Squeezing

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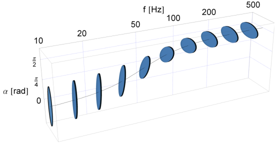

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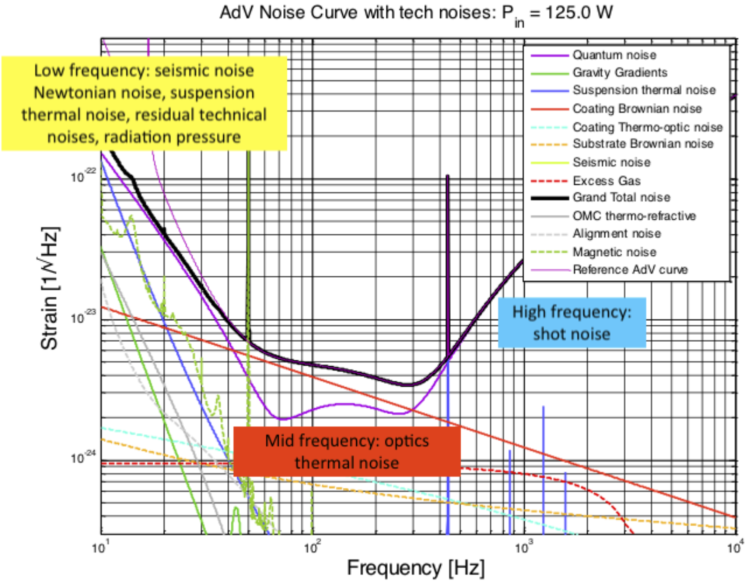
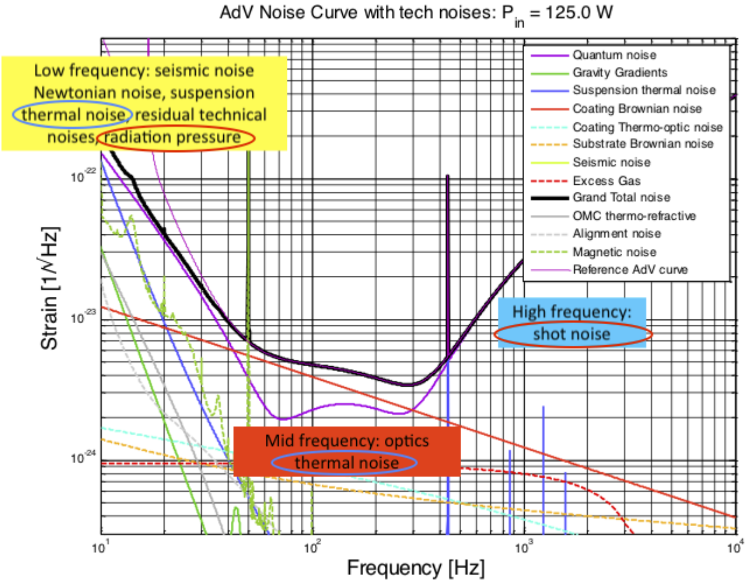
Fundamental noises in Advanced Virgo design

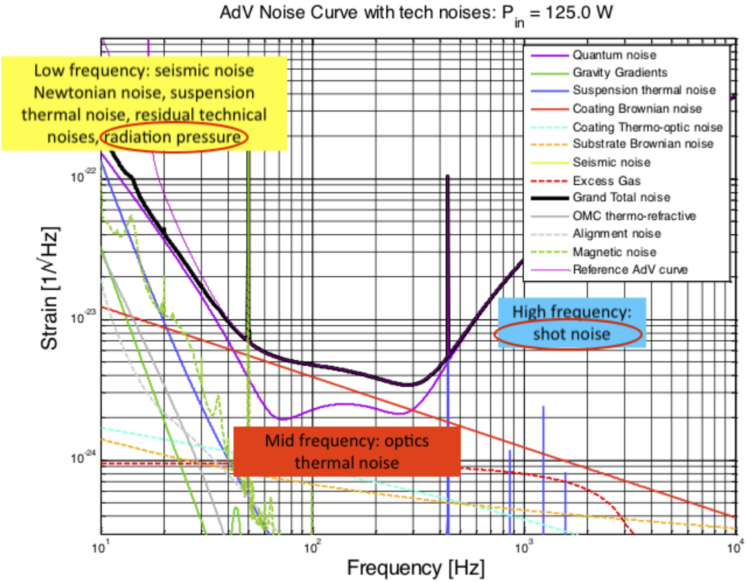


QM limits



AdV Noise Curve with tech noises: P_{in} = 125.0 W
Low frequency: seismic noise Newtonian noise, suspension thermal noise, residual technical noises, radiation pressure
Quantum noise
Gravity Gradients
Suspension thermal noise
Coating Brownian noise
Coating Thermo-optic noise
Substrate Brownian noise
Seismic noise
Excess Gas
Grand Total noise
OMC thermo-refractive
Alignment noise
Magnetic noise
Reference AdV curve
High frequency: shot noise
Mid frequency: optics thermal noise
Strain [1/√Hz]
Frequency [Hz]
10^{-22}
10^{-23}
10^{-24}
10^{1}
10^{2}
10^{3}
10^{4}

Optical quantum noises



Mitigation of fundamental noises

- Classical approach: act on experimental parameters which determine the scaling of fundamental noises, e.g.
  - optical power (shot noise and radiation pressure noise)
  - arm length (shot noise, radiation pressure noise, all thermal noises),
  - mirror mass (radiation pressure noise, suspension thermal noise).
  - test mass temperature (all thermal noises)
  - materials for mirror substrate and coatings (mirror thermal noise) and suspensions (suspension thermal noise)
- Quantum approach: modify quantum properties of the system, e.g.
  - inject squeezed states of light (shot noise, radiation pressure noise)
    - already successfully implemented in GW detectors
  - replace macroscopic test masses (mirrors) with quantum test masses (ultracold atoms)
    - not yet implemented in GW detectors so far

Optical quantum noise from a semiclassical model

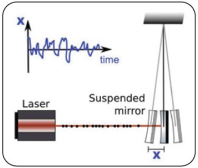
- Lght interacts with the detector in discrete quanta (photons)
- Optical power for mean number N_γ of photons in a time interval T:

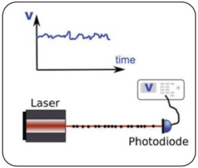
$$P = \frac{N_\gamma \hbar \omega_L}{T}$$

- Without correlations photons obey Poisson's statistics
- Photons number variance $\Delta N_\gamma = \sqrt{N_\gamma} \rightarrow$ optical power fluctuations

$$\Delta P = \frac{\sqrt{N_\gamma} \hbar \omega_L}{T} = \sqrt{\frac{\hbar \omega_L}{T} P}$$

- In GW detector such limit has two complementary aspects: *shot noise* and *radiation pressure noise*





Shot noise: signal fluctuations at output port

- In the absence of a GW, the output optical power is given by a $P = P_{circ} \sin^2(\phi_0)$, thus

$$(\Delta P)_{shot} = |\sin\phi_0| \sqrt{\frac{\hbar\omega_L}{T} P_{circ}}$$

- The output power due to a GW is $P = P_{circ} \sin^2[\phi_0 + \Delta\phi(t)]$, thus

$$(\Delta P)_{GW} \simeq 2P_{circ} \sin(\phi_0)\cos(\phi_0)\Delta\phi(t)$$

and finally

$$SNR = \frac{(\Delta P)_{GW}}{(\Delta P)_{shot}} = 2\sqrt{\frac{P_{circ}T}{\hbar\omega_L}}\Delta\phi(t)|\cos\phi_0|$$

Shot noise

- For an interferometer with F-P arm cavities and power recycling cavity, we have

$$(\Delta\phi)_{FP} = h_0 L \frac{4\mathcal{F}\omega_L}{\pi c\sqrt{1+(f_{gw}/f_p)^2}}$$

- The equivalent strain noise spectral density is deduced from SNR:

$$S_n(f) = T\left(\frac{h_0}{SNR}\right)^2$$

Finally the shot noise spectral density is:

$$\sqrt{S_{n,sn}(f)} = \frac{1}{8L\mathcal{F}}\sqrt{\frac{4\pi\hbar\lambda_L c[1+(f/f_p)^2*}{P_{circ}}}$$

Radiation pressure noise

- A beam of photons back-reflected from a mirror applies a pressure on the mirror itself.
- If the number of photons fluctuates, the magnitude of radiation pressure fluctuates too.
- Back-reflection corresponds to a change $2\vec{p}$ in the photon momentum.
- Since the photon energy is $|\vec{p}|c$, the force exerted by a beam with power P on the mirror is $F = 2P/c$ and its variance is $\Delta F = 2\Delta P/c$

$$\Delta F = 2\sqrt{\frac{\hbar\omega_L P}{c^2 T}}$$

Radiation pressure noise

- Such a stochastic force acts on a mirror which is free to move in the horizontal plane, thus $F = M\ddot{x} \to \tilde{F}(f) = -M(2\pi f)^2\tilde{x}(f)$ and

$$|\Delta\tilde{x}| = \frac{\Delta\tilde{F}}{M(2\pi f)^2}$$

- The length change of an arm is $\Delta L = 2|\Delta x|$ and for the full interferometer we must sum the two contributions

$$(\Delta L)_{rp} = \frac{4}{M(2\pi f)^2}\sqrt{\frac{\hbar\omega_L P}{c^2 T}}$$

Radiation pressure noise

- For a simple Michelson interferometer the effect of a GW on the single arm length is $(\Delta L)_{gw} = h_0 L$, and

$$SNR = \frac{h_0 L}{4} M (2\pi f)^2 \sqrt{\frac{c^2 T}{\hbar \omega_L P_{circ}}}$$

- In a Michelson interferometer with F-P arm cavities each photon undergoes $N = 2\mathcal{F}$ reflections on average in the cavity band;
- we finally obtain

$$SNR = \frac{\pi h_0 L}{8\mathcal{F}} M (2\pi f)^2 \sqrt{\frac{c^2 T [1 + (f/f_p)^2]}{\hbar \omega_L P_{circ}}}$$

$$\sqrt{S_{n,rp}(f)} = \frac{16\sqrt{2}\mathcal{F}}{ML(2\pi f)^2} \sqrt{\frac{\hbar P_{circ}}{2\lambda_L c [1 + (f/f_p)^2]}}$$

