Controls in Interferometric Detectors

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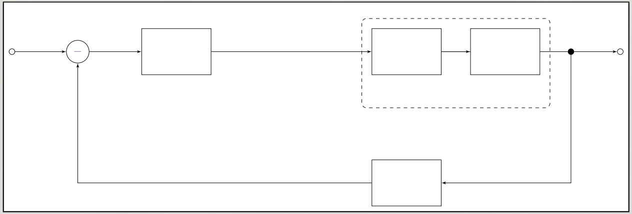
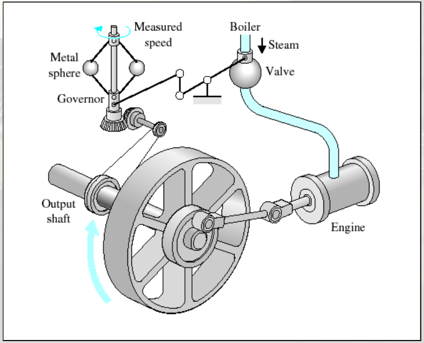
Background image: *West Arm tube of the Virgo interferometer*

What is a Control System?

A Control System is an ensemble of interconnected components designed to bring a physical process in a desired state:

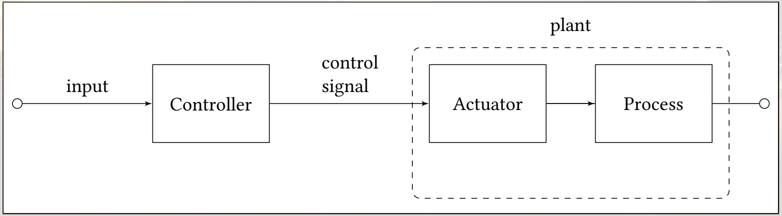
- Control Systems have been used for a long time, for both new and long-standing processes
  - motivation and goal may be constant over time
  - methodology and implementation can evolve rapidly
- Several types with different requirements and purposes
- Described by block diagrams





Watt's "Flyball Governor" (late 1700s)

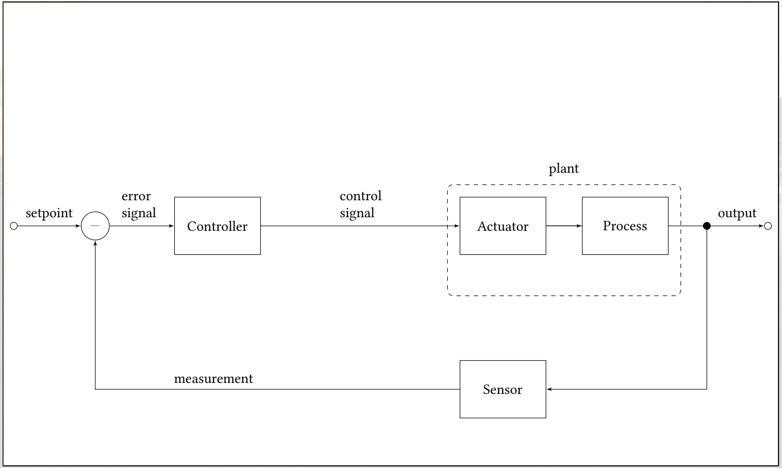
The basic one: Open Loop Control



Open Loop Control:

- It drives the Process to a certain state
- It is *not* error-based
- No reading is done on the system

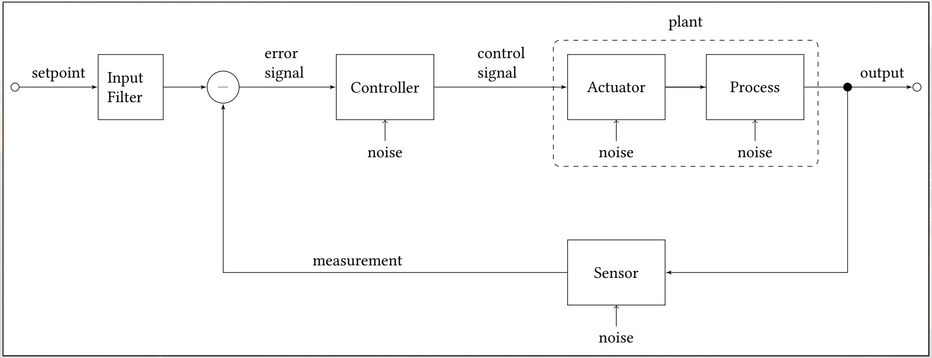
The important one: Feedback Control



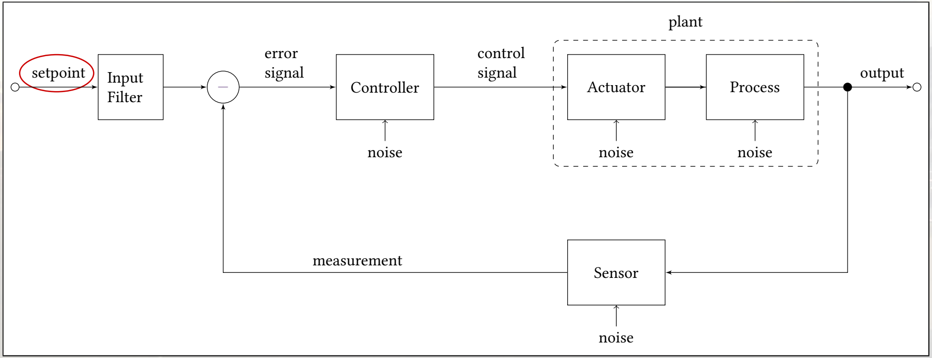
Feedback Control:

- It *is* error-based
- It reads back the behavior of the plant
- It defines the working point of the system

Feedback System: Terminology

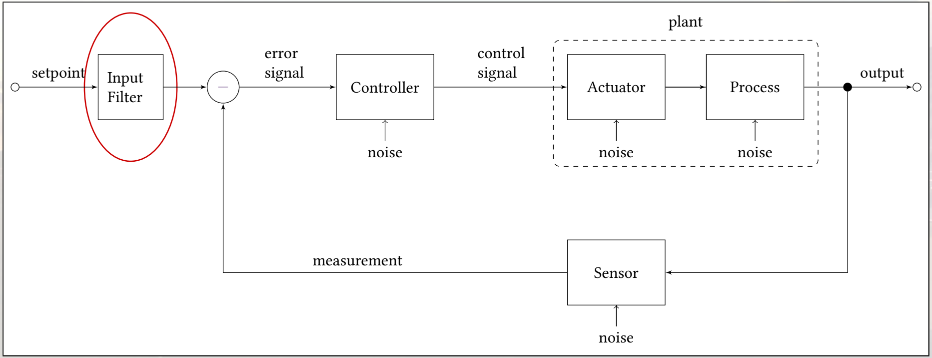


Feedback System: Terminology



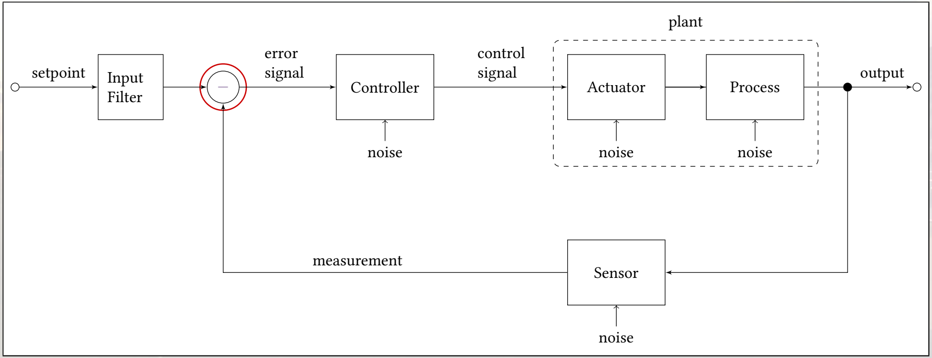
Setpoint: the desired value of your system

Feedback System: Terminology



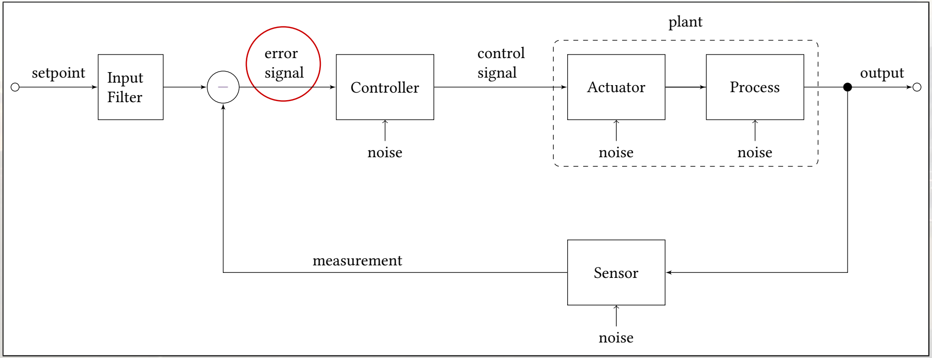
Input Filter: transducer from physical quantity to electrical/digital signal

Feedback System: Terminology



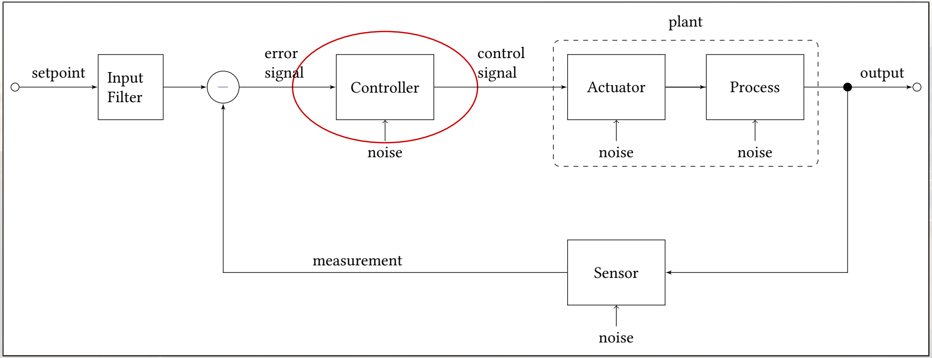
Summation Point: comparison between Setpoint and Measurement

Feedback System: Terminology



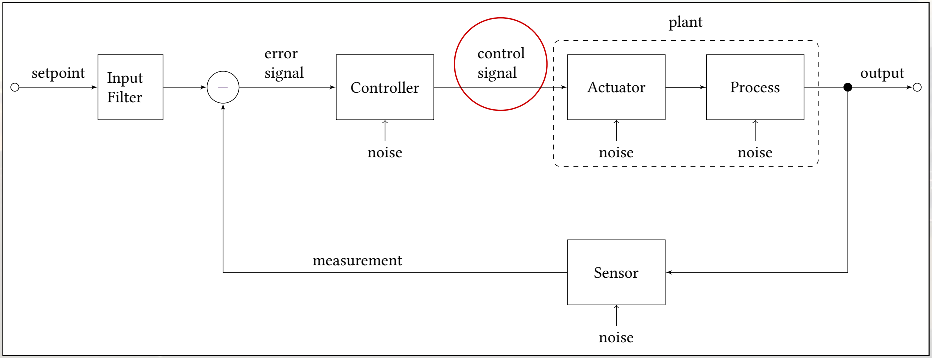
Error Signal: quantity that must be kept to zero

Feedback System: Terminology



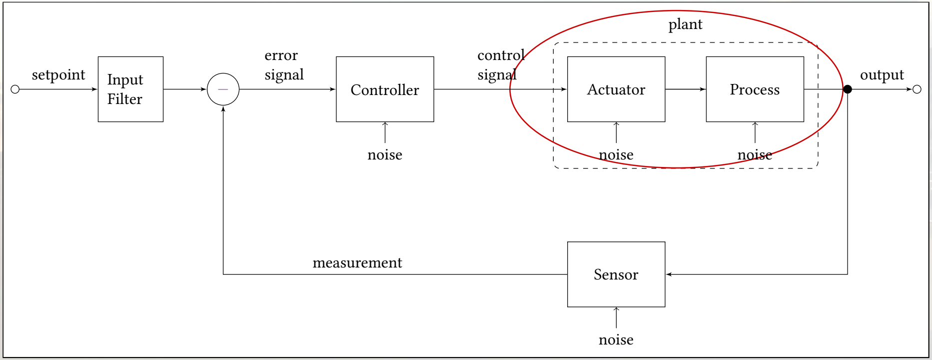
Controller: physical/software equipment that determines how to achieve the Setpoint

Feedback System: Terminology



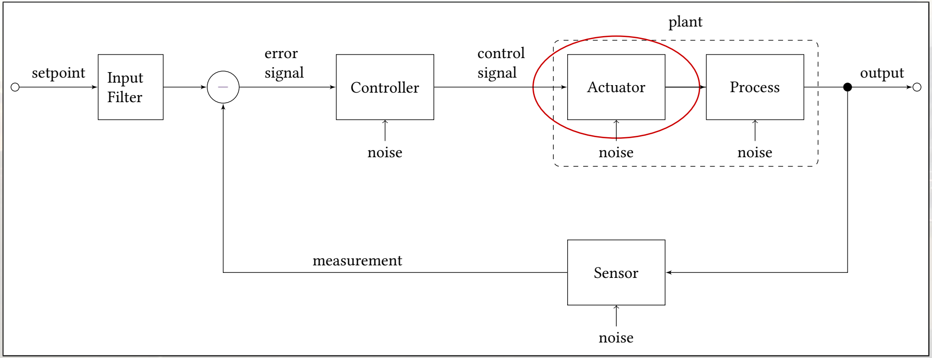
Control Signal: output of Controller which is sent to the machinery that actually *drives* the system

