Noise control in a space-based observatory

– Be aware, end mirrors shall not move! –

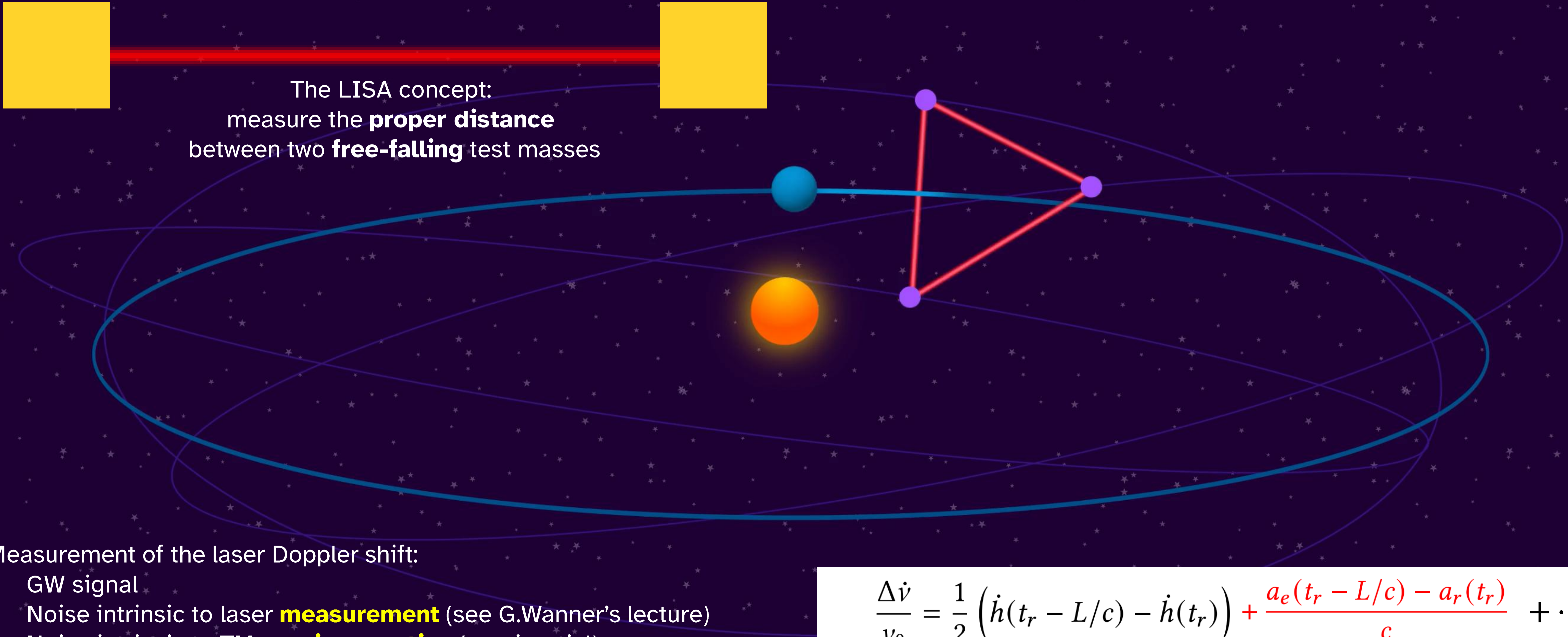
– They shall be free-falling in space –

– They shall follow a geodetic trajectory –

Lorenzo Sala – University of Trento, LISA GRS Team

25 May 2026 – Erice (TP), Ettore Majorana Center for Scientific Culture
School on Technologies in GW Detection 2026

LISA's need for nearly perfect free-fall



The LISA concept:
measure the **proper distance**
between two **free-falling** test masses

Measurement of the laser Doppler shift:

- GW signal
- Noise intrinsic to laser **measurement** (see G.Wanner's lecture)
- Noise intrinsic to TM **spurious motion** (non inertial)

$$\frac{\Delta\dot{\nu}}{\nu_0} = \frac{1}{2}\left(\dot{h}(t_r - L/c) - \dot{h}(t_r)\right) + \frac{a_e(t_r - L/c) - a_r(t_r)}{c} + \cdots$$

[Fractional frequency variation, overly simplified "single-link" LISA]

[This is overly simplified, as this does not consider laser noise, TDI, etc. / see G.Wanner's lecture]

*Image credits: quantamagazine.org

LISA's need for nearly perfect free-fall

The LISA concept:
measure the **proper distance**
between two **free-falling** test masses

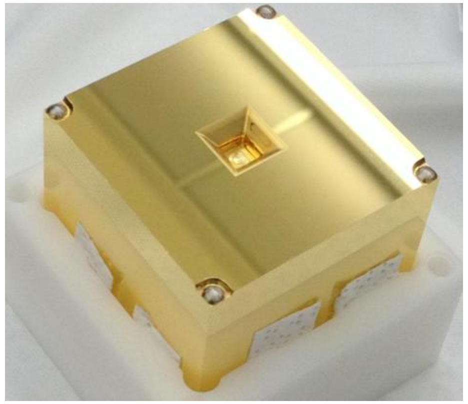
	Characteristic	Requirement
🟨	Free-fall purity	$\delta_a(1\,\mathrm{mHz}) \lesssim 3\times10^{-15}\,\mathrm{m\,s^{-2}}/\sqrt{\mathrm{Hz}}$
🟥	Displacement sensitivity	$\delta_x(1\,\mathrm{mHz}) \lesssim 15\times10^{-12}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$

$$\boxed{S_{\mathrm{acc}}^{1/2}(f) < 2.5\,\mathrm{fm\,s^{-2}}/\sqrt{\mathrm{Hz}}\left[1+\left(\frac{0.4\,\mathrm{mHz}}{f}\right)^2\right]^{1/2}\left[1+\left(\frac{f}{8\,\mathrm{mHz}}\right)^4\right]^{1/2}}$$

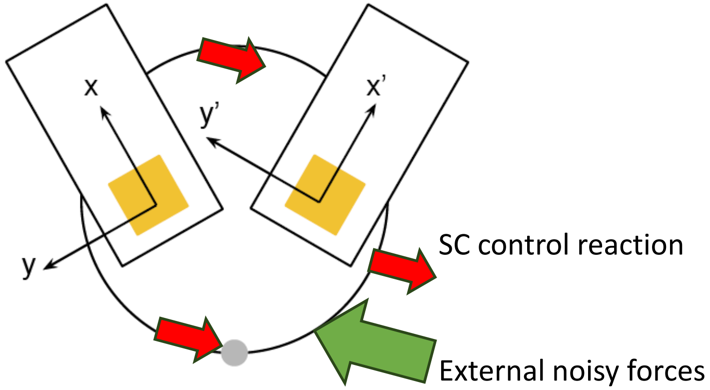
What can disturb the free-fall?

Forces and disturbances:

- Solar radiation pressure
- Actuation noise
- Brownian (molecular) noise
- Stray electrostatic noise
- TM-MOSA-S/C coupling noise
- Magnetic noise
- Laser pressure noise
- Temperature-induced noise
- Gravitational noise



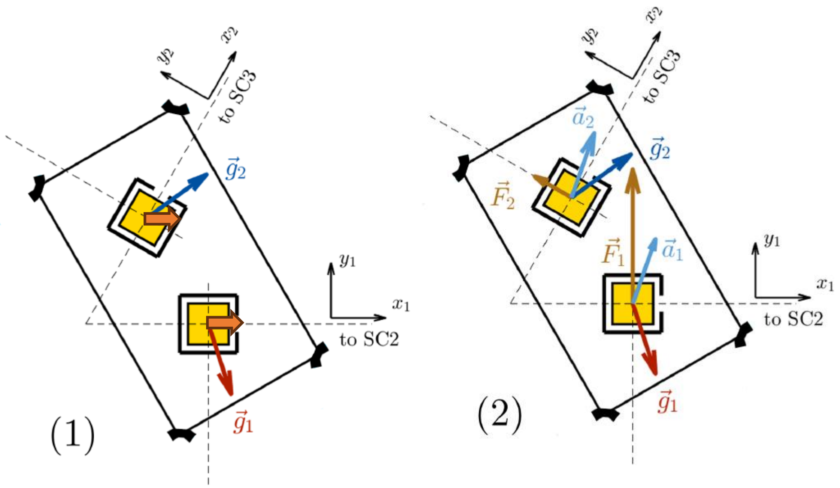
Get rid of external forces and deal with internal ones.



- External forces (like the solar radiation pressure) push on the SC.
  → Solution: use Test Masses as inertial reference, and apply forces to the SC.
  ? How?

- This is a single-SC problem.
  - Measure the relative SC-to-TM position on all DOFs,
  - Apply in-loop counteracting forces/torques to the SC (µN cold-gas thrusters)
  → Keep the SC-to-TM position fixed.

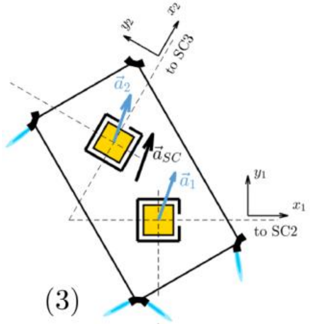
- Keep in mind a few things (will be relevant later):
  1. There are **internal** forces as well.
  2. Forces may be **quasi-DC**, as well as **noisy**.
  3. Any **application** of force/torque is noisy.
     - Thrusters are noisy → they induce additional SC jitter noise.
     - Actuating along the science axis is not viable for LISA.
     → We would end up bumping into a TM, as there is TM differential acceleration.



Credits: Vittorio Chiavegato

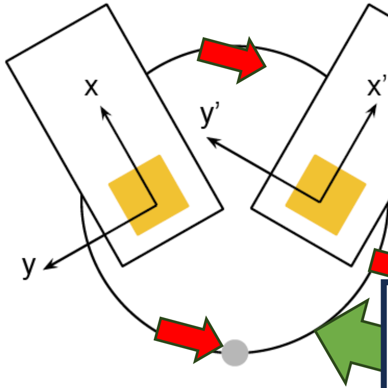
$$\begin{cases} \bar{a}_1 = \bar{g}_1 + F_1\hat{y}_1/M_1 \\ \bar{a}_2 = \bar{g}_2 + F_2\hat{y}_2/M_2 \\ \bar{a}_1 = \bar{a}_2 \end{cases} \Rightarrow \quad \Delta\bar{g} = \frac{F_1}{M_1}\hat{y}_1 + \frac{F_2}{M_2}\hat{y}_2$$

It is possible to find a solution,
if $\hat{y}_1$ and $\hat{y}_2$ are **not collinear**.
→ We need actuation along TMy.
(see W.Weber's lecture)



In the next slides: we need **LOW-NOISE** actuation!
(we don't want to be the source of noise…)

Get rid of external forces and deal with internal ones.



- External forces (like the solar radiation pressure) push on the SC.
  → Solution: use Test Masses as inertial reference, and apply forces to the SC.
  ? How?

- This is a single-SC problem.
  - Measure the relative SC-to-TM position on all DOFs,

he SC (µN cold-gas thrusters)

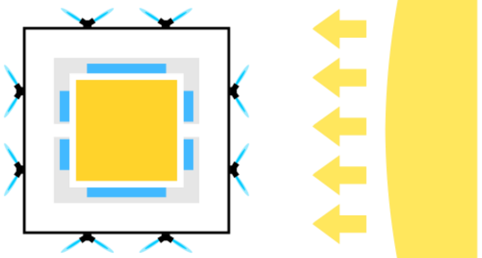
Easy calculation:

- Solar radiation intensity $I_0 = 1.3\ \mathrm{kW/m^2}$
- SC surface $20\ \mathrm{m^2}$, SC mass $2000\ \mathrm{kg}$
- Solar radiation variation: ~ 100 ppm over timescales of mins

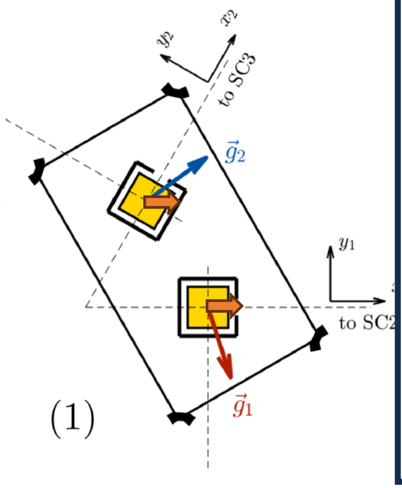
$a = \frac{F}{M} = \frac{2A}{Mc} I \sim 70\ \mathrm{nm/s^2}$ with 100ppm fluctuation over a few minutes

You can calculate the corresponding PSD…
That's **much higher** than $\mathrm{fm\ s^{-2}/Hz^{-1/2}}$.

The autocorrelation decays over timescales of minutes.
$R_a(t) = \delta a^2 e^{-|t|/\tau} \rightarrow S_a(f) = 2\delta a^2 \tau / (1 + (2\pi f \tau)^2)$
$S_a^{1/2} \sim 0.1\ \mathrm{nm\ s^{-2}/Hz^{-1/2}}$



SC jitter noise.
for LISA.
re is TM differential acceleration.

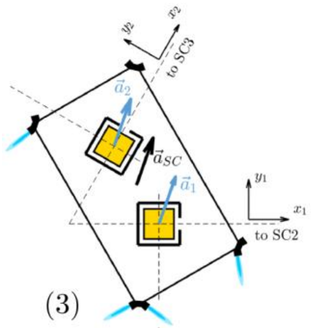


Credits: Vittorio Chiavegato

In the next slides: we need **LOW-NOISE** actuation!
(we don't want to be the source of noise…)

$\hat{y}_2$

It is possible to find a solution,
if $\hat{y}_1$ and $\hat{y}_2$ are **not collinear**.
→ We need actuation along TMy.
(see W.Weber's lecture)



TM actuation.

- Electrostatic actuation is applied with rectangular electrodes facing the TM. It is applied along all DOFs.
- It shares its electrodes with capacitive sensing.
- Main purpose: control the TM position along all non-x DOFs

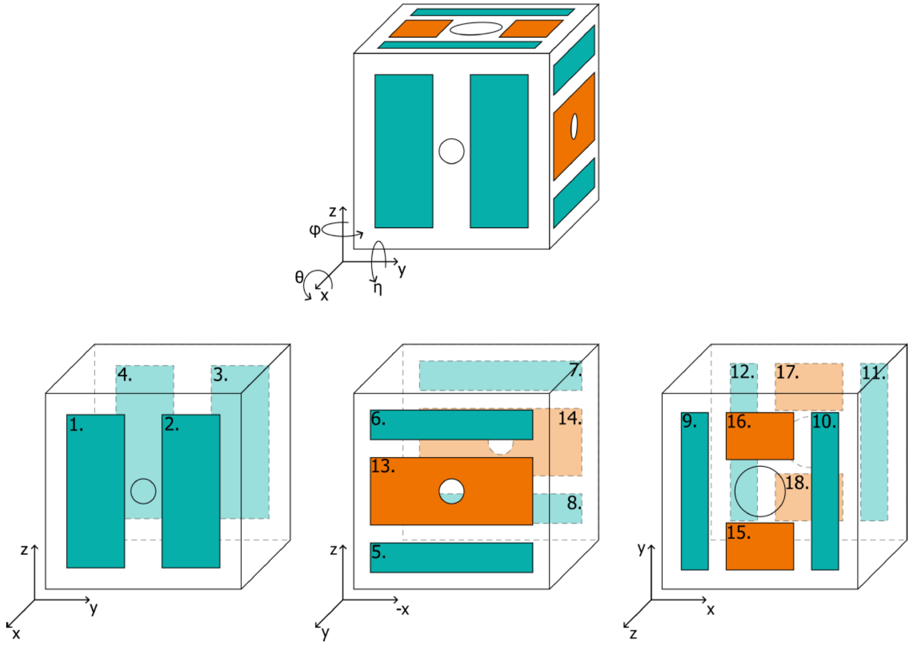
- Science constraints:
  - Apply **low-noise $(\mathrm{fm/s^2/Hz^{-1/2}})$ actuation** (force+torque) along all DOFs, except for x.
  - Do not apply actuation along x, and minimize noise leakage from other DOFs.
  - Do not induce any **TM potential** variation.
  - Do not introduce command-dependent **stiffness**.

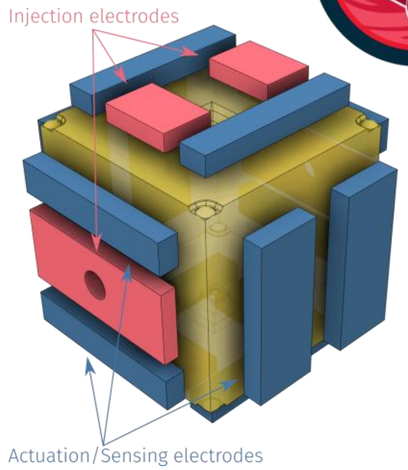
$$F_q = \frac{1}{2}\sum_i \frac{\partial C_i^*}{\partial q} V_i^2 - V_{TM}\sum_i \frac{\partial C_i}{\partial q} V_i + \frac{1}{2}\frac{\partial C_T}{\partial q} V_{TM}^2,$$

$$V_{TM} = \frac{Q + \Sigma_i V_i C_i}{C_T}$$

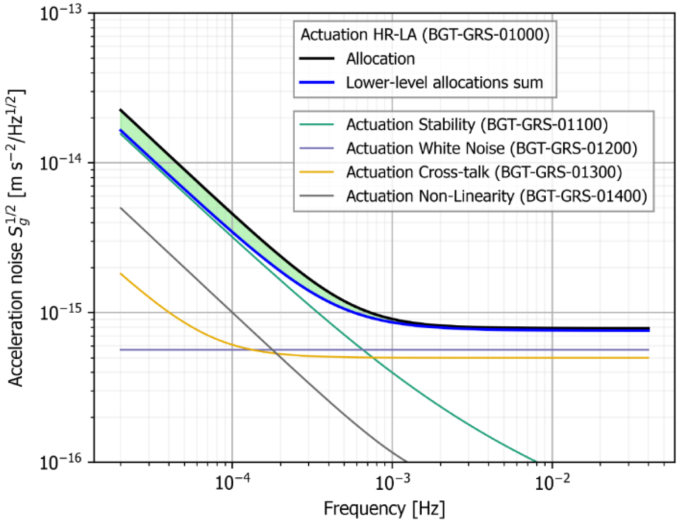
Important: electrostatic actuation is always positive, **it can only "pull"**

Important: unbalanced voltages yield **a net VTM** (unwanted!!)



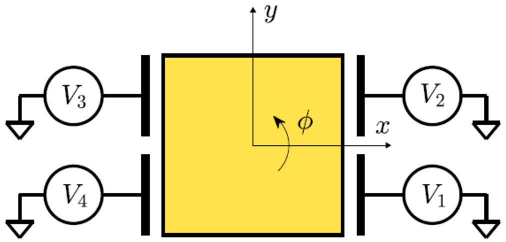


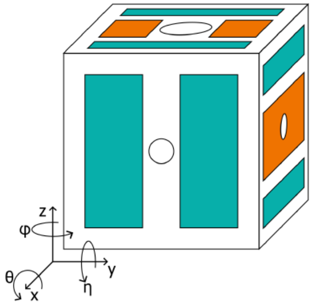
Credits: Davide Dal Bosco

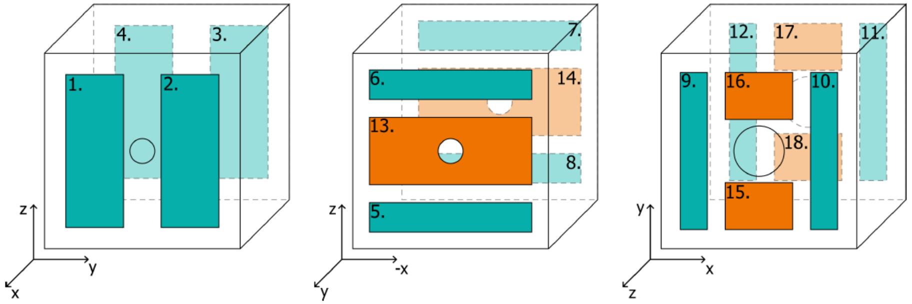


DC electrostatic actuation. Why not?

$$F_q = \frac{1}{2}\sum_i \frac{\partial C_i^*}{\partial q} V_i^2 - V_{TM}\sum_i \frac{\partial C_i}{\partial q} V_i + \frac{1}{2}\frac{\partial C_T}{\partial q} V_{TM}^2, \qquad V_{TM} = \frac{Q+\Sigma_i V_i C_i}{C_T}$$







Suppose that we need to apply a net phi torque

$$N_\phi = \frac{1}{2}\left|\frac{\partial C_X^*}{\partial \phi}\right| (V_{1\phi}^2 - V_{2\phi}^2)$$

Applying either $V_{1\phi}$ on EL1 and EL3 ($+\phi$ rotation),
or $V_{2\phi}$ on EL2 and EL4 ($-\phi$ rotation).

We could apply quasi-DC (slowly-varying if needed) voltages on the electrodes. As $|\partial_\phi C_X^*| \sim 3\text{pF/rad}$, to have a torque (per unit inertia) $\gamma_\phi = 1\ \text{nrad/s}^2$ (why? wait a few slides), we would need a voltage of roughly $0.5\ \text{V}$.

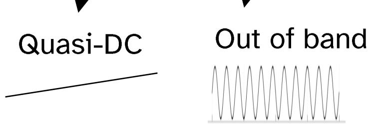
As this relation is quadratic,

$$\delta N_\phi \propto V_\phi\, \delta V_\phi \;\Rightarrow\; S_{N_\phi}(f) \propto V_\phi^2\, S_{V_\phi}(f),$$

meaning that **any** electrical noise (e.g. patch potentials) would **linearly** transfer into torque noise.