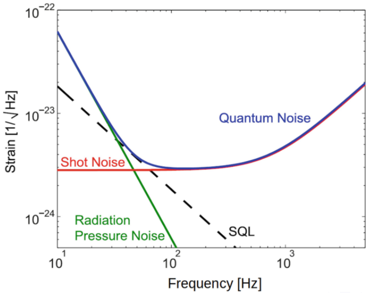
Standard quantum limit

- Total quantum noise: uncorrelated sum of SN and RPN:

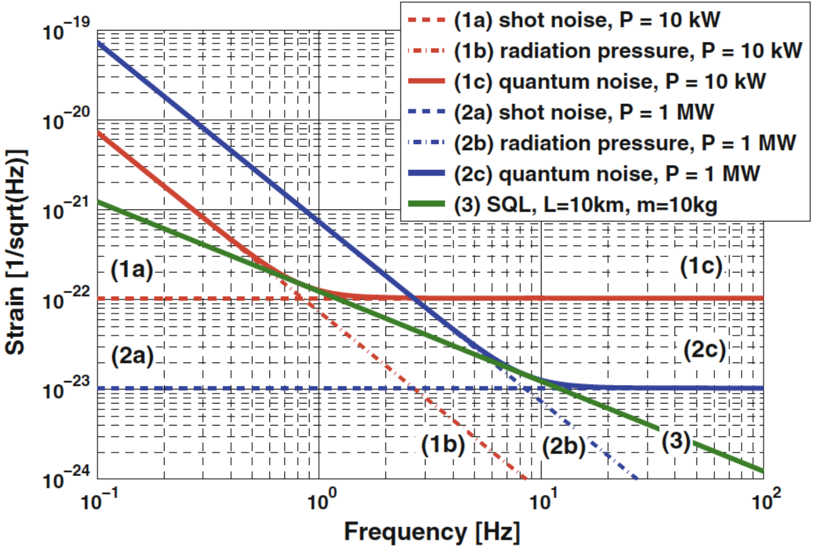
$$S_{n,qn}(f) = S_{n,sn}(f) + S_{n,rp}(f)$$

- The *standard quantum limit* (SQL) is the minimum value of $S_{n,qn}(f)$.
- SQL occurs at the frequency where shot noise equals radiation pressure noise:

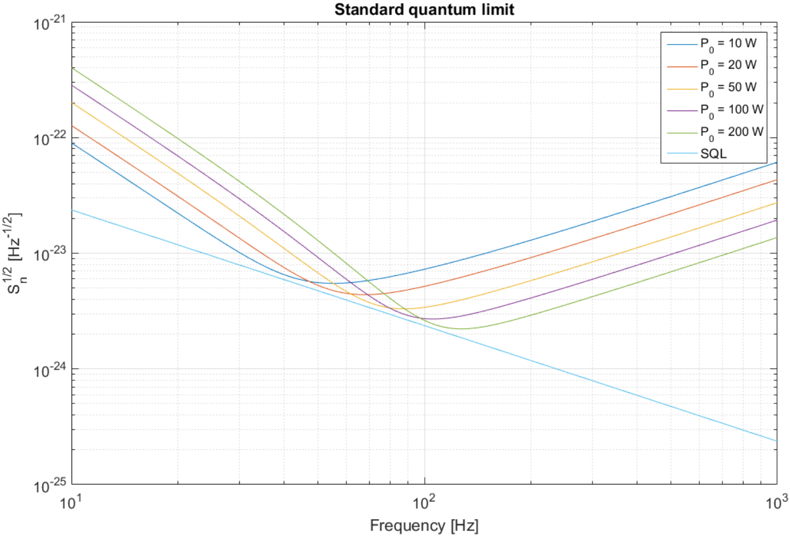
$$\sqrt{S_{SQL}(f)} = \frac{1}{2\pi f L}\sqrt{\frac{8\hbar}{M}}$$



Standard quantum limit - Michelson interferometer

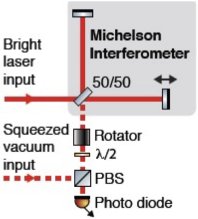


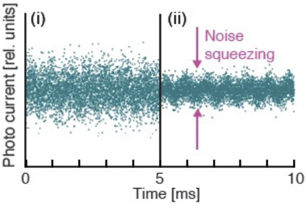
Standard quantum limit - Michelson + FR arm cavities



Quantum theory of light in interferometric GW detectors

- Optical interferometers for GW detection usually operate at dark fringe
- Vacuum fluctuations enter the dark port of the interferometer
- Quantum noise can be seen as vacuum fluctuations of the optical field
    - Quantum mechanical formalism to describe vacuum fluctuations
    - Tools for measuring quantum fluctuations
    - Methods to generate and manipulate quantum states of light (squeezing)





Quantum fluctuations of the EM field and expectation values

- In canonical quantization the e.m. field is expanded in terms of creation $\hat{a}^{\dagger}_{\omega}$ and annihilation $\hat{a}_{\omega}$ operators of photons at frequency ω.
- In the case of interferometric GW detectors it is more convenient to expand the electromagnetic field in terms of the sidebands photon creation operator $\hat{a}^{\dagger}_{\omega_0 \pm \Omega}$ around the carrier laser frequency ω_0;
  - these can be directly related to sidebands generated by the gravitational wave.

Expectation values

- In this framework the quantum electric field operator $\hat{E}(x, y, z, t)$ for a laser beam traveling in the "z" direction can be written as

$$\hat{E}(x, y, z) = u(x, y, z)\sqrt{\frac{\hbar\omega_0}{c\epsilon_0}}[\hat{X}_1(z, t)\cos\omega_0 t + \hat{X}_2(z, t)\sin\omega_0 t]$$

where ϵ_0 is the vacuum permittivity, the mode shape function $u(x, y, z)$ is normalized as $\int dxdyu(x, y, z) = 1$ and the quadrature operators $X_{1,2}(z, t)$ can be written in the Fourier space as

$$\hat{X}_{1,2}(z, t) = \int_0^\infty \frac{d\Omega}{2\pi}[\hat{X}_{1,2}(\Omega)\exp(-i\Omega t + kz) + \hat{X}^\dagger_{1,2}(\Omega)\exp(i\Omega t - kz)].$$

Expectation values

- The hermitian operators

$$\hat{X}_1(\Omega) = \frac{\hat{a}_{\omega_0+\Omega} + \hat{a}^{\dagger}_{\omega_0-\Omega}}{\sqrt{2}}$$

$$\hat{X}_2(\Omega) = \frac{\hat{a}_{\omega_0+\Omega} - \hat{a}^{\dagger}_{\omega_0-\Omega}}{i\sqrt{2}}$$

  coherently create a photon at the sideband $-\Omega$ and annihilate a photon at the sideband $+\Omega$.
- This so-called *Two Photons* formalism is particularly suitable for dealing with two-photon systems as in the case of squeezed light

Expectation values

- The two operators $\hat{X}_1(\Omega)$ and $\hat{X}_2^\dagger(\Omega)$ do not commute, namely

$$[\hat{X}_1(\Omega), \hat{X}_2^\dagger(\Omega')] = 2i\delta(\Omega - \Omega')$$

  which implies that X_1 and X_2 cannot be estimated simultaneously with infinite accuracy, as their variances $\Delta X_{1,2}^2$ must fulfil the uncertainty principle

$$\Delta X_1^2(\Omega)\Delta X_2^2(\Omega) \geq 1$$

- In other words, the phase and amplitude fluctuations of the electromagnetic field can never cancel out simultaneously without contradicting quantum mechanics.

Expectation values

- If the field amplitude is much larger than the fluctuations, one can decompose the quadrature operator into a steady state operator $\hat{X}_{1,2}$ plus a time dependent fluctuation operator $\delta\hat{X}_{1,2}(t)$ such that

$$< sl|\hat{X}_{1,2}|sl >= X_{1,2} \qquad\qquad < sl|\delta\hat{X}_{1,2}(t)|sl >= 0$$

  where $X_{1,2}(t)$ is the average value of the quadrature amplitude and $|sl>$ the state of light.

- The quadrature fluctuations associated to $\delta\hat{X}_{1,2}$ are described by the double side power spectral density

$$S_{\delta X_a \delta X_b}(\Omega) = \frac{1}{\pi} < sl|\delta\hat{X}_a(\Omega)\delta\hat{X}_b^\dagger(\Omega) + \delta\hat{X}_b(\Omega)\delta\hat{X}_a^\dagger(\Omega)|sl >$$

States of light

- There are many ways to represent the quantum state $|State>$ of light.
- The most popular is to specify the number n_ω of photons for each mode at frequency ω of the electromagnetic field. These are called *Fock states*.
- Due to the uncertainty principle, if the number of photons is exactly known the phase of the electric field is completely undetermined.
- Moreover, for the Fock states the expectation value of the electric field vanishes:

$$< n_1, n_2, ..|E(x, y, z, t)|n_1, n_2, .. > \equiv < \bar{n}|E(x, y, z, t)|\bar{n} > = 0$$

  and this holds even in the classical limit $\bar{n} \rightarrow \infty$.
- For these reasons Fock states are inappropriate to describe a laser field, which is actually used in experiments.

Coherent states

- The *coherent states* $|\alpha>$, where $\alpha = |\alpha| \exp[i\phi]$ is a complex number, are the most appropriate choice to describe the classical field of a laser beam.
- for such states the expectation value of the electric field

$$< \alpha|E(x, y, z, t)|\alpha >=$$

$$= u(x, y, z)\sqrt{\frac{\hbar\omega_0}{c\epsilon_0}}[|\alpha| \cos\phi \cos(\omega t) + |\alpha| \sin\phi \sin(\omega t)]$$

takes the same form of a classical field with amplitude $|\alpha|$

$$|\alpha| = \sqrt{\frac{2P_{las}}{\hbar\omega}}$$

phase ϕ and power P_{las}.