Feedback System: Terminology



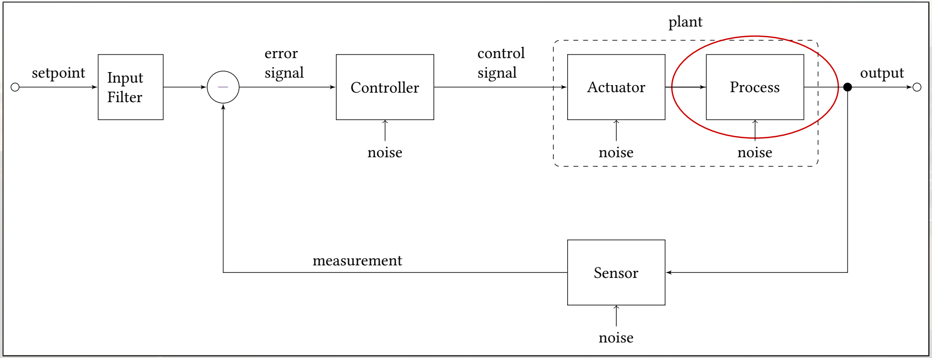
Plant: the physical system of interest

Feedback System: Terminology



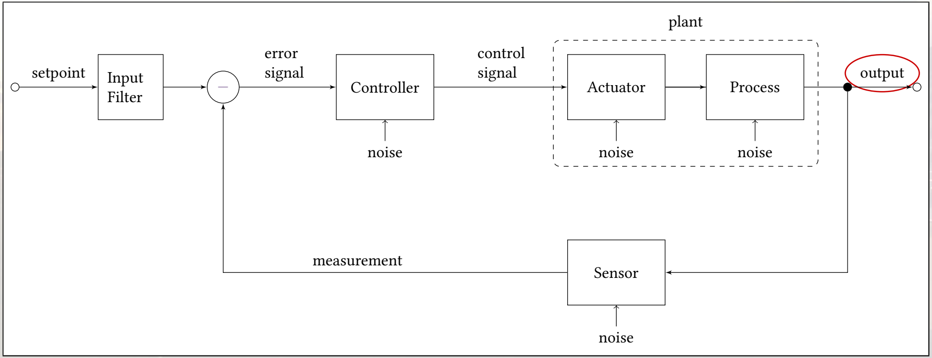
Actuator: the equipment which changes the Plant's behaviour according to the designer's desire

Feedback System: Terminology



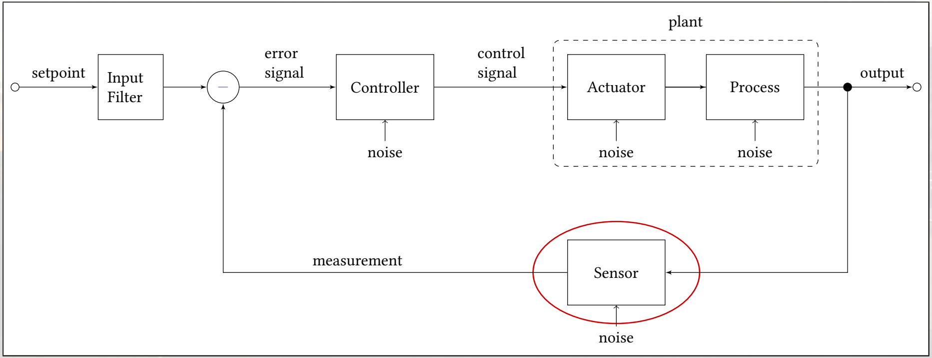
Process: the physical process one is interested in

Feedback System: Terminology



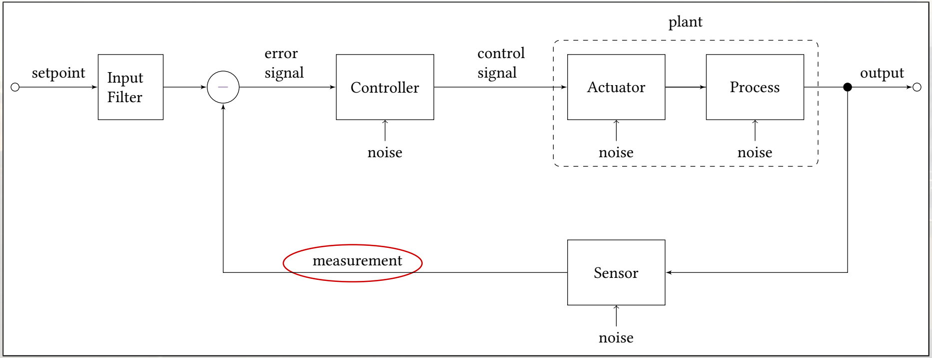
Output: outcome of the control chain

Feedback System: Terminology



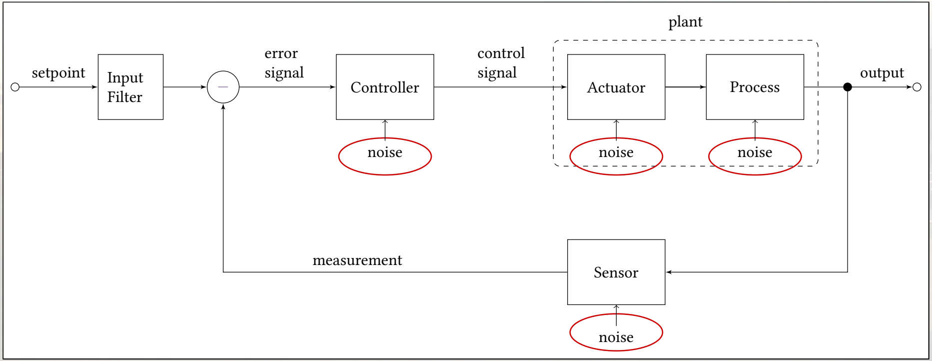
Sensor: equipment which reads the physical variable to be controlled

Feedback System: Terminology



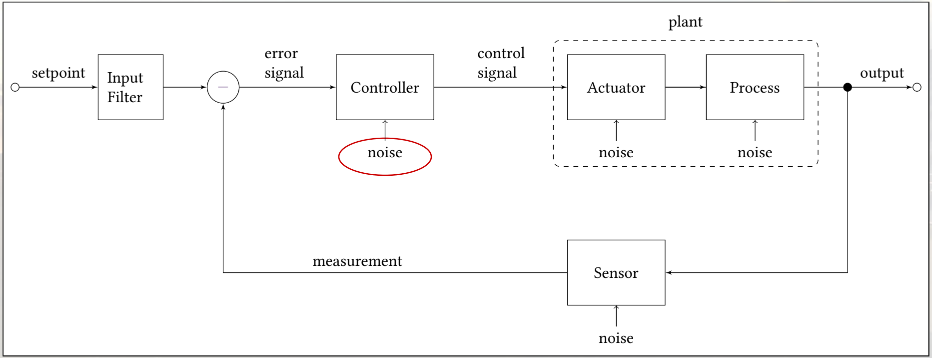
Measurement: the physical variable to be controlled, after a possible conversion to electrical signal

Feedback System: Terminology



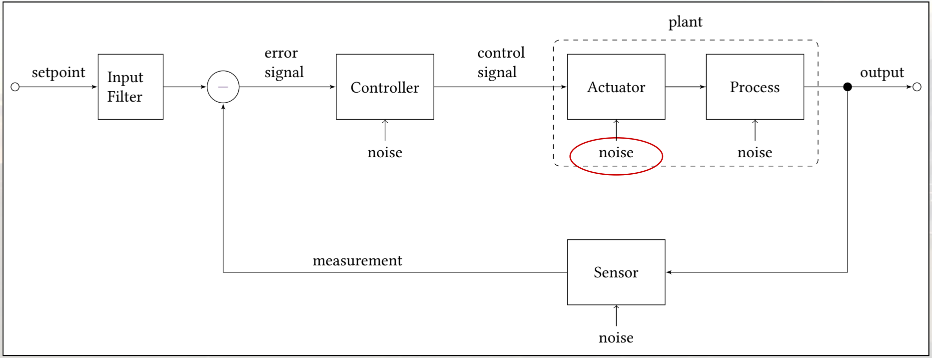
Noises: cause of disturbance in the system; they may be (among others):

Feedback System: Terminology



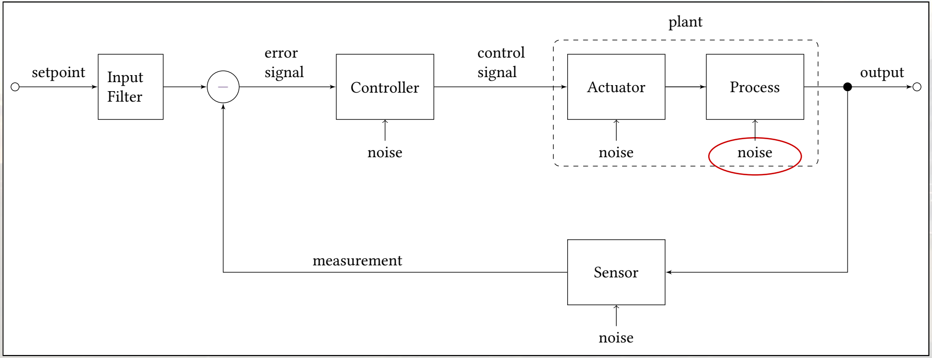
Control Noise: imperfect output of the controller (digital/numerical noise, mistuned gain, badly placed poles/zeros, etc...)

Feedback System: Terminology



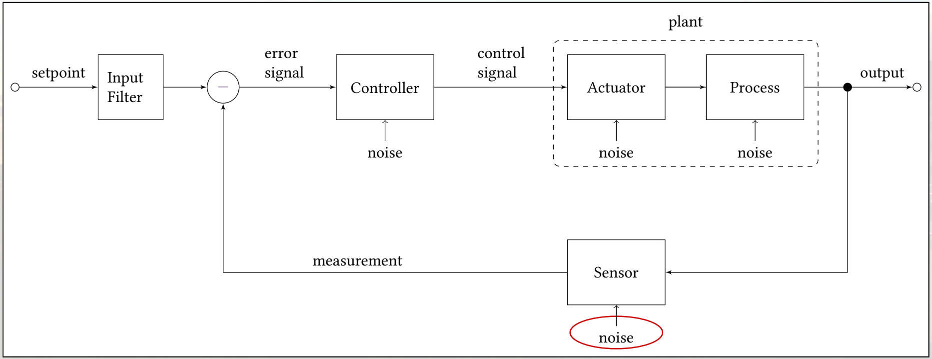
Actuator Noise: imperfect conversion from signal to real output (e.g. DAC noise), fabrication defects, physical set-up (asymmetry, unbalance), etc...

Feedback System: Terminology



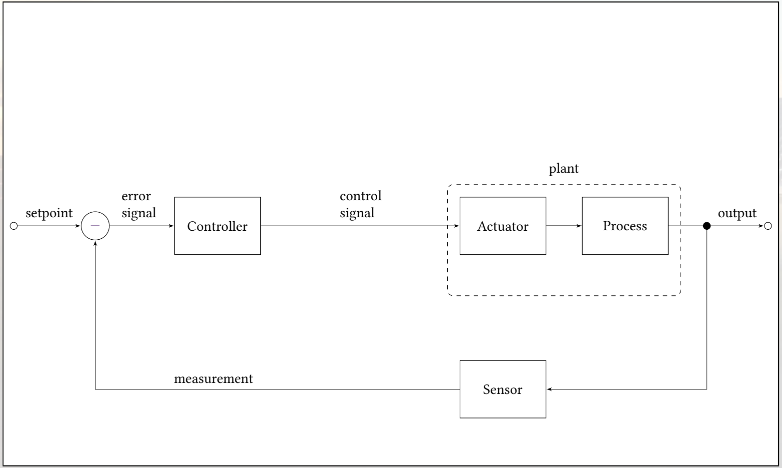
Process Noises: all kind of disturbances in the physical process (fabrication defects, unforeseen behaviour, mechanical faults, external environment, etc...)

Feedback System: Terminology



Sensor Noises: imperfect reading of the physical variable (sensor calibration, working conditions, environmental couplings, shot noise, ADC noise, electronic/dark noise, etc...)