→ The choice is: audio-frequency (N*16Hz) actuation.
(272Hz for phi actuation)
(exploiting the quadratic nature of electric force)

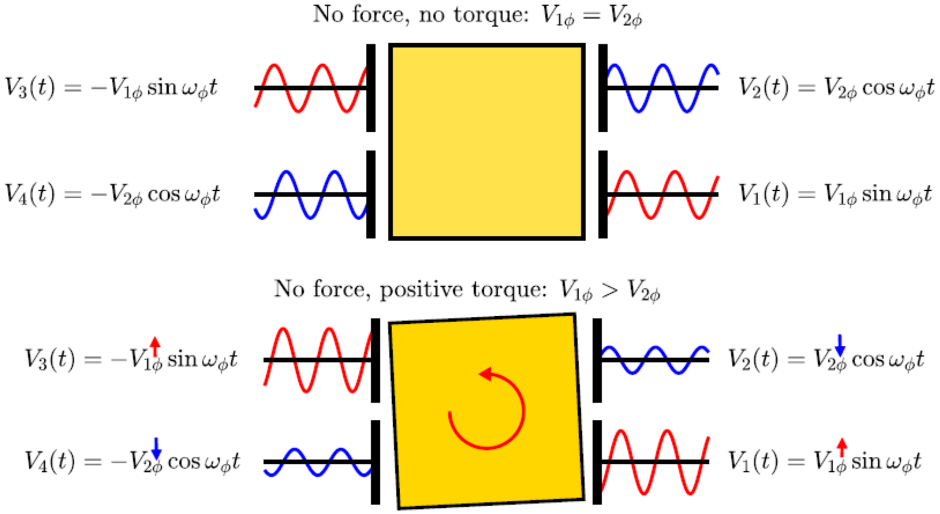
$$N_\phi = \frac{1}{2}\left|\frac{\partial C_X^*}{\partial \phi}\right| \left(V_{1\phi}\sin\omega_\phi t\right)^2 = \frac{1}{4}\left|\frac{\partial C_X^*}{\partial \phi}\right| V_{1\phi}^2\left(1-\cos 2\omega_\phi t\right)$$



Audio-frequency electrostatic actuation.

$$F_q = \frac{1}{2}\sum_i \frac{\partial C_i^*}{\partial q} V_i^2 - V_{TM}\sum_i \frac{\partial C_i}{\partial q} V_i + \frac{1}{2}\frac{\partial C_T}{\partial q} V_{TM}^2,$$

$$V_{1\phi/2\phi} = \sqrt{\frac{I_{zz}(\gamma_{\phi 0} \pm \gamma_{\phi c})}{\left|\frac{\partial C_x^*}{\partial \phi}\right|}}$$



- Commanding zero force(torque) means commanding balanced voltages.
- Commanding a net force(torque) means unbalancing voltages.
- This is valid for all DOFs.

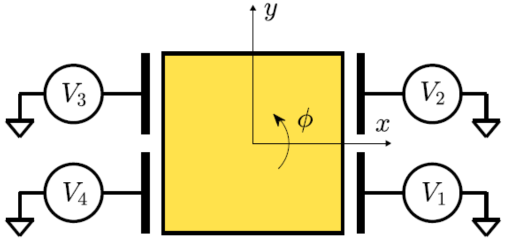
- The parameter γ_{ϕ_0} is known as *authority*. It is fixed in the actuation scheme. It is the maximum (and –minimum) commandable force(torque) $-\gamma_{\phi_0} < \gamma_{\phi_c} < +\gamma_{\phi_0}$

- Waveforms are chosen so that:
  1. Their sum is zero. No induced TM potential.
  2. They are orthogonal on different DOFs: this reduces electrical crosstalk. e.g. $\int_{\text{cycle}} \chi_q(t)\chi_p(t)dt = 0 \ \forall q, p$
  3. This scheme also ensures *constant stiffness*.

- For LISA (wait for the reason behind this): $g_{x_0} = 0$, $g_{y_0} \sim 1\text{nm/s}^2$, $\gamma_{\phi_0} \sim 1\text{nrad/s}^2$

Credits: Vittorio Chiavegato

A critical example of actuation noise (gain fluctuations)



Intuitive approach

Assume we want to apply phi torque but no x force.
We “pull” with EL1 and EL3. This yields both torque and force.

$$\begin{cases} N_\phi = R^*_\phi (F_{EL1} + F_{EL3}) \\ F_x = F_{EL1} - F_{EL3} \end{cases}$$

We choose nominally $F_{EL1} = F_{EL3}$ so that $F_x = 0$ (nominally!!!)

With $F_{CM13} = (F_{EL1} + F_{EL3})/2$, $F_{\Delta 13} = F_{EL1} - F_{EL3}$, if we had force fluctuations δF_{EL1}, δF_{EL3}, the leading order would be

$$\begin{cases} N_\phi = R^*_\phi (2F_{CM13}) + R^*_\phi (2\delta F_{CM13}) \\ F_x = \delta F_{\Delta 13} \end{cases}$$

If there is a (non-correlated!) unbalance $\delta F_{\Delta 13}$,
we involuntarily apply force while applying torque!

The larger the torque we want to apply, the larger the noise!
(this is also a reason why we do not actuate x force)

Rigorous approach

Assume we want to apply phi torque but no x force.
We unbalance the “pull force” on all electrodes, with AC actuation.
Assume $\alpha(t) = \delta V(t)/V$ is a fluctuating actuator gain.

$$V_{1\phi/2\phi} = \sqrt{\frac{I_{zz}(\gamma_{\phi 0} \pm \gamma_{\phi c})}{\left|\frac{\partial C^*_x}{\partial \phi}\right|}}$$

$$R^\star_\phi = \left| \frac{I_{zz}}{M_{\mathrm{TM}}} \frac{\partial C^*_X}{\partial x} \Big/ \frac{\partial C^*_X}{\partial \phi} \right| \sim 33\,\mathrm{mm}$$

$$g_x = 2a\alpha + \frac{1}{2}\sum_j a_j \alpha_j,$$

$$\begin{cases} a = 2g_{x_c} \\ a_1 = +g_{x_c} + g_{x_0} + R^*_\phi \gamma_{\phi_c} + R^*_\phi \gamma_{\phi 0} \\ a_2 = +g_{x_c} + g_{x_0} + R^*_\phi \gamma_{\phi_c} - R^*_\phi \gamma_{\phi 0} \\ a_3 = -g_{x_c} - g_{x_0} + R^*_\phi \gamma_{\phi_c} - R^*_\phi \gamma_{\phi 0} \\ a_4 = -g_{x_c} - g_{x_0} + R^*_\phi \gamma_{\phi_c} + R^*_\phi \gamma_{\phi 0} \end{cases}$$

As we apply no force along x, $g_{x_c} = g_{x_0} = 0$.
After calculations, noise along x depends on: uncorrelated noise, the *authority* and the *commanded* force.

$$S_g(f) = R^{*2}_\phi \left(\gamma^2_{\phi_c,\mathrm{DC}} + \gamma^2_{\phi 0} \right) S_{\alpha_{\mathrm{UC}}}(f)$$

→ **The larger the authority and the force command, the larger the actuation noise**

? Question: how do we *decide* the authority?

Quasi-DC forces, gravitational compensation

For any force, do never expect it has no spatial gradient!
Thanks to the control loop, the distance TM-to-SC is almost fixed, so that the system is effectively linear.

$$g = g|_{x_0} + \left.\frac{\partial g}{\partial x}\right|_{x_0} (x - x_0) + \cdots$$

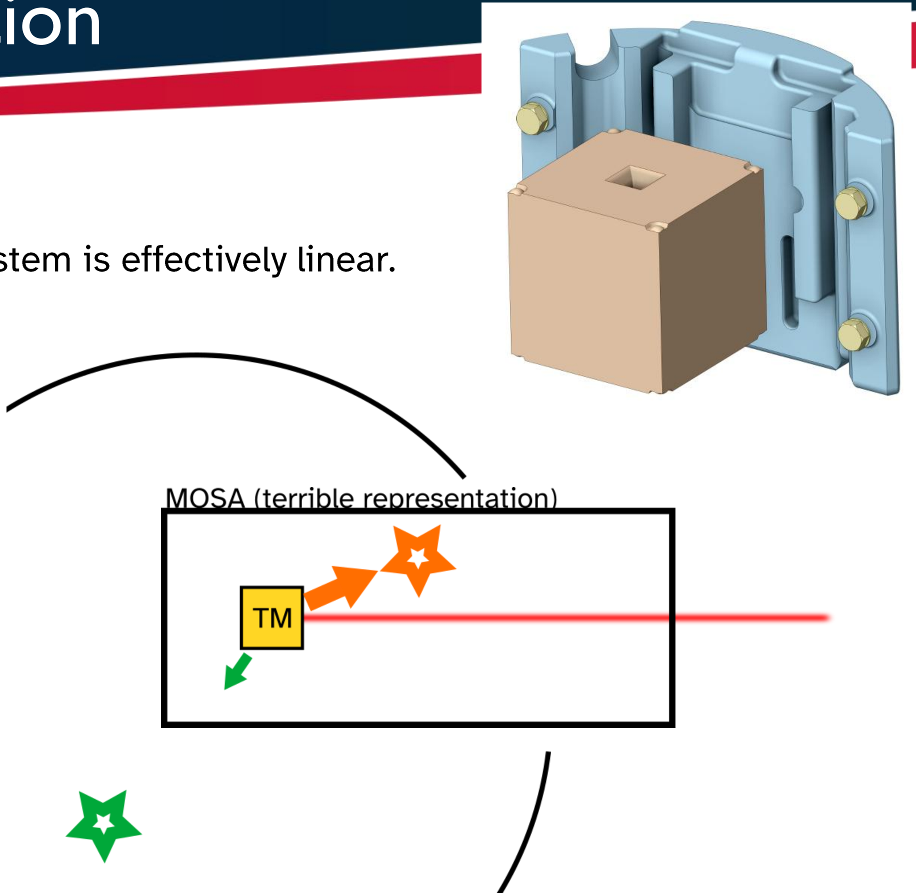
Quasi-DC force

Stiffness $\omega \equiv -\partial g/\partial x|_{x_0}$

DC forces (and torques) may be: gravitational, electrical, thermal,…
→ We need to **compensate** known forces/torques.
→ We need a strategy for fuel **depletion**.
→ We need to electrically **compensate what remains,** with actuation.

→ We need to compensate tens of $\mathrm{nm(rad)/s^2}$, down to $\mathrm{pm/s^2}$ and $\mathrm{nrad/s^2}$ to alleviate the burden on electrostatic actuation.

The (uncompensated) DC force/torque level sets the authority, which sets the actuation noise.
Unnecessarily large authority leads to unnecessarily large noise!



Do not think that balancing is trivial, as gravitational fields move with the MOSA!

Quasi-DC forces, gravitational compensation

For any force, do never expect it has no spatial gradient!
Thanks to the control loop, the distance TM-to-SC is almost fixed, so that the system is effectively linear.

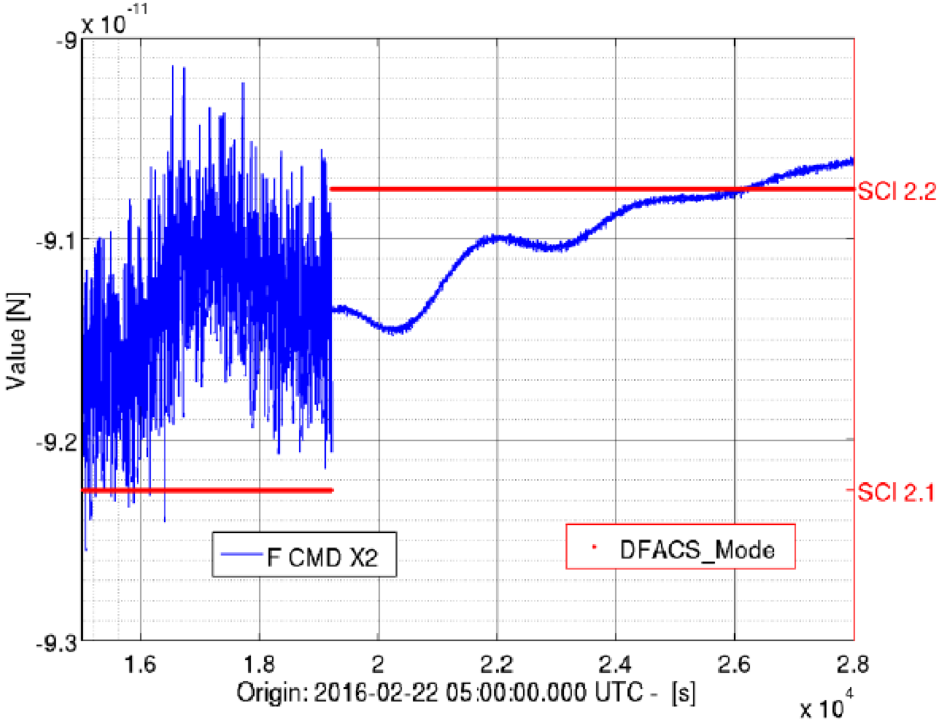
$$g = g|_{x_0} + \frac{\partial g}{\partial x}\bigg|_{x_0} (x - x_0) + \cdots$$

Quasi-DC force

Stiffness $\omega \equiv -\partial g/\partial x|_{x_0}$

DC forces (and torques) may be: gravitational, electrical, thermal,…
→ We need to **compensate** known forces/torques.
→ We need a strategy for fuel **depletion**.
→ We need to electrically **compensate what remains,** with actuation.

→ We need to compensate tens of $\mathrm{nm(rad)/s^2}$, down to $\mathrm{pm/s^2}$ and $\mathrm{nrad/s^2}$ to alleviate the burden on electrostatic actuation.



The (uncompensated) DC force/torque level sets the authority,
which sets the actuation noise.
Unnecessarily large authority leads to unnecessarily large noise!

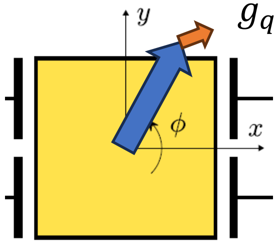
Quasi-DC forces, stiffness

For any force, do never expect it has no spatial gradient!
Thanks to the control loop, the distance TM-to-SC is almost fixed, so that the system is effectively linear.

$$g = g|_{x_0} + \left.\frac{\partial g}{\partial x}\right|_{x_0} (x - x_0) + \cdots$$

Quasi-DC force

Stiffness $\omega \equiv -\partial g/\partial x|_{x_0}$



$$g_q = g_q\Big|_{\bar{x}_0} - \sum_p \omega^2_{qp}(p - p_0) + \cdots$$

In general, stiffness is a matrix:

$$g_q = g_q\Big|_{\text{eq.}} - \sum_p \omega^2_{qp}(p - p_{\text{eq}})$$

$$S_{g_q,g_r}(f) = \sum_{p,s} \omega^2_{qp}\omega^2_{rs} S_{\delta p,\delta s}(f) \Rightarrow S_{g_q}(f) = \sum_p \left(\omega^2_{qp}\right)^2 S_{\delta p}(f) + \cancel{\sum_{p\neq s} \omega^2_{qp}\omega^2_{rs} S^{\frac{1}{2}}_{\delta p}(f) S^{\frac{1}{2}}_{\delta s}(f) \rho_{\delta p,\delta s}(f)}$$

Stiffness and the constant-stiffness scheme

For any force, do never expect it has no spatial gradient!
Thanks to the control loop, the distance TM-to-SC is almost fixed, so that the system is effectively linear.

$$g = g|_{x_0} + \left.\frac{\partial g}{\partial x}\right|_{x_0}(x - x_0) + \cdots$$

Quasi-DC force

Stiffness $\omega \equiv -\partial g/\partial x|_{x_0}$

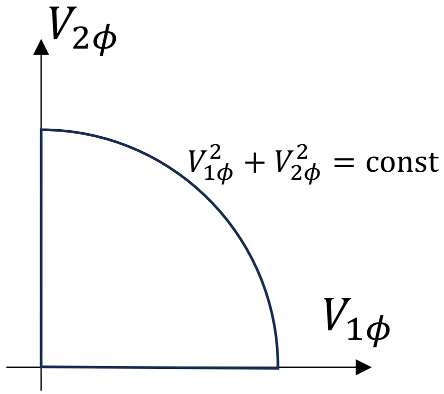
$$g_x = \cancel{g_{x_c}} + \left[\cancel{\frac{\left|\frac{\partial^2 C_X^*}{\partial x^2}\right|}{\left|\frac{\partial C_X^*}{\partial x}\right|}} g_{x0} + \frac{I_{zz}}{M}\frac{\left|\frac{\partial^2 C_X^*}{\partial x^2}\right|}{\left|\frac{\partial C_X^*}{\partial \phi}\right|}\gamma_{\phi 0} - \frac{4I_{zz}}{MC_T}\frac{\left|\frac{\partial C_X}{\partial x}\right|^2}{\left|\frac{\partial C_X^*}{\partial \phi}\right|}\gamma_{\phi 0} \right] x$$

Stiffness may be gravitational, and electrostatic/actuation.

→ The gravitational part may be partially compensated, if needed
(but generally, it’s not needed. It’s more important to focus on DC F/N)

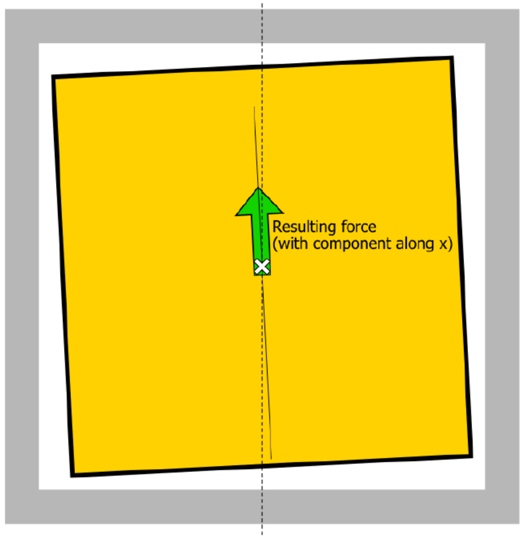
→ The electrostatic part may, in principle, depend on the command.
Example: think of an electric field.
The actuation scheme allows stiffness to only depend on authority (hence to be constant)
this alleviates post-processing needs

$$\begin{cases} \gamma_{\phi_0} \propto V_{1\phi}^2 + V_{2\phi}^2 \\ \gamma_{\phi_c} \propto V_{1\phi}^2 - V_{2\phi}^2 \end{cases}$$



Actuation crosstalk

At first approximation, electrostatic forces apply to the TM perpendicularly to **its** faces.
This means that forces applied along y and z may leak into force along x,
if the TM is not perfectly aligned with the science X axis.



$$F_x \sim -\delta\phi \, F_y + \delta\eta \, F_z$$

This effect is known as actuation crosstalk, and generally applies on all DOFs in matrix form,

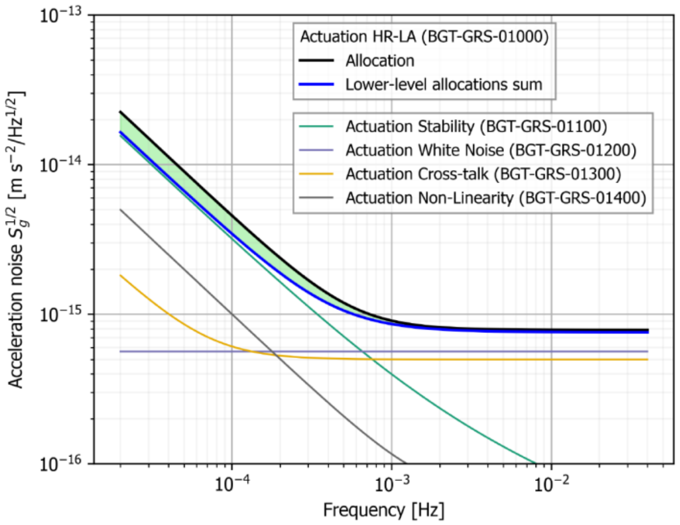
$$g_q(t) = \sum_p \beta_{qp} g_{p_c}(t)$$

thus, (assuming no cross-coherence between commanded forces)

$$S_{g_q}(f) = \sum_p \beta_{qp}^2 \, S_{g_{p_c}}(f)$$

Because of this, the force/torque noise along non-x DOFs **must** be constrained.

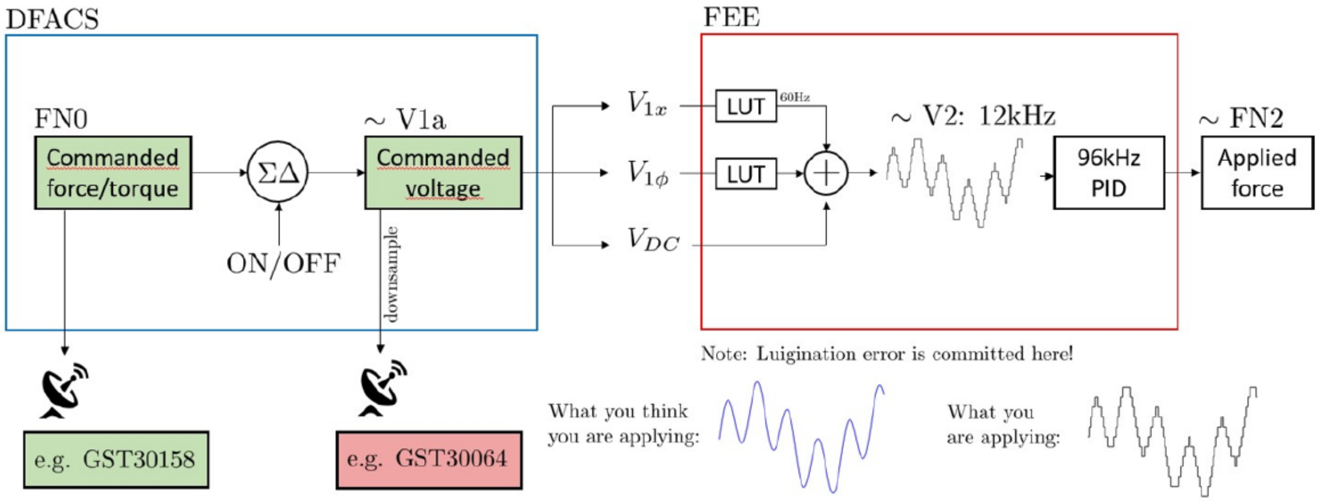
Also geometrical imperfections, as well as FEE unbalances, may yield crosstalk.
(we don't go into such detail…)



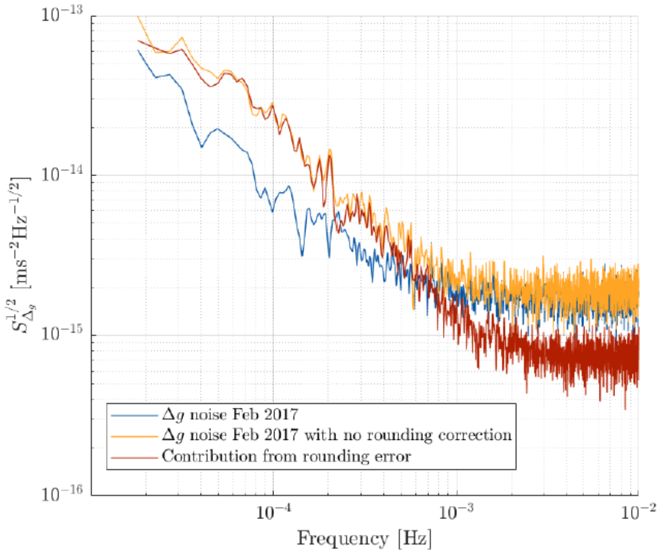
Need for precision on actuation waveforms

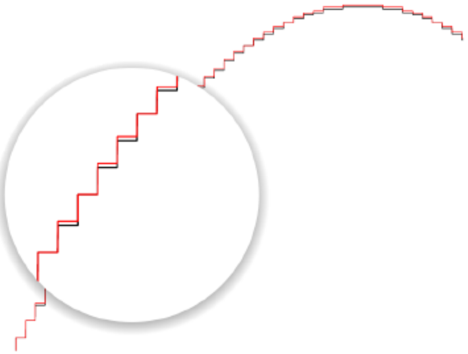
Audio-frequency carriers with ~0.5V amplitude must be almost perfectly **sinusoidal**.