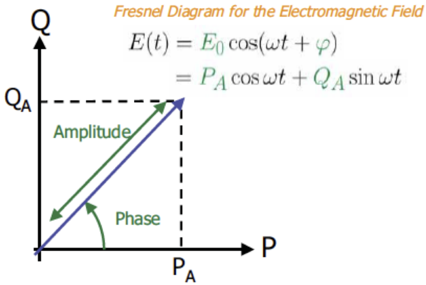
Coherent states

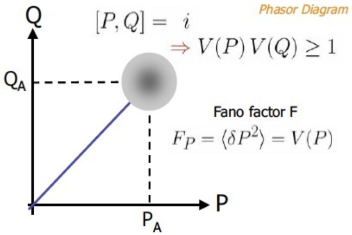
- For coherent states the quantum fluctuation are equally distributed on the two quadrature $X_{1,2}$:

$$\Delta X_1^2 =< \alpha|\delta\hat{X}_1^2|\alpha >=< \alpha|\delta\hat{X}_2^2|\alpha >= \Delta X_2^2 = 1$$

  which shows that a coherent state satisfies the lower limit of the uncertainty condition.
- Coherent states are *minimum uncertainty* states.
- Due to the uncertainty principle, a quantum state cannot be represented in the phase space (X_1, X_2) with a single point as for a classical state, but rather with a probability distribution.

Classical fields vs coherent states





Coherent states

- Coherent states are constructed by applying the unitary displacement operator $D(\alpha_\omega)$
$$\hat{D}(\alpha_\omega) = \exp(\alpha \hat{a}_\omega^\dagger - \alpha^* \hat{a}_\omega)$$
to the Fock vacuum state $|0>$, i.e. the state with zero photons.
$$|\alpha_\omega >= \hat{D}(\alpha_\omega)|0>$$
- Since the quantum operator $\hat{D}(\alpha_\omega)$ is unitary, the expectation values can be calculated using the Fock vacuum state $|0>$.
- The operators $\hat{X}_{1,2}$ transform as
$$\hat{X}_{1,2} \rightarrow \hat{D}^\dagger(\alpha_\omega)\hat{X}_{1,2}\hat{D}(\alpha_\omega)$$
- This leads to the double side power spectral densities
$$S_{\delta X_1 \delta X_1}(\Omega) = S_{\delta X_2 \delta X_2}(\Omega) = 1 \qquad S_{\delta X_1 \delta X_2}(\Omega) = 0$$

Squeezed states

- Squeezed states are minimum uncertainty states with unbalanced quadrature variance.
- They can be generated from the vacuum sate $|0>$ as

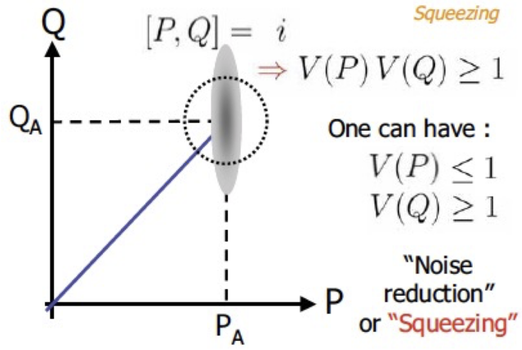
$$|\alpha, \xi, \phi >= \hat{D}(\alpha)\hat{S}(\xi, \theta)|0>$$

where $\hat{S}(\xi, \theta)$ is the squeezing unitary operator

$$\hat{S}(\xi, \theta) = \exp\left[\frac{1}{2}\xi e^{-2i\theta}\hat{a}^2 - \frac{1}{2}\xi e^{2i\theta}\hat{a}^{\dagger 2}\right]$$

- The uncertainty area of a squeezed state appears as an ellipse rotated by an angle θ, with a major axis variance enhanced by factor $\exp(\xi)$ and minor axis variance suppressed by factor $\exp(-\xi)$ with respect to the corresponding coherent state.

Squeezed states



- As for the case of the coherent states, the expectation values of quantum field and fluctuations can be estimated using the Fock vacuum state after replacing the classical quadratures $\hat{X}_{1,2}$ with the squeezed quadratures

$$\hat{X}_{1,2} \rightarrow \hat{S}^{\dagger}\hat{X}_{1,2}\hat{S}$$

which using the Baker-Hausdorf theorem leads to expressions

$$\Delta X_1^2 = \exp(2\xi)Sin^2(\theta) + \exp(-2\xi)Cos^2(\theta)$$
$$\Delta X_2^2 = \exp(2\xi)Cos^2(\theta) + \exp(-2\xi)Sin^2(\theta)$$

- In the particular case $\theta = 0$ this formula reads

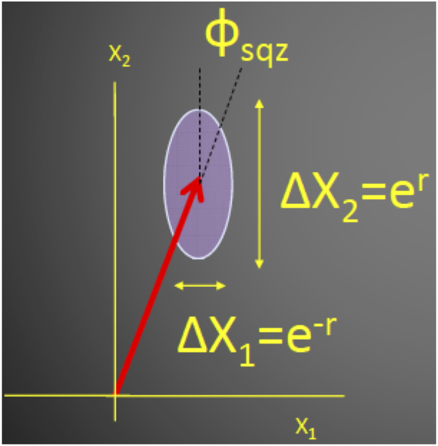
$$\Delta X_1 = \exp(-\xi) \qquad \Delta X_2 = \exp(+\xi)$$

where we see that the variance of the quadrature X_1 is *'squeezed'* by factor $\exp(-\xi)$ with respect the coherent state, while the quadrature X_2 is *'antisqueezed'* by a factor $\exp(\xi)$.

- The opposite occurs for a squeezing angle $\theta = \pi/2$.

Squeezed states

- Squeezing factor r describes level of squeezing and anti-squeezing
- Squeezing angle ϕ_{sqz} describes which quadrature is squeezed



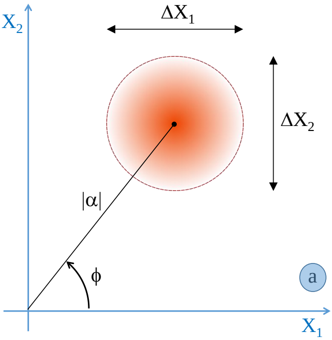
Squeezed states

- In the general case, with an appropriate rotation of the quadrature axis it is always possible to define a new set of quadratures $Y_{1,2}$ for which these conditions hold.
- It is common use to define the squeezing factor of a quadrature in decibels as

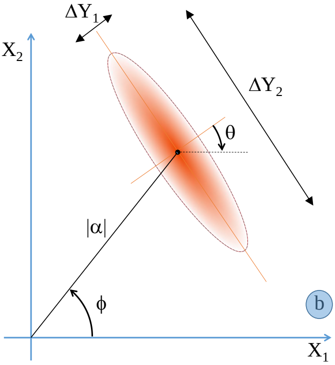
$$dB_{sqz} = 20 \log_{10} \frac{\Delta X_{sqz}}{\Delta X_{coh}}$$

where ΔX_{sqz} and ΔX_{coh} are the standard deviations of the squeezed and the coherent state quadratures, respectively.

Comparison of coherent and squeezed states

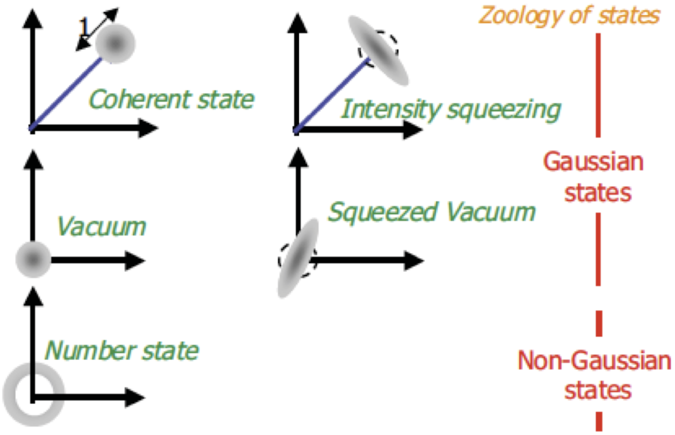


Coherent states



Squeezed state

Quantum states of light



Quantum states characterisation: balanced homodyne detector

- Detecting quantum states of light implies measuring the variance of a given quadrature of the field.
- The most common method to estimate the expectation value of the electric field quadratures is the *balanced homodyne* detection
- The vacuum electric field $\hat{E}(t)$ and a bright *Local Oscillator (LO)* field $\hat{E}^{LO}(t)$

$$\hat{E}^{LO} = u(x,y,z)\sqrt{\frac{\hbar\omega_0}{c\epsilon_0}}\left\{\left[|\alpha|\cos\zeta + \delta\hat{X}_1^{LO}\right]\cos(\omega_0 t) + \left[|\alpha|\sin\zeta + \delta\hat{X}_2^{LO}\right]\right.$$

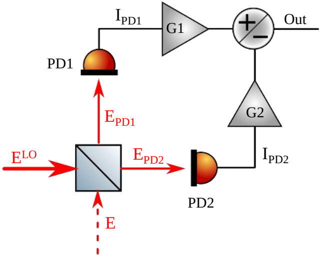
with power P_{LO}, classical amplitude $|\alpha| = \sqrt{2P^{LO}/\hbar\omega_0}$ and selectable phase ζ, combine at a beamsplitter with 50% splitting ratio, generating the two fields $\hat{E}_{PD1,2}(t)$ on the photodiodes:

$$\hat{E}_{PD1,2}(t) = \frac{\hat{E}^{LO}(t) \pm \hat{E}(t)}{\sqrt{2}}$$

Quantum states characterisation: balanced homodyne detector

- To leading order in $\hat{X}_{1,2}$ and $\delta\hat{X}_{1,2}^{LO}$ and assuming $|\alpha|$ much higher than the $\hat{E}(t)$ amplitude the power $\hat{P}_{PD1,2}$ impinging the detection photodiodes is

$$\hat{P}_{PD1,2} = \frac{1}{4}\{|\alpha|^2 + 2|\alpha|[(\delta\hat{X}_1^{LO} \pm \hat{X}_1)\cos\zeta + (\delta\hat{X}_2^{LO} \pm \hat{X}_2)\sin\zeta]\}$$

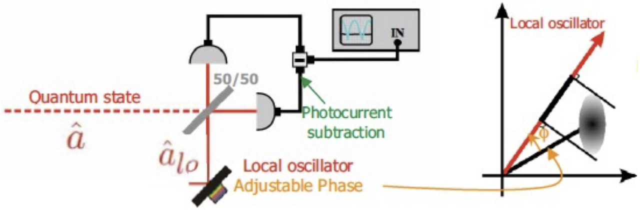


Quantum states characterisation: balanced homodyne detector

- Therefore the induced differential photocurrent

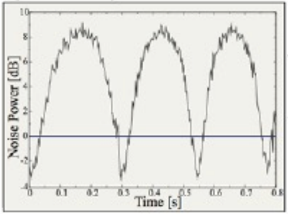
$$I_{PD2} - I_{PD2} \propto |\alpha| [\cos\zeta \cdot X_1 + \sin\zeta \cdot X_2]$$

can be used as an estimator of the quadratures $X_{1,2}$ amplitudes.

