar{P}\pi\sin(\phi)$

$$\rightarrow \phi = \arctan\left(\frac{Q}{I}\right)$$

beat note (exaggerated!)

reference in demodulation



- In experiments:
  - numerical correspondent
  - instead of integral, a sum over a few points is sufficient
- If the heterodyne frequency is not constant and known (like in LISA):
  → find the heterodyne frequency first… (a detail for another day…)

Test mass interferometer (TMI)

- **Test mass interferometer**

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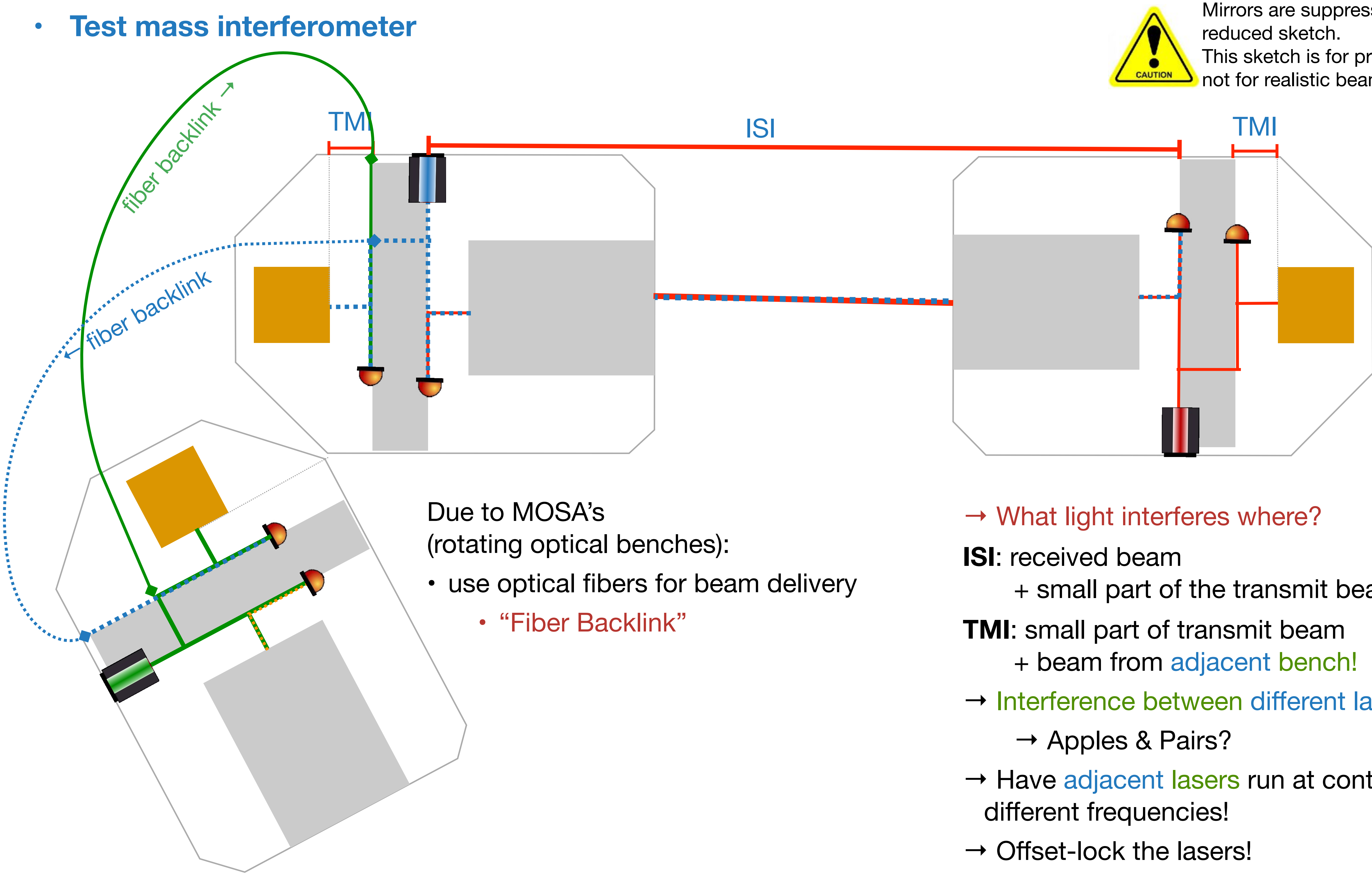
This sketch is for principal ideas, not for realistic beam paths.



Due to MOSA's (rotating optical benches):

- use optical fibers for beam delivery
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→ What light interferes where?