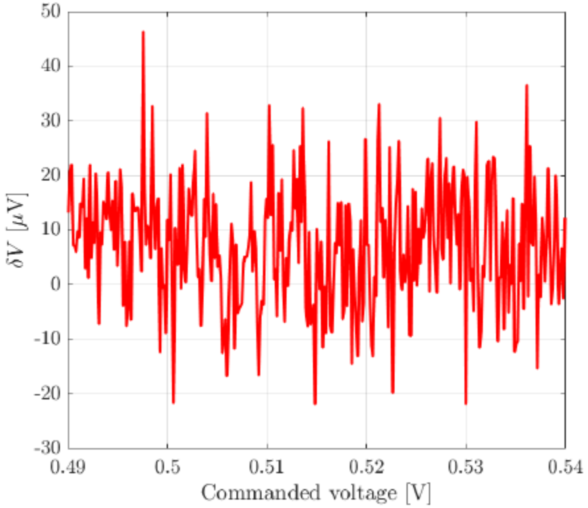
→ 16-bit digitization (153µV LSB)
→ Sigma-Delta loops for effective LSB reduction
→ Apparently not enough for LPF's ultra-low noise.
It needed post-processing techniques (see below)



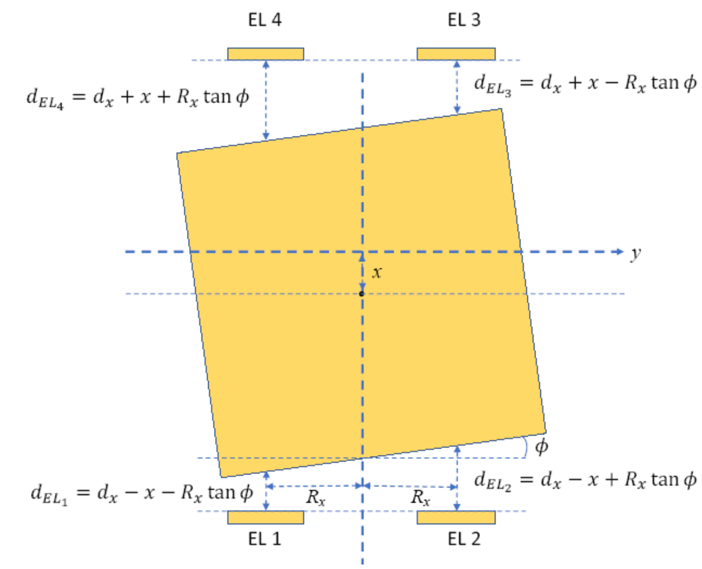
Note: Luigination error is committed here!







Electrode housing and capacitive sensing



The same electrodes serve a 6-DOF sensing purpose (all directions, all angles).

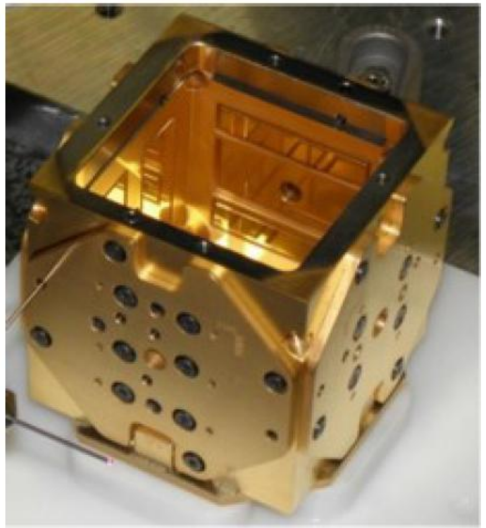
A 98kHz, 0.6V voltage bias is applied through injection electrodes,
and sensed with 12 electrodes.

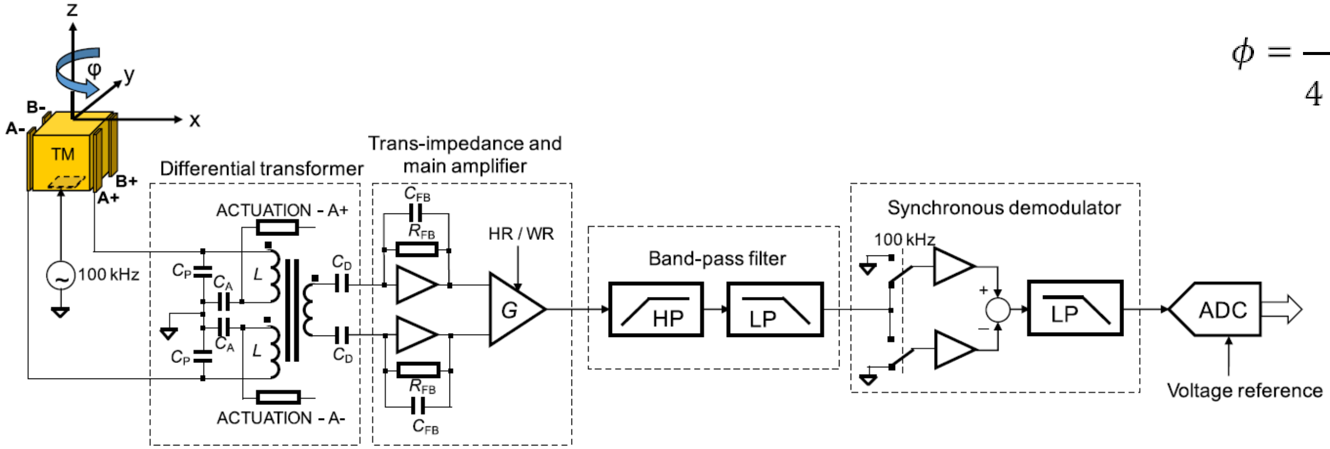
- With wide gaps (3-4mm) this corresponds to a sensitivity $\sim \mathrm{aF/Hz^{-1/2}}$, corresponding to a displacement sensitivity $\sim \mathrm{nm/Hz^{-1/2}}$, and rotation sensitivity $\sim 100\ \mathrm{nrad/Hz^{-1/2}}$
- Compare with an interferometric sensitivity $\sim 30\ \mathrm{fm/Hz^{-1/2}}$ and $\sim 100\ \mathrm{prad/Hz^{-1/2}}$

- **Why gaps so large??**
  As usual, to reduce noise (Brownian and stray electrostatics)
  We will come back to this.

$$x = \frac{1}{4\left|\frac{\partial C_x}{\partial x}\right|}(\Delta C_{1x} + \Delta C_{2x})$$

$$\phi = \frac{1}{4\left|\frac{\partial C_x}{\partial \phi}\right|}(\Delta C_{1x} - \Delta C_{2x})$$

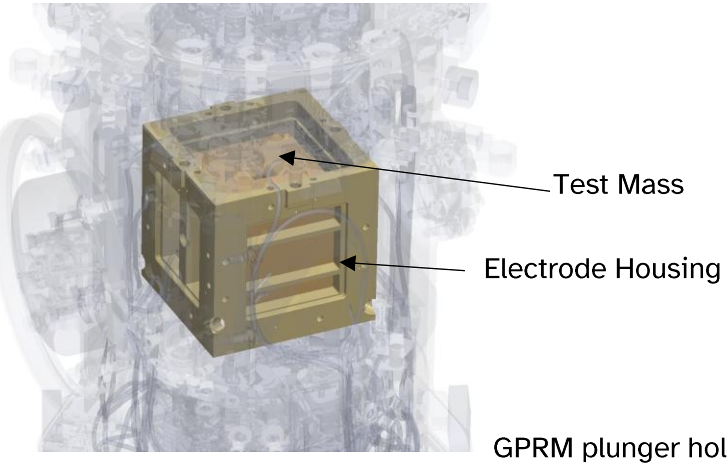


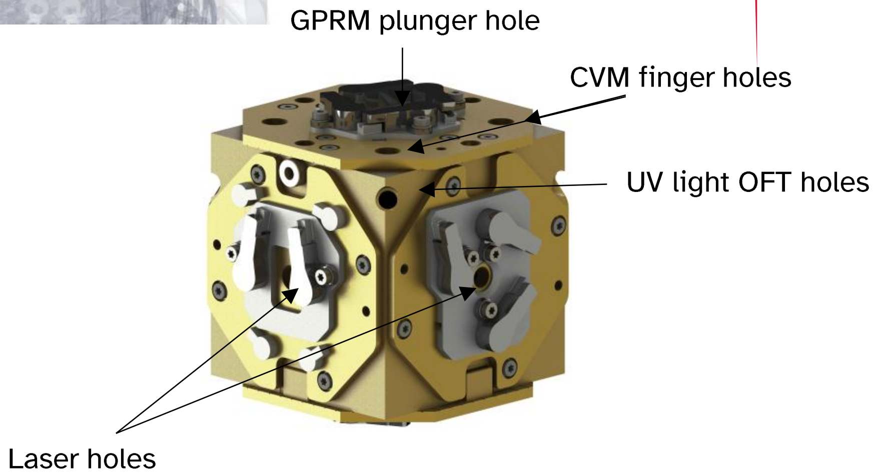


The LISA Gravitational Reference ~~Sensor~~ System

– Intentional, so we take a break from calculations –

The LISA GRS system





Credits: Carlo Zanoni

The LISA Gravitational Reference System (GRS):

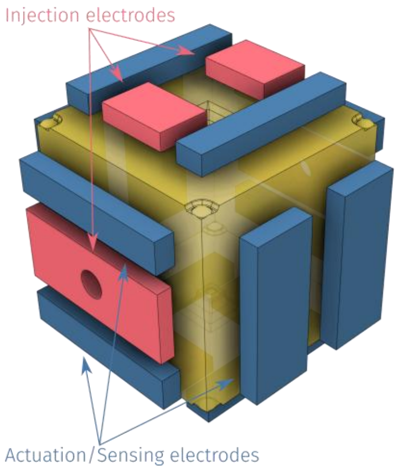
1. **Provides the geodesic (free-falling) reference** TMs.
2. **Provides the end-mirror** TMs for sub-pm interferometry.
3. Drives the spacecraft dynamics with TM position sensing.

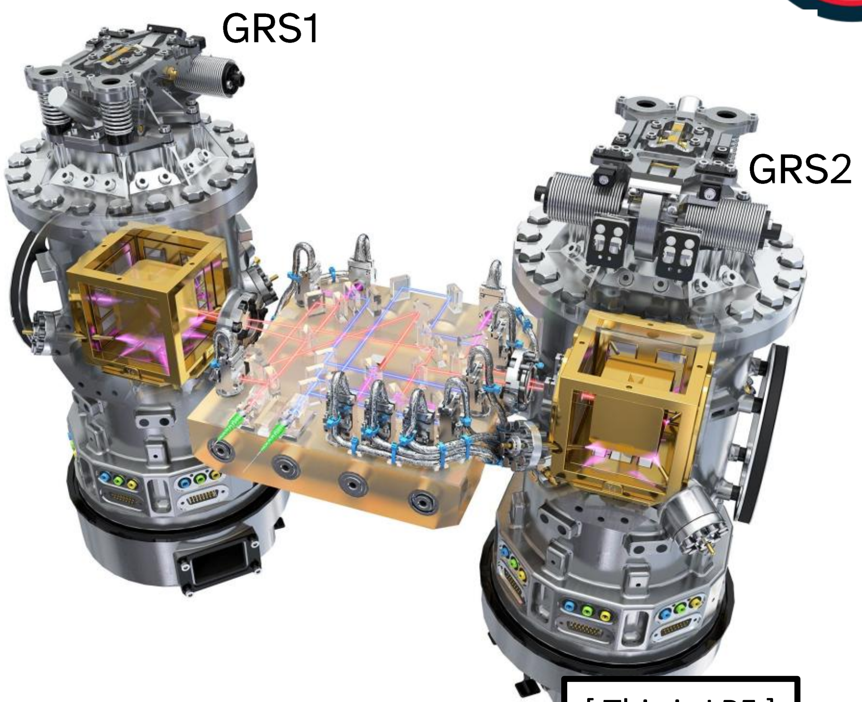
4. Limits all possible sources of unwanted force below the required level. → **Stray force shield.**

5. Acts as a **vacuum system**, on-ground and in-flight.

The Gravitational Reference System

- Residual **acceleration** noise down to **fm s^{-2}/$Hz^{1/2}$** at mHz frequencies.
  PRL **116**, 231101 (2016), PRL **120**, 061101 (2018), **arXiv:2405.05207**
- Audio-frequency 6(5)-DOF TM electrostatic actuation **(forces+torques)**.
  PRD **109**, 102009 (2024)
- Wide(mm)-gap TM capacitive **positioning** sensing. PRD **96**, 062004 (2017)



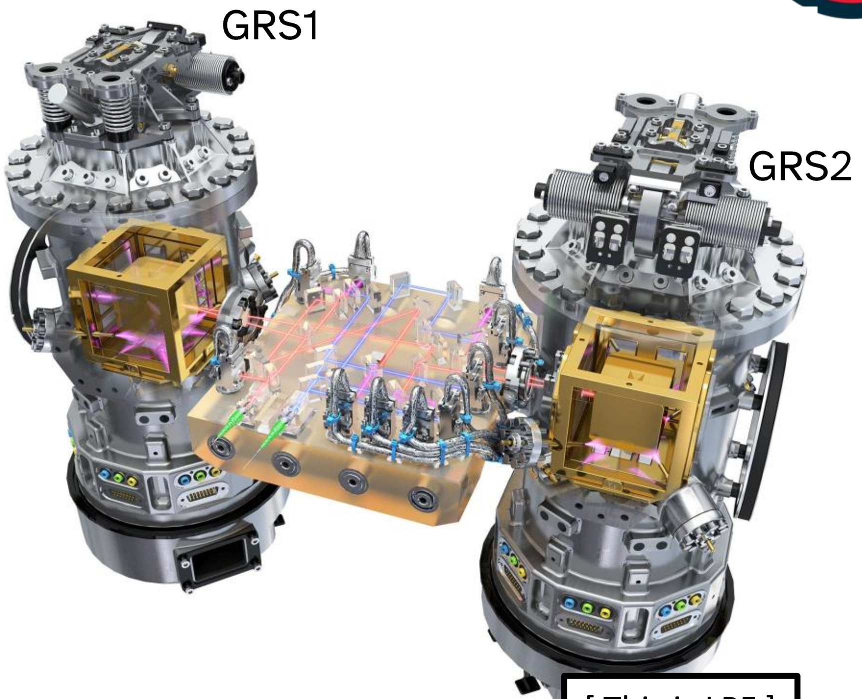


[This is LPF]

Note: pictures and citations referring to LPF's GRS

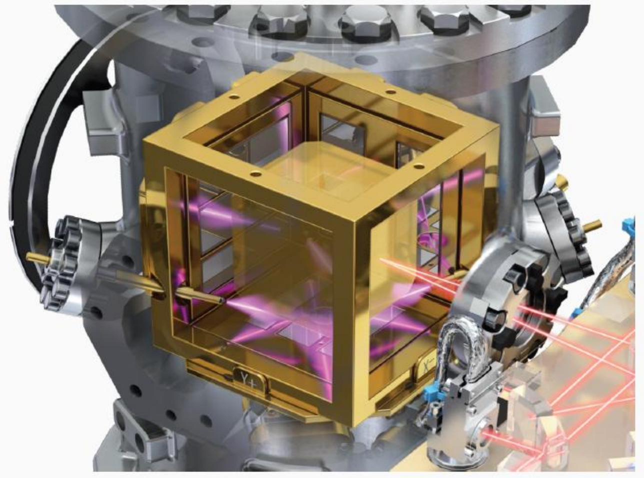
The Gravitational Reference System

- Residual **acceleration** noise down to **fm s^{-2}/$Hz^{1/2}$** at mHz frequencies. PRL **116**, 231101 (2016), PRL **120**, 061101 (2018), **arXiv:2405.05207**
- Audio-frequency 6(5)-DOF TM electrostatic actuation **(forces+torques)**. PRD **109**, 102009 (2024)
- Wide(mm)-gap TM capacitive **positioning** sensing. PRD **96**, 062004 (2017)
- Contact-free **UV-light** Charge Management System, control charge-induced force noise. PRL **118**, 171101 (2017) – PRD **107**, 062007 (2023)



[This is LPF]

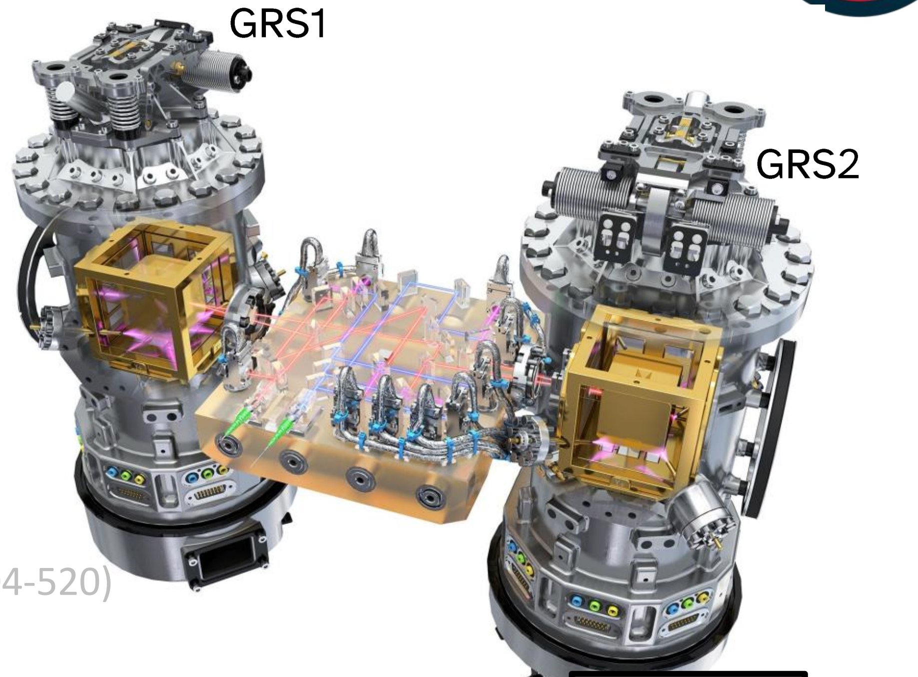
Note: pictures and citations referring to LPF's GRS



The Gravitational Reference System

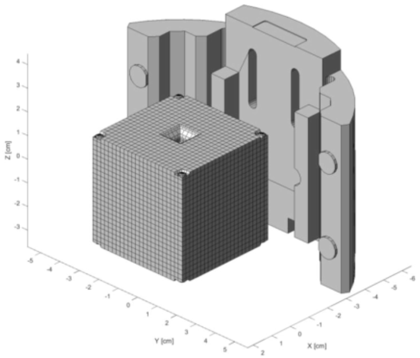
- Residual **acceleration** noise down to **fm $s^{-2}/Hz^{1/2}$** at mHz frequencies. PRL **116**, 231101 (2016), PRL **120**, 061101 (2018), **arXiv:2405.05207**
- Audio-frequency 6(5)-DOF TM electrostatic actuation **(forces+torques)**. PRD **109**, 102009 (2024)
- Wide(mm)-gap TM capacitive **positioning** sensing. PRD **96**, 062004 (2017)
- Contact-free **UV-light** Charge Management System, control charge-induced force noise. PRL **118**, 171101 (2017) – PRD **107**, 062007 (2023)

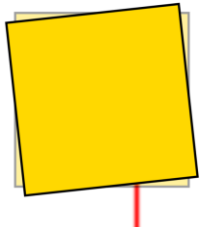
- Sub-nN static gravitational **compensation.** CQG **33** (2016) 235015
- TM **launch-lock/venting** and **grabbing/release** mechanisms. (ASR **67** (2021) 504-520)
- Vacuum system handling.
- Environmental diagnostics. MNRAS **486**, 3368–3379 (2019), MNRAS **494**, 3014–3027 (2020), Astropart. Phys. vol. **98** (2018)

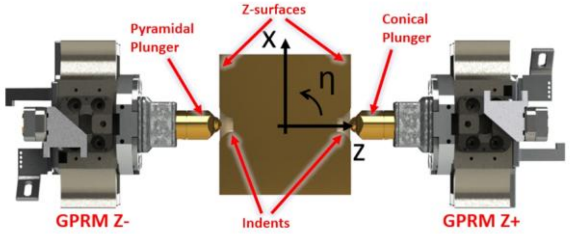


[This is LPF]

Note: pictures and citations referring to LPF's GRS





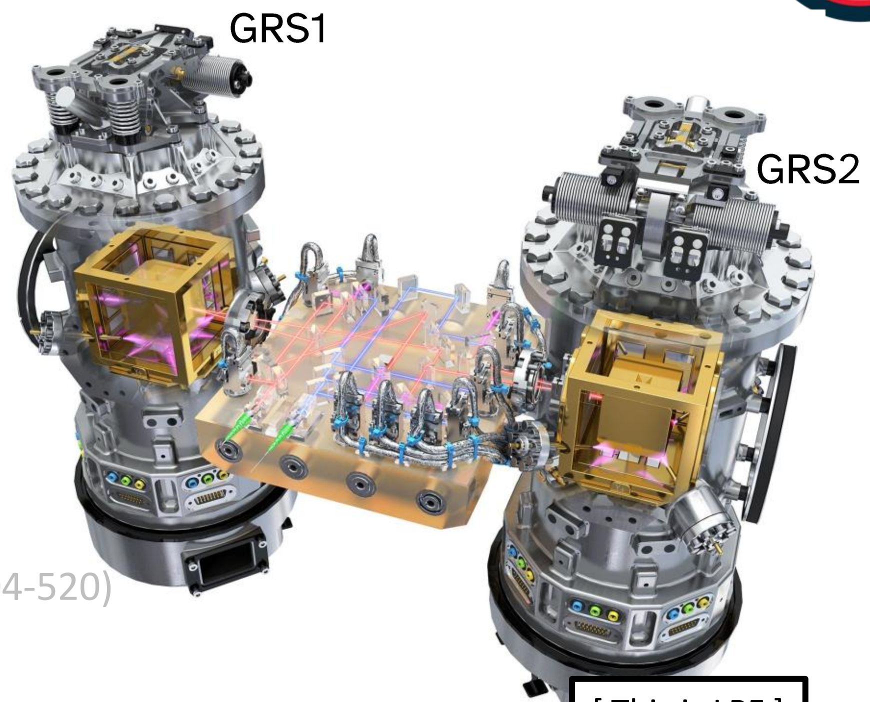


The Gravitational Reference System

- Residual **acceleration** noise down to **fm $s^{-2}/Hz^{1/2}$** at mHz frequencies.
  PRL **116**, 231101 (2016), PRL **120**, 061101 (2018), **arXiv:2405.05207**
- Audio-frequency 6(5)-DOF TM electrostatic actuation **(forces+torques)**.
  PRD **109**, 102009 (2024)
- Wide(mm)-gap TM capacitive **positioning** sensing. PRD **96**, 062004 (2017)
- Contact-free **UV-light** Charge Management System, control charge-induced force noise. PRL **118**, 171101 (2017) – PRD **107**, 062007 (2023)