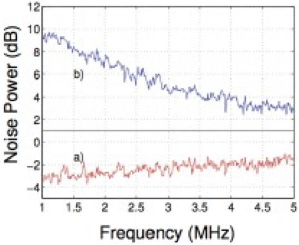


Quantum states characterisation: balanced homodyne detector

- In particular X_1 is obtained for the LO phase set to $\zeta = 0$ while X_2 for $\zeta = \pi/2$.
- the LO fluctuations $\delta X_{1,2}$ of the LO do not appear in the equation.
- This remarkable result is valid only if the beam splitting ratio is exactly 50:50 so that equation above holds.



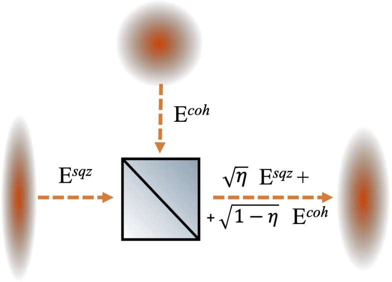
Variance



TF : Noise spectrum

Effect of losses

- Any optical loss on the path of the squeezed light mixes the squeezed vacuum with classical vacuum
- We can model the optical loss as a beamsplitter
- A fraction $\sqrt{1-\eta}$ of the incoming squeezed light field E_{sqz} is reflected by the beamsplitter
- At the same time, the $\sqrt{\eta}$ transmitted fraction combines with a fraction $\sqrt{1-\eta}$ of the uncorrelated coherent vacuum field E_{coh} entering the beamsplitter from the unused port.



Effect of losses

- The output quadrature $\delta\hat{X}_{1,2}^{Tot}$ results

$$\delta\hat{X}_{1,2}^{Tot} = \sqrt{\eta}\delta\hat{X}_{1,2}^{sqz} + \sqrt{1-\eta}\hat{X}_{1,2}^{coh}$$

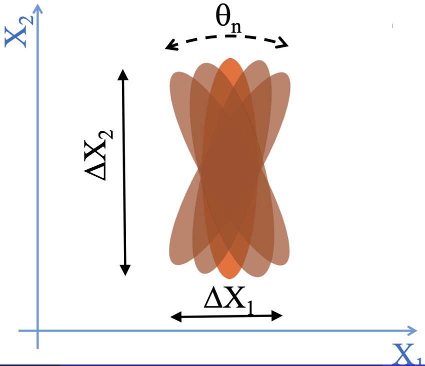
- The actual quadrature variance in presence of losses can then be calculated with the previous expression and the use of displacement operator.
- The case of phase squeezing corresponds to $(\theta = 0)$ for which the quadrature variances become:

$$\Delta X_{1,2}^2 = 1 + \eta[\exp(\mp 2\xi) - 1].$$

- This shows that, in the presence of losses, the resulting field is not in a *minimum uncertainty state*, i.e. $\Delta X_1 \Delta X_2 > 1$.
- Moreover, the squeezed quadrature is more affected by losses than the antisqueezed quadrature.

Jitter of squeezing angle

- The quadrature variance $\Delta X_{1,2}^2$ depends on the squeezing angle θ.
- In real experiments θ fluctuates around an average value θ_0 due to mechanical vibrations.
- The actual value of $\Delta X_{1,2}^2$ is the mean value averaged over the rapid fluctuations of $\theta_n(t)$.



Jitter of squeezing angle

- Expanding around the angle of phase squeezing $\theta_0 = 0$ for small fluctuations $\theta_n(t) << 1$, and combining equations on field fluctuations one gets:

$$\Delta X_1^2 \approx [1 + \eta(e^{2\xi} - 1)]\theta_n^2 + (1 - \theta_n^2)[1 + \eta(e^{-2\xi} - 1)]$$

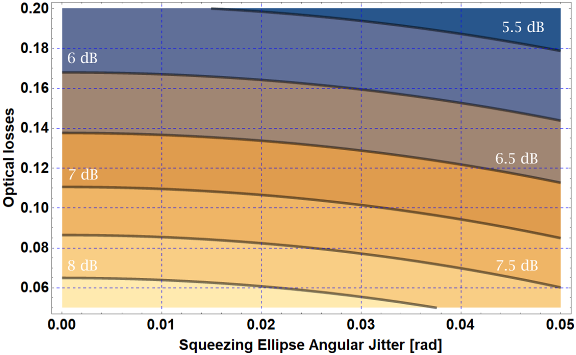
$$\Delta X_2^2 \approx [1 + \eta(e^{2\xi} - 1)](1 - \theta_n^2) + \theta_n^2[1 + \eta(e^{-2\xi} - 1)]$$

  where θ_n^2 is the mean value of $\theta_n(t)^2$.

- This shows that the jitter of squeezing angle has larger impact on the quadrature with lower variance, i.e. ΔX_1;
- the higher is the antisqueezing level $e^{2\xi}$, the higher is the squeezing degradation

Combined effect of optical losses and jitter of squeezing angle

Measurable squeezing level versus overall optical losses and rms jitter of the squeezing ellipse angle, for a given value of the original squeezing level



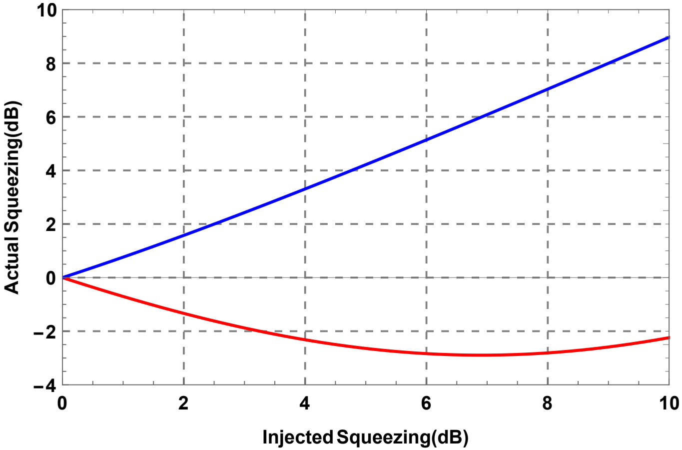
Non ideal detector

Accuracy of squeezing level estimation with BHD is limited by technical noise.

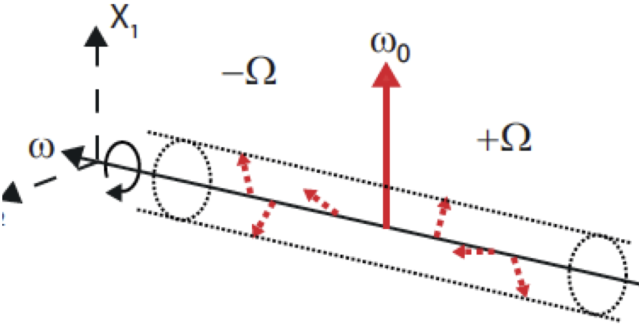
- *Unbalanced arms*. Imperfect balance of the two HD arms produces coupling of LO technical noise $\delta X_{1,2}^{LO}$ onto the output signal,
  - optomechanical (non-ideal splitting ratio, angular jitter of laser beams, light scattering)
  - electronical (photodiodes with different quantum efficiency, unbalanced electronics for the 2 channels) and
  - in general more effective at low frequency
  - quantified by the Common Mode Rejection *CMMR*: fraction of LO power present in the differential signal output, $\sim 80\,\text{dB}$ in the best cases.
- *Electronic noise*. Up to 30 dB clearance with respect the quantum shot noise in the best case for audio band.
- *Limited detection efficiency*. Equivalent to optical losses. Main loss source from anti-reflective coatings and quantum efficiency of photodetectors. Up to 99.5 % in the best case at 1.06 μm; much worse at 2 μm

Non ideal detector

Measured squeezing level vs input squeezing level

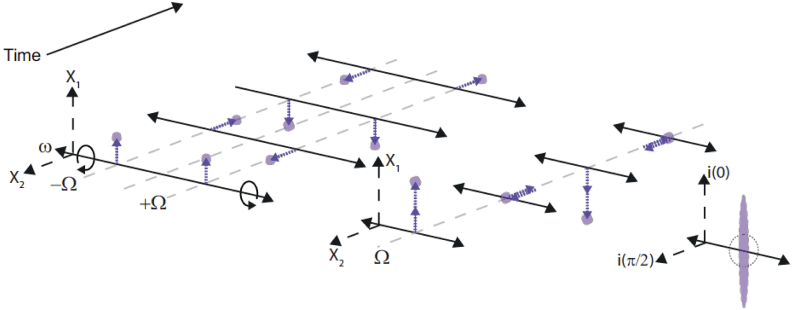


Quantum sideband picture:
uncorrelated sidebands $\Rightarrow$ coherent states



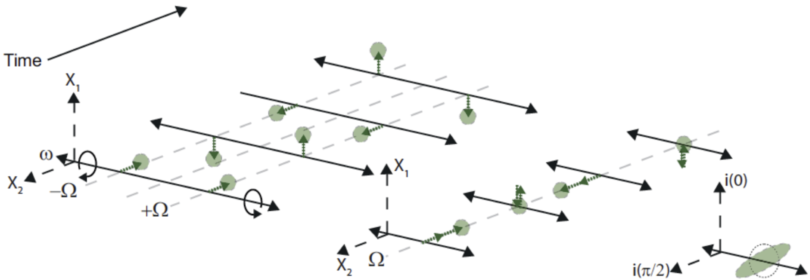
Generation of non-classical states of light