ISI: received beam
+ small part of the transmit beam

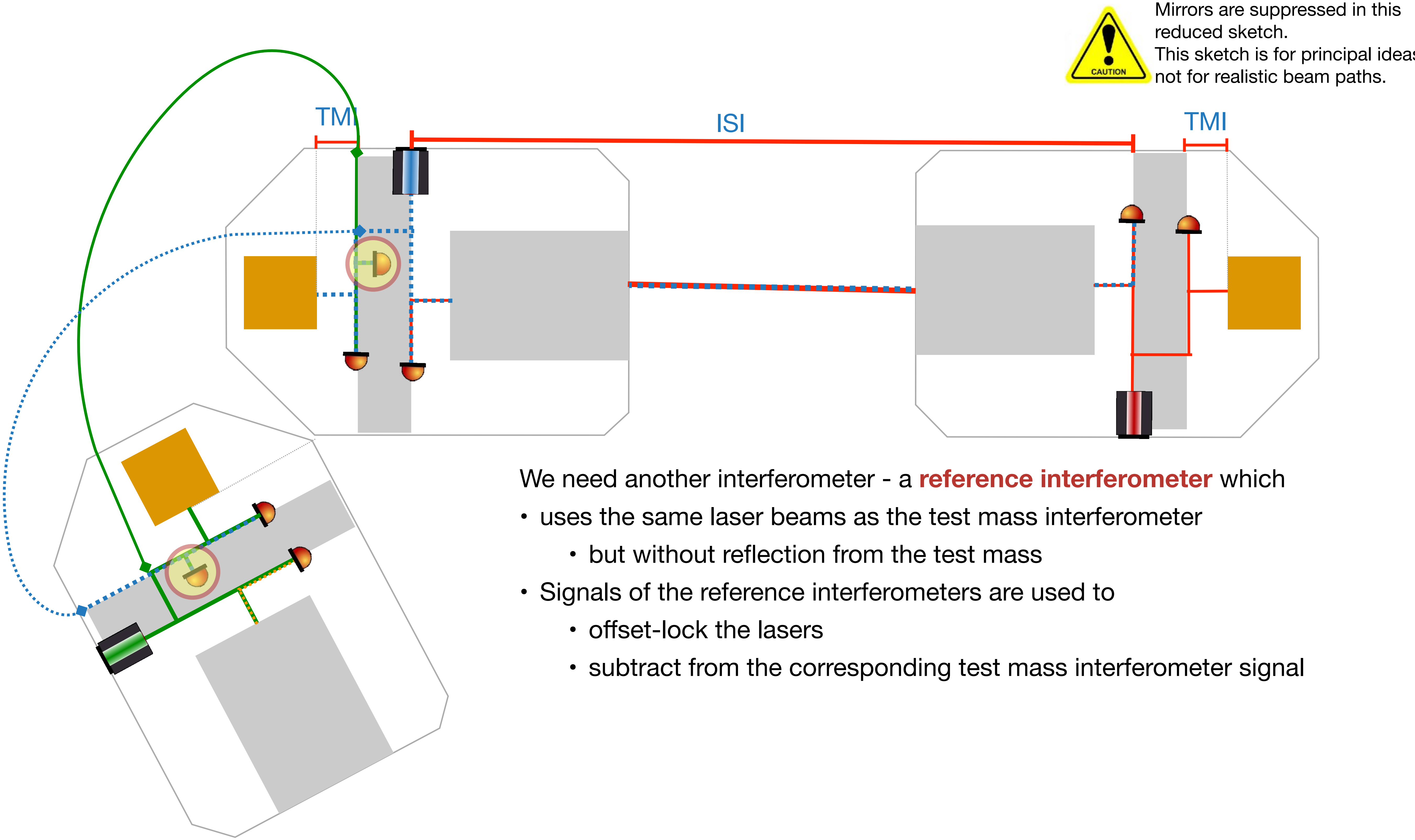
TMI: small part of transmit beam
+ beam from adjacent bench!

→ Interference between different lasers?
→ Apples & Pairs?

→ Have adjacent lasers run at controlled, different frequencies!

→ Offset-lock the lasers!
→ How?

Reference Interferometer



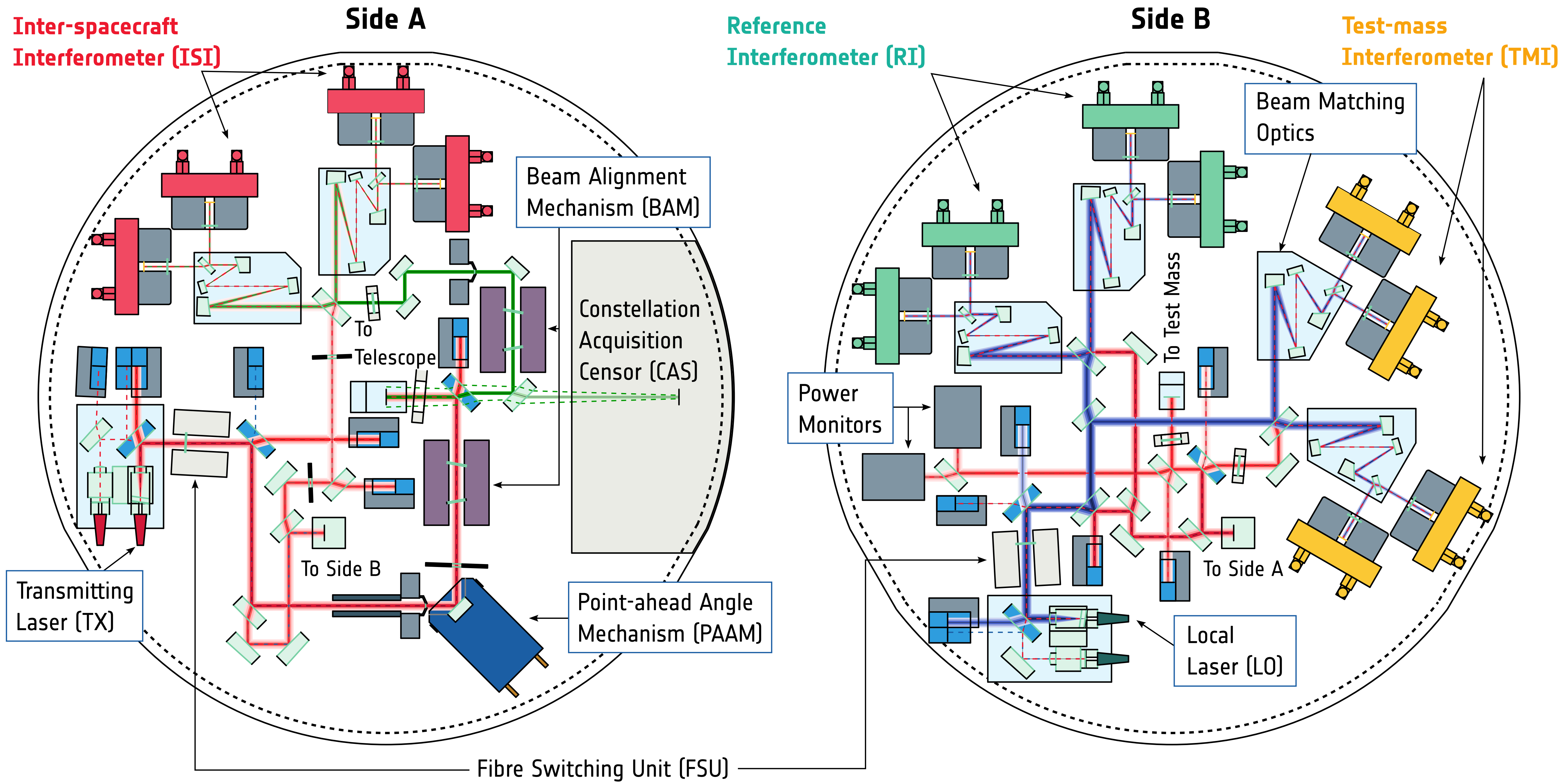
We need another interferometer - a **reference interferometer** which