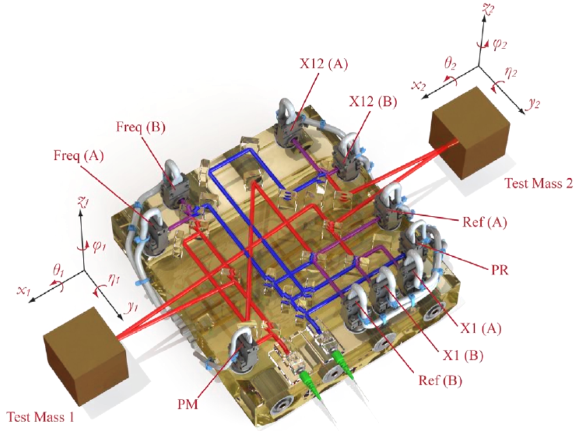
- Sub-nN static gravitational **compensation.** CQG **33** (2016) 235015
- TM **launch-lock/venting** and **grabbing/release** mechanisms. (ASR **67** (2021) 504-520)
- Vacuum system handling.
- Environmental diagnostics. MNRAS **486**, 3368–3379 (2019), MNRAS **494**, 3014–3027 (2020), Astropart. Phys. vol. **98** (2018)

- **++** Tested on LPF thanks to
  the first sub-pm local **interferometer** ever flown in space.
  PRL **126**, 131103 (2021), PRD **106**, 082001 (2022), PRD **109**, 042003 (2024)



[This is LPF]

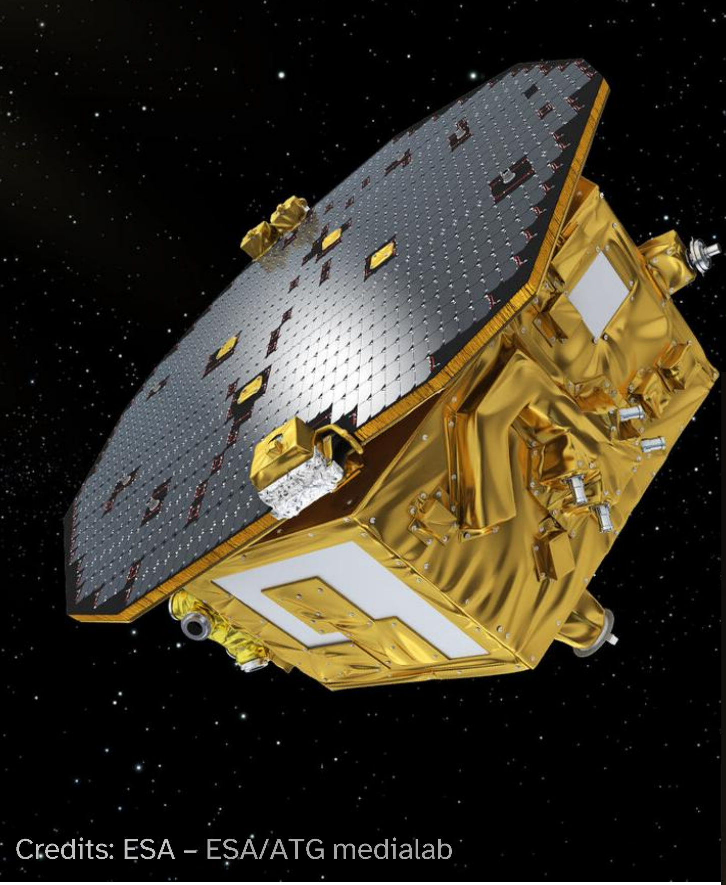
Note: pictures and citations referring to LPF's GRS



UNIVERSITÀ DI TRENTO

The LISA Pathfinder Mission

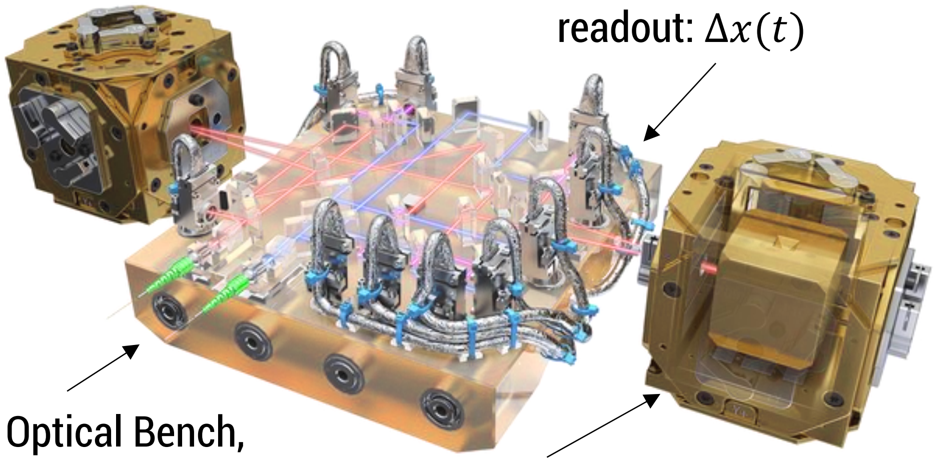
Launch to L1: December **2015**
End of operations: July **2017**



Credits: ESA – ESA/ATG medialab

2015-12-03, 04:04:00 UTC

lisa pathfinder



Electrode Housing

Interferometric IFO readout: $\Delta x(t)$

Optical Bench, four interferometers

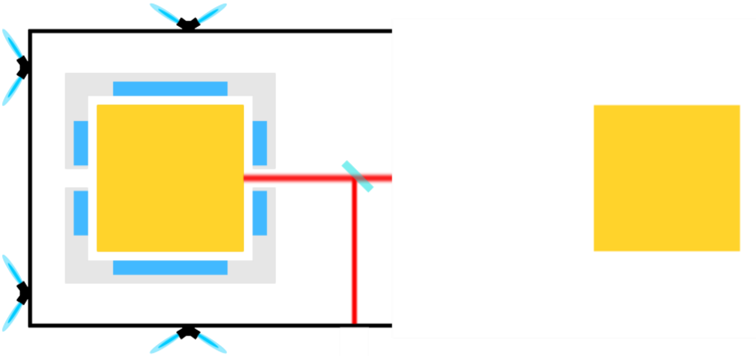
Capacitive (GRS) readout, 6 DOF

UNIVERSITÀ DI TRENTO

ASI INFN

Force noise testing: LISA Pathfinder

- The LISA noise performance was investigated in the **LISA Pathfinder (LPF)** mission, in 2015-2017
  - Demonstration mission: hardware testing, drag-free control technology testing, S/C environment testing…
  - Main scientific measurement: the **out-of-loop differential <u>acceleration</u>** between two LISA-like TMs, along a 38cm arm.



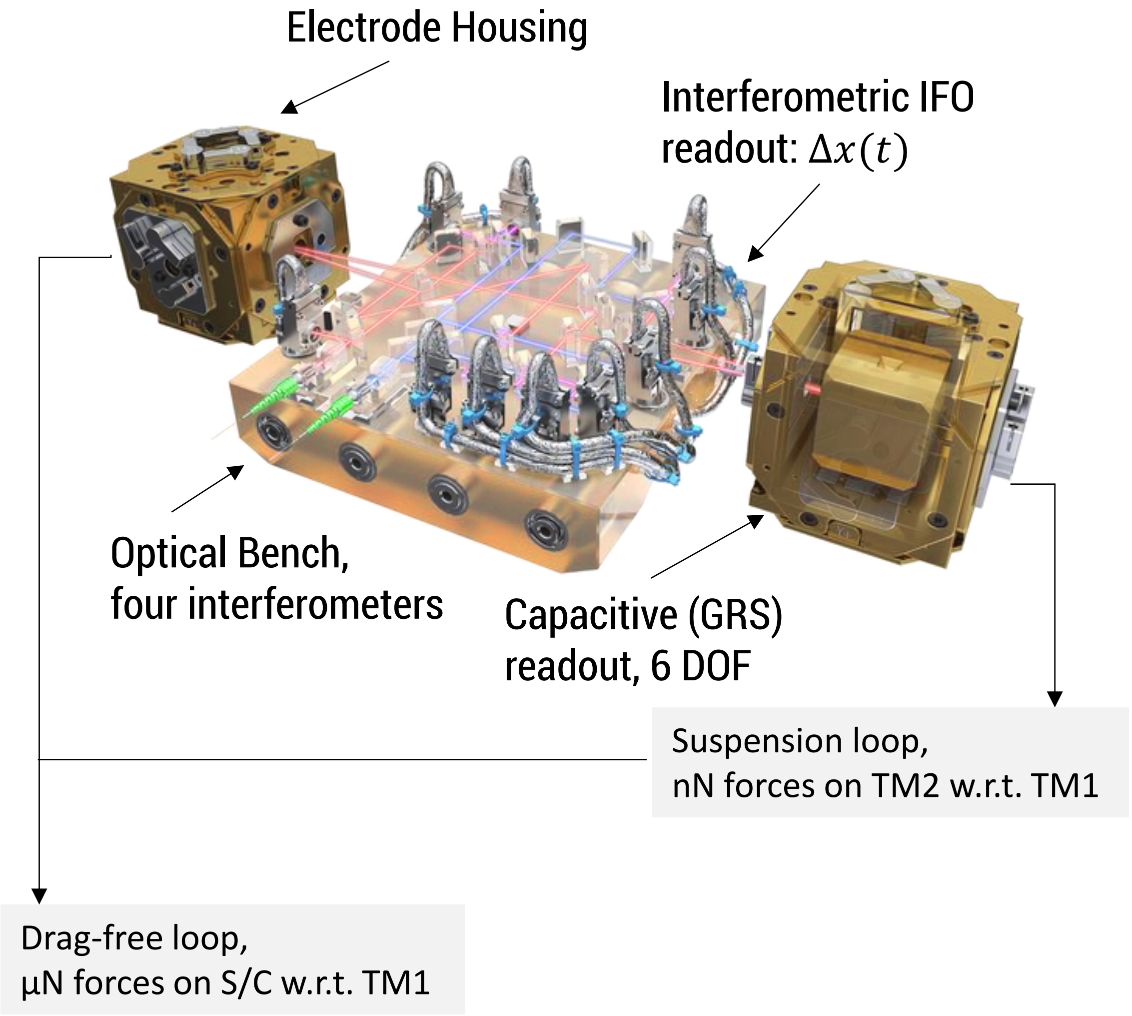
  - Positioning: **sub-pm** heterodyne IFO.

$$\Delta g(t) = \Delta\ddot{x}(t) - g_c(t) - g_\Omega(t) + \left[\omega_2^2\Delta x(t) + \Delta\omega_{12}^2 x_1(t)\right]$$

Applied control forces

Inertial forces

Coupling to S/C motion



Electrode Housing

Interferometric IFO readout: $\Delta x(t)$

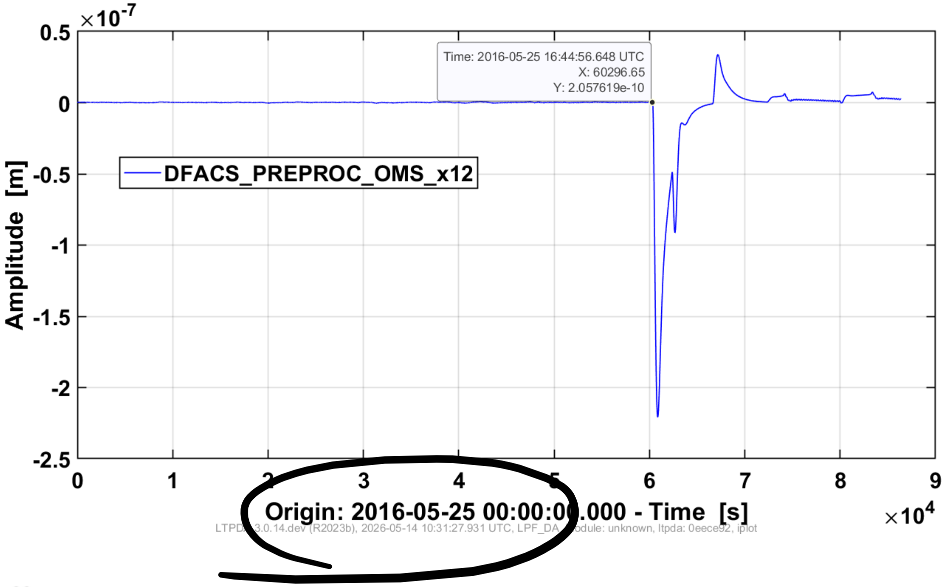
Optical Bench, four interferometers

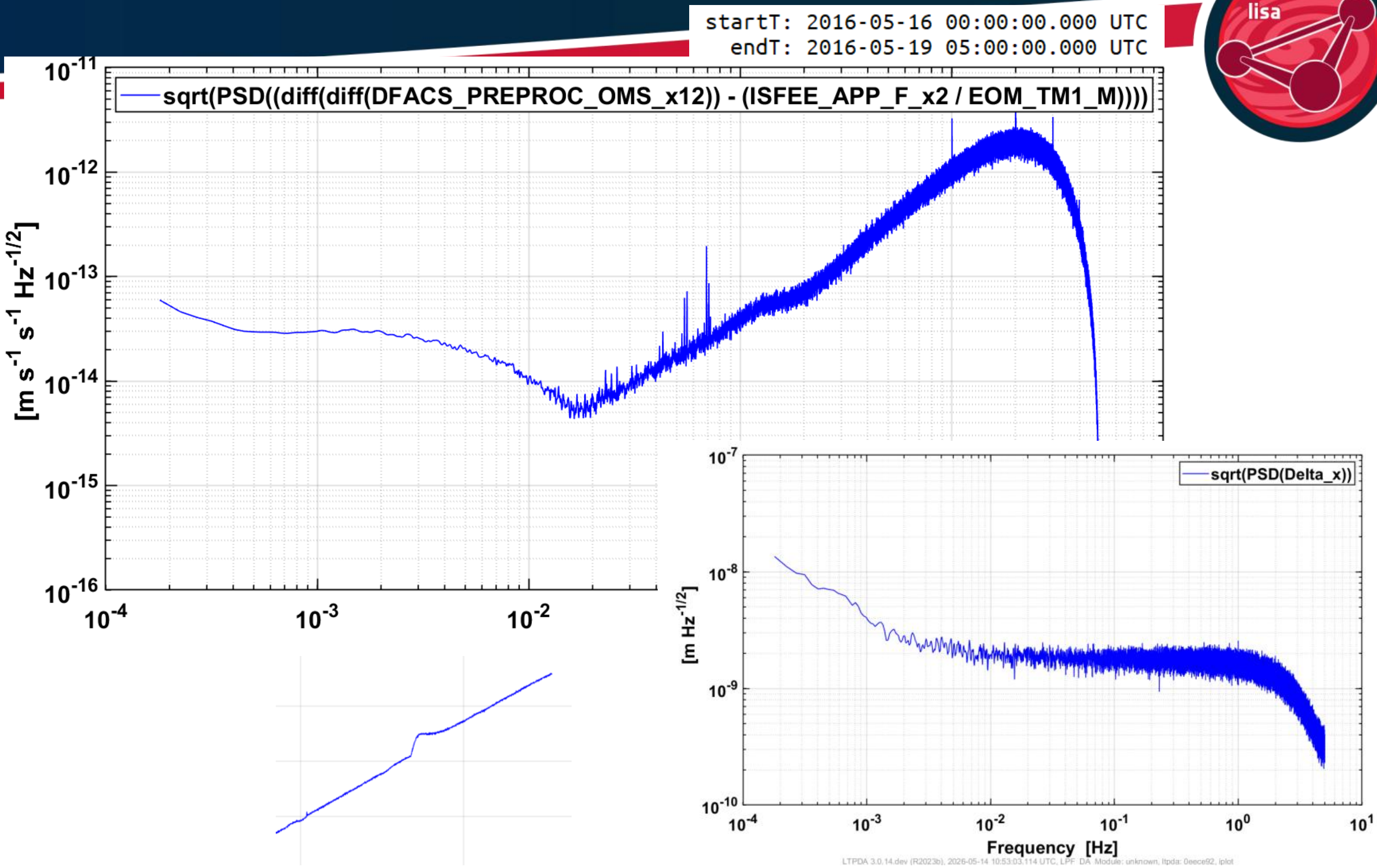
Capacitive (GRS) readout, 6 DOF

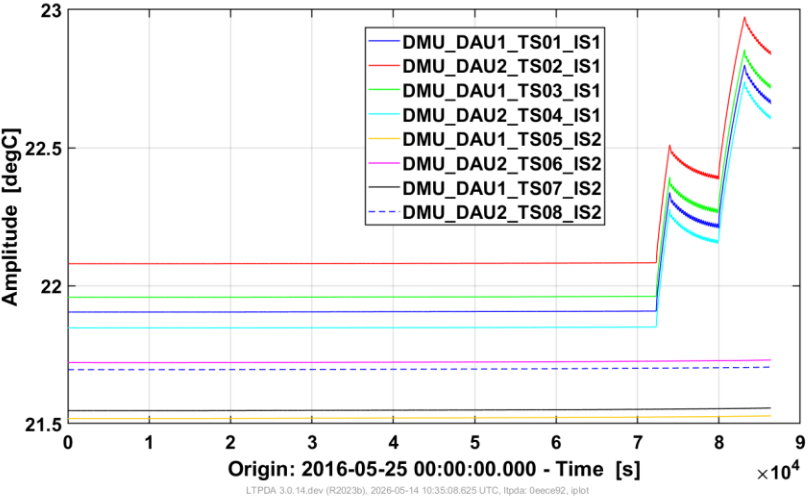
Suspension loop, nN forces on TM2 w.r.t. TM1

Drag-free loop, μN forces on S/C w.r.t. TM1

LPF exactly 10 years ago





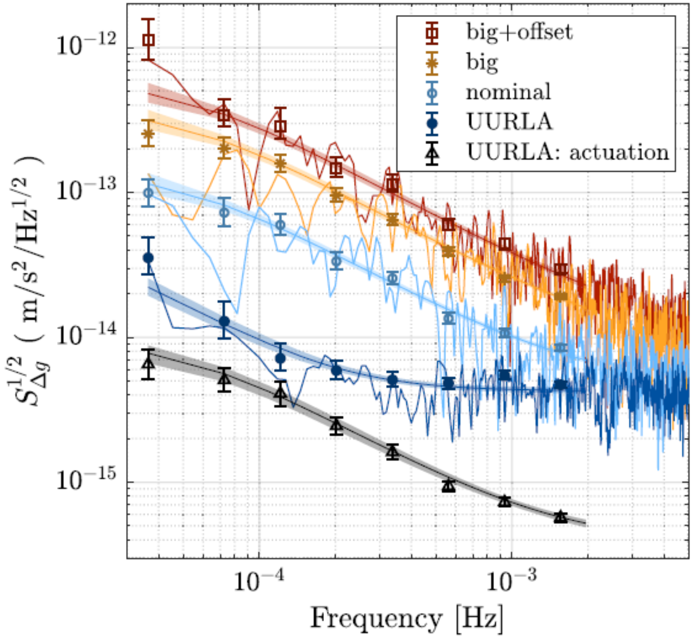


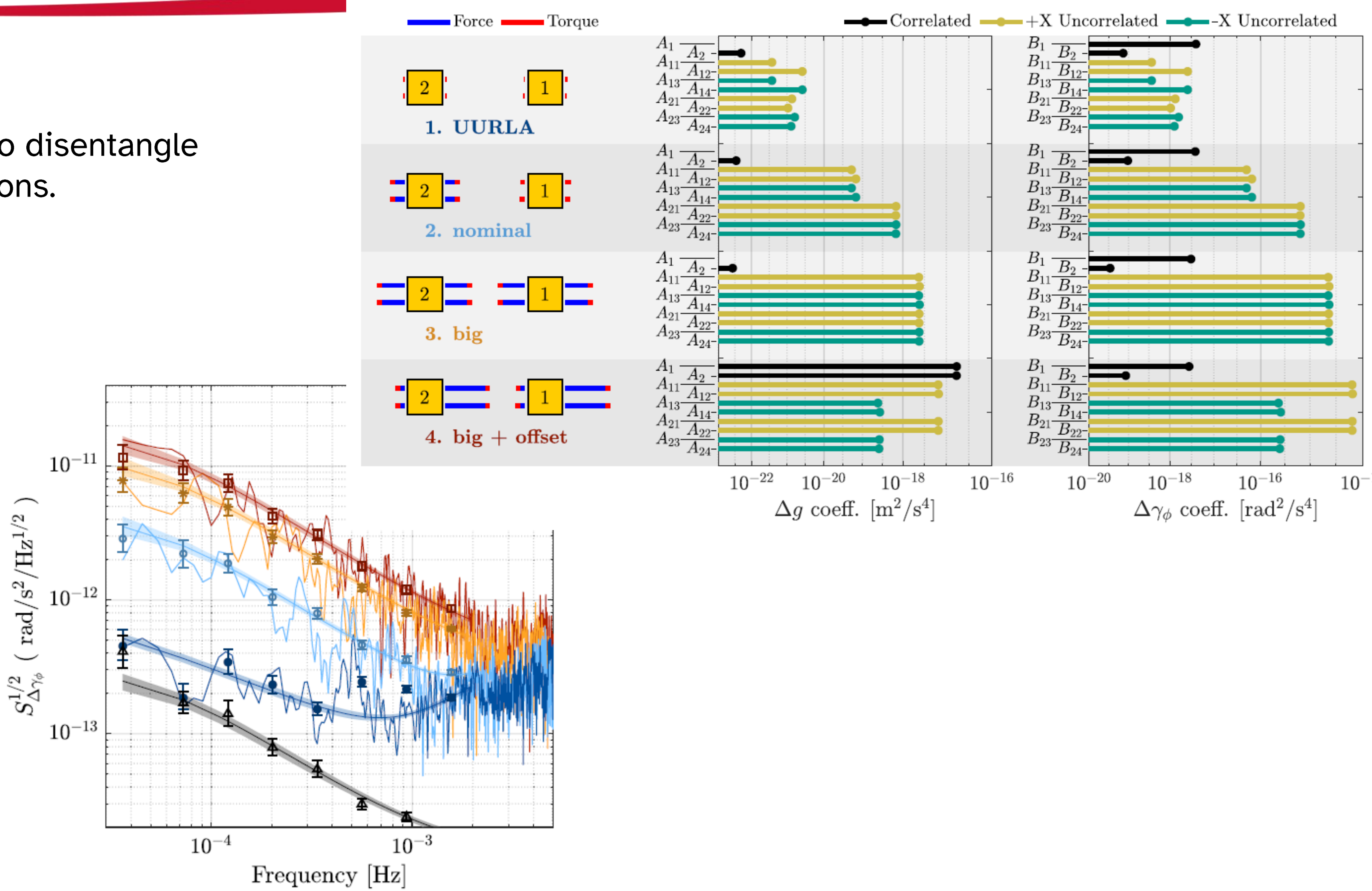
2016-05-25T00:00:00	2016-05-25T08:00:00	28800		Empty investigation of 8 hours duration.
2016-05-25T08:00:00	2016-05-25T16:00:00	28800	NOISE	Empty investigation of 8 hours duration.
2016-05-25T16:45:00	2016-05-25T16:45:35	35	OLF FORCES STOP	Set all open loop forces and torques on SC and TM1 and TM2 to 0
2016-05-25T16:46:00	2016-05-25T16:46:05	5	CHARGE TONE STOP	Set all AC and DC voltages on all TM1 and TM2 electrodes to 0.
2016-05-25T16:47:00	2016-05-25T16:47:20	20	MAXF DEFAULT	Reset all max forces and torques to MIB default
2016-05-25T17:16:00	2016-05-25T18:29:20	4400	TRANSITION TO SCI12	Transition from Custom Mode Matched Stiffness to SCI12 Unmatched Stiffness.
2016-05-25T18:30:00	2016-05-25T18:30:50	50	MAXF 200pN	Setting max force on x2 (b_x2) to 200 pN and on phi1 (b_phi1) to 2.00 pN*m and on phi2 (b_ph
2016-05-25T20:00:00	2016-05-27T01:00:00	53090	TM1 THERMAL EXCITATIONS	Thermal excitations into IS heaters 1 and 2 with final 1mHz and 0.5mHz modulations in phase.
2016-05-25T20:00:00	2016-05-27T01:00:00	51310	TM1 THERMAL EXCITATIONS	Thermal excitations into IS heaters 1 and 2 with final 1mHz and 0.5mHz modulations in phase.
2016-05-27T01:00:00	2016-05-28T06:00:00	35090	TM2 THERMAL EXCITATIONS	Thermal excitations into IS heaters 3 and 4 with final 1mHz and 0.5mHz modulations in phase.
2016-05-27T01:00:00	2016-05-28T06:00:00	69310	TM2 THERMAL EXCITATIONS	Thermal excitations into IS heaters 3 and 4 with final 1mHz and 0.5mHz modulations in phase.