Squeezing in sideband picture:
amplitude correlated sidebands ⇒ phase-squeezed states



Generation of non-classical states of light

Squeezing in sideband picture:
phase correlated sidebands ⇒ amplitude-squeezed states

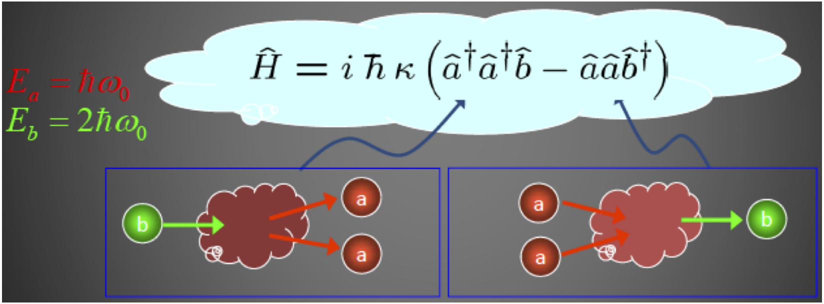


Generation of non-classical states of light

- Generation of non-classical states of light requires photon correlations
- This is generally possible in the presence of a system providing nonlinear interactions with the optical field
- Already demonstrated by various methods:
  - Interaction with dilute atomic ensembles
  - Nonlinear interactions in optical fibers
  - Hybrid methods, e.g. atomic vapours confined in optical fibers
  - Current control in semiconductor lasers
  - Pondero-motive squeezing by opto-mechanical couplings
  - **Parametric down-conversion in nonlinear crystals**: best performance so far

Parametric down-conversion

- A pump photon with frequency ω_p on a dielectric with a $\chi^{(2)}$ second-order nonlinear susceptibility
- generates two new photons; a signal photon of frequency ω_s and an idler photon at frequency ω_i
- $\omega_p = \omega_i + \omega_s$ for energy conservation.
- idler and signal photons have same frequency and polarisation in degenerate PDC.



Parametric down-conversion

- $\chi^{(2)}$ non-linearity is usually weak: significant polarization of the medium is required, a
  - High values of the optical pump energy density (high power pulsed lasers or crystal into an optical resonator).
  - Phase matching for momentum conservation: $k_p = k_s + k_i$ by temperature / wavelength tuning and/or periodically poling the nonlinear crystal.
- With type I phase matching, where signal and idler are degenerate both in frequency and polarization, the time-dependent Hamiltonian of the PDC process can be written as:

$$\hat{H} = \hbar\omega_s \hat{a}^\dagger \hat{a} + \hbar\omega_p \hat{b}^\dagger \hat{b} + i\hbar\chi^{(2)}(\hat{a}\hat{a}\hat{b}^\dagger - \hat{a}^\dagger \hat{a}^\dagger \hat{b})$$

where $\hat{b}$ is the pump field and $\hat{a}$ is the signal field.

Parametric down-conversion

- Assuming the pump to be a strong classical field, $\hat{b}$ and $\hat{b}^\dagger$ can be approximated by the classical operators $\beta e^{-i\omega_p t}$ and $\beta^* e^{-i\omega_p t}$;
- in the interaction picture, the Hamiltonian of the parametric process can be approximated as

$$\hat{H}(t) = i\hbar\chi^{(2)}[\beta^* \hat{a}\hat{a}e^{i(\omega_p - 2\omega_s)t} - \beta\hat{a}^\dagger\hat{a}^\dagger e^{-i(\omega_p - 2\omega_s)t}].$$

  Considering that $\omega_p = 2\omega_s$ this simplifies into

$$\hat{H}(t) = i\hbar\chi^{(2)}(\beta^*\hat{a}\hat{a} - \beta\hat{a}^\dagger\hat{a}^\dagger).$$

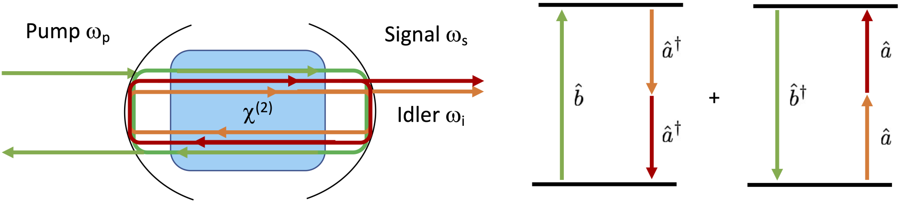
- The unitary evolution of the input signal under this Hamiltonian is

$$U = e^{-\frac{i}{\hbar}\hat{H}t} = e^{\chi^{(2)}(\beta^*\hat{a}\hat{a} - \beta\hat{a}^\dagger\hat{a}^\dagger)}$$

  which has the same form of the **squeezing operator**

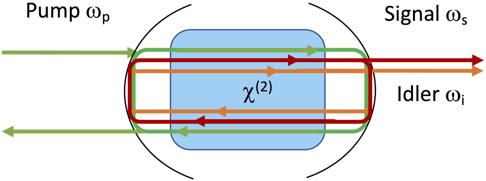
Optical parametric oscillator

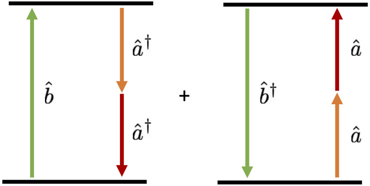
- In an optical parametric oscillator (OPO) the PDC process takes place inside a cavity,
- so far the most efficient source of squeezed light.
- signal and idler fields are usually simultaneously resonant, thus enhancing the non-linear process.



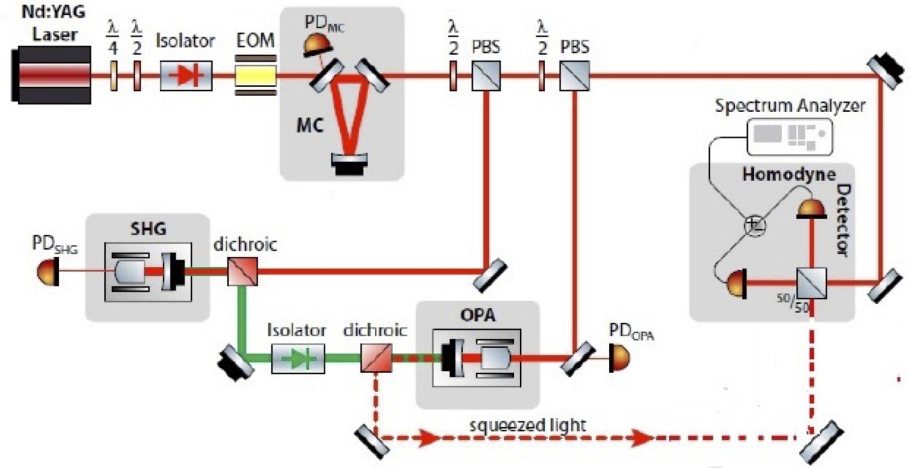
Optical parametric oscillator

- Degenerate system: the optical parametric oscillation occurs only above a threshold value of the pump optical power.
- In an deal, lossless system, the produced squeezing at threshold would be infinite.
- With a bright beam seeding the signal and idler modes, the critical point disappears, and the OPO becomes a squeezing, phase-sensitive amplifier.



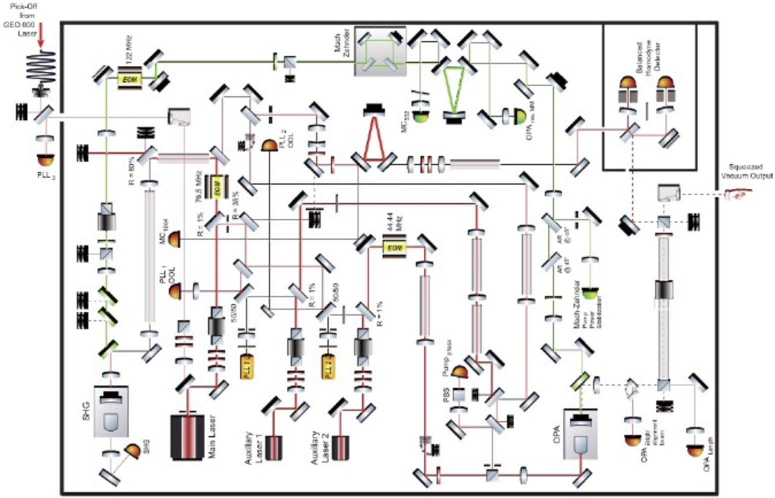


Essential diagram



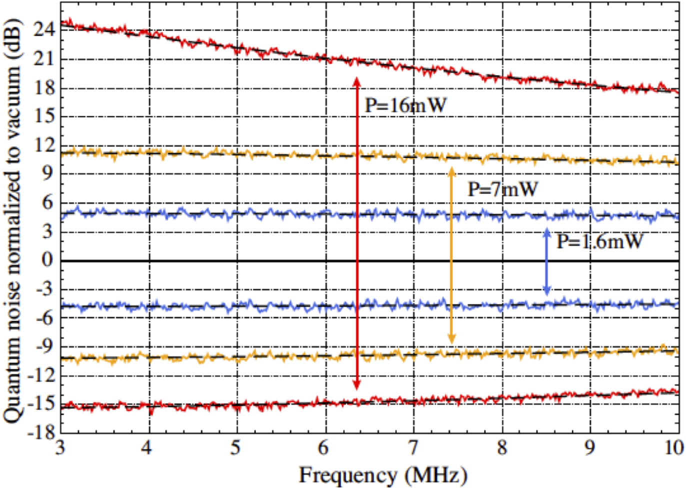
Experimental setup for squeezed light generation

AEI squeezer - Hannover



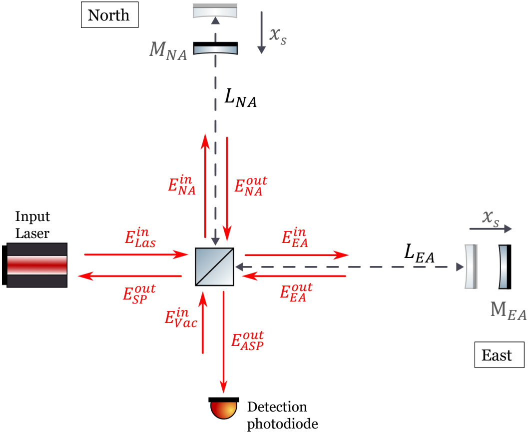
Experimental results - squeezed light generation

AEI squeezer - Hannover



Quantum noise in a Michelson interferometer

- Michelson interferometer with slight arm length asymmetry $\Delta L << L_{NA,EA}$ for DC-readout.
- A GW would produce a thin modification $x_s << L$ of the two arm lengths in counterphase to each other.