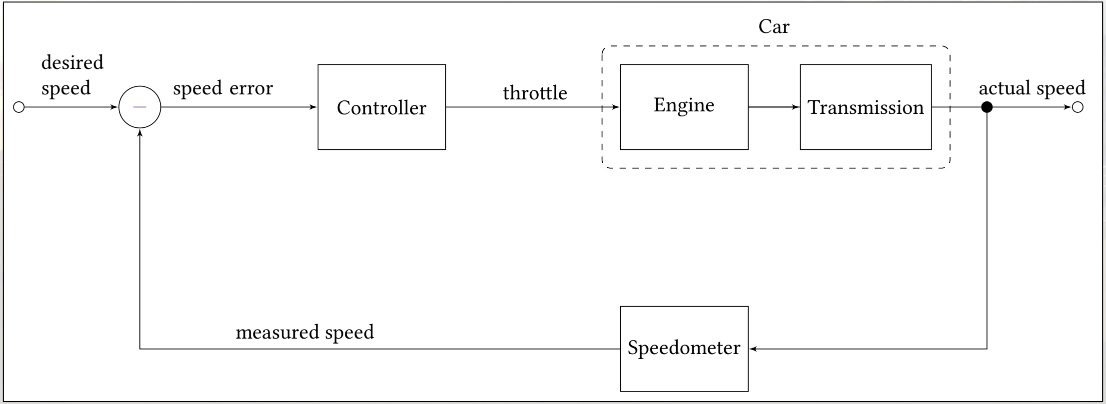
Requirements for a Feedback Control



Requirements:

- **Stability**: the overall system must be stable
- **Tracking**: the output must track the input signal
- **Regulation**: system must not overreact to disturbance inputs
- **Robustness**: these goals must be met even in case in inaccurate modeling or changes in dynamics or environment

Example of a Feedback Control: the Speed of a Car



SISO LTI Systems (1)

Most of the basic techniques used in the field have been developed for the most common (but not unique) kind of system: SISO LTI Systems.

- **SISO**: Single Input Single Output
  - there is one independent input, controlled with one output (*not* one actuator)
- **LTI**: Linear Time Invariant
  - the output is proportional to the input, the system properties do *not* change over time

This is because most of the dynamic systems of interest are described by Constant-Coefficient Linear Differential Equations.

Modelization of more complex systems is of course possible, but it requires different/more complex techniques.

SISO LTI Systems (2)

Two consequences arise from the properties of LTI systems, which are fundamental for the mathematical approach to control systems:

- A linear system response obeys the principle of superposition: if a system has an input which is expressed as a sum of signals, then the response of the system is the sum of the individual responses to the respective signals
- The response of a LTI system can be expressed as the convolution of the input with the unit impulse response of the system

Impulse Response & Transfer Functions (1)

The idea of impulse response comes from Paul Dirac: very intense but very short forces can be described by the mathematical impulse:

$$\delta(t) = 0, \quad t \neq 0$$
$$\int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

So, for the principle of superposition, the response of a continuous input $u(t)$ can be described as the sum of the responses of impulses $h(\tau_i)$:

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, u(t-\tau)\, d\tau$$

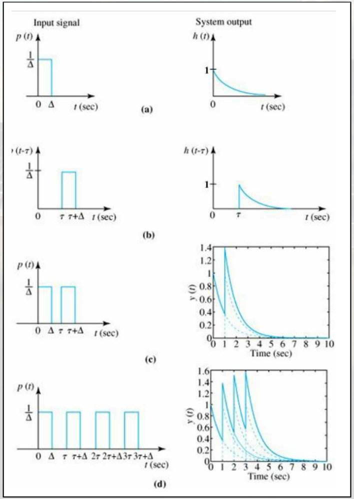
This is the convolution integral

Impulse Response & Transfer Functions (2)

- There is an immediate consequence if the input $u(t) = e^{st}$, where s can be complex: $s = \sigma + i\omega$
- The output $y(t)$ will be $H(s)\, e^{st}$, exponential as well:

$$H(s) = \int_0^{+\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

This is the Laplace Transform of the unit impulse response $h(t)$, and it is a pure amplitude factor.



Impulse Response & Transfer Functions (3)

Definition:

$$H(s) = \frac{Y(s)}{U(s)}$$

is the Transfer Function of the system, i.e. the transfer gain from *input* to *output*, and it is the ratio between the Laplace Transforms of the *output* and the *input*.

Frequency Response (1)

A natural way to use the exponential response of LTI systems is finding their frequency response, which is the response to a sinusoid.

This is because, thanks to Euler's relation and the principle of superposition, we can use a sinusoid as the sum of two exponentials:

$$A\cos(\omega t) = \frac{A}{2}\left(e^{j\omega t} + e^{-j\omega t}\right)$$

So, if we let $s = j\omega$, we immediately have the response to a sinusoid:

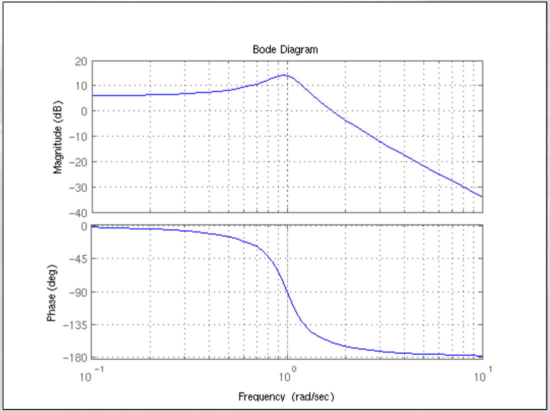
$$\begin{aligned} y(t) &= \frac{A}{2}\left[H(j\omega)\, e^{j\omega t} + H(-j\omega)\, e^{-j\omega t}\right] \\ &= AM\cos(\omega t + \varphi) \end{aligned}$$

Frequency Response (2)

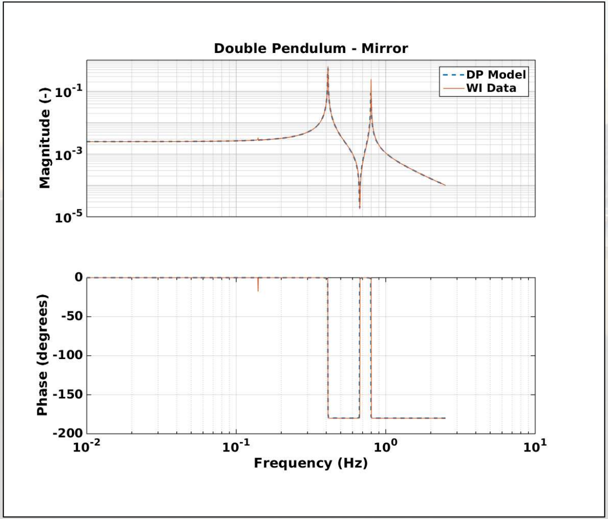
The Transfer Function $H(j\omega)$ is a complex number, which can be represented in the *magnitude*-and-*phase* notation:

$$M = |H(j\omega)|$$
$$\varphi = \angle H(j\omega)$$

- Transfer Functions are plotted with a Bode Diagram, which shows both Magnitude and Phase as a function of the frequency
- Bode diagrams describe everything: plants, actuators, controllers, etc...



Example: mechanical TF of a mirror of Advanced Virgo



Designing a Controller (1)

Among others, one advantage of the Laplace Transform is that *differentiation* and *integration* in the time domain become *multiplication* and *division* in Laplace's domain:

$$\mathcal{L}\left\{\frac{df}{dt}\right\} \propto sF(s)$$

$$\mathcal{L}\left\{\int_0^t f(\xi)\, d\xi\right\} = \frac{1}{s}F(s)$$

Remember that we want to control systems described by differential equations!

Designing a Controller (2)

This allows the description of a controller or a component ("system") as a rational function, which is much easier to handle:

$$F(s) = K \frac{\prod_{i=1}^{m} (s - z_i)}{\prod_{i=1}^{n} (s - p_i)}$$

- z_i are the zeros of the system
- p_i are the poles of the system

Designing a Controller (3)

Without going into any detail, designing a controller (a *loop filter*) means designing the controller's transfer function in order to have the global transfer function of the system (the *Closed Loop Transfer Function*) meet the requirement which were described before:

- stability, tracking, regulation, robustness

In order to do so, the loop filter's transfer function must *compensate* the structures which are present in the actual system, depicted as poles and zeros, with its own poles and zeros, which have to be placed in a sensible way.

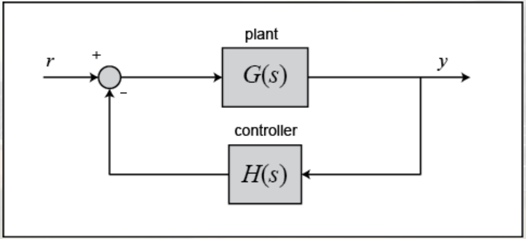
Designing a Controller (4)

- Closed-Loop Transfer Function: the overall transfer function from *output* to *input*

$$\mathrm{CLTF} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

- Open-Loop Transfer Function: the transfer function without the feedback path

$$\mathrm{OLTF} = G(s) \cdot H(s)$$



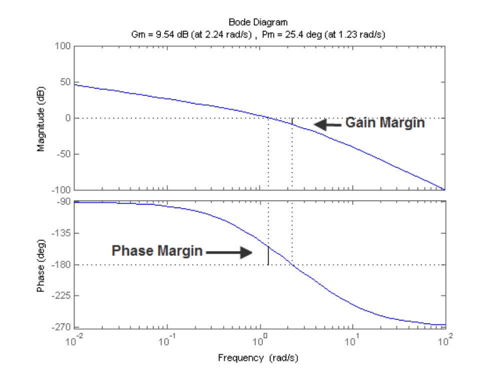
Simplified Loop Diagram

The OLTF is important for the stability of the loop

Designing a Controller (5)

A few words on the concept of stability:

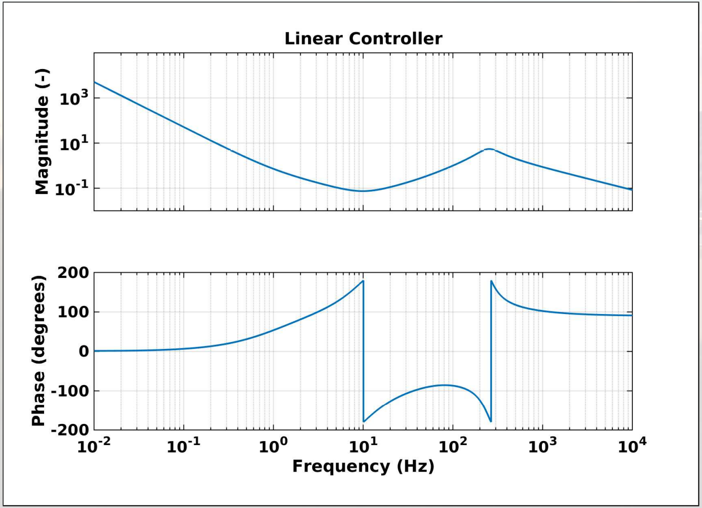
- a control system must be stable
- the response has to
  - correct the errors
  - not saturate the actuator
  - not damage the plant
  - bring to the desired state
- definitions:
  - Gain Margin: gain at $\varphi = -\pi$
  - Phase Margin: $\varphi + \pi$ where $G = 1$



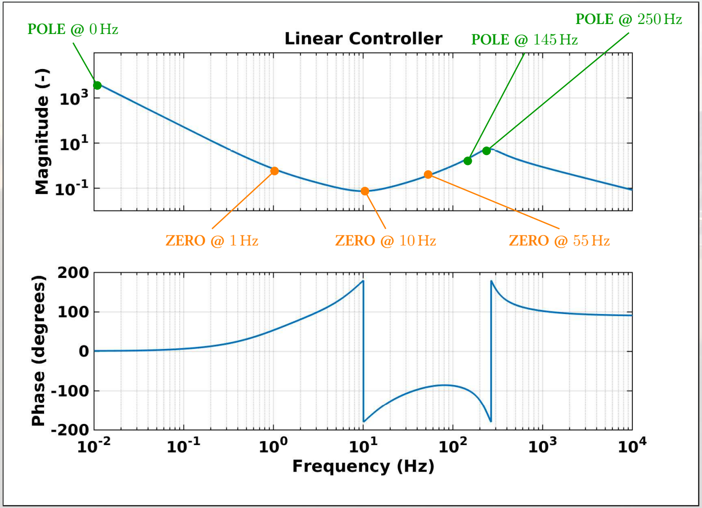
Open-Loop Transfer Function

Having a controller with enough phase is mandatory for stability!

Example: transfer function of the control filter for a single arm



Example: transfer function of the control filter for a single arm



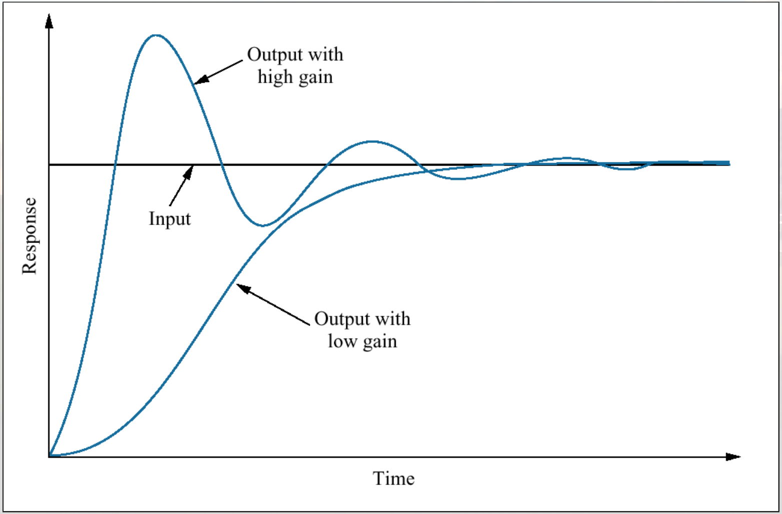
PID Controllers (1)

The standard type of controller relies on three different components, which must be built with different ways of pole-zero placements, also to maintain stability.