UNIVERSITÀ DI TRENTO

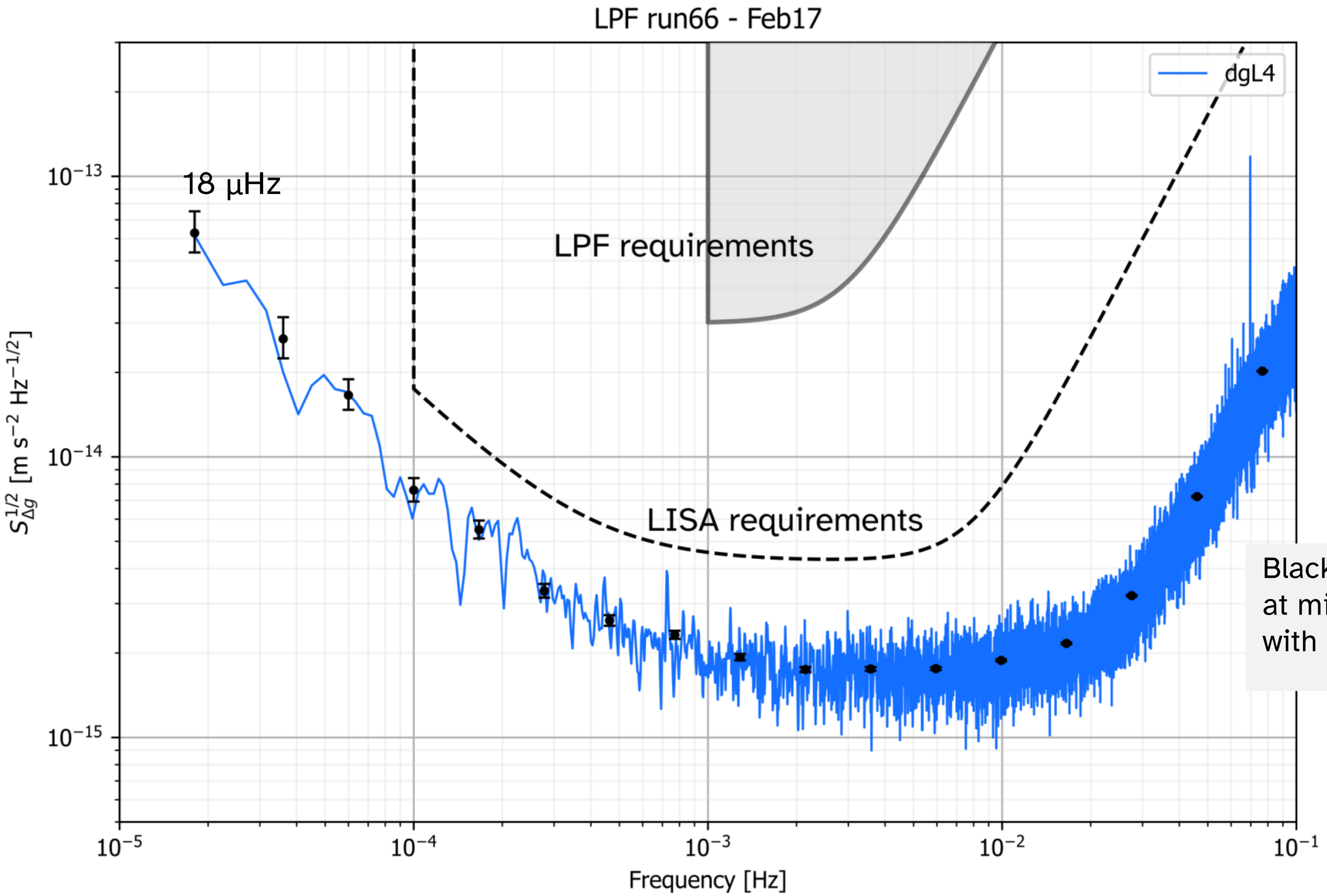
LPF **exactly** 10 years ago

May 2016: **actuation experiments**.
Change authorities, change offsets to disentangle parameters and make noise predictions.



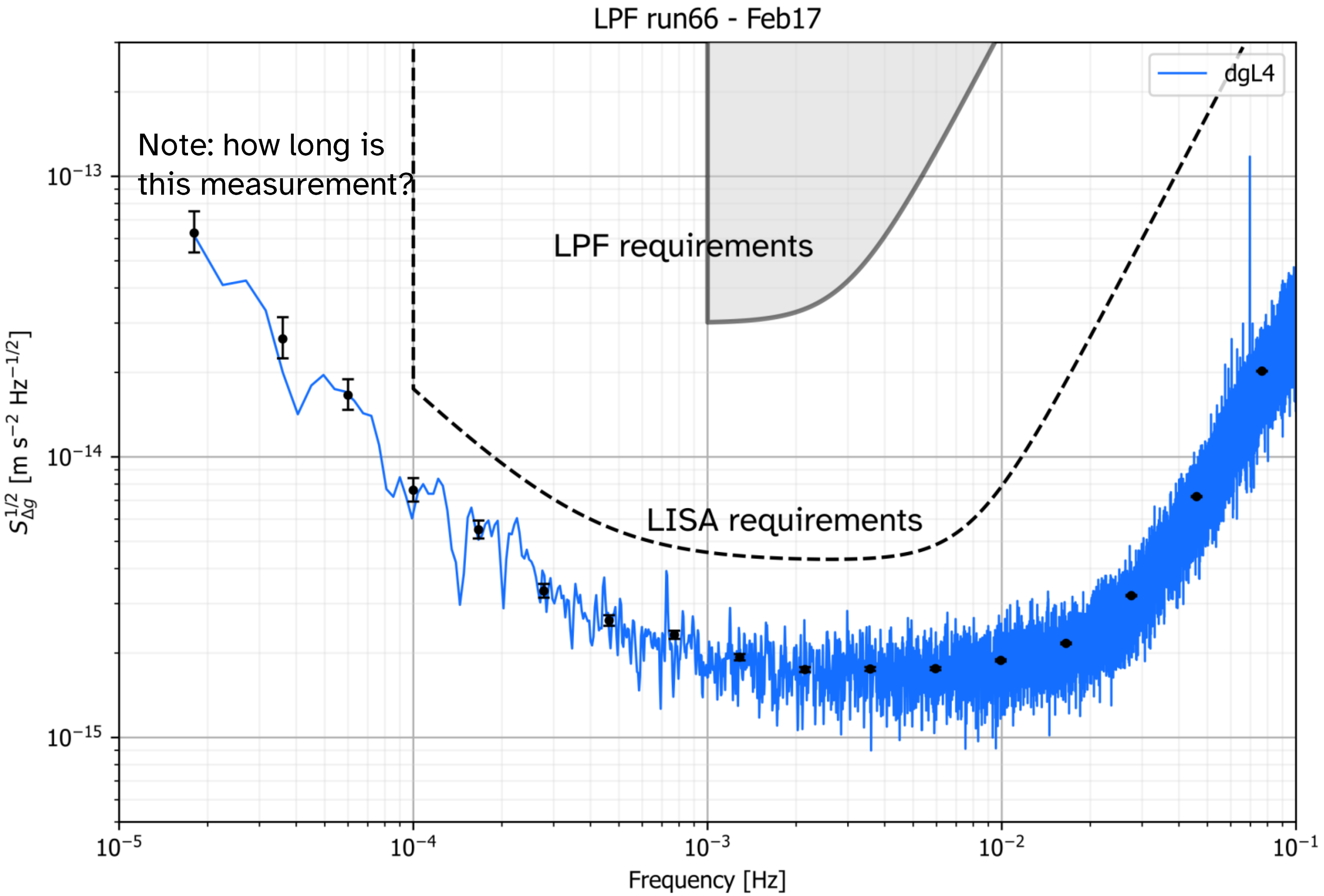


Results from LISA Pathfinder



Black points: Bayesian PSD estimation at minimally-correlated frequencies, with optimal periodogram length

That’s not straightforward…

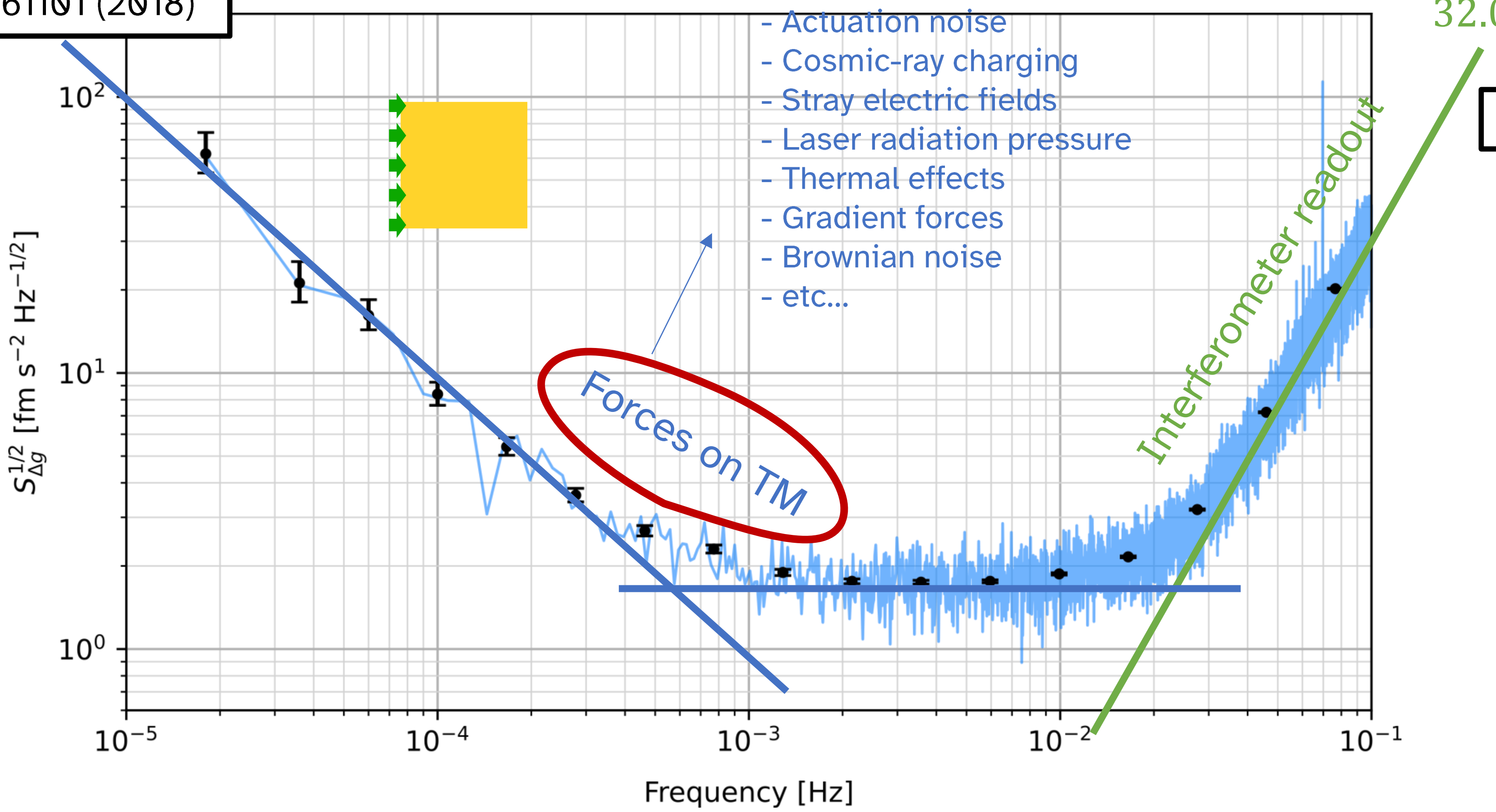


Note: how long is this measurement?

1. Measured dgL0.
2. Deglitching.
3. Subtraction of centrifugal force.
4. Decorrelation of cross-talk “debumping”
5. Decorrelation of Euler force

Noise characterization

PRL **120**, 061101 (2018)



Force noise sources:
- Actuation noise
- Cosmic-ray charging
- Stray electric fields
- Laser radiation pressure
- Thermal effects
- Gradient forces
- Brownian noise
- etc…

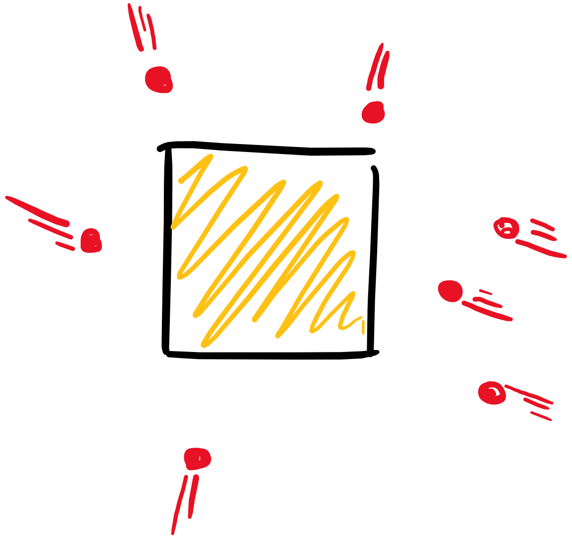
$32.0^{+2.4}_{-1.7}$ fm Hz$^{-1/2}$

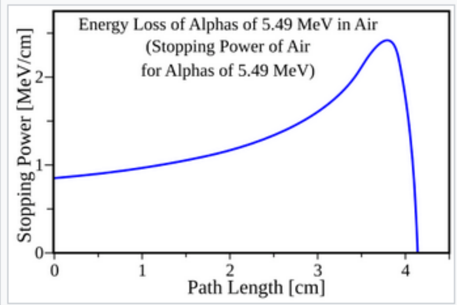
PRL **126**, 131103 (2021)

What if the TM charges up?

– Spoiler: it’s not a question, it does charge up –

– How do we prevent that? –





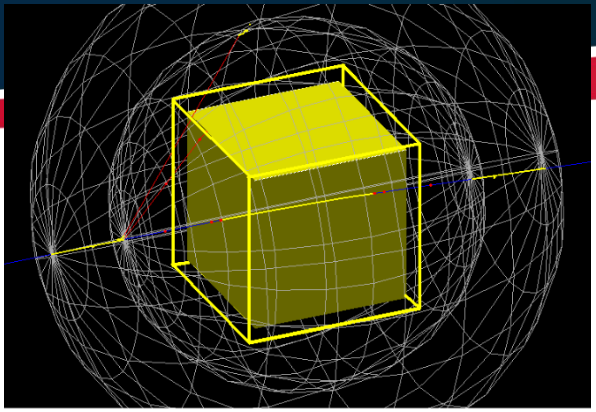
Bragg curve of 5.49 MeV alpha particles in air

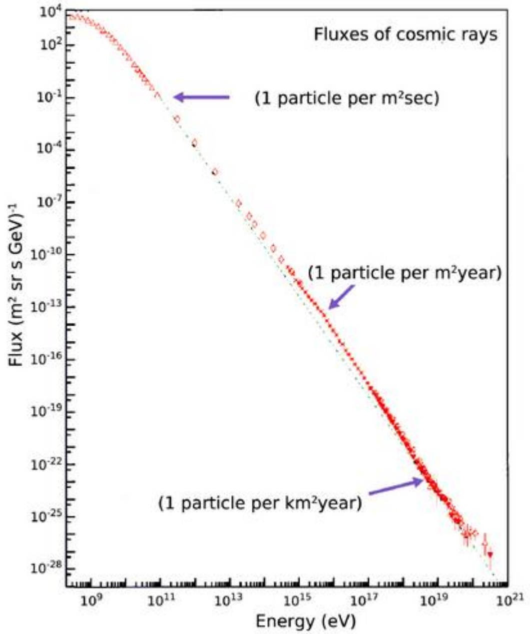
TM Charging

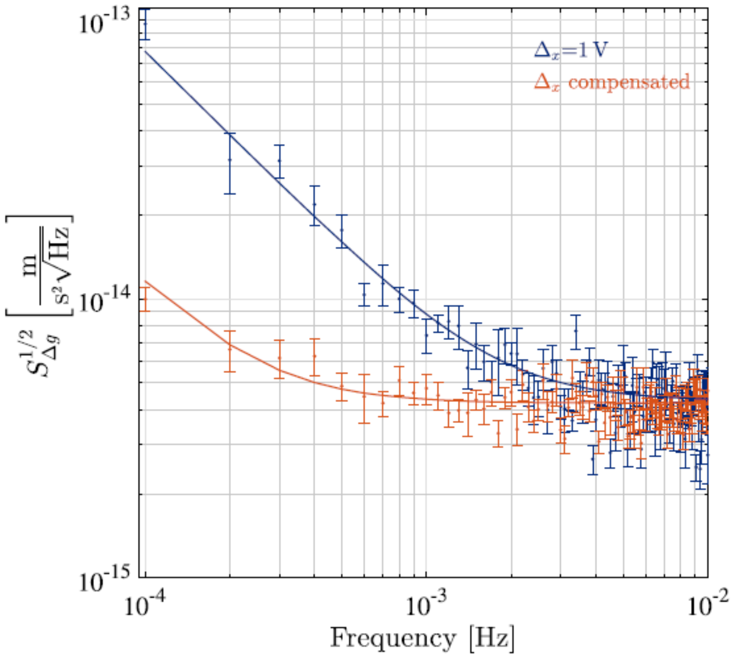
The TM is an isolated conductor in a **space environment with cosmic rays**, of course **it charges up**!!

- If isolated, the TM charges up with a rate $\lambda \sim 25\ \mathrm{e/s}$.
- Moreover, discrete-charge interactions induce **charging noise**.

→ We **must limit** all source of noise.



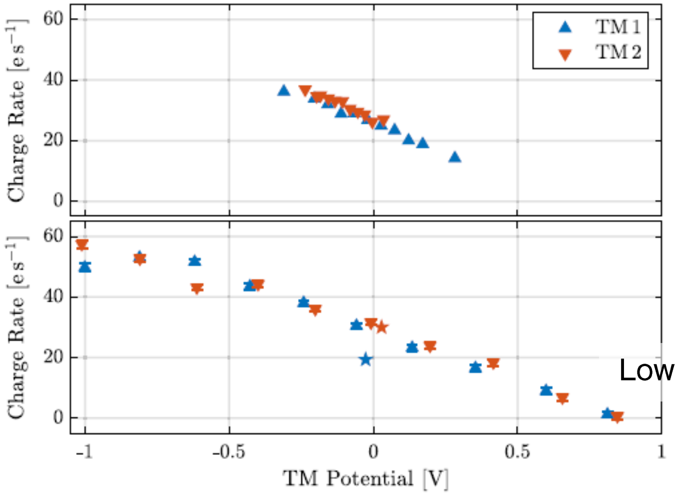




$$F_Q = -\frac{Q}{C_T}\left|\frac{\partial C_X}{\partial x}\right|\Delta_x + \frac{1}{2}\frac{Q^2}{C_T^2}\left|\frac{\partial^2 C_T}{\partial x^2}\right|(x - x_{\mathrm{null}Q})$$

Note that this equation describes multiple effects:

1) Quasi-**static charge** interacts with **random voltage** fluctuations.
2) Quasi-**static voltage** interacts with **random charge** noise.
3) Random charge noise produces force if TM is off-centered.



Coupling of charged TM with noisy stray bias

$$S_g^{\Delta_x}(f) \simeq \left[\frac{1}{M_{TM}}\frac{\partial C_x}{\partial x}\frac{Q_{TM}}{C_{tot}}\right]^2 S_{\Delta_x}(f)$$

Coupling of residual DC stray bias with noisy TM charge

$$S_g^{Q_{TM}}(f) \simeq \left[\frac{1}{M_{TM}}\frac{\partial C_x}{\partial x}\frac{\Delta_x}{C_{tot}}\right]^2 S_{Q_{TM}}(f)$$

Low-energy electrons!!!