Quantum noise in a Michelson interferometer

- The electric field operator

$$\hat{E}(x, y, z, t) = u(x, y, z)\sqrt{\frac{\hbar\omega_0}{\epsilon_0 c}}\hat{E}(z, t)$$

for input laser field $\hat{E}^{in}_{Las}$ and input vacuum field $\hat{E}^{in}_{Vac}$ can be written in terms of *Amplitude* (‘*A*’) and *Phase* (‘*P*’) quadratures.

$$\hat{E}^{in}_{Las}(z, t) = [\sqrt{2P_{Las}/\hbar\omega_0}+\delta\hat{X}^{A}_{Las}(z, t)]\cos(\omega_0 t)+\delta\hat{X}^{P}_{Las}(z, t)\sin(\omega_0 t)]\Big\}$$

and

$$\hat{E}^{in}_{Vac}(z, t) = \delta\hat{X}^{A}_{Vac}(z, t)\cos(\omega_0 t) + \delta\hat{X}^{P}_{Vac}(z, t)\sin(\omega_0 t)$$

where the amplitude quadrature is defined as the one in phase with the input laser beam whose power and angular frequency are P_{Las} and ω_0 respectively.

Quantum noise in a Michelson interferometer

- Amplitude and the phase fluctuations are supposed to be dominated by quantum fluctuations and thus are represented by the quadrature operators $\delta\hat{X}^{A}_{Las,Vac}(t)$ and $\delta\hat{X}^{P}_{Las,Vac}(t)$
- The two input fields combine in the 50:50 beamsplitter generating the two fields at the arms input

$$E^{in}_{NA,EA}(t) = \frac{E^{in}_{Las}(t) \mp E^{in}_{Vac}(t)}{\sqrt{2}}$$

which after reflection on the test masses return to the beamsplitter input in the form

$$E^{out}_{NA,EA}(t) = E^{in}_{NA,EA}(t - 2\tau_{NA,EA} \mp 2x_s/c)$$

where $\tau_{NA,EA} \equiv L_{NA,EA}/c$ is the time required for light to cover the unperturbed arm length. After recombination these two beams produce the field $E^{ASP}_{out}(t)$ at the output of the beamsplitter

Quantum noise in a Michelson interferometer

- To leading order in x_s/c and $\Delta L/c$ and $\delta\hat{X}$

$$\hat{E}^{ASP}_{out}(t) = \frac{E^{out}_{NA}(t) - E^{out}_{EA}(t)}{\sqrt{2}} =$$

$$= \left[\sqrt{\frac{2P_{Las}}{\hbar\omega_0}}\frac{\omega_0}{c}[2x_s(t-\tau)+\Delta L] + \delta\hat{X}^P_{Vac}(t-2\tau)\right]\times$$

$$\times \sin(\omega_0 t) - \delta\hat{X}^A_{Vac}(t-2\tau)\cos(\omega_0 t)$$

- we assume $\omega_0 L_{NA}/c = n\pi$
- the dependence from the amplitude $\delta\hat{X}^A_{Las}$ and phase $\delta\hat{X}^P_{Las}$ fluctuations of the input laser field have completely disappeared.
- This is a direct consequence of having chosen the unperturbed length almost equal for the two arms ($\Delta L/L_{NA} << 1$).

- The detection photodiode measures the expectation value of the power operator $\hat{P}(t) = \epsilon_0 c \int dxdy |\hat{E}^{ASP}_{out}(x, y, z, t)|^2$ of the *ASP* beam
- At leading order in $\delta\hat{X}$ and x_s and averaging over time the rapid oscillations at frequency ω_0 and $2\omega_0$ we obtain

$$\hat{P}(t) = \frac{P_{Las}\omega_0^2 \Delta L}{c^2}\left[\Delta L + 4x_s(t-\tau) + \sqrt{\frac{2\hbar\omega_0}{P_{Las}}}\frac{c}{\omega_0}\delta\hat{X}^P_{Vac}(t-2\tau)\right]$$

- This shows that phase fluctuations $\delta\hat{X}^P_{Vac}$ of the vacuum field generate a sensing noise called *quantum shot noise* competing with the displacement signal x_s contribution.

Quantum noise in a Michelson interferometer

- Mirrors are also subject to a displacement generated by radiation pressure force $F^{RP}_{NA,EA}$ which act on the mirrors.
- To leading order on the quantum fluctuation operators:

$$\hat{F}^{RP}_{NA,EA}(t) = 2\epsilon_0 \int dxdy|\hat{E}_{NA,EA}(t-\tau,x,y,z)|^2 =$$

$$\approx \frac{P_{las}}{c}\left\{1 + \sqrt{\frac{2\hbar\omega_0}{P_{las}}}\left[\delta\hat{X}^A_{Las}(t-\tau)) \mp \delta\hat{X}^A_{Vac}(t-\tau))\right]\right\}$$

for mirrors with equal mass M the displacement induced by the first two terms in equation produce an undetectable mirror common motion while the last term generate a differential displacement noise which adds to the signal contribution.

Quantum noise in a Michelson interferometer

- The detected power in the Fourier space then becomes

$$\hat{P}(\Omega) = 4P_{Las}\left(\frac{\omega_0 \Delta L}{c}\right)^2 \Bigg[e^{i\Omega\tau} x_s(\Omega) +$$

$$-e^{2i\Omega\tau}\sqrt{\frac{\hbar}{2M\Omega^2}}\left[\sqrt{K_{Mi}}\delta\hat{X}^A_{Vac}(\Omega) + \frac{\delta\hat{X}^P_{Vac}(\Omega)}{\sqrt{K_{Mi}}}\right]\Bigg]$$

where we have omitted the not relevant static term and

$$K_{Mi} = \frac{4\omega_0 P_{Las}}{c^2 M\Omega^2}$$

Quantum noise in a Michelson interferometer

- For a perfectly balanced interferometer (same arm length and mirror mass) input laser quantum fluctuations $\delta\hat{X}^{A,P}_{Las}$ do not contribute to quantum noise which is entirely generated from vacuum fluctuations $\delta\hat{X}^{P}_{Vac}$ (shot noise) and $\delta\hat{X}^{A}_{Vac}$ (quantum radiation pressure noise) entering from the dark port.
- Using the relationship $x_s(\Omega) = Lh(\Omega)/2$ between gravitational wave amplitude h and induced displacement x_s the quantum noise equivalent strain power spectral density becomes

$$S_{h_n h_n}(\Omega) = \frac{2\hbar}{ML^2\Omega^2}\left[K_{Mi} + \frac{1}{K_{Mi}}\right]$$

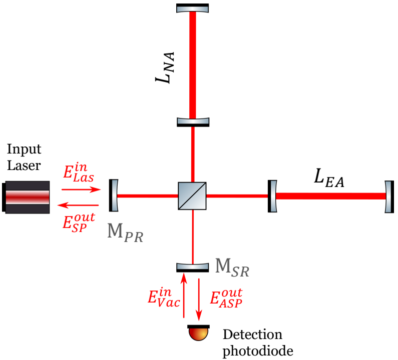
which takes the minimum value $S^{SQL}_{h_n h_n}(\Omega)$ called *Standard Quantum Limit* (SQL)

$$S^{SQL}_{h_n h_n}(\Omega) = \frac{4\hbar}{ML^2\Omega^2} \equiv h^2_{SQL-Mi}$$

when $K_{Mi} = 1$.

Quantum noise in a power recycled interferometer

- Optimal scale factor requires an effective arm length of the order of the GW wavelength, i.e. several hundred km for audio band GWs
- This is achieved by incorporating Fabry-Pérot resonators in the Michelson arms
- The effective optical power sensed by test masses is enhanced by a factor $4/T_{IM}$ where T_{IM} is the input mirror power transmittivity.



Quantum noise in a power recycled interferometer

- A further optical resonator is realized by inserting a *Power Recycling Mirror (PRM)* between the laser source and the beamsplitter, giving a power enhancement $4/T_{PR}$ at BS input.
- Symmetric port back-reflected light does not contain any gravitational wave induced signal or any quantum noise generator
- Quantum noise power spectral density takes the form

$$S_{h_n h_n}(\Omega) = \frac{4\hbar}{ML^2\Omega^2}\left[K_{PR} + \frac{1}{K_{PR}}\right] \equiv \frac{h_{SQL}^2}{2}\left[K_{PR} + \frac{1}{K_{PR}}\right]$$

where

$$K_{PR} = \frac{2\gamma_{arm}}{\Omega^2(\Omega^2 + \gamma_{arm}^2)}\frac{8\omega_0}{MLc}\frac{2P_{BS}}{T_{IM}}$$

- γ_{arm} is the half-width-half-maximum bandwidth of the arm cavity and $P_{BS} = 4P_{Las}/T_{PR}$ the enhanced power at the BS input.
- The multiplicative factor "2" that appears in the equations derives from the presence of two masses in each arm and therefore the mass of the mirror M must be replaced by the reduced mass $M/2$.