If $e(t)$ is the error signal, we have:

- Proportional: $y(t) \propto P e(t)$ $\Longrightarrow$ Gain
- Integral: $y(t) \propto \int_0^t e(\tau)\, d\tau$ $\Longrightarrow$ Pole
- Derivative: $y(t) \propto \frac{de(t)}{dt}$ $\Longrightarrow$ Zero

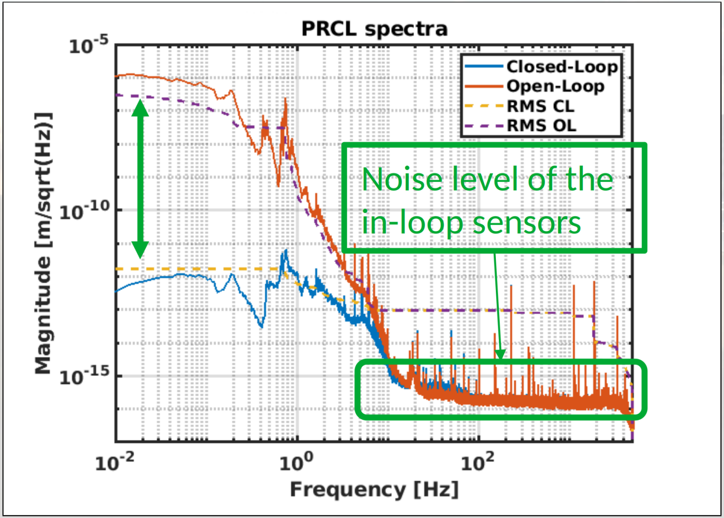
PID Controllers (2)



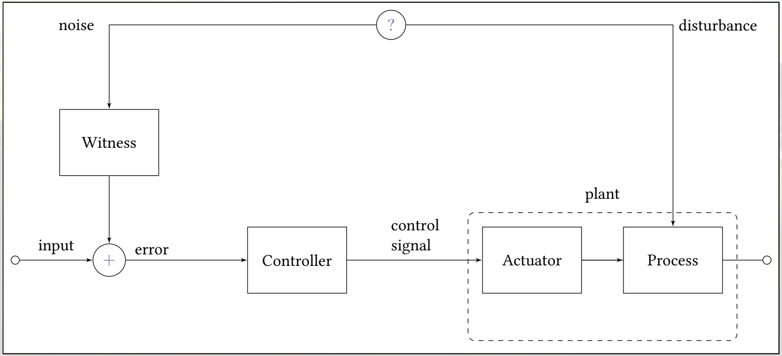
PID Controllers (3)

Controller	Response Time	Overshoot	Error
On-Off	Smallest	Highest	Large
Proportional	Small	Large	Small
Integral	Decreases	Increases	Zero
Derivative	Increases	Decreases	Small Change

What Happens when a Loop is Closed



A Different Kind: Feedforward Control

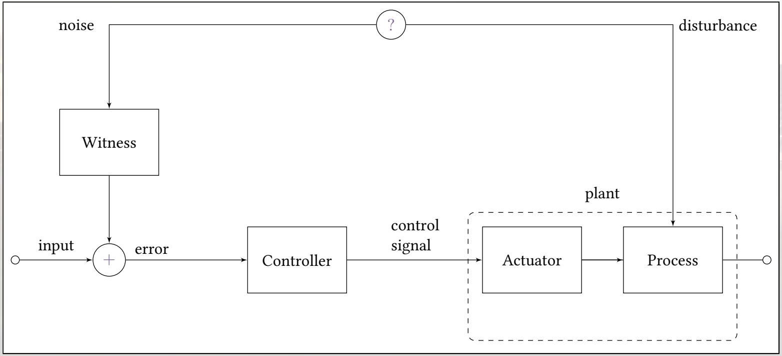


Feedforward:

- It reduces the *effect of an external disturbance*
- It needs a *witness*
- It is less constrained than feedback
- It needs very accurate modeling
- It is not error based

It can *proactively* anticipate a disturbance, or *subtract* a known, external, one

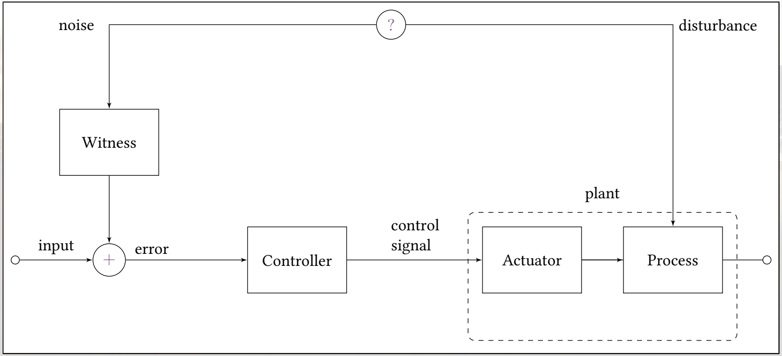
Feedforward Control: the Witness



Feedforward needs a *witness* of the noise to be reduced:

- The system must be set up in a way to be able to read the external disturbance *independently*
- Such witness must possibly carry no other information than the one needed for this subtraction
- The witness must be reliable over time

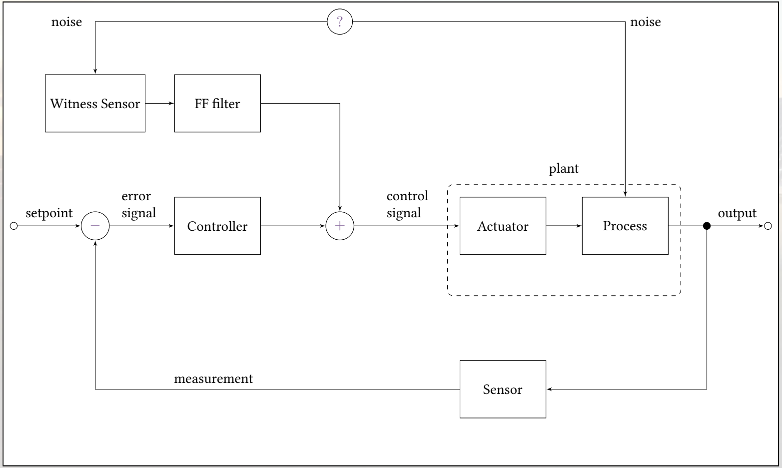
Feedforward Control: Operating Conditions



Fewer constraints than Feedback Control:

- There are no requirements (phase margin, etc...)
- A feedforward is not "stable" or "unstable"
- The witness and the model define the performance
- The effect is the *reduction* or *amplification* of noise

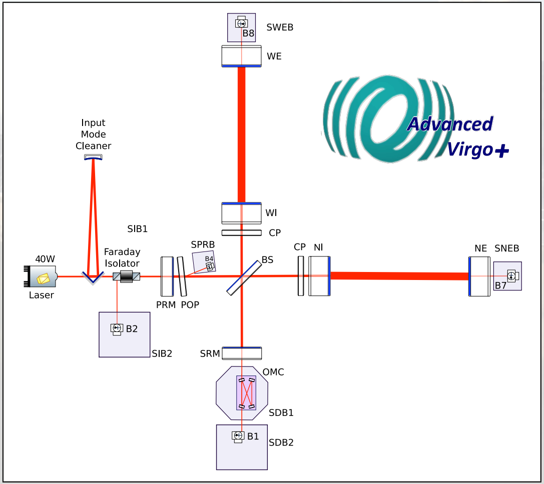
Feedforward inside a Feedback Control



- Reduction of an external disturbance from a *in-loop* variable
- The relationship between noise and witness must be well known
- The relationship between noise and actuator must be well known
- A precise model is needed in order to build a performing filter

Advanced Virgo +: a Gravitational Wave Interferometer

- Very complex opto-mechanical system, with several suspended mirrors
- Narrow operating point, defined by the *resonance condition* of several optical cavities
- Longitudinal and angular controls both important
- It has many auxiliary systems, but it must be considered a single entity

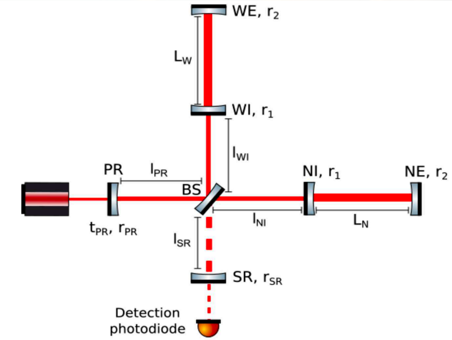


Global Working Point

The detector provides a good sensitivity only if all the main components are positioned and orientated in a very precise relative microscopic position. These relative positions build the so called **degrees of freedom**.

Operational conditions:

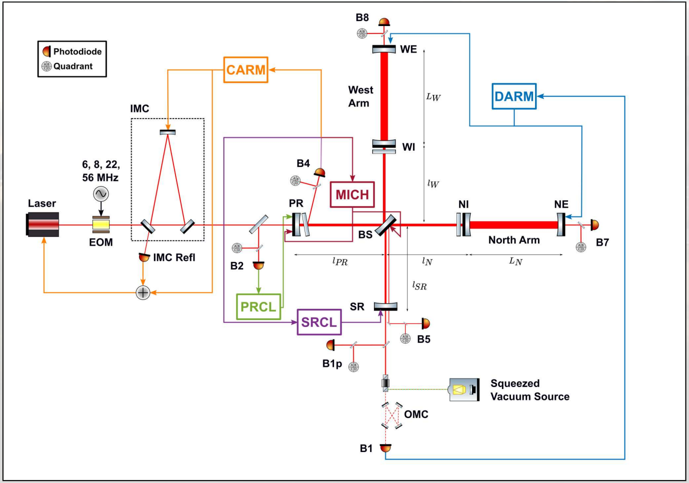
- Arm cavities on resonance
- Recycling cavities on resonance
- Michelson on destructive interference
- Mirrors aligned with respect to the beam



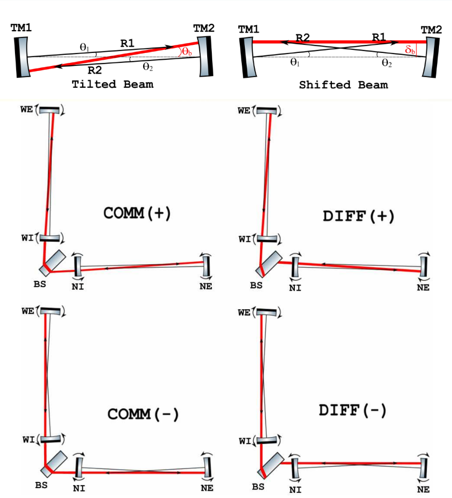
The working point has to be kept for a long time (to ensure a high duty cycle) and with good accuracy (to ensure good enough performances) for scientific data taking

Global Longitudinal Degrees of Freedom

- $\mathtt{DARM} = L_N - L_W$, the length difference of the long Fabry-Perot arm cavities, sensitive to gravitational waves
- $\mathtt{CARM} = \frac{L_N + L_W}{2}$, the common, average length of the long Fabry-Perot arm cavities
- $\mathtt{MICH} = l_N - l_W$, the length difference of the short arms of the Michelson, it defines the interference condition
- $\mathtt{PRCL} = l_{PR} + \frac{l_N + l_W}{2}$, the Power Recycling cavity length
- $\mathtt{SRCL} = l_{SR} + \frac{l_N + l_W}{2}$, the Signal Recycling cavity length



Global Angular Degrees of Freedom



We have the following *eighteen* angular degrees of freedom (TX is pitch, TY is yaw):

- Cavities: DIFF+, DIFF−, COMM+, COMM− (TX, TY for each of them)
- PR mirror: TX, TY; X, Y (beam pointing)
- BS mirror: TX, TY
- SR mirror: TX, TY

They can be related to a single specific mirror, or to an optical cavity as an ensemble.

What is a Cavity and What is its Resonance Condition?

Optical resonator: allows the light to circulate in a closed path. When it is on resonance the optical path is a multiple of the wavelength λ (1064 nm in current interferometers):

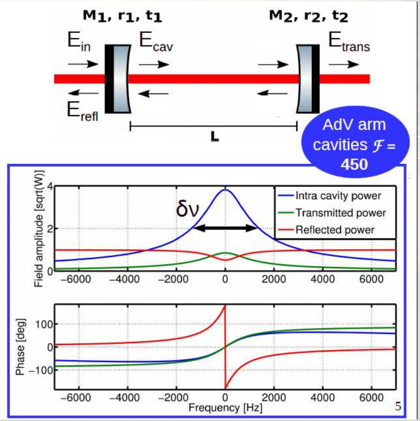
- **Resonance**: maximum power resonating inside the cavity

$$\delta\Phi \propto (\nu \cdot \delta L + L \cdot \delta\nu)$$

- **Finesse**: it quantifies the quality factor of a cavity

$$\frac{P_{\mathrm{cav}}}{P_{\mathrm{in}}} \approx \frac{2 \cdot \mathcal{F}}{\pi}$$

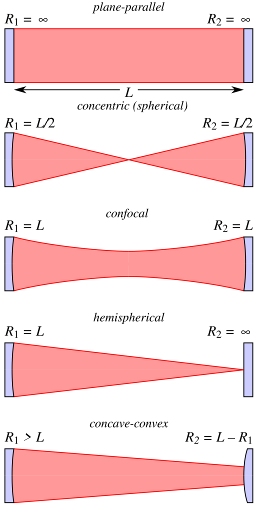
$$\mathcal{F} \approx \frac{\pi\sqrt{r_1 r_2}}{1 - \sqrt{r_1 r_2}}$$



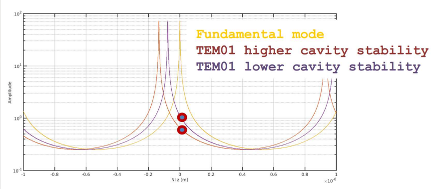
What is the Stability of a Cavity?