Another shot noise (charge noise…)

Note, **charging rate** (yields charge build-up)
and **effective charging rate** (yields shot noise)
are different things!!

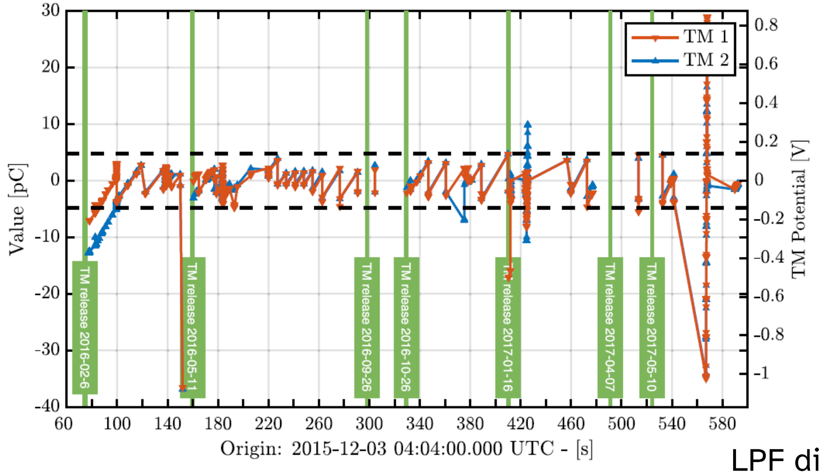
$$S_{Q_{\mathrm{TM}}} = \frac{2\sum_q q^2\lambda_q}{(2\pi f)^2} = \frac{2e^2\lambda_{\mathrm{eff}}}{(2\pi f)^2} \qquad \lambda_{\mathrm{eff}} = \sum_j j^2\lambda_j$$

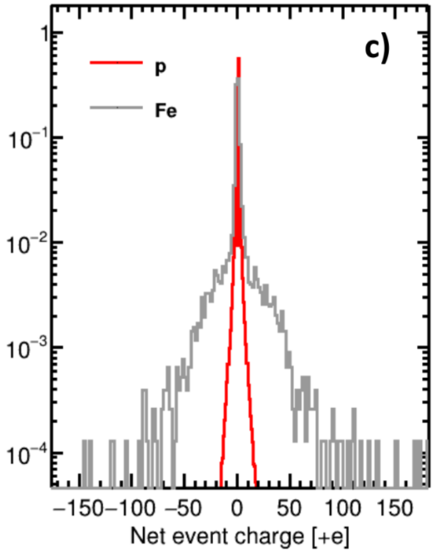
	TM1	TM2	
$\dot{q}$	+22.9	+24.5	es^{-1}
λ_{eff}	1060 ± 90	1360 ± 130	s^{-1}
$\lambda_{\mathrm{eff}(1+2)}$[a]	2200 ± 260		s^{-1}

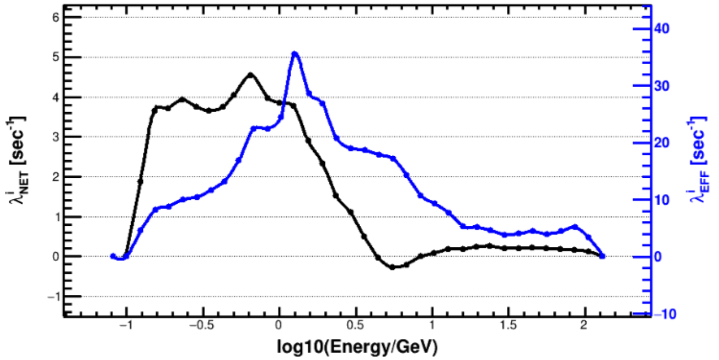
[a] Determined from fit to Δg with $\Delta_x = 1$ V.

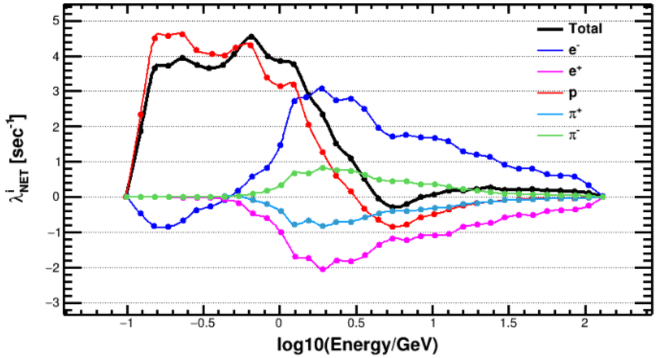
Need to:

1. **Control TM charge** (continuous discharge, fast discharge,…)
2. **Compensate** spurious quasi-static voltages Δ_x (order of mV)
3. Have low-noise on applied quasi-static DC fields.





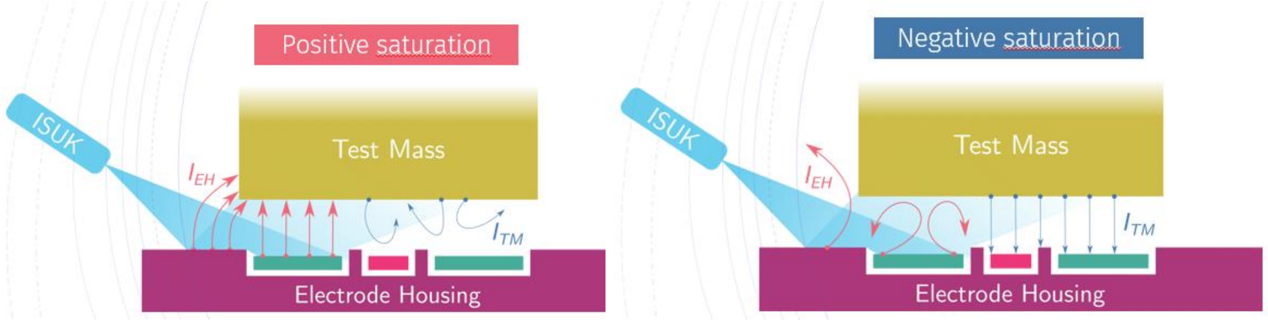




LPF discharge strategy:
measure charge before runs → discharge → measure again. This interrupts observation.
Can we do any better/differently?

Keeping the TM neutral: UV discharge

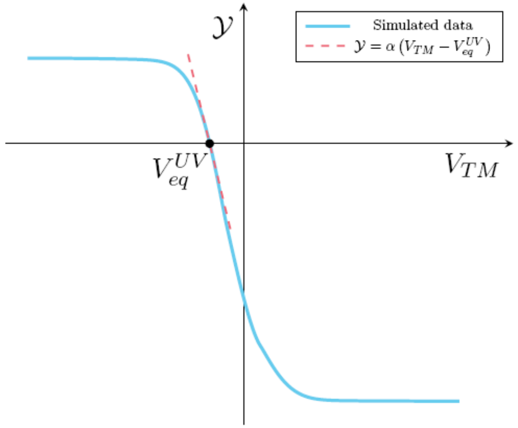
Discharge the TM with photoelectric effect!
UV light from UV LEDs.

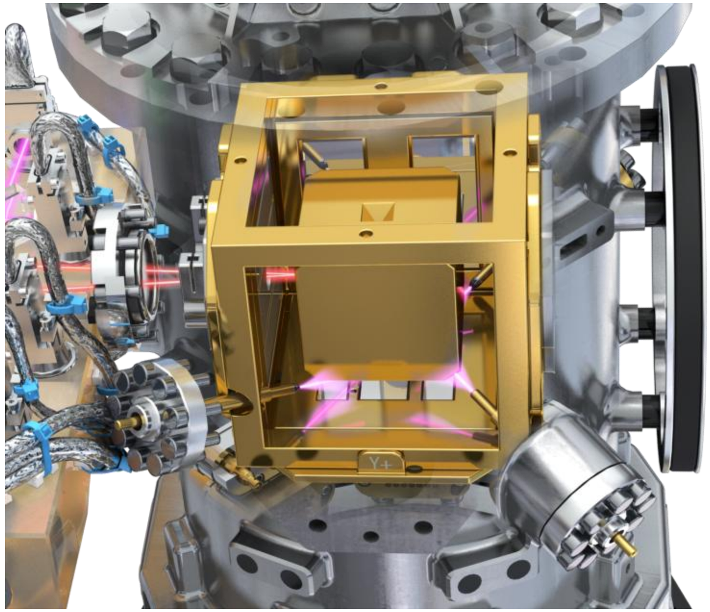


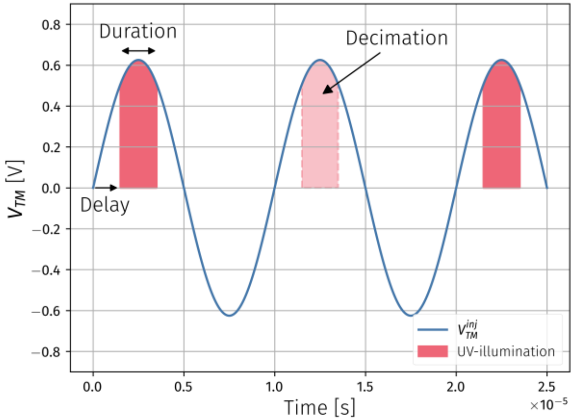
Credits: Davide Dal Bosco

$$\frac{\mathrm{d}V_{TM}}{\mathrm{d}t} \simeq -\frac{V_{TM}-V_{eq}^{env}}{\tau_{env}} - \frac{V_{TM}-V_{eq}^{UV}}{\tau_{UV}}$$
$$\simeq -\frac{V_{TM}-V_{eq}}{\tau}$$

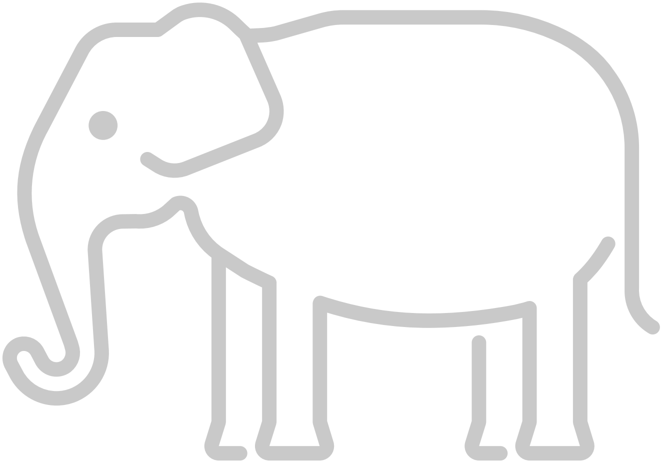
$$\tau_{UV} = -\frac{1}{\alpha}\frac{C_{tot}}{eP_{UV}} \geq 0$$



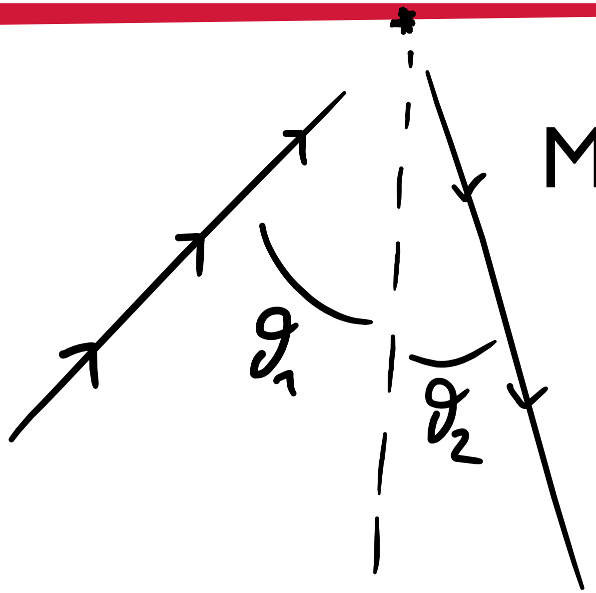




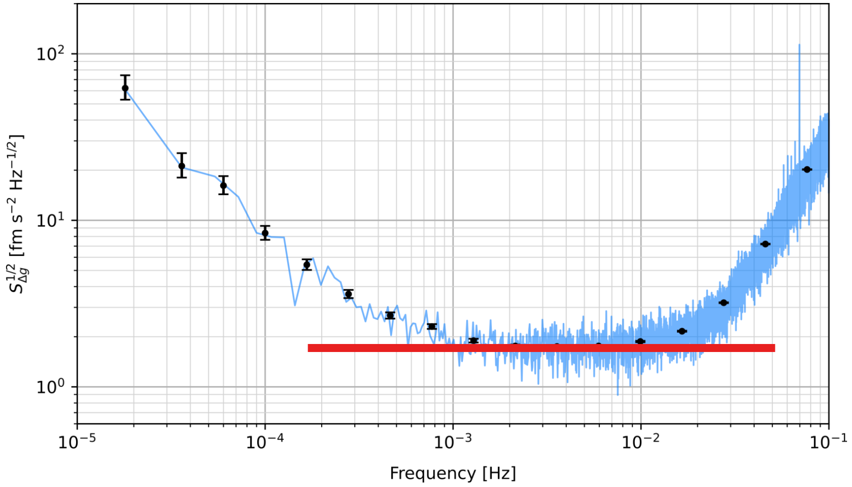
Clever solution: sync with sensing bias!
Not on LPF...



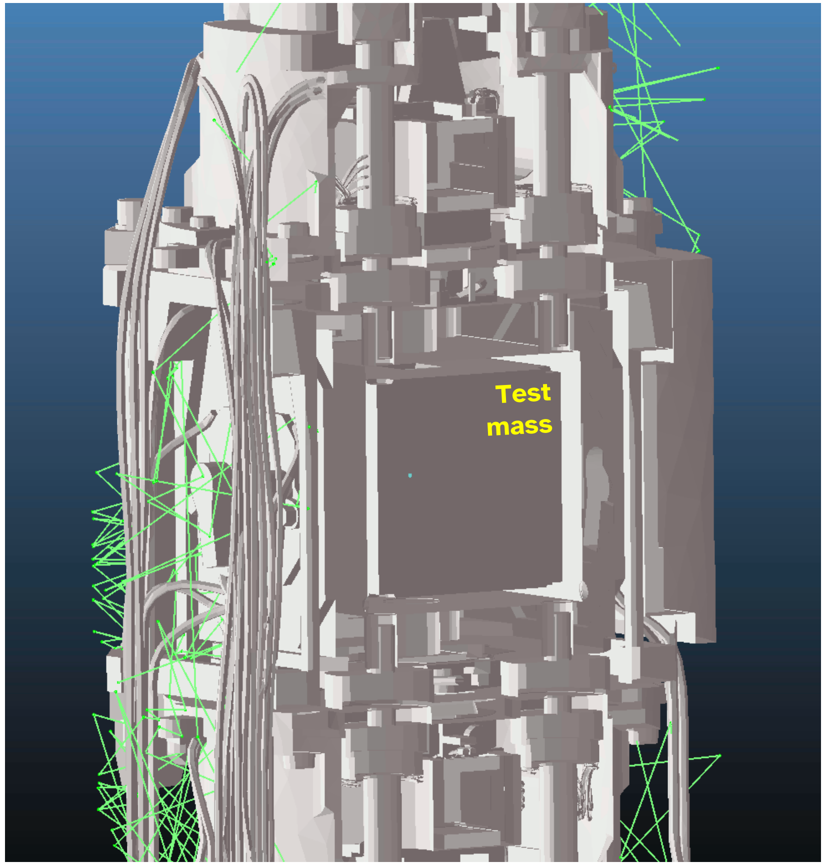
The elephant in the room



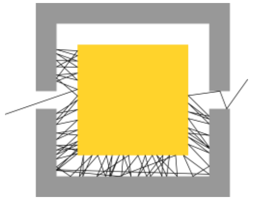
Molecules.



Brownian noise



Gas flow simulation with Molflow+
GRS XZ plane cross section at y=+24mm.



- **Random hit "shot noise"** by residual gas molecules
  [Physics Letters A 374 (2010) 3365–3369]
- Manifestation of the "fluctuation-dissipation theorem": **white noise** in the LISA frequency band.
- Strongly dependent on molecular dynamics and **system geometry**
  [PRL 103, 140601 (2009)]
- Depends on **pressure** → engineering-level requirement

$$S_g^{\mathrm{Br}} = \alpha_{\mathrm{B}}\, p \left(1+\frac{\pi}{8}\right) \frac{s_{\mathrm{TM}}^2}{M_{\mathrm{TM}}^2}\left(\frac{512\, m_0 k_B T}{\pi}\right)^{1/2} = \left[1.7\,\mathrm{fm\,s^{-2}}/\sqrt{\mathrm{Hz}}\right]^2 \times \left(\frac{p}{2\,\mu\mathrm{Pa}_{\mathrm{H_2O}}}\right)$$

Noise PSD

Geometric amplification (LISA GRS EH)

Pressure

TM mass

TM size

Molecule mass

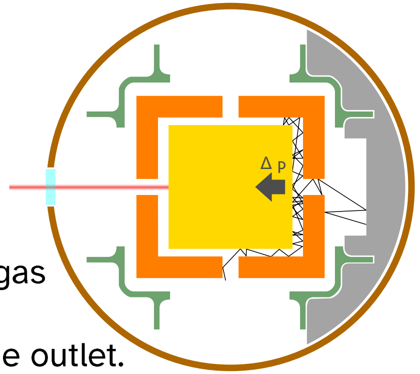
Temperature
("mild" dependency here)
(stronger dependency hidden in p)

- → For LISA, this translates into a **pressure requirement** at the beginning of the science operations (1.5 yr after launch)

$$\boxed{P < 2\mu\mathrm{Pa}}$$

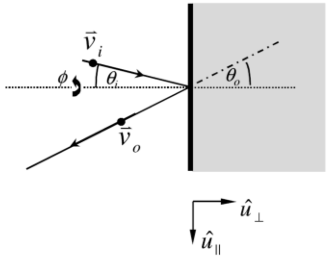
Understanding Brownian noise

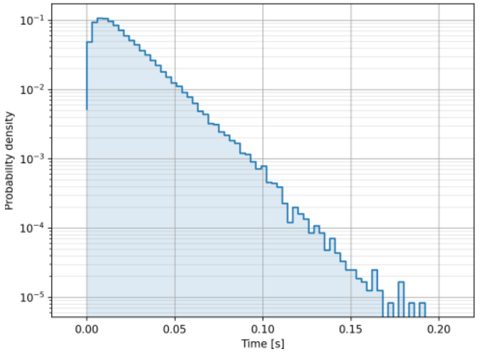
“Intuitive” microscopic approach



Molecules constantly (well, not constantly…) outgas from surfaces within the GRS.
Each molecule follows a path before reaching the outlet.
During its path, it **transfers momentum** to the TM faces.
That’s a “shot” noise!

Assume that a TM is surrounded by an infinite gas volume (independent hits)



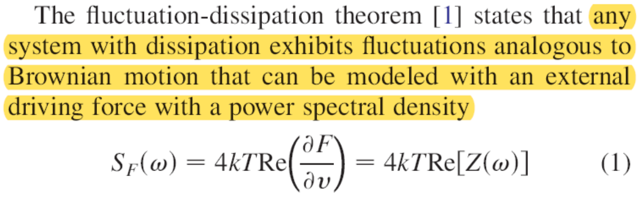


The average “shot” noise is calculable, assuming the correct distribution of the inward and outward particle direction.

$$S_F = 2\int_Z dx\,dy\,S_\perp + 2\int_X dy\,dz\,S_\parallel + 2\int_Y dx\,dz\,S_\parallel$$
$$= ps^2\left(\frac{512\,m_0 k_B T}{\pi}\right)^{1/2}\left(1+\frac{\pi}{8}\right)$$

Another (macroscopic) approach

From the macroscopic point of view, the presence of gas acts as a **damping** factor, with impedance $Z(\omega) = \beta$.
The fluctuation-dissipation theorem applies, yielding a calculable force noise.



The fluctuation-dissipation theorem [1] states that any system with dissipation exhibits fluctuations analogous to Brownian motion that can be modeled with an external driving force with a power spectral density

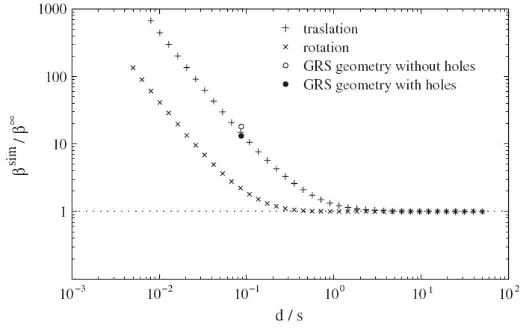
$$S_F(\omega) = 4kT\mathrm{Re}\left(\frac{\partial F}{\partial v}\right) = 4kT\mathrm{Re}[Z(\omega)] \qquad (1)$$

The real case

The TM is surrounded by the EH.
Microscopically → Hits by the same molecule are not independent, hence correlation must be considered.
Macroscopically → The damping coefficient β is larger as the system couples with the external environment.

Solution: **simulations** and **measurements**!!

We find an **amplification factor**, strongly dependent on the **gap size**.
→ We must have a large gap.