Quantum noise in a dual-recycled interferometer

- A *Signal Recycling Mirror* (SRM) at the AS port gives a frequency dependent enhancement of the signal sidebands. item Depending on the resonance frequency of the resulting *Signal Recycling Cavity* (SRC), we can have two different operating regimes
  - *Tuned configuration* the SRC is kept resonant with the interferometer carrier frequency ω_0, broadening the arms cavity bandwidth.
  - *Detuned configuration* carrier frequency is no longer resonant in the SRC and at least one of the two signal sideband is resonant in the arm cavity. Quadratures rotate of an angle related to the cavity detuning $\Delta = \omega_0 L_{SR}/c|_{mod 2\pi}$. SN and RPN generators become a combination of $\delta\hat{X}^A_{Vac}(t)$ and $\delta\hat{X}^P_{Vac}(t)$ giving rise to a non vanishing cross correlation spectral density that can be used to overcome the SQL in a narrow frequency range.
- The configuration with power and the signal recycling mirrors is called *Dual Recycling* (DR) configuration Routinely used in GEO 600, LIGO and Virgo

Quantum noise in a dual-recycled interferometer

Expected quantum noise power spectral density for a lossless Dual Recycled interferometer and homodyne readout with angle ζ:

$$S_{h_n h_n}(\Omega) = \frac{(\cos\zeta,\ \sin\zeta)\mathbf{T}\mathbf{T}^T(\cos\zeta,\ \sin\zeta)^T}{(\cos\zeta,\ \sin\zeta)\mathbf{s}\mathbf{s}^T(\cos\zeta,\ \sin\zeta)^T}$$

where the elements of the 2x2 matrix **T** and the vector **s** are

$$s_1 = -\frac{\sqrt{2K_{PR}}}{h_{SQL}^{PR}} t_{sr}(1 + r_{sr}e^{2i\Theta})\sin(\Delta)$$

$$s_2 = -\frac{\sqrt{2K_{PR}}}{h_{SQL}^{PR}} t_{sr}(-1 + r_{sr}e^{2i\Theta})\cos(\Delta)$$

$$T_{11} = T_{22} = e^{2i\Theta}\left[(1 + r_{sr}^2)\left(\cos(2\Delta) + \frac{1}{2}\cdot K_{PR}\sin(2\Delta)\right) - 2r_{sr}\cos(2\Theta)\right]$$

.

Quantum noise in a dual-recycled interferometer

$$T_{12} = -r_{sr}^2 \cdot [\sin(2\Delta) + K_{PR} \sin^2(\Delta)] e^{2i\Theta}$$

$$T_{21} = r_{sr}^2 \cdot [\sin(2\Delta) - K_{PR} \cos^2(\Delta)] e^{2i\Theta}$$

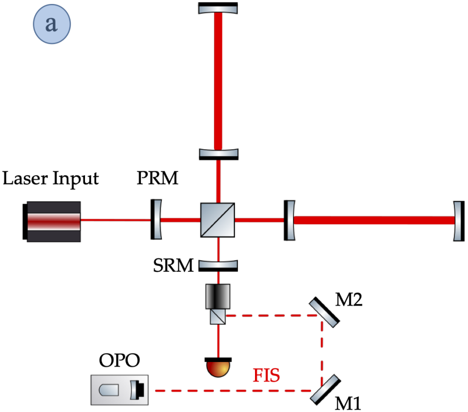
where

$$\Theta = \frac{\Omega L_{sr}}{c} + \arctan\left[\frac{\Omega}{\gamma_{arms}}\right]$$

and r_{sr} t_{sr} respectively the amplitude reflectivity and transmissivity of the SR recycling mirror.

Frequency independent squeezing injection

- Quantum shot and back-action noise are generated by a linear combination of the vacuum field quadrature operators $\delta\hat{X}_{Vac}^{P,A}$.
- By injecting into the dark port of the interferometer a vacuum squeezed field in place of the coherent vacuum field we can modify quantum noise in ITF.



Frequency independent squeezing injection

- In presence of squeezing the operators $\delta\hat{X}^P_{Vac}$ and $\delta\hat{X}^A_{Vac}$ are formally replaced with the new operators

$$\delta\hat{X}^P_{Vac} \rightarrow \hat{S}^\dagger\delta\hat{X}^P_{Vac}\hat{S} = \delta\hat{X}^P_{Vac}[\cosh\xi - \sinh\xi\cdot\cos 2\theta] - \delta\hat{X}^A_{Vac}\sinh\xi\cdot\sin 2\theta$$

$$\delta\hat{X}^A_{Vac} \rightarrow \hat{S}^\dagger\delta\hat{X}^A_{Vac}\hat{S} = \delta\hat{X}^A_{Vac}[\cosh\xi + \sinh\xi\cdot\cos 2\theta] - \delta\hat{X}^P_{Vac}\sinh\xi\cdot\sin 2\theta$$

leading to a modification of the back-action and shot noise value which also become partially correlated for $\theta \neq 0, \pi/2$.

Frequency independent squeezing injection

- Quantum noise PSD for a Michelson interferometer in the presence of squeezing takes the form

$$S_{h_n h_n}(\Omega) = \frac{2\hbar}{ML^2\Omega^2}\left[K_{Mi} + \frac{1}{K_{Mi}}\right][\cosh 2\xi + \cos 2(\Phi + \theta)\sinh 2\xi]$$

where

$$\Phi(\Omega) = \cot^{-1}[K_{Mi}(\Omega)]$$

- Similarly for a tuned dual recycled configuration by replacing mirror mass M i by reduced mass $M/2$, and K_{Mi} by K_{TDR}

$$K_{TDR} = \frac{2\gamma_{ITF}}{\Omega^2(\Omega^2 + \gamma_{ITF}^2)}\frac{8\omega_0}{MLc}\frac{2P_{BS}}{T_{IM}}$$

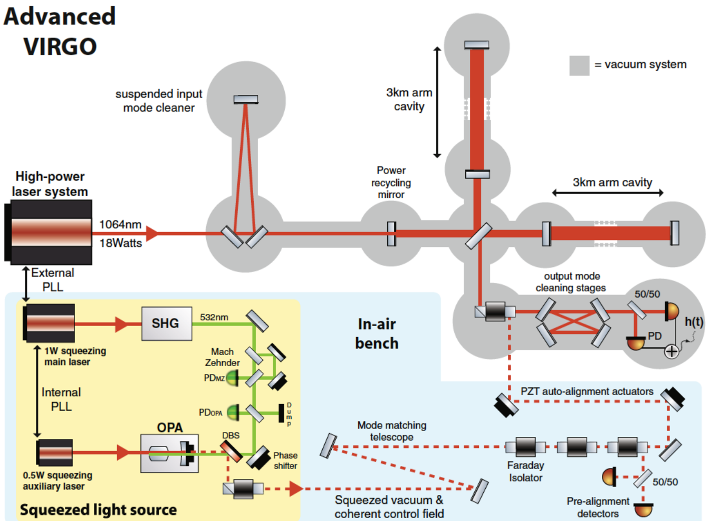
where the detector bandwidth γ_{ITF} is

$$\gamma_{ITF} = -\frac{c}{2L}\log\left(\frac{r_{IM} + r_{sr}}{1 + r_{IM}r_{sr}}\right)$$

whith r_{sr} , r_{IM} the amplitude reflectivity of the signal recycling and the arm input mirror respectively.

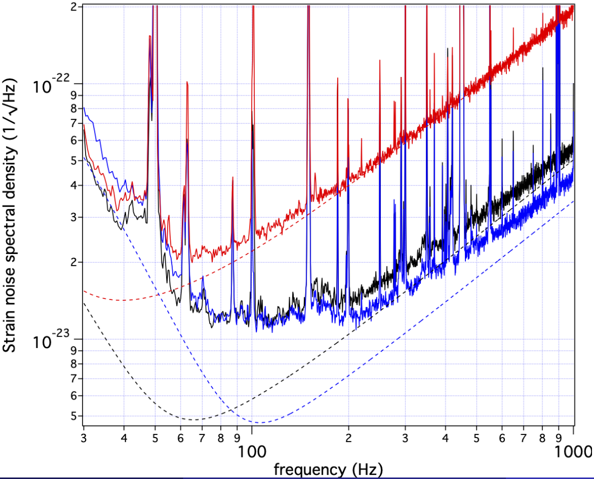
Experimental results - FIS injection

Apparatus for frequency-independent squeezed light injection in Virgo during the O3 science run.



Experimental results - FIS injection

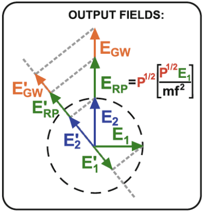
Effect of frequency-independent squeezed light injection in Virgo during the O3 science run. Black: no squeezing; red: phase squeezing ($\theta = 0$); blue: amplitude squeezing ($\theta = \pi/2$)

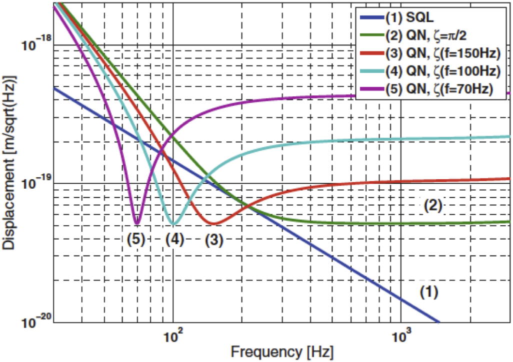


Frequency independent squeezing injection

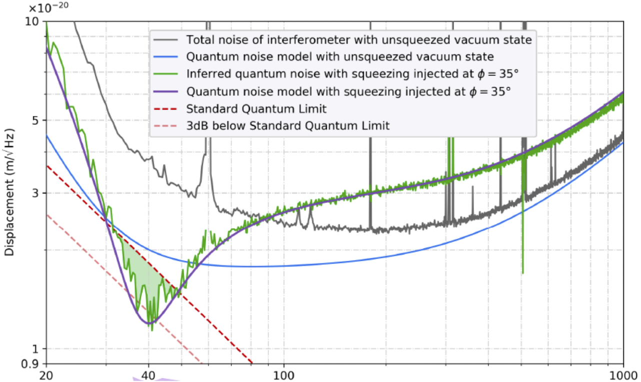
- So far the squeezing ellipse rotation angle θ is constant over the whole frequency band.
- Angle should be about $\theta \approx 0$ to reduce shot noise by a factor $\exp(-2\xi)$ at the cost of a RPN increase by a factor $\exp(2\xi)$
- RPN is usually smaller than dominant low-frequency technical noises
- The net effect is a detector sensitivity improvement at high frequency where shot noise dominates, leaving unaffected the low frequency sensitivity.
- For $\theta \approx 3\pi/4$ the correlations between the shot and back action noise induced by squeezed light allows to surpass the standard quantum limit by a factor $\exp[-2\xi]$.
- However this value of θ would produces a reduction of the quantum noise power spectral density only within a narrow band, and an increase elsewhere

FIS injection - phase angle tuning





Effect of frequency-independent squeezed light injection in LIGO with angle tuning during the O3 science run.