The stability of a cavity depends on three parameters: the two radii of curvature (R_1 and R_2) and the length L. A cavity is stable when the intracavity beam is periodically refocused. If the cavity is unstable, the beam size will grow without limit, eventually growing larger than the size of the cavity mirrors and being lost.

$$g_i = 1 - \frac{L}{R_i} \quad ; \text{if} \quad 0 < g_1 g_2 < 1 \quad \Rightarrow \quad \text{stable cavity}$$



The stability of a cavity sets also the capability of the cavity to “clean” the resonating beam from modes different from the fundamental mode.

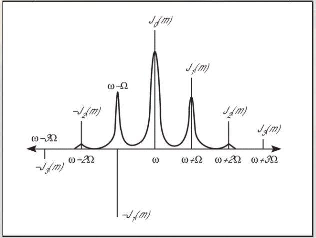


Phase Modulation

- Technique: phase-modulate the laser light
- Use of a EOM (electro-optical modulator), as a Pockels cell: a crystal with a tunable optical length via a driven voltage
- The EOM is driven with a sinusoidal signal which is converted in a variation of phase of the transmitted laser beam
- Generation of radio-frequency sidebands at frequencies $(\omega \pm \Omega)$

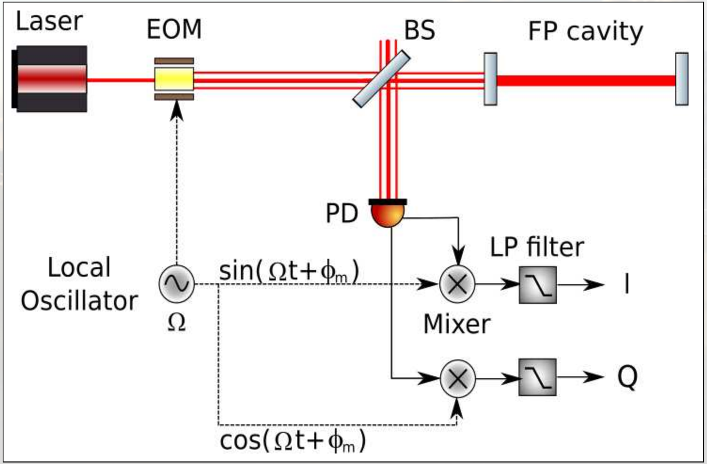
$$E_{\mathrm{inc}} = E_0 \, \mathrm{e}^{-i(\omega t + \beta \sin \Omega t)}$$

$$E_{\mathrm{inc}} \simeq E_0 \left[\mathrm{e}^{-i\omega t} + \frac{\beta}{2} \, \mathrm{e}^{-i(\omega + \Omega)} - \frac{\beta}{2} \, \mathrm{e}^{-i(\omega - \Omega)} \right]$$



The Pound-Drever-Hall Technique (1)

- Technique used in the ’80s to stabilize a laser using a resonant cavity’s length as reference; it can be used the other way around: stabilize a resonant cavity length’s using a laser as reference
- It is the main technique used in GW interferometers
- It gives an error signal: the beat note between carrier and non-resonant sidebands; it is then demodulated at the same frequency to select the term of interest



The Pound-Drever-Hall Technique (2)

After the EOM, we have:

$$\begin{aligned} E_{\text{inc}} &= E_0\, \mathrm{e}^{i(\omega t+\beta \sin \Omega t)} \\ &\approx E_0 \left[J_0(\beta) + 2iJ_1(\beta) \sin(\Omega t)\right] \mathrm{e}^{i\omega t} \\ &= E_0 \left[J_0(\beta)\, \mathrm{e}^{i\omega t} + J_1(\beta)\, \mathrm{e}^{i(\omega+\Omega)t} - J_1(\beta)\, \mathrm{e}^{i(\omega-\Omega)t}\right] \end{aligned}$$

where ω is the carrier's angular frequency, Ω is the EOM's modulation angular frequency, β is the modulation depth and J_0 and J_1 are Bessel's functions; therefore, the two sidebands have the angular frequency $(\omega \pm \Omega)$.

If the modulation depth is small, all the power is in the carrier field and in the *first order* sidebands; this means that

$$P_0 \approx J_0^2(\beta)\, P_0 + 2J_1^2(\beta)\, P_0$$

The Pound-Drever-Hall Technique (3)

The reflected field in a Fabry-Perot cavity has the form

$$E_\mathrm{r} = \frac{-r_\mathrm{I} + r_\mathrm{E}\,\mathrm{e}^{-i\phi}}{1 - r_\mathrm{I} r_\mathrm{E}\,\mathrm{e}^{-i\phi}} E_0 \equiv \varrho\left(\omega\right) E_0$$

where ϕ is the round trip phase.

Therefore, our reflected field is

$$\begin{aligned} E_\mathrm{r} = {} & E_0 \varrho\left(\omega\right) J_0\left(\beta\right) \mathrm{e}^{i\omega t} \\ & + E_0 \varrho\left(\omega + \Omega\right) J_1\left(\beta\right) \mathrm{e}^{i(\omega+\Omega)t} \\ & - E_0 \varrho\left(\omega - \Omega\right) J_1\left(\beta\right) \mathrm{e}^{i(\omega-\Omega)t} \end{aligned}$$

The Pound-Drever-Hall Technique (4)

The reflected power has three terms:

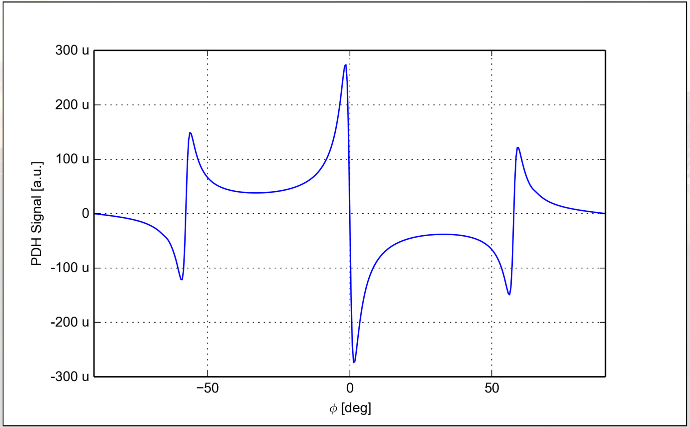
$$P_{\mathrm{DC}} \equiv P_{\mathrm{c}} |\varrho(\omega)|^2 + P_{\mathrm{s}} \left[|\varrho(\omega+\Omega)|^2 + |\varrho(\omega-\Omega)|^2 \right]$$
$$P_{\mathrm{I}} \equiv 2\sqrt{P_{\mathrm{c}} P_{\mathrm{s}}} \Re \left[\varrho(\omega) \varrho^*(\omega+\Omega) - \varrho^*(\omega) \varrho(\omega-\Omega) \right] \cos(\Omega t)$$
$$P_{\mathrm{Q}} \equiv 2\sqrt{P_{\mathrm{c}} P_{\mathrm{s}}} \Im \left[\varrho(\omega) \varrho^*(\omega+\Omega) - \varrho^*(\omega) \varrho(\omega-\Omega) \right] \sin(\Omega t)$$

which are the *DC*, *In-phase* and *Quadrature-phase* components respectively; the mixer can extract one of the RF terms (I for example), so the resulting error signal is

$$\varepsilon = 2\sqrt{P_{\mathrm{c}} P_{\mathrm{s}}} \Re \left[\varrho(\omega) \varrho^*(\omega+\Omega) - \varrho^*(\omega) \varrho(\omega-\Omega) \right]$$

The peculiarity of this error signal is that it is a bipolar signal for both the carrier and the sidebands, and it has a steep linear zero-crossing exactly at the resonance; also, the carrier and the sidebands can be recognized easily as they have opposite signs for the zero-crossing.

The Pound-Drever-Hall Technique (5)

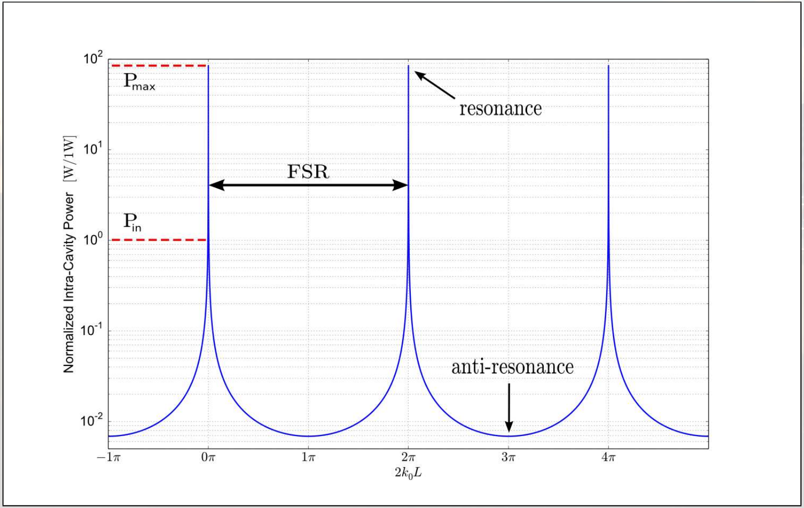


Approximated error signal expression:

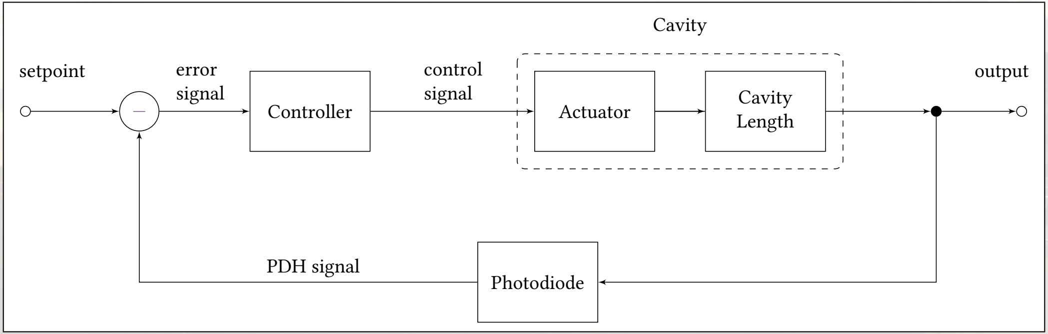
$$\varepsilon \approx -16\sqrt{P_c P_s}\frac{\mathcal{F}}{\lambda_0}\delta L$$

- Linear in the cavity length variation δL
- Dependent on Finesse $\mathcal{F}$ of the cavity
- Narrow linear range in the proximity of the resonance

The Pound-Drever-Hall Technique (6)

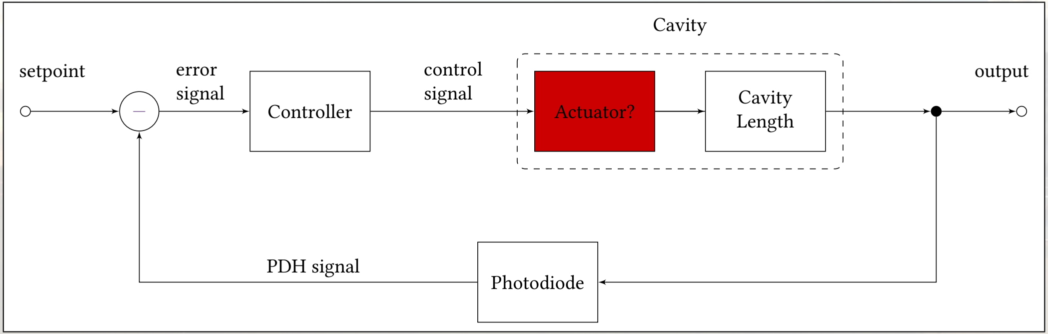


Length Control as a Feedback System



- This is the generic control scheme
- Several different implementations based on it

Length Control as a Feedback System

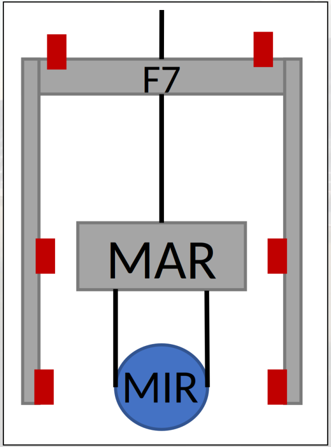


- This is the generic control scheme
- Several different implementations based on it

What do we use to control the cavity length?