- uses the same laser beams as the test mass interferometer
  - but without reflection from the test mass
- Signals of the reference interferometers are used to
  - offset-lock the lasers
  - subtract from the corresponding test mass interferometer signal

Bill Weber’s slide from yesterday…

see slide “Selected <<little enginering details>> en route to LISA science” by Bill Weber

A comparison with the official planning (2023)



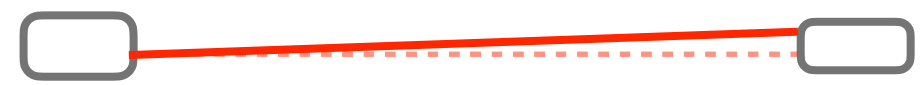
[1]: ESA-SCI-DIR-RP-002 (red book)

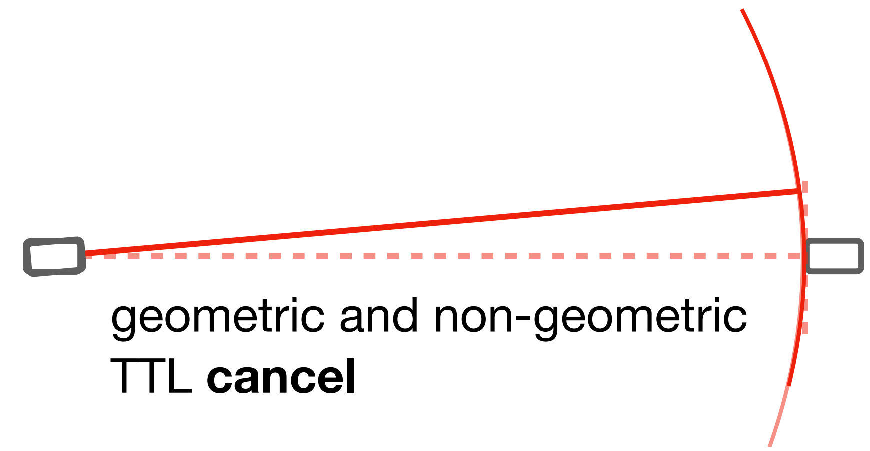
Beam alignment mechanism??

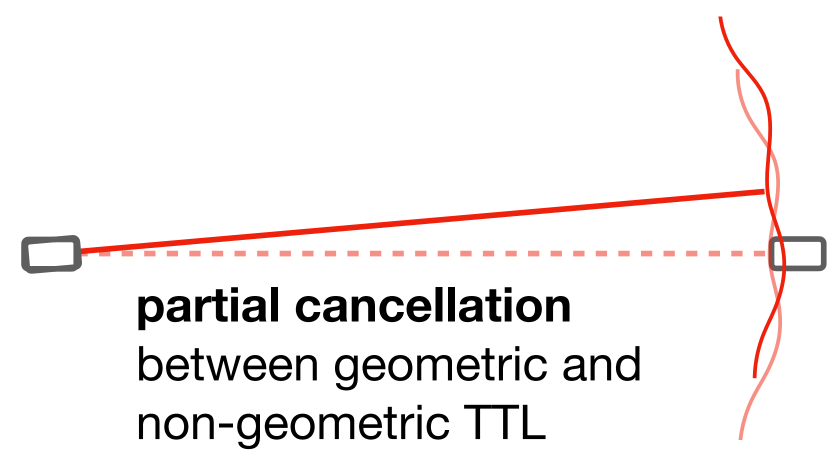
- **aim of the interferometer:**
  - observe distance variation
- **animation: SC-tilts**
  - → beam propagates longer distance
  - → interferometer would read phase change
  - → misinterpreted as distance variation
- **assume this dynamically:**
  - → angular spacecraft jitter → displacement readout noise