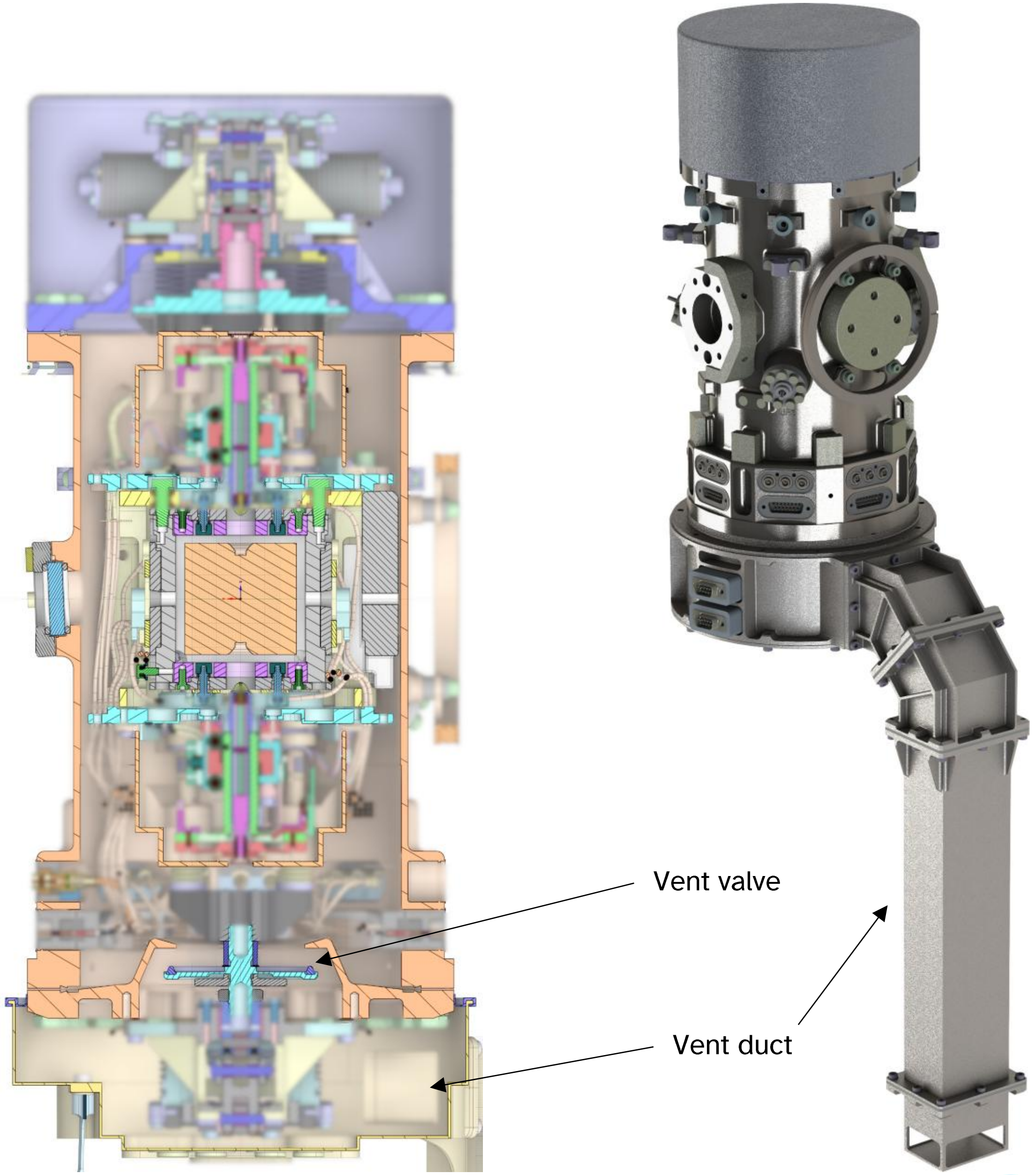
How to deal with that?

- **<u>First. Have a separate vacuum chamber.</u>**
- Pressure depends on: outgassing rate, pumping speed, and external pressure

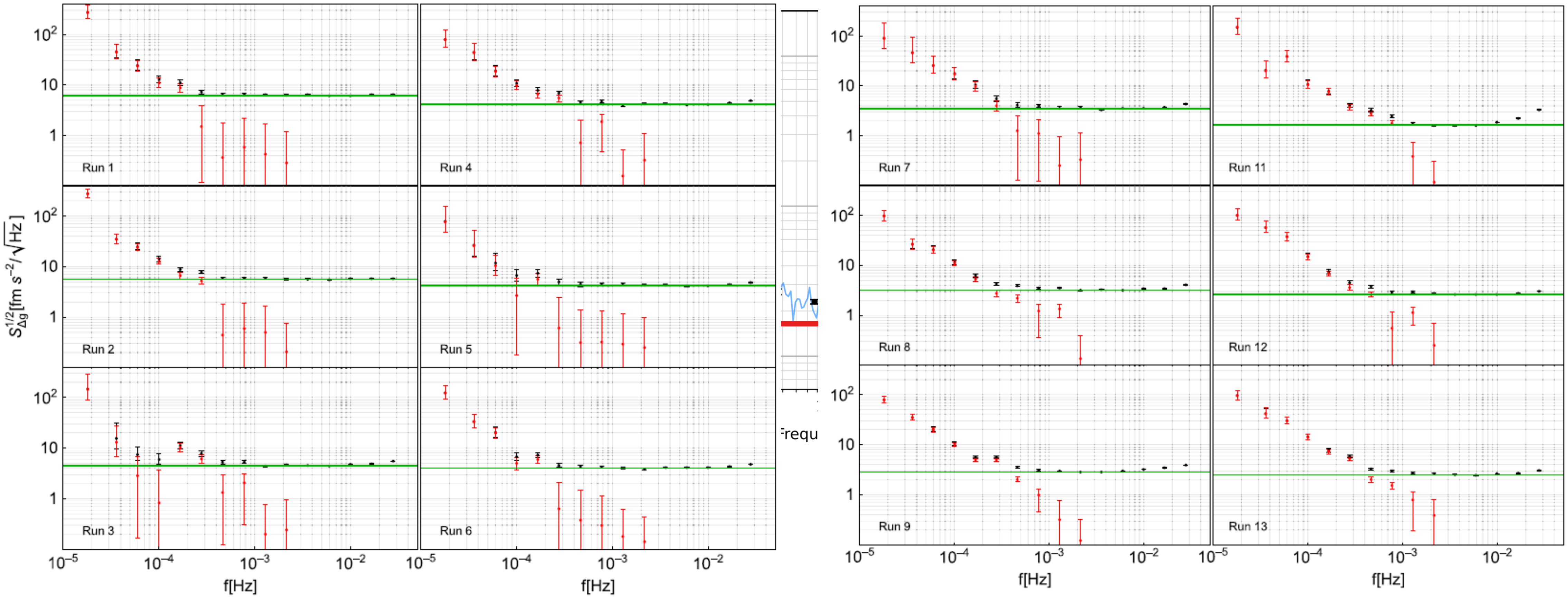
$$p = Q/S + p_{out}$$

  → System/geometric: maximize S

  → Physical properties: minimize Q
- **Vent valve** to:
  - <u>perform</u> surface conditioning and bakeout pre-flight
  - <u>preserve</u> during on-ground storage
  - <u>open</u> once in orbit (one-shot mechanism)
- **Vent duct** to:
  - <u>preserve</u> vacuum and surface properties during storage
  - <u>shield</u> the inner GRS from outer S/C outgassing
  - <u>reduce</u> field of view on external S/C surfaces
  - <u>guarantee</u> a pumping speed >30L/s
- **Bakeout**, surface conditioning to:
  - <u>reduce</u> outgassing rate

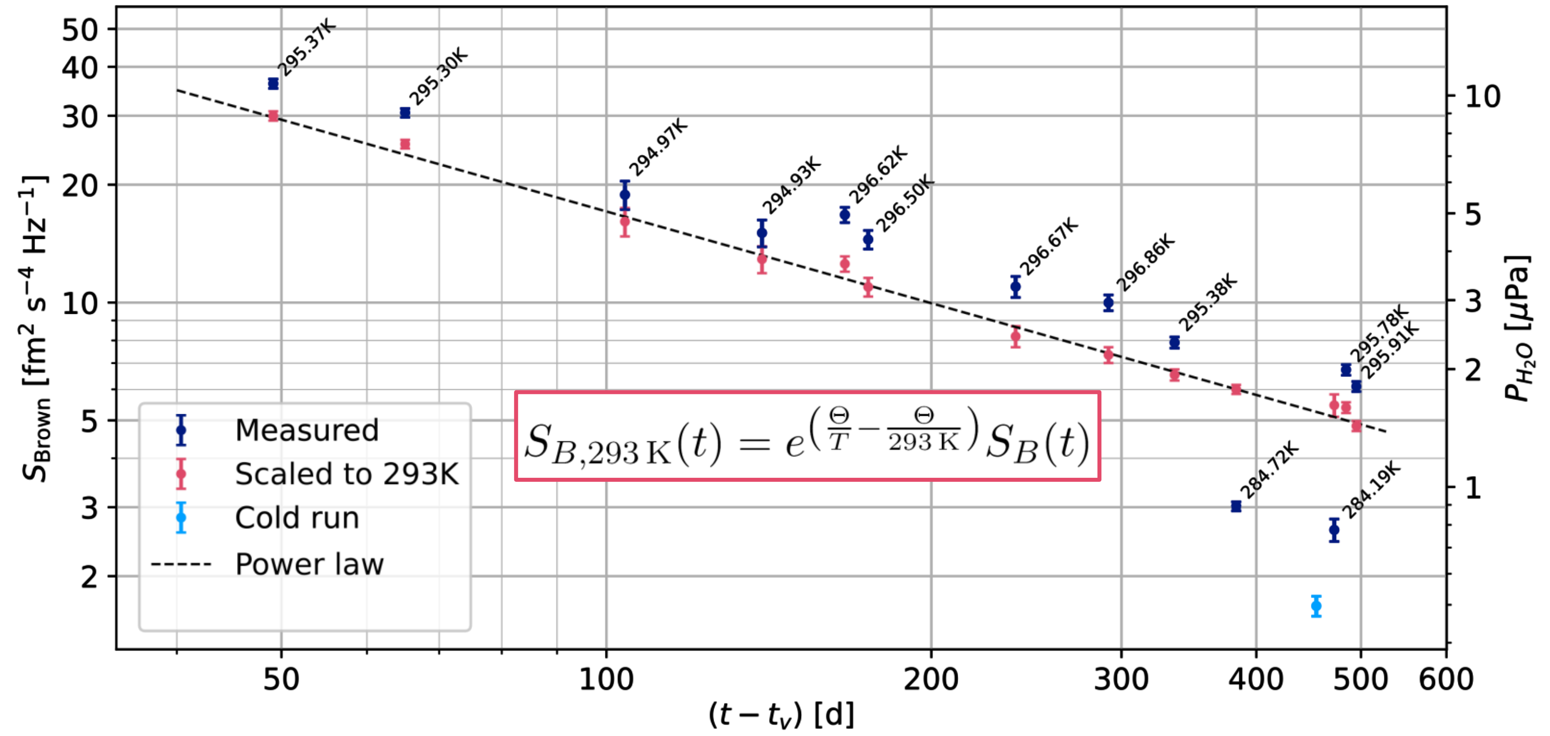


Measurements from LISA Pathfinder



Measurements from LISA Pathfinder

Evolution of the Brownian noise PSD in time



- Brownian noise decreased consistently over time (about 1.5 yr venting).
- Brownian **dominates** mHz acceleration noise.

- Reached “H_2O equiv. pressure” **2 μPa**.
- Reached LISA pressure spec.
- **Dependence on T** clearly identified.

- Consistent with water outgassing. However, incompatible with standard outgassing “water-film on metal”.

$$S_B(T, t) = a\, e^{\left(\frac{\Theta}{T} - \frac{\Theta}{293\,\mathrm{K}}\right)} \left(\frac{t_0 - t_{\mathrm{vent}}}{t - t_{\mathrm{vent}}}\right)^{\gamma}$$

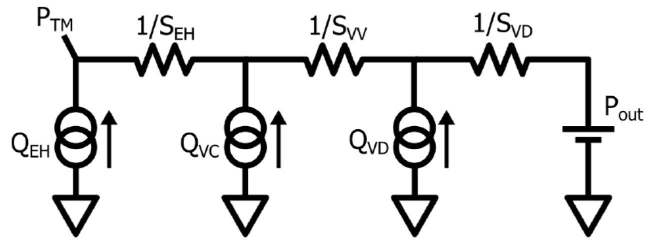
$$\Theta = (7.0 \pm 0.2)\ \mathrm{kK}$$

$$\gamma = (0.80 \pm 0.02)$$

UNIVERSITÀ
DI TRENTO

Pumping speed optimization / equivalent circuit

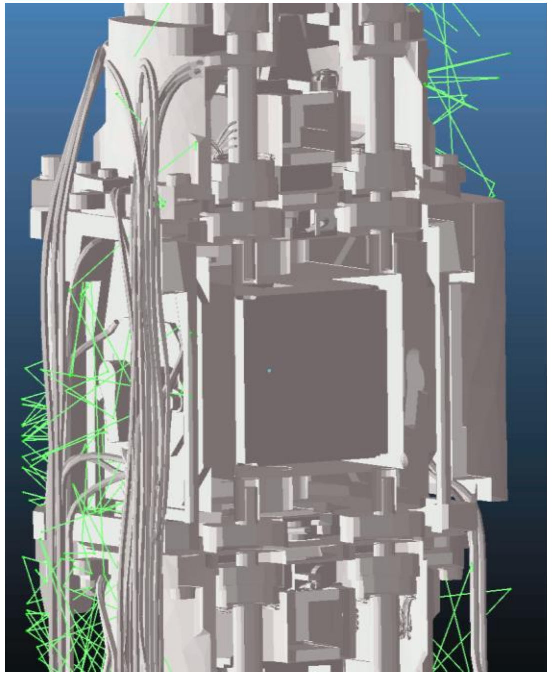
- **Equivalent vacuum circuits**:
  Approximate a vacuum system with an equivalent electrical circuit,
  → apply Thévenin/Norton
  → solve with software for electronics

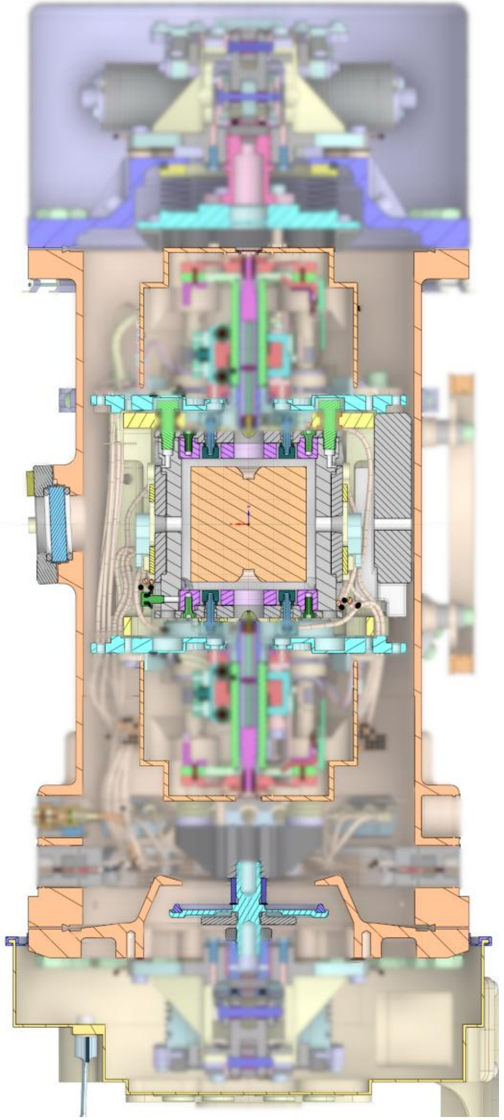


- **Molecular simulations**:
  sub-divide the system into smaller sources, implement the correct molecular emission properties, extract $\partial P_{TM}/\partial Q_k$:

$$P_{TM} = \sum_k \frac{\partial P_{TM}}{\partial Q_k} Q_k + P_{out}$$

→ Simulate emission Q_k, extract pressure P_{TM}.
→ Extract geometrical parameter $\partial P_{TM}/\partial Q_k$.
→ Implement design changes (duct and inner geometry) to **minimize $\boldsymbol{\partial P_{TM}/\partial Q_k}$**
→ Provide forecasts for end-to-end tests (given Q_k)



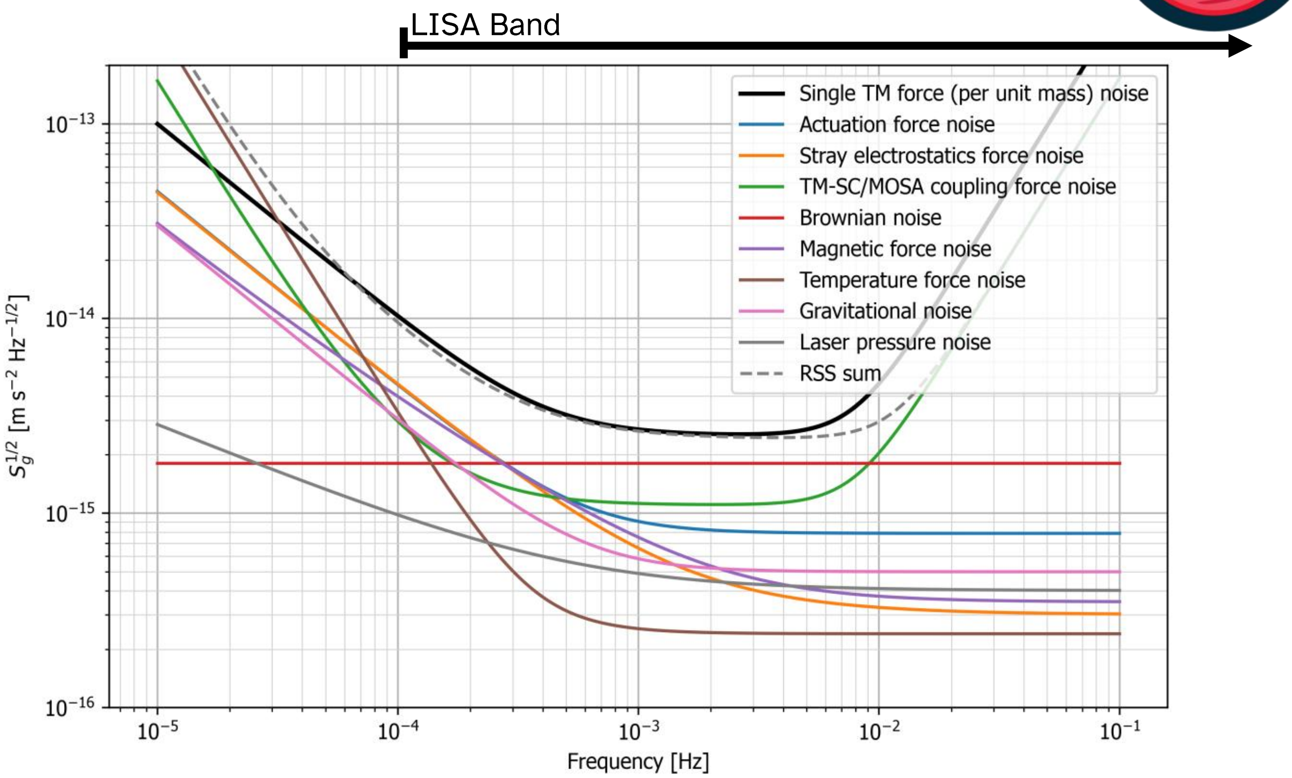


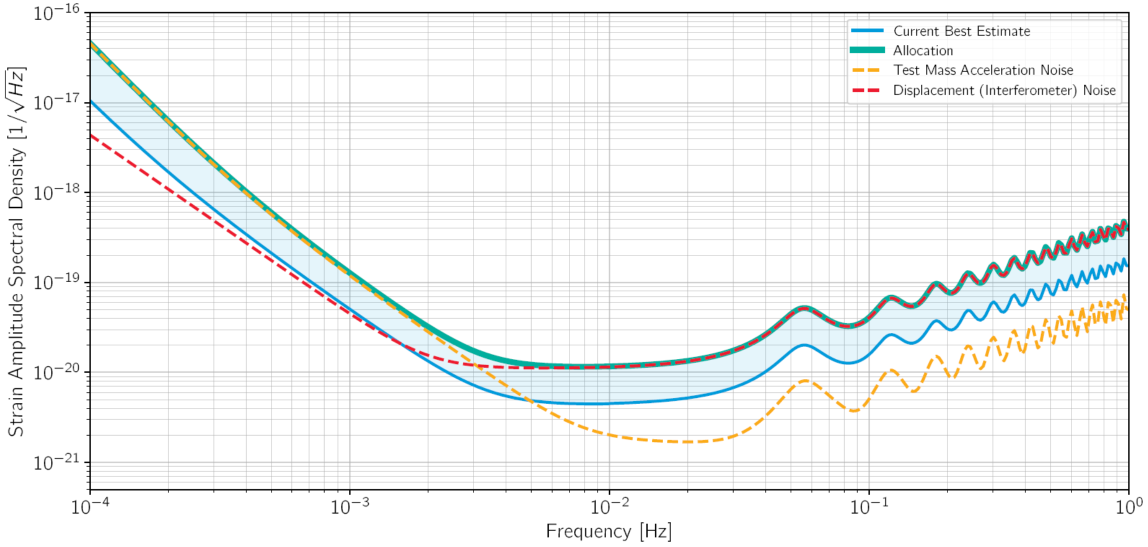
The LISA free fall breakdown

Free-fall disturbances – a breakdown

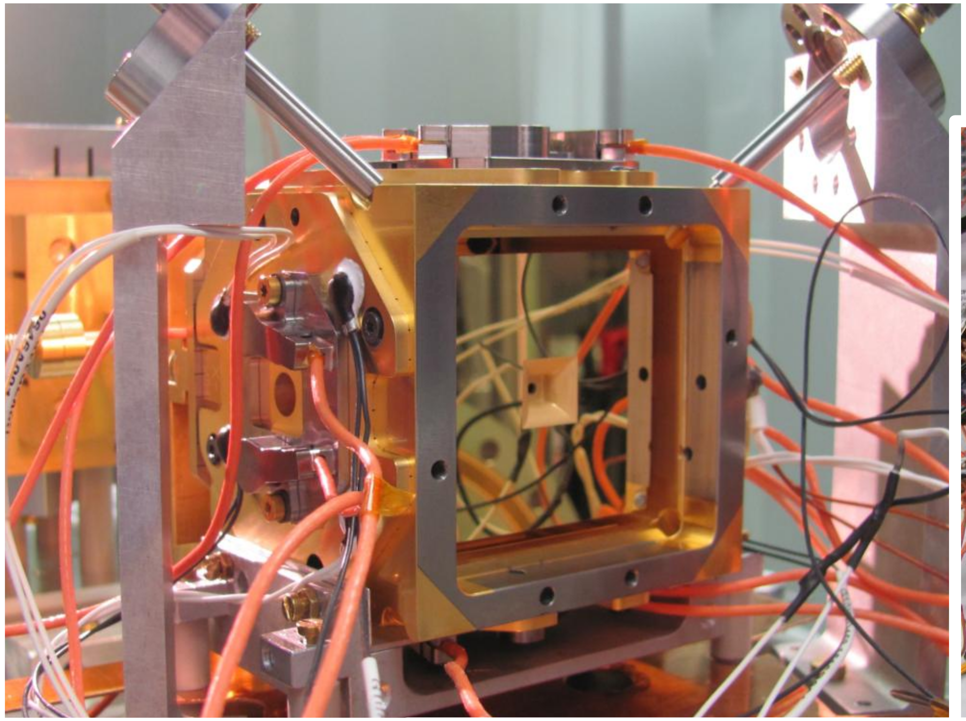
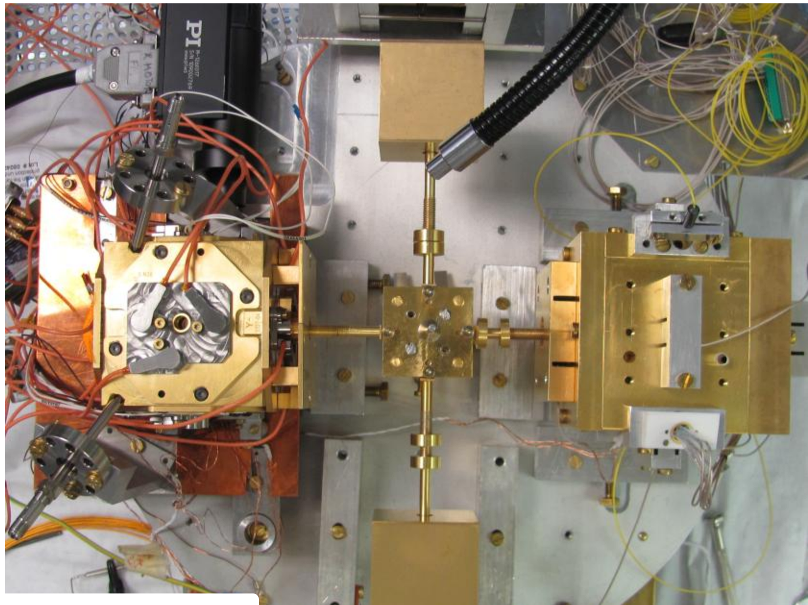
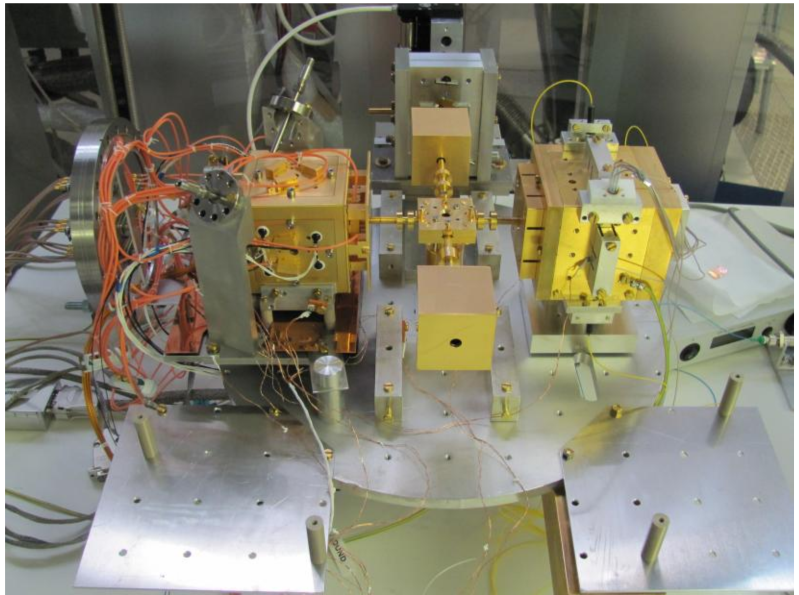
We budget for a list of known (and modeled) effects.
The top-level entry **“Single TM force per unit mass”**
is broken down into:

1. Actuation force noise
2. Stray electrostatics force noise
3. Brownian noise
4. TM-SC/MOSA coupling force noise
5. Temperature force noise
6. Magnetic force noise
7. Laser pressure noise
8. Gravitational noise

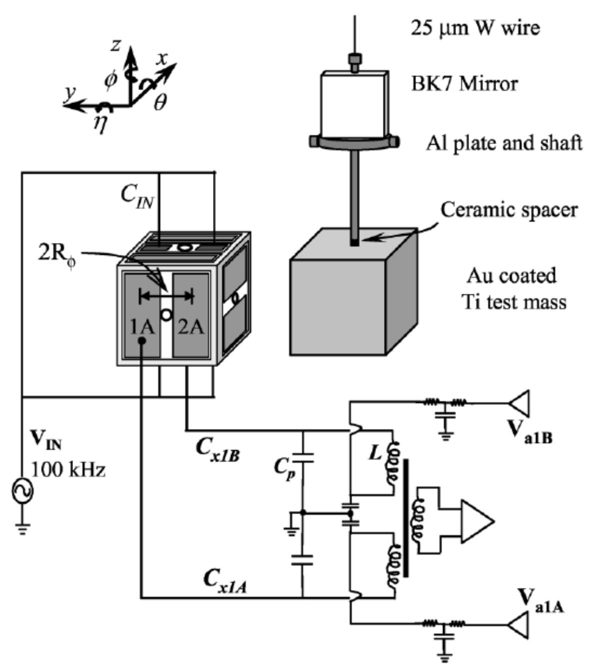


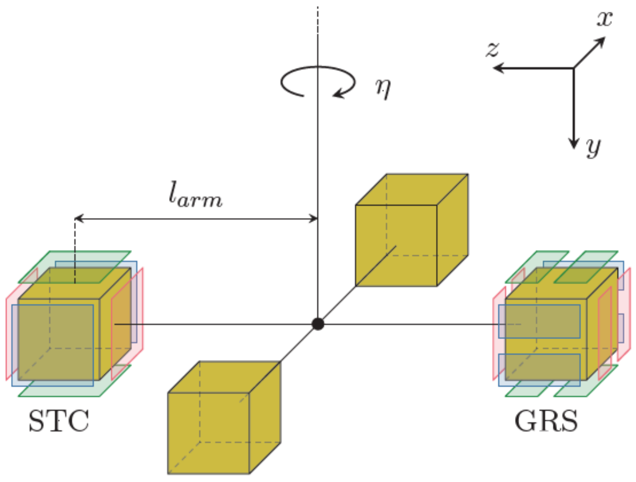


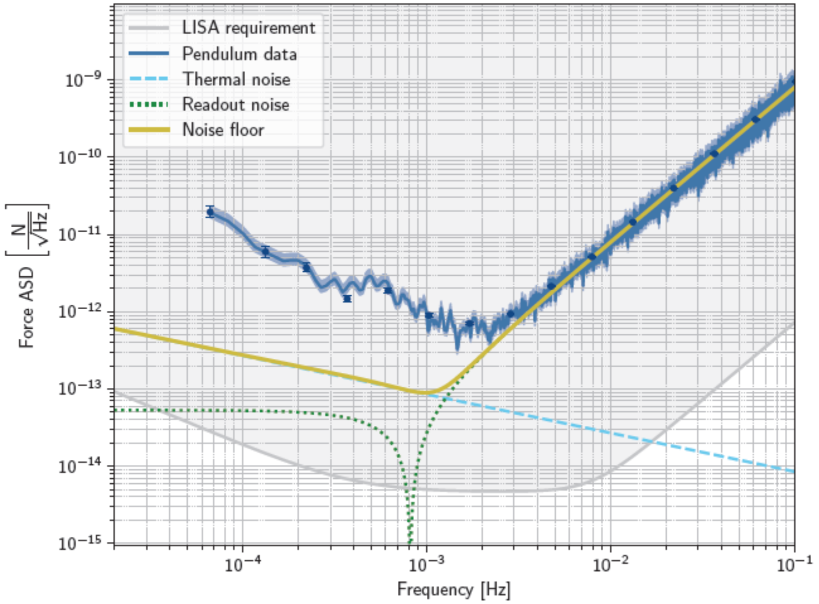
GRS testing at UniTrento / INFN (not exaustive...)

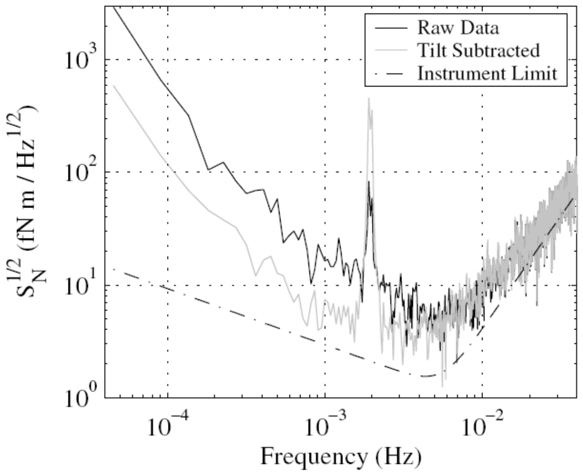





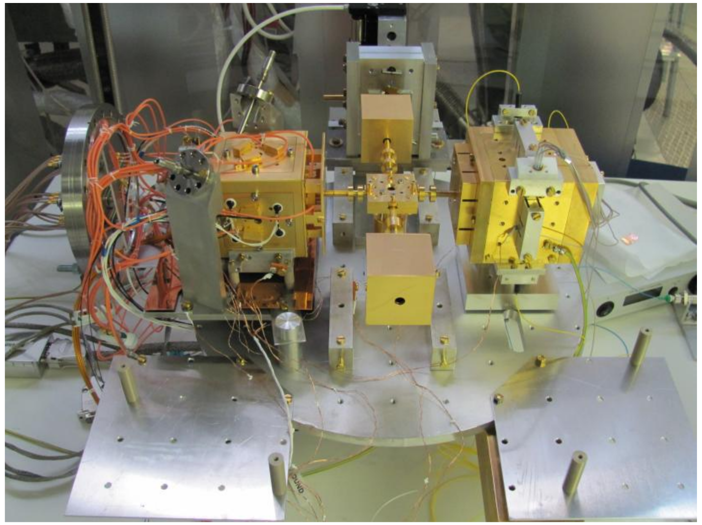

GRS testing at UniTrento / INFN











References (just a few…)

LISA Redbook

LISA Pathfinder (acceleration noise)
Results during operations: PRL 116, 231101 (2016)
Noise results: PRL 120, 061101 (2018)
Analyses: PRD 110, 042004 (2024)

Brownian noise
Modeling: PhysRevA 374 (2010) 3365–3369
Lab results: PRL 103, 140601 (2009)
LPF results: PRD 110, 042004 (2024)

Actuation noise
Actuation noise: PRD 109, 102009 (2024)

Noise
Magnetic: PRL 134, 071401 (2025)
Thermal: MNRAS 486, 3368 (2019)
LPF thrusters: PRD 99, 122003 (2019)
LPF glitches: PRD 106, 062001 (2022)
LPF drag-free: PRD 99, 082001 (2019)

Electrostatic
Stray electrostatics: PRL 108, 181101 (2012)
Charging: PRL 118, 171101 (2017), PRD 107, 062007 (2023)
Discharge: PRD 98, 062001 (2018)

There are many more references!
Contact me lorenzo.sala@unitn.it

Thanks!

UNIVERSITÀ
DI TRENTO

Additional slides (if time is not over yet...)

LPF Physical models and noise projection

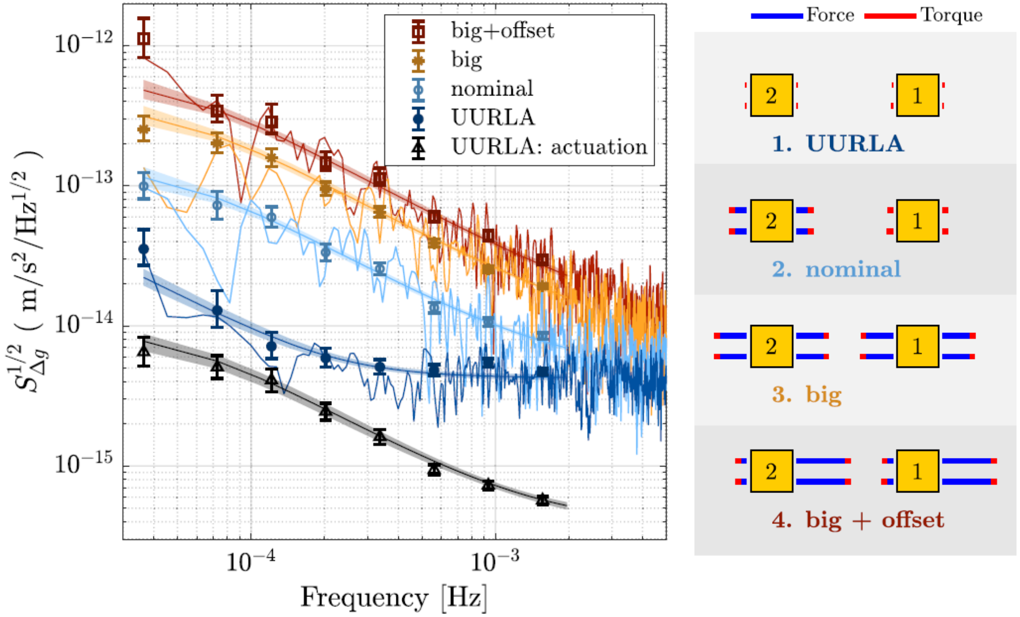
- **Actuation noise**
    - Actuation stability noise
    - Actuation white noise
    - Actuation crosstalk noise
    - Actuation nonlinearity noise
- **Stray Electrostatics noise**
    - Electrostatic field noise and DC Charge
    - Electrode V noise and DC bias
    - Charge noise and DC bias
    - Charge noise and TM position
    - Continuous Discharge Noise
    - Sensing bias noise and TM position
    - Sensing Amplifier Acceleration Noise
- **Brownian noise**
- **Temperature noise**
    - Mean T noise
    - Thermal Gradient Noise
- **Laser radiation pressure**

→ **Analysis:**

❖ **Decorrelation** of synchronous time series

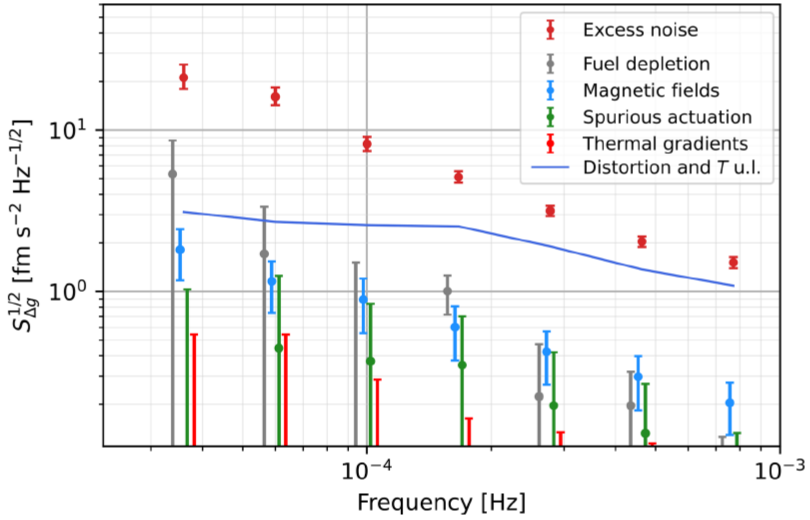
❖ Dedicated **experiments**

❖ **Estimations**



↑ LPF actuation experiments with modified authority

← LPF post-processing time series decorrelation



Cross-correlations

- Can we find any relevant cross-correlation between the available telemetries?
  - If the telemetry is available → **Decorrelation**
  - If no telemetry is available → **Estimation**

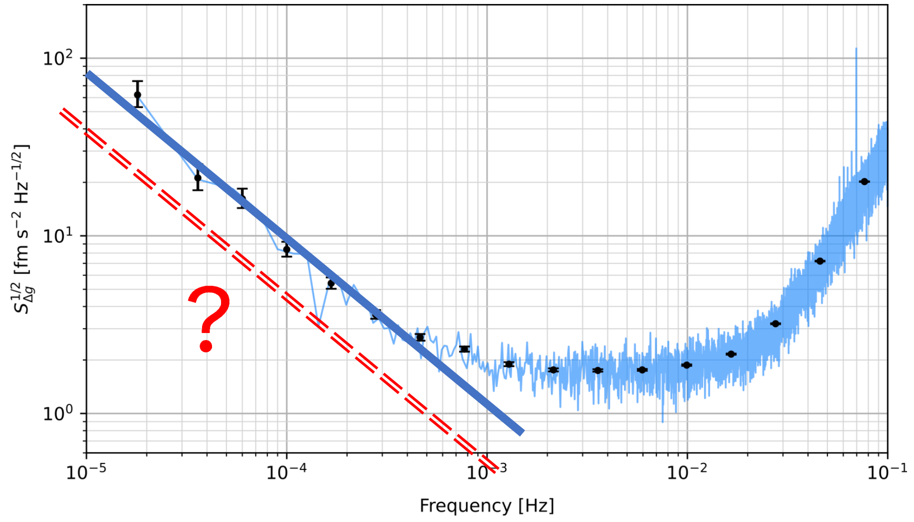
Measured differential acceleration

$$\Delta g(t) = \Delta g_0(t) + \sum_{i}^{N} \alpha_i y_i(t)$$

Residual non-correlating acceleration

susceptibilities

Disturbances



Magnetic interaction (three axes)

$$\Delta g(t) = \frac{\chi L^3}{M\mu_0}(\partial_x \boldsymbol{B}_{\mathrm{DC},2} - \partial_x \boldsymbol{B}_{\mathrm{DC},1}) \cdot \boldsymbol{B}(t)$$

Thermal effects

$$\begin{cases} \Delta g_{T\ \mathrm{mean}}(t) & = \alpha_T \overline{T}(t) \\ \Delta g_{T\ \mathrm{grad}}(t) & = \alpha_{\Delta T_1} \Delta T_1(t) + \alpha_{\Delta T_2} \Delta T_2(t) \end{cases}$$

Instrument distortion

$$\Delta g = -\omega_d^2(\Delta x_{\mathrm{GRS}} - \Delta x_{\mathrm{OMS}})$$

Imperfect digitization

$$\Delta g(t) = \sum_{i\in[1,8]} \alpha_i \, \delta V_{x,i}(t) + \beta \, t \, (\delta\Delta_{x,2} - \delta\Delta_{x,1})$$

Fuel depletion

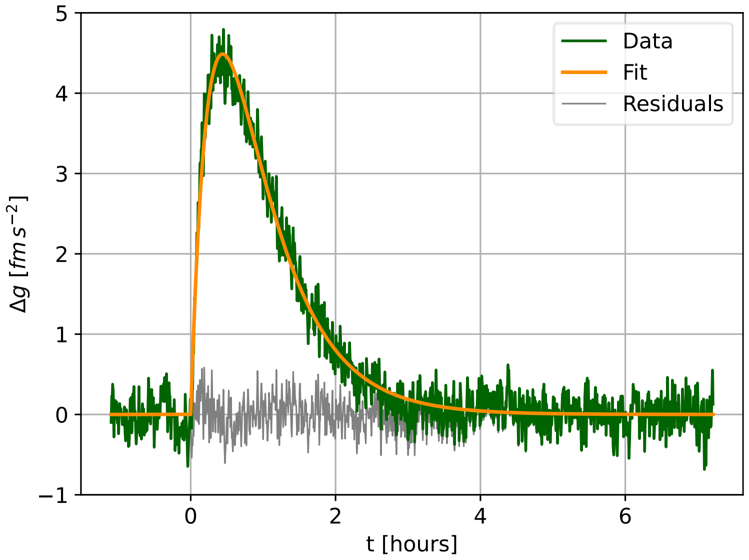
Actuation: gain fluctuations

DC voltage fluctuations

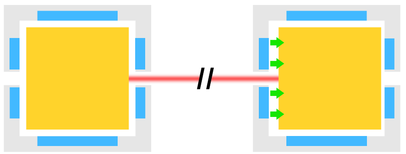
Laser radiation pressure

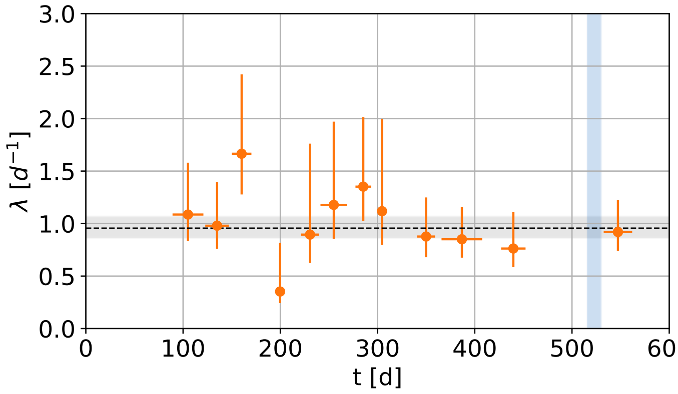
Random charging

LPF glitches: description

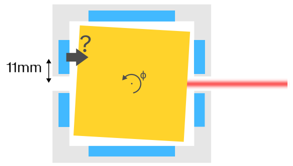


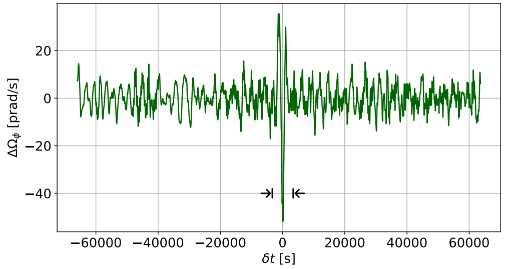
- Impulse-carrying events (98 events + 334 cold) detected during the whole mission, rate in ordinary runs: $\lambda = 0.96^{+0.11}_{-0.09}\ d^{-1}$
- Poisson-distributed, occurrence rate stable for **1.2 yr**.



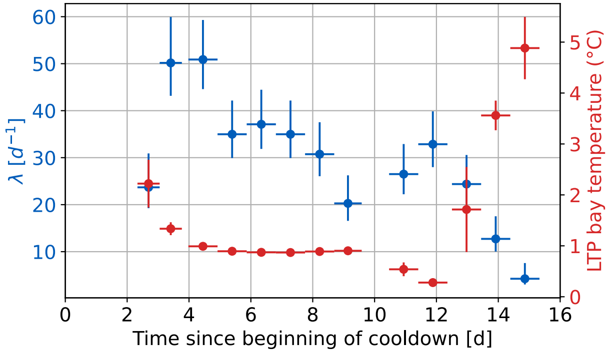


- **Real forces** pushing the TM(s),
  → Duration **between 1s and 5h.**
  → Transferred impulse **between 10fm/s and 1nm/s.**
- No torque associated, except for a few events.

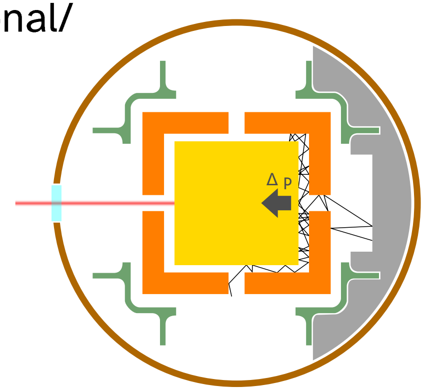




- Increased rate during 0°C runs.



- **Hypotheses**:
  - No clear smoking gun, but some sources excluded: thermal/magnetic/gravitational/ some kind of electrostatics.
  - A possibility: outgassing



- see PRD **106**, 062001 (2022) -