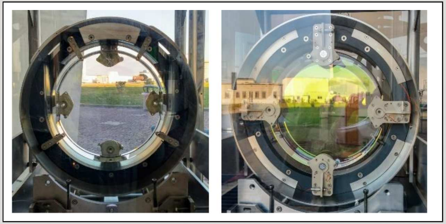
Applying Forces to Mirrors (1)

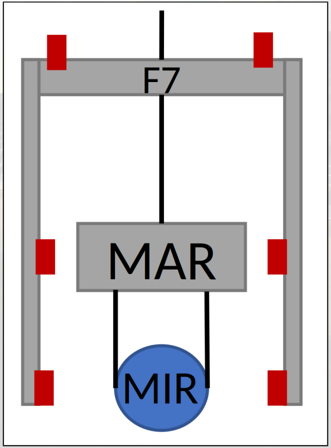
- It is obvious that we cannot touch the mirrors in any way, so we must use electromagnetic actuators (but beware of the noise!)
- The strategy of Advanced Virgo: coil-magnet pairs
- Many of them: on Mirrors, Marionettas, Filter7, etc...
  - 1 passive element (*magnet*) which is attached to the mirror
  - 1 active element (*coil*) attached to the last stage of the actuation cage



Applying Forces to Mirrors (2)

- We also have to control the angular position of the mirror, not only the longitudinal one
- The strategy of Advanced Virgo: 4 coil-magnet pairs on mirrors, 8 on marionettas
- Alignment control is done *from marionettas*
- By applying different corrections to each magnet it is possible to actuate in the *longitudinal*, *pitch*, and *yaw* directions

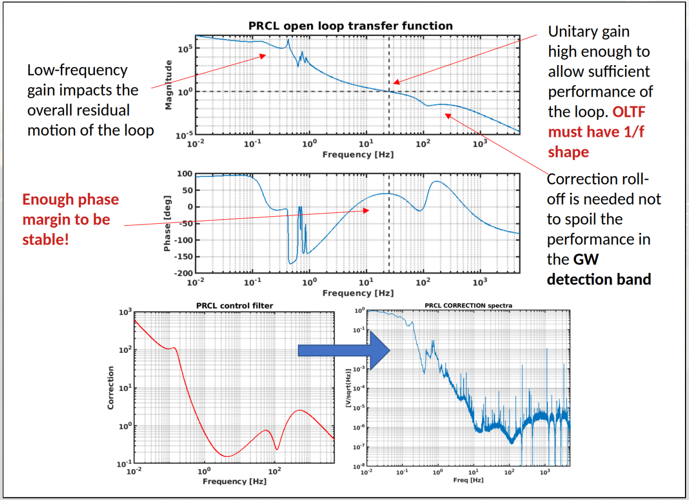




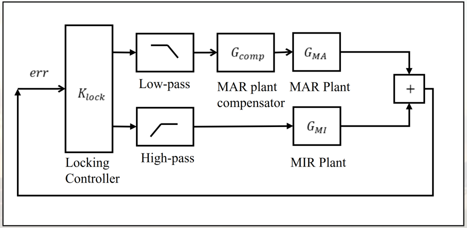
Design of a Loop Filter

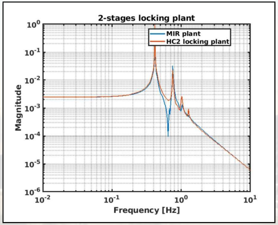
The main sources of noise of a given control scheme arise from:

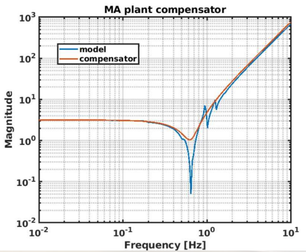
- Sensing noise (the goodness of the physical sensor, and of the signal extracted from it)
- Actuation noise (force necessary to compensate the residual motion of the mirrors)
- Coupling noise from other degrees of freedom

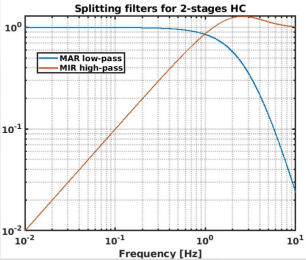


Reducing Control Noise: Reallocating the Correction





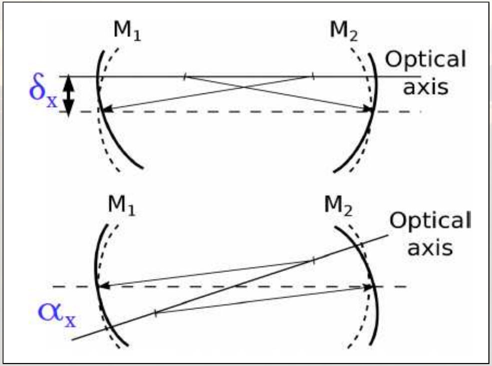


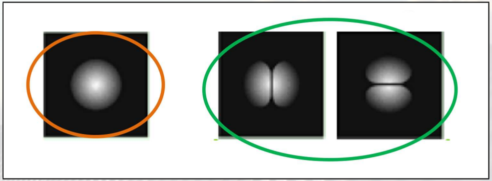


- The basic correction distribution strategy used in Virgo consists in properly reallocate the locking correction among the actuation points of the bottom stage (MAR + MIR)
- This is done by sending the low-frequency part of the actuation force to the marionetta, thus reducing by several orders of magnitude the correction needed at the mirror level
- To do so, a compensation of the mechanics, together with a proper splitting filters design, are needed.

Alignment in a Cavity

When a misalignment in a cavity is present, the generation of higher order modes (HOMs) occurs.





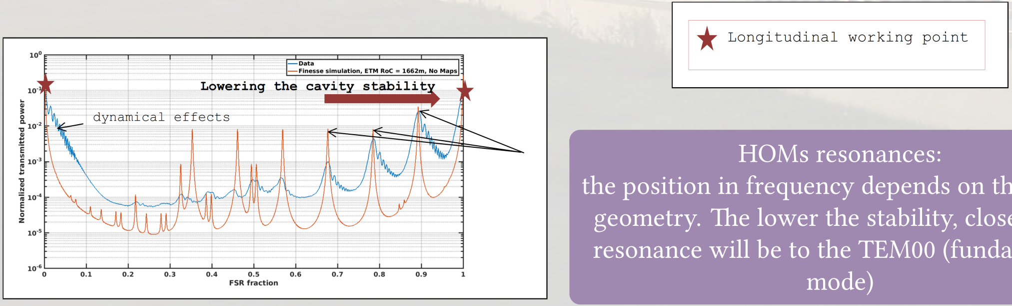
$$E(x+\delta_x) \approx A \cdot \left[H_0(x) + \frac{\delta_x}{w_0} \cdot H_1(x) \right]$$

$$E(x+\alpha_x) \approx A \cdot \left[H_0(x) + i \cdot \frac{\alpha_x}{\theta_d} \cdot H_1(x) \right]$$

Effect of a Misalignment

The main effect of a misalignment in a cavity is:

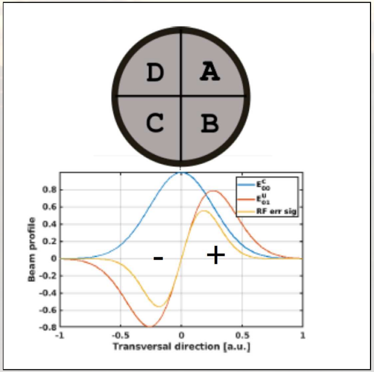
- **Geometrical length variation** of the cavity, due to the displacement of the cavity axis (easily recoverable with the longitudinal control)
- **Power loss** due to the generation of HOMs (the cavity does not let circulate the HOMs, accordingly to finesse and cavity stability)
- **Strong modification of the longitudinal error signal** (for low stability cavities)



HOMs resonances:
the position in frequency depends on the cavity geometry. The lower the stability, closer their resonance will be to the TEM00 (fundamental mode)

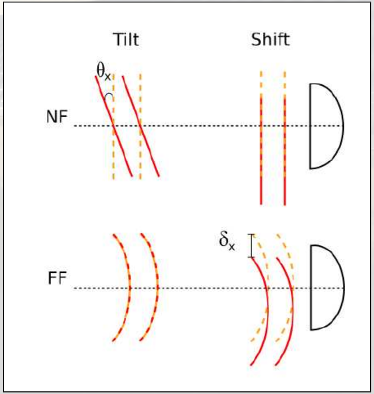
How to Sense Misalignments?

The sensing of a misalignment can be performed by taking advantage of the generation of the TEM 01/10 itself, by using a quadrant split photo-detector. Analogously to the longitudinal error signal, the alignment error signal comes from the beating of the TEM00 of the carrier and the 01 of the sidebands and vice versa



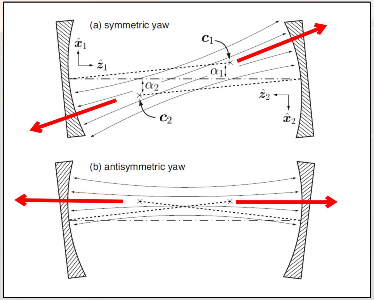
$$S_{\text{hor}} = (A+B) - (D+C)$$
$$S_{\text{ver}} = (A+D) - (B+C)$$

In order to disentangle the tilt and the shift of the cavity axis two QPDs will be used to image two different position of the cavity axis (Near Field, i.e. the waist position, and Far Field)



Sidle-Sigg Radiation Pressure Effect on Alignment

In a high power cavity the laser beam acts as a **spring** between the two mirrors (torsion pendula), modifying the overall opto-mechanical transfer function.



The laser beam applies a torque *against* the misalignment, making the system **harder** to be misaligned

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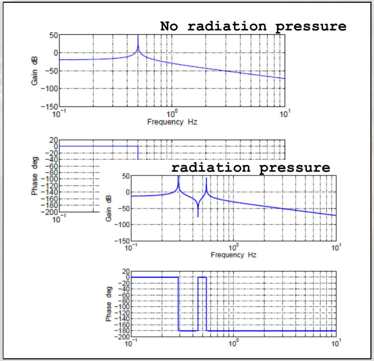
The laser beam applies a torque *towards* the misalignment, making the system **softer** to be misaligned

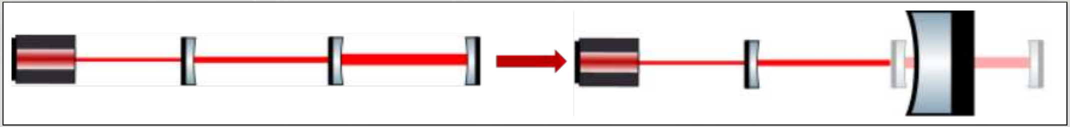


What is a Lock Acquisition?

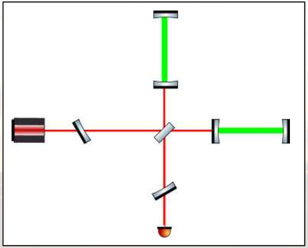
- When the interferometer is not controlled, the mirrors are free to move; their typical low frequency peak-to-peak motion can span as much as one wavelength ($\approx 1\,\mu$m), *if* the *Local Controls* (which use auxiliary lasers and Position Sensing Devices) are engaged, otherwise the motion is much higher

- Lock Acquisition: how to bring the system from a complete uncontrolled state to the final one, when all distances are tuned to the correct working point and the alignment of all mirrors is under control with the needed precision

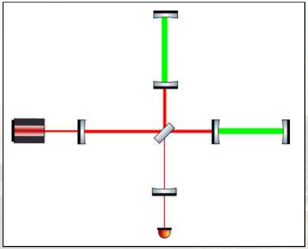
- A *serial* procedure must be identified, and the control progressively acquired using one or more techniques: not everything can be brought under control at the same time! Three macroscopic steps.



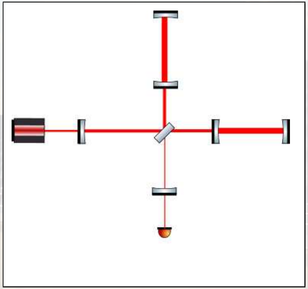
Advanced Detectors' Lock Acquisition in a Nutshell



Misalign the PR and SR mirror in order to have only the CARM and DARM DoFs. Lock DARM and CARM using an auxiliary laser (green beam).
2 DoFs: DARM and CARM



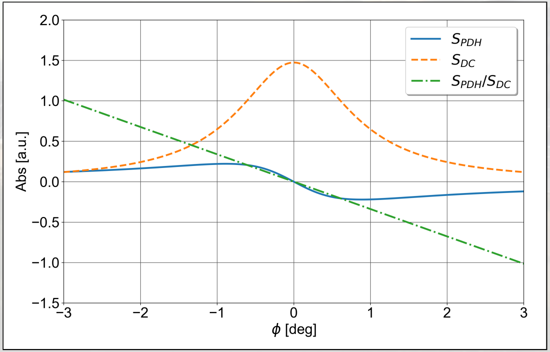
Move the resonance of the arms far from the IR resonance (arms invisible to the DRMI). Realign PR and SR mirrors and lock the central area DoFs.
3 DoFs: MICH, PRCL and SRCL



Bring back the arms to the resonance of the IR, having all the DoFs controlled.
5 DoFs: CARM, DARM, MICH, PRCL and SRCL

Step 1: Normalizing PDH Signals for Single Arms Control

- Normalizing the PDH error signals with the transmitted power increases the linear range
- This comes at the cost of a noisier signal
- In some cases, it is not guaranteed it is sufficient for the PDH lock to succeed

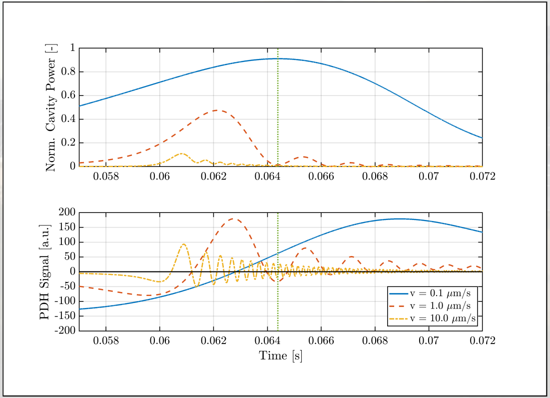


Step 1: *Guided Lock* of a Single Arm Cavity (1)

There are several upper thresholds for the cavity velocity, which is the relative velocity between the two mirrors:

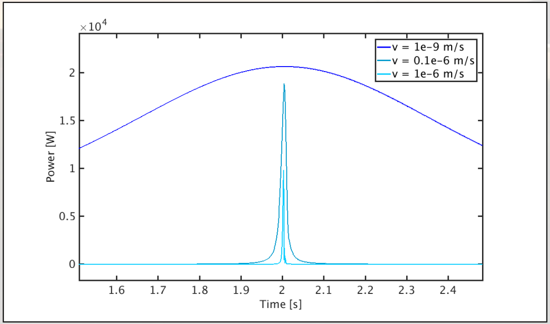
- response time of the feedback loop, which is related to the loop bandwidth
- the maximum force the actuators can do in a finite time
- the finite time needed by the laser field to build up inside the cavity

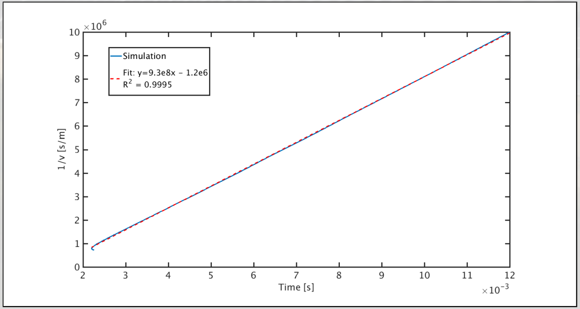
If such speed is above such thresholds the loop cannot be closed as the linear region is too narrow and a dynamical ringing effect arises.