→ that's **"Tilt-to-length" (TTL) coupling noise**

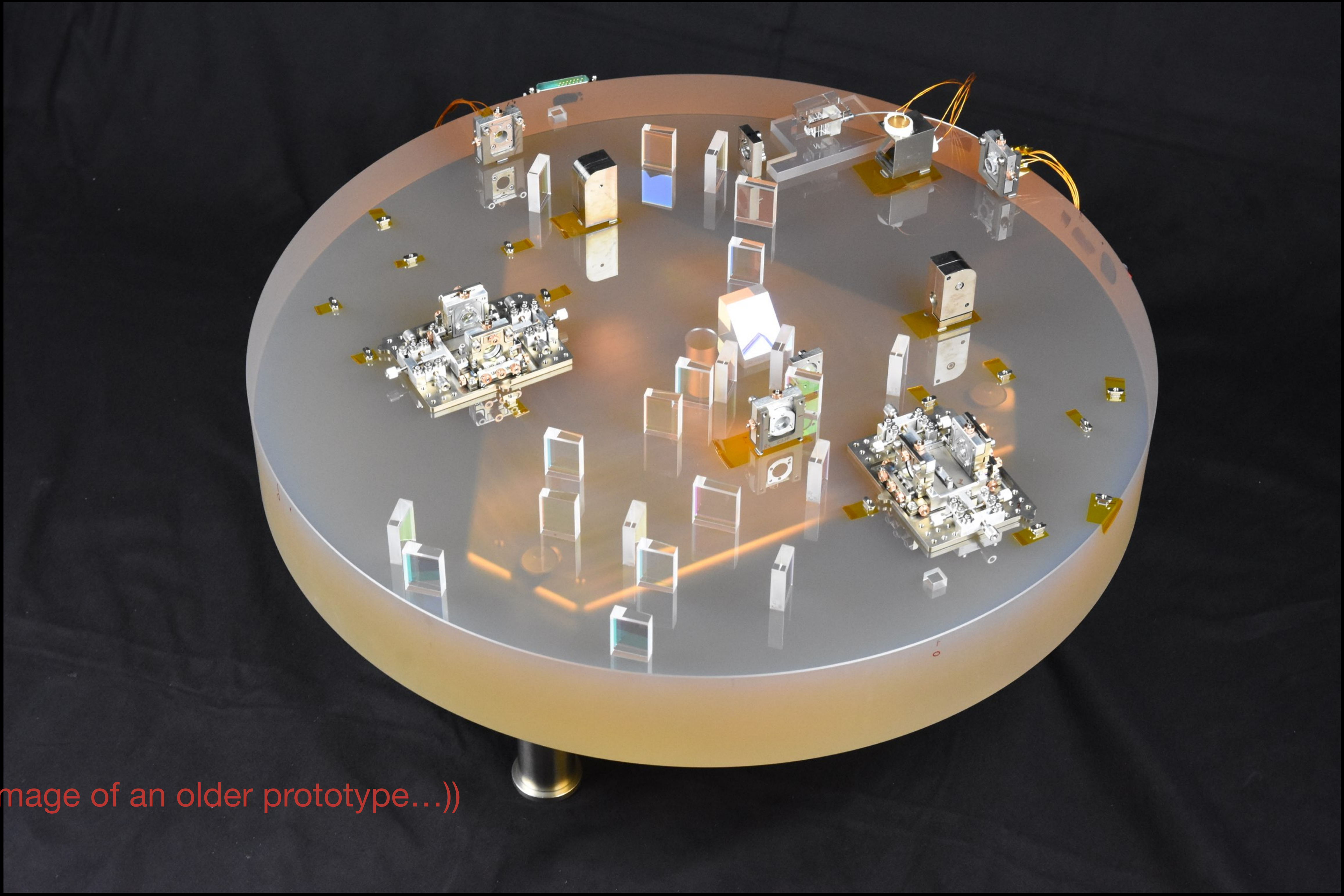
- let's look again:
  - Would this indeed cause displacement readout phase noise?
    - very large propagation distance
      - → wavefront rotates roughly around its curvature centre
      - → perfect wavefront? → small effect (no!)
      - → realistic wavefront with wavefront errors? → significant effect! (**YES**! - but not what you expect from geometry…)
  - → Lesson learned:
    - → TTL has two different types of contributions:
      - → "geometric TTL" = changes in the optical pathlength
      - → "non-geometric TTL" = phase changes in the absence of geometric TTL







Zerodur optical bench



((image of an older prototype...))