Frequency-dependent squeezing injection

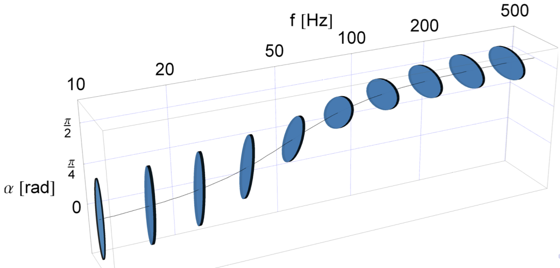
- Let's assume a squeezing angle $\theta(\Omega)$ with the following frequency dependence

$$\theta(\Omega) = \arctan(K_{Mi}(\Omega))$$

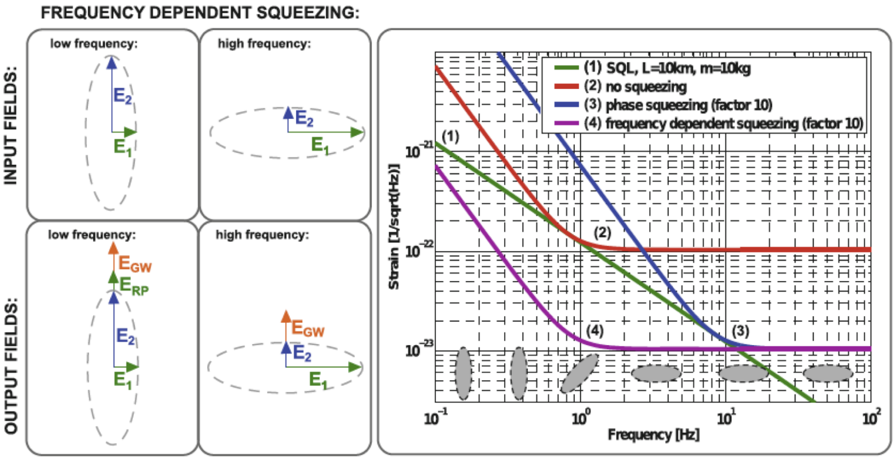
- quantum noise for a Michelson interferometer takes the form

$$S_{h_n h_n}(\Omega) = \frac{2\hbar}{ML^2\Omega^2}\left[K_{Mi} + \frac{1}{K_{Mi}}\right]e^{-2\xi}$$

which corresponds to a suppression of the standard quantum noise by a factor $e^{-2\xi}$ over the whole frequency band.



Frequency-dependent squeezing injection

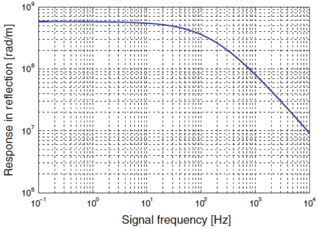


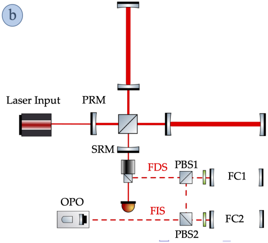
Frequency-dependent squeezing injection

- To generate requency dependent squeezed light we can e.g. couple a FIS beam to a detuned Fabry-Perot (FP) cavity.
- With a losseless linear cavity the reflected field acquire a frequency dependent phase shift $\alpha(\Omega)$.

$$\alpha(\Omega) = \arg\left\{ \frac{r_{in} - r_{out} e^{2i(\Omega - \Omega_{fc})L/c}}{1 - r_{in} r_{out} e^{2i(\Omega - \Omega_{fc})L/c}} \right\}$$

where $\omega_0 + \Omega_{fc}$ and L_{fc} are the cavity resonant frequency and length, and r_{in}, r_{out} the input and output cavity mirrors amplitude reflectivity.





Frequency-dependent squeezing injection

- after reflection the sideband annihilation operators $\hat{a}_{\omega_0 \pm \Omega}$ becomes

$$\hat{a}^{ref}_{\omega_0 \pm \Omega} = \hat{a}_{\omega_0 \pm \Omega} e^{i\alpha(\pm\Omega)}$$

and the quadrature $\hat{X}_{1,2}(\Omega)$ change into $\hat{X}^{ref}_{1,2}(\Omega)$:

$$\hat{X}^{Ref}_1(\Omega) = e^{i\alpha_m}[\hat{X}_1(\Omega)\cos\alpha_p(\Omega) - \hat{X}_2(\Omega)\sin\alpha_p(\Omega)]$$

$$\hat{X}^{Ref}_2(\Omega) = e^{i\alpha_m}[\hat{X}_2(\Omega)\cos\alpha_p(\Omega) - \hat{X}_1(\Omega)\sin\alpha_p(\Omega)]$$

where

$$\alpha_{p,m}(\omega) = \frac{\alpha(\Omega) \pm \alpha(-\Omega)}{2}$$

Frequency-dependent squeezing injection

- Apart from an irrelevant phase factor, this shows that the reflected quadratures are rotated by the frequency dependent angle $\alpha_p(\Omega)$:

$$\alpha_p(\Omega) = \arctan\left[\frac{2\gamma_{fc}\Omega_{fc}}{\gamma_{fc}^2 - \Omega_{fc}^2 + \Omega^2}\right]$$

where γ_{fc} is the half width half maximum power cavity linewidth.

- If the parameters of the filter cavity are set such that

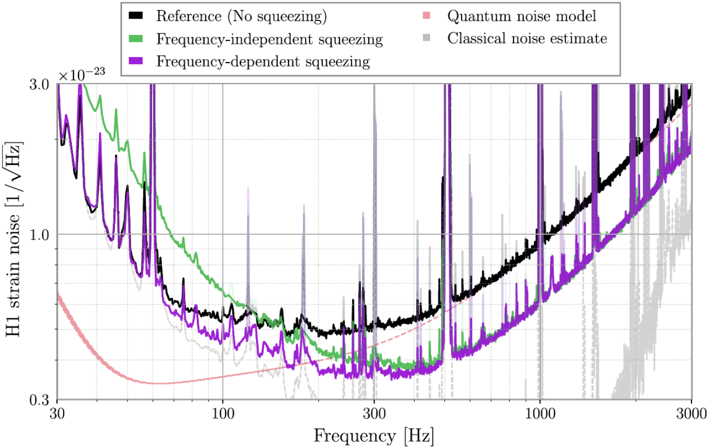
$$\Omega_{fc} = \gamma_{fc} = \Omega\sqrt{\frac{K_{Mi}}{2}} \equiv \frac{\Omega_{SQL}}{\sqrt{2}}$$

the condition above is fulfilled.

- Therefore by injecting the FIS beam reflected from the FC into the interferometer dark port a broadband quantum noise reduction

Experimental results - FDS injection

Effect of frequency-dependent squeezed light injection in LIGO during the O4 science run.



Open challenges in quantum optics for GW detection

- understand and beat mode mismatch to interferometer
- alternative architectures to avoid filter cavities (e.g. through EPR entanglement)
- Reach 10 FDS

References

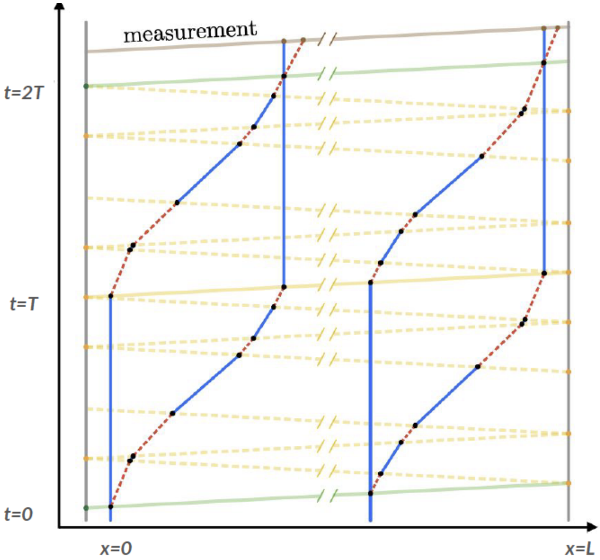
- S. L. Danilishin et al., *Advanced quantum techniques for future gravitational-wave detectors*, Living Reviews in Relativity 22, 2 (2019)
- F. Sorrentino and J. P. Zendri, *Squeezing and QM techniques in GW Interferometers*, in Handbook of Gravitational Wave Astronomy, Springer Nature Singapore Pte Ltd. 2022, C. Bambi et al. (eds.)

Matter-waves interferometry for GW detectors

- Replacing classical test masses (macroscopic suspended mirrors) with quantum test masses (ultracold atoms in free fall)
- Laser splits and recombines the atomic matter wave, phase shift proportional to spacetime area
- Advantages: no thermal noise, no radiation pressure noise
- Disadvantages: much higher shot noise (small number of atoms), much smaller bandwidth

Matter-waves interferometry for GW detectors

Measurement scheme



Matter-waves interferometry for GW detectors

Projected sensitivity