Step 1: *Guided Lock* of a Single Arm Cavity (2)

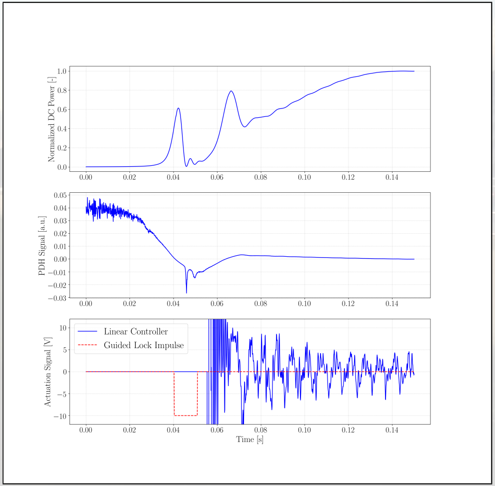
- The peak shape is function of the cavity velocity
- The rising time (already in the 10 to 40 % range of the transmitted power) is an estimator





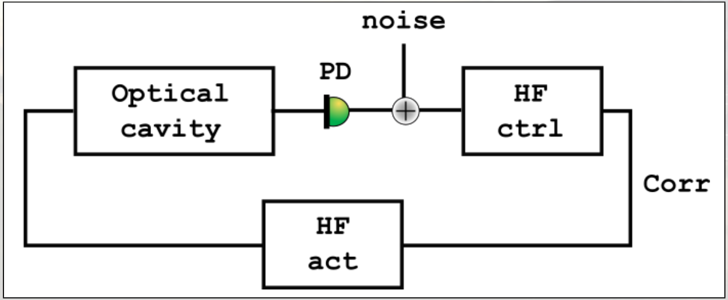
Step 1: *Guided Lock* of a Single Arm Cavity (3)

- The cavity velocity is constantly measured
- If needed, a pulsed actuation at maximum force is sent to slow down the cavity
- Once the cavity is in the linear range, the PDH lock is engaged



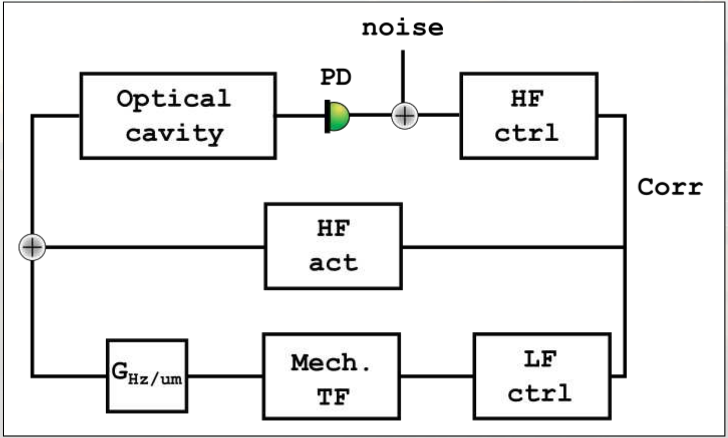
Step 1: Control of the Auxiliary Laser System (1)

- Novelty for *O4*, due to the SR mirror
- Frequency-doubled laser, used for controlling the cavities but away from the main laser resonance
- This uses the original PDH technique: the cavity is free-swinging and the laser frequency is locked to it



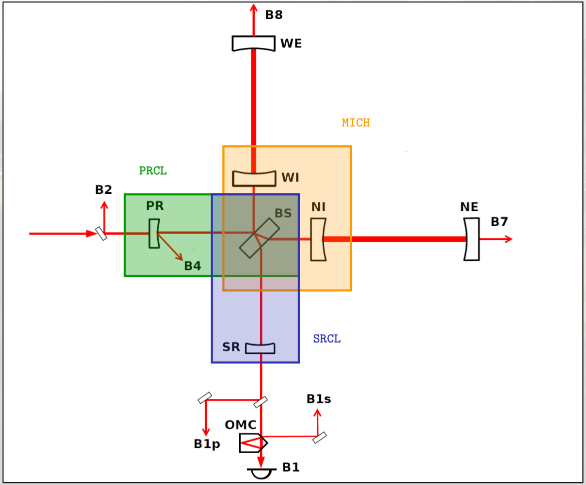
Step 1: Control of the Auxiliary Laser System (2)

- Later, a nested loop is engaged
- The correction signal is used as error signal for a usual PDH loop
- This low-frequency loop actuates on the mirrors instead, reducing the low-frequency residual motion

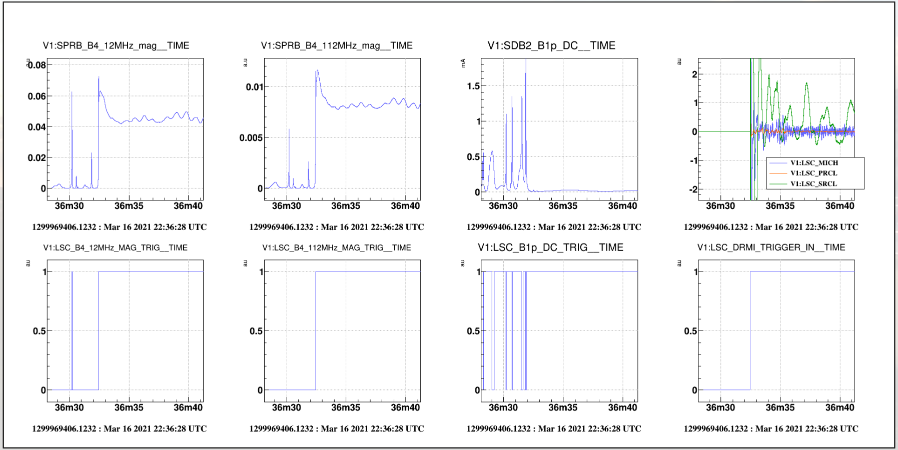


Step 2: *Coincidence Locking* for the DRMI (1)

- Novelty for *O4*, due to the SR mirror
- DRMI: Dual-Recycled Michelson Interferometer
- The three central degrees of freedom (MICH, PRCL and SRCL) are involved, with the arms locked with the green beam and out of the IR resonance
- Standard PDH for the lock, but the three DoFs must be engaged at the same time

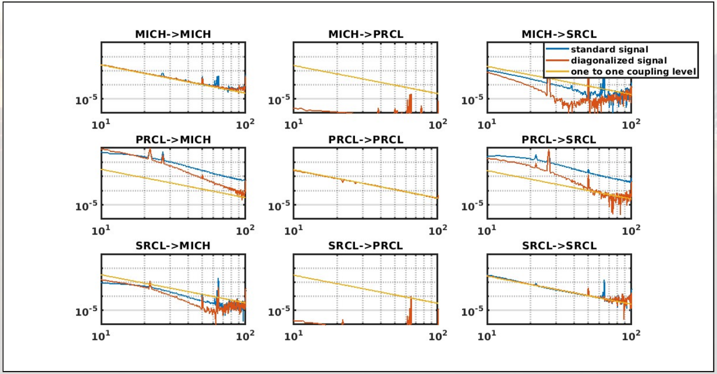


Step 2: *Coincidence Locking* for the DRMI (2)



- Several triggers, based on sidebands resonance and dark fringe condition
- The product of all triggers enables the three loops at once

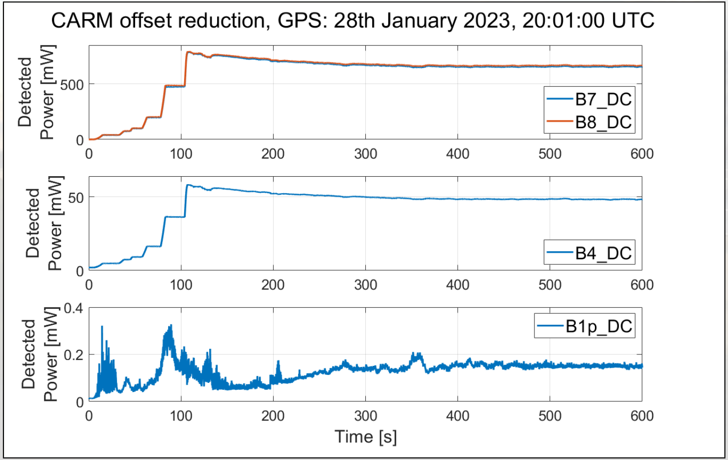
Step 2: *Diagonalizing the Sensing* of the DRMI



- The DRMI is a three-DoF system, that can be very coupled
- The inputs of the three loops can be made *non-diagonal* by mixing them
- In this way we can remove some of the couplings from the loops
- It is not a proper MIMO (Multiple Input Multiple Output) scheme, but it goes in that direction

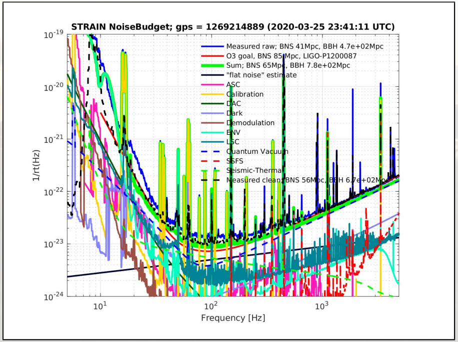
Step 3: CARM Offset Reduction

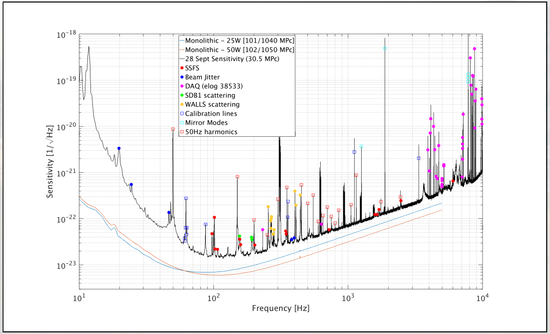
- Finally (*so to speak...*), we bring the arms back
- We progressively move back to the IR resonance, switching back from green beam control to the IR one
- This step, **the CARM offset reduction**, greatly increases the circulating power



Sensitivity Curve and Noises

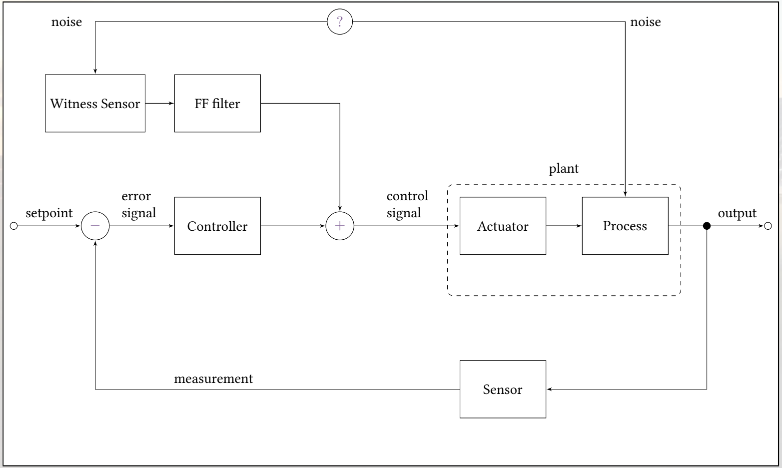
Once the interferometer is fully locked, the Noise Hunting begins:





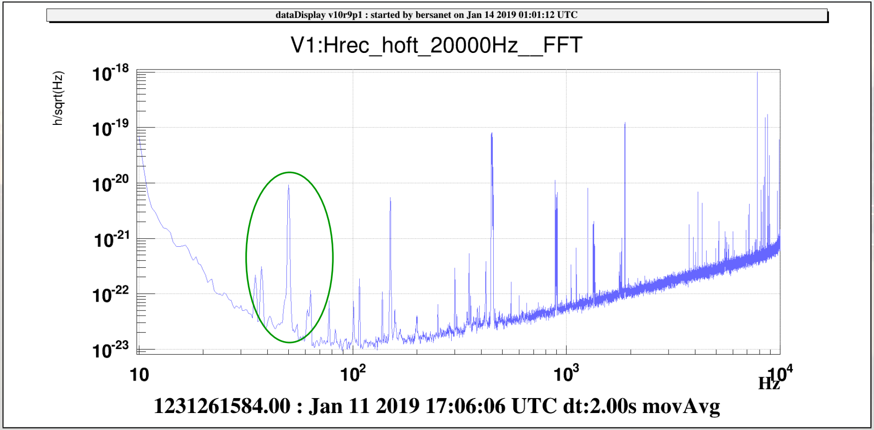
- Some noises are known and evaluated in the Noise Budget, others must be found
- Constant effort to reduce all known noises, and to find the source of the unknown ones

Feedforward inside a Feedback Control (*reprise*)



- Reduction of an external disturbance from a *in-loop* variable
- The relationship between noise and witness must be well known
- The relationship between noise and actuator must be well known
- A precise model is needed in order to build a performing filter

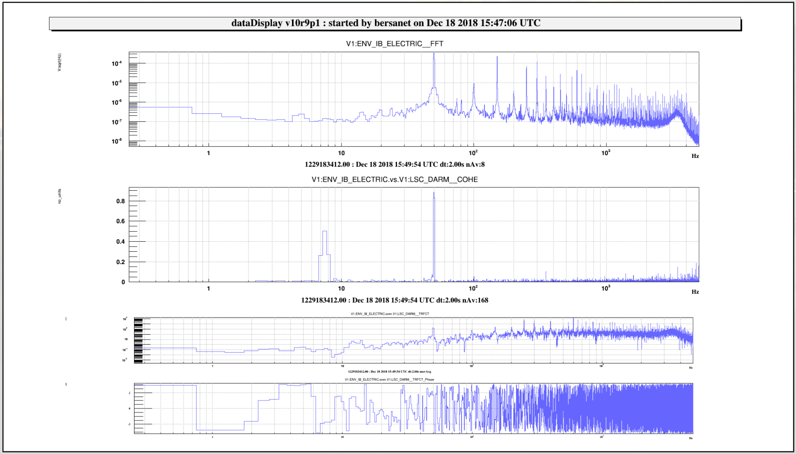
50 Hz Feedforward (1): Noise in $h(t)$



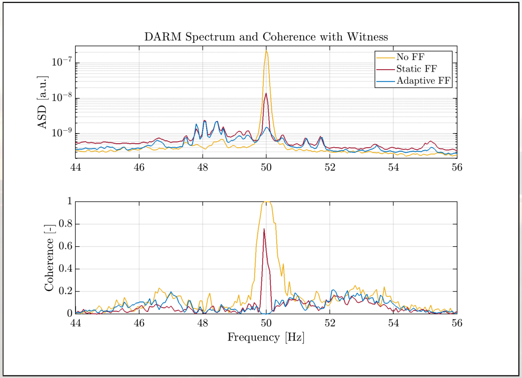
- Known source of noise
- Source is the mains lines
- Source can not be removed
- Effect can be subtracted

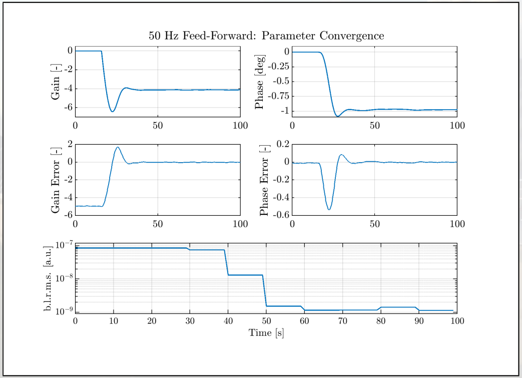
50 Hz Feedforward (2): Witness Sensor

- A reliable witness is one of the three phases of a probe of the IPS system
- The first step is the measurement of the TF between the witness and target (DARM)
- The second step is the measurement of the TF between target and actuator used for the subtraction



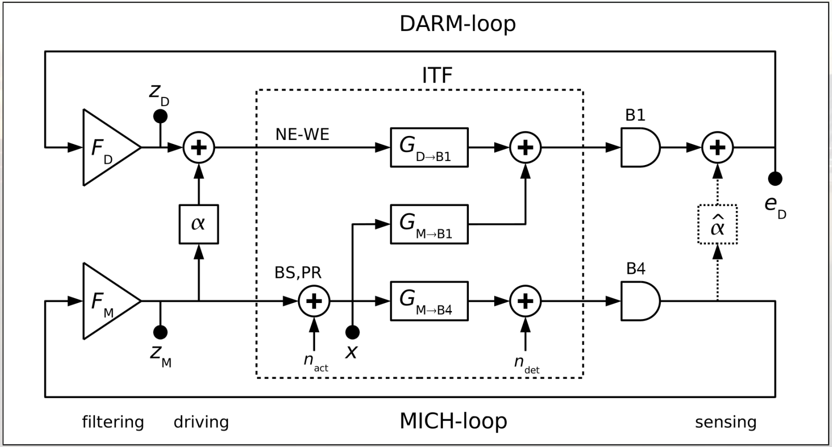
50 Hz Feedforward (3): Adaptive Control





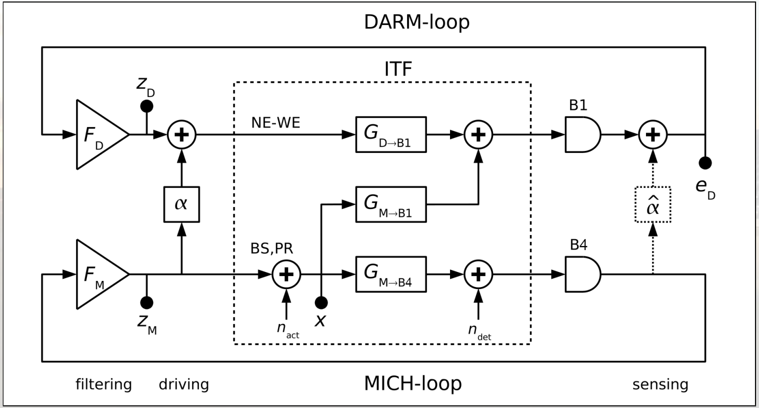
- A resonant gain filter was used
- The coupling was found to be not completely stationary in gain and phase
- Two feedback loops have been implemented to adapt the gain and the phase of the feedforward subtraction

MICH → DARM Coupling Subtraction (1)



- Coupling between longitudinal DOFs
- Broadband, frequency-dependent behavior
- Most of the coupling is *linear*
- Subtraction is possible

MICH $\longrightarrow$ DARM Coupling Subtraction (2)



- In principle the coupling factor is

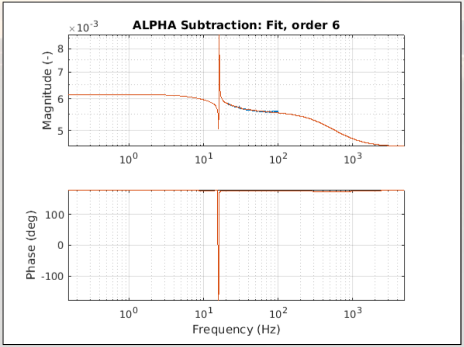
$$\alpha = -\frac{G_{M\to B1}}{G_{D\to B1}}$$

- But we cannot measure $G_{M\to B1}$ directly
- In the real interferometer, we have instead:

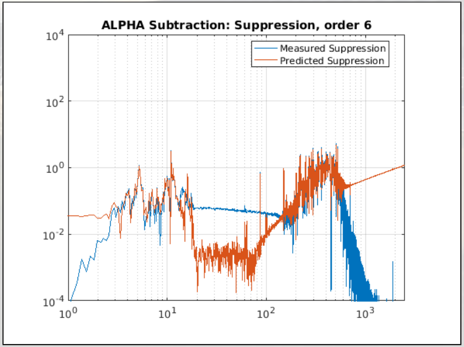
$$\alpha = -\frac{TF_{M\to B1}}{G_{Dcl} \cdot TF_{D\to B1}} = -\frac{TF_{M\to B1}\left(1 - TF_{Dpost\to Dpre}\right)}{TF_{D\to B1}}$$

MICH → DARM Coupling Subtraction (3)

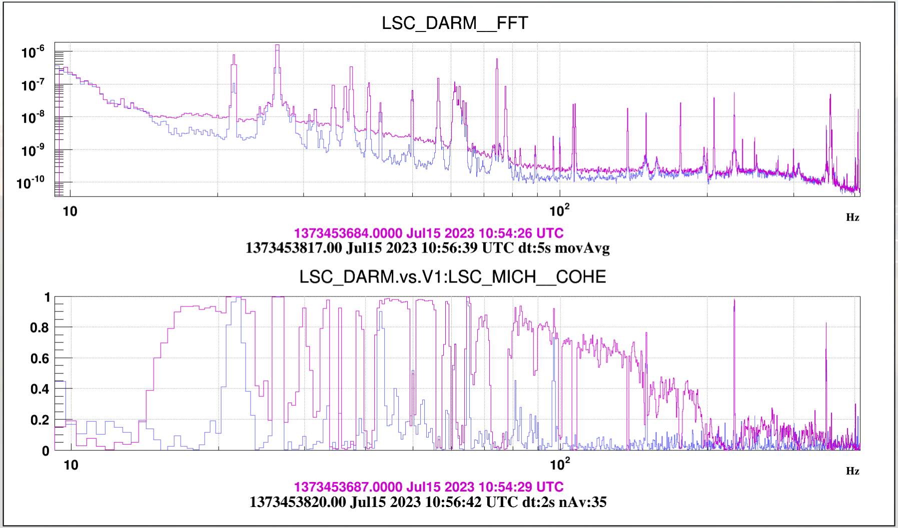
- Frequency-domain fit to find the feed-forward filter
- Filters of different orders are compared



- The predicted new suppression is computed and compared to the current one (if present)

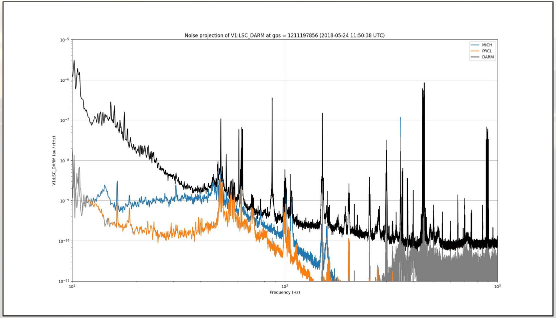


MICH → DARM Coupling Subtraction (4)



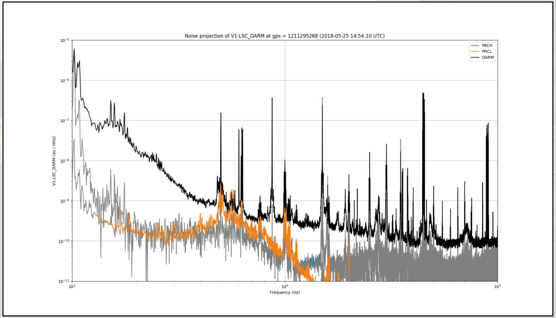
MICH → DARM Coupling Subtraction (5)

- Contribution from MICH is suppressed



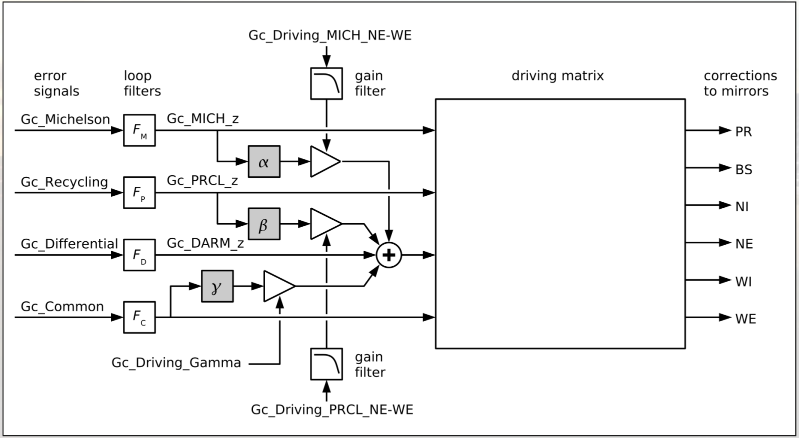
Old feedforward filter

- Coherence between DoFs is reduced



New feedforward filter

MICH → DARM Coupling Subtraction (6)



- General approach, can be used for *any* coupling to DARM
- But the coupling must be linear
- The performance depends strongly on *stationarity over time*

What Comes Next?

- Controlling a laser interferometer is a challenging topic, which makes extensive use of control techniques, from more standard to more exotic ones
- The current status is that of a mature field, with solid experience in *classical* controls
- For upgraded interferometers already, and for the future generation for sure, the use of more modern controls will become a necessity: MIMO schemes, state-space, *h*-infinity, machine learning, full AI


Thank You for the Attention