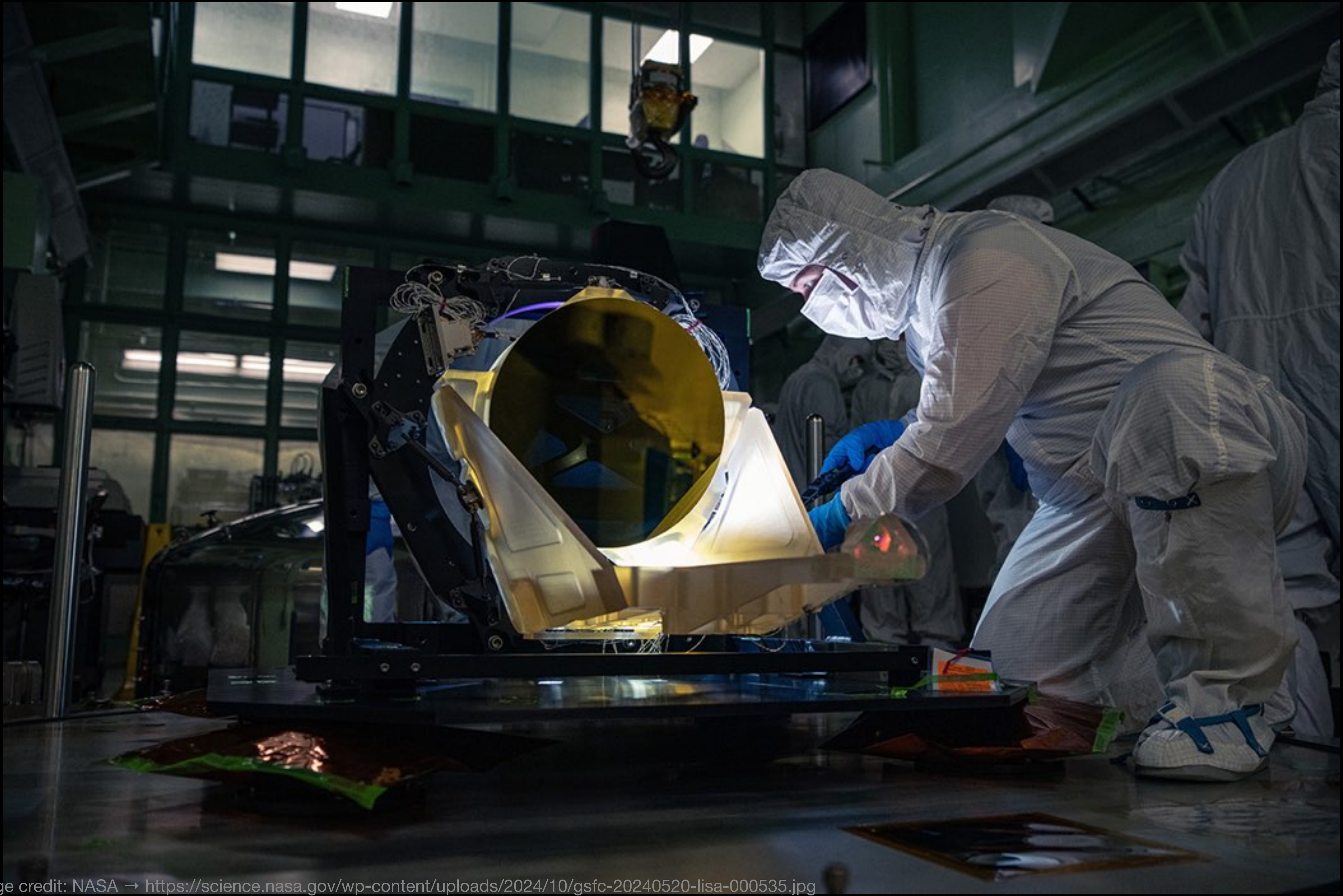
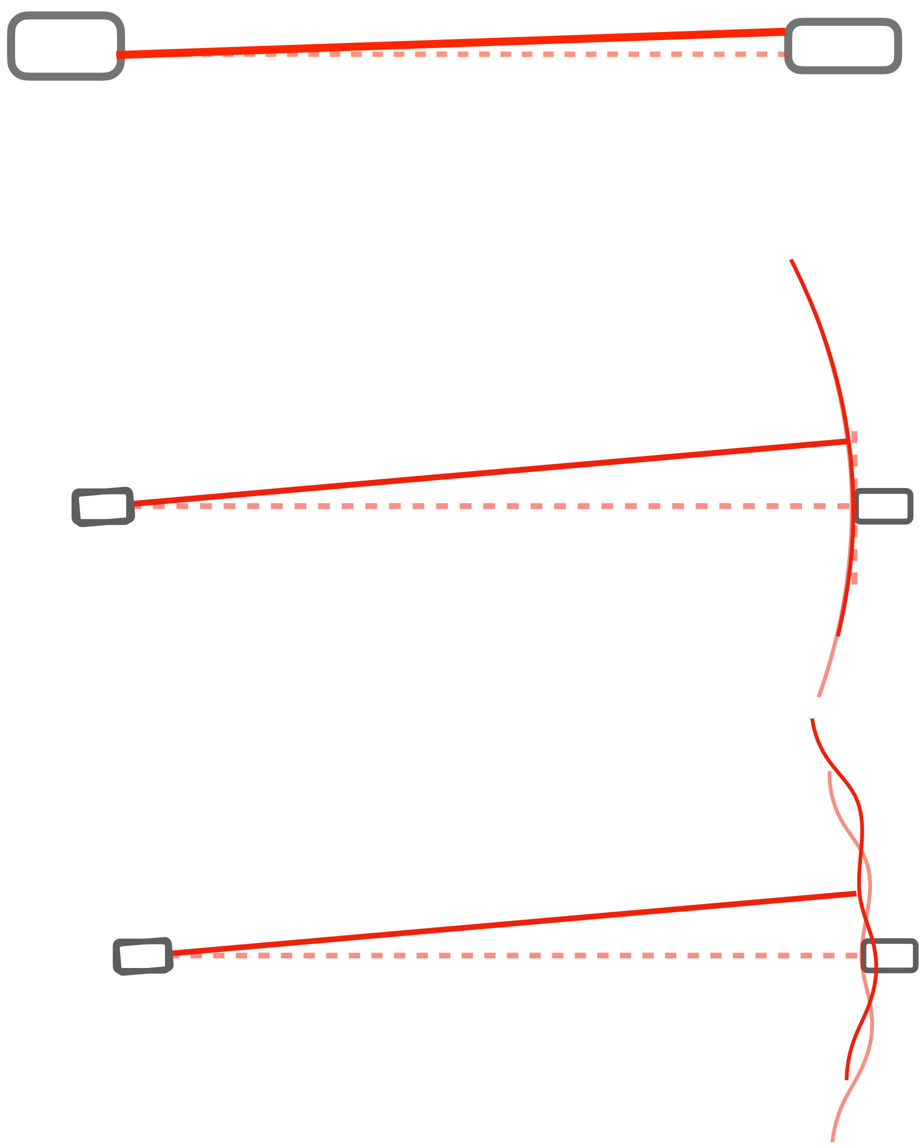
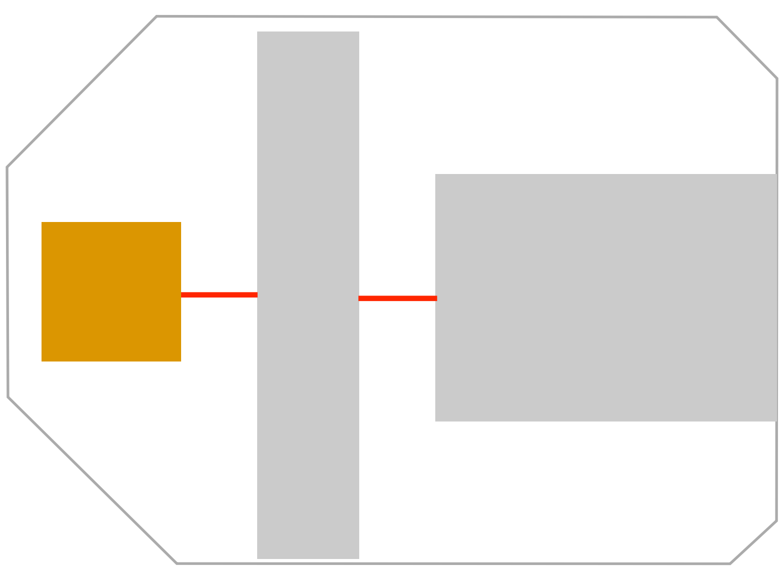


Image credit: NASA → https://science.nasa.gov/wp-content/uploads/2024/10/gsfc-20240520-lisa-000535.jpg

Beam alignment mechanism??

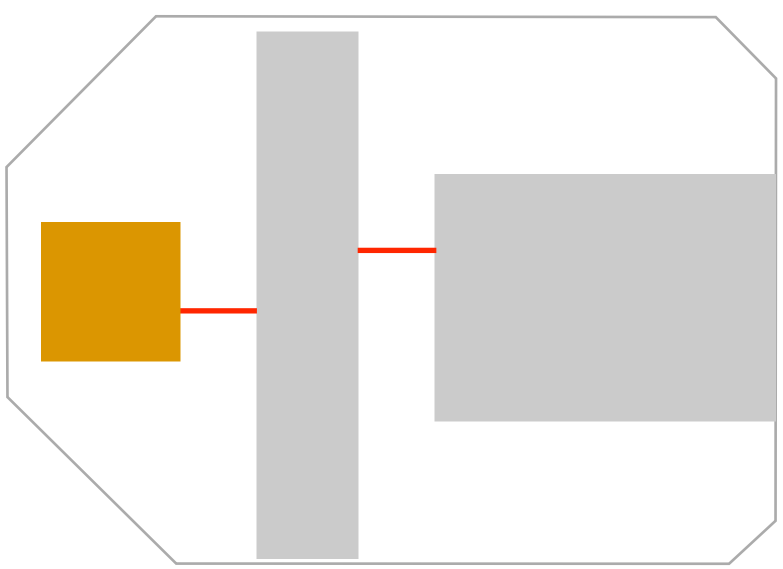
- **It's not only the spacecraft that will jitter in angles**
  - the MOSA's can jitter
    → optical bench + telescope relative to the free-floating test mass
  - the telescope can jitter relative to the optical bench
- **Magnitude of the coupling noise?**
  - depends on many, many things!
    - e.g. alignment of mirrors, beamsplitters, photodiodes! (and there are many)
    - wavefront properties, clipping effects
    - centres of rotations relative to the beam axes

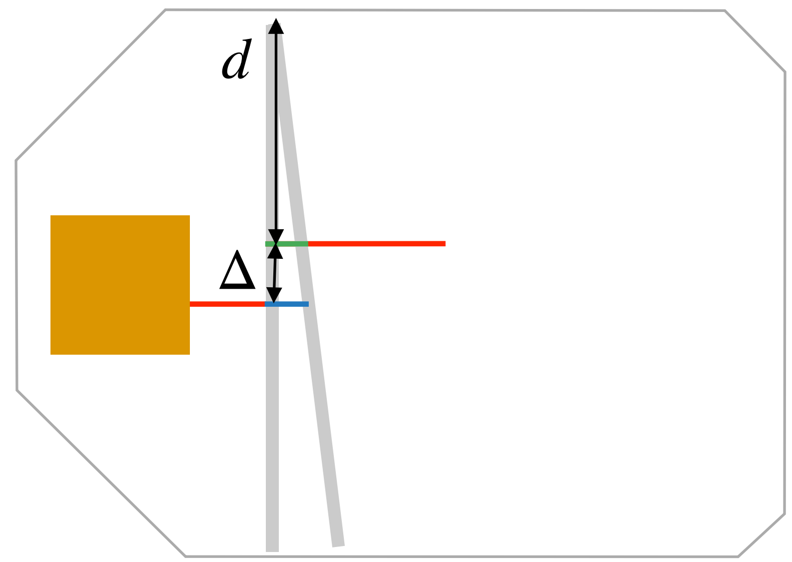




- **It's not only the spacecraft that will jitter in angles**
  - the MOSA's can jitter
    → optical bench + telescope relative to the free-floating test mass
  - the telescope can jitter relative to the optical bench
- **Magnitude of the coupling noise?**
  - depends on many, many things!
    - e.g. alignment of mirrors, beamsplitters, photodiodes! (and there are many)
    - wavefront properties, clipping effects
    - centres of rotations relative to the beam axes

One example, where a lateral shift of the beam axis generates a linear TTL effect.





$$L_{\text{ISI}} = -d\tan(\alpha) \approx -d\alpha$$

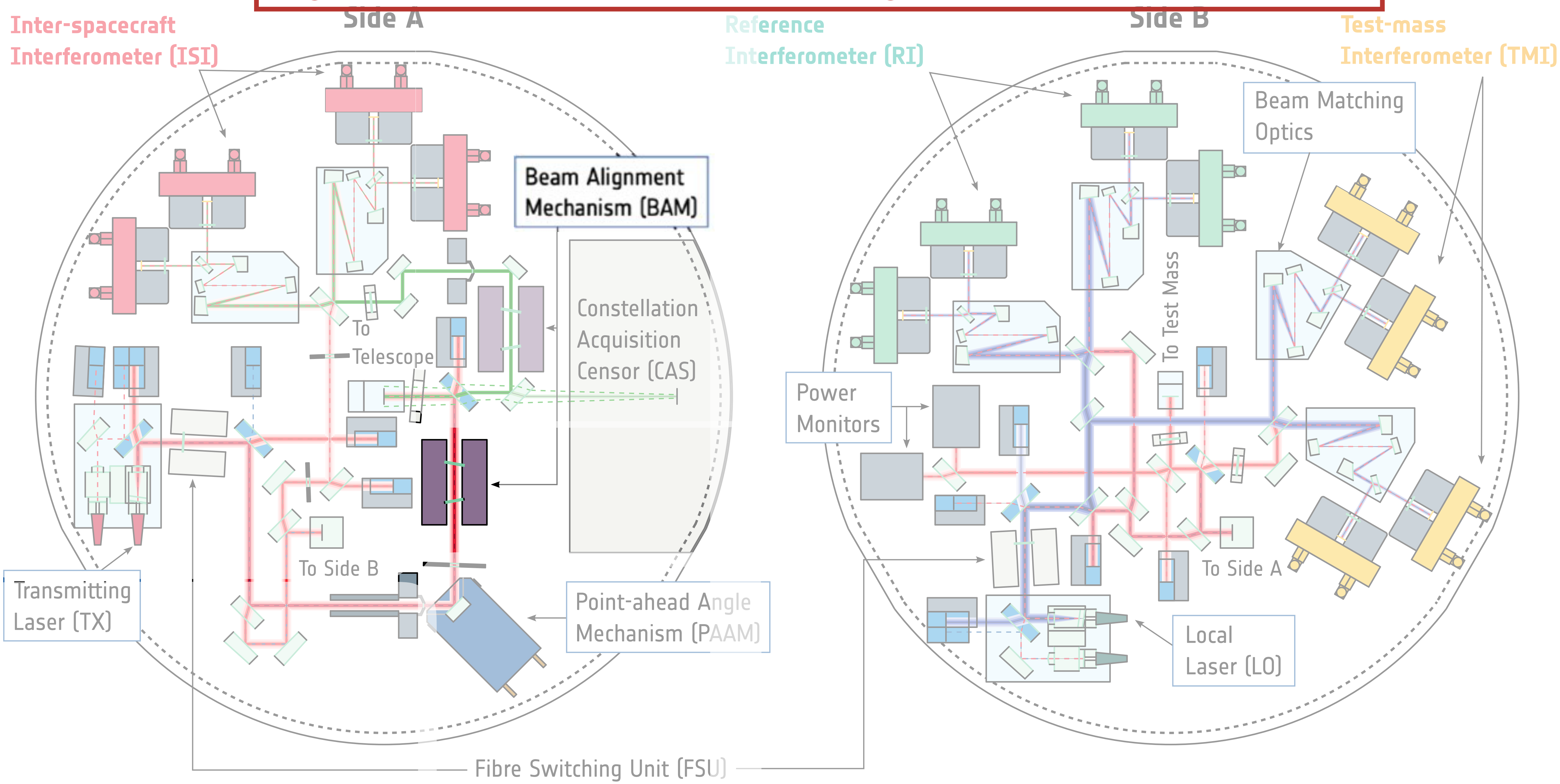
$$L_{\text{TMI}} \approx 2(d+\Delta)\alpha$$

$$L_{\text{ISI}} + \frac{1}{2}L_{\text{TMI}} = \Delta\alpha$$

We speak of a first-order TTL effect

A comparison with the official planning (2023)

Beam Alignment Mechanism (BAM): a device that parallel shifts a beam to generate a first-order TTL effect c*angular viration



[1]: ESA-SCI-DIR-RP-002 (red book)

BAM working principle: see Ada Uminska's slide 13 from https://www.youtube.com/watch?v=IDV99gOAjLM

LISA science

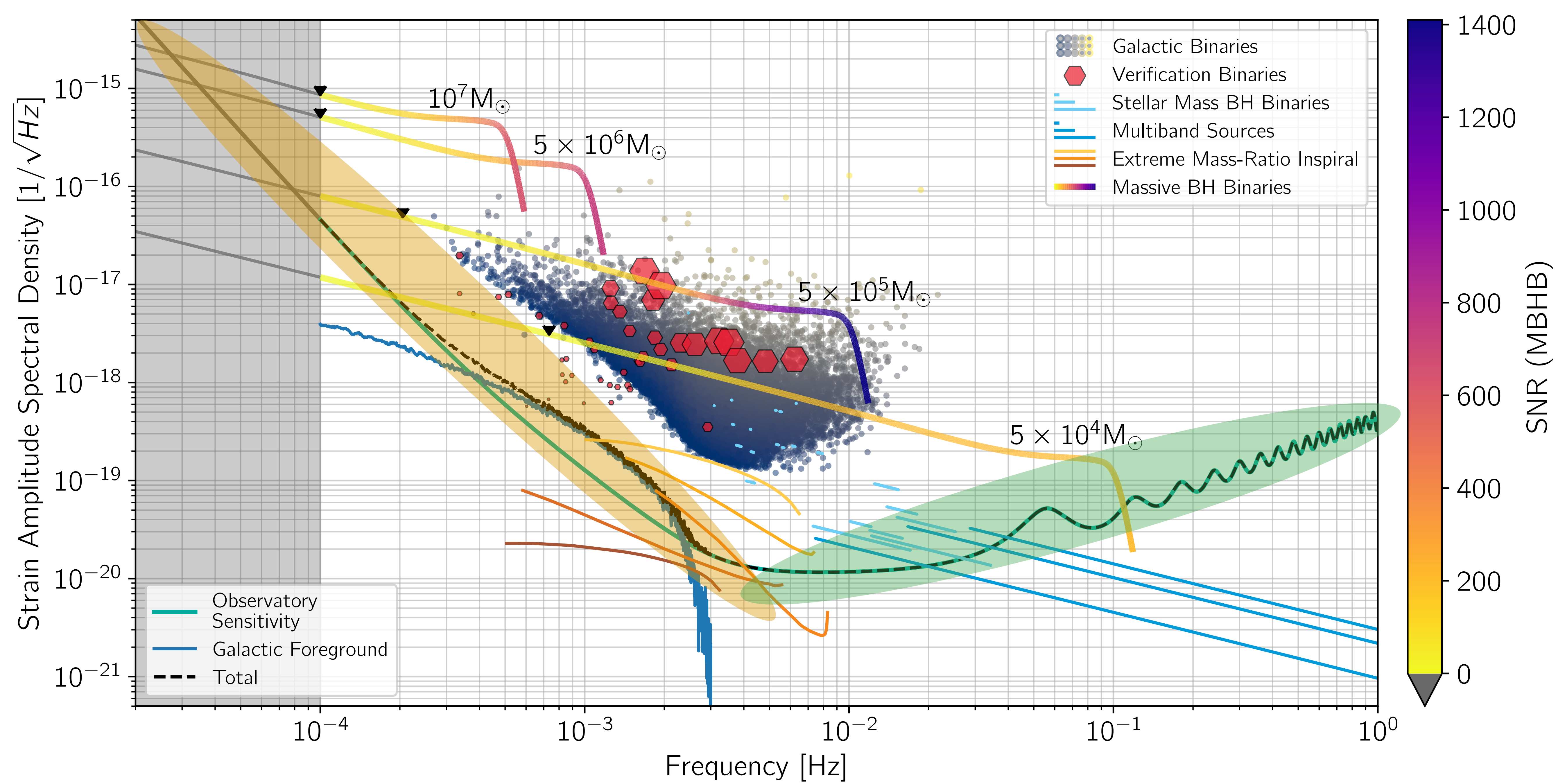
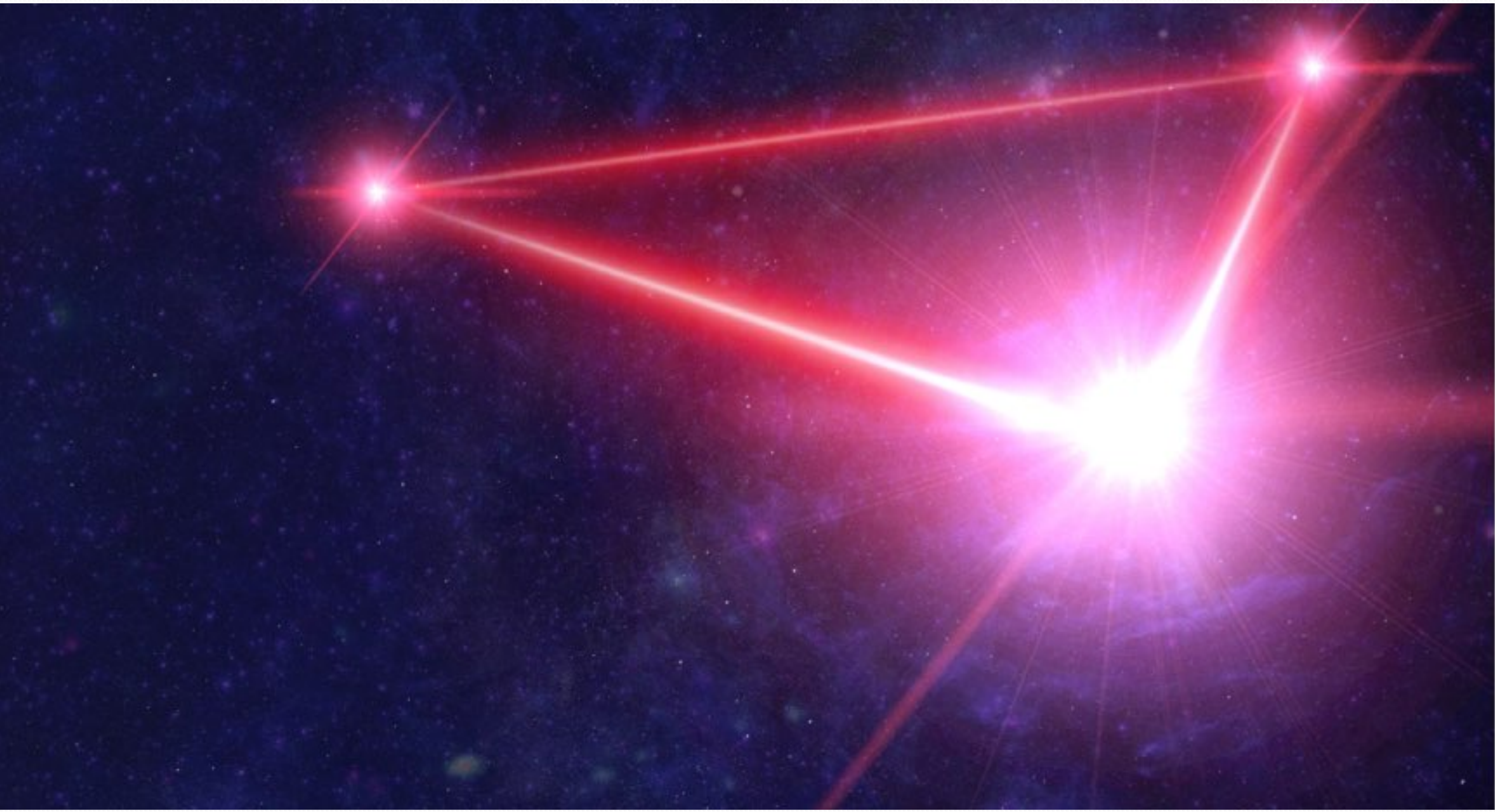


Image: https://www.cosmos.esa.int/web/lisa/lisa-redbook

Disclaimer

- This story is incomplete (!)
- There are other important aspects to the LISA story, not mentioned here (e.g., ranging, clock noise, locking in the ISI, …)
- In this story, we found reasons for some design choices and needed mechanisms
  - These reasons are not necessarily complete; other arguments could be made
- So, this story can serve as a starting point for asking more fun questions and for hunting for their answers…



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Thank you for your attention!